Ex 1.1 i)

a	b	$a \wedge b$	$\neg(a \wedge b)$	$\neg a$	$\neg b$	$\neg a \vee \neg b$	$\neg(a \wedge b) \Leftrightarrow \neg a \vee \neg b$
T	T	T	F	F	F	F	T
T	F	F	T	F	T	T	T
F	T	F	T	T	F	T	T
F	F	F	T	T	T	T	T

a	b	$a \vee b$	$\neg(a \vee b)$	$\neg a$	$\neg b$	$\neg a \wedge \neg b$	$\neg(a \vee b) \Leftrightarrow \neg a \wedge \neg b$
T	T	T	F	F	F	F	T
T	F	T	F	F	T	F	T
F	T	T	F	T	F	F	T
F	F	F	T	T	T	T	T

ii) $$\begin{aligned} x \in (A \cap B)^c &\Leftrightarrow x \in M \wedge x \notin (A \cap B) \\ &\Leftrightarrow x \in M \wedge \neg[x \in (A \cap B)] \\ &\Leftrightarrow x \in M \wedge \neg(x \in A \wedge x \in B) \\ &\Leftrightarrow x \in M \wedge [\neg(x \in A) \vee \neg(x \in B)] \\ &\Leftrightarrow [x \in M \wedge \neg(x \in A)] \vee [x \in M \wedge \neg(x \in B)] \\ &\Leftrightarrow (x \in M \wedge x \notin A) \vee (x \in M \wedge x \notin B) \\ &\Leftrightarrow x \in (A^c \cup B^c) \end{aligned}$$

Therefore, $(A \cap B)^c = A^c \cup B^c$

$$\begin{aligned} x \in (A \cup B)^c &\Leftrightarrow x \in M \wedge x \notin (A \cup B) \\ &\Leftrightarrow x \in M \wedge \neg[x \in (A \cup B)] \\ &\Leftrightarrow x \in M \wedge \neg(x \in A \vee x \in B) \\ &\Leftrightarrow x \in M \wedge [\neg(x \in A) \wedge \neg(x \in B)] \\ &\Leftrightarrow x \in M \wedge (x \notin A \wedge x \notin B) \\ &\Leftrightarrow (x \in M \wedge x \notin A) \wedge (x \in M \wedge x \notin B) \\ &\Leftrightarrow x \in A^c \cap x \in B^c \\ &\Leftrightarrow x \in (A^c \cap B^c) \end{aligned}$$

Therefore $(A \cup B)^c = A^c \cap B^c$

Ex 1.2 From the truth table in n propositional variables, we know how each variable leads to a true or false proposition. By taking the conjuctions of the variables or their negations, we can ensure that the proposition is true. For different conjunctions of variables to make the proposition be true, we can simpily disconjuct them to get the true compound proposition.

Ex 1.3 i) The logic operators we have learnt are $\{\wedge, \vee, \neg, \Rightarrow, \Leftrightarrow\}$. In order to prove $\{\wedge, \vee, \neg\}$ is a functionally complete collection of logic operators, we need to use $\{\wedge, \vee, \neg\}$ express $\{\Rightarrow, \Leftrightarrow\}$.

For $\Rightarrow$, we know $A \Rightarrow B \equiv \neg A \vee B$

For $\Leftrightarrow$, we know $A \Leftrightarrow B \equiv (A \Rightarrow B) \wedge (B \Rightarrow A) \equiv (\neg A \vee B) \wedge (\neg B \vee A)$

Therefore, $\{\wedge, \vee, \neg\}$ is a functionally complete collection of logic operators

ii) Since $\{\wedge, \vee, \neg\}$ is a functionally complete collection of logic operators, we need to use $\{\wedge, \neg\}$ express $\{\vee\}$.

$$A \vee B \equiv \neg\neg(A \vee B) \equiv \neg(\neg A \wedge \neg B)$$

Therefore, $\{\wedge, \neg\}$ is a functionally complete collection of operators.

iii) Since $\{\wedge, \vee, \neg\}$ is a functionally complete collection of logic operators, we need to use $\{\vee, \neg\}$ express $\{\wedge\}$.

$$A \wedge B \equiv \neg\neg(A \wedge B) \equiv \neg(\neg A \vee \neg B)$$

Therefore, $\{\vee, \neg\}$ is a functionally complete collection of operators.

Ex 1.4 i) $A \oplus B \equiv (A \wedge \neg B) \vee (\neg A \wedge B)$

ii) $A \vee B \equiv [\neg\neg(A \vee B)] \wedge \neg[\neg(A \oplus B) \wedge (A \oplus B)] \equiv [\neg(\neg A \wedge \neg B)] \wedge \neg[\neg(A \oplus B) \wedge (A \oplus B)]$

iii) Since $\vee$ can be expressed with $\{\wedge, \oplus, \neg\}$, the compound proposition with $\vee$ can also be expressed with $\{\wedge, \oplus, \neg\}$. From the definition of functionally complete, as well as Ex 1.3 i), we know every compound proposition can be expressed with $\{\wedge, \vee, \neg\}$. Moreover, $\vee$ can be expressed with $\{\wedge, \oplus, \neg\}$, therefore, we may write $\{\vee, \wedge, \neg\}$ as $\{\wedge, \wedge, \oplus, \neg, \neg\}$, which is $\{\wedge, \oplus, \neg\}$.

Ex 1.5 i)

A	B	$A \wedge B$	$\neg(A \wedge B)$	$A \mid B$
T	T	T	F	F
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

A	B	$A \vee B$	$\neg(A \vee B)$	$A \downarrow B$
T	T	T	F	F
T	F	T	F	F
F	T	T	F	F
F	F	F	T	T

ii) $A \downarrow A \equiv \neg(A \vee A) \equiv \neg A$

$(A \downarrow B) \downarrow (A \downarrow B) \equiv \neg[(A \downarrow B) \vee (A \downarrow B)] \equiv \neg(A \downarrow B) \equiv \neg\neg(A \vee B) = A \vee B$

iii) From the previous question, we know that

$\neg A \equiv A \downarrow A$

$A \vee B \equiv (A \downarrow B) \downarrow (A \downarrow B)$

From Ex 1.3 iii), we know $\{\vee, \neg\}$ is a functionally complete, which means every compound proposition can be expressed with $\{\vee, \neg\}$. Therefore, they can also be expressed with $\{\downarrow\}$. Hence $\{\downarrow\}$ is also functionally complete.

iv) From Ex 1.4 i) we know that

$A \oplus B \equiv (\neg A \wedge B) \vee (A \wedge \neg B)$

$\equiv (\neg A \wedge (A \vee B)) \vee (\neg B \wedge (A \vee B))$

$\equiv \neg (A \vee \neg (A \vee B)) \vee \neg (B \vee \neg (A \vee B))$

$\equiv [A \downarrow \neg (A \vee B)] \vee [B \downarrow \neg (A \vee B)]$

$\equiv [A \downarrow (A \downarrow B)] \vee [B \downarrow (A \downarrow B)]$

$\equiv \neg \neg \{[A \downarrow (A \downarrow B)] \vee [B \downarrow (A \downarrow B)]\}$

$\equiv \neg \{[A \downarrow (A \downarrow B)] \downarrow [B \downarrow (A \downarrow B)]\}$

$\equiv \{[A \downarrow (A \downarrow B)] \downarrow [B \downarrow (A \downarrow B)]\} \downarrow \{[A \downarrow (A \downarrow B)] \downarrow [B \downarrow (A \downarrow B)]\}$

V) $A | A \equiv \neg (A \wedge A) \equiv \neg A$

$(A|B)|(A|B) \equiv \neg [(A|B) \wedge (A|B)] \equiv \neg (A|B) \equiv \neg\neg (A \wedge B) \equiv A \wedge B$

Therefore, $\neg A \equiv A|A$ and $A \wedge B \equiv (A|B)|(A|B)$

From Ex 1.3 ii), we know $\{\wedge, \neg\}$ is a functionally complete, which means every compound proposition can be expressed with $\{\wedge, \neg\}$. Therefore, they can also be expressed with $\{|\}$

Hence $\{|\}$ is also functionally complete.

Ex 1.6 i) $x \in X \triangle Y \Leftrightarrow x \in (X \cup Y) \setminus (X \cap Y)$

$\Leftrightarrow x \in (X \cup Y) \wedge x \notin (X \cap Y)$

$\Leftrightarrow (x \in X \wedge x \notin Y) \vee (x \in Y \wedge x \notin X)$

$\Leftrightarrow (A(x) \wedge \neg B(x)) \vee (B(x) \wedge \neg A(x))$

A	B	¬A	¬B	A∧¬B	B∧¬A	(A∧¬B)∨(B∧¬A)	A⊕B	(A∧¬B)∨(B∧¬A)⇔A⊕B
T	T	F	F	F	F	F	F	T
T	F	F	T	T	F	T	T	T
F	T	T	F	F	T	T	T	T
F	F	T	T	F	F	F	F	T

Therefore $x \in X \triangle Y \Longleftrightarrow A(x) \oplus B(x)$

ii) $X \triangle Y = (X \cup Y) \setminus (X \cap Y)$

$= (X \setminus (X \cap Y)) \cup (Y \setminus (X \cap Y))$

$= [(X \setminus X) \cup (X \setminus Y)] \cup [(Y \setminus X) \cup (Y \setminus Y)]$

$= [\phi \cup (X \setminus Y)] \cup [(Y \setminus X) \cup \phi]$

$= (X \setminus Y) \cup (Y \setminus X)$

iii) $X^c \triangle Y^c = (X^c \setminus Y^c) \cup (Y^c \setminus X^c)$

$= (Y \cap X^c) \cup (X \cap Y^c)$

$= (Y \setminus X) \cup (X \setminus Y)$

$= X \triangle Y$