CS 763: Assignment 1

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Problem 3

The parametric (Euclidean) coordinates of a point on the hyperbola are $\boldsymbol{x} = (t, 1/t)$.

Hence. the homogeneous coordinates of $\boldsymbol{x}$ are given by $\boldsymbol{x_H} = (t, 1/t, 1)$.

Thus, applying the projective transformation matrix M to $\boldsymbol{x_H}$ gives the corresponding point on the image as,

$$\boldsymbol{x}'_{\boldsymbol{H}} = M\boldsymbol{x_H} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} t \\ 1/t \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1/t \\ t \end{bmatrix}$$

So, $\boldsymbol{x}'_{\boldsymbol{H}} = (1, 1/t, t) = \left(1/t, 1/t^2, 1\right)$ and that implies the parametric (Euclidean) coordinates of point in the image are $\boldsymbol{x}' = (1/t, 1/t^2)$.

The equation of the curve corresponding to points $\boldsymbol{x}'$ as t varies is $y = x^2$ which is a parabola.

Conclusion: The shape of the image is given by $y = x^2$ which is a **parabola**.

Problem 4

For a while, let us focus only on Camera P (corresponding to focal-length f_p and resolution s_p)

Assume our 3D coordinate system is as defined in my solution of Problem 5(a) *i.e.* assume that in this coordinate system defined with the pinhole of Camera P as origin, and the image plane at along $z = f_p$.

Also assume the direction vectors of l_1, l_2 and l_3 with respect to this coordinate system are $\vec{n}_1$, $\vec{n}_2$, $\vec{n}_3$ respectively. Note that this implies, $\vec{n}_1 \cdot \vec{n}_2 = 0$ and similar equalities.

Then, using the analysis done in Problem 5(a), the images of the vanishing point on the image plane in another 2D coordinate system with center of image plane (call O) as origin will be

$$\left(\frac{n_{1x} f_p}{n_{1z}}, \frac{n_{1y} f_p}{n_{1z}}\right), \left(\frac{n_{2x} f_p}{n_{2z}}, \frac{n_{2y} f_p}{n_{2z}}\right), \left(\frac{n_{3x} f_p}{n_{3z}}, \frac{n_{3y} f_p}{n_{3z}}\right)$$

respectively (call them points A, B, C respectively).

We observe that,

$$\overrightarrow{OA} \cdot \overrightarrow{OB} = \left(\frac{n_{1x} f_p}{n_{1z}}, \frac{n_{1y} f_p}{n_{1z}}\right) \cdot \left(\frac{n_{2x} f_p}{n_{2z}}, \frac{n_{2y} f_p}{n_{2z}}\right) = \frac{n_{1x} n_{2x} f_p^2 + n_{1y} n_{2y} f_p^2}{n_{1z} n_{2z}} = -f_p^2$$

Similarly, $\overrightarrow{OB} \cdot \overrightarrow{OC} = \overrightarrow{OC} \cdot \overrightarrow{OA} = -f_p^2$

Now, observe that,

$$\overrightarrow{OA}. \overrightarrow{BC} = \overrightarrow{OA} \cdot \overrightarrow{OC} - \overrightarrow{OA} \cdot \overrightarrow{OB} = -f_p^2 - (-f_p^2) = 0$$

This with the other two symmetric equalities imply that O is the **orthocenter** of $\triangle ABC$.

Now, the coordinates of A, B, C in pixel coordinate system are given. Assume the orthocenter of $\triangle ABC$ which is also the center of image plane to be (o_{1x}, o_{1y}) (this can be calculated using data from hypothesis). Since the resolution is s_p pixels per unit length, we get the relation for point A as

$$\left(\frac{n_{1x} f_p s_p}{n_{1z}}, \frac{n_{1y} f_p s_p}{n_{1z}}\right) = (p_{1x} - o_{1x}, p_{1y} - o_{1y}) \qquad [1]$$

We particularly note that we know $(p_{1x} - o_{1x}, p_{1y} - o_{1y})$ at this point.

So, in this pixel coordinate system, $\overrightarrow{OA} \cdot \overrightarrow{OB} = \overrightarrow{OB} \cdot \overrightarrow{OC} = \overrightarrow{OC} \cdot \overrightarrow{OA} = -f_p^2 s_p^2$. Thus since the values of all coordinates are known, we get the numeric value of $f_p s_p$ at this point.

Now again using equation $[1]$ and symmetric equations, we get the values of $\vec{n}_1$, $\vec{n}_2$ and $\vec{n}_3$.

Similarly, doing the whole process for second camera, gives the value of $f_q s_q$ and the direction vectors with respect to its 3D coordinate system (call them $\vec{m}_1$, $\vec{m}_2$ and $\vec{m}_3$).

Now, we also point out that since the cameras are related by the rotation matrix R, the following relations among directions hold:

$$[n_1 \mid n_2 \mid n_3] = R\,[m_1 \mid m_2 \mid m_3] \implies R = [n_1 \mid n_2 \mid n_3]\,[m_1 \mid m_2 \mid m_3]^{-1}$$

Since, all the values are known, and none of the matrices above is singular (because they are mutually perpendicular directions in 3D world) we can get the value of rotation matrix R.

Finally, I claim that t cannot be found, since vanishing points don't carry any information regarding translations *i.e.* a line translated without changing any direction has the same vanishing point.

Conclusion: R only can be determined. The products $f_p s_p$ and $f_q s_q$ can be determined but not their values individually. t cannot be determined with the information from the hypothesis.

Problem 5

Without loss of generality, we shall assume that the image plane is given by $z = d$ and the eye is at $(0, 0, 0)$ (origin). Hence, d is the (perpendicular) distance between eye (*or* pin-hole) and the image plane, which in case of a pin-hole camera is the camera constant.

Part (a)

Assume $L_1 \equiv \vec{a} + \lambda\vec{n}$ and $L_2 \equiv \vec{b} + \lambda\vec{n}$ be two parallel lines in $\mathbb{R}^3$ where $\vec{a}, \vec{b}, \vec{n} \in \mathbb{R}^3$. Now, for a point $\vec{p_1} = \vec{a} + \lambda_1\vec{n}$ on L_1 the perspective projection transformation gives the corresponding point on the image plane $\vec{q}_1$ as

$$\vec{q}_1 = \left(d \cdot \frac{a_x + \lambda_1 n_x}{a_z + \lambda_1 n_z}, d \cdot \frac{a_y + \lambda_1 n_y}{a_z + \lambda_1 n_z} \right)$$

Now as $\lambda_1 \to \infty$, the point $\vec{q}_1$ converges towards the vanishing point of L_1 (lets call it $\vec{r}_1$). Then,

$$\vec{r}_1 = \lim_{\lambda_1 \to \infty} \vec{q}_1 = \left(\frac{n_x d}{n_z}, \frac{n_y d}{n_z} \right)$$

A similar analysis for L_2 gives the the vanishing point of L_2 (lets call it $\vec{r}_2$) as,

$$\vec{r}_2 = \lim_{\lambda_2 \to \infty} \left(d \cdot \frac{b_x + \lambda_2 n_x}{b_z + \lambda_2 n_z}, d \cdot \frac{b_y + \lambda_2 n_y}{b_z + \lambda_2 n_z} \right) = \left(\frac{n_x d}{n_z}, \frac{n_y d}{n_z} \right)$$

We, observe that $\vec{r}_1 = \vec{r}_2$ and hence any two parallel lines have an intersection point, which is the vanishing point itself.

Part (b)

Suppose we now assume three pairs of lines $L_i \equiv \vec{a}_i + \lambda_i\vec{n}_i$ and $M_i \equiv \vec{b}_i + \lambda_i\vec{n}_i$ for $i \in \{1, 2, 3\}$ in R^3 which are coplanar on a 3D plane. Since they all lie on the same plane, we get the condition

$$[\vec{n}_1\ \vec{n}_2\ \vec{n}_3] = \vec{n}_1 \cdot (\vec{n}_2 \times \vec{n}_3) = 0$$

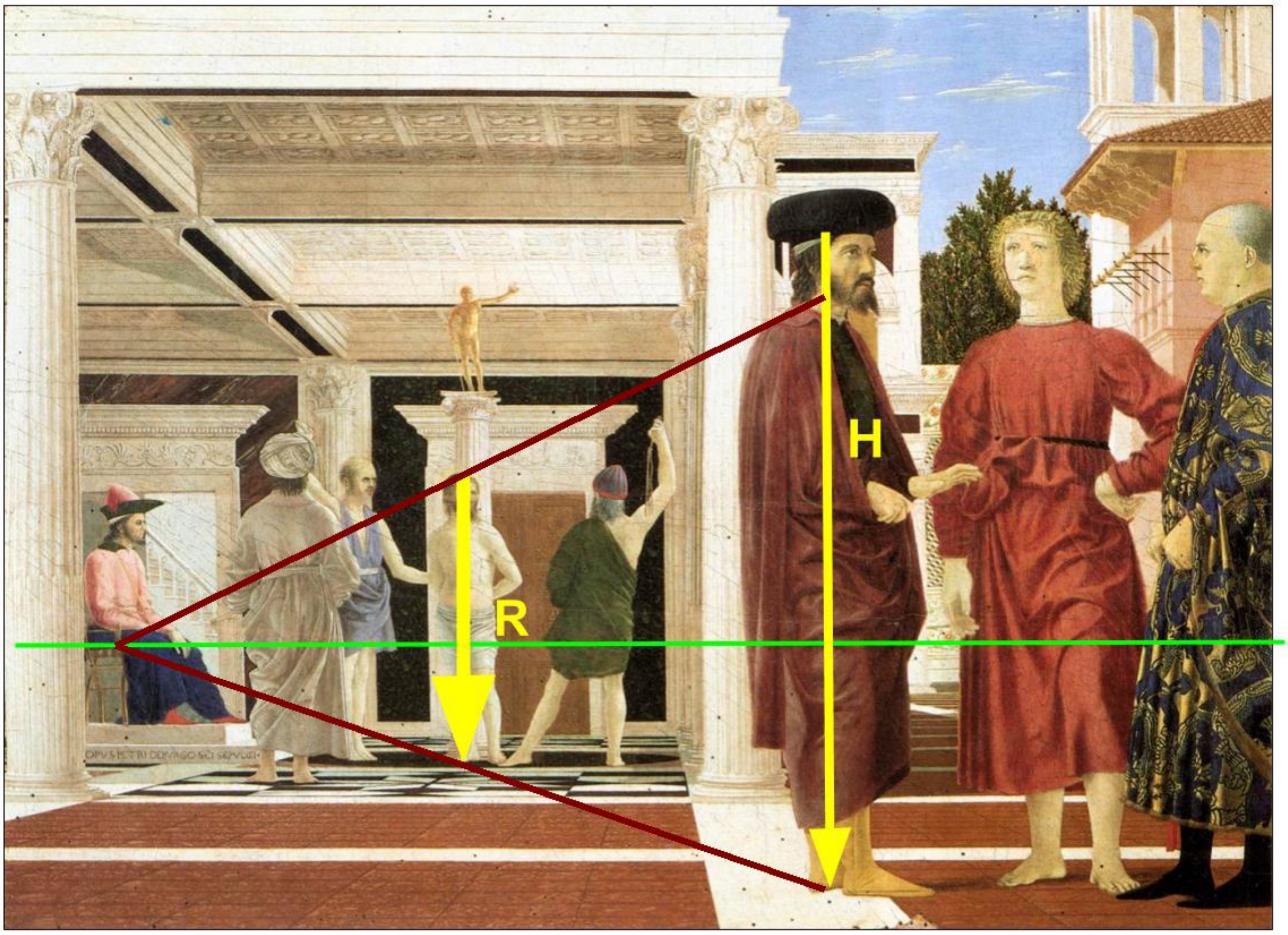
where $[.]$ denotes the scalar triple product.

Now from the analysis above for part (a) with $d = 1$ (we can do this without loss of generality since z-axis can always be rescaled accordingly while preserving any collinearity present), the vanishing points for the lines with directions corresponding to $\vec{n}_i$ in homogeneous coordinates are $(n_{ix}, n_{iy}, n_{iz}) = \vec{n}_i$ themselves for $i \in \{1, 2, 3\}$.

The line passing through the vanishing points of 2nd and 3rd pairs of lines is $\vec{n}_2 \times \vec{n}_3$. Since we know that $\vec{n}_1 \cdot (\vec{n}_2 \times \vec{n}_3) = 0$, we conclude that vanishing point of 1st pair of lines passes through the above line.

Hence proved that the three vanishing points are collinear in the image plane.

Problem 6



[All coordinates are x from left to right and y from top to bottom. Also a very reasonable assumption we make here is that the lines from head to feet for each of Christ and Person are parallel.]

Consider the line joining the feet of person with coordinates $P_f \approx (740, 795)$ and feet of Christ $C_f \approx (420, 685)$ and call it L_1. Assume the point of intersection of L_1 with the horizon (the green line) as $K \approx (114.55, 580)$. Also, consider another line L_2 which passes through K and forehead of Christ $C_h \approx (420, 430)$. Now assume the point of intersection of this line L_2 and the line along which H is measured (as in diagram) is called $Q \approx (740, 272.857)$. We also denote the top forehead point of person as $P_h \approx (740, 215)$.

With all the defined points in hand, we observe that the lines L_1 and L_2 are parallel because of the fact that they intersect at the horizon. This point of intersection, K is the vanishing point of lines along this direction. So, it is necessarily true that

$$P_f Q = R = 180 \text{ cm}$$

Using the measured (pixel) coordinates of various points defined above we get,

$$\frac{P_f P_h}{P_f Q} = \frac{H}{R} \approx \frac{(795 - 215) \text{ px}}{(795 - 272.857) \text{ px}} \implies P_f P_h = H \approx 199.9453 \text{ cm}$$

Thus the height of the person is nearly 199.95 cm (small errors if any might be due to inaccurate measurement of pixel coordinates of various points).