

# Project Part-1

After feature extraction -

Training size -  $12000 \times 2$  784

Test data size =  $2000 \times 2$

① Estimation parameters -

One class  $\mu = [\mu_1, \mu_2]$   $\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}$  } np.cov  
 Class 1 -  $\mu = [\mu_1, \mu_2]$   
 Class 0 -  $\mu = [\mu_1, \mu_2]$

Hint: The features are independent.

Class 0 - Avg, standard deviation

$\mu, \sigma \leftarrow 1\text{-D Gaussian}$   $\downarrow$   $\downarrow$  1-D Gaussian

Class 1 - Avg

$\downarrow$   
1-D

Std

$\downarrow$   
1-D Gaussian

$$\begin{bmatrix} \sigma_{11} & 0 \\ 0 & \sigma_{22} \end{bmatrix}$$

784

$\downarrow$   
1

② Naive Bayes -

$$P(Y|X) \sim P(X|Y) \cdot P(Y)$$

From the data

$$P(X_1|Y) \cdot P(X_2|Y) \cdot P(Y)$$

$$P(Y=0|x) > P(Y=1|x) \rightarrow \text{Class 0}$$

otherwise Class = 1

③ Logistic Regression - Weights w  $P(Y|x)$

Step 1: Initialize w randomly.

Step 2:  $z = \sigma(w^T x)$   $\sigma(t) = \frac{1}{1 + e^{-t}}$

Step 3: Calculate the cost function.

Log-likelihood func<sup>n</sup>

$$L(w) = \sum_{i=1}^n y^{(i)} \log z^{(i)} + (1 - y^{(i)}) \log(1 - z^{(i)})$$

$\leq 1e^{-8}$

Step 4: Gradient-descent

iteration  $w_{t+1}^j = w_t^j + \eta \frac{\partial L(w)}{\partial w^j}$

np.random

$$\frac{\partial L(w)}{\partial w^j} = \frac{\partial L(w)}{\partial z} \cdot \frac{\partial z}{\partial p} \cdot \frac{\partial p}{\partial w^j} \Rightarrow \text{Using chain rule.}$$

$$\frac{\partial L(w)}{\partial z} = \frac{y}{z} - \frac{(1-y)}{1-z} \quad \text{--- (1)}$$

$$\frac{\partial z}{\partial p} = z(1-z) \quad \text{--- (2)}$$

$$\frac{\partial p}{\partial w^j} = x^j \quad \text{--- (3)}$$

$$\frac{\partial L(w)}{\partial w^j} = \frac{y(1-z) - z(1-y)}{z(1-z)} \cdot z(1-z) \cdot x^j$$

$$= (y - z) x^j$$

Prediction  $z = \sigma(w^T x)$

$z > 0.5 \rightarrow \text{Class 1}$

otherwise  $\rightarrow \text{Class 0}$

Covariance matrix -

$$\frac{1}{600} \begin{bmatrix} x_1 & x_2 \end{bmatrix} \Rightarrow \text{Class 0}$$

$$\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} \sigma_{11} & 0 \\ 0 & \sigma_{22} \end{bmatrix}$$

$$12000 \begin{bmatrix} x_1 & x_2 \end{bmatrix}$$

- Training data

$$6000 \begin{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \end{bmatrix}$$

Class 0

$$600 \begin{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \end{bmatrix}$$

Class 1

$\mu, \sigma, \Sigma$

$\mu, \sigma, \Sigma$