mathexam — 数学类考试出题宏包*

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摘要

这是按照西南大学考试模版格式, 为数学类考试出题的 X∃LᴬTEX 模版. 它在MiKTeX以及 TeXLive下都能正常工作. 创作过程中, 本模板吸收了暨南大学的考试模版 jnuexam中的代码, 但略有改进. 此外, 还借鉴了 exam 宏包中关于总页码显示的代码. 本模板与它们最大的区别在于使用了 datatool 来计算总题目数、总分, 这为进一步开发依据题库出题奠定了基础.

1 简明使用教程

基本上, 使用本宏包mathexam, 你只需要下载mathexam.sty 并将其放到你的工作目录, 然后在你的主文件中通过\usepackage{mathexam} 即可使用它. 关于使用的实际例子, 你可以参考mathexam-main.tex. 所有这些文件都可以在模版 发布页下载.

最终排版效果可以参考mathexam-main.pdf 以及mathexam-main-answer.pdf.

2 选项、命令以及环境

2.1 基本选项

选项可以通过传递给文档类或者宏包的形式启用.

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bsphack bsphack esphackesphack**showans** 选项实际上是一个开关, 如果没有该选项则不显示答案.

bsphack bsphack esphackesphack**a3paper** 选项是为制作 A3 考试卷子提供的选项, 即将两页 A4 纸打印到一页. 此外, 该选项还会自动生成侧边学生填写信息以及在最后一页添加草稿纸.

bsphack bsphack esphackesphack**fixlast** 该选项只在**a3paper** 选项启用时才有作用, 这是为了修复试卷总页码为奇数页时, 在 A3 纸上显示时的格式错误.

例如:

`\documentclass[showans]{article}` 则表示把**showans** 选项传给**article** 类.

`\usepackage[showans]{mathexam}` 将**showans** 选项传递给**mathexam** 宏包.

2.2　试卷基本信息

`\university`　　　`\university` 命令用来输出学校名称, 它有主参数 (即用`{}` 写的参数) `{`⟨*university name*⟩`}`.

`\school`　　　`\school` 命令用来输出学院名称, 它有主参数 `{`⟨*school name*⟩`}`.

`\course`　　　`\course` 命令用来输出课程名称, 它有主参数 `{`⟨*course name*⟩`}`.

`\AorB`　　　`\AorB` 命令用来设置试卷是 A 卷、B 卷 (当然 C 卷等也可以), 它有一个主参数 `{`⟨*Capital letter*⟩`}`.

`\semester`　　　`\semester` 命令用来设置试卷第几学期, 它有一个主参数 `{`⟨*semester number*⟩`}`. 事实上, 本模版会自动根据出题时间计算正确的学期, 如果不正确可以用 `\semester` 命令修改.

`\finalmiddle`　　　`\semester` 命令用来设置试卷是期中还是期末, 它有一个主参数 `{`⟨*final or middle*⟩`}`.

`\totaltime`　　　`\totaltime` 命令用来设置考试的总时间, 它有一个主参数 `{`⟨*number of minutes*⟩`}`, 表示多少分钟.

`\openclose`　　　`\openclose` 命令用来设置试卷是开卷还是闭卷, 它有一个主参数 `{`⟨*open or close*⟩`}`.

`\degree`　　　`\degree` 命令用来设置考试学生的学位, 本科、硕士研究生、博士研究生等. 它有一个主参数 `{`⟨*degree name*⟩`}`.

`\totalstu`　　　`\totalstu` 命令用来设置学生人数, 它有一个主参数 `{`⟨*number of total students*⟩`}`.

`\major`　　　`\major` 用来设置学生的专业, 它有一个主参数 `{`⟨*major name*⟩`}`.

`\grade`　　　`\grade` 用来设置学生的年级, 它有一个主参数 `{`⟨*grade number*⟩`}`.

\examiner	\examiner 用来设置出题者, 它有一个主参数 {⟨*name of examiner*⟩}.
\director	\director 用来设置教研室主任, 它有一个主参数 {⟨*name of director*⟩}.
\dean	\dean 用来设置主管院长, 它有一个主参数 {⟨*name of dean*⟩}.

2.3 生成试卷头

\makehead \makehead 用来根据以上信息生成试卷的页眉、页脚以及表头. 它不带参数.

2.4 判断题打勾打叉

\ture

\false 这两个命令分别对应于判断题的答案是正确的和错误的.

2.5 填空题的下划线

\fillin \fillin[⟨*space length*⟩]{⟨*answer*⟩} 命令用来出填空题, 它将在答案下面加横线. 它有一个可选参数 [⟨*space length*⟩], 默认为 [⟨*1em*⟩]; 还有一个主参数 {⟨*answer*⟩}, 即填空题的答案.

\fillout \fillout{⟨*answer*⟩} 命令也是用来出填空题, 它也将在答案下面加横线, 与\fillin 的区别是横线将延长到行末.

2.6 选择题的答案

\pickout \pickout 命令用来写选择题的答案, 它有一个主参数 {⟨*captial letter*⟩}, 即答案的字母. 它会自动用点填充题目与答案之间的空隙, 并把答案用括号括起来.

2.7 答案表格

\answertable \answertable 可以为选择题或者填空题生成答题表格, 这方便批阅. 它有一个可选参数 [⟨*height*⟩] 指定答题表格中各行的高度, 默认为 [⟨*1em*⟩]. 另外, 它还有两个主参数 {⟨*total number of answer*⟩}, {⟨*number of answer in each line*⟩}, 即总共的答案个数以及每行的答案个数.

2.8 修改证明题或解答题的答案提示

\solutionname   \solutionname 用来设置解答或证明中的开头文字, 它有一个主参数, {⟨*name of proof*⟩}, 默认为 [⟨解⟩], 你也可以用 \renewcommand{\solutionname}{ } 来修改为 [⟨证⟩].

2.9 评分

\score   \score 命令用来在解答过程中给出评分, 它有一个主参数 {⟨*score number*⟩}, 即一个数字表示给分多少.

2.10 答案隐藏

\answer   \answer 命令可以用来书写答案, 它有一个主参数 {⟨*contents*⟩}, 表示具体的答案内容. 答案将在showans 选项未启用时隐藏.

2.11 辅路数据

\makedata   \makedata{⟨*title*⟩} 用来生成附录标题, 其下面可以写一些用到的公式、数据等.

2.12 草稿纸

\caogaozhi   \caogaozhi 命令没有参数, 它会在a3paper 选项启用时在试卷末尾增加一张草稿纸.

2.13 环境

abcd   abcd 环境用来输出选择题的四个选项, 每个选项用\item 命令来书写, 因此这个环境类似通常的列表环境, 但是会自动根据答案的长度选择排列成四、二、一行.

makepart   makepart 环境会生成每个部分的标题, 它有三个参数, 格式为 [⟨*contents*⟩]{⟨*title*⟩}[⟨*score/question*⟩], 即第一个可选参数为标题的说明, 如果省略, 默认会根据第三个参数是否为零以及本部分小题的个数和总分. 当然你也可以手动指定. 第二个参数就是这部分的标题, 第三个参数默认为0, 若大于零则表示这部分每小题或者每空的分值. 例如选择题、判断题的每小题以及填空题点每空都是一样的分.

如果第三个参数为零, 则此时需要为每个小题指定分数. 具体方法参考problem 环境的使用.

本环境还会在环境结束时根据每小题分值自动计算这部分的小题总数以及这部分的总分, 并利用datatool 宏包写入到数据库, 你可以查看\jobname.dat 文件, 其中记录了具体的数据. 这里\jobname.tex 就是你的主文件. problem 环境用来输出题目, 这包括各种题型. 它有一个可选参数 [⟨*score number*⟩], 表示本小题的分值.

problem

solution
solution 环境用来产生解答题或者证明题的答案, 它有两个可选参数, [⟨*skip height*⟩]、[⟨*solution name*⟩]. 第一个表示在答案所占空白高度的基础上增加或者减少多少高度, 例如 [⟨*10em*⟩] 表示增加 10 行, [⟨*-10em*⟩] 则表示减少 10 行. 第二个可选参数默认为\solutionname, 表示证明或者解答的开头文字.

rmk
rmk 环境是为了在证明或者解答中增加一些注记, 例如不同的解法, 评分说明等. 这在参考答案中会显示出来, 但是不占据试卷的答题空白高度.

3　题型举例

我们将在这节用具体例子说明上面的命令、环境怎么使用.

3.1　判断题

```
\begin{makepart}{      }[2]
\begin{problem}
                              3    4,              5. \true
\end{problem}
\end{makepart}
```

这将生成判断题部分的标题, 而且用小括号说明本部分:(共 1 题, 每题 2 分, 共计 2 分). 每个problem 环境对应着一个小题, \true 这种该小题的答案为真.

3.2　选择题

```
\begin{makepart}{      }[2]
\begin{problem}
                              3    4,              \pickout{C}
\begin{abcd}
```

```
 \item 7;
 \item 6;
 \item 5;
 \item 4.
\end{abcd}
\end{problem}
\end{makepart}
```

这会生成选择题部分的标题, 类似前面的判断题, 会自动使用括号说明本部分小题的总数、分值情况. 每个小题用 problem 环境出题, 其中正确答案用 \pickout 命令输出, 而备选项用 aabcd 环境输出, 每个选项用 \item 输出.

3.3 填空题

```
\begin{makepart}{        }
\begin{problem}[2]
                                       3    4,              \fillin{5}.
\end{problem}
\end{makepart}
```

类似地, 这里给出了一个填空题, 设置了本题的分值为 2 分. 这是因为一个小题往往有多个空, 故没有用统一设置分值的方法. 如果用 \fillout{C} 则答案下的横线将填充到行末.

3.4 计算题

```
\begin{makepart}{        }
\begin{problem}[10]
                                       3    4,              ?
\begin{solution}[10em]
           ,                                        .\score{5}
           $\sqrt{3^3+4^2}=5$.\score{5}
\end{solution}
\end{problem}
\end{makepart}
```

由于没有给定第三个可选参数, `makepart` 环境将生成本部分的标题, 且根据本部分的小题数和总分值, 自动生成标题说明: (共 1 题, 共计 10 分).

每个问题用`problem` 环境给出, 环境后的可选参数 [⟨*10*⟩] 表示本小题 10 分.

相应的答案用`solution` 环境给出, [⟨*10em*⟩] 表示在试卷隐藏答案时, 答案的空白高度将在答案的高度基础上增加`10em`.

最后, 答案中的`\score` 命令表示这步的分值.

3.5 证明题

```
\begin{makepart}{        }
\renewcommand{\solutionname}{  }
\begin{problem}[10]
                              3    4.                  5.
\begin{solution}[10em]
          ,                                    .\score{5}
        $\sqrt{3^3+4^2}=5$.\score{5}
\end{solution}
\end{problem}
\end{makepart}
```

这里和前面计算题完全类似, 只是我们用`\renewcommand{\solutionname}{  }` 更改了本部分的解答开头文字都为 "证".

4   源码参考

```
1 ⟨*package⟩
2 \NeedsTeXFormat{LaTeX2e}[2005/12/01]
3 \ProvidesPackage{mathexam}
4 [2020/06/21 v.2.0.0                    ]
5 \NeedsTeXFormat{LaTeX2e}[1996/06/01]
6 \ProvidesPackage{mathexam}[2020/06/24 A package for create math examination
7 v2.1.0]
8 \RequirePackage{mathtools, amssymb, amsthm}
9 \RequirePackage[contents={}]{background}
10 \RequirePackage{ctex}
```

```latex
11 \RequirePackage{geometry}
12 \RequirePackage{tabularx}
13 \RequirePackage{refcount, fancyhdr}
14
15 \RequirePackage{calc}
16 \RequirePackage{tikzpagenodes}
17 \usetikzlibrary{calc}
18 \RequirePackage{eso-pic}
19 \RequirePackage{etoolbox,xparse,  multido, ifthen}
20 \RequirePackage{zhnumber}
21 \RequirePackage{datatool}
22 \IfFileExists{\jobname.dat}{
23   \DTLloaddb{\jobname}{\jobname.dat}
24   }{
25 \DTLnewdb{\jobname}
26 }
27
28 \newif\ifmathexam@showans\mathexam@showansfalse %
29 \newif\ifsidebyside \sidebysidefalse %       A3
30 \newif\ifmathexam@fixlast\mathexam@fixlastfalse %
31 \DeclareOption{showans}{ \mathexam@showanstrue}
32 \DeclareOption{fixlast}{
33   \mathexam@fixlasttrue
34 }
35 \DeclareOption{a3paper}{\sidebysidetrue}
36
37 \ProcessOptions\relax
38
39 \newcommand{\university}[1]{\def\mathexam@value@university{#1}}
40 \newcommand{\school}[1]{\def\mathexam@value@school{#1}}
41 \newcommand{\course}[1]{\def\mathexam@value@course{#1}}
42 \newcommand{\AorB}[1]{\def\mathexam@value@AorB{#1}}
43 \newcommand{\semester}[1]{\def\mathexam@value@semester{#1}}
44 \newcommand{\finalmiddle}[1]{\def\mathexam@value@finalmiddle{#1}}
45 \newcommand{\totaltime}[1]{\def\mathexam@value@totaltime{#1}}
```

```latex
46 \newcommand{\openclose}[1]{\def\mathexam@value@openclose{#1}}
47 \newcommand{\degree}[1]{\def\mathexam@value@degree{#1}}
48 \newcommand{\totalstu}[1]{\def\mathexam@value@totalstudent{#1}}
49 \newcommand{\major}[1]{\def\mathexam@value@major{#1}}
50 \newcommand{\grade}[1]{\def\mathexam@value@grade{#1}}
51 \newcommand{\examiner}[1]{\def\mathexam@value@examiner{#1}}
52 \newcommand{\director}[1]{\def\mathexam@value@director{#1}}
53 \newcommand{\dean}[1]{\def\mathexam@value@dean{#1}}
54
55 \newcommand{\mathexam@value@semester}{%
56 \ifnum\the\month<9\ifnum\the\month>2 {2}\fi\else {1}\fi%
57 }%
58 \newcommand{\mathexam@lasttwoofyear}[1]{%#1 is the offset
59   \expandafter\mathexam@getlasttwo\number\numexpr\year+(#1)\relax\relax
60 }
61 \def\mathexam@getlasttwo#1#2#3#4\relax{#3#4}
62 \def\tobecalculate{\mbox{??}}
63 \def\totalnumpages{%
64   \@ifundefined{mathexam@totalpages}{%
65     \tobecalculate%
66     }{%
67     \mathexam@totalpages%
68   }%
69 }
70 \newcommand{\mathexam@barfill}{%
71   \leavevmode\xleaders\hb@xt@1em{\hss | \hss }\hfill\kern\z@%
72 }
73 \newcommand{\mathexam@barfilltext}[1]{~\rotatebox[origin=c]{270}{#1}~}
74
75 \newlength\myleft
76 \newlength\myinner
77 \newlength\myouter
78 \newlength\mytop
79 \newlength\mybottom
80 \newlength\myhead
```

```latex
81 \setlength\myleft{.75in}
82 \setlength\myinner{1in}
83 \setlength\myouter{.4in}
84 \setlength\mytop{.2in}
85 \setlength\myhead{.3in}
86 \setlength\mybottom{.6in}
87
88 \newgeometry{
89   top=\mytop,
90   inner=\myinner,
91   outer=\myouter,
92   bottom=\mybottom,
93   headheight=\myhead,
94   includeheadfoot,
95   twoside
96 }
97
98 \newlength\lefttable
99 \setlength{\lefttable}{(\textheight-11\ccwd-\tabcolsep*15)/17}
100
101 \newcommand{\myheaderright}{%
102   \zihao{5}(       \mathexam@value@AorB      \ifmathexam@showans           \fi)
103 }
104 \newcommand{\myheader}{\zihao{5}\mathexam@value@university            }
105 \newlength\headertextlen
106 \newcommand{\makehead}{%
107   \pagestyle{plain} %plain
108   \settowidth{\headertextlen}{\myheader}
109   \ifsidebyside{
110     \AddEverypageHook{%
111       \ifthenelse{\isodd{\value{page}}}%
112       {
113         %% The header line
114         \AddToShipoutPictureBG*{
115          % Add background picture to every page/ *version for current page
```

```
116        \begin{tikzpicture}[overlay,remember picture]
117          \draw [line width=1pt ]
118          ($(current page text area.north west)-(.6\myinner,0pt)$)
119          to
120          ($(current page text area.north east)$);
121          \draw [line width=1pt]
122          ($(current page footer area.south west)-(.6\myinner,0pt)$)
123          to
124          ($(current page footer area.south east)$);
125        \end{tikzpicture}
126      }
127      \newgeometry{
128        top=\mytop,
129        inner=\myinner,
130        outer=\myouter,
131        bottom=\mybottom,
132        headheight=\myhead,
133        includeheadfoot,
134        twoside
135      }
136      \backgroundsetup{
137        color=black,
138        angle=90,
139        scale=1,
140        opacity=1,
141        position={-\myinner+\myleft,-\textheight/2},
142        vshift=8pt,
143        hshift=12.5pt,
144        contents={
145          \begin{tabular}{c|c|c|c|c|c|c|c|c|c|c|c}
146            \hline                  &
147            \hspace*{3\lefttable}  &
148                                    &
149            \hspace*{3\lefttable}  &
150                                    &
```

```
151            \hspace*{2\lefttable}  &
152                                   &
153            \hspace*{2\lefttable}  &
154                                   &
155            \hspace*{3\lefttable}  &
156                                   &
157            \hspace*{3\lefttable} \\
158            \hline
159            \multicolumn{12}{c}{
160              \mathexam@barfill\mathexam@barfilltext{  }
161              \mathexam@barfill\mathexam@barfilltext{  }
162              \mathexam@barfill\mathexam@barfilltext{  }
163            \mathexam@barfill}    \\
164            \hline
165          \end{tabular}
166        }
167      }
168      }{
169      \restoregeometry
170      \AddToShipoutPictureBG*{
171        % Add background picture to every page/ *version for current page
172        \begin{tikzpicture}[overlay,remember picture]
173          \draw [line width=1pt ]
174          ($(current page text area.north west)-0.8*(\myleft,0)$)
175          to
176          ($(current page text area.north east)-0.8*(\myleft,0)$);
177          \draw [line width=1pt]
178          ($(current page footer area.south west)-.8*(\myleft,0)$)
179          to
180          ($(current page footer area.south east)-0.8*(\myleft,0)$);
181        \end{tikzpicture}
182      }
183      }
184      \BgMaterial
185  }}\fi
```

```latex
186    \ifthenelse{\value{page}=1}{
187      %% The header
188      \newgeometry{
189        top=\mytop,
190        inner=\myinner,
191        outer=\myouter,
192        bottom=\mybottom,
193        headheight=\myhead,
194        includeheadfoot,
195        twoside
196      }
197      \begin{center}
198        {\zihao{-2}\heiti\mathexam@value@university\quad
199        \mathexam@value@school}\\[2em]
200        {%
201 \zihao{-3}\heiti  \mathexam@value@course    \quad
202          \mathexam@value@AorB    \ifmathexam@showans         \fi%
203        }\\[2em]
204      \end{center}
205      \zihao{-4}
206      \newcolumntype{P}{>{\centering\arraybackslash}X}
207      \newcolumntype{L}{>{\flushleft\arraybackslash}X}
208      \begin{tabularx}{\linewidth}{|*{12}{P|}}
209        \hline
210        \multicolumn{8}{|c}{
211          20\mathexam@lasttwoofyear{-1}    20\mathexam@lasttwoofyear{0}
212 \qquad   \mathexam@value@semester
213        }                                                        &
214        \multicolumn{4}{|c|}{\mathexam@value@finalmiddle\quad    } \\
215        \hline
216        \multicolumn{2}{|c}{         }                           &
217        \multicolumn{2}{|c}{\mathexam@value@totaltime     }      &
218        \multicolumn{2}{|c}{         }                           &
219        \multicolumn{1}{|c}{\mathexam@value@openclose}           &
220        \multicolumn{2}{|c|}{         }                          &
```

```
221        \mathexam@value@degree                                    &
222                                                                   &
223        \mathexam@value@totalstudent                              \\
224        \hline
225        \multicolumn{3}{|c}{              }                        &
226        \multicolumn{6}{|c|}{\mathexam@value@major}                &
227                                                                   &
228        \multicolumn{2}{c|}{\mathexam@value@grade    }            \\
229        \hline
230                                                                   &
231         &     &     &     &     &     &     &     &     &     \\
232        \hline
233                                                                   &
234         &     &     &     &     &     &     &     &     &     \\
235        \hline
236                                                                   &
237         &     &     &     &     &     &     &     &     &     \\
238        \hline
239     \end{tabularx}
240
241     \noindent \zihao{5}          :                          %
242                                                     %
243                                               0; %
244                                           ;%
245                                     %
246          ; %
247                                                              ;%
248                                                         %
249     \begin{center}
250       \setlength\fboxsep{1em}
251       \setlength\fboxrule{1pt}
252       \fbox{%
253 \zihao{4}\heiti{%
254           :                              %
255                       }
```

14

```
256          }
257      \end{center}
258      \ifnum0\DTLcolumncount{\jobname}>2%
259        \DTLsumforkeys{\jobname}{problem}{\totalproblems}%
260      \else%
261        \let\totalproblems\tobecalculate%
262      \fi%
263      \ifnum0\DTLcolumncount{\jobname}>2%
264        \DTLsumforkeys{\jobname}{score}{\totalscores}%
265      \else%
266        \let\totalscores\tobecalculate%
267      \fi%
268      \noindent\zihao{-4}            %
269      \ifnum0\DTLcolumncount{\jobname}>2%
270        \textbf{\DTLrowcount{\jobname}}%
271      \else%
272        \textbf{\tobecalculate}%
273      \fi%
274              \textbf{\totalnumpages}      \textbf{\totalproblems}          %
275          \textbf{\totalscores}                                            %
276                                                        %
277
278    }\ignorespace}%
279
280  \fancypagestyle{plain}{
281    \ifsidebyside
282      \renewcommand{\headrulewidth}{0pt}
283      \renewcommand{\footrulewidth}{0pt}
284    \else
285      \renewcommand{\headrulewidth}{0.8pt}
286      \renewcommand{\footrulewidth}{0.8pt}
287    \fi
288    \settowidth{\headertextlen}{\myheader}
289    \fancyhf{}
290    \fancyhead[CE]{
```

```
291        \mathexam@value@course              \mathexam@value@AorB
292      \ifmathexam@showans          \fi
293    }
294    \fancyhead[CO]{\makebox[2\headertextlen][s]{\myheader}}
295    \fancyhead[RO]{\ifthenelse{\value{page}=1}{}{\myheaderright}
296    }
297    %% The footer
298    \cfoot{
299      \ifthenelse{\value{page}=1}{%\isodd{\value{page}}
300        \zihao{-5}
301        \begin{tabular*}{\linewidth}{@{\extracolsep{\fill}}clclclc}
302                    :                        &
303          \mathexam@value@examiner            &
304                        :                    &
305          \mathexam@value@director            &
306                :                            &
307          \mathexam@value@dean                &
308          \the\year    \the\month    \the\day    \\
309          \multicolumn{7}{c}{
310            \mathexam@value@AorB    \quad%
311                \thepage    ,    \totalnumpages    %
312          }
313        \end{tabular*}
314        }{
315            \thepage    ,    \totalnumpages    %
316      }
317    }
318 }%
319
320 %% \makedata     /
321 \def\solutionname{   }
322 \newcounter{problem}
323 \newcounter{mypart}
324 \newcounter{score}
325 \newcounter{prescore}
```

```latex
326 \newcounter{totalblanks}
327 \setcounter{mypart}{0}
328 \setcounter{problem}{0}
329 \NewDocumentEnvironment{makepart}{O{%
330     \ifnum0\DTLrowcount{\jobname}<\numexpr\themypart\else%
331     \edtlgetrowforvalue{\jobname}{1}{\themypart}%
332     \dtlgetentryfromcurrentrow{\totalprobinpart}{2}%
333     \dtlgetentryfromcurrentrow{\totalscoreinpart}{3}%
334     \fi%
335      \@ifundefined{totalprobinpart}{\tobecalculate}{\totalprobinpart}      %
336     \edef\blanknamegiven{#2}\edef\blanknameset{        }%
337     \ifnum#3>0   \ifx\blanknamegiven\blanknameset{}   \else{}   \fi{#3}   ,\fi%
338         \@ifundefined{totalscoreinpart}{\tobecalculate}{\totalscoreinpart}    %
339 }%
340   m%
341   O{0}%
342   }{
343   \noindent\par
344   \stepcounter{mypart}
345   \setcounter{problem}{0}
346   \setcounter{score}{0}
347   \setcounter{prescore}{#3}
348   \setcounter{totalblanks}{0}
349   \noindent\zihao{-4}\chinese{mypart}  #2%
350   \if\relax\detokenize{#1}\relax\else(#1)\fi%
351   \par%
352   \phantomsection
353   \addcontentsline{toc}{section}{\chinese{mypart}  #2}
354   }{
355   \ifnum\thetotalblanks>0
356     \addtocounter{score}{\the\numexpr(\thetotalblanks-\theproblem)*\theprescore}
357   \fi
358   \ifnum\DTLrowcount{\jobname}>\numexpr\themypart
359     \dtlexpandnewvalue
360     \edtlgetrowforvalue{\jobname}{1}{\themypart}
```

```latex
361     %this seems weird just because update is not expand the value
362     \dtlremoveentryincurrentrow{1}
363     \dtlappendentrytocurrentrow{mypart}{\themypart}
364     \dtlremoveentryincurrentrow{2}
365     \dtlappendentrytocurrentrow{problem}{\theproblem}
366     \dtlremoveentryincurrentrow{3}
367     \dtlappendentrytocurrentrow{score}{\thescore}
368     \dtlrecombine
369   \else
370     \ifnum\DTLrowcount{\jobname}=\numexpr\themypart
371       \dtlexpandnewvalue
372       \edtlgetrowforvalue{\jobname}{1}{\themypart}
373       %this seems weird just because update is not expand the value
374       \dtlremoveentryincurrentrow{1}
375       \dtlappendentrytocurrentrow{mypart}{\themypart}
376       \dtlremoveentryincurrentrow{2}
377       \dtlappendentrytocurrentrow{problem}{\theproblem}
378       \dtlremoveentryincurrentrow{3}
379       \dtlappendentrytocurrentrow{score}{\thescore}
380       \dtlrecombine
381     \else
382       \DTLnewrow{\jobname}
383       \dtlexpandnewvalue
384       \DTLnewdbentry{\jobname}{mypart}{\themypart}
385       \DTLnewdbentry{\jobname}{problem}{\theproblem}
386       \DTLnewdbentry{\jobname}{score}{\thescore}
387     \fi
388     \fi
389   \setcounter{prescore}{0}
390 \par}
391 \newcommand{\centertext}{%
392   \leavevmode\xleaders\hb@xt@.25em{\hss - \hss }\hfill\kern\z@%
393 }
394 \newcommand{\makedata}[1]{
395   \noindent\centertext~{\heiti\zihao{4}    \quad#1~\centertext}\par
```

```
396    \smallskip\ignorespaces\noindent
397 }
398 %% problem/solution     /
399 \newcounter{choice}
400 \NewDocumentEnvironment{problem}{O{0}}{
401    \setcounter{choice}{0}
402    \stepcounter{problem}
403    \noindent\arabic{problem}.\,\ignorespaces
404    \ifnum#1>0($#1'$)\addtocounter{score}{#1}\fi
405    }{
406    \addtocounter{score}{\theprescore}
407    \par
408 }
409 %% showans
410 \newcommand{\answer}[1]{\ifmathexam@showans#1\else\phantom{#1}\fi}
411
412 %%
413 \newcommand{\cdotfill}{%
414    \leavevmode\xleaders\hbox to 0.5em{\hss$\cdot$\hss}\hfill\kern0pt\relax
415 }
416 \newcommand{\true}{%
417    \unskip\nobreak\cdotfill(\makebox[1.5em]{\answer{$\checkmark$}})%
418 }%
419 \newcommand{\false}{%
420    \unskip\nobreak\cdotfill(\makebox[1.5em]{\answer{\sffamily x}})%
421 }%
422 %%
423 \newcommand{\ulinefill}[1]{%
424    \xleaders\hbox{\uline{\vphantom{#1}\kern1pt}}\hfill\kern0pt%
425 }
426 \newcommand{\fillin}[2][1em]{%
427    \stepcounter{totalblanks}
428    \uline{\hspace{#1}\answer{#2}\hspace{#1}}
429 }
430 \newcommand{\fillout}[1]{%
```

```
431    \stepcounter{totalblanks}
432    \allowbreak\hbox{}\nobreak\ulinefill{#1}\uline{\answer{#1}}\ulinefill{#1}
433 }
434 %%
435 \newcommand{\pickout}[1]{%
436    %\addtocounter{score}{\theprescore}
437    \unskip\nobreak\cdotfill(\makebox[1.5em]{\answer{#1}})
438 }
439 \newlength{\my@item@len}
440 \newcommand\my@item@temp{%
441    \unskip\cr\stepcounter{choice}(\Alph{choice})%
442 }
443 \newcommand\my@item@box{%
444    \hfill\egroup\hfill\hbox to \my@item@len\bgroup
445    \stepcounter{choice}(\Alph{choice})\ignorespaces
446 }
447 \newcommand\my@item@par{%
448    \par\stepcounter{choice}(\Alph{choice})\ignorespaces
449 }
450 \NewDocumentEnvironment{abcd}{+b}{
451    \unskip
452    \setlength{\parindent}{0pt}%
453    \setlength{\parskip}{0pt}%
454    %\setcounter{choice}{0}%
455    \let\item=\my@item@temp
456    \settowidth{\my@item@len}{\vbox{\halign{##\hfil\cr #1\crcr}}}%
457    \setcounter{choice}{0}%
458    \ifdim\my@item@len>0.486\linewidth
459      \setlength{\my@item@len}{\linewidth}%
460      \let\item=\my@item@par
461      #1\par
462    \else
463      \ifdim\my@item@len>.243\linewidth
464        \setlength{\my@item@len}{0.5\linewidth}%
465      \else
```

```
466      \setlength{\my@item@len}{0.25\linewidth}%
467    \fi
468    \let\item=\my@item@box
469    \par\bgroup #1 \hfill\egroup\par
470  \fi
471 }{}
472
473 %% \score
474 \newcommand{\score}[1]{%
475  \ifmmode%
476    \tag*{$\cdots\cdots$(#1\,    )}
477  \else%
478    \cdotfill(#1\,    )\par\noindent
479  \fi
480 }
481
482 \newlength\ansheight
483 \newcounter{cnt}
484 \newcommand{\ansskip}[1]{
485  \setcounter{cnt}{0}
486  \whiledo {\value{cnt} <100}
487  {
488    \vspace*{.01#1}\goodbreak
489    \stepcounter{cnt}
490  }
491 }
492 \newbox{\ansbox}
493 \NewDocumentEnvironment{solution}{O{0em} O{\solutionname} +b}
494 { \savebox{\ansbox}{
495  \parbox[b]{\linewidth}{#3}}
496  \settoheight{\ansheight}{\usebox\ansbox}
497  \ifmathexam@showans
498    \par\noindent\textbf{#2}:~#3\qed\par
499  \else
500    \addtolength{\ansheight}{#1}
```

```
501     \ansskip{\ansheight}
502   \fi
503 }{\par}
504
505 \NewDocumentEnvironment{rmk}{+b}{
506   \ifmathexam@showans
507     \par\noindent\textbf{  }: #1\par
508   \fi
509 }
510
511 %% ----------------------------------------------------------------------
512 %%            \answertable
513 %% ----------------------------------------------------------------------
514
515 \gdef\answer@lines@temp{}%
516 \newcommand{\answer@lines@add}[1]{%
517   \xdef\answer@lines@temp{\answer@lines@temp#1}%
518 }
519
520 \newrobustcmd{\answer@number@hided}[1]{    } %    PDFLaTeX
521 \newrobustcmd{\answer@cell@strut}[1]{\parbox[c][#1][c]{2em}{\hbox{    }}}
522
523 \newcounter{answer@col}
524 \newcounter{answer@row}
525 \newcounter{answer@total}
526
527 \newcommand{\answer@lines}[3]{%
528   % #1
529   % #2
530   % #3
531   \setcounter{answer@row}{(#2-1)/#3+1}%
532   \begingroup
533   \let\hline=\relax  \let\\=\relax %
534   \gdef\answer@lines@temp{}%
535   \setcounter{answer@total}{1}%
```

```
536    \whileboolexpr{%
537      test{\ifnumgreater{\value{answer@row}}{0}}
538    }{%
539    \addtocounter{answer@row}{-1}%
540    \answer@lines@add{\answer@number@hided}%
541    \setcounter{answer@col}{1}%
542    \unlessboolexpr{%
543      test{\ifnumgreater{\value{answer@col}}{#3}}%
544    }{%
545    \answer@lines@add{&}%
546    \ifnumgreater{\value{answer@total}}{#2}{}{%
547        \answer@lines@add{\arabic{answer@total}}%
548      }%
549      \stepcounter{answer@col}%
550      \stepcounter{answer@total}%
551    }%
552    \answer@lines@add{\\ \hline \answer@cell@strut{#1}}%
553    \setcounter{answer@col}{1}%
554    \unlessboolexpr{
555      test{\ifnumgreater{\value{answer@col}}{#3}}
556    }{%
557        \answer@lines@add{&}%
558        \stepcounter{answer@col}%
559      }%
560      \answer@lines@add{\\ \hline}%
561    }%
562    \endgroup
563    \answer@lines@temp
564 }
565
566 \newcommand{\answertable}[3][1em]{%
567    \noindent
568                    ,                    .\par
569    \noindent\begin{tabularx}{\linewidth}{|c|*{#3}{P|}}
570      \hline
```

```latex
571    \answer@lines{#1}{#2}{#3}
572  \end{tabularx}%
573  \par\vspace{0.8em}%
574 }
575
576 \newcommand{\caogaozhi}{%
577  \begin{tikzpicture}[%
578    remember picture,overlay,font=\sffamily\fontsize{100pt}{100pt}\selectfont%
579    ]%
580    \node[text=lightgray!20, rotate=45] at (current page text area.center)%
581    {  \quad   \quad   };
582 \end{tikzpicture}}
583 \ifsidebyside
584  \preto{\@enddocumenthook}{%
585    \clearpage
586    \pagestyle{empty}
587    \caogaozhi
588    \clearpage
589    \caogaozhi
590    \addtocounter{page}{-2}
591  }
592  \RequirePackage{pgfpages}
593  \ifmathexam@fixlast
594    \preto{\@enddocumenthook}{
595      %insert an enpty page for odd total page
596      \clearpage
597      \thinspace
598    }
599  \fi
600  \pgfpagesuselayout{2 on 1}[a3paper, border shrink=5mm,landscape]
601 \fi
602
603 \preto{\@enddocumenthook}{
604  \if@filesw
605    \immediate\write\@mainaux
```

```
606     {\string\gdef\string\mathexam@totalpages{\arabic{page}}}}%
607   \fi
608   \DTLsavedb{\jobname}{\jobname.dat}
609   \DTLsavetexdb{\jobname}{\jobname.dbtex}
610   \IfFileExists{\jobname.dat}{}{\DTLdisplaydb{\jobname}}
611 }
612 \ifmathexam@showans{%
613    \ifnum\shellescape=1
614      \immediate\write18{cp \jobname.pdf \jobname-        .pdf}
615    \else
616      \message{^^J Warning: *********************^^J}
617      \message{^^J      `xelatex -shell-escape \jobname.tex`%
618                       ^^J}
619      \message{^^J *****************************^^J}
620    \fi
621  }
622 \fi
623 ⟨/package⟩
624 ⟨*maintex⟩
625 \documentclass[cs4size]{article}
626 \usepackage[a3paper]{mathexam} %showans
627 \usepackage[colorlinks,linkcolor=cyan,ocgcolorlinks]{hyperref}
628 \usepackage{caption}
629 \input{main}
630 ⟨/maintex⟩
631 ⟨*mainanstex⟩
632 \documentclass[cs4size]{article}
633 \usepackage[showans]{mathexam} %showans
634 \usepackage[colorlinks,linkcolor=cyan,ocgcolorlinks]{hyperref}
635 \usepackage{caption}
636 \input{main}
637 ⟨/mainanstex⟩
638 ⟨*main⟩
639 \university{        }
640 \school{             }
```

```
641 \course{          (A)(2)}
642 \AorB{A}
643 \finalmiddle{    }
644 \totaltime{120}
645 \openclose{    }
646 \degree{    }
647 \totalstu{70}
648 \major{              }
649 \grade{2020}
650 \examiner{}
651 \director{}
652 \dean{}
653 \DeclareMathOperator{\sech}{sech}
654 \DeclareMathOperator{\arctanh}{arctanh}
655 \begin{document}
656 \makehead
657 \begin{makepart}{          }[3]
658   \begin{problem}
659        $f(x,y)=\sqrt{x^2+y^2}$    $(0,0)$   \pickout{B}
660     \begin{abcd}
661     \item        ;
662     \item              ;
663     \item              ;
664     \item
665     \end{abcd}
666   \end{problem}
667   \begin{problem}
668     $f(x,y)=\sqrt{(x-1)^2+y^2}$           $2x+y-1=0$           :\pickout{A}
669     \begin{abcd}
670     \item $\frac{1}{\sqrt{5}}$;
671     \item $\frac{2}{\sqrt{5}}$;
672     \item $1$;
673     \item $2$.
674     \end{abcd}
675   \end{problem}
```

```latex
676    \begin{problem}
677         $D:x^2+y^2\leq2^2$                    $\iint_D\sqrt{4-x^2-y^2}dxdy$      %
678      \pickout{C}
679      \begin{abcd}
680      \item $16\pi$;
681      \item $8\pi$;
682      \item $\frac{16\pi}{3}$;
683      \item $\frac{8\pi}{3}$.
684      \end{abcd}
685    \end{problem}
686    \begin{problem}
687
688      $\Omega=\left\{ (x,y,z): \frac{x^2}{a^2}
689      +\frac{y^2}{b^2}+\frac{z^2}{c^2}\leq1 \right\}$
690                  $\iiint_\Omega\sqrt{1- \frac{x^2}{a^2}
691      -\frac{y^2}{b^2}-\frac{z^2}{c^2}}dV$
692      \pickout{B}
693      \begin{abcd}
694      \item $\frac{\pi^2}{4}$;
695      \item $\frac{\pi^2abc}{4}$;
696      \item $\pi^2$;
697        $\pi^2abc$.
698      \end{abcd}
699    \end{problem}
700    \begin{problem}
701         $\gamma$          $x^2+y^2+z^2=1$          $y=x$           .
702            $\int_\gamma\sqrt{2y^2+z^2}ds$      \pickout{C}
703      \begin{abcd}
704      \item $0$;
705      \item $\pi$;
706      \item $2\pi$;
707      \item $\pi/2$.
708      \end{abcd}
709    \end{problem}
710 \end{makepart}
```

```latex
\begin{makepart}{        }[3]
  \begin{problem}
        $f(u)$                 .        $F(t)=\iiint_{x^2+y^2+z^2\leq t^4}
    f(x^2+y^2+z^2)dxdydz$,      $F'(1)=$\fillin{$8\pi f(1)$}.
  \end{problem}
  \begin{problem}
            $z=z(x,y)$                                .
        $z$         $P(1,1,1)$                $\vec{l}=(1,1)$          $=$\fillin{$-2$}.
  \end{problem}
  \begin{problem}
      $n$                  $f(x_1,\ldots,x_n)=f(r)$
    $\Delta f=\sum_{i=1}^n \frac{\partial^2 f}{\partial x_i^2}$
                  $=$\fillin{$f''(r)+(n-1)f'(r)/r$}.
  \end{problem}
  \begin{problem}
        $z=f(x,y)$        $P(1,1)$            .
      $\lim\limits_{\substack{x\to1\\y\to1}}
    \frac{f(x,y)-x-2y+3}{\ln(1+(x-1)^2+(y-1)^2)}=\pi$,
          $dz|_{(1,1)}=$\fillin{$dx+2dy$}.
  \end{problem}
  \begin{problem}
                $z=2x^2+3y^2-1$          $4x+6y+z-1=0$
    \fillin{4x+6y+z+6=0}.
  \end{problem}
\end{makepart}
\begin{makepart}{       }[5]
  \renewcommand{\solutionname}{  }
  \begin{problem}
                $f(x,y)$
    $\Omega \subset \mathbb{R}^2$                  $P_0\in\Omega$          .
        $\epsilon$-$\delta$                  :    $P_0(0,0)\in \Omega$    $f$%
            $df(P_0)$.
    \begin{solution}[6em]
                $a,b\in \mathbb{R}$,      $df(P_0)=adx+bdy$. \score{1}\\
        $\epsilon$-$\delta$                  :
```

```
746              $\epsilon>0$,            $\delta>0$,
747        $(x,y)\in \mathring{U}(P_0,\delta)$,
748         $0<\sqrt{x^2+y^2}\leq \delta$,\score{3}\\
749              $a,b\in \mathbb{R}$,       :
750      \[
751        \left\lvert \frac{f(x,y)-f(0,0)-ax-by}{\sqrt{x^2+y^2}} \right\rvert
752 \leq\epsilon.
753        \score{2}
754      \]
755    \end{solution}
756  \end{problem}
757  \begin{problem}
758              $f=f(x,y)$, $g=g(x,y)$              $\Omega$           .
759               $f$, $g$                 .
760    \begin{solution}
761                                        :\\
762         $f=f(x,y)$               $\Omega$            ,\score{1}\\
763      $g=g(x,y)$    $\Omega$                .\score{1}\\
764        $g$    $\Omega$          \score{1}\\
765           $(\xi,\eta)\in \Omega$,
766      \[
767        \iint_\Omega f(x,y)g(x,y)dxdy=f(\xi,\eta)\iint_\Omega g(x,y)dxdy.
768        \score{3}
769      \]
770    \end{solution}
771  \end{problem}
772 \end{makepart}
773 \clearpage
774 \begin{makepart}{      }[10]
775  \renewcommand{\solutionname}{  }
776  \begin{problem}
777      $z^2=x^2+y^2$         $x^2+y^2=4x$                      .
778    \begin{solution}
779           ,                                .
780                  ,                 $Oxy$              $(x-2)^2+y^2\leq 2^2$,
```

```
781                          $(x,y,z=\sqrt{x^2+y^2})$,
782        $(x,y)\in D=\left\{ (x,y):(x-2)^2+y^2\leq2^2 \right\}$. \score{2}
783
784
785        \[
786          dS=\sqrt{1+z_x^2+z_y^2}=\sqrt{2}dxdy,\score{3}
787        \]
788             ,
789        \begin{align}
790          I&=2\iint_D \sqrt{2}dxdy\score{3}\\
791          &=2\sqrt{2}\cdot \pi 2^2=8\sqrt{2}\pi. \score{2}
792        \end{align}
793      \end{solution}
794   \end{problem}
795   \begin{problem}
796        $\Sigma$           $ \frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$
797             ,              $I=\iint_\Sigma zdxdy$.
798     \begin{solution}
799                 $\Sigma$                          ,              $\Sigma_+$, $\Sigma_-$.
800           $Oxy$                              $D_{xy}:\frac{x^2}{a^2}+\frac{y^2}{b^2}
801        \leq 1$.
802             ,
803        \[
804          \Sigma_\pm:\left(x,y,z^\pm=\pm c\sqrt{1-\frac{x^2}{a^2}
805          -\frac{y^2}{b^2}}\right), \quad (x,y)\in D_{xy}.\score{2}
806        \]
807                                   $\vec{n}_\pm=(-z^\pm_x,-z^\pm_y,1)$.
808                  ,                 ,                          .
809        \score{(2)}
810
811                              ,
812        \begin{align*}
813          I&=\iint_{\Sigma_+}zdxdy+\iint_{\Sigma_-}zdxdy\\
814          &=\iint_{D_{xy}}(0,0,z^+)\cdot\vec{n_+}dxdy
815          -\iint_{D_{xy}}(0,0,z^-)\cdot\vec{n}_-dxdy\\
```

```
816        &=2\iint_{D_{xy}} c\sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}} dxdy
817        =2abc\iint_{u^2+v^2\leq1}\sqrt{1-u^2-v^2}dudv\score{2}\\
818        &=2abc\int_0^{2\pi}d\theta\int_0^1 r\sqrt{1-r^2}dr
819        =2\pi abc\int_0^1\sqrt{1-t}dt\score{2}\\
820        &=2\pi abc
821 \left. \left( -\frac{2}{3}(1-t)^{3/2} \right) \right\rvert_{t=0}^1
822        =\frac{4\pi abc}{3}.\score{2}
823      \end{align*}
824    \end{solution}
825  \end{problem}
826 \end{makepart}
827 \clearpage
828 \begin{makepart}{      }
829  \renewcommand{\solutionname}{  }
830  \begin{problem}[10]
831              $n$($n\geq3$)        ,      $n$                .
832    \begin{solution}
833                        $n$                $n$      ;\score{2}\\
834                                         $\theta_i$, $i=1,2,\ldots n$,
835        $\theta_i\in(0,\pi)$(          $n\geq3$).        ,
836      \[
837        S=\sum_{i=1}^n \frac{1}{2}R^2\sin\theta_i,\score{1}
838      \]
839        $R>0$          .
840      \[
841        \Theta=\sum_{i=1}^n\theta_i-2\pi=0.\score{1}
842      \]
843
844      \[
845        L(\theta_1,\theta_2,\ldots,\theta_n,\lambda)
846        =S-\lambda\Theta
847        =\sum_{i=1}^n\left( \frac{1}{2}R^2\sin\theta_i
848        -\lambda\theta_i \right)+\lambda2\pi.\score{1}
849      \]
850              $\nabla L=0$
```

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```
851        \[
852          \begin{cases}
853            R^2\cos\theta_i/2-\lambda=0,\quad i=1,2,\ldots,n\\
854            \sum_{i=1}^n\theta_i-2\pi=0.
855          \end{cases}\score{2}
856        \]
857            $\theta_i\in(0,\pi)$,
858        $\cos\theta_i\in(-1,1)$                    .
859        \[
860          \cos\theta_i=2\lambda/R^2, \quad i=1,2,\ldots,n,
861          \implies\theta_1=\theta_2=\cdots=\theta_n.\score{2}
862        \]
863
864        \[
865          \sum_{i=1}^{n}\theta_i=2\pi\implies \theta_1=\theta_2
866  =\cdots=\theta_n=2\pi/n.
867        \]
868                       ,        $n$            $n$      .
869                                              .
870                       .\score{1}
871        \begin{rmk}
872                                  $\theta_i$        ,          .
873        \end{rmk}
874      \end{solution}
875    \end{problem}
876  \end{makepart}
877  \clearpage
878  \begin{makepart}[   1    ,         10    ,      30    ]{      }
879    \begin{problem}[30]
880            ,
881                    $A$            $1$                      $z$
882      $A$              \emph{     }(tractrix).      $z$                      .
883              $z$                          \emph{    }(tractricoid).
884              $Oxy$-                        \emph{    }(pseudosphere),
```

885 -1 ·   ,
886   Hilbert   1901
    %
887   .
888   1693   ·   .
889   1678   (Trait\'e de la Lumi\`ere)
890   .
891   \begin{enumerate}
892   \item $z=z(x)$
893   \begin{equation}\label{eq:tractrix}
894   z'(x)=-\frac{\sqrt{1-x^2}}{x},\quad 0<x\leq 1.
895   \end{equation}
896   \eqref{eq:tractrix}
897   \begin{equation}\label{eq:para-tractrix}
898   x(t)=\sech t,\quad z(t)=t-\tanh t,\quad 0<t<+\infty.
899   \end{equation}
900   ,   .
901
902   ,
903   \[
904   \sech t=\frac{1}{\cosh t}=\frac{2}{e^{t}+e^{-t}},\quad\tanh t
905   =\frac{\sinh t}{\cosh t}=\frac{e^{t}-e^{-t}}{e^{t}+e^{-t}}.
906   \]
907   \item Σ .
908   \item Oxy- Ω .
909   \end{enumerate}
910   \begin{minipage}{\textwidth}
911   %\begin{mpost}
912   % u:=8pt;
913   % vardef exp primary x =(mexp(256)**x) enddef;
914   % %e=2.718;
915   % %vardef exp primary x= (e**x) enddef;
916   % vardef sinh primary x = save xx; xx=exp x; (xx-1/xx)/2 enddef;
917   % vardef cosh primary x= save xx; xx= exp x; (xx+1/xx)/2 enddef;
918   % vardef sech primary x = (1/cosh x) enddef;

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```
919    %  vardef csch primary x = (1/sinh x) enddef;
920    %  vardef tanh primary x =(sech(x)/csch(x)) enddef;
921    %  vardef f primary x = (( sech(x), x-tanh(x))) enddef;
922
923    %  vardef ParametricCurve(suffix f)(expr xmin, xmax, xinc)=
924    %  ( f(xmin)
925    %  for x=xmin+xinc step xinc until xmax:
926    %  hide(show(x); show(f(x));)
927    %  ..f(x)
928    %  endfor )
929    %  enddef;
930
931    %  pickup defaultpen;
932    %  pickup pencircle scaled 1pt;
933
934    %  drawarrow -4u*right--20u*right;
935    %  drawarrow -4u*up--20u*up;
936
937    %  path pat;
938    %  pat=ParametricCurve(f, 0.1, 3.05, 0.25) scaled 10u;
939    %  z0=10u*right;
940    %  t=4.5;
941    %  z1=point t of pat;
942    %  z2=(origin--20u*up) intersectionpoint%
943    %  (z1--(z1+10u*(direction t of pat)));
944
945    %  draw pat withcolor blue;
946    %  draw z1--z2 withcolor red;
947    %  undraw origin--z0;
948    %  draw origin--z0 withcolor red;
949
950    %  pickup defaultpen;
951    %  pickup pencircle scaled 3pt;
952    %  dotlabel.urt(btex $A$ etex, z0);
953    %  dotlabel.urt(btex $A$ etex, z1);
```

```
954        %   dotlabel("",z2);
955        %   label.bot(btex $x$ etex, 20u*right);
956        %   label.rt(btex $z$ etex, 20u*up);
957        %   dotlabel.llft(btex $O$ etex, origin);
958        %\end{mpost}
959        \hfill\includegraphics[scale=.8]{tractrix}\\
960        %\hfill\captionof{figure}{     }\label{fig:tractrix}
961     \end{minipage}
962     \begin{solution}
963       \begin{enumerate}
964          \item           ,              $A$           .        $A=(x,z)$,
965        $A$
966            \[
967              z'(x)=-\frac{\sqrt{1-x^2}}{x},\quad 0<x\leq1.
968            \]
969                        $z=z(x)$                \eqref{eq:tractrix}. \score{4}
970
971           $z(t)=z(x(t))$,
972          \begin{align*}
973            z'(t)&=z'(x)x'(t)=-\frac{\sqrt{1-[x(t)]^2}}{x(t)}\cdot x'(t)\\
974               &=-\sqrt{\cosh^2 t-1}\cdot \frac{-\sinh t}{\cosh^2t}
975               =\frac{\sinh^2 t}{\cosh^2t}=\tanh^2t.
976          \end{align*}
977                 ,
978            \[
979              z'(t)=1-\tanh't=1-\frac{1}{\cosh^2t}
980              =\frac{\sinh^2 t}{\cosh^2t}=\tanh^2t.
981            \]
982              , \eqref{eq:para-tractrix}                      \eqref{eq:tractrix}.
983          \score{2}
984
985              $x(0)=1, z(0)=0$               ,
986          \eqref{eq:para-tractrix}                    . \score(2)
987
988              ,                      $z=z(x)$      $z$                 .
```

```latex
        \[
          \begin{cases}
            x=\sech t\cos\theta,\\
            y=\sech t\sin\theta,\\
            z=t-\tanh t,
          \end{cases}\quad
          0\leq t<+\infty,\quad 0\leq \theta\leq 2\pi
          \score{2}
        \]
    \item
        \[
          X(r,\theta)=\left(r\cos\theta,r\sin\theta,z(r)\right),
          0<r\leq1, 0\leq \theta\leq 2\pi,
        \]
            ,
        \[
          dS=\lvert X_r\times X_\theta \rvert drd\theta
          =r\sqrt{1+[z'(r)]^2}drd\theta,\score{5}
        \]
            ,
        \[
          S=\int_0^{2\pi}d\theta\int_0^1 \lvert X_r\times X_\theta \rvert dr
          =2\pi\int_0^1r\sqrt{1+[z'(r)]^2}dr. \score{3}
        \]

          \eqref{eq:tractrix},
        \[
          \sqrt{1+z'^2(r)}=1/r,\implies S= 2\pi. \score{2}
        \]

        \textbf{     }:                 (a)
        \[
          X(t,\theta)
    =\left( \sech t\cos\theta,\sech t\sin\theta,t-\tanh t \right),
```

```
1024                \score{2}
1025          \]
1026
1027          \[
1028            dS=\lvert X_t\times X_\theta \rvert dt d\theta
1029            =\sech t\tanh t dt d\theta
1030            =\frac{\sinh t}{\cosh^2t} dt d\theta.\score{5}
1031          \]
1032              , $\Sigma$
1033          \begin{align*}
1034            S&=\int_0^{2\pi}d\theta\int_0^{+\infty}\sech t\tanh tdt \score{3}\\
1035             &=-2\pi\int_0^{+\infty} d(\sech t)
1036              =2\pi.\score{2}
1037          \end{align*}
1038        \item    \eqref{eq:para-tractrix}              ,
1039          \[
1040            X(t,\theta)=(\sech t\cos\theta,\sech t\sin\theta,t-\tanh t),\quad
1041            t\in[0,+\infty),\quad\theta\in[0,2\pi].
1042          \]
1043              ,
1044          \[
1045            J=\frac{\partial(x,y)}{\partial(t,\theta)}
1046    =-\sech^2 t\tanh t.\score{2}
1047          \]
1048
1049          \begin{align*}
1050            \lvert \Omega \rvert
1051            &=\iint_{x^2+y^2\leq 1}zdxdy
1052            =\int_0^{2\pi}d\theta\int_0^{+\infty}
1053            (t-\tanh t)\sech^2 t\tanh t dt,\score{5}\\
1054            \frac{\lvert \Omega \rvert}{2\pi}
1055            &=\int_0^1 (u\arctanh u-u^2) du,
1056            \quad(u=\tanh t, du=\sech^2 tdt)\\
1057            &=-\frac{1}{3}+\frac{1}{2}\int_0^1\arctanh u du^2\\
1058            &=-\frac{1}{3}+\frac{1}{2}
```

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```
1059      \left( \left. u^2\arctanh u \right\rvert_{u=0}^1
1060            -\int_0^1 \frac{u^2}{1-u^2}du \right),\quad
1061      (\arctanh'u=\frac{1}{1-u^2})\\
1062            &=-\frac{1}{3}+\frac{1}{2}
1063      \left( \left. u^2\arctanh u \right\rvert_{u=0}^1
1064        +1-\frac{1}{2}\int_0^1\left( \frac{1}{1-u}+\frac{1}{1+u} \right)
1065      du \right)\\
1066            &=\frac{1}{6}+\frac{1}{2}
1067      \left. \left( u^2\arctanh u
1068        -\frac{1}{2}\left( -\ln(1-u)+\ln(1+u) \right) \right)
1069      \right\rvert_{u=0}^1\\
1070            &=\frac{1}{6}+\frac{1}{2}\lim_{u\to1^-}\left( u^2\arctanh u
1071            -\sqrt{\frac{1-u}{1+u}} \right).\score{3}
1072          \end{align*}
1073             $u=\tanh t$,
1074          \begin{align*}
1075            \lim_{u\to1^-}\left(u^2\arctanh u+\ln\sqrt{\frac{1-u}{1+u}}\right)
1076        &=\lim_{t\to+\infty}\left(t\tanh^2 t
1077        +\ln\sqrt{\frac{1-\tanh t}{1+\tanh t}}\right)\\
1078        &=\lim_{t\to+\infty}\left( t\frac{(e^{t}-e^{-t})}{e^{t}+e^{-t}}
1079        -\ln(\cosh t+\sinh t) \right)\\
1080        &=\lim_{t\to+\infty}(\frac{t(e^{t}-e^{-t})}{e^{t}+e^{-t}}-t)=0.
1081          \end{align*}
1082
1083          \[
1084            \lvert \Omega \rvert=\pi/3.\score{2}
1085          \]
1086      \end{enumerate}
1087    \end{solution}
1088  \end{problem}
1089 \end{makepart}
1090 \end{document}
1091 ⟨/main⟩
```
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5 版本历史

v1.0.0

    General: 初始版本 1

v1.1.0

    General: 增加测验宏包; 改进 ans 环境 . . 1

v2.0.0

    General: SWU 版本 1

6 索引

    斜体数字表示相应条目描述的页码, 而下划线的数字表示表示相应条目定义的页码. 使用条目的页码用罗马数字表示.