CS 726: Homework 1 (Due 21 Jan, 2019)

Write your answers in the space provided. You are expected to solve each question on your own. Do not try to search the answers from any external sources, like the web. You are allowed to discuss a few questions with your classmates provided you mention their names.

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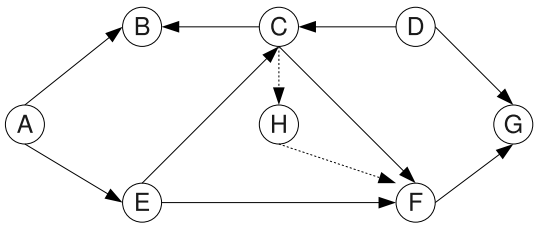
1. Consider the Bayesian Network represented by the following graph:



   (a) Assume that the vertex H and the dotted edges $C \to H$ and $H \to F$ are absent. State with reasons whether the following conditional independence statements hold:

   i. $A \perp\!\!\!\perp G \,|\, F$ ..1 `No since the A-E-F-C-D-G trail is not blocked`

   ii. $A \perp\!\!\!\perp D$ ..1 `Yes, since all trails are blocked by v-nodes`

   iii. $B \perp\!\!\!\perp G \,|\, C, F$ ..1 `No, since the B-A-E-C-D-G trail is not blocked as C is a v-node and in the conditioning set.`

   iv. $F \perp\!\!\!\perp D \,|\, C$ ..1 `No. The F-E-C-D trail is not blocked.`

   (b) Now add the vertex H along with the two dotted edges to the graph. Find the smallest set S ($G, H \notin S$) such that $G \perp\!\!\!\perp H \,|\, S$. Provide a proof if no such S exists. ..2 `D and F`

   (c) Draw a new minimal Bayesian network on variables C, D, E, F, G using the alphabetic variable order. ..3 `pa(D) = C, pa(E) = C,D, pa(F) = E,C,D, pa(G) = D,F`

2. Prove that if we factorize $\Pr(\mathbf{x})$ of a DAG as follows

$$\Pr(x_1 \ldots x_n) = \prod_{i=1}^{n} f_i(x_i, x_{\pi_i})$$

   where π_i denotes the indices corresponding to the parents of node i and $\sum_{x_i} f_i(x_i, x_{\pi_i}) = 1$ then $f_i(x_i, x_{\pi_i}) = \Pr(x_i \,|\, x_{\pi_i})$ ..2

3. You have been hired by Orange Corp to predict the value of its profits at the end of each of the next three quarters given its profits this quarter, number of visitors to its website, number of advertisors and the market share of its competition Smug, Inc. You decide to model this using a Bayesian network where for each of the four factors: profits P, visitors V, advertisors A and competition share C, you associate a variable corresponding to values of each of the factors in each quarter t. Assume that value of each factor in quarter t is directly dependant only on its value in the previous quarter. In addition, the value of profit P_t in quarter t is also directly influenced by the variables V_t, A_t and C_t.

   (a) Draw the Bayesian network over five quarters $t = 1, 2, \ldots, 5$. ..2

   (b) List two conditional independencies (CIs) you get by the D-separation rule which are not local CIs. ..2

4. Draw a BN with $O(n)$ edges where n is the number of variables such that the construction algorithm (based on an assumed variable order) will give rise to a BN with $O(n^2)$ edges with an adverserial ordering of vertices. ..4 A star with one node x_1 in the center and n-1 nodes around it with edges out from x_1 to each other x_i. If the variable order puts x_1 last, we will find that all but x_1 will have all preceding variables as parents.

Total: 19