SEMESTER 1

January 2020 Examination

CS370

Computation and Complexity

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Time allowed: 2 hours

Answer all **three** questions

All three questions carry equal marks

Instructions

	Yes	**No**
Log Books Allowed		**X**
Formula Tables Allowed		**X**
Other Allowed (*enter details*)		**X**

General (*enter details*)

1. [25 marks]

(a) Give self-contained definitions of the following concepts: A_{TM} (the acceptance problem), decidable set, recognisable set, recursively enumerable set. [8 marks]

(b) Prove: For every set A, A is recognisable iff A is recursively enumerable. [4 marks]

(c) Recall $\text{Halt}_{TM} = \{< M, w >: M$ is a TM s.t. $M(w)$ halts$\}$. Show Halt_{TM} is recognisable (give a recogniser). Show Halt_{TM} is recursively enumerable. Give an enumerator for Halt_{TM}. [9 marks]

(d) Prove: Halt_{TM} is undecidable (you can use the recursion theorem). [4 marks]

2. [25 marks]

(a) Give the statement of Rice's Theorem. Prove the theorem (you can use the recursion theorem). [6 marks]

(b) Use Rice's theorem to show that the set $S = \{\langle M \rangle : M$ is a TM s.t. $L(M) \neq \emptyset\}$ is undecidable. [6 marks]

(c) Prove: S is recognisable (construct a recogniser for it). [3 marks]

(d) Give the statement of the recursion theorem. Give a consequence of the recursion theorem. [3 marks]

(e) Give the statement of the fixed point theorem. [2 marks]

(f) Prove that $E_{\text{TM}} \leq_T \text{EQ}_{TM}$. [5 marks]

3. [25 marks]

(a) Give the definitions of the following sets: SAT, HAMPATH, SUBSET-SUM. Your definition should be self-contained (e.g. for CLIQUE, explain what a k-clique is). [6 marks]

(b) Show that HAMPATH is in NP (give a verifier). [3 marks]

(c) Consider the hierarchy of complexity classes: $P \subseteq NP, coNP \subseteq EXP \subset DEC \subset R.E. \subset LANG$, where DEC are the decidable sets, R.E. are the recognisable sets, and LANG is the class of all sets of strings. Place each of the following sets, *and its complement*, in the smallest complexity class that contains the set (e.g. write: set foo is in DEC and $\overline{\text{foo}}$ is in R.E.) [16 marks]

i. SAT

ii. E_{TM}

iii. $S = \{\langle M \rangle : M$ is a TM s.t. $M(1) =$ reject$\}$

iv. $T = \{\langle M \rangle : M$ is a TM s.t. $L(M)$ is finite$\}$

v. $U = \{\langle M \rangle : M$ is a TM s.t. $L(M)$ is in P$\}$

vi. $V = \{x \in \{0, 1\}^* : x$ starts with 001 $\}$

vii. $W = \{\langle M \rangle : M$ is a TM that accepts at most five strings $\}$

viii. $X = \{\langle M \rangle : M$ is a TM s.t. $M(0)[3000] =$ reject$\}$