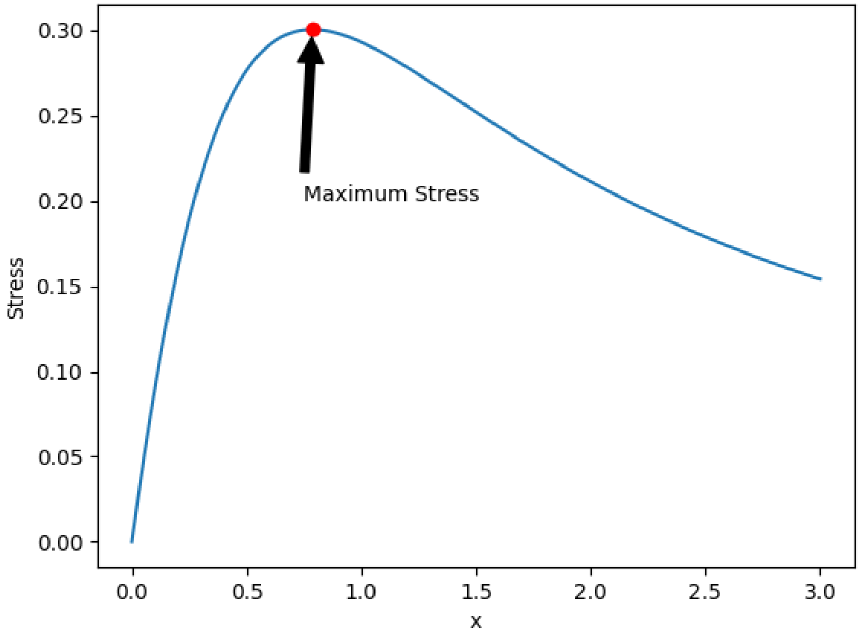
a) Graph of the Function Plotted:



c)

SI.NO	Search Method	Optimum Point	Final Interval of Uncertainty		Number of experiments	Maximum Stress	Final Interval Length
1	Unrestricted Search	0.8	NA	NA	8	0.300244	NA
2	Accelerated Step Size	0.8	NA	NA	5	0.300244	NA
3	Exhaustive	0.789474	0.6316	0.9474	19	0.300281	0.3158
4	Dichotomous	0.843969	0.74975	0.938187	8	0.299635	0.188437
5	Interval Halving	0.75000	0.6563	0.8438	9	0.300000	0.1875
6	Fibonacci	0.792135	0.775281	0.808989	10	0.300276	0.033708
7	Golden Section	0.772130	0.7082	0.7721	10	0.300242	0.0639

NOTE: Number of iterations are *pre-specified* for Fibonacci and Golden Section Methods.

Actual Maximum Value: ***0.3002830*** for the given function in range (0,3).

Actual Optimum point: ***0.786786***

Observations:

- Exhaustive search method uses the highest number of experiments and the largest maximum stress value at optimum point.
- Exhaustive search method has the largest final interval length, indicating this is not a very efficient search method

- Fibonacci method and Golden section method have the least final interval length, for a given set of experiments (n=10).
- Interval Halving method has the least number of experiments and least final interval length for 5% accuracy.
- Fibonacci method gave a good final interval size, but this experiment needs you to compute Fibonacci numbers.

Comparing methods for same number of iterations: 19

SI.NO	Search Method	Optimum Point	Final Interval of Uncertainty		Number of experiments	Maximum Stress	Final Interval Length	Accuracy (L_n/ L_0)
1	Exhaustive	0.789474	0.6316	0.9474	19	0.300281	0.3158	0.10526
2	Fibonacci	0.758427	0.785809	0.786696	19	0.300283	0.000887	0.000296
3	Golden Section	0.772130	0.7814	0.7872	19	0.300283	0.0058	0.001933

Observations:

- It can be seen that the for the same number of experiments, Fibonacci and Golden section methods have the least final interval length.
- As the number of experiments increase, both Fibonacci and Golden section methods produce the optimum point very close to the actual point of maximum stress.
- Fibonacci method is highly accurate for large number of experiments.

Comparing methods for same Accuracy: 5 %

SI.NO	Search Method	Optimum Point	Final Interval of Uncertainty		Number of experiments	Maximum Stress	Final Interval Length
1	Exhaustive	0.789474	0.6316	0.9474	19	0.300281	0.3158
2	Fibonacci	0.785714	0.714286	0.857143	8	0.300283	0.142857
3	Dichotomous	0.843969	0.74975	0.938187	8	0.299635	0.188437
4	Interval halving	0.75000	0.6563	0.8438	9	0.300000	0.1875

Observations:

- It can be seen that for same accuracy, Fibonacci method has taken least number of experiments, along with Dichotomous method.
- The length of final interval is the smallest for Fibonacci method.
- Fibonacci method gives the maximum stress for same accuracy among all other methods.
- Even though Exhaustive search method has larger number of experiments, the optimum point achieved is closer to the actual optima.

Assignment - 2

(1) Shear stress induced along z-axis :-

$$\frac{\tau_{xy}}{p_{max}} = -\frac{1}{2}\left[-\frac{1}{\sqrt{1+(\frac{z}{b})^2}} + \left\{2 - \frac{1}{1+(\frac{z}{b})^2}\right\} \times \sqrt{1+(\frac{z}{b})^2} - 2\frac{z}{b}\right] \quad -①$$

$$b = \left[\frac{2F}{\pi l} \times \frac{\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}}{\frac{1}{d_1} + \frac{1}{d_2}}\right]^{1/2}$$

$2b$ = width

p_{max} = max pressure

$$p_{max} = \frac{2F}{\pi b l}$$

$\nu_1 = \nu_2 = 0.3$

d_1 = const $\quad E_1$ = const

d_2 = const $\quad E_2$ = const

For the above given values, the equation ① is dependent solely on $(\frac{z}{b})$ ratio.

Assume $\frac{z}{b} = \lambda$

∴ Equation ① reduces to :-

$$\frac{\tau_{xy}}{p_{max}} = -\frac{1}{2}\left[-\frac{1}{\sqrt{1+\lambda^2}} + \left\{2 - \frac{1}{1+\lambda^2}\right\} \times \sqrt{1+\lambda^2} - 2\lambda\right\}$$

$$\boxed{f(\lambda) = \frac{0.5}{\sqrt{1+\lambda^2}} - \sqrt{1+\lambda^2}\left\{1 - \frac{0.5}{1+\lambda^2}\right\} + \lambda} \quad -②$$

Hence in order to find the location of max shear stress

Equation ② must be maximum,

(ii) Unrestricted Search:

* Note:- Converting the maximization problem into minimization problem, by taking the objective function as $\{-f(\lambda)\}$

$$\boxed{f(\lambda) = \frac{-0.5}{\sqrt{1+\lambda^2}} + \left\{1 - \frac{0.5}{1+\lambda^2}\right\}\sqrt{1+\lambda^2} + \lambda}$$

Initial guess :- $x_1 = 0$ $\qquad x_2 = x_1 + s = 0.1$

Step size :- $s = 0.1$

$$f(x_1) = \frac{-0.5}{\sqrt{1+0}} + \left\{1 - \frac{0.5}{1+0}\right\}\sqrt{1+0} - 0$$

$f_1 = f(x_1) = 0$

$$f(x_2) = \frac{-0.5}{\sqrt{1+0.1^2}} + \left\{1 - \frac{0.5}{1+0.1^2}\right\}\sqrt{1+0.1^2} - 0.1$$

$f_2 = f(x_2) = -0.0900496281$

Here $f_1 > f_2$, hence the minimum cannot lie $\lambda < \lambda_1$

$x_3 = x_1 + 2\lambda = 0 + 0.2 = 0.2$

$f_3 = f(x_3) = -0.1607$

* $f_2 > f_3$, new interval: $(x_2, 3)$

$x_4 = x_1 + 3\lambda = 0 + 0.3 = 0.3$

$f_4 = f(x_4) = -0.21379$

* $f_4 < f_3$

$x_5 = x_1 + 4\lambda = 0.4$

$f_5 = f(x_5) = -0.25144$

* $f_4 > f_5$

$x_6 = 0.5$ $\qquad f_6 = f(x_6) = -0.27639$

* $f_5 > f_6$

$x_7 = 0.6$

$f_7 = f(x_7) = -0.29130$

* $f_6 > f_7$

$x_8 = 0.7$

$f_8 = f(x_8) = -0.2985$

* $f_7 > f_8$

$x_9 = 0.8$

$f_9 = f(x_9) = -0.30024$

* $f_8 > f_9$

$x_{10} = 0.9$

$f_{10} = f(x_{10}) = -0.2979$

Now, we can see that $f_{10} > f_9$, the value of function is rising, hence, we have reached a minimum point in the case of minization problem,

Since this is a maximization problem, assuming that the function is unimodal, we have reached a maximum point of stress.

$\therefore$ The point is $\boxed{\lambda = 0.8}$

$\boxed{f(\lambda) = 0.30024}$

(ii) Unrestricted search with accelerated step size

$$f(\lambda) = \left(\frac{0.5}{\sqrt{1+\lambda^2}} - \sqrt{1+\lambda^2}\left\{1 - \frac{0.5}{1+\lambda^2}\right\} + \lambda\right) \times (-1)$$

Initial guess $x_1 = 0.0$ * Doubling the step size every iteration

Initial step $s = 0.1$

$x_1 = 0$ $f(x_1) = f_1 = 0$

$x_2 = 0 + 0.1 = 0.1$ $f(x_2) = f_2 = \cancel{-0.090049}$ $\underline{-0.090049}$

$x_3 = 0.2$ $f(x_3) = f_3 = \cancel{-0.21379}$ $\underline{-0.160776}$

$x_4 = 0.4$ $f(x_4) = f_4 = \cancel{-0.298576}$ $\underline{-0.251443}$

$x_5 = 0.8$ $f(x_5) = f_5 = \cancel{-0.251924}$ $\underline{-0.30024}$

$x_6 = 1.6$ $f(x_6) = f_6 = \underline{-0.2432027}$

We can see that $f_6 > f_5$, hence we have searched a minimum in the case of minization problem,

Hence it is at this point $\lambda = 0.8$, where we can find the maximum stress

$\therefore$ The approximate point of maximum stress $=$ $\boxed{\lambda = 0.8}$

$\boxed{f(\lambda) = 0.30024}$

(iii) Exhaustive search in interval $(0, 3)$ Accuracy $= 5\%$

$$f(\lambda) = (-1) \times \left\{\frac{0.5}{\sqrt{1+\lambda^2}} - \sqrt{1+\lambda^2}\left\{1 - \frac{0.5}{1+\lambda^2}\right\} + \lambda\right\}$$

Finding the number of iterations 'n'

$$\frac{L_n}{L_0} = \frac{2}{n+1} \qquad \frac{1}{2} \times \frac{L}{L_n} \le \frac{5}{100}$$

$$\frac{1}{n+1} \le \frac{5}{100}$$

$$n+1 \geq 20$$

$\therefore$ Minimum number $= 19$.

$\boxed{n \geq 19}$

Step size = $\frac{L_0}{n} = \frac{3-0}{19} = 0.15789$

$i=1$ $x_1 = 0.15789$ $f_1 = f(x_1) = -0.13326$

$i=2$ $x_2 = 0.315789$ $f_2 = f(x_2) = -0.22069$

$i=3$ $x_3 = 0.473684$ $f_3 = f(x_3) = -0.270964$

$i=4$ $x_4 = 0.6315789$ $f_4 = f(x_4) = -0.294320$

$i=5$ $x_5 = 0.7894736$ $f_5 = f(x_5) = -0.3002808$

$i=6$ $x_6 = 0.9473684$ $f_6 = f(x_6) = -0.295820$

Here @ $i=6$, we can observe that there is an increase in value of function, hence we stop here.

The interval :- $(0.6315789, 0.9473684)$

Optimum point = $\frac{0.6315789 + 0.9473684}{2}$

$= 0.789473$

$\therefore$ The point is $\boxed{\lambda = 0.789473}$

Considering this to be maximization problem $\boxed{f(\lambda) = 0.3002808}$

(iv) Dichotomous Search $(0, 3]$ 5% Accuracy

$$\frac{L_n}{L_0} = \frac{1}{2^{n/2}} + \frac{\delta}{L_0}\left\{1 - \frac{1}{2^{n/2}}\right\}$$

Assume $\delta = 0.001$

$L_0 = 3$

$$\frac{1}{2}\frac{L_n}{L_0} \leq \frac{5}{100} \longrightarrow \text{From Accuracy}$$

$$\frac{1}{2^{n/2}} + \frac{0.001}{3}\left\{1 - \frac{1}{2^{n/2}}\right\} \leq 0.1 \qquad \text{Let } 2^{n/2} = t$$

$$\frac{1}{t} + \frac{0.001}{3} - \frac{0.001}{3t} \leq 0.1$$

$$\left\{1 - \frac{0.001}{3}\right\}\frac{1}{t} \leq 0.1 - \frac{0.001}{3}$$

$$\frac{0.999667}{t} \leq 0.099667$$

$$t \geq 10.03$$

$$2^{n/2} \geq 10.03$$

Taking logarithm

$$n/2 \log 2 \geq \log 10.03$$

$$\frac{n}{2} \geq 3.326$$

$$n \geq 6.65 \qquad n \text{ has to be even}$$

so:- $\boxed{n = 8}$

$$x_2 = \frac{L_0}{2} + \frac{\delta}{2} = \frac{3}{2} + \frac{0.001}{2} = 1.505$$

$$x_1 = \frac{L_0}{2} - \frac{\delta}{2} = \frac{3}{2} - \frac{0.001}{2} = 1.495$$

$$f_2 = f(x_2) = (-1) \times \left\{\frac{0.5}{\sqrt{1+\lambda^2}} - \sqrt{1+\lambda^2}\left\{1 - \frac{0.5}{1+\lambda^2}\right\} + \lambda\right\}$$

$$= -0.251484$$

$f(x_p) = f_p = -0.252365$

$f_2 = -0.251484$

New interval :- $(0, 1.495)$

$x_3 = 0 + \frac{1.495-0}{2} - 0.0005 = 0.747$ $f(x_3) = -0.299949$

$x_4 = 0 + \frac{1.495-0}{2} + 0.0005 = 0.748$ $f(x_4) = -0.299967$

New interval :- $(0.747, 1.495)$

$x_5 = 0.747 + \frac{1.495-0.747}{2} - 0.0005 = 1.1205$ $f(x_5) = -0.23465$

$x_6 = 0.747 + \frac{1.495-0.747}{2} + 0.0005 = 1.1215$ $f(x_6) = -0.22914$

New interval :- $(0.747, 1.1205)$

$x_7 = 0.747 + \frac{1.1205-0.747}{2} - 0.0005 = 0.93325$ $f(x_7) = -0.2965068$

$x_8 = 0.747 + \frac{1.1205-0.747}{2} + 0.0005 = 0.93425$ $f(x_8) = -0.296459$

final interval :- $(0.747, 0.93325)$

$x_{opt} \simeq \frac{0.747 + 0.93325}{2} \simeq 0.840125$

$f(x_{opt}) \simeq -0.2997$

$\therefore$ The point of maximum stress is

$\boxed{\lambda^* = 0.840125}$ → from Dichotomous search

$\boxed{f(\lambda^*) = 0.2997}$

(vi) Fibonacci method

maximization problem can be converted into minimization problem

$$f(d) = (-1)\left\{ \frac{0.5}{\sqrt{1+d^2}} - \sqrt{1+d^2}\left\{1 - \frac{0.5}{1+d^2}\right\} + d \right\}$$

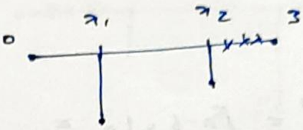
No of iterations: $n = 10$

Range = $(0, 3)$

$a = 0$, $b = 3$

$\boxed{L_0 = 3}$

we know that $\boxed{L_j^* = \frac{F_{n-j}}{F_n} L_0}$



* $L_2^* = \frac{F_{n-2}}{F_n} L_0 = \frac{F_8}{F_{10}} \times 3$

$= \frac{34}{89} \times 3 = 1.14606$

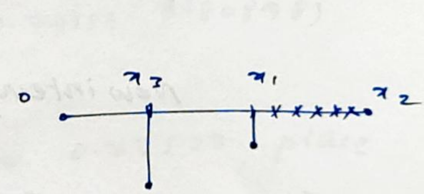
$x_1 = 0 + L_2^* = 0 + 1.14606 = 1.14606$

$x_2 = 3 - L_2^* = 3 - 1.14606 = 1.85393$

$f_1 = -0.282515$

$f_2 = -0.222234$

New interval $(0, 1.85393)$



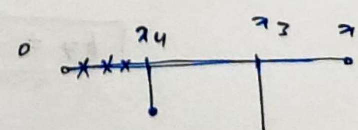
* $L_3^* = \frac{F_{n-3}}{F_n} L_0 = \frac{F_7}{F_{10}} \times 3$

$= \frac{21}{89} \times 3 = 0.70786$

$x_3 = 0 + L_3^* = 0.70786$

$f_3 = -0.298886$

New interval: $(0, 1.14606)$



* $L_4^* = \frac{F_{n-4}}{F_n} L_0 = \frac{F_6}{F_{10}} \times 3$

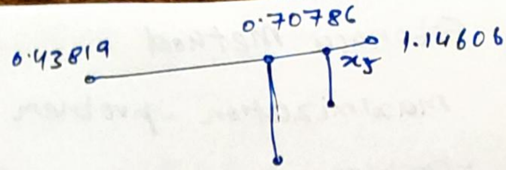
$= \frac{13}{89} \times 3 = 0.43819$

$x_4 = 0 + L_4^* = 0.43819$

$f_4 = -0.262322$

New interval: $(0.43819, 1.14606)$

$* \; L_5^* = \dfrac{F_{n-5}}{F_n} \times L_0$



$= \dfrac{F_5}{F_{10}} \times 3 = \dfrac{8}{89} \times 3 = 0.26966$

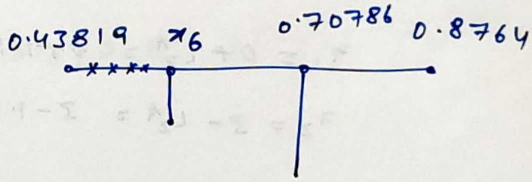
$x_5^* = 1.14606 - L_5^*$

$= 1.14606 - 0.26966$

$\boxed{x_5 = 0.8764}$ $\boxed{b_5 = -0.298764}$

New interval :- $(0.43819, 0.8764)$

$* \; L_6^* = \dfrac{F_{n-6}}{F_n} \times L_0 = \dfrac{F_4}{F_{10}} \times 3 = \dfrac{5}{89} \times 3 = 0.16853$



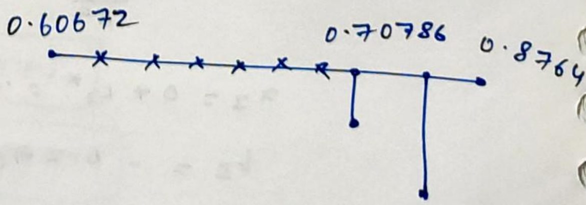
$x_6 = 0.43819 + L_6^*$

$\boxed{x_6 = 0.60672}$

$b_6 = -0.2920058$

New interval :- $(0.60672, 0.8764)$

$* \; L_7^* = \dfrac{F_{n-7}}{F_n} \times L_0 = \dfrac{F_3}{F_{10}} \times 3 = \dfrac{3}{89} \times 3 = 0.101123$



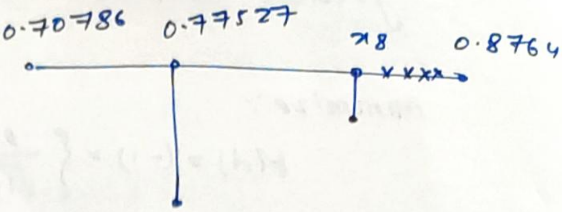
$x_7 = 0.8764 - L_7^*$

$\boxed{x_7 = 0.77527}$

$\boxed{b_7 = -0.3002582}$

New interval :- $(0.70786, 0.8764)$

$* \; L_8^* = \frac{F_{n-8}}{F_n} \times L_0 = \frac{F_2}{F_{10}} \times 3 = 0.067415$



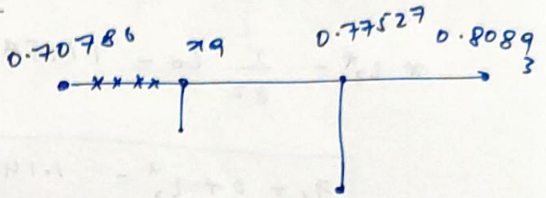
$x_8 = 0.8764 - L_8^*$

$\boxed{x_8 = 0.80893}$

$\boxed{t_8 = -0.300177}$

New interval :- $(0.70786, 0.80893)$

$* \; L_9^* = \frac{F_{n-9}}{F_n} \times L_0 = \frac{F_1}{F_{10}} \times 3 = 0.033707$



$x_9 = 0.70786 + L_9^*$

$\boxed{x_9 = 0.74156}$

$\boxed{t_9 = -0.299848}$

New interval :- $(0.74156, 0.80898)$

This is the final interval :- $(0.74156, 0.80898)$

There is an experiment ~~near~~ at to 0.77527, place an experiment near to obtain the optimum value that maximizes the stress.

The optimal point lies in region

$\lambda^* = 0.77527$

$\boxed{\lambda = (0.74156, 0.80898)}$

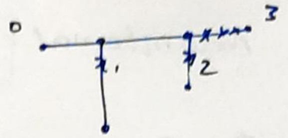
$\boxed{f(0.77527) = 0.3002582}$

(viii) Golden section method with $n = 10$

Minimize :-

$$f(d) = (-1) * \left\{ \frac{0.5}{\sqrt{1+d^2}} - \sqrt{1+d^2} \left\{ 0.1 - \frac{0.5}{1+d^2} \right\} + d \right\}$$

Range :- $(0, 3)$ $n = 10$ $\gamma = 1.618$



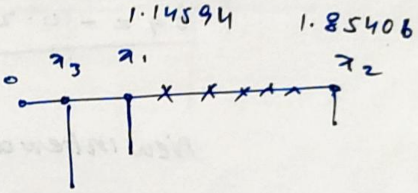
$L_0 = (0, 3) = 3$

* $L_2^* = \frac{1}{\gamma^2} L_0 = 1.14594$

$\boxed{x_1 = 0 + L_2^* = 1.14594}$ $f(x_1) = -0.282525$

$x_2 = 3 - L_2^* = 1.85406$ $f(x_2) = -0.222224$

New intervals (0, 1.85406)

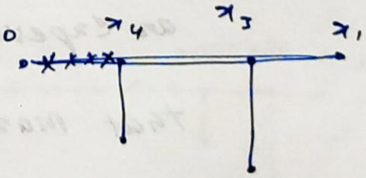


* $L_3^* = \frac{1}{\gamma^3} L_0 = 0.70824$

$x_3 = 0 + L_3^* = 0 + 0.70824$ $f(x_3) = -0.2989009$

$\boxed{x_3 = 0.70824}$

New interval : (0, 1.14594)



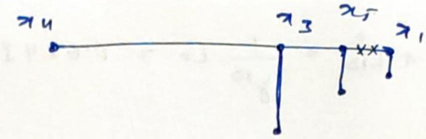
* $L_4^* = \frac{1}{\gamma^4} L_0 = 0.43773$

$\boxed{x_4 = 0 + L_4^* = 0.43773}$

$f(x_4) = -0.262097$

New interval :- (0.43773, 1.14594)

$* \; L_5^* = \frac{1}{\gamma^5} L_0 = 0.27053$



$x_5 = x_1 - L_5^*$

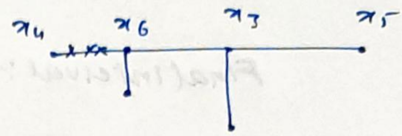
$= 1.14594 - 0.27053$

$\boxed{x_5 = 0.87541}$

$f(x_5) = -0.298795$

New interval:- $(0.43733, 0.87541)$

$* \; L_6^* = \frac{1}{\gamma^6} L_0 = 0.16720$



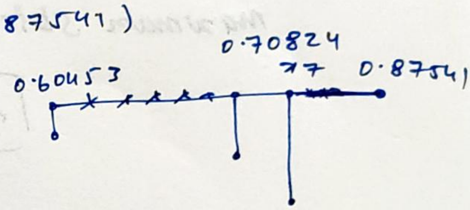
$x_6 = x_4 + L_6^*$

$= 0.43733 + 0.16720$

$\boxed{x_6 = 0.60453}$

$f(x_6) = -0.291780$

New interval: $(0.60453, 0.87541)$



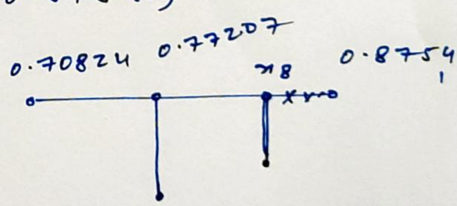
$* \; L_7^* = \frac{1}{\gamma^7} L_0 = 0.103340$

$x_7 = 0.87541 - L_7^*$

$\boxed{x_7 = 0.77207}$

$f(x_7) = -0.3002412$

New interval:- $(0.70824, 0.87541)$



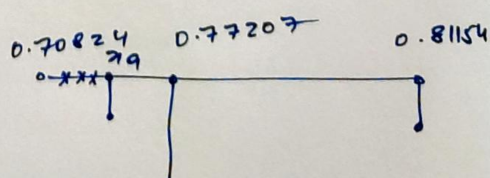
$* \; L_8^* = \frac{1}{\gamma^8} L_0 = 0.063869$

$x_8 = 0.875411 - 0.063869$

$\boxed{x_8 = 0.811541}$

$f(x_8) = -0.3001533$

New interval: $(0.70824, 0.811541)$



$* \; L_9^* = \frac{1}{\gamma^9} L_0 = 0.03947$

$x_9 = 0.70824 + 0.03947$

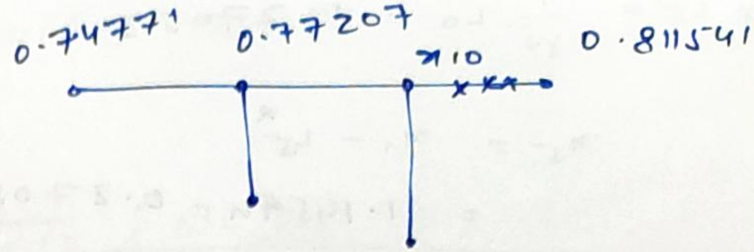
$\boxed{x_9 = 0.74771}$

$f(x_7) = -0.299962$

New interval:

$(0.74771, 0.811541)$

$* \; L_{10}^{*} = \frac{1}{\gamma^{10}} L_0 = 0.024396$



$x_{10} = 0.811541 - 0.024396$

$$\boxed{x_{10} = 0.787145}$$

$f(x_{10}) = -0.3002829$

Final interval :- $(0.74771, 0.787145)$

$x_{opt} \simeq 0.76742 \qquad f(x_{opt}) \simeq 0.300208$

$\therefore$ The final interval where we can find the point of maximum stress is $(0.74771, 0.787145)$

$$\boxed{x^{*}_{opt} \simeq 0.76742} \longrightarrow \text{Optimum point for max stress.}$$

$$\boxed{f(x^{*}_{opt}) = 0.3002086} \longrightarrow \text{maximum stress}$$

(v) Interval halving method

$(0,3)$ $\quad f(\lambda) = (-1) \times \left\{ \frac{0.5}{\sqrt{1+\lambda^2}} - \sqrt{1+\lambda^2} \left\{ 1 - \frac{0.5}{1+\lambda^2} \right\} + \lambda \right\}$

Accuracy = 5%

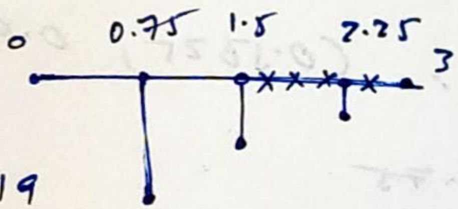
$L_0 = 3$

By virtue of Accuracy

$\frac{1}{2} \frac{L_n}{L_0} \le 0.05$ $\Rightarrow 2^{\frac{n-1}{2}} \geq 10$ $\Rightarrow n-1 \geq 6.6438$

$\Rightarrow \frac{1}{2^{\frac{n-1}{2}}} \leq \frac{1}{10}$ $\Rightarrow \frac{n-1}{2} \geq \frac{\log 10}{\log 2}$ $\quad n \geq 7.6438$

$\boxed{n = 9}$

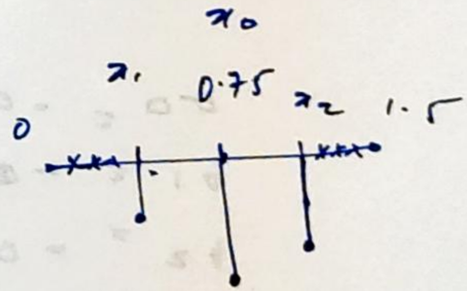


* $(0,3) = L_0$

$x_0 = 3/2 = 1.5$ $\quad f(x_0) = -0.2519$

$x_1 = 3/4 = 0.75$ $\quad f(x_1) = -0.3000$ $\quad f_2 > f_0 > f_1$

$x_2 = 9/4 = 2.25$ $\quad f(x_2) = -0.1939$



* New interval :- $(0, 1.5) = L_3$

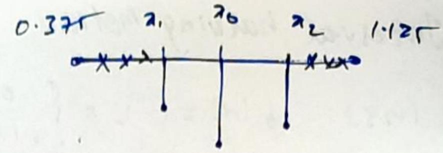
$x_0 = 0.75$ $\quad f(x_0) = -0.30000$

$x_1 = \frac{1.5}{4} = 0.375$ $\quad f(x_1) = -0.2433$

$x_2 = \frac{3}{4} \times 1.5 = 1.125$ $\quad f(x_2) = -0.28416$

$f_1 > f_0$ & $f_2 > f_0$

* New interval :- $(0.375, 1.125) = L_5$



$x_1 = 0.375 + \frac{0.75}{4} = 0.5625$

$x_2 = 0.375 + \frac{0.75}{4} \times 3 = 0.9375$

$x_0 = 0.75$

$f_0 = -0.30000$

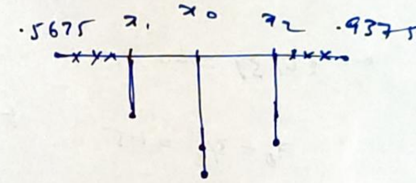
$f_1 = -0.28672$

$f_2 = -0.29630$

$f_1 > f_0$

$f_2 > f_0$

* New interval : $(0.5625, 0.9375) = L_7$



$x_0 = 0.75$

$x_1 = 0.5625 + 0.09375 = 0.65625$

$x_2 = 0.5625 + 0.28125 = 0.84375$

$f_0 = -0.30000$

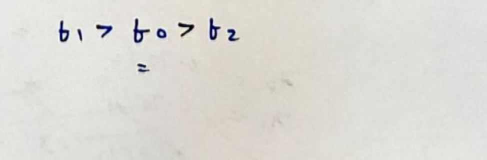
$f_1 = -0.29619$

$f_2 = -0.29964$

$f_1 > f_0$

$f_2 > f_0$

* New interval : $(0.65625, 0.84375) = L_9$



$x_0 = 0.75$

$x_1 = 0.703125$

$x_2 = 0.796875$

$f_1 > f_0 > f_2$

$f_0 = -0.300000$

$f_1 = -0.29870$

$f_2 = -0.300259$

Final interval : $(0.65625, 0.84375)$

optimum point : $\frac{1}{2}\{0.84375 + 0.65625\} = \frac{1.5}{2}$

$\boxed{\lambda^* = 0.75}$