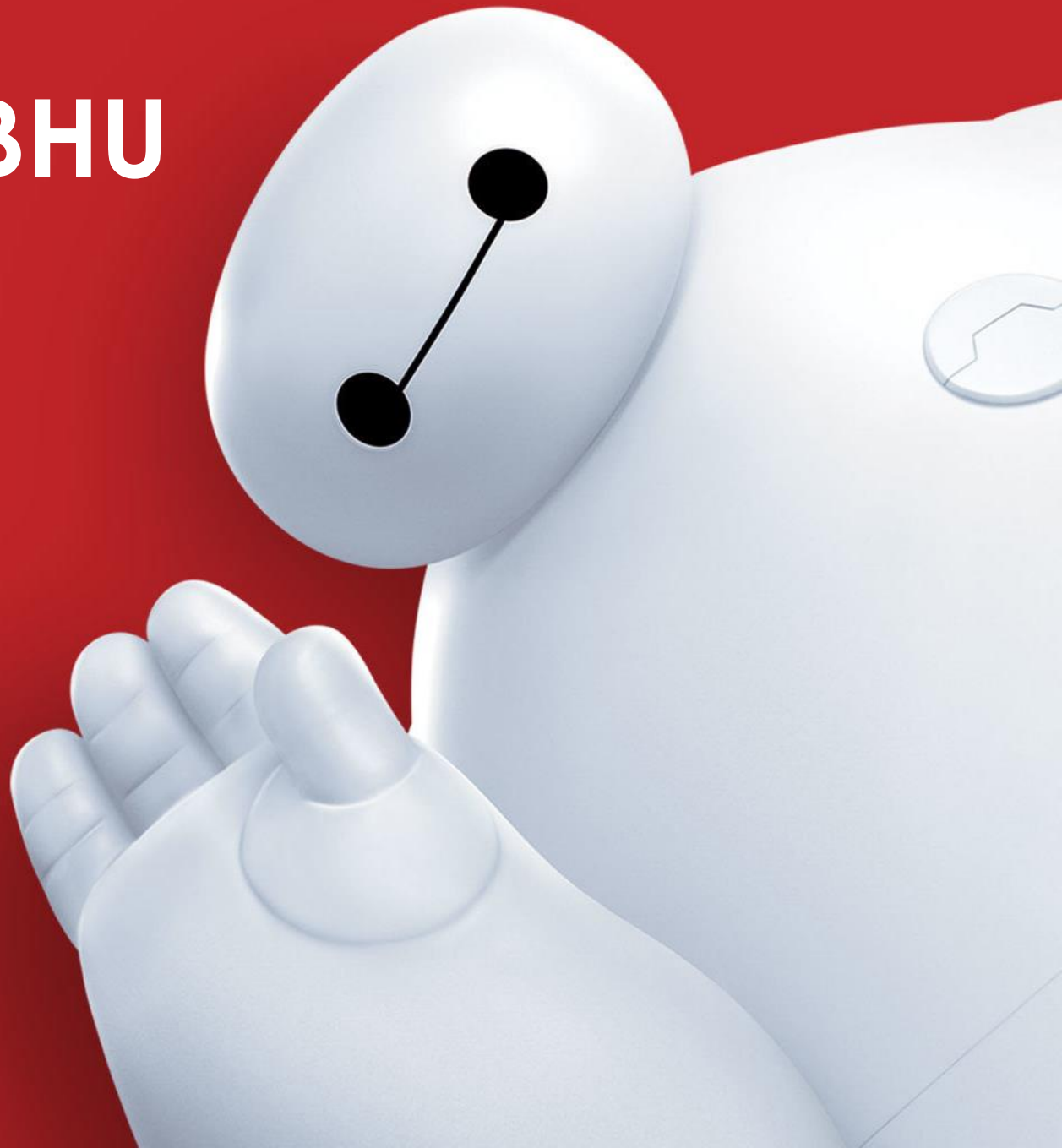
Robotics Club, IIT BHU

Presents

Dijkstra's Shortest Path Algorithm

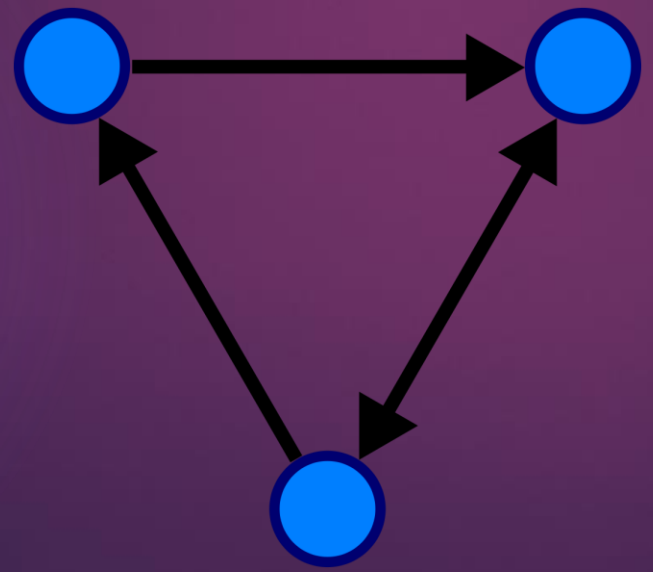


Dijkstra's Algorithm:

- Given a graph ,a source vertex and a destination vertex in graph, find shortest paths from source to destination.
- There can be more than one shortest path between two vertices in a graph.
- The shortest path may not pass through all the vertices.
- This algorithm can be used for directed as well as un-directed graphs

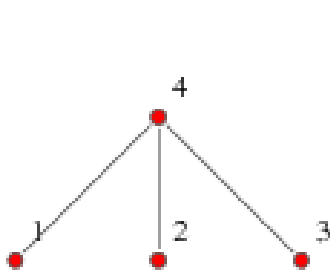
Graph:

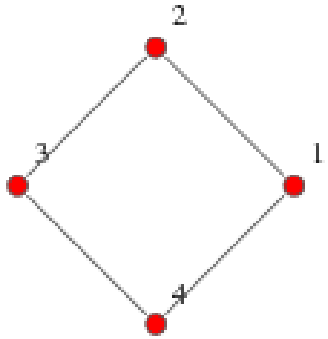
- Data Structure: A data structure is a particular way of organizing data in a computer so that it can be used effectively.
- Graph: A collection of entities that we call Nodes connected to each other through a set of Edges.

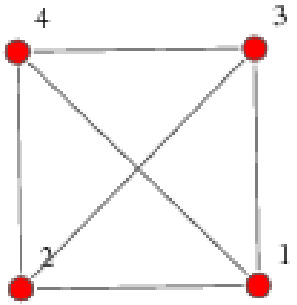


Adjacency Matrix:

- The adjacency matrix, sometimes also called the connection matrix, of a simple labeled graph is a matrix with rows and columns labeled by graph vertices, with a 1 or 0 in position according to whether and are adjacent or not.



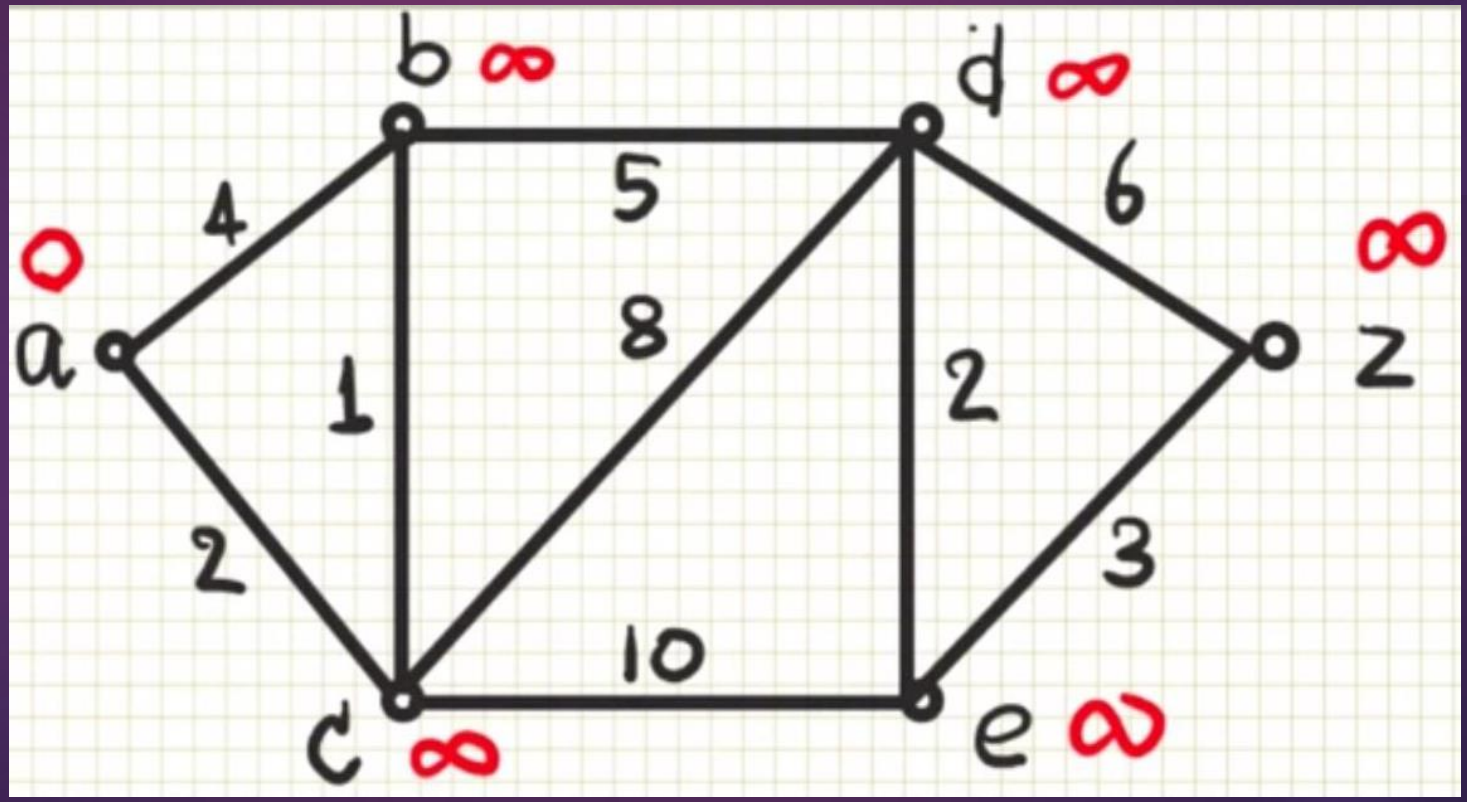




$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

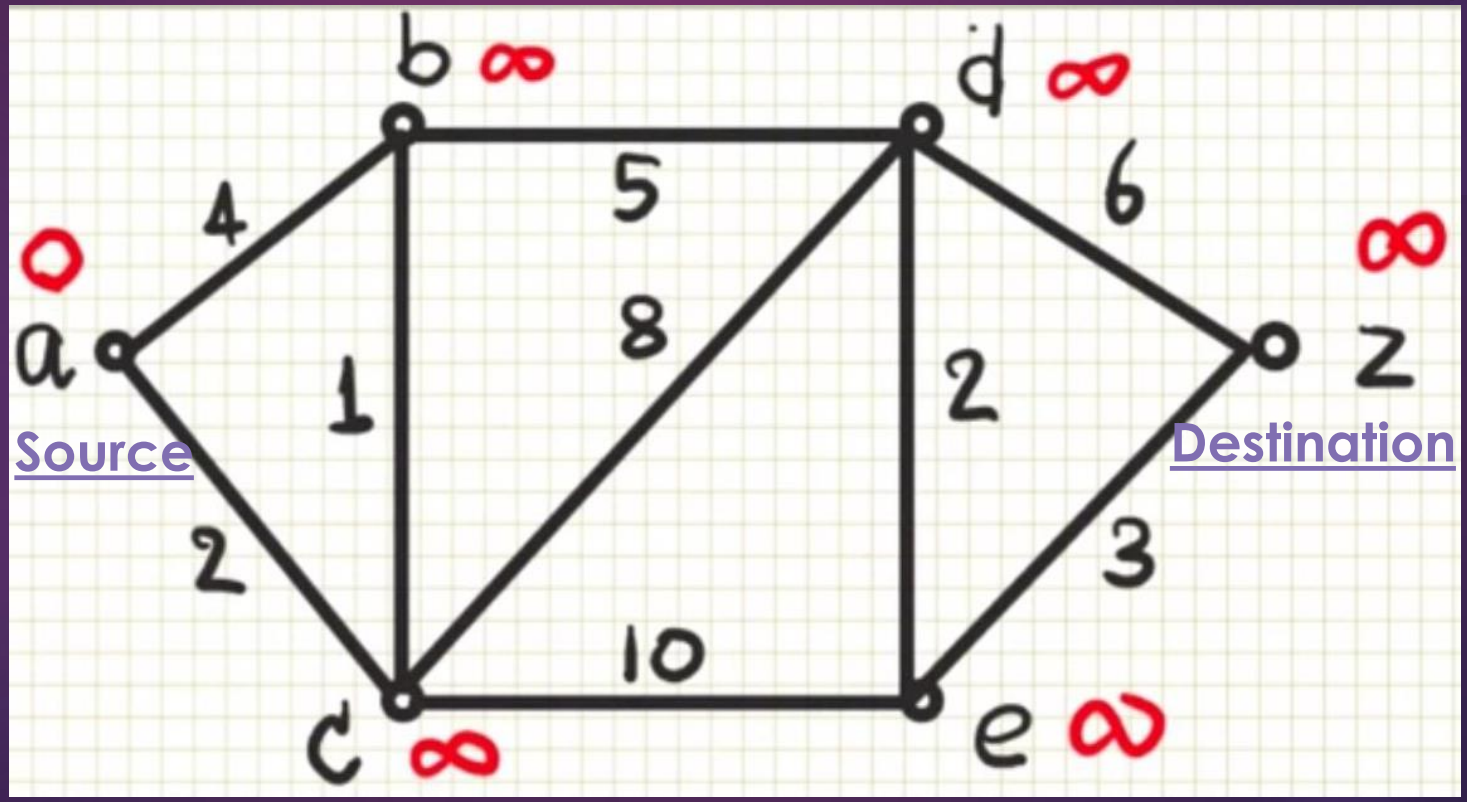
Dijkstra's Algorithm:

- Assign distance zero to Source and (infinite to all other vertices).



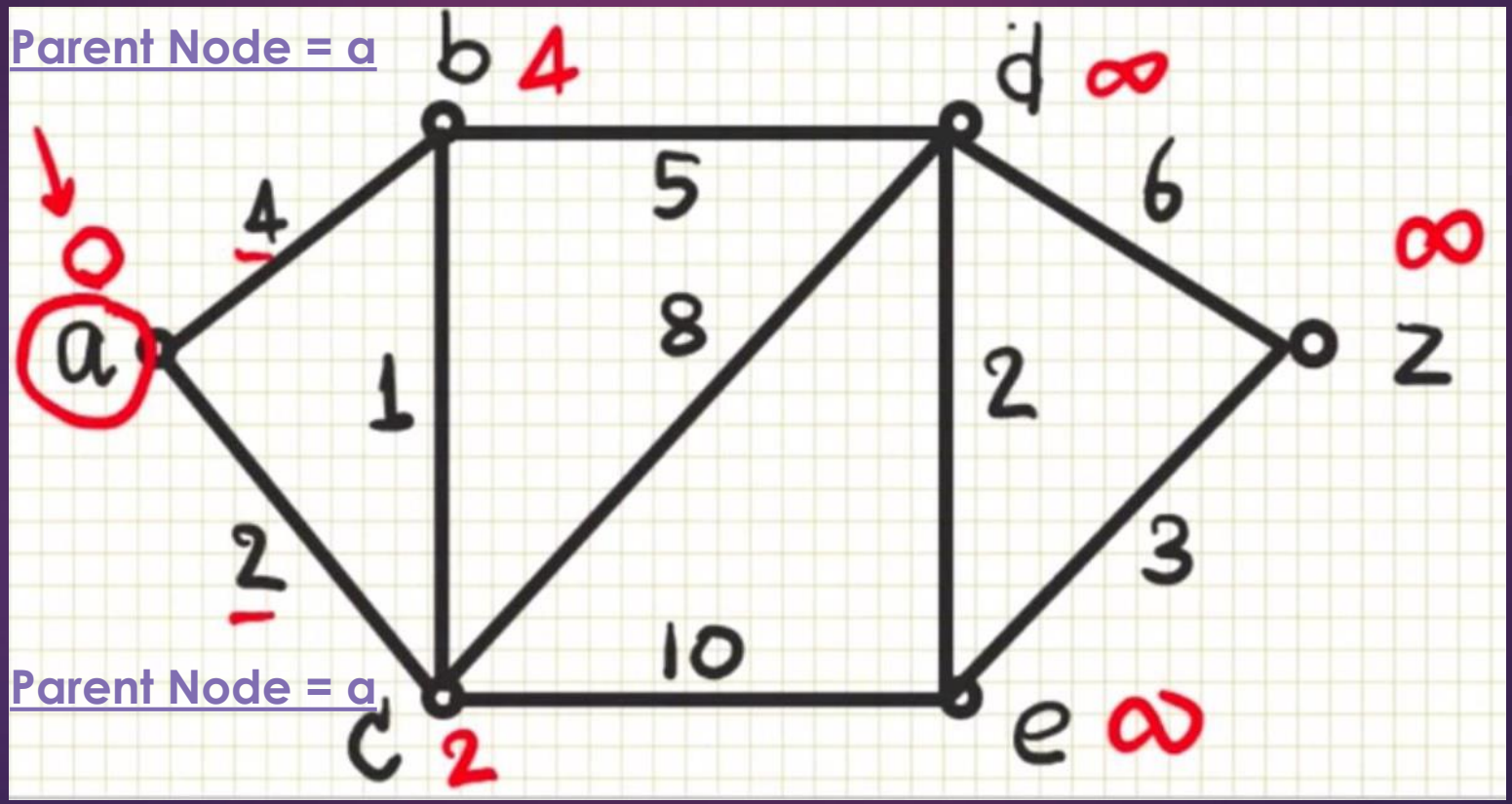
Dijkstra's Algorithm:

- Assign distance zero to Source and (infinite to all other vertices).



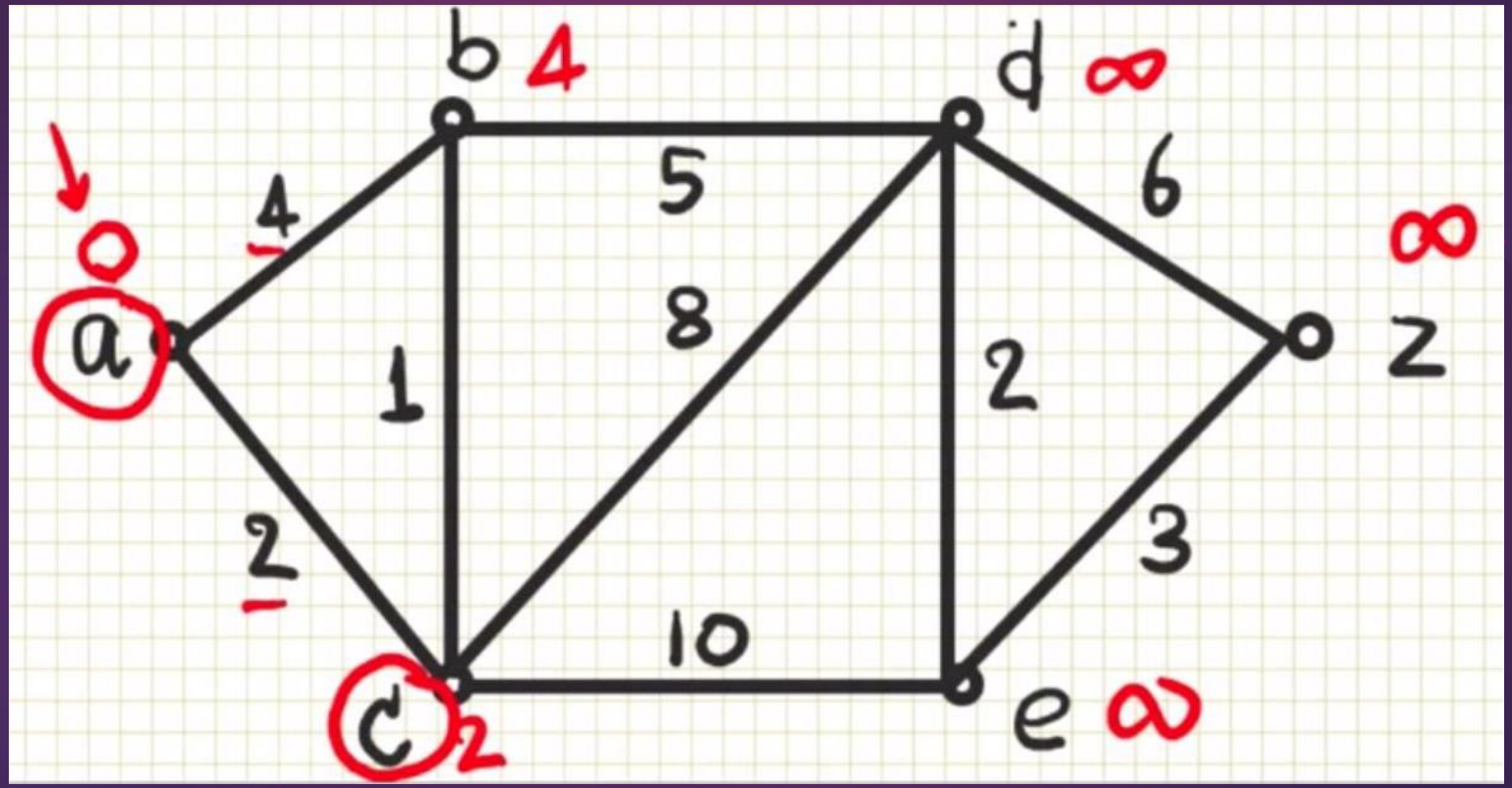
Dijkstra's Algorithm:

- The distances of Node B and Node C are updated as their distances (infinite) was more than the distance from the current Node A.
- Node A is marked as visited.



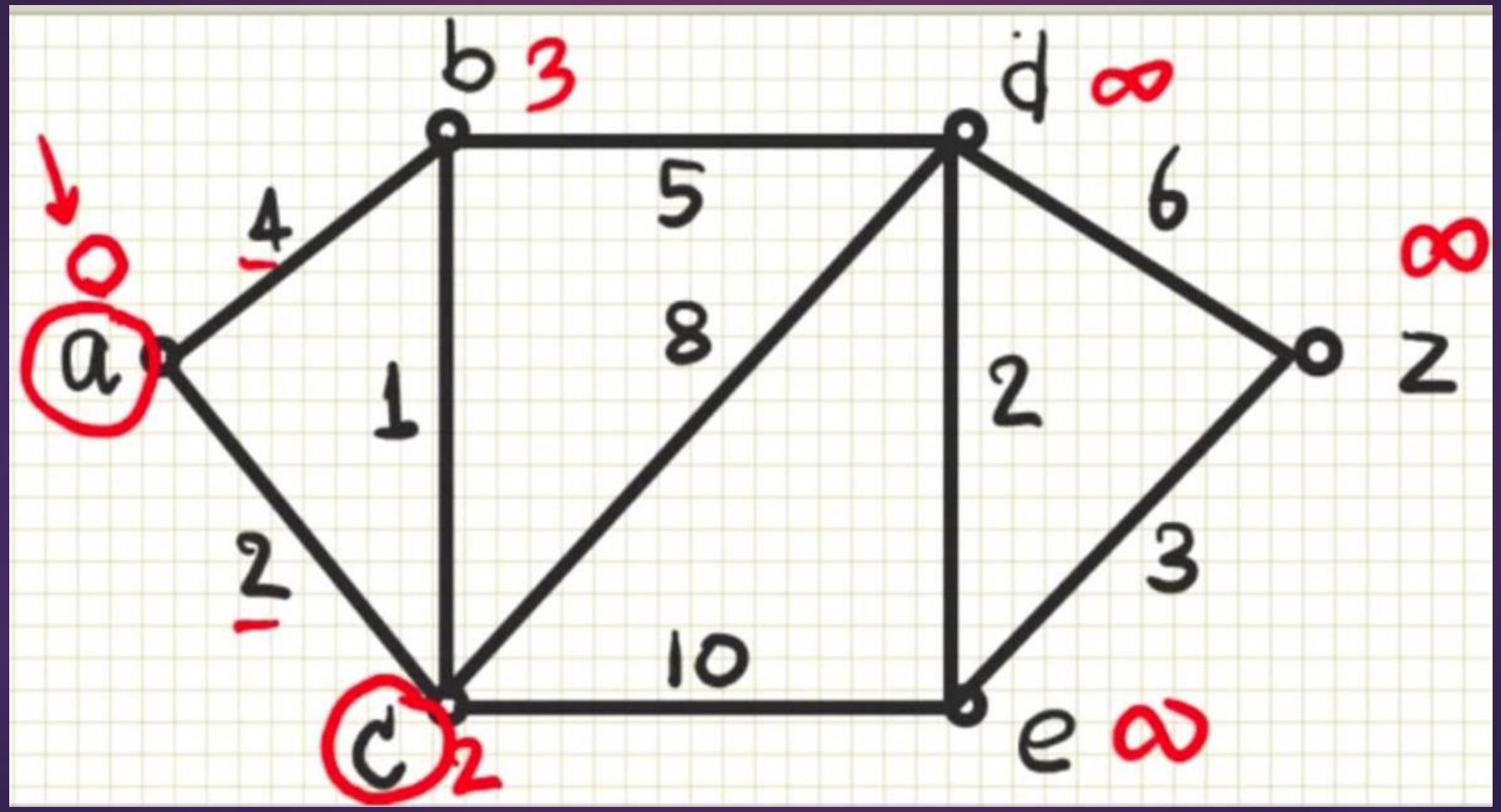
Dijkstra's Algorithm:

- Of all the nodes ,Node C Has the least distance.
- Node C is hence selected as the current node.



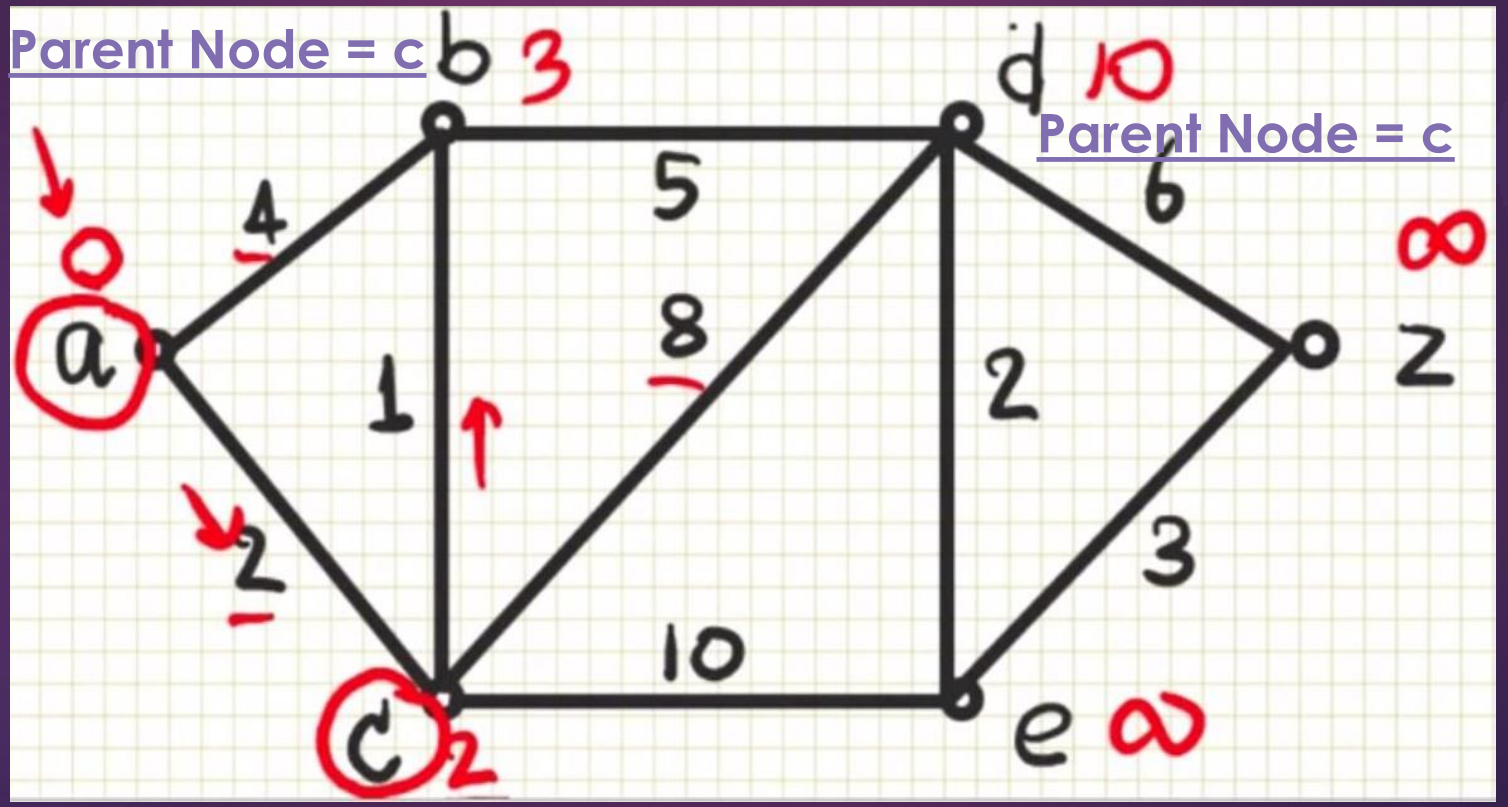
Dijkstra's Algorithm:

- Distance of Node B is updated.



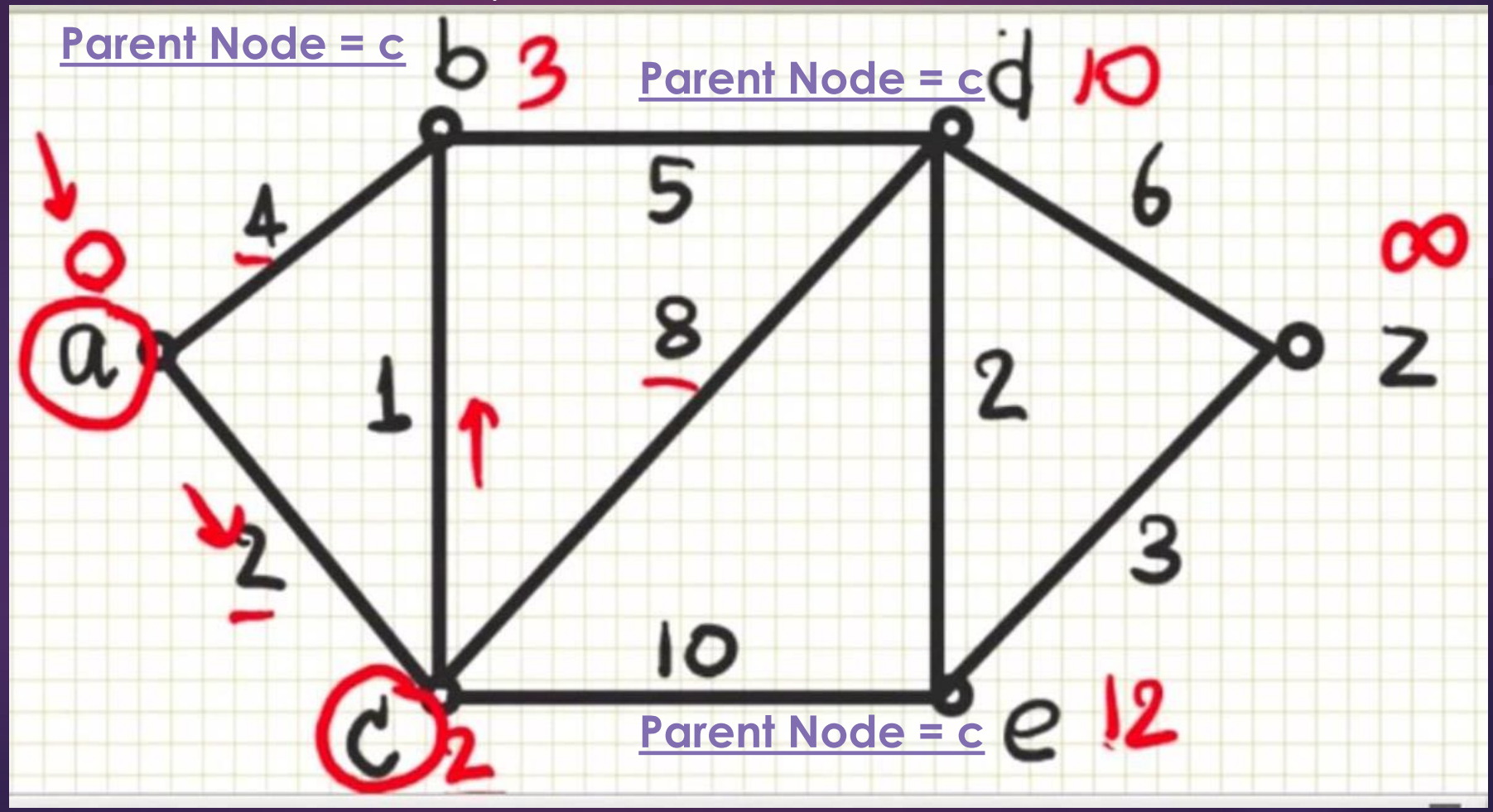
Dijkstra's Algorithm:

- Distance of Node B is updated.
- Distance of node D is updated here.



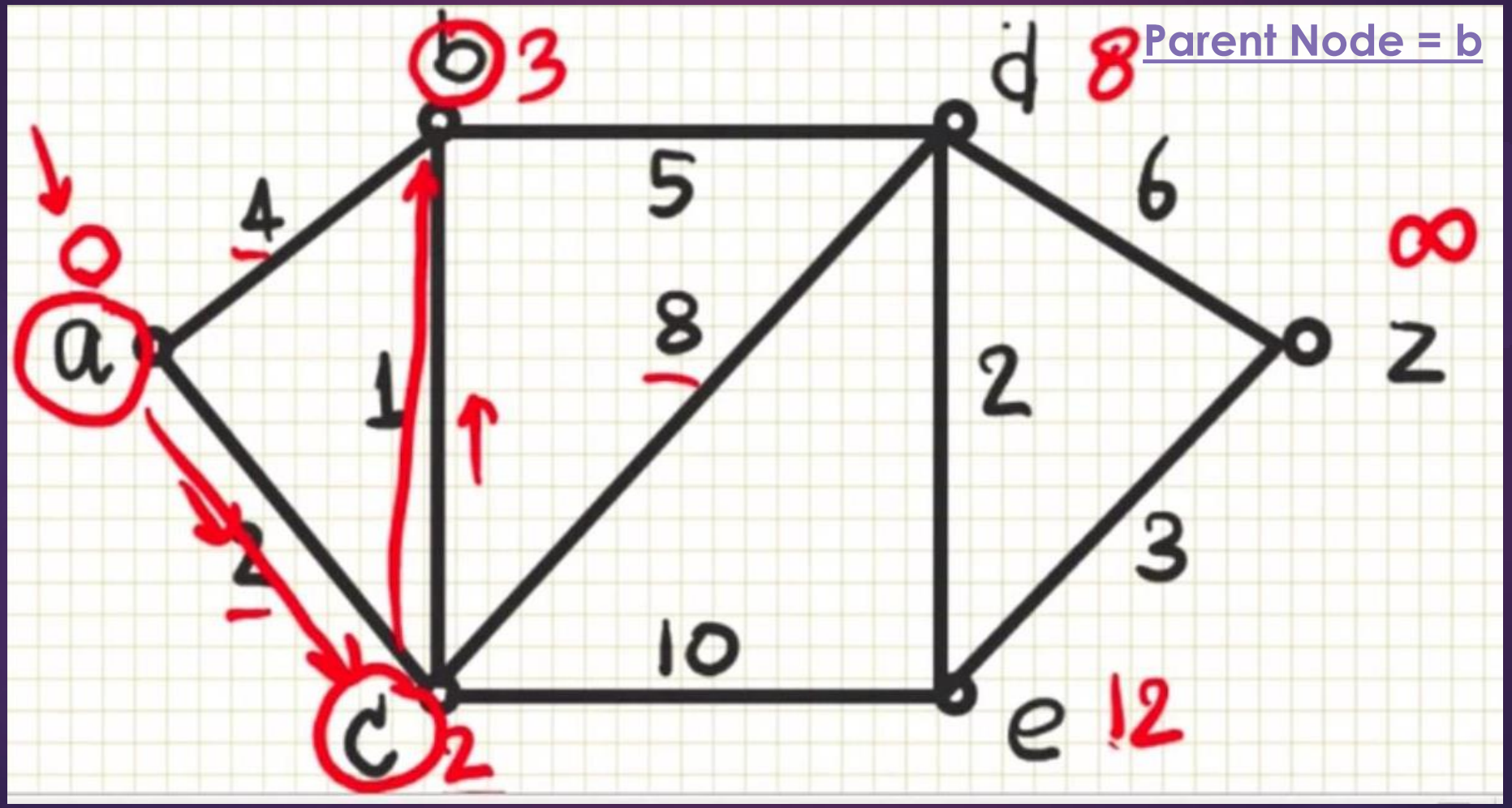
Dijkstra's Algorithm:

- Distance of Node B is updated.
- Distance of node D,E is updated here.

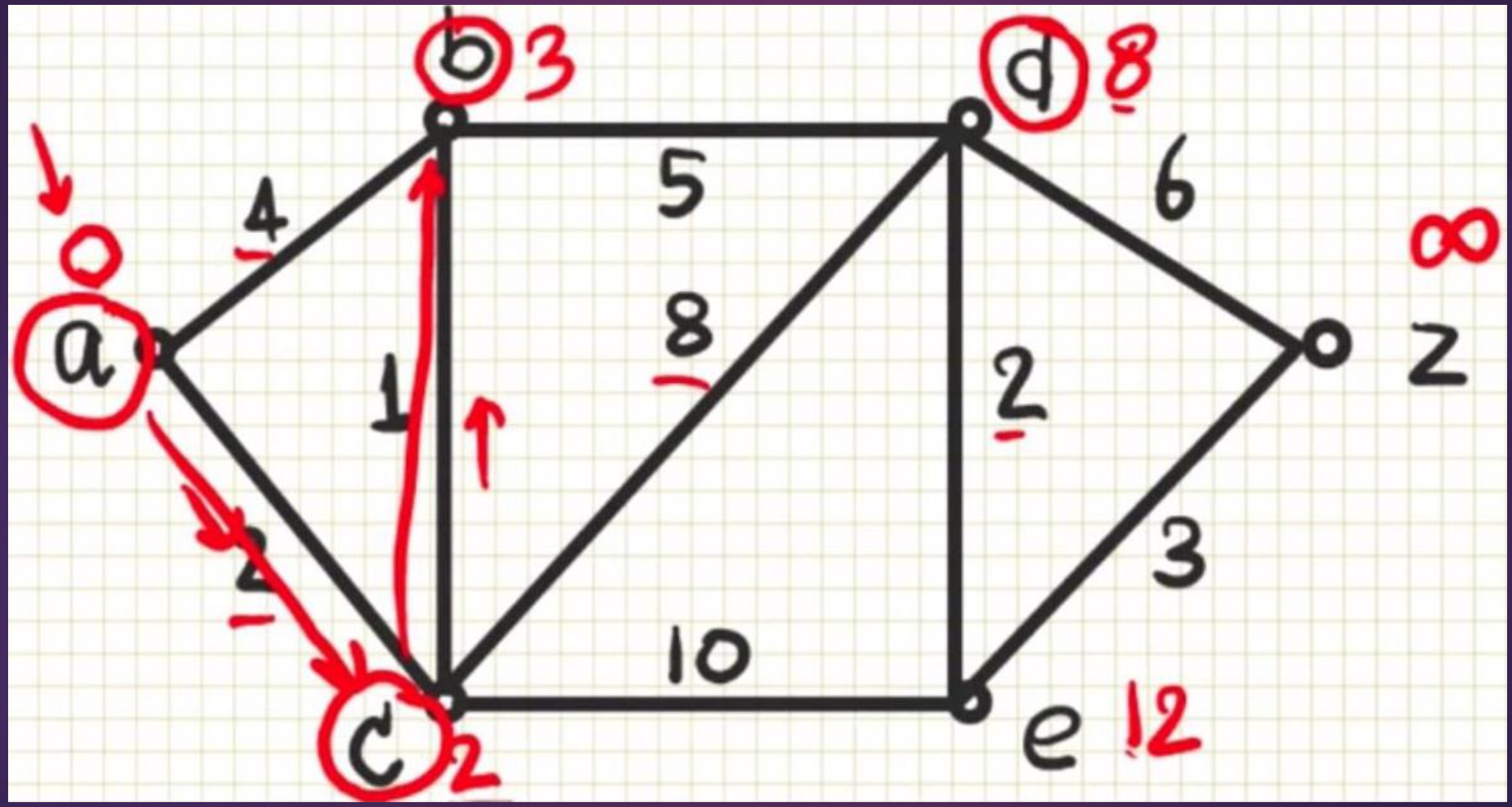


Dijkstra's Algorithm:

- Node D updated.

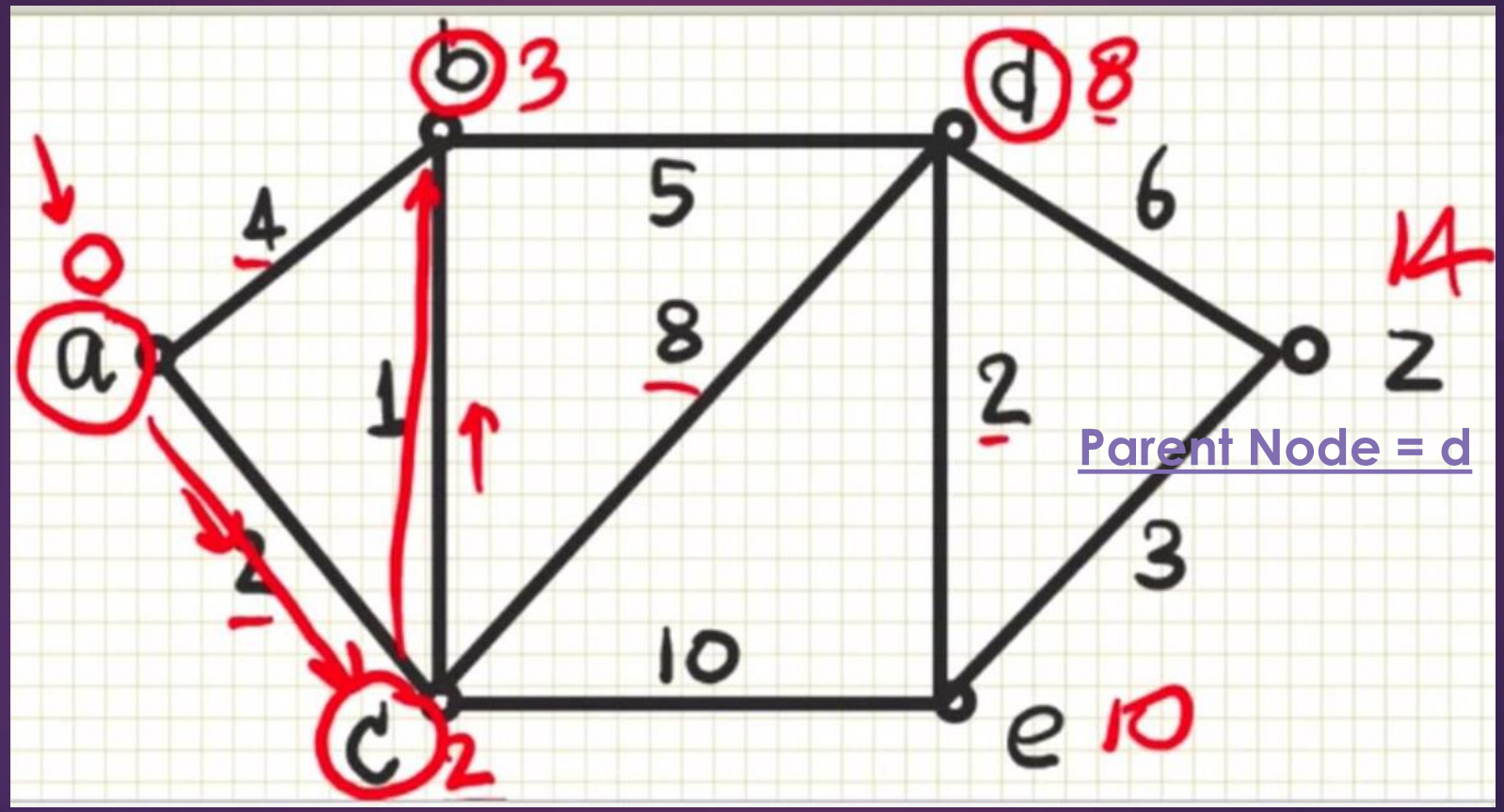


Dijkstra's Algorithm:



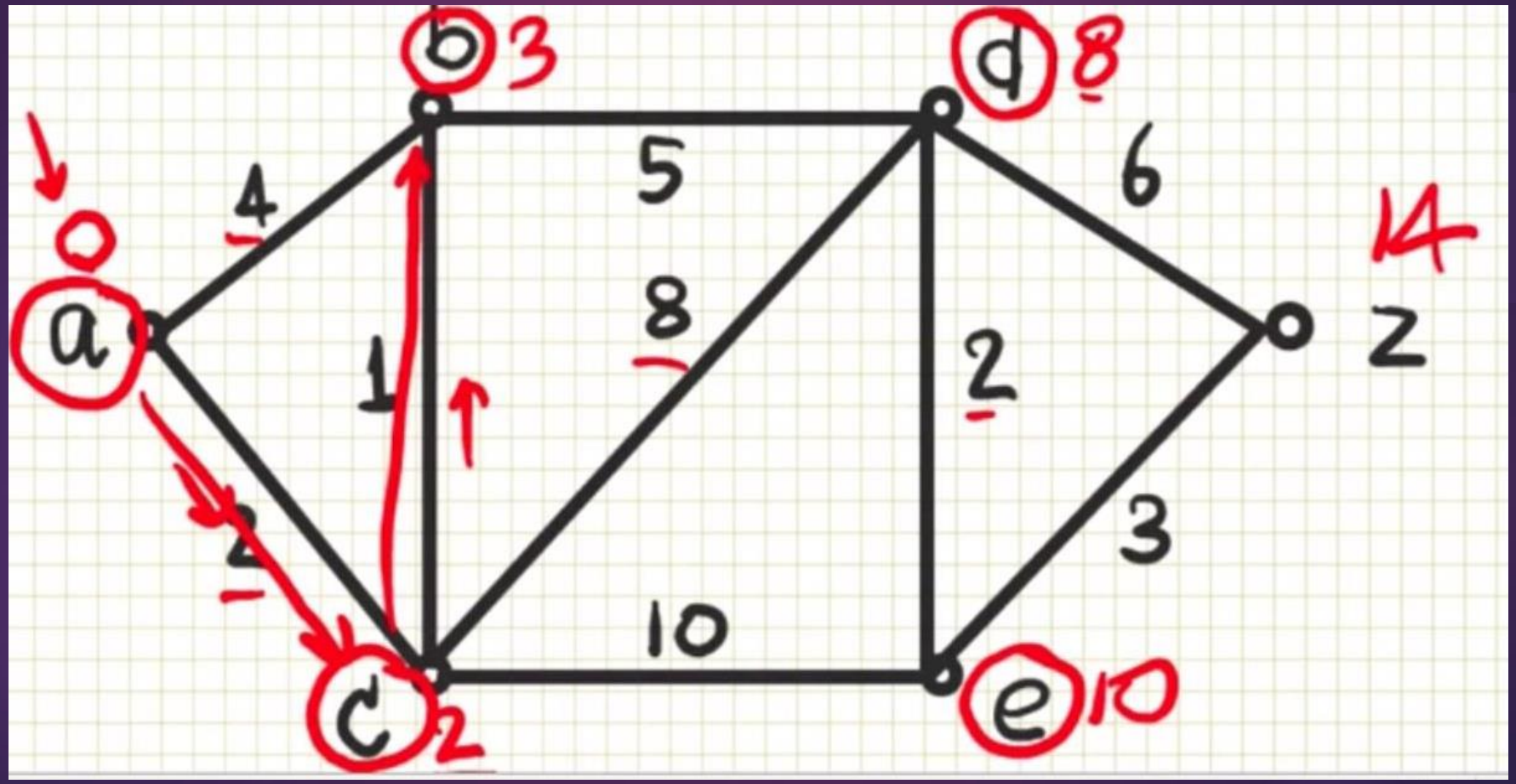
Dijkstra's Algorithm:

- Node E and Node Z updated.

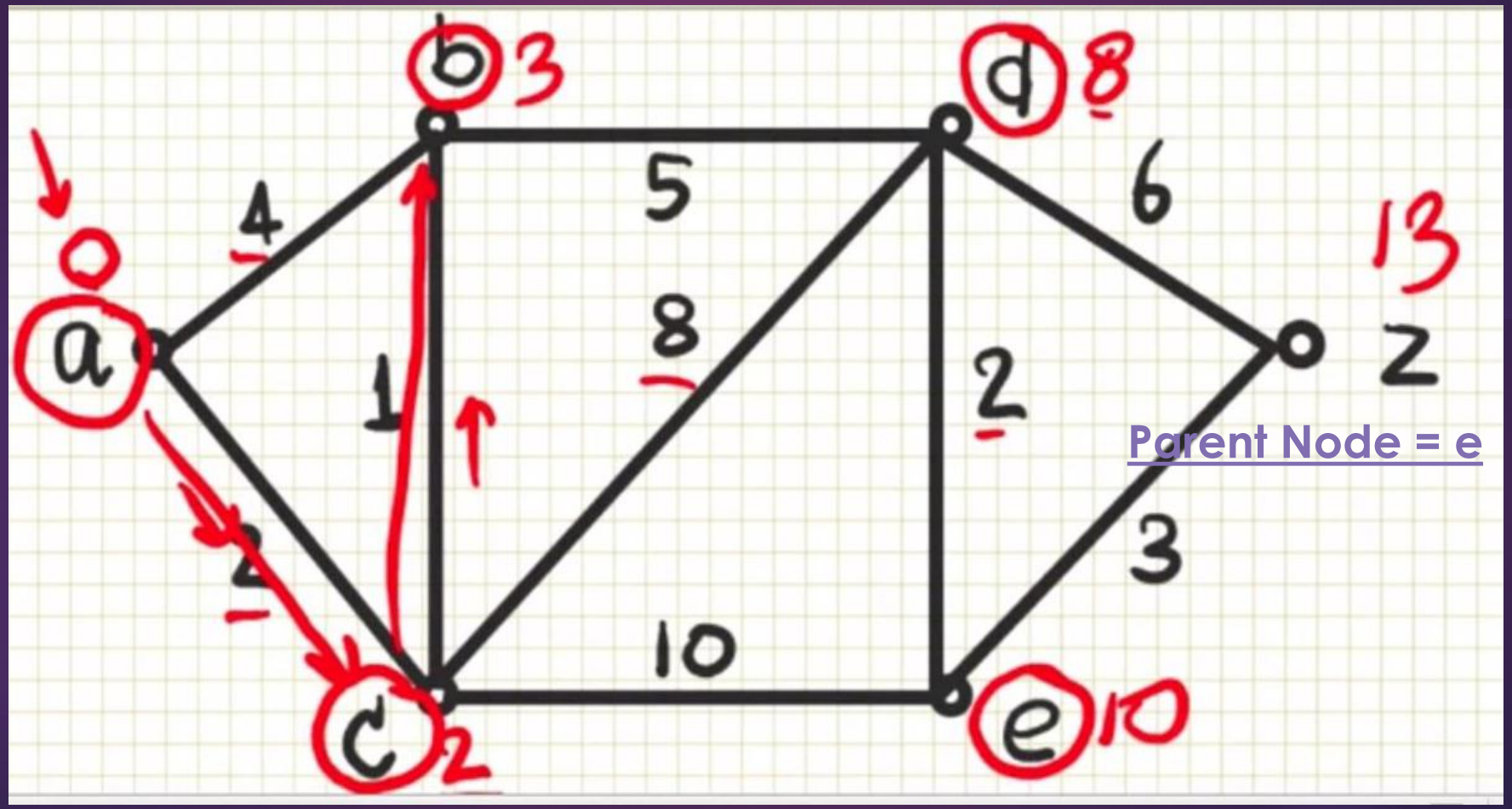


Dijkstra's Algorithm:

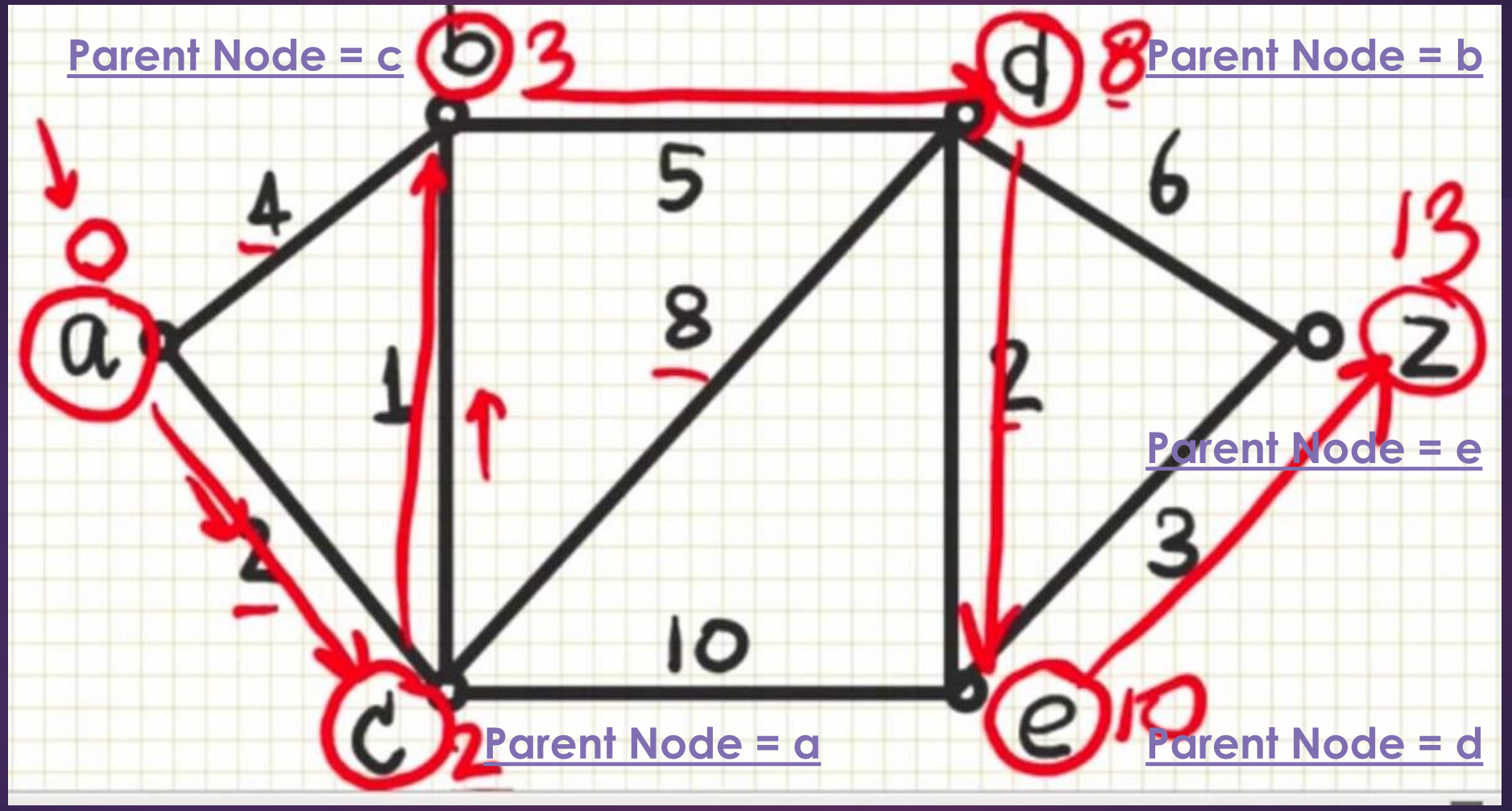
- Node E is taken as current node after Marking node D as visited.



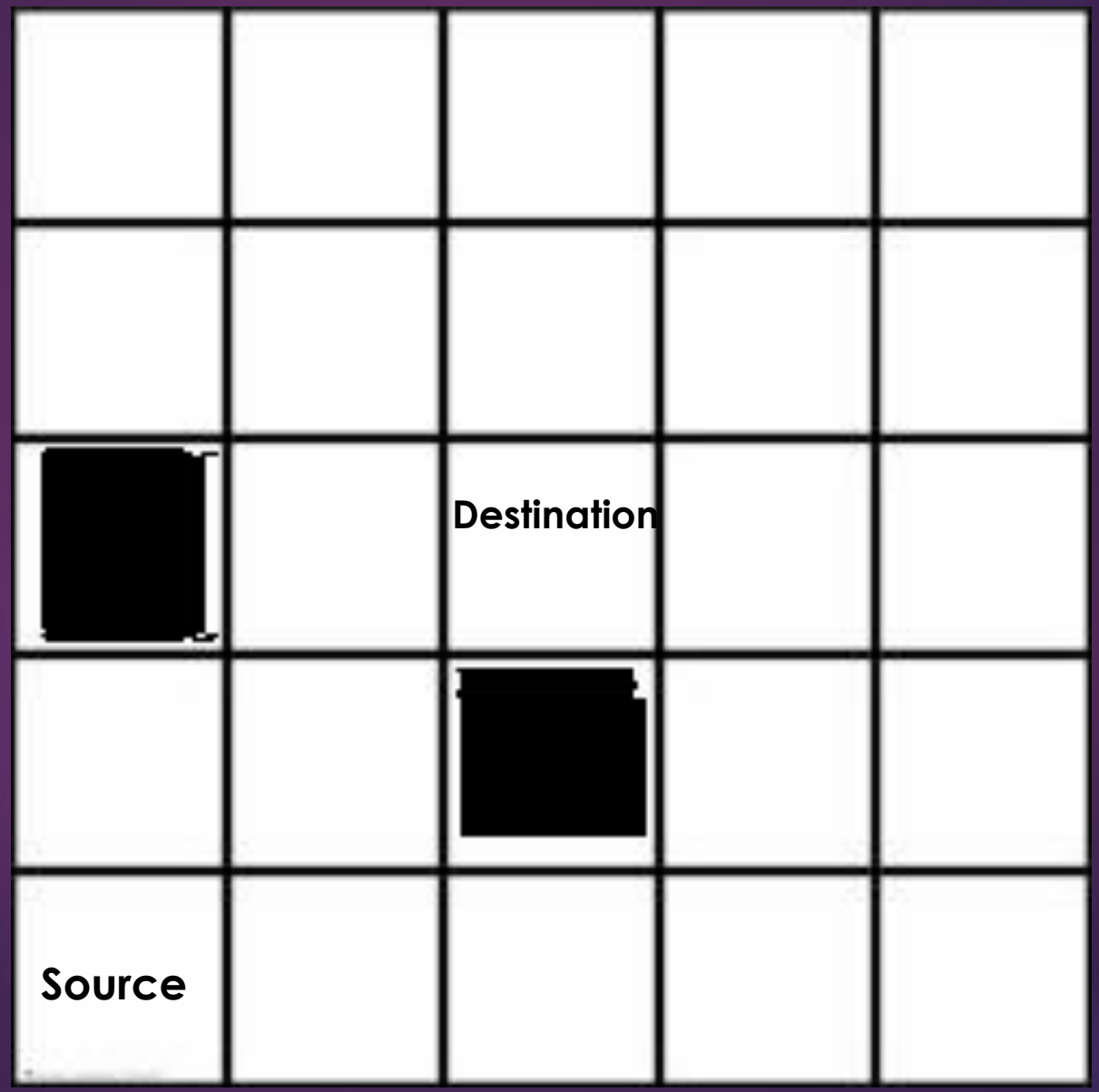
Dijkstra's Algorithm:



Dijkstra's Algorithm:



Dijkstra's Algorithm:



Dijkstra's Algorithm:

Graph:

1	1	1	1	1
1	1	1	1	1
0	1	1	1	1
1	1	0	1	1
1	1	1	1	1

Dijkstra's Algorithm:

d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf		d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
d = 0 v = 0 **P = 0**	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf

Dijkstra's Algorithm:

d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
d = 1 v = 0 **P = 5**	d = Inf v = 0 P = Inf		d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
d = 0 v = 0 **P = 0**	**d = 1** v = 0 **P = 5**	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf

Dijkstra's Algorithm:

d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
d = 1 v = 0 **P = 5**	d = Inf v = 0 P = Inf		d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
d = 0 **v = 1** **P = 0**	**d = 1** v = 0 **P = 5**	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf

Dijkstra's Algorithm:

d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
d = 1 **v = 1** **P = 5**	**d = 2** v = 0 **P = 4**		d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
d = 0 **v = 1** **P = 0**	**d = 1** v = 0 **P = 5**	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf

Dijkstra's Algorithm:

d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
d = 1 **v = 1** **P = 5**	**d = 2** v = 0 **P = 4**		d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
d = 0 **v = 1** **P = 0**	**d = 1** **v = 1** **P = 5**	**d = 2** v = 0 **P = 10**	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf

Dijkstra's Algorithm:

d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
	**d = 3** v = 0 **P = 9**	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
d = 1 **v = 1** **P = 5**	**d = 2** **v = 1** **P = 4**		d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
d = 0 **v = 1** **P = 0**	**d = 1** **v = 1** **P = 5**	**d = 2** v = 0 **P = 10**	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf

Dijkstra's Algorithm:

d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
	**d = 3** v = 0 **P = 9**	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
d = 1 **v = 1** **P = 5**	**d = 2** **v = 1** **P = 4**		d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
d = 0 **v = 1** **P = 0**	**d = 1** **v = 1** **P = 5**	**d = 2** **v = 1** **P = 10**	**d = 3** v = 0 **P = 15**	d = Inf v = 0 P = Inf

Dijkstra's Algorithm:

d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
d = Inf v = 0 P = Inf	**d = 4** v = 0 **P = 8**	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
	**d = 3** **v = 1** **P = 9**	**d = 4** v = 0 **P = 8**	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
d = 1 **v = 1** **P = 5**	**d = 2** **v = 1** **P = 4**		d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
d = 0 **v = 1** **P = 0**	**d = 1** **v = 1** **P = 5**	**d = 2** **v = 1** **P = 10**	**d = 3** v = 0 **P = 15**	d = Inf v = 0 P = Inf

Dijkstra's Algorithm:

d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
d = Inf v = 0 P = Inf	**d = 4** v = 0 **P = 8**	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
	**d = 3** **v = 1** **P = 9**	**d = 4** v = 0 **P = 8**	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
d = 1 **v = 1** **P = 5**	**d = 2** **v = 1** **P = 4**		**d = 4** v = 0 **P = 20**	d = Inf v = 0 P = Inf
d = 0 **v = 1** **P = 0**	**d = 1** **v = 1** **P = 5**	**d = 2** **v = 1** **P = 10**	**d = 3** **v = 1** **P = 15**	**d = 4** v = 0 **P = 20**

Dijkstra's Algorithm:

d = Inf v = 0 P = Inf	**d = 5** v = 0 **P = 7**	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
d = 5 v = 0 **P = 7**	**d = 4** **v = 1** **P = 8**	**d = 5** v = 0 **P = 7**	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
	**d = 3** **v = 1** **P = 9**	**d = 4** v = 0 **P = 8**	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
d = 1 **v = 1** **P = 5**	**d = 2** **v = 1** **P = 4**		**d = 4** v = 0 **P = 20**	d = Inf v = 0 P = Inf
d = 0 **v = 1** **P = 0**	**d = 1** **v = 1** **P = 5**	**d = 2** **v = 1** **P = 10**	**d = 3** **v = 1** **P = 15**	**d = 4** v = 0 **P = 20**

Dijkstra's Algorithm:

Current Node = End Node

=> Re-trace Parent

d = Inf v = 0 P = Inf	**d = 5** v = 0 **P = 7**	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
d = 5 v = 0 **P = 7**	**d = 4** **v = 1** **P = 8**	**d = 5** v = 0 **P = 7**	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
[redacted]	**d = 3** **v = 1** **P = 9**	**d = 4** v = 0 **P = 8**	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
d = 1 **v = 1** **P = 5**	**d = 2** **v = 1** **P = 4**	[redacted]	**d = 4** v = 0 **P = 20**	d = Inf v = 0 P = Inf
d = 0 **v = 1** **P = 0**	**d = 1** **v = 1** **P = 5**	**d = 2** **v = 1** **P = 10**	**d = 3** **v = 1** **P = 15**	**d = 4** v = 0 **P = 20**

Dijkstra's Algorithm:

d = Inf v = 0 P = Inf	**d = 5** v = 0 **P = 7**	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
d = 5 v = 0 **P = 7**	**d = 4** **v = 1** **P = 8**	**d = 5** v = 0 **P = 7**	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
	**d = 3** **v = 1** ← **P = 9**	**d = 4** v = 0 **P = 8**	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
d = 1 **v = 1** **P = 5**	**d = 2** **v = 1** **P = 4**		**d = 4** v = 0 **P = 20**	d = Inf v = 0 P = Inf
d = 0 **v = 1** **P = 0**	**d = 1** **v = 1** **P = 5**	**d = 2** **v = 1** **P = 10**	**d = 3** **v = 1** **P = 15**	**d = 4** v = 0 **P = 20**

Dijkstra's Algorithm:

Final Path:
5 -> 4 -> 9 -> 8 -> 13

d = Inf v = 0 P = Inf	**d = 5** v = 0 **P = 7**	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
d = 5 v = 0 **P = 7**	**d = 4** **v = 1** **P = 8**	**d = 5** v = 0 **P = 7**	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
[redacted]	**d = 3** **v = 1** **P = 9**	**d = 4** v = 0 **P = 8**	d = Inf v = 0 P = Inf	d = Inf v = 0 P = Inf
d = 1 **v = 1** **P = 5**	**d = 2** **v = 1** **P = 4**	[redacted]	**d = 4** v = 0 **P = 20**	d = Inf v = 0 P = Inf
d = 0 **v = 1** **P = 0**	**d = 1** **v = 1** **P = 5**	**d = 2** **v = 1** **P = 10**	**d = 3** **v = 1** **P = 15**	**d = 4** v = 0 **P = 20**