Problem 1:

$P(C=1 \mid R=1, H=1, M=0)$

Rule

Bayes Rule $= \dfrac{P(H=1 \mid C=1, R=1, M=0) \cdot P(C=1 \mid R=1, M=0)}{P(H=1 \mid R=1, M=0)}$

Law of Conditional Probability $= \dfrac{P(H=1 \mid C=1, M=0) \cdot P(C=1 \mid R=1, M=0)}{P(H=1 \mid R=1, M=0)}$

Bayes Rule Variant $= \dfrac{P(R=1 \mid C=1, M=0) \cdot P(C \mid M) \cdot P(H=1 \mid C=1, M=0)}{P(R \mid M) \cdot P(H=1 \mid R=1, M=0)}$

Bayes Rule AND $P(C \mid M) = P(C)$ $= \dfrac{P(R=1 \mid C=1, M=0) \cdot P(C=1) \cdot P(H=1 \mid C=1, M=0)}{\cancel{P(R \mid M)} \cdot \frac{P(R \mid H, M) \cdot P(H \mid M)}{\cancel{P(R \mid M)}}}$

Law of Total Probability $= \dfrac{P(R=1 \mid C=1, M=0) \cdot P(C=1) \cdot P(H=1 \mid C=1, M=0)}{\frac{P(R=1)}{P(M=0)}}$

Calculations:

$P(R=1) =$ Law of total probability

$P(R=1 \mid C=0, M=0) + P(R=1 \mid C=1, M=0) = 0.9 + 0.05 = 0.95$

$P(M=0) = 1 - P(M=1)$

$= 1 - \dfrac{P(M=1 \mid V=1) \cdot P(V=1)}{P(V=1 \mid M=1)} \rightarrow = 1$

$= 1 - (0.0001)(0.999)$

$= 0.999$

Final answer $= \dfrac{(0.9) \cdot (0.05) \cdot (0.6)}{\frac{0.999}{0.95}} = \dfrac{0.027}{1.052} = 0.0256$

Problem 2: $P(M=1 \mid H=1, C=0) =$

Rule

$= \dfrac{P(H=1 \mid C=0, M=1) \cdot P(M=1 \mid C=0)}{P(H=1 \mid C=0)}$ Variant of Bayes Rule

$= \dfrac{P(H=1 \mid C=0, M=1) \cdot P(M=1)}{P(H=1 \mid C=0)}$ $P(M \mid C) = P(M)$

Calculations:

$P(M=1) = \dfrac{P(M=1 \mid V=1) \cdot P(V=1)}{P(V=1 \mid M=1)}$

Bayes Rule

$1 = P(V=1 \mid M=1)$

$= 0.999 \cdot 0.0001$

$P(H=1 \mid C=0) =$

$P(H=1 \mid C=0, M=1) +$

$P(H=1 \mid C=0, M=0)$

$= 0.93 + 0.02 = 0.95$

Final answer $= \dfrac{0.98 \cdot 0.0001 \cdot 0.999}{0.95} = 0.000103$