Lectures 20 and 21 – Number Theory II

Summary

These lectures were on modular arithmetic

We solved the beer barrel problem by distinguishing between "threven", "throdd", and "thweird" integers.

We defined $\mathbb{Z}_n = \{0, 1, 2, \ldots, n-1\}$, where $k \in \mathbb{Z}_n$ refers to the set $k = \{i \in \mathbb{Z} \mid i = j \cdot n + k\}$.

We defined addition, subtraction, multiplication, and, if possible, division in $\mathbb{Z}_n$.

We showed how to use the Euclidean Algorithm to find multiplicative inverses in $\mathbb{Z}_n$, and showed that every non-zero element of $\mathbb{Z}_p$ has a multiplicative inverse if p is prime.

Exercises on Lectures 20 and 21

1. Billy and Bobby will get a prize if the dolls they knock down sum to exactly 50.

How many different combinations of dolls would get them the prize?

[Hint: The guy in the checkered suit is not a doll.]

2. Fill in addition and multiplication tables for $\mathbb{Z}_{12}$.

   List the elements with multiplicative inverses modulo 12.

3. $7 \cdot 5$ modulo 12.

4. Compute 2^{10}, 2^{100}, and 2^{1000} modulo 12.

5. Solve $5x = 7$ modulo 12.

6. Fill in addition and multiplication tables for $\mathbb{Z}_{13}$.

   List the elements with multiplicative inverses modulo 13.

7. $7 \cdot 5$ modulo 13.

8. Compute 2^{10}, 2^{100}, and 2^{1000} modulo 13.

9. Solve $5x = 7$ modulo 13.

10. Define a relation R on $\mathbb{Z}$ by setting $(n, m) \in R$ if $3 \mid n - m$.

    Show that R is an equivalence relation.

    Describe the equivalence classes.

11. Compute the multiplicative inverse of 13 modulo 1776.

12. Compute the multiplicative inverse of 1776 modulo 1999.

13. Solve $5x \equiv 7$ modulo 1999.

14. Solve $7x \equiv 5$ modulo 1999.

15. Solve $8x \equiv 25$ modulo 1999.

16. Solve $16x \equiv 10$ modulo 1999.