1a.

(a) $f(x) = e^x - x^2 + 3x - 2 = 0$ for $0 \le x \le 1$ cont. ✓

$a_1 = 0$ and $b_1 = 1$ so $f(a_1) = -2$ $f(b_1) = 2.71828$

Condition Satisfied ✓

$\rightarrow$ So, $a_2 = a_1 = 0$

$b_2 = p_1 = 0.5$

$p_1 = \frac{0+1}{2} = 0.5$

$f(0.5) = 0.898721 > 0$ like $f(b_1)$

$p_2 = \frac{0+0.5}{2} = 0.25$

$f(0.25) = -0.028475 < 0$ like $f(a_2)$

$\rightarrow$ $a_3 = p_2 = 0.25$

$b_3 = b_2 = 0.5$

$p_3 = \frac{0.5+0.25}{2} = 0.375$

1b. Please look at the file named bisection.m to see the code and use HW7_script.m to run it.

I put in the following input:

```
f=@(x) -x.^2+exp(x)+3*x-2;
a=0;
b=1;
tol=0.00001;

p=bisection(f,a,b,tol)
```

And got the following output:

```
>> HW7_script

p =

   0.257530212402344


iter =

    17
```

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2.

2 (i) $x^3-2x+1=0$

$2x = x^3+1$

$x = \frac{1}{2}(x^3+1)$

$x=g(x) \rightarrow g(x)=\frac{1}{2}(x^3+1)$

$g(\frac{1}{2}) = \frac{1}{2}((\frac{1}{2})^3+1)$

$= \boxed{0.5625 = p_1}$

$g(0.5625) = \frac{1}{2}((0.5625)^3+1)$

$= \boxed{0.588989 = p_2}$

$g(0.588989) = \frac{1}{2}((0.588989)^3+1)$

$= \boxed{0.602163 = p_3}$

$g(0.602163) = \frac{1}{2}((0.602163)^3+1)$

$= \boxed{0.609172 = p_4}$

(ii) $x^3-2x+1=0$

$x^3 = 2x-1$

$x = \frac{2x-1}{x^2}$

$x = \frac{2}{x} - \frac{1}{x^2}$

$x=g(x) \rightarrow g(x)=\frac{2}{x}-\frac{1}{x^2}$

$g(\frac{1}{2}) = \frac{2}{0.5} - \frac{1}{0.5^2}$

$= \boxed{0 = p_1}$

$g(0) = \frac{2}{0} - \frac{1}{0^2}$ → undef.

Not an appropriate method because approx. is undefined

(iii) $x^3-2x+1=0$

$x^3 = 2x-1$

$x^2 = \frac{2x-1}{x} = 2-\frac{1}{x}$

$x = \pm\sqrt{2-\frac{1}{x}}$

$x=g(x) \rightarrow g(x) = +\sqrt{2-\frac{1}{x}}$

$g(0.5) = \sqrt{2-\frac{1}{0.5}}$

$= \boxed{0 = p_1}$

$g(0) = \sqrt{2-\frac{1}{0}}$ ← undef.

Not an appropriate method because approximation is undefined

(iv) $x^3-2x+1=0$

$x^3 = 2x-1$

$x = \pm\sqrt[3]{1-2x}$

$x=g(x) \rightarrow g(x) = -\sqrt[3]{1-2x}$

$g(0.5) = -\sqrt[3]{1-2(0.5)}$

$= \boxed{0 = p_1}$

$g(0) = -\sqrt[3]{1-2(0)}$

$= \boxed{-1 = p_2}$

$g(-1) = -\sqrt[3]{1-2(-1)}$

$= \boxed{-1.44225 = p_3}$

$g(-1.44225) = -\sqrt[3]{1-2(-1.44225)}$

$= \boxed{-1.57197 = p_4}$

→ Most appropriate methods are parts (i) and (iv).

3a.

3a) $f(x) = x^3+3x^2-1=0$ for $-3 \le x \le -2$

$f'(x) = 3x^2+6x$ Start with $p_0 = -2.5$

$p_1 = -2.5 - \frac{f(-2.5)}{f'(-2.5)} = -2.62319$

$p_2 = -2.62319 - \frac{f(-2.62319)}{f'(-2.62319)} = -2.70672$

3b. Please look at the file named newtons.m to see the code and use HW7_script.m to run it.

I put in the following input:

```
f=@(x) x.^3+3*x.^2-1;
deriv=@(x) 3*x.^2+6*x;
x0=-2.5;
```

```
tol=0.00001;
iter=25;

[p,maxiter]=newtons(f,deriv,x0,tol,iter)
```

And got the following output:

```
p =

  Columns 1 through 4

  -3.066666666666666  -2.900875603864734  -2.879719904423836  -
2.879385324669270

  Column 5

  -2.879385241571822 <- answer


maxiter =

     4
```

When inputting a different initial guess such as -3, the following was outputted:

```
p =

  -2.888888888888889  -2.879451566951567  -2.879385244836671
-2.879385241571816 <- answer


maxiter =

     3
```

So changing the initial guess did change the answer because for Newton's method, the whole answer depends on the initial p0 value that was chosen. The closer the value to the actual solution, the faster and more accurate the results.