Homework 6, Due: Thursday, 2/21

This assignment is due on **Thursday, February 21**, by 11:59 PM. Your assignment should be well-organized, typed (or neatly written and scanned) and saved as a .pdf for submission on Canvas. You must show all of your work to receive full credit. For problems requiring the use of MATLAB code, remember to include your output in the main body of your solution .pdf and to also submit your .m-files on Canvas as a part of your completed assignment. Your code should be appropriately commented to receive full credit.

Problems

1 Consider the matrix

$$\mathsf{A} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

(a) (6 points) Find the first three iterations obtained by applying the Power method to A with $\mathbf{x}^{(0)} = (1, -1, 2)^\mathsf{T}$. At each iteration, give the approximate value of the dominant eigenvalue and corresponding estimate of the associated eigenvector.

(b) (4 points) Continue using the Power method with $\mathbf{x}^{(0)} = (1, -1, 2)^\mathsf{T}$ to approximate the most dominant eigenvalue of A. Iterate until a tolerance of 10^{-4} is achieved or until the number of iterations exceeds 25. You may do this numerically using `power_method.m`. Report your solution (approximated eigenvalue and associated eigenvector) and the number of iterations it took the algorithm to converge.

(c) (4 points) Use Wielandt deflation and the results of part (b) to approximate the second most dominant eigenvalue of A. Iterate until a tolerance of 10^{-4} is achieved or until the number of iterations exceeds 25. Once you find the deflated matrix, you may do this numerically using `power_method.m`. Report your solution (approximated eigenvalue and associated eigenvector) and the number of iterations it took the algorithm to converge.

2 (6 points) Apply two iterations of the QR method without shifting to the following matrix:

$$\mathsf{A} = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

Note: For any of the above problems for which you use MATLAB to help you solve, you must submit your code/.m-files as part of your work. Your code must run in order to receive full credit. If you include any plots, make sure that each has a title, axis labels, and readable font size, and include the final version of your plots as well as the code used to generate them.