CS605
5 March 2025

DEFINITION 3.5

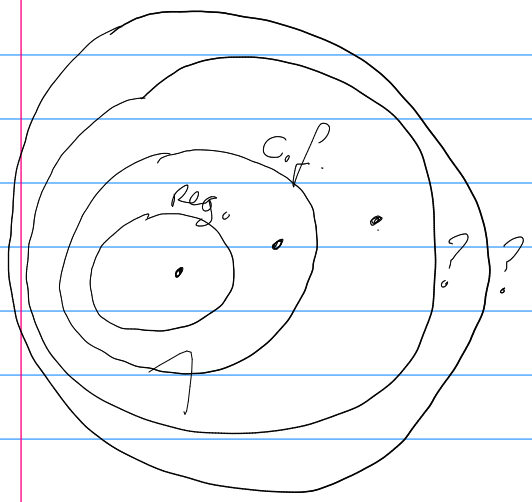
Call a language ***Turing-recognizable*** if some Turing machine recognizes it.[1]

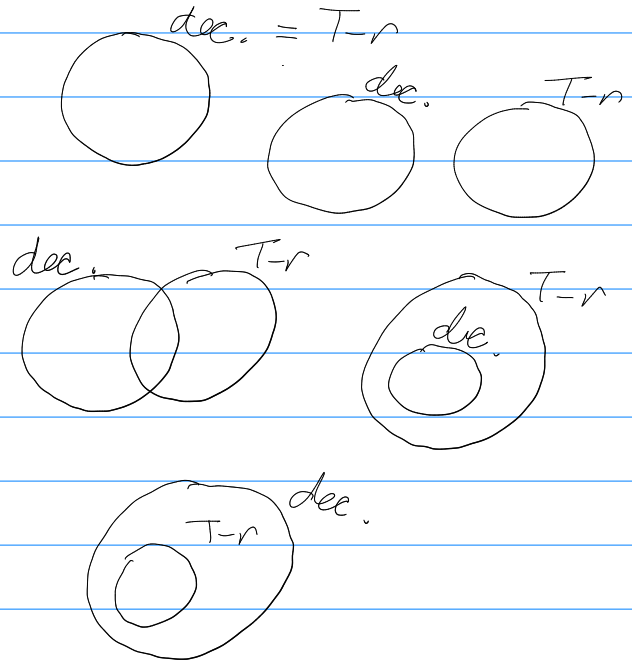
When we start a Turing machine on an input, three outcomes are possible. The machine may *accept*, *reject*, or *loop*. By ***loop*** we mean that the machine simply does not halt. Looping may entail any simple or complex behavior that never leads to a halting state.

A Turing machine M can fail to accept an input by entering the q_{reject} state and rejecting, or by looping. Sometimes distinguishing a machine that is looping from one that is merely taking a long time is difficult. For this reason we prefer Turing machines that halt on all inputs; such machines never loop. These machines are called ***deciders*** because they always make a decision to accept or reject. ~~A decider that recognizes some language also is said to ***decide*** that language.~~

DEFINITION 3.6

Call a language ***Turing-decidable*** or simply ***decidable*** if some Turing machine decides it.[2]

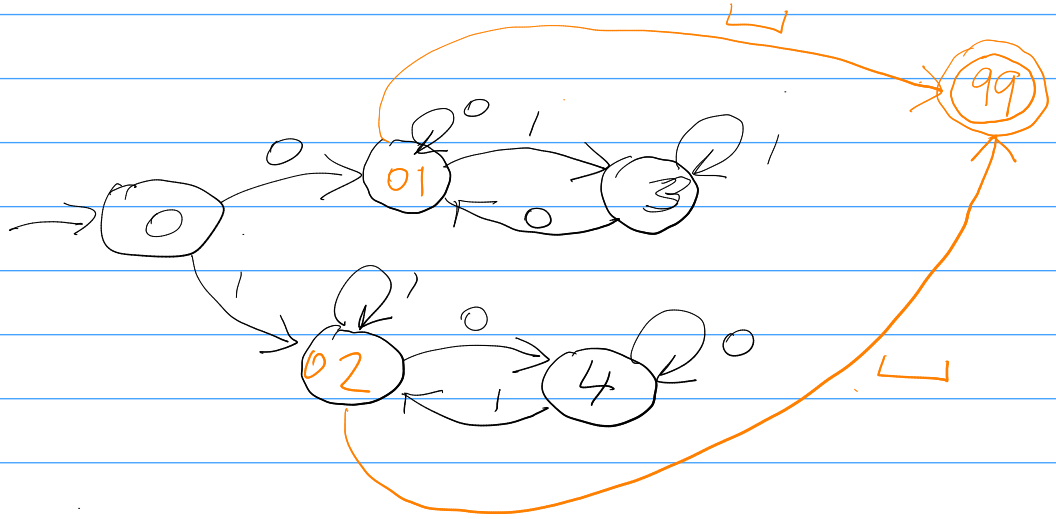




Examples of Turing machines

$L = \{ w : w \in \{0,1\}^*, w \text{ begins and ends with the same symbol} \}$

Aside: FA to do the same job:

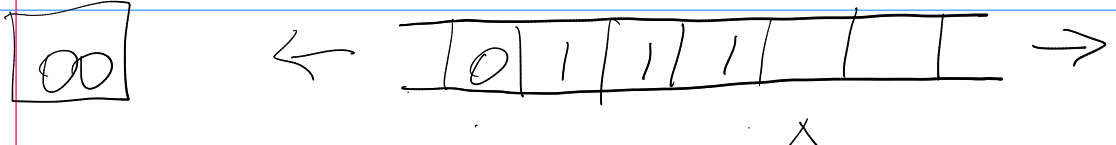


δ

00,	␣,	50,	␣,	R
00,	0,	01,	0,	R
00,	1,	02,	1,	R
01,	0,	01,	0,	R
01,	1,	03,	1,	R
02,	0,	04,	0,	R
02,	1,	02,	1,	R
03,	0,	01,	0,	R
03,	1,	03,	1,	R
04,	0,	04,	0,	R
04,	1,	02,	1,	R
01,	␣,	99,	␣,	R
02,	␣,	99,	␣,	R
03,	␣,	50,	␣,	R
04,	␣,	50,	␣,	R

Recogniser

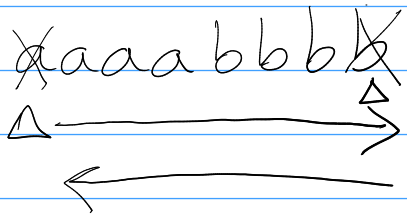
Decider



Examples of Turing machines

$L = \{ a^n b^n : n \geq 0 \} = \{ e, ab, aabb, aaabbb, \ldots \}$

TM idea :



δ :

00	a	01	␣	R	
00	b	50	b	R	# input b
00	␣	99	␣	R	
01	a	01	a	R	Loop to the last symbol
01	b	01	b	R	
01	␣	02	␣	L	
02	a	50	a	R	# aa
02	b	03	␣	L	
02	␣	50	␣	R	# a
03	a	03	a	L	Loop to first non-blank symbol
03	b	03	b	L	
03	␣	00	␣	R	

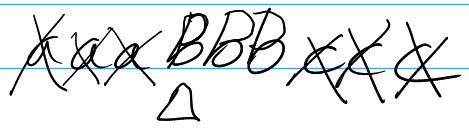
Recogniser

decider

Examples of Turing machines

$L = \{a^n b^n c^n : n \geq 0\}$

Idea:



00, a, 01, _, R
00, B, 05, B, R
00, _, 99, _, R

Recogniser

01, a, 01, a, R
01, b, 02, B, R
01, B, 01, B, R

02, b, 02, b, R
02, c, 02, c, R
02, _, 03, _, L

03, c, 04, _, L

04, a, 04, a, L
04, b, 04, b, L
04, B, 04, B, L
04, c, 04, c, L
04, _, 00, _, R

05, B, 05, B, R
05, _, 99, _, R

Decider

00, c, 50, c, R # c
00, b 50 b' R # b
01, _, 50 _ R # a
01, c' 50 c R # ac

02 a 50 a R # aba
02 B 50 B R #

Complete rest yourselves.