

# Introduction to Queues

## Definition

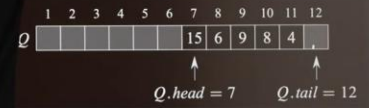
A linear data structure that follows **FIFO** (First In, First Out) principle.

## Key Analogy

Waiting line at a ticket counter.

## Parameters

1. **Q.head**: Position of the front element.
2. **Q.tail**: Position where the next element will be inserted.



# Introduction to Queues

## Definition

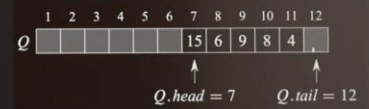
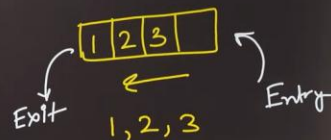
A linear data structure that follows **FIFO** (First In, First Out) principle.

## Key Analogy

Waiting line at

## Parameters

1. **Q.head**: Position of the front element.
2. **Q.tail**: Position where the next element will be inserted.



## Queue Operations (Regular)

### Insertion: Enqueue(Q, x)

Insert an element  $x$  at the tail/rear/end of the queue.

### Pseudo Code

Initialize:

Q[0...N-1] // array of size N

head = 0

tail = 0

ENQUEUE(Q, x)

1 if tail == N:

2     error "Queue Overflow"

3 else:

4     Q[tail] =  $x$

5     tail = tail + 1

**Queue Operations (Regular)**

**Insertion: Enqueue(Q, x)**

Insert an element  $x$  at the tail/rear/end of the queue.

**Pseudo Code**

array of size N

ENQUEUE(Q, x)

1 if tail == N:

2     error "Queue Overflow" ✓

3 else:

4     Q[tail] =  $x$

5     tail = tail + 1

*Handwritten notes:*

- $n = 4$
- Diagram of an array of size 4: 

0	1	2	3
1	2	3	4

 with arrows pointing to each element from below, labeled  $h$  and  $t$ .
- $h = 0$
- $t = 0, 1, 2, 3, 4$
- $3, 4$
- $\rightarrow 1, 2, 3, 4, 5$
- Queue is full*  $\begin{cases} t = 4 \\ n = 4 \end{cases}$

for filling 5 it will be overflow

# Queue Operations (Regular)

## Deletion: Dequeue(Q)

Removes an element from the head/front of the queue.

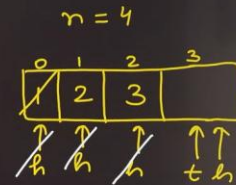
### Pseudo Code

**Dequeue (Q)**

```
1 if head == tail:
    error "Queue Underflow"
else:
    x = Q[head]
    head = head + 1
    return x
```

$h = 0$   
 $t = 3$

$Q[h] = 1$   
↓  
Return  $x$



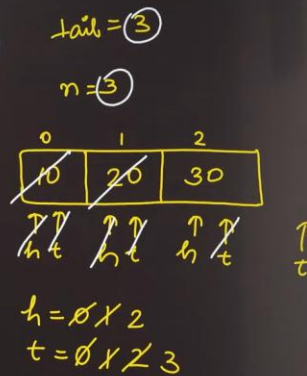
$h = 1$   
 $x = Q[h] = 2$   
 $h = h + 1 = 2$   
Return  $x$

# Queue Operations (Regular)

## Example

Step by step dry run:

- Initial state:  $head = 0$ ,  $tail = 0$ ,  $n = 3$
- Enqueue (10), Enqueue (20), Enqueue (30)
- Dequeue () → Return (10)
- Enqueue (40), Enqueue (50)
- Dequeue () → overflow
- Enqueue (70) → Return (20)



## Circular Queue

Why?

Regular queue wastes space when tail reaches end

### Definition

A Circular Queue is a linear data structure that follows **FIFO** (First In, First Out) but connects the last position back to the first position to form a circle.



+ 5 &gt;



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## Queue Operations (Circular)

### Insertion: Enqueue(Q, x)

Insert an element x at the tail of the queue.

### Pseudo Code

Initialize:

```
Q[0...N-1] // array of size N
```

```
head = 0
```

```
tail = 0
```

ENQUEUE(Q, x)

```
1 if (tail + 1) % N == head:
```

```
2     error "Queue Overflow"
```

```
3 else:
```

```
4     Q[tail] = x
```

```
5     tail = (tail + 1) % N
```



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## Queue Operations (Circular)

### Deletion: Dequeue(Q)

Removes an element from the head of the queue.

### Pseudo Code

#### Dequeue(Q)

```

1 if head == tail: 3 ≠ 0 → 3 + 1 = 4 % 4
2   error "Queue Underflow" (3 + 1) % 4
3 else:
4   x = Q[head]
5   head = (head + 1) % N
6   return x
  
```

$(t+1) \% n == h$   
→ overflow.



1. enq(1) ✓
  2. enq(2) ✓
  3. enq(3) ✓
  4. enq(4) ✗
  5. dequeue() ✓
  6. enq(4) ✓
  7. dequeue() ✓
  8. dequeue() ✓
  9. dequeue() ✓
  10. dequeue() ✗
- Handwritten notes on the right side of the slide showing a sequence of enqueue and dequeue operations. Some operations are marked with checkmarks (✓) and others with crosses (✗). A calculation  $t = (t+1) \% n = 4 \% 4 = 0$  is shown.

## Queue Operations (Circular)

### Example

Step by step dry run:

Initial state: head = 0, tail = 0, n = 5

Enqueue(10), Enqueue(20), Enqueue(30), Enqueue(40)

3. Dequeue()

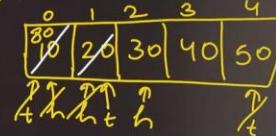
Enqueue(50), Enqueue(60)

Enqueue(70) → overflow.

Dequeue() ✓

Enqueue(80), Enqueue(90) → overflow.

$(t+1) \% n == h$  → overflow.  
 $h == t$  → underflow.



$h = 0 \neq 2$   
 $t = 0 \neq 3 \neq 4 \neq 1$   
 $1 == 2 \neq$

$(t+1) \% 5$   
 $5 \% 5 = 0 \neq h$

## Other Variations of Queues

### Priority Queue

A special type of queue where each element has a priority value.

Two types, Max-Priority & Min-Priority.

### Operations

- $\text{insert}(Q, x, \text{priority}) \rightarrow$  Add element with given priority
- $\text{delete}(Q) \rightarrow$  Remove element with highest (or lowest) priority
- $\text{isEmpty}(Q), \text{isFull}(Q) \rightarrow$  Usual checks.

max-priority  
 $\rightarrow$  highest priority element will be deleted

min-priority  
 $\rightarrow$  lowest priority element will be deleted

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## Queue Operations (Priority)

### Example Min-Priority

Step by step dry run (n = 10):

1. Initial state: Queue = [ ]

2. Insert(10, priority=3) = [(10, 3)]

3. Insert(20, priority=1) = [(20, 1), (10, 3)]

4. Insert(30, priority=2) = [(20, 1), (30, 2), (10, 3)]

5. Delete() = [(30, 2), (10, 3)]

6. Insert(40, priority=0) = [(40, 0), (30, 2), (10, 3)]

7. Insert(50, priority=4) = [(40, 0), (30, 2), (10, 3), (50, 4)]

8. Delete() = [(30, 2), (10, 3), (50, 4)]

9. Insert(70, priority=1)



1. Always maintain the priority.  $\rightarrow$  Increasing order of priority.

Insertion

$O(n)$

$O(\log n)$

using heaps

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## Queue Operations (Priority)

Priority

run ( $n = 10$ ):

Queue = [ ]

priority=3 = [(10, 3)]

priority=1 = [(20, 1), (10, 3)]

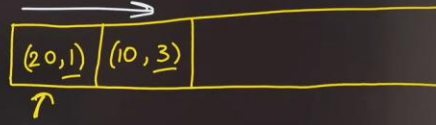
priority=2 = [(20, 1), (30, 2), (10, 3)]

priority=0 = [(40, 0), (30, 2), (10, 3)]

priority=4 = [(40, 0), (30, 2), (10, 3), (50, 4)]

priority=2 = [(70, 1), (30, 2), (10, 3), (50, 4)]

priority=1 = [(70, 1), (30, 2), (10, 3), (50, 4)]



Insertion

$O(n)$

$O(\log n)$

using heap

①. Always maintain the priority. → Increasing order of priority.