Pattern Recognition Machine Learning

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1

distance metric $d(x, y) = (x - y)^2, x, y \in \mathbb{R}$
1. $d(x, y) = d(y, x) \implies symmetric$
2. $d(x, y) >= 0$ and $d(x, y) = 0 \iff x = y$
3. $d(x, y) = (x - y)^2 = x^2 + y^2 - 2xy$
$> (x - 0)^2 + (y - 0)^2$
$> d(x, 0) + d(0, y)$, if x and y are of oppsite signs

Triangle inequality not satisfied $\implies d(x, y) = (x - y)^2$ is not a metric on real line.

2

x_0 is an accumulation point of $A \subset (X, d)$.i.e. every neighbourhood of x_0 contains atleast one point of A distinct from x_0. Let $B(x_0, r) = \{x \in X : d(x, x_0) < r\}$ be the open ball centered around x_0 of radius r, i.e $\forall r \in \mathbb{R}, \exists$ atleast one $y \in A$ such that $y \in B(x_0, r)$ and $x_0 \neq y$
$\therefore$ Neighborhood of x_0 contains infinitely many points of A.

3

$x = (x_1, x_2, ...x_n), y = (y_1, y_2, ...y_n), d(x, y) = max_i\ |y_i - x_i|$
Let $\{x^{(n)}\}$be the cauchy sequence in $\mathbb{R}^n$.$\forall \epsilon > 0, \exists N \in \mathbb{N}$ such that $d(x^{(m)}, x^{(n)}) < \epsilon$ for $m, n > N$.
$d(x^{(m)}, x^{(n)}) = max_i\ |x_i{}^{(n)} - x_i{}^{(m)}| < \epsilon \implies \forall i$ and $m, n > N, |x_i{}^{(n)} - x_i{}^{(m)}| < \epsilon$.i.e, Every i^{th} component of sequence $\{x^{(n)}\}$ forms a cauchy sequence of real numbers. Since cauchy sequences are bounded every sequence of the real numbers has a monotone subsequence. By the Monotone Convergence Theorem, we have that the subsequence converges. Since the subsequence converges, cauchy sequence also converges. Hence (X, d) is complete.

4

Let $y = m_1x$ and $y = m_2x$ be the straight lines passing through the origin. Union of points on these two lines $V = \{(x, y); y = m_1x \text{ or } y = m_2x\}$. Let $X_1 = (x_1, y_1), X_2 = (x_2, y_2) \in V$ such that $y_1 = m_1x_1, y_2 = m_2x_2 \therefore X_3 = X_1 + X_2 = (x_1 + x_2, y_1 + y_2) = (x_1 + x_2, m_1x_1 + m_2x_2) \notin V$.i.e, X_3 lies neither on $y = m_1x$ nor on $y = m_2x$. Hence V is not a subspace of $\mathbb{R}^2$

5

$M = \{v_1 = (1,1,1), v_2 = (0,0,2)\}$ containstwo linearly independent vectors in $\mathbb{R}^3$.
Dimension of vector space formed by span of M is 2.
span of M = $\{\alpha_1 v_1 + \alpha_2 v_2; \alpha_1, \alpha_2 \in \mathbb{R}\}$

6

$(X_1, \|.\|_1), (X_2, \|.\|_2)$ are normed spaces.Product space $X = X_1 \text{x} X_2$.i.e, any element in X can be written as $x = (x_1, x_2), x_1 \in X_1, x_2 \in X_2$
$\|x\| = \max(\|x_1\|_1, \|x_2\|_2)$. clearly
1. $\|x\| >= 0$ and $\|x\| = 0 \iff x_1 = 0$ and $x_2 = 0 .i.e, x = 0$
2. $\|\alpha x\| = \max(\|\alpha x_1\|_1, \|\alpha x_2\|_2) = |\alpha| * \max(\|x_1\|_1, \|x_2\|_2) = |\alpha|\|x\|$
3. Let $x = (x_1, x_2), y = (y_1, y_2), z = x + y = (x_1 + y_1, x_2 + y_2)$

$$
\begin{aligned}
\|z\| = \|x + y\| &= \max(\|x_1 + y_1\|_1, \|x_2 + y_2\|_2) \\
&< \max(\|x_1\|_1, \|x_2\|_2) + \max(\|y_1\|_1, \|y_2\|_2) \\
&< \|x\| + \|y\|
\end{aligned}
$$

Hence X is a normed space.

7

$X = \mathbb{C}^{2x2}$ vector space of complex 2x2 matrices and $T : X \to X$ and $T(x) = bx$, b is fixed, $b \in X$
1. $T(\alpha x) = b(\alpha x) = \alpha bx = \alpha T(x)$
2. $T(x + y) = b(x + y) = bx + by = T(x) + T(y)$

Hence T is linear

8

$\langle x, u\rangle >= \langle x, v\rangle, \forall x \implies \langle x, u - v\rangle = 0, \forall x \implies u - v = 0 \implies u = v$