

## Appendix B: Formulation of the Benchmark Problems

Table 8

Unconstrained bi-objective problems.

Problem	Objective functions	n	Variable bounds	Comments
Schaffer	$f_1(x) = x^2$ $f_2(x) = (x - 2)^2$	1	$-10^5 \leq x \leq 10^5$	<i>convex</i>
Fonseca	$f_1(\vec{x}) = 1 - e^{-\sum_{i=1}^n (x_i - \frac{1}{\sqrt{n}})^2}$ $f_2(\vec{x}) = 1 - e^{-\sum_{i=1}^n (x_i + \frac{1}{\sqrt{n}})^2}$	3	$-4 \leq x_i \leq 4$	<i>nonconvex</i>
Kursawe	$f_1(\vec{x}) = \sum_{i=1}^{n-1} \left( -10e^{(-0.2 * \sqrt{x_i^2 + x_{i+1}^2})} \right)$ $f_2(\vec{x}) = \sum_{i=1}^n ( x_i ^a + 5 \sin(x_i)^b)$	3	$-5 \leq x_i \leq 5$	<i>nonconvex</i>
ZDT1	$f_1(\vec{x}) = x_1$ $f_2(\vec{x}) = g(\vec{x})[1 - \sqrt{x_1/g(\vec{x})}]$ $g(\vec{x}) = 1 + 9 \left( \sum_{i=2}^n x_i \right) / (n - 1)$	30	$0 \leq x_i \leq 1$	<i>convex</i>
ZDT2	$f_1(\vec{x}) = x_1$ $f_2(\vec{x}) = g(\vec{x}) \left[ 1 - (x_1/g(\vec{x}))^2 \right]$ $g(\vec{x}) = 1 + 9 \left( \sum_{i=2}^n x_i \right) / (n - 1)$	30	$0 \leq x_i \leq 1$	<i>nonconvex</i>
ZDT3	$f_1(\vec{x}) = x_1$ $f_2(\vec{x}) = g(\vec{x}) \left[ 1 - \sqrt{\frac{x_1}{g(\vec{x})}} - \frac{x_1}{g(\vec{x})} \sin(10\pi x_1) \right]$ $g(\vec{x}) = 1 + 9 \left( \sum_{i=2}^n x_i \right) / (n - 1)$	30	$0 \leq x_i \leq 1$	<i>convex</i> <i>disconnected</i>
ZDT4	$f_1(\vec{x}) = x_1$ $f_2(\vec{x}) = g(\vec{x})[1 - (x_1/g(\vec{x}))^2]$ $g(\vec{x}) = 1 + 10(n - 1) + \sum_{i=2}^n [x_i^2 - 10 \cos(4\pi x_i)]$	10	$0 \leq x_1 \leq 1$ $-5 \leq x_i \leq 5$ $i = 2, \dots, n$	<i>nonconvex</i>
ZDT6	$f_1(\vec{x}) = 1 - e^{-4x_1} \sin^6(6\pi x_1)$ $f_2(\vec{x}) = g(\vec{x})[1 - (f_1(\vec{x})/g(\vec{x}))^2]$ $g(\vec{x}) = 1 + 9[(\sum_{i=2}^n x_i)/(n - 1)]^{0.25}$	10	$0 \leq x_i \leq 1$	<i>nonconvex</i> <i>nonuniformly spaced</i>

Table 9  
Constrained test bi-objective problems.

Problem	Objective functions	Constraints	n	Variable bounds
Oszycza2	$f_1(\vec{x}) = -(25(x_1 - 2)^2 +$ $(x_2 - 2)^2 +$ $(x_3 - 1)^2(x_4 - 4)^2 +$ $(x_5 - 1)^2)$ $f_2(\vec{x}) = x_1^2 + x_2^2 +$ $x_3^2 + x_4^2 + x_5^2 + x_6^2$	$g_1(\vec{x}) = 0 \leq x_1 + x_2 - 2$ $g_2(\vec{x}) = 0 \leq 6 - x_1 - x_2$ $g_3(\vec{x}) = 0 \leq 2 - x_2 + x_1$ $g_4(\vec{x}) = 0 \leq 2 - x_1 + 3x_2$ $g_5(\vec{x}) = 0 \leq 4 - (x_3 - 3)^2 - x_4$ $g_6(\vec{x}) = 0 \leq (x_5 - 3)^3 + x_6 - 4$	6	$0 \leq x_1, x_2 \leq 10$ $1 \leq x_3, x_5 \leq 5$ $0 \leq x_4 \leq 6$ $0 \leq x_6 \leq 10$
Tanaka	$f_1(\vec{x}) = x_1$ $f_2(\vec{x}) = x_2$	$g_1(\vec{x}) = -x_1^2 - x_2^2 + 1 +$ $0.1 \cos(16 \arctan(x_1/x_2)) \leq 0$ $g_2(\vec{x}) = (x_1 - 0.5)^2 +$ $(x_2 - 0.5)^2 \leq 0.5$	2	$-\pi \leq x_i \leq \pi$
Constr_Ex	$f_1(x) = x_1$ $f_2(x) = (1 + x_2)/x_1$	$g_1(\vec{x}) = x_2 + 9x_1 \geq 6$ $g_2(\vec{x}) = -x_2 + 9x_1 \geq 1$	2	$0.1 \leq x_1 \leq 1.0$ $0 \leq x_2 \leq 5$
Srinivas	$f_1(\vec{x}) = (x_1 - 2)^2 +$ $(x_2 - 1)^2 + 2$ $f_2(\vec{x}) = 9x_1 - (x_2 - 1)^2$	$g_1(\vec{x}) = x_1^2 + x_2^2 \leq 225$ $g_2(\vec{x}) = x_1 - 3x_2 \leq -10$	2	$-20 \leq x_i \leq 20$
Golinski	$f_1(\vec{x}) = 0.7854x_1x_2^2(10x_3^2/3 +$ $14.933x_3 - 43.0934)$ $-1.508x_1(x_6^2 + x_7^2) +$ $7.477(x_6^3 + x_7^3)$ $+0.7854(x_4x_6^2 + x_5x_7^2)$ $f_2(\vec{x}) = \frac{\sqrt{(\frac{745.0x_4}{x_2x_3})^2 + 1.69 \cdot 10^7}}{0.1x_6^3}$	$g_1(\vec{x}) = \frac{1.0}{x_1x_2^2x_3} - \frac{1.0}{27.0} \leq 0$ $g_2(\vec{x}) = \frac{1.0}{x_1x_3^2x_4} - \frac{1.0}{27.0} \leq 0$ $g_3(\vec{x}) = \frac{x_3^4}{x_2x_3^2x_6^4} - \frac{1.0}{1.93} \leq 0$ $g_4(\vec{x}) = \frac{x_5^4}{x_2x_3x_7^4} - \frac{1.0}{1.93} \leq 0$ $g_5(\vec{x}) = x_2x_3 - 40 \leq 0$ $g_6(\vec{x}) = x_1/x_2 - 12 \leq 0$ $g_7(\vec{x}) = 5 - x_1/x_2 \leq 0$ $g_8(\vec{x}) = 1.9 - x_4 + 1.5x_6 \leq 0$ $g_9(\vec{x}) = 1.9 - x_5 + 1.1x_7 \leq 0$ $g_{10}(\vec{x}) = f_2(\vec{x}) \leq 1300$  $a = 745.0x_5/x_2x_3$ $b = 1.575 \cdot 10^8$ $g_{11}(\vec{x}) = \frac{\sqrt{a^2 + b}}{0.1x_3^3} \leq 1100$	7	$2.6 \leq x_1 \leq 3.6$ $0.7 \leq x_2 \leq 0.8$ $17.0 \leq x_3 \leq 28.0$ $7.3 \leq x_4 \leq 8.3$ $7.3 \leq x_5 \leq 8.3$ $2.9 \leq x_6 \leq 3.9$ $5.0 \leq x_7 \leq 5.5$

Table 10  
Problems with more than two objectives.

Problem	Objective functions	Constraints	n	Variable bounds
Viennet2	$f_1(\vec{x}) = \frac{(x_1-2)^2}{2} + \frac{(x_1+1)^2}{13} + 3.0$ $f_2(\vec{x}) = \frac{(x_1+x_2-3)^2}{36} + \frac{(-x_1+x_2+2)^2}{8} - 17$ $f_3(\vec{x}) = \frac{(x_1+2x_2-1)^2}{175} + \frac{(2x_2+x_1)^2}{17} - 13$		2	$-4.0 \leq x_i \leq 4.0$
Viennet3	$f_1(\vec{x}) = 0.5x_1^2 + x_2^2 + \sin(x_1^2 + x_2^2)$ $f_2(\vec{x}) = \frac{(3x_1-2x_2+4)^2}{8} + \frac{(x_1-x_2+1)^2}{27} + 15$ $f_3(\vec{x}) = \frac{1}{x_1^2+x_2^2+1} - 1.1\exp(-x_1^2 - x_2^2)$		2	$-3.0 \leq x_i \leq 3.0$
Viennet4	$f_1(\vec{x}) = \frac{(x_1-2)^2}{2} + \frac{(x_2+1)^2}{13} + 3$ $f_2(\vec{x}) = \frac{(x_1+x_2-3)^2}{175} + \frac{(2x_2-x_1)^2}{17} - 13$ $f_3(\vec{x}) = \frac{(3x_1-2x_2+4)^2}{8} + \frac{(x_1-x_2+1)^2}{27} + 15$	$g_1(\vec{x}) = -x_2 - 4x_1 + 4 \geq 0$ $g_2(\vec{x}) = x_1 + 1 \geq 0$ $g_3(\vec{x}) = x_2 - x_1 + 2 \geq 0$	2	$-4.0 \leq x_i \leq 4.0$
Water	$f_1(\vec{x}) = 106780.37(x_2 + x_3) + 61704.67$ $f_2(\vec{x}) = 3000x_1$ $f_3(\vec{x}) = \frac{305700*2289x_2}{(0.06*2289)^{0.65}}$ $f_4(\vec{x}) = 250 * 2289x_2$ $\exp(-39.75x_2 + 9.9x_3 + 2.74)$ $f_5(\vec{x}) = 25 \frac{1.39}{(x_1x_2)+4940*x_3-80}$	$g_1(\vec{x}) = 1 - \frac{0.00139}{(x_1x_2)} + 4.94x_3 - 0.08 \geq 0$ $g_2(\vec{x}) = 1 - \frac{0.000306}{(x_1x_2)} + 1.082x_3 - 0.0986 \geq 0$ $g_3(\vec{x}) = 5000 - \frac{12.307}{(x_1x_2)} + 4.9408x_3 + 4051.02 \geq 0$ $g_4(\vec{x}) = 16000 - \frac{2.09}{(x_1x_2)} + 8046.33x_3 - 696.71 \geq 0$ $g_5(\vec{x}) = 10000 - \frac{2.138}{(x_1x_2)} + 7883.39x_3 - 705.04 \geq 0$ $g_6(\vec{x}) = 2000 - \frac{0.417}{(x_1x_2)} + 1721.26x_3 - 136.54 \geq 0$ $g_7(\vec{x}) = 550 - \frac{0.164}{(x_1x_2)} + 631.13x_3 - 54.48 \geq 0$	3	$0.01 \leq x_1 \leq 0.45$ $0.01 \leq x_2 \leq 0.10$ $0.01 \leq x_3 \leq 0.10$