

**MA622 Pattern Recognition and Machine Learning**

**Assignment-1**

Date: 02-02-2016

1. Does  $d(x, y) = (x - y)^2$  define a metric on the set of real numbers? Justify your answer.
2. If  $x_0$  is an accumulation point of a set  $A \subset (X, d)$ , show that any neighborhood of  $x_0$  contains infinitely many points of  $A$ .
3. Let  $X$  be the space of all ordered  $n$  tuples  $x = (\xi_1, \xi_2, \dots, \xi_n)$  of real numbers and  $d(x, y) = \max_i |\xi_i - \eta_i|$  where  $y = (\eta_1, \eta_2, \dots, \eta_n)$ . Show that  $(X, d)$  is complete.
4. Is the union of two straight lines passing through the origin a subspace of  $\mathbb{R}^2$ ? Justify your answer.
5. Find the span of  $M$  where  $M = \{(1, 1, 1), (0, 0, 2)\}$ . Find the dimension of the resulting vector space.
6. If  $(X_1, || \cdot ||_1)$  and  $(X_2, || \cdot ||_2)$  are normed spaces show that the product space  $X = X_1 \times X_2$  is a normed space where the norm is defined as  $||x|| = \max(||x_1||_1, ||x_2||_2)$ .
7. Let  $X$  be the vector space of all complex  $2 \times 2$  matrices and define  $T : X \rightarrow X$  by  $Tx = bx$  where  $b \in X$  is fixed and  $bx$  denotes the usual product of matrices. Show that  $T$  is linear.

8. If in an inner product space  $\langle x, u \rangle = \langle x, v \rangle$  for all  $x$ , show that  $u = v$ .
9. Show that the annihilator  $M^\perp$  of a set  $M \neq \emptyset$  in an inner product space  $X$  is a closed subspace of  $X$ .

## Notes

- Assignment has to be written in latex.
- All the files related with the assignment should be saved in a single folder and send to [sumitra@iist.ac.in](mailto:sumitra@iist.ac.in).
- Last date of submission: 16-02-2016.
- **As far as assignments are concerned, students are expected to observe academic honesty and integrity. Though the students can collaborate and discuss, copying directly other students' assignment or allowing your own assignment to be copied constitute academic dishonesty and is highly discouraged.**