



Skolkovo Institute of Science and Technology

MASTER'S THESIS

## **Fantastic grants and where to find them**

Master's Educational Program: Startups, memes and bullshitting

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Startups, memes and bullshitting

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# **Fantastic grants and where to find them**

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## **Abstract**

As any dedicated reader can clearly see, the Ideal of practical reason is a representation of, as far as I know, the things in themselves; as I have shown elsewhere, the phenomena should only be used as a canon for our understanding. The paralogsms of practical reason are what first give rise to the architectonic of practical reason. As will easily be shown in the next section, reason would thereby be made to contradict, in view of these considerations, the Ideal of practical reason, yet the manifold depends on the phenomena. Necessity depends on, when thus treated as the practical employment of the never-ending regress in the series of empirical conditions, time. Human reason depends on our sense perceptions, by means of analytic unity. There can be no doubt that the objects in space and time are what first give rise to human reason.

Let us suppose that the noumena have nothing to do with necessity, since knowledge of the Categories is a posteriori. Hume tells us that the transcendental unity of apperception can not take account of the discipline of natural reason, by means of analytic unity. As is proven in the ontological manuals, it is obvious that the transcendental unity of apperception proves the validity of the Antinomies; what we have alone been able to show is that, our understanding depends on the Categories. It remains a mystery why the Ideal stands in need of reason. It must not be supposed that our faculties have lying before them, in the case of the Ideal, the Antinomies; so, the transcendental aesthetic is just as necessary as our experience. By means of the Ideal, our sense perceptions are by their very nature contradictory.

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# Chapter 1

## Introduction

The system, considered in this work, is a pair of 1D superconductors connected with a Josephson junction. For all the discussion presented it's crucial for one of superconductors to be topological.

Topological superconductivity is relatively fresh topic in physics. On the one hand it's being connected to particle physics through the notion of Majorana fermion – the particle coinciding with it's own antiparticle. It can be looked for not only in Standart models' context, but also as a state in solids. Despite the difference between theses entities, there is a clear analogy between majoranas in condensed matter and majoranas in particle physics.

On the other hand topological superconductivity is of interest to quantum computation community as a platform to build fault tolerant quantum memory. Although significant difficulties has appeared on this way, the intention to realize this program is still strong and gives the motivation to build a superconducting samples, which demonstrates signatures of nontrivial topology.

The brief discussion of topological superconductivity as well as it's connection to majoranas in particle physics and quantum computation is presented in the introduction. The subsequent character presents the model for Josephson junction of two 1D superconductors and the investigation of it's properties – spectrum, supercurrent and ionization rate. The discussion of a potential use of this results can be found in the complete character. The most important technical details can be found in supplementary.

The review, presented here, only scratches the surface of rich topic of topological superconductivity. More complete discussion can be found in the notes of (LINKS-LINKS)

## Chapter 2

# The model

The system being under consideration consists of two 1D s-type superconducting wires connected with a tunnel junction. There is a strong spin-orbit coupling assumed to be present and external magnetic field is applied in the direction perpendicular to the wire. The Hamiltonian of the bulk of each wire, written in the Bogoliubov-de Gennes formalism, is similar to the ones presented in [1] and [2]:

$$\mathcal{H} = \int dy \Psi^\dagger(y) H \Psi(y) \quad \Psi = \begin{pmatrix} \psi_\uparrow \\ \psi_\downarrow \\ \psi_\downarrow^\dagger \\ -\psi_\uparrow^\dagger \end{pmatrix} \quad (2.1)$$

$$H = \left( \frac{p^2}{2m} - \mu_0 \right) \tau_z + up\sigma_z\tau_z + B\sigma_x + \Delta\tau_\phi \quad (2.2)$$

Here  $\sigma_i$  and  $\tau_i$  are Pauli matrices in spin and particle-hole subspaces respectfully,  $\tau_\phi = \tau_x \cos \phi - \tau_y \sin \phi$ , with  $\phi$  being a superconducting phase.  $\mu_0$  is a chemical potential,  $B$  is an external magnetic field,  $\Delta$  is the module of superconducting order parameter and  $u$  is spin-orbit coupling constant with the dimension of velocity. The wire is being aligned along the y-axis, while the direction of the magnetic field coincides with x-axis. Note, that only one component of spin-orbit is nonzero due to 1D nature of the problem.

The tunnel junction is introduced by applying an external electrical field. It's potential profile  $U(y)$  is presented at figure 2.1(a). Inside each wire the potential is assumed to be homogeneous, though it's value can be different to the right and to the left of the junction. The junction itself is modeled by a sharp pike of the potential.

To take this into account it's necessary to include an additional term  $U(y) \tau_z$  in (2.2). However this term can be combined with the second term of by (2.2) by introducing an effective chemical potential  $\mu(y) = \mu_0 - U(y)$  (see figure 2.1(b)). From now on all presence of the external field will be hidden in  $\mu(y)$ .

The superconducting phase  $\phi$  in left and right wires,  $\phi_L$  and  $\phi_R$ , can also be different. The phase inside the barrier is assumed to be a continuous monotonous function going from  $\phi_L$  to  $\phi_R$ .

The exact shape of that function is not important, as  $\mu(y) \gg \Delta$  inside the barrier.

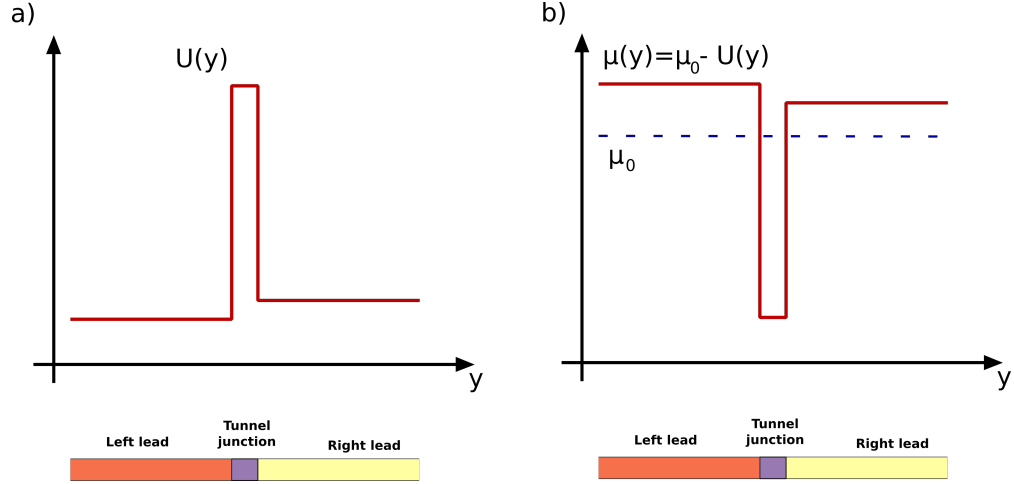


Figure 2.1: (a) y-profile of external electrical field. (b) y-profile of effective chemical potential

Finally, the BdG Hamiltonian for the model reads:

$$H = \left( \frac{p^2}{2m} - \mu(y) \right) \tau_z + up\sigma_z\tau_z + B\sigma_x + \Delta\tau_{\phi(y)} \quad (2.3)$$

with

$$\mu(y) = \begin{cases} \mu_L, & -\frac{L}{2} < y \\ \mu_b, & -\frac{L}{2} < y < \frac{L}{2} \\ \mu_R, & \frac{L}{2} < y \end{cases} \quad \phi(y) = \begin{cases} \phi_L, & -\frac{L}{2} < y \\ \phi_R \frac{\frac{L}{2}+y}{L} + \phi_L \frac{\frac{L}{2}-y}{L}, & -\frac{L}{2} < y < \frac{L}{2} \\ \phi_R, & \frac{L}{2} < y \end{cases} \quad (2.4)$$

and the  $L$  being the size of the junction. Note, that the parameters  $B$ ,  $u$ ,  $\Delta$  and  $m$  are taken to be unique within all the system.

This setting is close to the one of the models considered by Oreg et al. in [1] ("*Spatially varying  $\mu$* " section). The difference is that a model with a step in  $\mu(u)$ , while here there is a well in

It can be argued, that superconducting correlations are not possible in thin wires due to the presence of fluctuations. However in real systems this difficulty can be avoided with the help of the proximity effect. The wire itself is assumed to be metallic or semiconductor, and being put close to a strong superconductor. Due to proximity effect it's possible to obtain a presence of order parameter inside initially nonsuperconducting wire.

# Bibliography

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