MATH 317: SPRING 2016 **EXAM 1**

RULES

- No books or notes or calculators allowed.
- No bathroom breaks until after you have completed and turned in your test.
- Out of consideration for your classmates, do not make disturbing noises during the exam.
- **Phones and other electronic devices must be off or in silent mode.**

Cheating will not be tolerated. If there is any indication that a student may have given or received unauthorized aid on this test, the case will be handed over to the ISU Office of Judicial Affairs. When you finish the exam, please sign the following statement acknowledging that you understand this policy:

"On my honor as a student I, ________________________________, have neither given nor received unauthorized aid on this exam." (print name clearly)

Signature: ____ Date: 2016-02-19

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1. Let $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ and $\mathbf{y} = (y_1, y_2, \ldots, y_n)$ be vectors in $\mathbb{R}^n$.

(3pts) (a) Define the dot product of $\mathbf{x}$ and $\mathbf{y}$.

(3pts) (b) What does it mean to say that $\mathbf{x}$ and $\mathbf{y}$ are *parallel*?

(4pts) (c) What does it mean to say that $\mathbf{x}$ and $\mathbf{y}$ are *orthogonal*?

(4pts) (d) Suppose $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ are also vectors in $\mathbb{R}^n$. What does it mean to say that $\mathbf{x}$ is a *linear combination* of the vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$?

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(10pts) 2. Recall Prop. 2.1 of our text says that the dot product satisfies the following properties: for all $\mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathbb{R}^n$ and $c \in \mathbb{R}$,

1. $\mathbf{x} \cdot \mathbf{y} = \mathbf{y} \cdot \mathbf{x}$;
2. $\mathbf{x} \cdot \mathbf{x} = \|\mathbf{x}\|^2 \geq 0$, with equality if and only if $\mathbf{x} = 0$;
3. $(c\mathbf{x}) \cdot \mathbf{y} = c(\mathbf{x} \cdot \mathbf{y})$;
4. $\mathbf{x} \cdot (\mathbf{y} + \mathbf{z}) = \mathbf{x} \cdot \mathbf{y} + \mathbf{x} \cdot \mathbf{z}$.

Prove that if $\mathbf{x}$ is orthogonal to each of the vectors $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k$, then $\mathbf{x}$ is orthogonal to every linear combination of the vectors $\mathbf{v}_1, \dots, \mathbf{v}_k$. (For full credit, identify places in your proof where properties from the list above are used; to refer to these properties, use the letters given above.)

(5pts) 3. State a theorem about existence and uniqueness of solutions to the system $A\mathbf{x} = \mathbf{b}$. You may state more than one theorem if you wish, but quality is better than quantity. Only write what you know is true and **carefully state your assumptions**. If you make a broad statement that, in fact, only applies under a narrow set of conditions, and you leave out those conditions, then you will not receive very much credit. (Use only the space provided below.)

4. For the given matrix, write down the letters corresponding to true statements in each case. Select from the following statements:

   A. For every $\mathbf{b} \in \mathbb{R}^m$, $A\mathbf{x} = \mathbf{b}$ is consistent.
   B. For every $\mathbf{b} \in \mathbb{R}^m$, $A\mathbf{x} = \mathbf{b}$ is inconsistent.
   C. For every $\mathbf{b} \in \mathbb{R}^m$, $A\mathbf{x} = \mathbf{b}$ has exactly one solution.
   D. For every $\mathbf{b} \in \mathbb{R}^m$, $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions.
   E. There exists $\mathbf{b} \in \mathbb{R}^m$ such that $A\mathbf{x} = \mathbf{b}$ is consistent.
   F. There exists $\mathbf{b} \in \mathbb{R}^m$ such that $A\mathbf{x} = \mathbf{b}$ is inconsistent.
   G. There exists $\mathbf{b} \in \mathbb{R}^m$ such that $A\mathbf{x} = \mathbf{b}$ has exactly one solution.
   H. There exists $\mathbf{b} \in \mathbb{R}^m$ such that $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions.

(4pts) (a) If the matrix A has reduced echelon form $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$,

then which of the statements (a)–(h) above is true? (Select all that apply.)

A. B. C. D. E. F. G. H.

(4pts) (b) If the matrix A has reduced echelon form $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$,

then which of the statements (a)–(h) above is true? (Select all that apply.)

A. B. C. D. E. F. G. H.

(4pts) (c) If the matrix A has reduced echelon form $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$,

then which of the statements (a)–(h) above is true? (Select all that apply.)

A. B. C. D. E. F. G. H.

(4pts) (d) If the matrix A has reduced echelon form $\begin{bmatrix} 1 & 0 & -1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 3 & 0 & -2 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$,

then which of the statements (a)–(h) above is true? (Select all that apply.)

A. B. C. D. E. F. G. H.

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5. Let $A \in \mathbb{R}^{m\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{n\times m}$, and $\mathbf{b} \in \mathbb{R}^m$. For each statement below, either prove the claim or write FALSE and give a counter-example.

(5pts) (a) **Claim:** If $AB = I_m$, then a solution to $A\mathbf{x} = \mathbf{b}$, if it exists, is unique.

(5pts) (b) **Claim:** If $CA = I_n$, then a solution to $A\mathbf{x} = \mathbf{b}$, if it exists, is unique.

(5pts) (c) **Claim:** If $A\mathbf{x} = \mathbf{0}$ has only the trivial solution $\mathbf{x} = \mathbf{0}$, then for every $\mathbf{b} \in \mathbb{R}^m$ there is exactly one solution to $A\mathbf{x} = \mathbf{b}$.

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6. Let $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

(8pts) (a) Find A^{-1}.

(5pts) (b) Use your answer to (a) to solve $A\mathbf{x} = \mathbf{b}$.

(2pts) (c) Use your answer to (b) to express $\mathbf{b}$ as a linear combination of the columns of A. (Fill in the blanks with your answers.)

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \underline{\qquad} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + \underline{\qquad} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \underline{\qquad} \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

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