Ollscoil Mhá Nuad
Maynooth University

AUTUMN
2022-2023

CS370

Computation and Complexity

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Time allowed is 2 hours

Answer **any three** out of four questions

All questions carry equal marks

[25 marks]

1 (a) What is a Turing machine, Draw a diagram of a Turing machine and label the key elements it possesses. The mathematical description of the Turing machine contains the following terms: [5 marks]

Alphabet
States
Initial state
Accepting state

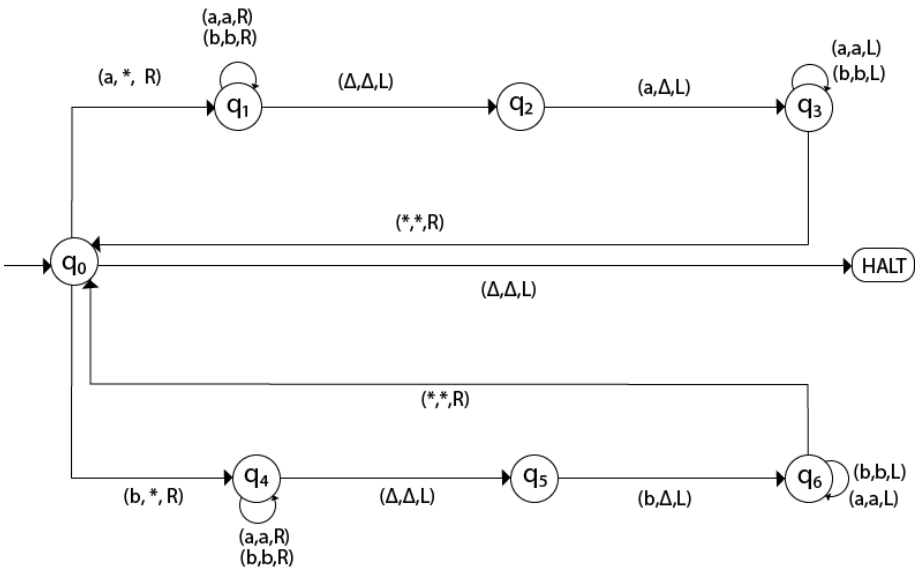
Explain what they mean

How does a Turing machine compare to a modern computer device?

(b) Explain the Halting problem with the aid of a suitable graphic. Then explain what the terms Decidability and Recognizable mean with reference to the Halting problem. [5 marks]

(c) Explain what **Time complexity** and **Space Complexity** mean. Use an appropriate example for each that illustrates the issue they describe. [5 marks]

(d) If we wish to construct a Turing Machine for checking the palindrome of the string of even length. Assume that the input string is ababbabaΔ The process is that the first symbol from the left of the head is read and then compared with the first symbol to the right of the head to check whether it is the same. This process is repeated for all the symbols. If any symbol is found not to match then the machine goes into the HALT state. A diagram of the machine is shown below. Write out the states of the machine at each point in time. [5 marks]



[25 marks]

2 (a) What is the P Versus NP problem? What happens if P=NP, how would this affect computing problems as we currently know them? [5 marks]

(b) Define the difference between "Brute Force search", "Approximation", and "Heuristics/Average-case complexity" [5 marks]

(c) Will Quantum computers be able to solve NP-problems? [5 marks]

(d) If a problem is described as NP complete what does that actually mean? [5 marks]

(e) What is the basic difference between a Register Machine and a Turing Machine? Define a Random-access Machine? [5 marks]

[25 marks]

3 (a) Define the sets Acc_{010}, EQ_{TM} and E_{TM}. [6 marks]

(b) Recall the set $Rej_{TM} = \{\langle M, w\rangle : M\ a\ TM\ such\ that\ M\ rejects\ on\ input\ w\}$ [9 marks]

i. Describe an element in the set and one not in the set. You should provide pseudocode for any *TM* you discuss.

ii. Prove Rej_{TM} is recognisable

(c) Prove Rej_{TM} is undecidable [10 marks]

i. Prove Rej_{TM} is undecidable via a proof by contradiction.

ii. Is its compliment $\overline{Rej_{TM}}$ recognisable? Why?

[25 marks]

4 (a) What is the difference between a decider and a recogniser? [6 marks]

(b) State why Rices Theorem cannot be applied to the following sets: [4 marks]

i. $Rej_{TM} = \{\langle M, w\rangle : M \text{ is a TM such that } M \text{ rejects on input } w\}$

ii. $S = \{\langle M\rangle : M \text{ ends with empty tape}\}$

(c) Consider two sets *A* and *B* which are both i.e. both A and B have their own deciders *M* and *N* respectively. Prove the following sets are also decidable: [8 marks]

i. $A \cup B$

ii. $\bar{A}$

(d) Consider the following sets: [7 marks]

$$A_{TM} = \{\langle M, w\rangle : M \text{ a TM } s.t. \ M \text{ accepts on input } w\}$$
$$Halt_{TM} = \{\langle M, w\rangle : M \text{ a TM } s.t. \ M \text{ halts on input } w\}$$

Perform the following Turing Reduction

$$Halt_{TM} \leq_T A_{TM}$$