CS 726: Homework 2 (Due Feb 11 , 2019)

Write your answers in the space provided. You are expected to solve each question on your own. Do not try to search the answers from any external sources, like the web. You are allowed to discuss a few questions with your classmates provided you mention their names.

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1. Consider a distribution over four variables $V = \{x_1, x_2, x_3, x_4\}$ such that $\forall\ i, j,\ x_i \perp\!\!\!\perp x_j$ but no other CI holds. Indicate what happens when you attempt to draw a network via each of the three methods below

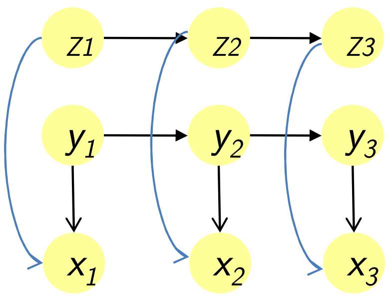
   (a) BN using the fixed order algorithm. ..2 `if you use order` $\{x_1, x_2, x_3, x_4\}$`, no edge` $x_1 \rightarrow x_2$`. Subsequently all` x_i `will be connected to all previous variables as parents.`

   (b) MRF using the Markov blanket method. ..2 `skeleton will consist of all edges.`

   (c) MRF using the pairwise conditional independence method ..2 `Fully connected graph.`

   (d) Give a numerical example of such a distribution over three variables. ..3
   `Consider the case of three variables. Let` $\Pr(x_1 = x_2|x_3 = 0) = 0.2$ `and` $\Pr(x_1 \neq x_2|x_3 = 0) = 0.8$, $\Pr(x_1 = x_2|x_3 = 1) = 0.8$ `and` $\Pr(x_1 \neq x_2|x_3 = 1) = 0.2, \Pr(x_3 = 0) = 1/2$.

2. For the Bayesian network G below, perform the following operations



   (a) Convert it into a undirected graphical model H. ..2

   (b) Identify the clique potentials in the undirected graph above? ..1

   (c) List two CIs that holds in G but do not hold in H. ..2

   (d) If you now convert the undirected graph you created above to another BN G' using the variable ordering algorithm, does there exist a variable order where $G' = G$? If not, for what ordering of variables in H would you add minimum extra edge? ..3

3. In class we defined a distribution $P(x_1, x_2, x_3)$ as follows:

$$\begin{aligned} \Pr(x_1, x_2, x_3) &= 0.5 \ \text{ if } \ x_1 = x_2 = x_3 \\ &= 0 \ \text{ otherwise.} \end{aligned} \tag{1}$$

   For this distribution $x_1 \perp\!\!\!\perp x_2|x_3$ but say we change the distribution slightly to

$$\begin{aligned} \Pr(x_1, x_2, x_3) &= \frac{1-\epsilon}{2} \ \text{ if } \ x_1 = x_2 = x_3 \\ &= \frac{\epsilon}{6} \ \text{ otherwise.} \end{aligned} \tag{2}$$

   For what values of ϵ is $x_1 \not\perp\!\!\!\perp x_2|x_3$? ..3 `We look for conditions under which` $P(x_1 = 1|x_2 = 1, x_3 = 1) = P(x_1 = 1|x_2 = 0, x_3 = 1)$`. That is,` $\frac{\frac{1-\epsilon}{2}}{\frac{1-\epsilon}{2}+\frac{\epsilon}{6}} = \frac{\frac{\epsilon}{6}}{\frac{\epsilon}{6}+\frac{\epsilon}{6}}$ `This holds only for` $\epsilon = 0$ `and for` $\epsilon = \frac{3}{4}$`. The latter corresponds to the case where the distribution is uniform over all 8 possible combinations. Students who point this point should get +2 extra credit.`

Total: 20