Ollscoil Mhá Nuad
Maynooth University

SEMESTER 1
2023-2024

CS370
Computation and Complexity

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Time allowed: 2 hours

Answer at least ***three*** questions

Your mark will be based on your best ***three*** answers

All questions carry equal marks

Instructions

	Yes	No	N/A
Formulae and Tables book allowed *(i.e. available on request)*	X		
Formulae and Tables book required *(i.e. distributed prior to exam commencing)*			
Statistics Tables and Formulae allowed *(i.e. available on request)*			X
Statistics Tables and Formulae required *(i.e. distributed prior to exam commencing)*			X
Dictionary allowed *(supplied by the student)*		X	
Non-programmable calculator allowed			X
Students required to write in and return the exam question paper		X	

[25 marks]

1 (a) Draw a diagram of a Turing machine and label the key elements it has. [10 marks]

The mathematical description of the Turing machine contains the following terms:

Alphabet
States
Initial state
Accepting state

Explain what each of these terms mean

Give one difference between a Turing machine and a modern computer device?

(b) For any three problems of your choice, describe those problems as sets. [6 marks]

(c) Give an explanation of the difference between the **Decidability** and **Undecidability** of any Decision problem. [3 marks]

(d) Let S_5 be the set of binary strings of length at least 5, i.e. [6 marks]

$$S_5 = \{x \in \{0,1\}^* : |x| \geq 5\}$$

(I) Build a recogniser for S_5 that is not a decider for S_5.
(II) Is S_5 decidable?
(III) If you had the option to have access to either a decider or a recogniser when solving membership of a set, which would you choose and why?

[25 marks]

2 (a) Briefly describe the Halting problem with the aid of a suitable graphic. (i.e. draw a block diagram that shows the inputs and outputs from a Turing machine and uses arrows to illustrate the flow of control of the program code). [9 marks]

If it is stated that the Halting problem is undecidable. Can it be proven by contradiction that this is the case?

(b) Consider the set $L_5 = \{\langle M\rangle : M \text{ is a TM such that } |L(M)| \geq 5\}$. [4 marks]

Describe this set in English. Then, for the following TMs whose inputs are binary strings, state whether they are elements of L_5 or not:

i. $M_1(x)$ = " if $|x| \leq 4$ accept, else reject."
ii. $M_2(x)$ = "if x = 00000 accept, else reject.
iii. $M_3(x)$ = "loop."

(c) In computing, what does the reduction of a problem mean? Support your answer using two different examples. [6 marks]

(d) Consider the following sets: [6 marks]
(a) $A_{TM} = \{\langle M,w\rangle : M$ is a TM and w a string so that $M(w) = acc\}$
(b) $\mathrm{Rej}_{010} = \{\langle M,w\rangle : M$ is a TM that rejects input 010.$\}$

Perform the following Turing reductions
(a) $A_{TM} \leq_T Halt_{TM}$.
(b) $A_{TM} \leq_T \mathrm{Rej}_{010}$

[25 marks]

3 (a) Define what an oracle machine is and how oracle machine can be used to solve decision problems. [5 marks]

(b) State Rice's theorem and explain why it is important. [5 marks]

(c) State Why Rice's theory cannot be applied to the following sets: [9 marks]

(a) $A_{TM} = \{\langle M,w\rangle : M$ is a TM and w a string so that $M(w) = acc\}$
(b) $L = \{\langle M,w\rangle : M$ is a TM such that M decides $Halt_{TM}\}$
(c) $L = \{\langle M,w\rangle : M$ is a TM such that M's first two tape cells are read only once$\}$

(d) An interesting question about Turing machines is whether they can reproduce themselves. A Turing machine cannot be defined in terms of itself, but can it still somehow print its own source code? With reference to the recursion theorem, explain why this is true. [6 marks]

[25 marks]

4 (a) Write definitions for the following terms: [10 marks]

(I) PTime
(II) EXPtime
(III) BPPtime
(IV) PSPACE
(V) EXPSPACE

(b) Whether P=NP or P!=NP is something that appears frequently in complexity theory. Explain what this means for the problems that we are attempting to solve using computers? [8 marks]

(c) Give the details behind two complex problems where the time required to solve them could become infeasible. Use suitable diagrams to illustrate your answer. [7 marks]