Homework 2, Due: Thursday, 1/24

This assignment is due on **Thursday, January 24**, by 11:59 PM. Your assignment should be well-organized, typed (or neatly written and scanned) and saved as a .pdf for submission on Canvas. You must show all of your work to receive full credit. For problems requiring the use of MATLAB code, remember to include your output in the main body of your solution .pdf and to also submit your .m-files on Canvas as a part of your completed assignment. Your code should be appropriately commented to receive full credit.

Problems

$\boxed{1}$ In lecture we discussed the number of floating-point operations (i.e., multiplications/divisions and additions/subtractions) required to compute Gaussian elimination and back substitution for solving the linear system

$$\begin{aligned} E_1 : \; a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= a_{1,n+1} \\ E_2 : \; a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= a_{2,n+1} \\ &\vdots \\ E_n : \; a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n &= a_{n,n+1} \end{aligned}$$

Consider instead the following hybrid elimination algorithm: First, apply Gaussian (forward) elimination to reduce the system to upper-triangular form. Then perform reverse (backward) elimination, starting with the nth equation and eliminating the coefficients multiplying x_n in each of the first $n-1$ rows. After this is completed, use the $(n-1)$st equation to eliminate the coefficients multiplying x_{n-1} in the first $n-2$ rows, and so on. The system will eventually appear as the reduced diagonal system

$$\left[\begin{array}{cccc|c} \widehat{a}_{11} & 0 & \cdots & 0 & \widehat{a}_{1,n+1} \\ 0 & \widehat{a}_{22} & \ddots & \vdots & \widehat{a}_{2,n+1} \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & \widehat{a}_{nn} & \widehat{a}_{2,n+1} \end{array}\right]$$

and the solution can be obtained by setting

$$x_i = \frac{\widehat{a}_{i,n+1}}{\widehat{a}_{ii}}$$

for each $i = 1, 2, \ldots, n$.

(a) (6 points) Show that this method requires

$$\frac{n^3}{3} + \frac{3n^2}{2} - \frac{5n}{6} \quad \text{multiplications/divisions}$$

and

$$\frac{n^3}{3} + \frac{n^2}{2} - \frac{5n}{6} \quad \text{additions/subtractions.}$$

(b) (4 points) Make a table comparing the required operations for Gaussian elimination + back substitution (Algorithm 6.1) and the hybrid method for $n = 3, 10, 50, 100$. Note that you may write a MATLAB code to compute your table entries.

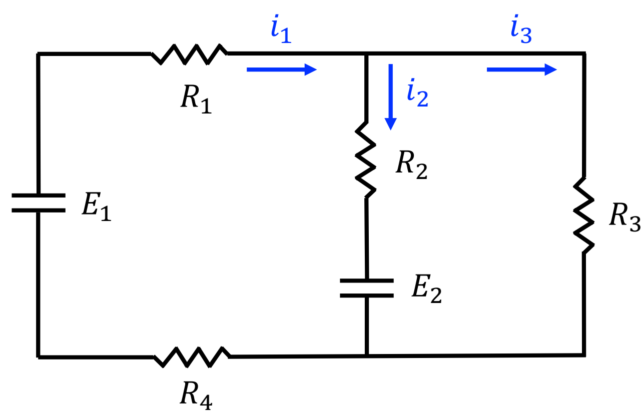
(c) (6 points) Write a MATLAB **function** implementing the hybrid method for solving linear systems. Name your function `hybrid.m`. List the input and output variables of your function, and insert comments to describe what each line of the code does. Note that you may use the `gauss_elim.m` function posted on Canvas as a starting point for modification.

Demonstrate that your `hybrid.m` function works by using it to solve the linear system

$$\begin{aligned} x_1 + x_2 + 3x_4 &= 4 \\ 2x_1 + x_2 - x_3 + x_4 &= 1 \\ 3x_1 - x_2 - x_3 + 2x_4 &= -3 \\ -x_1 + 2x_2 + 3x_3 - x_4 &= 4 \end{aligned}$$

which has the solution $x_1 = -1$, $x_2 = 2$, $x_3 = 0$, and $x_4 = 1$.

$\boxed{2}$ The following circuit has four resistors and two voltage sources. The resistors are R_1, R_2, R_3, and R_4 ohms; the voltage sources are E_1 and E_2 volts; and the currents are i_1, i_2, and i_3 amps.



Kirchhoff's laws give rise to the linear system

$$\begin{aligned} (R_1 + R_4)i_1 + R_2 i_2 &= E_1 + E_2 \\ (R_1 + R_4)i_1 + R_3 i_3 &= E_1 \\ i_1 - i_2 - i_3 &= 0 \end{aligned}$$

(a) (4 points) Use Gaussian elimination without pivoting to find i_1, i_2, and i_3 when $E_1 = 12$ volts, $E_2 = 10$ volts, $R_1 = 2$ ohms, $R_2 = 2$ ohms, $R_3 = 4$ ohms, and $R_4 = 1$ ohm.

(b) (4 points) If the resistances are changed to $R_1 = 0.001$ ohms, $R_2 = 3.333$ ohms, $R_3 = 4.002$ ohms, and $R_4 = 0.012$ ohms, find the currents i_1, i_2, and i_3 using Gaussian elimination and 3-digit chopping arithmetic.

(c) (6 points) Repeat part (b) using Gaussian elimination with partial pivoting and three-digit chopping arithmetic. Does partial pivoting improve the answer to part (b)? Explain why or why not.

$\boxed{3}$ (6 points) Suppose that

$$\begin{aligned} 2x_1 + x_2 + 3x_3 &= 1 \\ 4x_1 + 6x_2 + 8x_3 &= 5 \\ 6x_1 + \alpha x_2 + 10x_3 &= 5 \end{aligned}$$

with $|\alpha| < 10$. For which of the following values of α will there be no row interchange required when solving this system using scaled partial pivoting?

(a) $\alpha = 6$

(b) $\alpha = 9$

(c) $\alpha = -3$

Show all work, and explain your reasoning.

Note: For any of the above problems for which you use MATLAB to help you solve, you must submit your code/.m-files as part of your work. Your code must run in order to receive full credit. If you include any plots, make sure that each has a title, axis labels, and readable font size, and include the final version of your plots as well as the code used to generate them.