1a)

$$\text{1a)}\quad y^{(1)} = Ax^{(0)} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$$

$$\|y^{(1)}\|_\infty = 4 \quad \text{so} \quad x^{(1)} = \frac{y^{(1)}}{4} = [0.75, 0.25, 1]^T$$

$$y^{(2)} = Ax^{(1)} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 0.75 \\ 0.25 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.75 \\ 2.25 \\ 3 \end{bmatrix}$$

$$\|y^{(2)}\|_\infty = 3 \quad \text{so} \quad x^{(2)} = \frac{y^{(2)}}{3} = [0.916667, 0.75, 1]^T$$

$$y^{(3)} = Ax^{(2)} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 0.916667 \\ 0.75 \\ 1 \end{bmatrix} = \begin{bmatrix} 3.58333 \\ 3.41667 \\ 3.66667 \end{bmatrix}$$

$$\|y^{(3)}\|_\infty = 3.66667 \quad \text{so} \quad x^{(3)} = \frac{y^{(3)}}{3.66667} = [0.97727, 0.93181, 1]^T$$

1b) I inputted the following, which can be found in the HW6_script.m file, into the power method function found in power_method.m:

```
A1=[2 1 1; 1 2 1; 1 1 2];
x0=[1; -1; 2];
tol=0.0001;
maxiter=150;
[lambda,v,iter] = power_method(A1,x0,tol,maxiter)
```

And got the following output:

```
>> HW6_script

lambda =

   3.999908450059507


v =

   0.999994277997757
   0.999982833993271
   1.000000000000000


iter =

     9
```

1c)

1c) $\lambda_1 = 3.999908$ $\quad V^{(1)} = [0.999994, 0.999983, 1]^T$

$x = \frac{1}{3.999908} \cdot [2, 1, 1]^T = [0.500011, 0.250006, 0.250006]^T$

$V^{(1)} x^T = [0.999994, 0.999983, 1]^T \cdot [0.500011, 0.250006, 0.250006]$

$$= \begin{bmatrix} 0.500008 & 0.250005 & 0.250005 \\ 0.500002 & 0.250002 & 0.250002 \\ 0.500011 & 0.250006 & 0.250006 \end{bmatrix}$$

$$B = A - \lambda_1 V^{(1)} x^t = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} - 3.999908 \begin{bmatrix} \; \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ -0.999964 & 1.00002 & 0.000016 \\ -0.999998 & -0.000001 & 0.999999 \end{bmatrix} \quad \text{so} \quad B' = \begin{bmatrix} 1.00002 & 0.000016 \\ -0.000001 & 0.999999 \end{bmatrix}$$

Plug into matlab for power method

I inputted the following, which can be found in the HW6_script.m file, into the power method function found in power_method.m:

```
A2=[1.00002 0.000016; -0.000001 0.999999];
x0=[1; 1]; %choose nonzero initial vector
tol=0.0001;
maxiter=150;
[lambda,v,iter] = power_method(A2,x0,tol,maxiter)
```

And got the following output:

```
>> HW6_script

lambda =

   1.000036000000000

v =
   1.000000000000000
   0.999962001367951


iter =

     1
```

$\lambda_2 = 1.600036$ and $(B' - 3I)w^{(2)'} = 0$, $w^{(2)'} = [1, 0.999962]^T$

$w^{(2)'} = [0, 1, 0.999962]^T$

$v^{(2)} = (\lambda_2 - \lambda_1) w^{(2)} + \lambda_1 (x^t w^{(2)}) v^{(1)}$

$= (1.600036 - 3.999908)([0, 1, 0.999962]^T)$

$+ 3.999908 [[0.500011, 0.250006, 0.250006] \cdot [0, 1, 0.999962]^T]$

$\cdot [0.999994, 0.999983, 1]^T = [-2.999908, 2.999908, 0]^T$

2)

Iteration 1

2. $P = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ so $P \cdot A^{(1)} = \begin{bmatrix} 4\cos\theta - \sin\theta & -\cos\theta + 3\sin\theta & -\sin\theta \\ -4\sin\theta - \cos\theta & \sin\theta + 3\cos\theta & -\cos\theta \\ 0 & -1 & 2 \end{bmatrix}$

$-4\sin\theta - \cos\theta = 0 \Rightarrow \tan\theta = -\frac{1}{4} \Rightarrow \sin\theta = \frac{-\sqrt{17}}{17}$ and $\cos\theta = \frac{4\sqrt{17}}{17}$

Substitute into P matrix to get rotation matrix:

$P_2 = \begin{bmatrix} \frac{4\sqrt{17}}{17} & -\frac{\sqrt{17}}{17} & 0 \\ \frac{\sqrt{17}}{17} & \frac{4\sqrt{17}}{17} & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow$ Now find $P_2 A^{(1)} \Rightarrow \begin{bmatrix} \sqrt{17} & -\frac{7\sqrt{17}}{17} & \frac{\sqrt{17}}{17} \\ 0 & \frac{11\sqrt{17}}{17} & -\frac{4\sqrt{17}}{17} \\ 0 & -1 & 2 \end{bmatrix} = A_2^{(1)}$

This A_2 matrix has the form: $\begin{bmatrix} z_1 & q_1 & r_1 \\ 0 & x_2 & y_2 \\ 0 & b_3 & a_3 \end{bmatrix}$ so $P_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_3 & \sin\theta_3 \\ 0 & -\sin\theta_3 & \cos\theta_3 \end{bmatrix}$

$\sin\theta_3 = \frac{b_3}{\sqrt{x_2^2 + b_3^2}}$ and $\cos\theta_3 = \frac{x_2}{\sqrt{x_2^2 + b_3^2}}$ where $x_2 = \frac{11\sqrt{17}}{17}$, $b_3 = -1$

$= -0.3509$ $\qquad = 0.9363$

Substitute into

$P_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.9363 & -0.3509 \\ 0 & 0.3509 & 0.9363 \end{bmatrix}$

$R^{(1)} = A_3^{(1)} = P_3 \cdot A_2^{(1)} = \begin{bmatrix} \ \end{bmatrix} \cdot \begin{bmatrix} \sqrt{17} & -\frac{7\sqrt{17}}{17} & \frac{\sqrt{17}}{17} \\ 0 & \frac{11\sqrt{17}}{17} & -\frac{4\sqrt{17}}{17} \\ 0 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 4.1231 & -1.69775 & 0.2425 \\ 0 & 2.8488 & -1.6191 \\ 0 & 0 & 1.5321 \end{bmatrix}$

$$Q^{(1)} = P_2^T \cdot P_3^T = \begin{bmatrix} \frac{4\sqrt{17}}{17} & 0.227086 & 0.085106 \\ \frac{-\sqrt{17}}{17} & 0.908344 & 0.340423 \\ 0 & -0.3509 & 0.9363 \end{bmatrix}$$

$$A^{(2)} = R^{(1)} \cdot Q^{(1)} = \begin{bmatrix} 4.592920 & -0.472934 & 0 \\ -0.472934 & 3.108760 & -0.232083 \\ 0 & -0.232083 & 1.298319 \end{bmatrix}$$

2nd iteration

Plug $A^{(2)}$ into MatLab to find $R^{(2)}$ and $Q^{(2)}$

I inputted the following, which can be found in the HW6_script.m file, into the power method function found in power_method.m:

```
A3=[4.592920 -0.472934 0; -0.472934 3.108760 -0.232083; 0 -0.232083
1.298319];
[Q,R]=qr(A3)
```

And got the following output:

```
>> HW6_script

Q(2) =

  -0.994740355236037  -0.102132215405104   0.007786927592403
   0.102428636937547  -0.991861644073824   0.075623100639199
                   0   0.076022954373109   0.997106067782352


R(2) =

  -4.617204857352119   0.788872584529170  -0.023771945346377
                   0  -3.052801642790319   0.328896272040326
                   0                   0   1.277010916751468
```