Ollscoil Mhá Nuad
Maynooth University

SEMESTER 1
2022-2023

CS370

Computation and Complexity

Dr. P. Healy, Dr. J. Timoney, Mr. J. Duffin, Mr. Dara MacConville

Time allowed: 2 hours

Answer **any three** out of four questions

All questions carry equal marks

Instructions

	Yes	No	N/A
Formulae and Tables book allowed *(i.e. available on request)*		X	
Formulae and Tables book required *(i.e. distributed prior to exam commencing)*		X	
Statistics Tables and Formulae allowed *(i.e. available on request)*		X	
Statistics Tables and Formulae required *(i.e. distributed prior to exam commencing)*		X	
Dictionary allowed *(supplied by the student)*	X		
Non-programmable calculator allowed	X		
Students required to write in and return the exam question paper		X	

[25 marks]

1 (a) Draw a diagram of a Turing machine and label the key elements it has. [10 marks]

The mathematical description of the Turing machine contains the following terms:

Alphabet
States
Initial state
Accepting state

Explain what each of these terms mean

Give one difference between a Turing machine and a modern computer device?

(b) What is the difference between a decider and a recogniser? [6 marks]

(c) State why Rice's Theorem cannot be applied to the following sets: [4 marks]

i. $Rej_{TM} = \{\langle M, w\rangle : M\ is\ a\ TM\ such\ that\ M\ rejects\ on\ input\ w\}$

ii. $S = \{\langle M\rangle :\ M\ ends\ with\ empty\ tape\}$

(d) Parity bits are used in data communication as a means of detecting certain transmission errors (an error occurs when a transmitted '0' is received as a '1' or vice versa). [5 marks]

Consider an odd parity Turing Machine that does the following:

Counts the number of '1's in a binary string.

If there is an odd number, append a '0' to the end of the string.

Otherwise, append a '1' to the end of the string to assure that the total number of '1's is odd.

This can be done in a single left-to-right scan of the input with 3 states. The states are:

1. An even number of '1's have been scanned so far.
2. An odd number of '1's have been scanned so far.
3. finished processing the input

If the instructions are given as

(1, 0, 0, 1, R)
(1, 1, 1, 2, R)
(1, B, 1, 3, R)
(2, 0, 0, 2, R)
(2, 1, 1, 1, R)
(2, B, 0, 3, R)

Write out in pseudocode to explain clearly what the machine is doing here.

[25 marks]

2 (a) In complexity theory, a reduction is a transformation of one problem into another problem. How does this lead to the idea of using reductions to prove undecidability? [6 marks]

Then, explain how this is associated with Rice's theorem.

(b) Consider the set RejTM={⟨ M , w⟩ : M is a TM such that M rejects on input w}. [19 marks]

i. Describe an element in the set and one not in the set. You should provide pseudocode for any TM you discuss.

ii. Prove RejTM is recognisable

iii. Prove RejTM is undecidable via a proof by contradiction

iv. Is the compliment of RejTM recognisable? Explain your answer.

[25 marks]

3 (a) The recursion theorem states that some Turing Machines (TM) can reproduce their own descriptions. It is implied that we can turn any TM into an equivalent one that has this property. We can implement self-referential programs in any sufficiently strong programming languages. Thus, any program can refer to its own description. Elaborate on these statements for the Robots building robots dilemma. [5 marks]

(b) Consider two sets A and B where both A and B have their own deciders M and N respectively. [5 marks]

Prove that the set A ∪ B is also decidable.

(c) Consider the following sets: [8 marks]
ATM = {⟨ M ,w⟩ : M is a TM such that M accepts on input w}
Halt TM = {⟨ M ,w⟩ : M is a TM such that M halts on input w}
Perform the following Turing Reduction:

Halt TM ≤T ATM.

(d) Briefly describe the Halting problem with the aid of a suitable graphic. (I.e. draw a block diagram that shows the inputs and outputs from a Turing machine and uses arrows to illustrate the flow of control of the program code) [3 marks]

(e) Explain what **Time complexity** and **Space Complexity** mean. Use an appropriate example for each that illustrates the issue they describe. [4 marks]

[25 marks]

4 (a) What is understood by the P Versus NP problem? [3 marks]

(b) Define the difference between "Brute Force search", and "Heuristics" for problem solving. [3 marks]

(c) Briefly explain any one of the hardest problems in NP and say why they are so difficult to solve. [6 marks]
What happens if P=NP, how would this affect computing problems as we currently know them?

(d) The work of Turing is still relevant to the field of AI. Give two short arguments that support or contradict this statement. [5 marks]

(e) A set of three nodes in an undirected graph is called a triangle if for every pair of nodes in the set, there is an edge in the graph connecting them. [8 marks]

Let:

$$\text{TRIANGLE} = \{\langle \text{G} \rangle : \text{G is an undirected graph which contains a triangle}\}$$

Show: $TRIANGLE \in P$.

i.e. describe a polynomial time algorithm deciding TRIANGLE.

(hint: recall time analysis)