CS605
4 February 2025

See website for information: https://www.cs.nuim.ie/~tomn/teaching/cs605/

To contact the lecturer, email tomn@cs.nuim.ie with "605" anywhere in the subject line.

The CS605 hours for the first week (Tue 4th to Fri 7th 09:15-17:00)
- 09:15 to 15:00 lectures
- 15:00 to 17:00 labs, when a demonstratror will be available to help you with your FA/PDA/TM programming

The CS605 hours for each Wednesday (except study week/Easter) starting 12th February 2025:

- 09:15 to 14:00 lectures
- 14:00 to 16:00 break (room used for other students)
- 16:00 to 18:00 lab with a demonstrator

$A = \{a, b\}$ $B = \{0, 1\}$ $C = \{x, y, z\}$

$A \times B \times C = \{(a,0,x), (a,0,y), (a,0,z), (a,1,x), \ldots\}$

$A = \{a, b\}$ $B = \{0, 1\}$ $C = \{\}$

$A \times B \times C = \{(a,0,\{\}), (b,0,\{\}), \ldots\}$

$\{(a), (a,a)\} \circ \{(a), (a,a)\} = \{(a,a), (a,a,a), \cancel{(a,a,a)}, (a,a,a,a)\}$

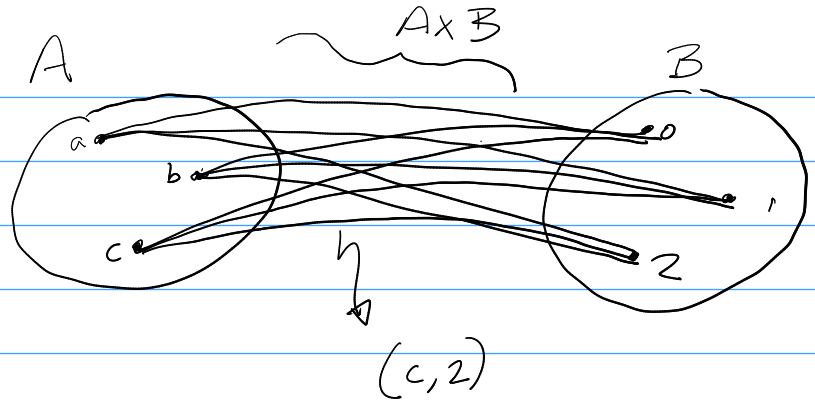
$\{(a), (a,a)\} \times \{(a), (a,a)\} = \{((a),(a,a)), ((a),(a)), ((a,a),(a)), ((a,a),(a,a))\}$

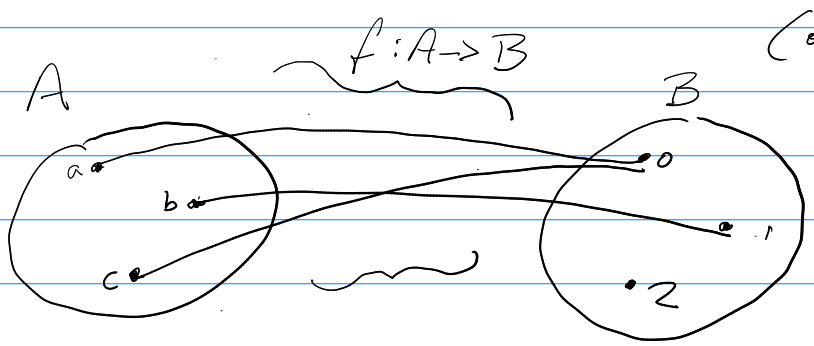
$A = \{a, b\}$ $B = \{0, 1\}$ $A \times B = \{(a,0),(a,1),(b,0),(b,1)\}$

$2^{A \times B} = \{\{\}, \{(a,0)\}, \ldots \{(a,1),(b,1)\}, \ldots A \times B\}$

$|2^{A \times B}| = 16$

Any element of $2^{A \times B}$ is a relation on sets A and B.





(one of the possible ones)

$sum4 : \mathbb{N}_4 \times \mathbb{N}_4 \to \mathbb{N}_4$

$sum4 = \{((0,0),0),((0,1),1), \ldots ((1,2),3) \ldots\}$

$A = \{a,b\}$ $B = \{0,1\}$ $A \times B = \{(a,0),(a,1),(b,0),(b,1)\}$

$C = \{(a,1),(b,0)\} \subseteq A \times B$ so C is a relation on A and B.

C can also be written as a function

$F: A \times B \Rightarrow \{T,F\}$

$F = \{((a,0),F),((a,1),T),((b,0),T),((b,1),F)\}$

```
bool f(int a, int b){
  . . .
}
```

$\{\} \neq \{\{\}\} \neq \{e\}$

$|\{\{\}\}| = 1 \quad |\{\}| = 0 \quad |\{e\}| = 1$

$|e| = 0$

$\Sigma = \{0, 1, a\} \quad \Sigma^* = \{e, 0, 1, a, 00, 01, 0a, 10, 11, 1a, a0, a1, aa, 000, 001, \ldots\ldots\}$

$\Sigma = \{\} \quad \Sigma^* = \{e\}$

$\Sigma = \{a\} \quad \Sigma^* = \{e, a, aa, aaa, aaaa, \ldots\ldots\}$

$\Sigma^* = \{a^n, n \geq 0\} \qquad a^0 = e$

$\Sigma = \{0, 1\} \quad \Sigma^* = \{e, 0, 1, 00, 01, 10, 11, 000, \ldots\}$

Any subset of Σ^* is a language over Σ, e.g.

$L = \{\}$

$L = \{e\}$

$L = \{e, 0\}$

$L = \{0, 1, 00, 1111\}$

$L = \{w : w \text{ begins with } 0\} = \{0, 00, 01, 000, 001, 010, 011, 0000, \ldots,\}$

$L = \Sigma^*$

What is 2^{Σ^*} ? $(\Sigma = \{0, 1\})$

It is the set of all languages over Σ.

$2^{\Sigma^*} = \{\{\}, \{e\} \ldots \{001, 11100, 1111111111\}, \ldots\}$

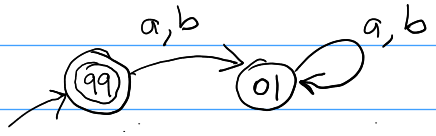
$2^{\Sigma^*} = \{\{\}, \{e\}, \{0\}, \{1\}, \{00\}, \{01\}, \ldots\ldots$
$\{e, 0\}, \{e, 1\}, \{e, 00\}, \ldots\ldots$

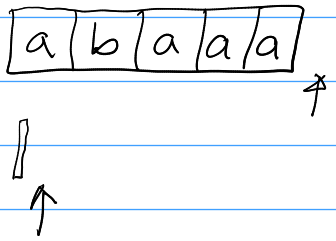
$\ldots\ldots \Sigma^*\}$

https://mumachines.cs.nuim.ie/

$\Sigma = \{a, b\}$

$L = \{e\}$ $\quad L \subseteq \Sigma^*$





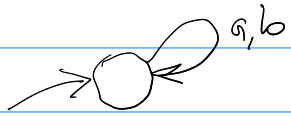
99, a, 01
99, b, 01
01, a, 01
01, b, 01

$L = \Sigma^*$

ab

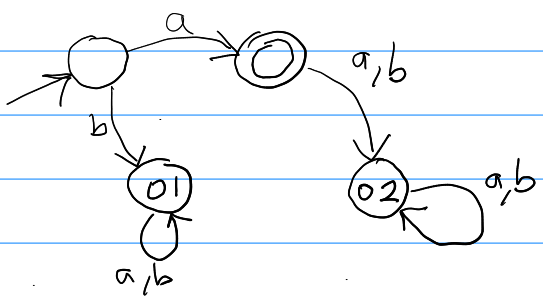
99, a, 99
99, b, 99

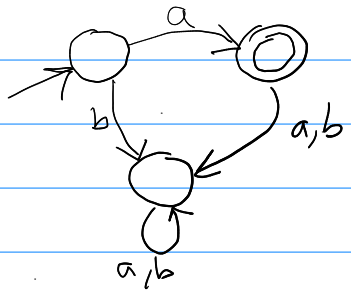
$L = \{\}$



00, a, 00
00, b, 00

$L = \{a\}$





00, a, 99
00, b, 01
01, a, 01
01, b, 01
99, a, 02
99, b, 02
02, a, 02
02, b, 02

00, a, 99
00, b, 01
01, a, 01
01, b, 01
99, a, 01
99, b, 01

Examples for you to try on Tuesday afternoon

```
definition='{w: w ∈ {a, b}*, w contains zero or more as and no bs}',
description=(
    'This language consists of all binary words that contain '
    'some as (zero or more) and do not have any bs, e.g. e and '
    'and aaa, but not b and not aabab.'
    ),


definition='{w: w ∈ {a, b}*, w contains at least one b}',
description=(
    'This language consists of all binary words that have at '
    'least one b, e.g. b and aabab, but not e and not aaa.'
    ),


definition='{w: w ∈ {0, 1}*, w begins with a 1}',
description=(
    'This language consists of all binary words that begin '
    'with a 1, e.g. 10 and 110, but not e and not 01.'
    ),


definition='{w: w ∈ {0, 1}*, w does not begin with a 1}',
description=(
    'This language consists of all binary words that do not '
    'begin with a 1, e.g. e and 01, but not 10 and not 110.'
    ),


definition='{w: w ∈ {0, 1}*, w ends with a 1}',
description=(
    'This language consists of all binary words that end with '
    'a 1, e.g. 01 and 101, but not e and not 110.'
    ),

definition='{w: w ∈ {0, 1}*, w does not end with a 1}',
    description=(
        'This language consists of all binary words that do not '
        'end with a 1, e.g. e and 110, but not 01 and not 101.'
        ),

definition='{w: w ∈ {0, 1}*, w has exactly one 1}',
description=(
    'This language consists of all binary words that have '
    'exactly one 1, e.g. 1 and 010, but not 0 and not 101.'
    ),


definition='{w: w ∈ {0, 1}*, w does not have exactly one 1}',
description=(
    'This language consists of all binary words that have zero '
    '1s or more than one 1, e.g. 0 and 101, but not 1 and not '
    '010.'
    ),

definition='{w: w ∈ {0, 1}*, w contains an even number of 1s}',
description=(
    'This language consists of all words that contain an even '
    'number of 1s and any number of 0s, e.g. e and 1010, but '
    'not 111 and not 0100.'
    ),

definition='{w: w ∈ {0, 1}*, w contains an odd number of 0s}',
description=(
    'This language consists of all words that contain an odd '
    'number of 0s and any number of 1s, e.g. 0 and 0100, but '
    'not 00 and not 1010.'
    ),
```