Ma2201/CS2022 Quiz 0011

Discrete Mathematics

A Term, MMXVII

Print Name: ____________________

Sign: ____________________

1. (**6 points**) Let X, Y, and Z be sets. Prove that $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$ using the double inclusion method.

♣ *We first show* $X \cap (Y \cup Z) \subseteq (X \cap Y) \cup (X \cap Z)$. *Let* $a \in X \cap (Y \cup Z)$, *so* $a \in X$ *and* $a \in Y \cup Z$. *Since* $a \in Y \cup Z$ *there are two cases.*

If $a \in Y$, *then* $a \in X \cap Y$. *Since* $a \in X \cap Y$, $a \in (X \cap Y) \cup (X \cap Z)$, *as required.*

If $a \in Z$, *then* $a \in X \cap Z$. *Since* $a \in X \cap Z$, $a \in (X \cap Y) \cup (X \cap Z)$, *as required.*

In either case, $a \in (X \cap Y) \cup (X \cap Z)$, *so* $X \cap (Y \cup Z) \subseteq (X \cap Y) \cup (X \cap Z)$.

Next we show $(X \cap Y) \cup (X \cap Z) \subseteq X \cap (Y \cup Z)$.

Let $b \in (X \cap Y) \cup (X \cap Z)$, *so there are two cases.*

If $b \in X \cap Y$ *then* $b \in X$ *and* $b \in Y$. *Since* $b \in Y$, $b \in Y \cup Z$, *and so* $b \in X \cap (Y \cup Z)$, *as required.*

If $b \in X \cap Z$ *then* $b \in X$ *and* $b \in Z$. *Since* $b \in Z$, $b \in Y \cup Z$, *and so* $b \in X \cap (Y \cup Z)$, *as required.*

So in either case, $b \in X \cap (Y \cup Z)$ *and* $(X \cap Y) \cup (X \cap Z) \subseteq X \cap (Y \cup Z)$.

Since $X \cap (Y \cup Z) \subseteq (X \cap Y) \cup (X \cap Z)$ *and* $(X \cap Y) \cup (X \cap Z) \subseteq X \cap (Y \cup Z)$, *we have proved* $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$ ♣

2. (**4 points**) Let $D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $E = \{0, 2, 4, 6, 8\}$ and $O = \{1, 3, 5, 7, 9\}$, and let them be ordered as usual in $\mathbb{N}$. Let $E \times D \times O$ be ordered lexicographically.

a) What are the 5 elements following $(5, 0, 5)$ in $E \times D \times O$?

♣ *[What's the matter with you?* $(5, 0, 5) \notin E \times D \times O$! *Oh, true, use* $(4, 0, 5)$.*] The five elements following* $(4, 0, 5)$ *are* $(4, 0, 7), (4, 0, 9), (4, 1, 1), (4, 1, 3), (4, 1, 5)$. ♣

b) What is the 101'st element of $E \times D \times O$ in lexicographic order (starting from the 0'th, $(0, 0, 1)$)?

♣ *To determine the rightmost digit, divide* 101 *by* $|O| = 5$: $101 \div 5 = 20\ R\ 1$, *so the rightmost digit is* 3.

To determine the middle digit, divide 20 *by* $|D| = 10$: $20 \div 10 = 2\ R\ 0$, *so the middle digit is* 0.

To determine the leftmost digit, divide 2 *by* $|E| = 5$: $2 \div 5 = 0\ R\ 2$, *so the leftmost digit is* 4.

It is $(4, 0, 3)$.

♣