

Homework 7, Due: Thursday, 2/28

This assignment is due on **Thursday, February 28**, by 11:59 PM. Your assignment should be well-organized, typed (or neatly written and scanned) and saved as a .pdf for submission on Canvas. You must show all of your work to receive full credit. For problems requiring the use of MATLAB code, remember to include your output in the main body of your solution .pdf and to also submit your .m-files on Canvas as a part of your completed assignment. Your code should be appropriately commented to receive full credit.

Problems

- [1] Consider the root-finding problem of solving $e^x - x^2 + 3x - 2 = 0$ for $0 \leq x \leq 1$.
- (a) (4 points) Denoting by p_1 the midpoint of the initial interval, use the Bisection method to analytically (by hand) find the root estimate p_3 .
- (b) (4 points) **BONUS:** Use the Bisection method to find a solution accurate to within 10^{-5} . Do this numerically by writing a MATLAB code to perform the Bisection method iterations. Submit any code/.m-files used as part of your work.
- [2] (4 points) Let $f(x) = x^3 - 2x + 1$. To solve $f(x) = 0$, consider the following four fixed-point problems $x = g(x)$ with $p_0 = \frac{1}{2}$:
- | | |
|---|---|
| (i) $x = g_1(x) = \frac{1}{2}(x^3 + 1)$ | (iii) $x = g_3(x) = \sqrt{2 - \frac{1}{x}}$ |
| (ii) $x = g_2(x) = \frac{2}{x} - \frac{1}{x^2}$ | (iv) $x = g_4(x) = -\sqrt[3]{1 - 2x}$ |

Derive each fixed-point method, and compute p_1 , p_2 , p_3 , and p_4 for each method. Which methods seem to be appropriate? Explain.

- [3] Consider the root-finding problem of solving $x^3 + 3x^2 - 1 = 0$ for $-3 \leq x \leq -2$.
- (a) (4 points) Starting with $p_0 = -2.5$, use Newton's method to analytically (by hand) find the root estimate p_2 .
- (b) (4 points) **BONUS:** Starting with $p_0 = -2.5$, use Newton's method to find a solution accurate to within 10^{-5} . Do this numerically by writing a MATLAB function to perform the Newton's method iterations. Submit any code/.m-files used as part of your work. Does changing your initial guess p_0 affect the results of the algorithm? Discuss.

Note: For any of the above problems for which you use MATLAB to help you solve, you must submit your code/.m-files as part of your work. Your code must run in order to receive full credit. If you include any plots, make sure that each has a title, axis labels, and readable font size, and include the final version of your plots as well as the code used to generate them.