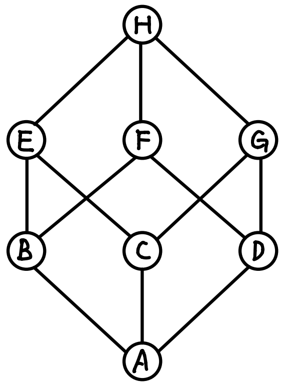
VE203 Final 周维凯 518021911039

Ex1. $P(M)=\{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$

Let $A=\phi$, $B=\{a\}$, $C=\{b\}$, $D=\{c\}$, $E=\{a,b\}$, $F=\{a,c\}$, $G=\{b,c\}$, $H=\{a,b,c\}$



Ex2. If $n>10$, let G be the graph with n vertices and e edges.

By corollary 3.3.5, we have $e \le 3n-6$. ①

G' is the graph with n vertices and $\frac{n(n-1)}{2}-e$ edges.

Similarily, we have $\frac{n(n-1)}{2}-e \le 3n-6$ ②

①+②, we have $\frac{n(n-1)}{2} \le 6n-12$

$$n^2-13n+24 \le 0$$

$$\frac{13-\sqrt{73}}{2} \le n \le \frac{13+\sqrt{73}}{2} \approx 10.77$$

Since $n \in N$, $n \le 10$, which is contradict to $n>10$.

Therefore, $n \le 10$.

Ex3. The chromatic number is 3.

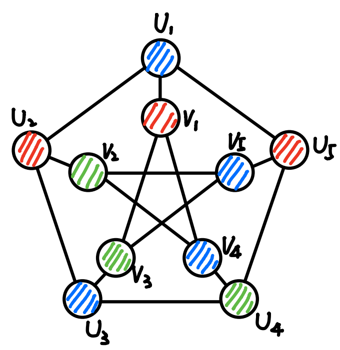
On the right, it is 3-coloring, with V_1, U_2 and U_5 red; V_2, V_3 and U_4 green; V_4, V_5, U_1 and U_3 blue.

If we try to use 2 colors, we can consider U_1, U_2, U_3, U_4 and U_5.

If U_1 is color A, then U_2 should be color B, U_3 color A, U_4 color B, U_5 color A. However, U_1 and U_5 are adjacent.

Therefore, it cannot be 2-colorable.

Therefore, chromatic number is 3.



Ex4. The crossing number of the Petersen graph is 2.

We can delete the edge between V_2 and V_4, and the edge between V_3 and V_5

After deleting, the corresponding planar representation is given in ②

If we delete only one edge, we need to consider three different cases.

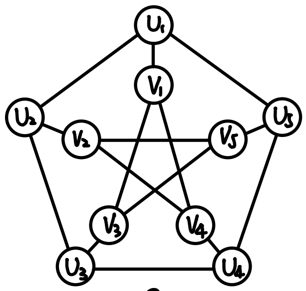
i) the outer edge, i.e. the edge between U_1 and U_2. Then it has the subgraph shown in ③. And ③ has the homeomorphic graph shown in ④ by example 3.1.13. Then by Kuratowski's theorem, we know that if we delete one outer edge cannot have planar representation.

ii) the edge linking outer and inner, i.e. the edge between U_1 and V_1. Similarily, it has the subgraph shown in ③. Therefore, it cannot have planar representation.

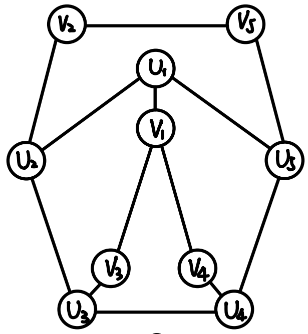
iii) the inner edge, i.e. the edge between V_2 and V_5. Also, we can find a subgraph whose homeomorphic is ⑤. Therefore, it cannot have planar representation.

So only deleting one edge does not satisfy.

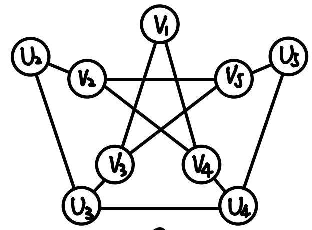
Hence the crossing number is 2.



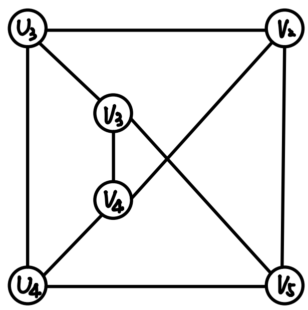
①



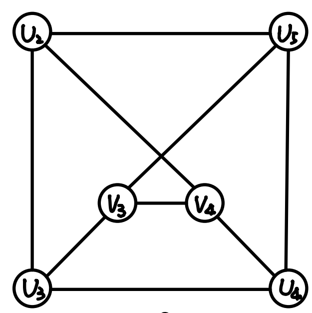
②



③



④



⑤

Ex5. I and III are isomorphic.

We can find φ: I → III by setting

φ: I(A) → III(B), I(B) → III(C), I(C) → III(H), I(D) → III(J), I(E) → III(G), I(F) → III(E), I(G) → III(D), I(H) → III(I), I(I) → III(A), I(J) → III(F)

Then we can have I = {{A, C}, {A, D}, {A, F}, {B, D}, {B, E}, {B, G}, {C, E}, {C, H}, {D, I}, {E, J}, {F, G}, {G, H}, {H, I}, {I, J}, {J, F}}

Then φ_*I = {{B, H}, {B, J}, {B, E}, {C, J}, {C, G}, {C, D}, {H, G}, {H, I}, {J, A}, {G, F}, {E, D}, {D, I}, {I, A}, {A, F}, {F, E}}

= III

Therefore, φ is a graph isomorphism and I and III are isomorphic.

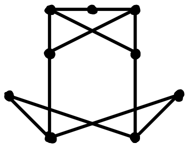
I and II, II and III are not isomorphic.

In II, we can find B-C-H-G-B of length 4.

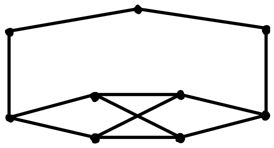
However, in I and III, they do not have circuit of 4.

Therefore, I and II, II and III are not isomorphic.

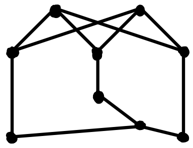
Ex6.



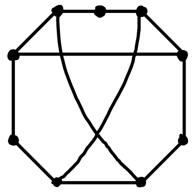
A'



B'

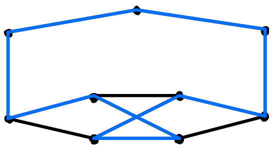


C'

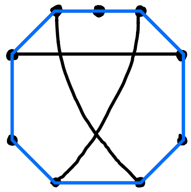


D'

B' and D' have hamiltonian circuit. The circuit is in blue in the following graphs.

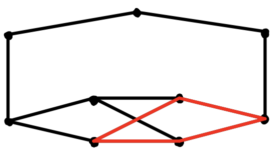


B'



D'

In B', we can find a simple circuit of length 4, show in red, while in D', we cannot have such a circuit. Therefore, B' and D' are not isomorphic.



B'