

Measuring the Rydberg Constant using a Diffraction Grating

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Abstract—The theoretical Rydberg constant was estimated to be $1.04 \pm 0.02 \times 10^7 \text{ m}^{-1}$ via emission-line spectroscopy of an hydrogen lamp using a spectroscope and a 80 lines/mm diffraction grating. Due to anomalous results the final measured value for the Rydberg constant was $6.950000 \times 10^6 \text{ m}^{-1}$ with a percentage difference of 58%. Red, aqua and violet emission lines with wavelengths around 656.279nm, 486.135nm and 410.173nm respectively were used to measure the Rydberg constant; $1.1 \pm 0.007 \times 10^7 \text{ m}^{-1}$, $9.8 \pm 0.2 \times 10^6 \text{ m}^{-1}$ and $6.7 \pm 0.3 \times 10^6 \text{ m}^{-1}$ respectively for each wavelength. The expected value for the Rydberg Constant would be about $1 \times 10^7 \text{ m}^{-1}$. The measurement of the Rydberg constant for the violet emission line was concluded as anomalous for being significantly lesser than the expected value.

I. INTRODUCTION

SPECTROSCOPY is a "non-invasive" technique and is one of the most powerful tools available to physicists to study cryogenically cooled crystals, superhot plasma, stars, semiconductors, etc. A spectrometer is an instrument used to study the light spectrum produced by a light source. It uses a lens or mirror to produce a collimated beam of light which is dispersed via a grating or a prism, which is then focused and viewed with an eyepiece or an electronic detector as shown in Fig(1).[1]

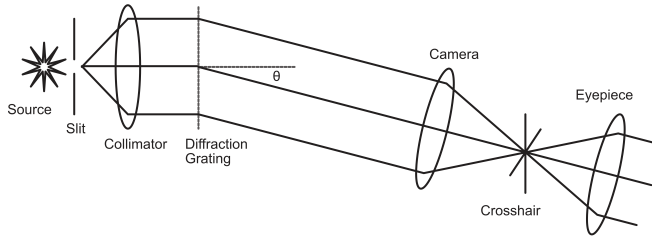


Fig. 1. A ray diagram of the components in a spectrometer. The light from the source passes through the slits and is collimated by the collimator lens. The diffraction grating deviates the incident light by an angle θ , which is then brought to focus by the camera and an eyepiece lens.

The spectrum can range all the way from the longest infrared to gamma-rays. In the case of optical light, the spectrum generally takes one of two forms:

- *Continuous Spectra* - where the spectra is a slowly varying function of the wavelength, usually emitted by hot solids e.g. tungsten lamp filaments.
- *Emission Line Spectra* - the emissions are concentrated at a few specific wavelengths, usually emitted by hot or electrically excited gasses.

In an emission line spectra, the ratio of the intensities of the spectral lines tell the temperature of the gas and the line widths can give the density of the emitting gas, or the density of the free electrons in an electrical discharge.

The Rydberg Constant, R_∞ represents the wavenumber of the lowest-energy photon capable of exciting the atom from its ground state. It has been measured to be $1.09737316 \times 10^7 \text{ m}^{-1}$ [2].

II. THEORY

The transition wavelengths in hydrogen are given by the equation:

$$\frac{1}{\lambda} = -R_\infty \left(\frac{1}{n^2} - \frac{1}{p^2} \right) \quad (1)$$

Where R_∞ is the Rydberg constant and, n and p are integers representing the energy levels of the initial and final states of the atom respectively.

The Balmer series for an hydrogen atom are given by Eq(1) with $p = 2$ and $n = 3, 4, 5 \dots$. Stating that the atom is dropping down from the n^{th} energy level to the second energy level, emitting a photon of wavelength, λ . Due to the Balmer transition wavelengths being in the visible part of the spectrum, the values of the wavelengths can be calculated using a standard colour spectrum chart.[3]

By passing the light emitted through a spectrometer, we can consider the diffraction grating to be a large number N of equally spaced narrow slits with a slit separation d . This causes the emission lines to diffract and appear as fringes when observed through the eyepiece. The principle maximum of a given wavelength, λ of light occurs when the phase difference is an integer multiple m of 2π , i.e. the path difference is divisible by the wavelength.

$$\text{phase difference} = \text{path difference} \times \frac{2\pi}{\lambda} = 2\pi m. \quad (2)$$

Given that the path difference is $d \sin(\theta)$ [4], we can rewrite the equation as:

$$d \sin(\theta) = m\lambda, \quad (3)$$

where d is the slit separation in the diffraction grating, m is the order number, λ is the wavelength and θ is the angle at which the maximum is relative to the normal of the grating.

It is important to note that the Rayleigh Criterion[5] does apply in order to resolve two distinct diffraction patterns i.e. the two diffraction patterns are resolved when the first minimum of one diffraction pattern coincides at the principle maximum of the other.

III. METHOD

A. Calibrating the Spectrometer

In order to minimise uncertainty in the measured angles and make the emission line more clear, it is necessary to calibrate and focus the spectrometer. The camera and the eyepiece were focused onto an image approximately 8m away in order to have the object to be perceived as being at infinity. Next we used a sodium lamp, which is monochromatic and positioned it at the slit and aligned with the crosshair. After turning the sodium lamp on, we focused the slit width and the focus of the collimator to produce a clear central maximum at the crosshair.

In order to ensure the orientation of the diffraction grating was normal to the incident light from the collimator, the turntable had to be accurately positioned. The telescope was first turned 90° from its initial angle 141.45° to 51.45° . A mirror was placed in the holder on the turntable and was rotated until the reflected light from the collimator was aligned with the crosshair, which would mean the mirror was angled at 45° , the angle reading for the turntable was 6.2267° . The turntable was then rotated by 45° in the opposite direction to 51.2667° , so that it was normal to the incident light. The mirror was then removed.

B. Spectroscopy

Between diffraction gratings of 80 lines/mm and 300 lines/mm, the 80 lines/mm with slit separation of 1.25×10^{-5} mm was chosen. This was primarily due to the Balmer emission lines being closer together than the 300 lines/mm which would produce more data points. The 80 lines/mm diffraction grating was placed on the mount on the turntable and the sodium lamp was replaced with the hydrogen lamp and turned on.

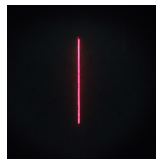


Fig. 2. The central maximum of the spectral lines aligned with the crosshair of the telescope, formed from an hydrogen lamp diffracted through a 80 lines/mm diffraction grating.

The central maximum when aligned with the crosshair, as shown in Fig(2), was at the reading $141.517 \pm 0.008^\circ$. The angles for all visible orders of the visible wavelengths - red, aqua and violet, were measured.

IV. RESULTS

Using the set of measured angles for each order, the wavelength of each spectral line was calculated using Eq(3), by plotting the sin of the angle, θ against the order number, m . The gradient of each plot, Fig(3), was then multiplied by the slit separation of the diffraction grating, d to obtain λ .

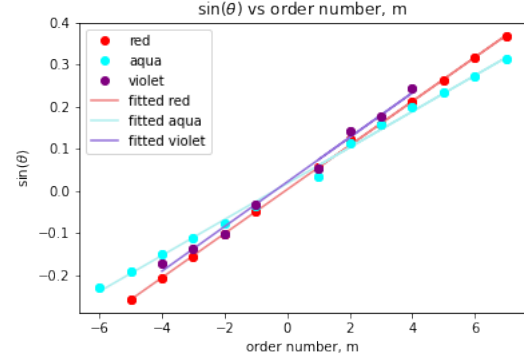


Fig. 3. A plot of $\sin(\theta)$ vs the order number, m . A scatter plot for red, aqua and violet were fitted using a linear fit. The gradients for red, aqua and violet were 0.0522525 ± 0.0000001 , 0.0426885 ± 0.0000005 and 0.052872 ± 0.000004 respectively.

The wavelengths for the visible emission lines were:

Red	: $663.103 \pm 0.002\text{nm}$
Aqua	: $541.732 \pm 0.007\text{nm}$
Violet	: $670.97 \pm 0.05\text{nm}$.

The wavelengths for red was as expected but aqua and violet were significantly greater than expected. However, it was decided not to discard them for the moment and calculate the Rydberg constant for each of the values.

The theoretical R_∞ is calculated using Eq(1) with the final state of the energy level, n for red, aqua and violet are 3, 4 and 6 respectively with the initial state being $p = 2$. The theoretical Rydberg constant estimates for each wavelength were:

Red	: $1.086 \pm 0.007 \times 10^7 \text{m}^{-1}$
Aqua	: $9.8 \pm 0.2 \times 10^6 \text{m}^{-1}$
Violet	: $6.7 \pm 0.3 \times 10^6 \text{m}^{-1}$.

The Rydberg constant value for violet was significantly lower than the expected value so it was decided to discard this value for calculating the estimate. Using the values for red and aqua, which were around the expected values, the average estimate for R_∞ was calculated to be $1.04 \pm 0.02 \times 10^7 \text{m}^{-1}$.

Using the equation:

$$x = \left(\frac{1}{n^2} - \frac{1}{p^2} \right) \quad (4)$$

as a substitute into Eq(1), the Rydberg constant could be calculated using the relationship:

$$\frac{1}{\lambda} = -R_\infty x. \quad (5)$$

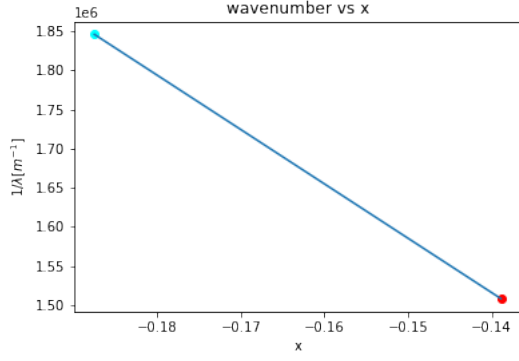


Fig. 4. Wavenumber $1/\lambda$ vs x , Eq(4), plot. This plot contains red and aqua but does not contain violet as it was discarded. The gradient of the fitted line is -6950477.446657944 which gives R_∞ of 6950000m^{-1} . The uncertainty of this value is unknown as the polyfit, $\text{deg} = 1$ is unable to provide the covariance matrix for 2 values.

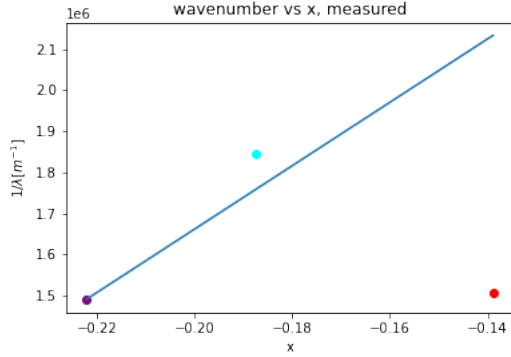


Fig. 5. Wavenumber $1/\lambda$ vs x , Eq(4), plot. This plot contains red, aqua and violet. The gradient of the fitted line is 7706086.08988688 which gives R_∞ of -7700000m^{-1} with an uncertainty of $\pm 2 \times 10^{13}\text{m}^{-1}$. The large uncertainty is due to the anomalous value of violet.

The best value between the two plots Fig(4) and Fig(5), is Fig(4) as the anomalous nature of violet does not skew the value of R_∞ . Therefore, the final measured value for R_∞ is $6.950000 \times 10^6\text{m}^{-1}$.

V. DISCUSSION

The final estimate for the Rydberg constant was as expected and agreed with the expected value with a percentage difference of 5.2%. However, the final measured value for the Rydberg constant had a percentage uncertainty of 58%. The value for R_∞ present in Fig(4) has been fitted with only two points which is not enough to produce a reliable linear fit. The contribution to this issue was the anomalous wavelength value for violet, which is closer to red, could have been caused by human error i.e. skipped orders of maximums. This is highly likely as the violet spectral lines were quite faint which meant that any other spectral lines next to it would easily bleed over it. Some precautions that could have been taken to avoid this would have been to calculate the theoretical angle between successive maximums prior to measuring the angles.

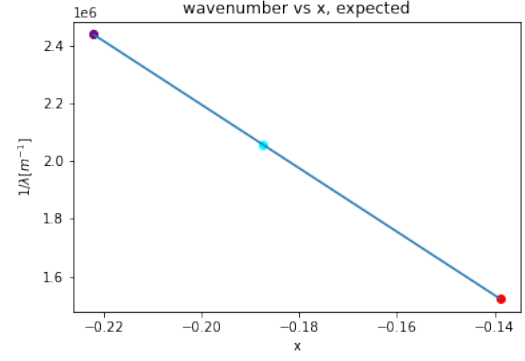


Fig. 6. Wavenumber $1/\lambda$ vs x , Eq(4), plot. This plot contains the expected red, aqua and violet points. The gradient of the fitted line is -10970995.378802722 which gives R_∞ of $10970995.378802722\text{m}^{-1}$

The expected values for the wavelength of red, aqua and violet were 656.279nm , 486.135nm and 410.1734nm [6] respectively. Fig(6) shows the expected plot.

In order to increase the accuracy of this experiment, it would be beneficial to have a spectrometer with better calibration. This would minimise any uncertainties due to the spectrometer. A brighter lamp would also increase the visibility of fainter emission lines.

VI. CONCLUSION

The theoretical Rydberg constant was estimated to be $1.04 \pm 0.02 \times 10^7\text{m}^{-1}$ with a percentage difference of 4% to the expected value $1.09737316 \times 10^7\text{m}^{-1}$. However, the measured value, due to anomalous results was $6.950000 \times 10^6\text{m}^{-1}$ with a percentage difference of 58%. This was obtained by measuring the wavelengths of the red, aqua and violet emission lines in the hydrogen emission line spectra which were $663.103 \pm 0.002\text{nm}$, $541.732 \pm 0.007\text{nm}$ and $670.97 \pm 0.05\text{nm}$ respectively and plotting them against the energy level transitions to obtain the negative gradient i.e. R_∞ . It was noted that the wavelength for violet as being quite anomalous, as the measured wavelength was closer to the wavelength of red. The expected wavelengths were 656.279nm , 486.135nm and 410.1734nm respectively. The calculated Rydberg constant estimates were $1.086 \pm 0.007 \times 10^7\text{m}^{-1}$, $9.8 \pm 0.2 \times 10^6\text{m}^{-1}$ and $6.7 \pm 0.3 \times 10^6\text{m}^{-1}$ respectively.

It was concluded that the possible errors that resulted in the high values for aqua and violet were either a calibration error or skipped orders of maximums due to poor visibility from either low intensity or being unresolved by the Rayleigh Criterion.

VII. ACKNOWLEDGEMENT

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