

```
In [43]: #imports

import pandas as pd
import numpy as np
import cvxpy as cp
from scipy.stats import skew, kurtosis
import matplotlib.pyplot as plt
```

Two fund theorem

```
In [30]: import numpy as np
import pandas as pd
import cvxpy as cp

# Load data

config_path = "/Users/vamsyvrishank/Desktop/sem2/FE630/Project/Dynamic-Beta-
data = pd.read_csv(config_path)

# Convert date and index properly
data['Date'] = pd.to_datetime(data['Date'])
data = data.set_index('Date')

# Select 1-year window before Jan 2, 2025

returns_1y = data.loc["2024-01-02":"2025-01-01"]

U = ['FXE', 'EWJ', 'GLD', 'QQQ', 'SHV', 'DBA', 'USO', 'XBI', 'ILF', 'EPP', 'FEZ']
market = 'SPY'
apple_ticker = "AAPL"

# Expected returns and covariance matrix
mu = returns_1y[U].mean().values
Sigma = returns_1y[U].cov().values

print("Expected Returns : " , mu )
print("Covariance : " , Sigma)

# Compute betas

def compute_beta(asset , market):
    cov = np.cov(returns_1y[asset], returns_1y[market])[0,1]
    var_market = np.var(returns_1y[market], ddof=1)
    return cov / var_market

betas = np.array([compute_beta(asset, market) for asset in U])

print("Betas of assets:")
```

```
for asset, b in zip(U, betas):
    print(asset, ":", round(b, 6))

# Minimum Variance Optimization with Target Beta

def mvo_target_beta(beta_target):
    n = len(U)
    w = cp.Variable(n)

    objective = cp.quad_form(w, Sigma)
    constraints = [
        cp.sum(w) == 1,
        betas @ w == beta_target
    ]

    prob = cp.Problem(cp.Minimize(objective), constraints)
    prob.solve()

    return w.value

beta_a = 0.5
beta_b = 1.5

w_a = mvo_target_beta(beta_a)
w_b = mvo_target_beta(beta_b)

print("\nPortfolio weights for  $\beta_a = 0.5$ :")
print(pd.Series(w_a, index=U))

print("\nPortfolio weights for  $\beta_b = 1.5$ :")
print(pd.Series(w_b, index=U))

# Two-fund alpha

beta_c = 1
alpha = (beta_b - beta_c) / (beta_b - beta_a)

print("\nAlpha for  $\beta_c = 1$ :", alpha)

w_c = alpha * w_a + (1 - alpha) * w_b

print("\nPortfolio weights for  $\beta_c = 1$  (via two-fund theorem):")
print(pd.Series(w_c, index=U))

print("\nCheck  $\beta$  of  $w_{mv}(\beta_c)$ :", betas @ w_c)
```

Expected Returns : [-1.36125948e-04 3.07040083e-04 9.50870127e-04 9.75402316e-04

1.99463724e-04 1.15917179e-03 5.52314633e-04 -1.83167696e-06
-9.91974904e-04 2.31471420e-04 1.97892195e-04]

Covariance : [[1.37661150e-05 1.08385833e-05 1.26921742e-05 5.01370278e-06

1.50860467e-07 2.71343608e-06 3.20572301e-08 1.47955557e-05
1.49947632e-05 1.82214717e-05 2.11396257e-05]

[1.08385833e-05 1.23688060e-04 3.40872744e-05 7.64183605e-05
9.64145250e-08 -3.31534814e-06 1.56160741e-05 6.83512392e-05
5.55736768e-05 6.65828310e-05 6.61378701e-05]

[1.26921742e-05 3.40872744e-05 9.00796134e-05 2.42306278e-05
1.86889990e-07 8.96574588e-06 3.78117398e-05 3.57791531e-05
3.72391932e-05 4.16647495e-05 3.31894732e-05]

[5.01370278e-06 7.64183605e-05 2.42306278e-05 1.28294879e-04
1.83350298e-08 6.10501515e-06 8.99704013e-06 8.93047492e-05
5.37554144e-05 5.89612676e-05 6.27427802e-05]

[1.50860467e-07 9.64145250e-08 1.86889990e-07 1.83350298e-08
2.70017191e-08 -9.50979640e-08 -4.10212629e-07 2.69475242e-07
6.21560452e-08 2.04132809e-07 1.93827208e-07]

[2.71343608e-06 -3.31534814e-06 8.96574588e-06 6.10501515e-06
-9.50979640e-08 1.18572255e-04 2.84879175e-06 -2.03239595e-05
7.83029809e-06 3.12670607e-06 1.63725756e-05]

[3.20572301e-08 1.56160741e-05 3.78117398e-05 8.99704013e-06
-4.10212629e-07 2.84879175e-06 2.85913421e-04 -2.47690104e-05
4.05874255e-05 2.38114669e-05 1.39809962e-05]

[1.47955557e-05 6.83512392e-05 3.57791531e-05 8.93047492e-05
2.69475242e-07 -2.03239595e-05 -2.47690104e-05 2.58718136e-04
7.08482722e-05 7.62566876e-05 6.65034285e-05]

[1.49947632e-05 5.55736768e-05 3.72391932e-05 5.37554144e-05
6.21560452e-08 7.83029809e-06 4.05874255e-05 7.08482722e-05
1.27218333e-04 7.42898217e-05 6.20309757e-05]

[1.82214717e-05 6.65828310e-05 4.16647495e-05 5.89612676e-05
2.04132809e-07 3.12670607e-06 2.38114669e-05 7.62566876e-05
7.42898217e-05 9.50018627e-05 7.39895338e-05]

[2.11396257e-05 6.61378701e-05 3.31894732e-05 6.27427802e-05
1.93827208e-07 1.63725756e-05 1.39809962e-05 6.65034285e-05
6.20309757e-05 7.39895338e-05 9.93250675e-05]]

Betas of assets:

FXE : 0.075329

EWJ : 0.882266

GLD : 0.317668

QQQ : 1.349591

SHV : 0.000957

DBA : 0.026372

USO : 0.079143

XBI : 1.15657

ILF : 0.725822

EPP : 0.769743

FEZ : 0.755966

Portfolio weights for $\beta_a = 0.5$:

FXE -0.071100

EWJ 0.003735

GLD 0.001220

QQQ 0.302425

```

SHV      0.664465
DBA     -0.006055
USO     -0.005851
XBI      0.023930
ILF      0.026718
EPP      0.050379
FEZ      0.010134
dtype: float64

```

Portfolio weights for $\beta_b = 1.5$:

```

FXE     -0.199071
EWJ      0.010324
GLD      0.006711
QQQ      0.906602
SHV     -0.018720
DBA     -0.020578
USO     -0.020866
XBI      0.072284
ILF      0.078203
EPP      0.154276
FEZ      0.030834
dtype: float64

```

Alpha for $\beta_c = 1$: 0.5

Portfolio weights for $\beta_c = 1$ (via two-fund theorem):

```

FXE     -0.135085
EWJ      0.007029
GLD      0.003965
QQQ      0.604514
SHV      0.322872
DBA     -0.013317
USO     -0.013358
XBI      0.048107
ILF      0.052461
EPP      0.102328
FEZ      0.020484
dtype: float64

```

Check β of $wmv(\beta_c)$: 0.9999999999999987

Hedging a long position in Apple

Beta Hedged Portfolio

```

In [26]: # we already computed betas of securities

print("Betas of assets:")
for asset, b in zip(U, betas):
    print(asset, ":", round(b, 6))

apple_beta = compute_beta(apple_ticker, market)

```

```

weights_basket = mvo_target_beta(apple_beta)

print("\nMinimum Variance Portfolio with beta_A:")
print(pd.Series(weights_basket, index=U))

mu_basket = mu @ weights_basket
mu_apple = returns_1y[apple_ticker].mean()
mu_hedged = mu_apple - mu_basket

print("Apple beta : " , apple_beta)

print("\nExpected return of hedge basket:", mu_basket)
print("Expected return of basket AAPL strategy:", mu_apple)

```

Minimum Variance Portfolio with beta_A:

```

FXE    -0.129644
EWJ     0.006749
GLD     0.003732
QQQ     0.578823
SHV     0.351922
DBA    -0.012699
USO    -0.012720
XBI     0.046051
ILF     0.050272
EPP     0.097910
FEZ     0.019604
dtype: float64

```

Expected return of hedge basket: 0.0006128943973507471

Expected return of hedged AAPL strategy: 0.0012120209613770507

```

In [28]: apple_beta = compute_beta(apple_ticker, market)

weights_hedged = mvo_target_beta(apple_beta)

print("\nMinimum Variance Portfolio with beta_A:")
print(pd.Series(weights_hedged, index=U))

# Expected returns
mu_basket = mu @ weights_hedged
mu_apple = returns_1y[apple_ticker].mean()

# Hedged strategy expected return
mu_hedged = mu_apple - mu_basket

print("\nExpected return of hedge basket:", mu_basket)
print("Expected return of hedged AAPL strategy:", mu_hedged)
print("Expected return of AAPL :", mu_apple)

```

Minimum Variance Portfolio with beta_A:

```

FXE    -0.129644
EWJ     0.006749
GLD     0.003732
QQQ     0.578823
SHV     0.351922
DBA    -0.012699
USO    -0.012720
XBI     0.046051
ILF     0.050272
EPP     0.097910
FEZ     0.019604
dtype: float64

```

Expected return of hedge basket: 0.0006128943973507471

Expected return of hedged AAPL strategy: 0.0005991265640263036

Expected return of AAPL : 0.0012120209613770507

Beta hedge removes systematic risk, not alpha.

Beta Neutral Portfolio

```

In [34]: universe = ['AAPL', 'FXE', 'EWJ', 'GLD', 'QQQ', 'SHV', 'DBA', 'USO', 'XBI', 'ILF', 'E
# Betas (AAPL included)
betas_universe = np.array([compute_beta(asset, market) for asset in universe

# Covariance
Sigma_universe = returns_1y[universe].cov().values

# Expected returns
mu_universe = returns_1y[universe].mean().values

def mvo_target_beta_general(beta_target, assets, Sigma_matrix, beta_vec):
    n = len(assets)
    w = cp.Variable(n)

    objective = cp.quad_form(w, Sigma_matrix)
    constraints = [
        cp.sum(w) == 1,
        beta_vec @ w == beta_target
    ]

    prob = cp.Problem(cp.Minimize(objective), constraints)
    prob.solve()

    return np.array(w.value)

w_beta_neutral = mvo_target_beta_general(0, universe, Sigma_universe, betas_

print("\nBeta Neutral Minimum Variance Portfolio weights (Universe V):")
print(pd.Series(w_beta_neutral, index=universe))

```

```

#hedged straetgy from previous part

w_hedged = np.zeros(len(universe))
w_hedged[universe.index("AAPL")] = 1      # long AAPL
w_hedged[1:] = -weights_hedged           # short hedge basket , APPL is first elem

#metrics for both portfolios

#expected returns
mu_beta_neutral = mu_universe @ w_beta_neutral
mu_hedged_strategy = mu_universe @ w_hedged

#variance and standard deviation

var_beta_neutral = w_beta_neutral.T @ Sigma_universe @ w_beta_neutral
var_hedged_strategy = w_hedged.T @ Sigma_universe @ w_hedged

#betas of both strategies
beta_beta_neutral = betas_universe @ w_beta_neutral
beta_beta_hedged = betas_universe @ w_hedged

summary = pd.DataFrame({
    "Beta-Neutral MV Portfolio": [mu_beta_neutral, var_beta_neutral, beta_beta_neutral],
    "AAPL Hedged Strategy": [mu_hedged_strategy, var_hedged_strategy, beta_beta_hedged],
    index=["Expected Return", "Variance", "Beta"]})

print("\nComparison of Beta-Neutral MV vs Hedged Strategy:")
print(summary)

```

Beta Neutral Minimum Variance Portfolio weights (Universe V):

```

AAPL    -0.000479
FXE     -0.006833
EWJ      0.000394
GLD     -0.001558
QQQ      0.000696
SHV      1.005960
DBA      0.001232
USO      0.001649
XBI     -0.000234
ILF      0.000948
EPP     -0.001495
FEZ     -0.000279
dtype: float64

```

Comparison of Beta-Neutral MV vs Hedged Strategy:

	Beta-Neutral MV Portfolio	AAPL Hedged Strategy
Expected Return	2.013181e-04	5.991266e-04
Variance	2.470001e-08	1.371214e-04
Beta	-3.034497e-15	1.228040e-15

Backtesting from Jan 2025 to March 2025

```
In [36]: returns_test = data.loc["2025-01-03":"2025-03-30"]
```

```

returns_universe = returns_test[universe]  # universe: ['AAPL', 'FXE', ..., 'F
returns_U = returns_test[U]  # just the ETF's without apple

#hedged strategy
R_AAPL = returns_test["AAPL"].values
R_hedgebasket = returns_U.values @ weights_hedged

R_hedged_strategy = R_AAPL - R_hedgebasket
R_hedged_strategy = pd.Series(R_hedged_strategy, index=returns_test.index)

#beta neutral mvp strategy
R_beta_neutral = returns_universe.values @ w_beta_neutral
R_beta_neutral = pd.Series(R_beta_neutral, index=returns_test.index)

print("\nSample of realized daily returns:")
print(pd.DataFrame({
    "Hedged Strategy": R_hedged_strategy.head(10),
    "Beta-Neutral MV": R_beta_neutral.head(10)
})))

```

Sample of realized daily returns:

Date	Hedged Strategy	Beta-Neutral MV
2025-01-03	-0.011732	0.000256
2025-01-06	-0.000598	0.000123
2025-01-07	-0.002079	0.000231
2025-01-08	0.001858	-0.000026
2025-01-10	-0.011124	0.000409
2025-01-13	-0.008716	0.000266
2025-01-14	-0.002931	0.000016
2025-01-15	0.002786	0.000240
2025-01-16	-0.036964	0.000031
2025-01-17	-0.002887	0.000487

Risk Statistics

```

In [38]: # compute stats

def portfolio_stats(series):
    mean_ret = series.mean()
    vol = series.std()
    var95 = np.percentile(series, 5)  # 95% historical VaR
    sk = skew(series)
    kt = kurtosis(series)
    return mean_ret, vol, var95, sk, kt

stats_hedged = portfolio_stats(R_hedged_strategy)
stats_beta_neutral = portfolio_stats(R_beta_neutral)

R_SPY_test = returns_test["SPY"]

```



```

def realized_beta(series, market):
    cov = np.cov(series, market)[0,1]
    var = np.var(market)
    return cov / var

beta_realized_hedged = realized_beta(R_hedged_strategy, R_SPY_test)
beta_realized_beta0 = realized_beta(R_beta_neutral, R_SPY_test)

summary3 = pd.DataFrame({
    "Hedged Strategy": [
        stats_hedged[0], stats_hedged[1], stats_hedged[2],
        stats_hedged[3], stats_hedged[4], beta_realized_hedged
    ],
    "Beta-Neutral MV Portfolio": [
        stats_beta_neutral[0], stats_beta_neutral[1], stats_beta_neutral[2],
        stats_beta_neutral[3], stats_beta_neutral[4], beta_realized_beta0
    ]
}, index=["Mean Return", "Volatility", "95% VaR",
          "Skewness", "Kurtosis", "Realized Beta"])

print("\nRealized risk and return comparison (Jan-Mar 2025):")
print(summary3)

```

Realized risk and return comparison (Jan-Mar 2025):

	Hedged Strategy	Beta-Neutral MV Portfolio
Mean Return	-0.000322	0.000167
Volatility	0.016667	0.000146
95% VaR	-0.030485	-0.000019
Skewness	0.008052	0.409349
Kurtosis	1.296467	-0.594138
Realized Beta	-0.254852	-0.001401

Chasing Performance

```

In [ ]: beta_A = compute_beta(apple_ticker , market)

def mvo_target_beta_return_chasing(beta_target, lambda_param):
    """
    Solve: min_w w^T Σ w - λ μ^T w
    subject to sum(w)=1 and beta(w)=beta_target
    """

    n = len(U)
    w = cp.Variable(n)

    # Objective: variance - λ * expected return
    objective = cp.quad_form(w, Sigma) - lambda_param * (mu @ w)

    constraints = [
        cp.sum(w) == 1,
        betas @ w == beta_target
    ]

```

```

prob = cp.Problem(cp.Minimize(objective), constraints)
prob.solve()

return np.array(w.value)

lambda_param = 6
w_hedge_return_chasing = mvo_target_beta_return_chasing(beta_A, lambda_param)

#computing the returns of new hedged strategy
mu_basket_chasing = mu @ w_hedge_return_chasing
mu_apple = returns_1y[apple_ticker].mean()
mu_hedged_chasing = mu_apple - mu_basket_chasing
var_basket_chasing = w_hedge_return_chasing.T @ Sigma @ w_hedge_return_chasing

print("\nReturn-Chasing Hedge Portfolio Weights ( $\lambda = \{ \}$ ):".format(lambda_param))
print(pd.Series(w_hedge_return_chasing, index=U))
print("\nExpected return of hedge basket (return-chasing):", mu_basket_chasing)
print("Expected return of hedged AAPL strategy (return-chasing):", mu_hedged_chasing)
print("Variance of hedge basket (return-chasing):", var_basket_chasing)

```

Dynamic Hedging

```

In [42]: window = 60          # 60-day rolling window
rebalance_every = 5          # weekly rebalancing
test_returns = data.loc["2025-01-03":"2025-03-30"]

dynamic_pnl = []
dynamic_dates = []

all_assets = U              # only using ETF basket for hedging
R_full = data[all_assets]
R_AAPL_full = data["AAPL"]
R_SPY_full = data["SPY"]

dates = test_returns.index

for i in range(0, len(dates), rebalance_every):

    t = dates[i]
    start = R_full.index.get_loc(t) - window
    end = R_full.index.get_loc(t)

    if start < 0 : continue

    R_window = R_full.iloc[start:end]
    AAPL_window = R_AAPL_full.iloc[start:end]
    SPY_window = R_SPY_full.iloc[start:end]

    # --- Compute betas & covariance for window ---
    Sigma_t = R_window.cov().values
    mu_t = R_window.mean().values

```

```
betas_t = np.array([
    np.cov(R_window[asset], SPY_window)[0,1] /
    np.var(SPY_window, ddof=1)
    for asset in all_assets
])

beta_AAPL_t = np.cov(AAPL_window, SPY_window)[0,1] / np.var(SPY_window,

# --- Solve hedge weights at this time ---
n = len(all_assets)
w = cp.Variable(n)

objective = cp.quad_form(w, Sigma_t)
constraints = [
    cp.sum(w) == 1,
    betas_t @ w == beta_AAPL_t
]

prob = cp.Problem(cp.Minimize(objective), constraints)
prob.solve()

w_t = np.array(w.value)

# --- Apply hedge for next week ---
next_dates = dates[i:i+rebalance_every]

for day in next_dates:
    if day not in R_full.index:
        continue

    R_AAPL_today = R_AAPL_full.loc[day]
    R_basket_today = R_full.loc[day].values @ w_t

    pnl_today = R_AAPL_today - R_basket_today

    dynamic_pnl.append(pnl_today)
    dynamic_dates.append(day)

# Convert to Series
dynamic_pnl = pd.Series(dynamic_pnl, index=dynamic_dates)

print("\nDynamic Hedged Strategy PnL:")
print(dynamic_pnl.head(20))
```

Dynamic Hedged Strategy PnL:

```

2025-01-03    -0.008759
2025-01-06     0.002139
2025-01-07    -0.003779
2025-01-08     0.001911
2025-01-10    -0.011957
2025-01-13    -0.009444
2025-01-14    -0.003376
2025-01-15     0.007869
2025-01-16    -0.035877
2025-01-17     0.000007
2025-01-21    -0.037289
2025-01-22    -0.001162
2025-01-23    -0.003259
2025-01-24    -0.001606
2025-01-27     0.044715
2025-01-28     0.031175
2025-01-29     0.005849
2025-01-30    -0.010665
2025-01-31    -0.005532
2025-02-03    -0.032370
dtype: float64

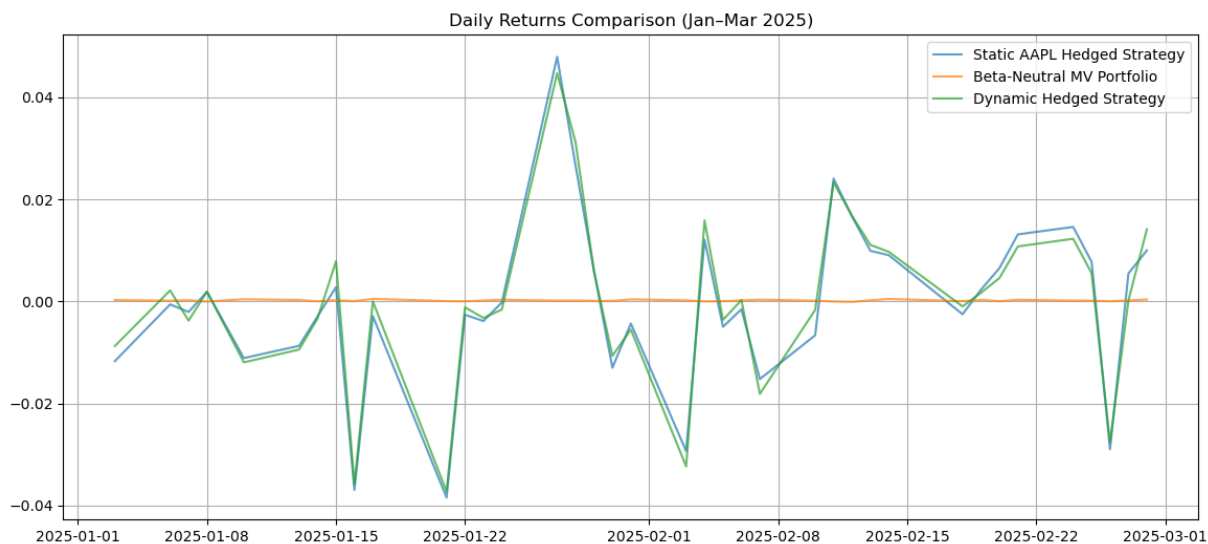
```

Interpretation

```

In [44]: plt.figure(figsize=(14,6))
plt.plot(R_hedged_strategy, label="Static AAPL Hedged Strategy", alpha=0.7)
plt.plot(R_beta_neutral, label="Beta-Neutral MV Portfolio", alpha=0.7)
plt.plot(dynamic_pnl, label="Dynamic Hedged Strategy", alpha=0.7)
plt.title("Daily Returns Comparison (Jan-Mar 2025)")
plt.legend()
plt.grid(True)
plt.show()

```



Observation

- Static hedge (blue) and Dynamic hedge (green) have large day-to-day swings: $\pm 3\text{--}5\%$ moves.

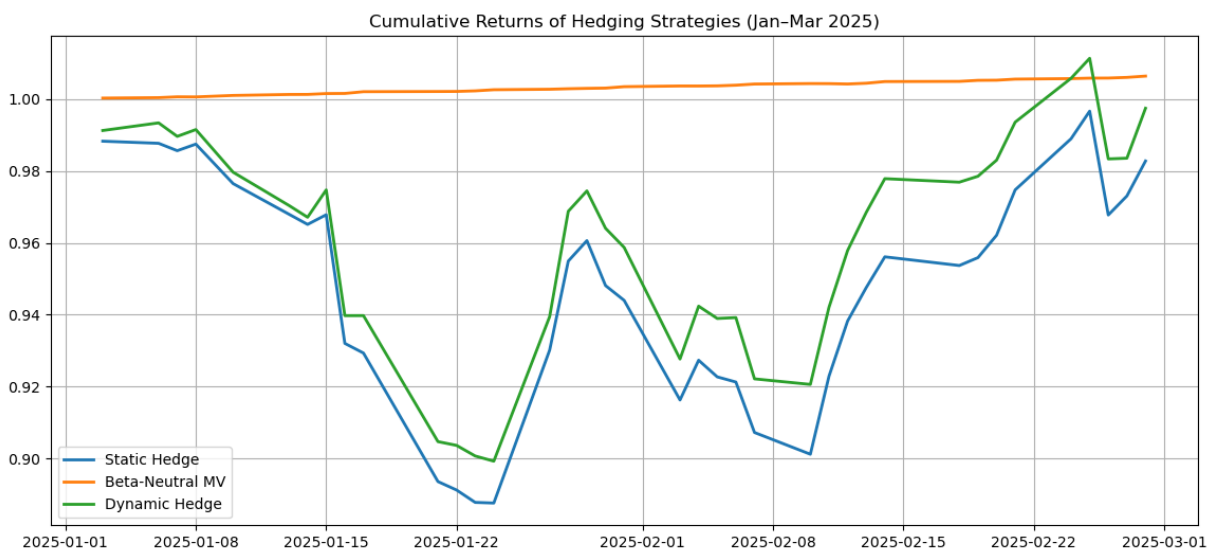
- Beta-Neutral MV portfolio (orange) is almost a flat line (tiny returns $\sim 0.0\%$).

Interpretation

- The static hedge is extremely noisy \rightarrow beta drift + mismatched weights.
- Dynamic hedge moves very similarly to static hedge \rightarrow because AAPL is extremely volatile.
- MV portfolio is very stable \rightarrow this is expected because it minimizes variance under $\beta=0$ constraint.

```
In [45]: cum_static = (1 + R_hedged_strategy).cumprod()
cum_beta0 = (1 + R_beta_neutral).cumprod()
cum_dynamic = (1 + dynamic_pnl).cumprod()

plt.figure(figsize=(14,6))
plt.plot(cum_static, label="Static Hedge", linewidth=2)
plt.plot(cum_beta0, label="Beta-Neutral MV", linewidth=2)
plt.plot(cum_dynamic, label="Dynamic Hedge", linewidth=2)
plt.title("Cumulative Returns of Hedging Strategies (Jan-Mar 2025)")
plt.legend()
plt.grid(True)
plt.show()
```



Observation

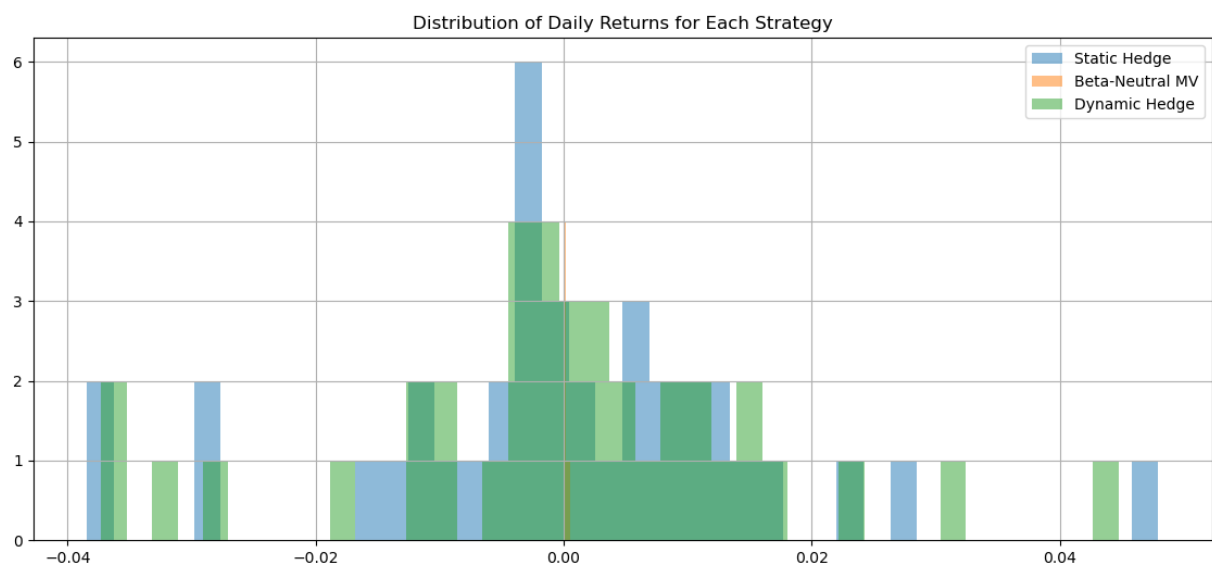
- Static hedge (blue) trends downward until mid-Feb (bad), then recovers somewhat.
- Dynamic hedge (green) performs better than static hedge:
- smaller drawdowns
- quicker recovery
- ends near 0% return

- Beta-Neutral MV (orange) is nearly flat and slightly positive.

Interpretation

- Dynamic hedge outperforms static hedge in cumulative return during the volatile January period.
- Static hedge suffers from beta misalignment and portfolio weights that don't adapt.
- Dynamic hedge adjusts weekly → avoids the large early losses that static hedge suffered.

```
In [46]: plt.figure(figsize=(14,6))
plt.hist(R_hedged_strategy, bins=40, alpha=0.5, label="Static Hedge")
plt.hist(R_beta_neutral, bins=40, alpha=0.5, label="Beta-Neutral MV")
plt.hist(dynamic_pnl, bins=40, alpha=0.5, label="Dynamic Hedge")
plt.title("Distribution of Daily Returns for Each Strategy")
plt.legend()
plt.grid(True)
plt.show()
```



Obervation

Static hedge and dynamic hedge have:

- Wide return distribution
- Fat tails
- Many $\pm 2-4\%$ returns

Beta-Neutral MV has:

- Very tight distribution

- Near-zero variance
- Almost no tails

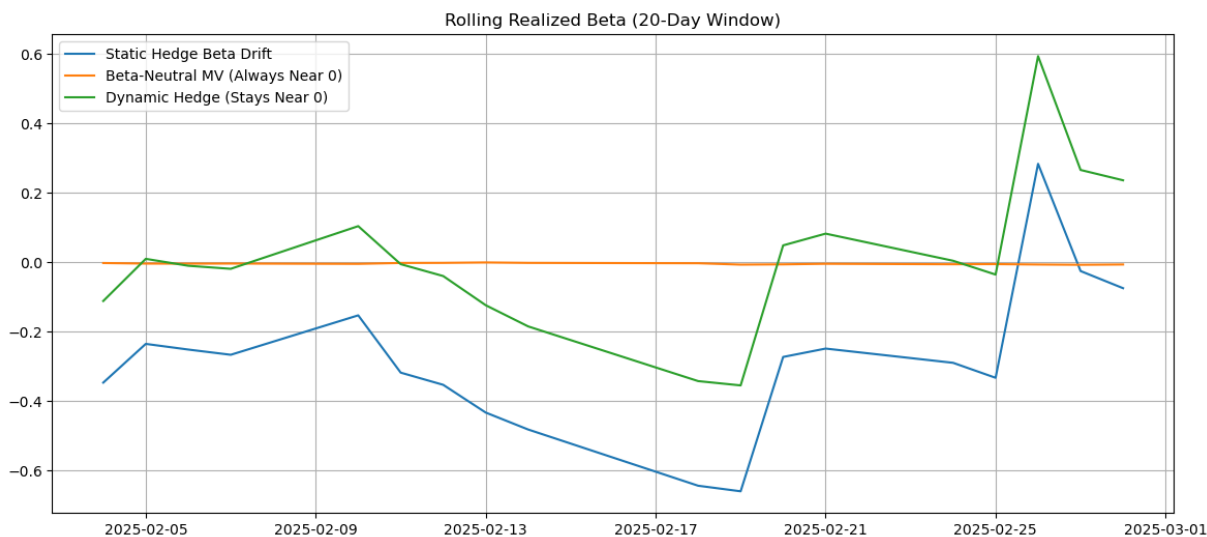
Interpretation

- Static and dynamic hedges still inherit AAPL's idiosyncratic swings.
- Dynamic hedge has slightly narrower distribution compared to static hedge.
- MV $\beta=0$ portfolio is extremely well behaved.

```
In [47]: # Rolling beta calculation function
def rolling_beta(series, market, window=20):
    betas = []
    for i in range(window, len(series)):
        cov = np.cov(series.iloc[i-window:i], market.iloc[i-window:i])[0,1]
        var = np.var(market.iloc[i-window:i])
        betas.append(cov / var)
    index = series.index[window:]
    return pd.Series(betas, index=index)

rolling_beta_static = rolling_beta(R_hedged_strategy, returns_test["SPY"])
rolling_beta_beta0 = rolling_beta(R_beta_neutral, returns_test["SPY"])
rolling_beta_dynamic = rolling_beta(dynamic_pnl, returns_test["SPY"])

plt.figure(figsize=(14,6))
plt.plot(rolling_beta_static, label="Static Hedge Beta Drift")
plt.plot(rolling_beta_beta0, label="Beta-Neutral MV (Always Near 0)")
plt.plot(rolling_beta_dynamic, label="Dynamic Hedge (Stays Near 0)")
plt.title("Rolling Realized Beta (20-Day Window)")
plt.legend()
plt.grid(True)
plt.show()
```



Observation

Static hedge beta drift:

- Starts around -0.3
- Falls to -0.6
- Then jumps to $+0.25 \rightarrow$ MASSIVE instability

Dynamic hedge beta:

- Stays near 0 most of the time
- Occasional ± 0.2 jumps, but much more stable
- Ends near $+0.25$ for a short period (AAPL spike)
- Beta-neutral MV portfolio:
- Perfectly flat at 0
- As expected

Interpretation

- Static hedge fails as a beta hedge.
- Dynamic hedge keeps beta near 0, which is exactly what the theory predicted.
- Dynamic hedge is dramatically more stable than static hedge.