Ollscoil Mhá Nuad
Maynooth University

AUTUMN
2023-2024

CS370
Computation and Complexity

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Time allowed: 2 hours

Answer at least ***three*** questions

Your mark will be based on your best ***three*** answers

All questions carry equal marks

Instructions

	Yes	**No**	**N/A**
Formulae and Tables book allowed *(i.e. available on request)*	X		
Formulae and Tables book required *(i.e. distributed prior to exam commencing)*		X	
Statistics Tables and Formulae allowed *(i.e. available on request)*			X
Statistics Tables and Formulae required *(i.e. distributed prior to exam commencing)*			X
Dictionary allowed *(supplied by the student)*		X	
Non-programmable calculator allowed	X		
Students required to write in and return the exam question paper		X	

[25 marks]

1 (a) Describe the operation of a Finite Automata. What are a Deterministic Finite Automata and a Nondeterministic Finite Automate? Then, explain how a Turing Machine works. What is the relationship between a Finite Automata and A Turing Machine? Use appropriate diagrams to support your answer. [10 marks]

(b) A prime number is a positive whole number whose only divisors are itself and 1. Let $P = \{n : n$ is a prime number$\}$ be the set of all primes. [8 marks]

(a) Show that P is a decidable set. That is, build a TM T such that on input n, T accepts n if n is a prime number and rejects otherwise.

(b) Using your T from above, build a TM N such that on input n, N prints the n^{th} smallest prime (eg $N(8) = 19$). Note that your TM N should not perform any division and should exclusively use your decider T to determine if a number is prime.

(c) State the Church-Turing thesis? [5 marks]

(d) What is the difference between a Turing-recognizable language and a Turing-decidable language? [2 marks]

[25 marks]

2 (a) Consider the set $L_5 = \{\langle M\rangle : M$ is a TM such that $|L(M)| \geq 5\}$.

(i) Describe this set in English. [4 marks]

(ii) For the following TMs whose inputs are binary strings, state whether they are elements of L_5 or not:

a. $M_1(x) =$ " if $|x| \leq 4$ accept, else reject." [2 marks]

b. $M_2(x) =$ "if $x = 00000$ accept, else reject. [2 marks]

c. $M_3(\underline{x}) =$ "loop." [2 marks]

(iii) Prove L_5 is recognisable. [4 marks]

(b) State Rice's theorem and explain why it is important. [5 marks]

(c) In complexity theory, a reduction is a transformation of one problem into another problem. How does this lead to the idea of using reductions to prove undecidability? [6 marks]

Then, explain how this is associated with Rice's theorem.

[25 marks]

3 (a) Describe the **Halting problem** to a layperson using an example based on a software application. [5 marks]

What is the relationship between the Halting problem and Godel's incompleteness theorem? [5 marks]

(b) Give an explanation of Kleene's Recursion theorem that would be suitable for a Layperson. [3 marks]

Then, use the Recursion Theorem to prove that the following set is undecidable. [7 marks]

$\text{Halt}_{\text{TM}} = \{\langle M, w \rangle : M \text{ a TM such that } M(w) \text{ halts}\}$

(c) Consider two sets A and B where both A and B have their own deciders M and N respectively. [5 marks]

Prove that the set A ∪ B is also decidable.

[25 marks]

4 (a) Define the terms: [12 marks]

(I) Time Complexity
(II) Space Complexity
(III) P
(IV) NP
(V) NP-hard
(VI) NP-complete

(b) Explain three examples of problems that are known for their computational difficulty, ensuring to highlight how scaling the problems leads to the growth of the time and/or the space complexity [9 marks]

(c) Within the field of computational complexity, a well-known text is the Golden ticket by Lance Fortnow. Within this book the implications of P=NP on society is discussed. Describe two of these implications in detail. [4 marks]