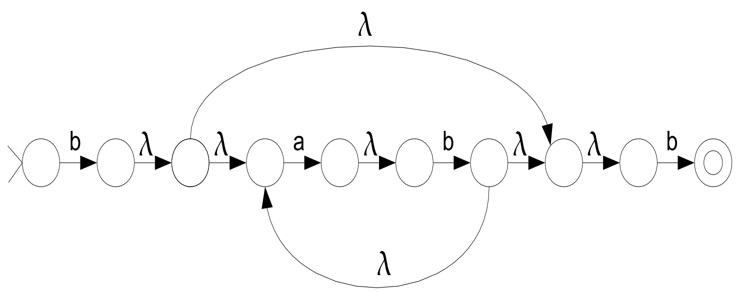
CS 3133 Foundations of Computer Science
C term 2018

Solutions for Homework 4

1. Use the technique from Section 6.1 in the book (i.e. constructing an NFA-λ for a given regular set by following the recursive definition of the regular set) to build the state diagram of an NFA-λ that accepts the language $\boldsymbol{b}(\boldsymbol{ab})^*\boldsymbol{b}$.

   Solution:

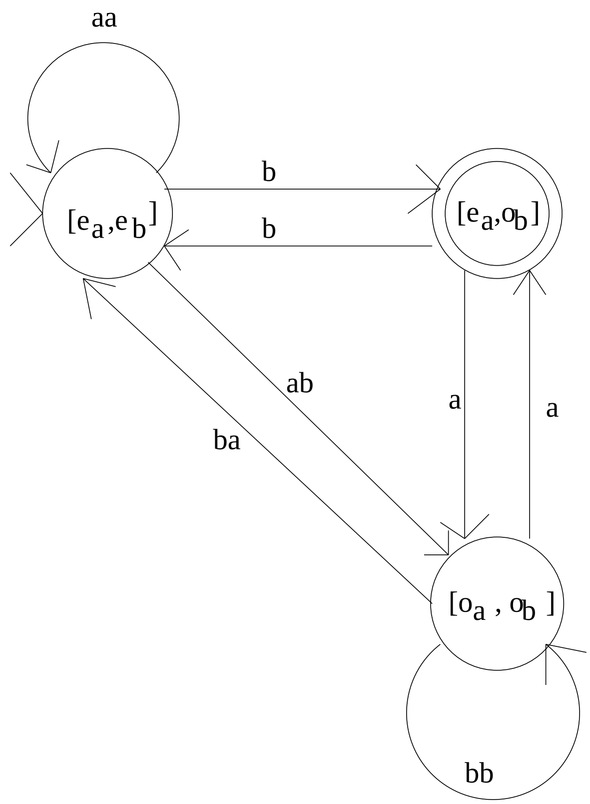
   

   (20 points)

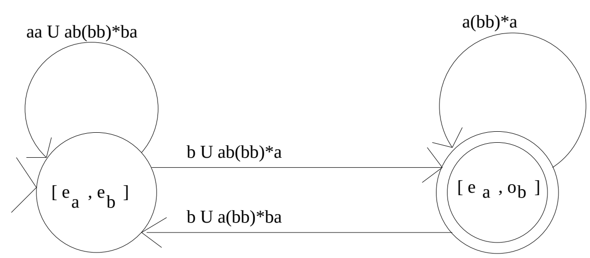
2. Exercise 3 on page 217. (There is an error in the book, it should say: The language of the DFA M in Example 5.3.7 consists of...)

   Solution:

   After deleting the state $[o_a, e_b]$ we get the following expression graph:



Finally after deleting the only remaining state $[o_a, o_b]$ that is not the starting state and the accepting state we get the final expression graph:



Thus in the second figure on page 195 we have

$$\boldsymbol{u} = \boldsymbol{aa} \cup \boldsymbol{ab}(\boldsymbol{bb})^*\boldsymbol{ba}, \boldsymbol{v} = \boldsymbol{b} \cup \boldsymbol{ab}(\boldsymbol{bb})^*\boldsymbol{a}, \boldsymbol{w} = \boldsymbol{a}(\boldsymbol{bb})^*\boldsymbol{a} \quad \text{and}$$

$$\boldsymbol{x} = \boldsymbol{b} \cup \boldsymbol{a}(\boldsymbol{bb})^*\boldsymbol{ba},$$

and the regular expression is

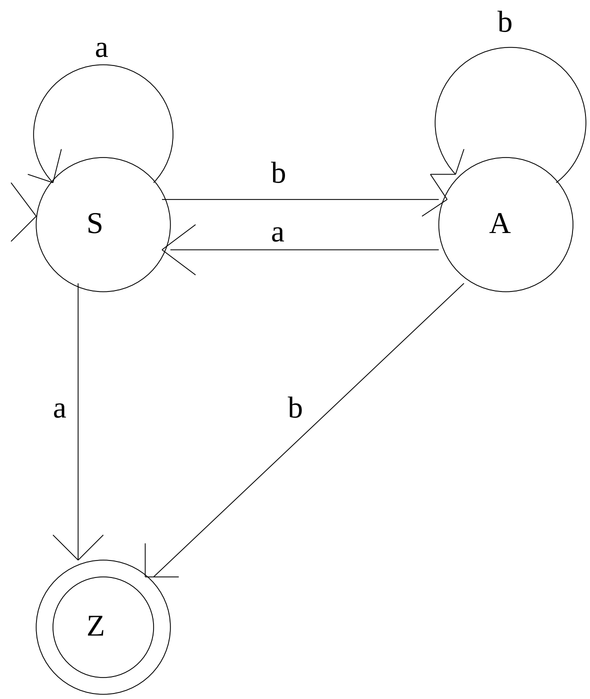
$$\boldsymbol{u}^*\boldsymbol{v}(\boldsymbol{w} \cup \boldsymbol{x}(\boldsymbol{u})^*\boldsymbol{v})^*.$$

(20 points)

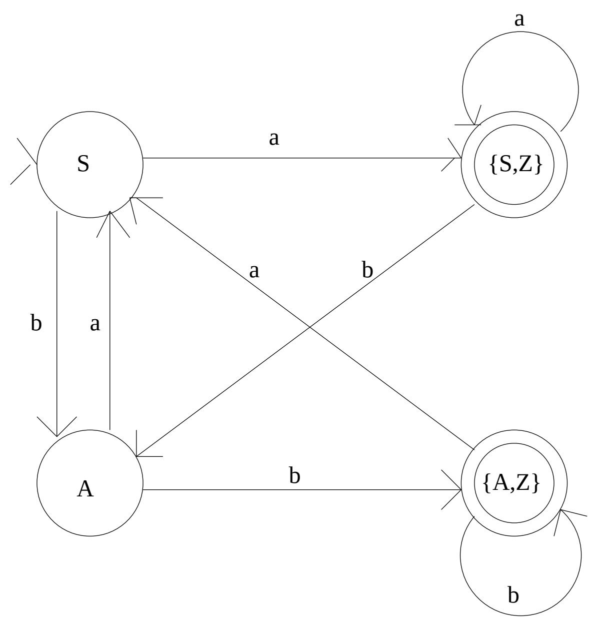
3. Exercise 4 on page 217.

   Solution:

   a.)



   b.)



c.)

$$\begin{aligned} &S \to aS|bA|aZ \\ &A \to bA|aS|bZ \\ &Z \to \lambda \end{aligned}$$

d.)

$$\begin{aligned} &S \to bA|aB \\ &A \to aS|bC \\ &B \to aB|bA|\lambda \\ &C \to aS|bC|\lambda \end{aligned}$$

where $\{S, Z\} = B$ and $\{A, Z\} = C$.

e.) $(\boldsymbol{a} \cup \boldsymbol{b}^+\boldsymbol{a})^*(\boldsymbol{a} \cup \boldsymbol{b}^+\boldsymbol{b})$.

(20 points)

4. Exercise 14.b. on page 218.

Solution: (with the pumping lemma) Let us assume indirectly that the language $L = \{a^n b^m | n < m\}$ is regular. This implies that L is accepted by some DFA. Let k be the number of states of the DFA. By the pumping lemma, every string $z \in L$ of length k or more can be decomposed into substrings u, v and x such that $length(uv) \leq k$, $length(v) > 0$ and $uv^ix \in L$ for all $i \geq 0$.

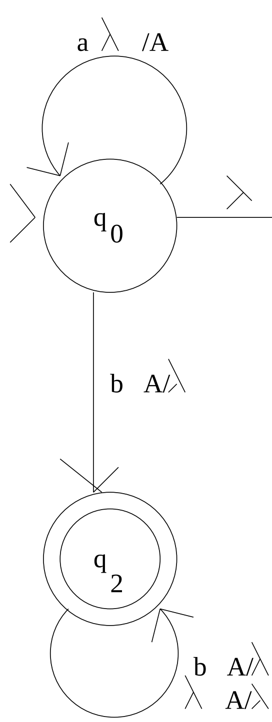
Consider the string $z = a^kb^{k+1}$. Clearly $z \in L$ and $length(z) \geq k$. Using the pumping lemma we decompose z into substrings u, v and x, where $0 < length(uv) \leq k$. Then v is a substring of the first a^k. But in this case uv^2x cannot be in L, since we have at least as many a's as b's, a contradiction. L is non-regular. (20 points)

5. Exercise 1 on page 247.

   Solution:

   a.) $L(M) = \{a^ib^j | i \geq j \geq 0\}$.

   b.)



   c.) Here are *all* computations for the string aab:

$$
\begin{array}{llll}
\quad [q_0, aab, \lambda] & \quad [q_0, aab, \lambda] & \quad [q_0, aab, \lambda] & \quad [q_0, aab, \lambda] \\
\vdash [q_0, ab, A] & \vdash [q_1, aab, \lambda] & \vdash [q_0, ab, A] & \vdash [q_0, ab, A] \\
\vdash [q_0, b, AA] & reject & \vdash [q_1, ab, A] & \vdash [q_0, b, AA] \\
\vdash [q_2, \lambda, A] & & \vdash [q_1, ab, \lambda] & \vdash [q_1, b, AA] \\
\vdash [q_2, \lambda, \lambda] & & reject & \vdash [q_1, b, A] \\
accept & & & \vdash [q_1, b, \lambda] \\
 & & & reject
\end{array}
$$

For the string abb:

$$
\begin{array}{lll}
\quad [q_0, abb, \lambda] & \quad [q_0, abb, \lambda] & \quad [q_0, abb, \lambda] \\
\vdash [q_0, bb, A] & \vdash [q_1, abb, \lambda] & \vdash [q_0, bb, A] \\
\vdash [q_2, b, \lambda] & reject & \vdash [q_1, bb, A] \\
reject & & \vdash [q_1, bb, \lambda] \\
 & & reject
\end{array}
$$

For the string aba:

$$
\begin{array}{lll}
\quad [q_0, aba, \lambda] & \quad [q_0, aba, \lambda] & \quad [q_0, aba, \lambda] \\
\vdash [q_0, ba, A] & \vdash [q_1, aba, \lambda] & \vdash [q_0, ba, A] \\
\vdash [q_2, a, \lambda] & reject & \vdash [q_1, ba, A] \\
reject & & \vdash [q_1, ba, \lambda] \\
 & & reject
\end{array}
$$

d.)

$$
\begin{array}{l}
\quad [q_0, aabb, \lambda] \\
\vdash [q_0, abb, A] \\
\vdash [q_0, bb, AA] \\
\vdash [q_2, b, A] \\
\vdash [q_2, \lambda, \lambda] \\
accept
\end{array}
$$

$$
\begin{aligned}
& [q_0, aaab, \lambda] \\
\vdash\ & [q_0, aab, A] \\
\vdash\ & [q_0, ab, AA] \\
\vdash\ & [q_0, b, AAA] \\
\vdash\ & [q_2, \lambda, AA] \\
\vdash\ & [q_2, \lambda, A] \\
\vdash\ & [q_2, \lambda, \lambda] \\
& \textit{accept}
\end{aligned}
$$

(20 points)