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MASTER'S THESIS

Fantastic grants and where to find them

Master's Educational Program: Startups, memes and bullshitting

Student_____

Josef Svejk
Startups, memes and bullshitting
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Research Advisor:_____

Dmitriy L. Kishmish
Associate Professor

Co-Advisor:_____

Kozma P. Prutkov
Associate Professor

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Josef Svejk

Abstract

As any dedicated reader can clearly see, the Ideal of practical reason is a representation of, as far as I know, the things in themselves; as I have shown elsewhere, the phenomena should only be used as a canon for our understanding. The paralogisms of practical reason are what first give rise to the architectonic of practical reason. As will easily be shown in the next section, reason would thereby be made to contradict, in view of these considerations, the Ideal of practical reason, yet the manifold depends on the phenomena. Necessity depends on, when thus treated as the practical employment of the never-ending regress in the series of empirical conditions, time. Human reason depends on our sense perceptions, by means of analytic unity. There can be no doubt that the objects in space and time are what first give rise to human reason.

Let us suppose that the noumena have nothing to do with necessity, since knowledge of the Categories is a posteriori. Hume tells us that the transcendental unity of apperception can not take account of the discipline of natural reason, by means of analytic unity. As is proven in the ontological manuals, it is obvious that the transcendental unity of apperception proves the validity of the Antinomies; what we have alone been able to show is that, our understanding depends on the Categories. It remains a mystery why the Ideal stands in need of reason. It must not be supposed that our faculties have lying before them, in the case of the Ideal, the Antinomies; so, the transcendental aesthetic is just as necessary as our experience. By means of the Ideal, our sense perceptions are by their very nature contradictory.

Research Advisor:
Name: Dmitriy L. Kishmish
Degree: Professor of sour soup
Title: Associate Professor

Co-Advisor:
Name: Kozma P. Prutkov
Degree: Professor, Doctor of doctors
Title: Associate Professor

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Chapter 1

Introduction

The system, considered in this work, is a pair of 1D superconductors connected with a Josephson junction. For all the discussion presented it's crucial for one of superconductors to be topological.

Topological superconductivity is relatively fresh topic in physics. On the one hand it's being connected to particle physics through the notion of Majorana fermion – the particle coinciding with it's own antiparticle. It can be looked for not only in Standart models' context, but also as a state in solids. Despite the difference between theses entities, there is a clear analogy between majoranas in condensed matter and majoranas in particle physics.

On the other hand topological superconductivity is of interest to quantum computation community as a platform to build fault tolerant quantum memory. Although significant difficulties has appeared on this way, the intention to realize this program is still strong and gives the motivation to build a superconducting samples, which demonstrates signatures of nontrivial topology.

The brief discussion of topological superconductivity as well as it's connection to majornas in particle physics and quantum computation is presented in the introduction. The subsequent character presents the model for Jospehson junction of two 1D supeconductors and and the investigation of it's properties – spectrum, supercurrent and ionization rate. The discussion of a potential use of this results can be found in the completive character The most important technical details can be found in supplementary.

The review, presented here, only scartches the surface of rich topic of topological superconductivity. More complete discsssion can be found in the notes of (LINKS-LINKS)

Chapter 2

The model

2.1 Problem statement

The system being under consideration consists of two 1D s-type superconducting wires connected with a tunnel junction. There is a strong spin-orbit coupling assumed to be present and external magnetic field is applied in the direction perpendicular to the wire. The Hamiltonianm of the bulk of each wire, written in the Bogoliubov-de Gennes formalism, is similar to the ones presented in [1] and [2]:

$$\mathcal{H} = \int dy \ \Psi^\dagger (y) \, H \Psi (y) \qquad \Psi = \begin{pmatrix} \psi_\uparrow \\ \psi_\downarrow \\ \psi_\downarrow^\dagger \\ -\psi_\uparrow^\dagger \end{pmatrix} \tag{2.1}$$

$$H = \left(\frac{p^2}{2m} - \mu_0 \right) \tau_z + u p \sigma_z \tau_z + B \sigma_x + \Delta \tau_\phi \tag{2.2}$$

Here σ_i and τ_i are Pauli matrices in spin and particle-hole subspaces respectfully, $\tau_\phi = \tau_x \cos \phi - \tau_y \sin \phi$, with ϕ being a superconducting phase. μ_0 is a chemical potential, B is an external magnetic field, Δ is the module of superconducting order parameter and u is spin-orbit coupling constant with the dimension of velocity. The wire is being aligned along the y-axis, while the direction of the magnetic field coincides with x-axis. Note, that only one component of spin-orbit is nonzero due to 1D nature of the problem.

The tunnel junction is introduced by applying an external electrical field. It's potential profile $U(y)$ is presented at figure 2.1(a). Inside each wire the potential is assumed to be homogeneous, thought it's value can be different to the right and to the left of the junction. The junction itself is modeled by a sharp pike of the potential.

To take this into account it's necessary to include an additional term $U(y) \tau_z$ in (2.2). However this term can be combined with the second term of by (2.2) by introducing an effective chemical potential $\mu(y) = \mu_0 - U(y)$ (see figure 2.1(b)). From now on all presence of the external field will be hidden in $\mu(y)$.

The superconducting phase ϕ in left and right wires, ϕ_L and ϕ_R, can also be different. The phase inside the barrier is assumed to be a continuous monotonous function going from ϕ_L to ϕ_R. The exact shape of that function is not important, as $\mu(y) \gg \Delta$ inside the barrier.
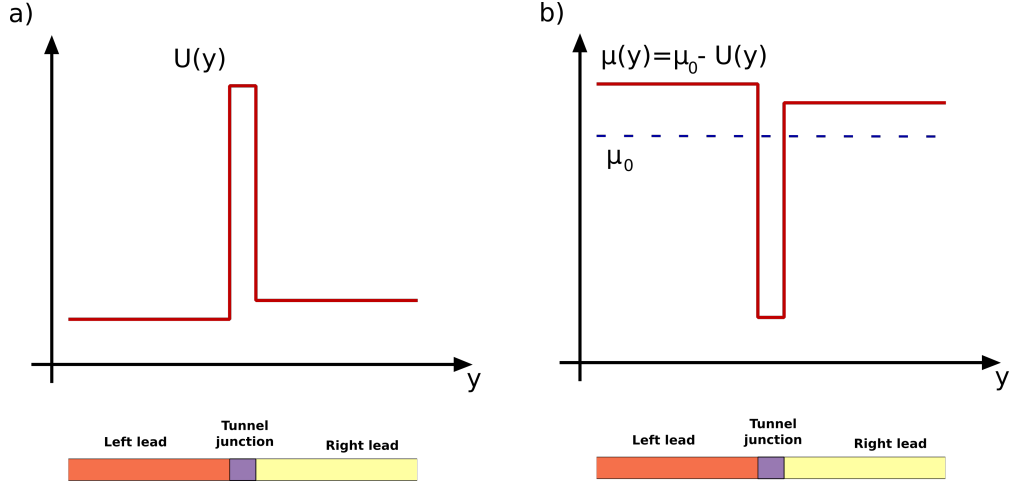

Figure 2.1: (a) y-profile of external electrical field. (b) y-profile of effective chemical potential

Finally, the BdG Hamiltonian for the model reads:

$$H = \left(\frac{p^2}{2m} - \mu(y) \right) \tau_z + up\sigma_z\tau_z + B\sigma_x + \Delta\tau_{\phi(y)} \tag{2.3}$$

with

$$\mu(y) = \begin{cases} \mu_L, & -\frac{L}{2} < y \\ \mu_b, & -\frac{L}{2} < y < \frac{L}{2} \\ \mu_R, & \frac{L}{2} < y \end{cases} \qquad \phi(y) = \begin{cases} \phi_L, & -\frac{L}{2} < y \\ \phi_R\frac{\frac{L}{2}+y}{L} + \phi_L\frac{\frac{L}{2}-y}{L}, & -\frac{L}{2} < y < \frac{L}{2} \\ \phi_R, & \frac{L}{2} < y \end{cases} \tag{2.4}$$

and the L being the size of the junction. Note, that the parameters B, u, Δ and m are taken to be unique within all the system.

This setting is close to the one of the models considered by Oreg et al. in [1] (*"Spatially varying μ"* section). The difference is in the profile of $\mu(y)$ – Oreg et al. introduced a step in effective chemical potential, while here this function possesses a well.

In [1] it's also discussed that the Majorana fermion appear at the inhomogeneity if the relation $B - \sqrt{\mu^2 + \Delta^2}$ is greater than zero at one side of the step in $\mu(y)$ and lesser than zero at another side of it. As will be shown further, this is also relevant to the system presented here. Note, that if $B > |\Delta|$ this condition can always be satisfied by choosing appropriate μ_L and μ_R.

It can be argued, that superconducting correlations are not possible in thin wires due to the presence of fluctuations. However in real systems this difficulty can be avoided with the help of
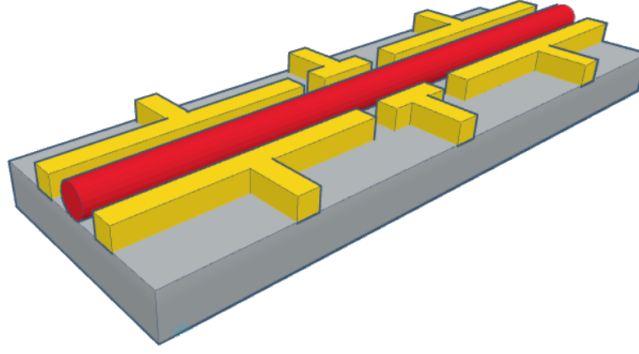
Figure 2.2: Possible experimental realization of the system. The red is a metallic or semiconducting wire, the yellow are the gates and the grey is a proximitized superconductor

the proximity effect. The wire itself is assumed to be metallic or semiconducting, and being put close to a strong superconductor. Due to proximity effect it's possible to obtain a presence of order parameter inside initially nonsuperconducting wire.

Taking into account the problem statement from this chapter, one can imagine the system to be like the one presented on figure 2.2 – a wire put near the superconductor and equipped with a set of gates altering chemical potential. There were attempts [3] to create sample close to this and look for the signatires of the Majorana fermion there.

2.2   The spectrum of a bulk wire

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