Homework 1 11/01/18

1a) $f(x) = e^x\cos(x)$ and $x_0 = 0$

$P_2(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2}(x-x_0)^2$

$P_2(x) = f(0) + f'(0)(x-0) + \frac{f''(0)}{2}(x-0)^2$

$f(0) = e^0 \cdot \cos(0) = 1$

$f'(x) = e^x\cos(x) - e^x\sin(x) = e^x(\cos x - \sin x)$

$f'(0) = e^0((\cos(0) - \sin(0)) = 1$

$f''(x) = e^x\cos(x) - e^x\sin(x) - (e^x\sin(x) + e^x\cos(x))$

$= e^x\cos(x) - e^x\sin(x) - e^x\sin(x) - e^x\cos(x) = -2e^x\sin(x)$

$f''(0) = -2e^0\sin(0) = 0$ $\quad f'''(x) = -2e^x(\sin x + \cos x)$

So, $P_2(x) = 1 + x + 0 = x + 1$

$P_2(0.5) = 1.5$ and $f(0.5) = 1.447$

Absolute error: $|f(0.5) - P_2(0.5)| = 0.053$

$R_2(x) = \frac{f^{(3)}(\xi(x))}{3!}\cdot(x-x_0)^3$ for $\xi(x) \in (0, 0.5)$

$= |-2e^{\xi(x)}(\sin(\xi(x)) + \cos(\xi(x)))/48|$ for $\xi(x) \in (0, 0.$

$\leq 2e^{0.5}(\sin 0.5 + \cos 0.5)/48 \approx 0.0932$

$\Rightarrow$ The upper bound derived using the error term for polynomials is larger than the absolute error.

1b) $\int_0^1 (x+1)dx \quad \frac{x^2}{2} + x\Big|_0^1 = \frac{1}{2} + 1 = \frac{3}{2} \approx 1.5$ so $\int_0^1 f(x)dx \approx 1.5$

$\int_0^1 |-2e^{\xi}(\sin\xi + \cos\xi)\cdot x^3/3!|\,dx = \frac{1}{3}\int_0^1 e^{\xi}(\sin\xi + \cos\xi)\cdot x^3\,dx$

$\leq \int_0^1 \frac{1}{3}\max|e^{\xi}(\sin\xi + \cos\xi)|\,|x^3|\,dx$

$\leq \int_0^1 \frac{e(\sin(1) + \cos(1))}{3}|x^3|\,dx$

$\leq \frac{e(\sin(1) + \cos(1))}{3}\int_0^1 |x^3|\,dx$

$\leq \frac{e(\sin(1) + \cos(1))}{3}\cdot\frac{(1)^4}{4} \approx 0.313$

Absolute error: $|\int_0^1 f(x)dx - \int_0^1 P_2(x)\,dx| = |\int_0^1 f(x)dx - 1.5|$

$\int_0^1 e^x\cos x\,dx = \frac{1}{2}e^x(\sin x + \cos x)\Big|_0^1$

$= \frac{1}{2}(e^1(\sin 1 + \cos 1) - e^0(\sin 0 + \cos 0)) = 1.378$

$|1.378 - 1.5| = 0.122$

2a) $p = e^{10} = 22026.4657948$

Normalized Decimal Form: $0.220264657948 \times 10^5$

6-digit chopping: 0.220264×10^5

6-digit rounding: 0.220265×10^5

(2b) Actual error $= p - p^* = 22026 - 22000$
$= 26$

Absolute error $= |p - p^*| = |22026 - 22000|$
$= 26$

Relative error $= \frac{|p - p^*|}{|p|}, p \neq 0 \Rightarrow \frac{26}{22026}$
$= 0.00118042$

(3a) The definition says that for a large n and a positive constant K, $|\alpha_n - \alpha| < Kn^{-p}$. Since q is less than p, we know that $n^{-p} < n^{-q}$. So, $Kn^{-p} < Kn^{-q}$ so $|\alpha_n - \alpha| < Kn^{-q}$ and therefore $\alpha_n = \alpha + O(n^{-q})$.

(3b)

	$1/n$	$1/n^2$	$1/n^3$	$1/n^4$
$n=$ 5	0.2	0.04	0.008	0.0016
10	0.1	0.01	0.001	0.0001
100	0.01	0.0001	0.000001	1×10^{-8}
1000	0.001	0.000001	1×10^{-9}	1×10^{-12}

The most rapid convergence is $O(\frac{1}{n^4})$

(4a) The first >> puts the entered numbers into a matrix called "A". The Size'(A) finds the dimensions of matrix A which is 3x5. A' will find the transpose of matrix A which will flip the dimensions of A so that the new matrix is 5x3.

a different row is indicated by the semicolons

(4b) A*B will produce a 2x3 matrix which is $\begin{bmatrix} 2 & 0 & 4 \\ 0 & 3 & 1 \end{bmatrix}$

inv(A) is the inverse matrix of A which is $\begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix}$

If you enter inv(B), there is an error because in order to take the inverse, the original matrix must be a square matrix.

4c. Format short rounds the answers to 4 decimal places (rounds $\frac{4}{3}$ to 1.3333 and $1.2345e^{-6}$ to 0.0031). Format long displays the answers to 15 decimal places. Format short e shows the answers in Euler's e form ($1.3333e^{0}$ and $3.06e^{-3}$). Format rat shows the answers in a fraction form ($\frac{4}{3}$ and $\frac{44}{14379}$).

4d. length(x) tells you the length of the largest array dimension. In this case, the length of the vector x is the number of elements which is 11. x(3) is the 3rd element in vector x which is 2. The for loop is computing the log of each of the elements in the vector x.

5. The algorithm and the MatLab command "eps" produced the same exact output even when I compared them using the "format long" command. The output was $2.2204e^{-16}$.