In lectures 5 and 6 we continued to discuss sets.
We discussed the double inclusion method of showing two sets are equal.
We discussed the cartesian products of sets and Lexicographic order.
We discussed Russel's Paradox.

1. Let $Y = \{1, 2\}$.

   Label each of the following as true or false.

   a. $\mathcal{P}(Y) \subseteq \mathcal{P}(\mathcal{P}(Y))$.

   b. $\emptyset \subseteq \mathcal{P}(\mathcal{P}(Y))$.

   c. $\emptyset \in \mathcal{P}(\mathcal{P}(Y))$.

   d. $\mathcal{P}(\emptyset) \in \mathcal{P}(\mathcal{P}(Y))$.

   e. $\mathcal{P}(\mathcal{P}(\mathcal{P}(Y)))$ has 16 elements.

2. For each of the following shade a venn diagram to illustrate the identity.

   a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

   b) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

   c) $\overline{A \cap B} = \overline{A} \cup \overline{B}$

   d) $\overline{A \cup B} = \overline{A} \cap \overline{B}$

3. Prove each of these identities with the double inclusion method:

   a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

   b) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

   c) $\overline{A \cap B} = \overline{A} \cup \overline{B}$

   d) $\overline{A \cup B} = \overline{A} \cap \overline{B}$

4. Let $\mathcal{E}$ be the set of even numbers $\mathcal{E} = \{n \in \mathbb{Z} \mid n = 2k, k \in \mathbb{Z}\}$, and $\mathcal{O}$ be the set of odd numbers $\mathcal{O} = \{n \in \mathbb{Z} \mid n = 2k + 1, k \in \mathbb{Z}\}$,

   Show that "odd times odd is odd" by using the double inclusion method to show $\mathcal{O} = \{n \in \mathbb{Z} \mid n = m_1 m_2; m_1, m_2 \in \mathcal{O}\}$.

5. Using the sets $\mathcal{E}$ and $\mathcal{O}$ defined above, show "odd times even is even" by using the double inclusion method to show $\mathcal{E} = \{n \in \mathbb{Z} \mid n = m_1 m_2; m_1 \in \mathcal{O}; m_2 \in \mathcal{E}\}$.

6. Using the sets $\mathcal{E}$ and $\mathcal{O}$ defined above, consider "even times even is even".

   Show that $\mathcal{E} \neq \{n \in \mathbb{Z} \mid n = m_1 m_2; m_1 \in \mathcal{E}; m_2 \in \mathcal{E}\}$ by showing that one of the implications fails.

   Prove the other one.

   In conclusion, try to make a more precise statement than "even times even is even"

7. Let $V = \{a, e, i, o, u\}$ be the set of vowels. How many elements are in $V \times V \times V$?

   What is the 0'th element in $V \times V \times V$ in lexicographic order.

   List the 50'th to 55'th elements of $V \times V \times V$ in lexicographic order.

   List the 100'th to 110'th elements of $V \times V \times V$ in lexicographic order.

8. Let $V = \{a, e, i, o, u\}$ be the set of vowels as before and let D be the subset of $V \times V \times V$ consisting of those elements no two of whose coordinates are equal. (so omitting things like (e, e, e) and (a, e, a).)

   What is the 0'th element in $V \times V \times V$ in lexicographic order.

   List the 50'th to 55'th elements of $V \times V \times V$ in lexicographic order.

   What happened to the third part of the question?

9. Let $D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and consider $X = D \times D \times D \times D$. Taking the usual ordering on the digits, and starting with the 0'th element in lexicographic order (that is $(0, 0, 0, 0)$. what is the seven hundred and seventy-seventh element of X in lexicographic order.

Please don't do this problem

In 1918, during the bad old days when students drank beer, a fraternity had six barrels of beer in the basement, just like this:

When the Dean of Students visited the house, they drank a certain number of complete barrels, leaving the rest untouched.

When the Provost visited the house, they again drank a certain number of complete barrels, leaving the rest untouched, but the amount of beer drunk was exactly twice as much as at the first party.

When the President of the University visited the house, the fraternity was embarrassed to discover that there was only one barrel left. What was the maximum amount of beer that the president could have drunk?

(A proper solution should not use computer, calculator, phone or even paper and pencil – certainly no beer.)