# CS 753: Assignment #1

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## Question 1

Assume that output of C (which is of length $2n$) is $z_1 \dots z_{2n}$. Then note that

$$y_i = \square \iff z_{2i} = \square \;\wedge\; z_{2i-1} = \square$$

Also,

$$P(z_i = \square) \;=\; \epsilon_1 P(z_{i-1} = \square) + \epsilon_0(1 - P(z_{i-1} = \square)) \quad \forall\, i > 1 \quad | \quad P(z_1 = \square) = \epsilon_0 \qquad \text{... eqn [A]}$$

$$P(y_i = \square) = P(z_{2i} = \square \;\wedge\; z_{2i-1} = \square) = P(z_{2i} = \square \mid z_{2i-1} = \square)\, P(z_{2i-1} = \square) = \epsilon_1 P(z_{2i-1} = \square)$$

### Part 1

$$P(y_1 = \square) = \epsilon_1 P(z_1 = \square) = \epsilon_0 \epsilon_1$$

### Part 2

Using equation [A],

$$P(z_2 = \square) \;=\; \epsilon_1 P(z_1 = \square) + \epsilon_0(1 - P(z_1 = \square)) \;=\; \epsilon_1\epsilon_0 + \epsilon_0(1 - \epsilon_0) = 2\epsilon_0\epsilon_1$$

Again using equation [A],

$$P(z_3 = \square) \;=\; \epsilon_1 P(z_2 = \square) + \epsilon_0(1 - P(z_2 = \square)) \;=\; \epsilon_1(2\epsilon_0\epsilon_1) + \epsilon_0(1 - 2\epsilon_0\epsilon_1) = \epsilon_0[1 + 2\epsilon_1(\epsilon_1 - \epsilon_0)]$$

And finally,

$$P(y_2 = \square) = \epsilon_1 P(z_3 = \square) = \epsilon_0\epsilon_1[1 + 2\epsilon_1(\epsilon_1 - \epsilon_0)]$$

## Part 3

Let $\beta$ denote the following limit,

$$\beta = \lim_{n\to\infty}(z_n = \square)$$

Then, applying limit on equation [A] gives,

$$\begin{aligned}\lim_{i\to\infty} P(z_i = \square) &= \epsilon_1 \lim_{i\to\infty} P(z_{i-1} = \square) + \epsilon_0(1 - \lim_{i\to\infty} P(z_{i-1} = \square)) \\ &\implies \beta = \epsilon_1\beta + \epsilon_0(1-\beta) \\ &\implies \beta = \frac{1}{2} \quad [\because 1 - \epsilon_1 = \epsilon_0]\end{aligned}$$

Thus this gives,

$$\lim_{n\to\infty} P(y_n = \square) = \epsilon_1 \lim_{n\to\infty} P(z_{2n-1} = \square) = \frac{\epsilon_1}{2}$$