Homework 6, Due: Thursday, 12/13

This assignment is due on **Thursday, December 13**, by 11:59 PM. Your assignment should be well-organized, typed (or neatly written and scanned) and saved as a .pdf for submission on Canvas. You must show all of your work to receive full credit. For problems requiring the use of MATLAB code, remember to also submit your .m-files on Canvas as a part of your completed assignment. Your code should be appropriately commented to receive full credit.

Problems

$\boxed{1}$ (6 points) Consider the initial value problem

$$y' = \frac{2}{t}y + t^2e^t, \qquad 1 \le t \le 2, \qquad y(1) = 0.$$

Use Theorem 5.4 to show that the initial value problem has a unique solution.

$\boxed{2}$ Consider Euler's method (Algorithm 5.1, pg. 267 of the text) for solving the initial value problem

$$y'(t) = f(t, y), \qquad a \le t \le b, \qquad y(a) = \alpha.$$

(a) (6 points) Write a MATLAB **function** implementing Euler's method. Name your function `eulers.m`. List the input and output variables of your function, and insert comments to describe what each line of the code does.

(b) (6 points) Use your function from part (a) to approximate the solution to

$$y' = \frac{2 - 2ty}{t^2 + 1}, \qquad 0 \le t \le 1, \qquad y(0) = 1$$

with $h = 0.5$, $h = 0.25$, and $h = 0.1$.

(c) (6 points) Compare your approximations from part (b) to the true solution

$$y(t) = \frac{2t + 1}{t^2 + 1}$$

by computing the absolute error at each mesh point t_i for each h. Make a plot of the absolute errors in each case, and discuss your results.

Note: For any of the above problems for which you use MATLAB to help you solve, you must submit your code/.m-files as part of your work. Your code must run in order to receive full credit. If you include any plots, make sure that each has a title, axis labels, and readable font size, and include the final version of your plots as well as the code used to generate them.