Ma2201/CS2022 Discrete Mathematics A Term, MMXVII
Quiz 0010

Print Name: _____
Sign: _____

1. (**2 points each**) Consider the set $C = \{\text{red}, \text{yellow}, \text{blue}, \text{green}, \text{black}\}$.
a) What is $|\mathcal{P}(\mathcal{P}(C))|$?

♣ *We have* $|\mathcal{P}(A)| = 2^{|A|}$ *for a finite set* A. *So* $|\mathcal{P}(C)| = 2^5 = 32$. *Thus* $|\mathcal{P}(\mathcal{P}(C))| = 2^{|\mathcal{P}(C)|} = 2^{32}$.

(You could write the answer easily in binary: 1,0000,0000,0000,0000,0000,0000,0000,0000 or in Hexadecimal: 1,0000,0000 but it is not worth your time to write $|\mathcal{P}(\mathcal{P}(C))|$ *in decimal, and it is not advisable to try two write out* $\mathcal{P}(\mathcal{P}(C))$ *itself, that is if you want to get done before A-term is quite over.)* ♣

b) If you order the colors alphabetically, so black < blue < green < red < yellow, then this gives an ordering on the power set $\mathcal{P}(C)$ with the 0'th and first elements being $\emptyset$ and $\{\text{black}\}$ respectively. What is the twenty-second element in $\mathcal{P}(C)$?

♣ *Converting 22 to binary we have, repeatedly dividing by 2,*

$$11R0, \quad 5R1 \quad 2R1 \quad 1R0 \quad 0R1$$

So in binary 22 is 10110 *with* 1*'s in the blue, green and yellow coordinates. So the twenty-second subset is* $\{yellow, blue, green\}$. ♣

c) Is there a way so re-order C so that the twenty-second subset is $\{\text{black}, \text{blue}\}$? Give such an ordering or explain why that is impossible.

♣ *Since 22 in binary has three 1's, every reordering will give a twenty-second subset with three elements.* ♣

d) List all the elements in the set $\{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}\} \cap \mathcal{P}(C)$.

♣ *All elements of* $\mathcal{P}(C)$ *are subsets of* C. *In* $\{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}\}$ *only* $\emptyset$ *is a subset of* C, *so* $\emptyset$ *the only element in the intersection*[1], *and so*

$$\{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}\} \cap \mathcal{P}(C) = \{\emptyset\} \quad ♣$$

e) List all the elements in $\{X \in \mathcal{P}(C) \mid |X| = 1\}$.

♣ *Here we want subsets with exactly one element, that is just one color. Remember, the elements in the power set are sets of colors, not colors, so* $\{blue\} \in \mathcal{P}(C)$, *but* $blue \notin \mathcal{P}(C)$.

$$\{X \in \mathcal{P}(C) \mid |X| = 1\} = \{\{black\}, \{blue\}, \{green\}, \{red\}, \{yellow\}\} \quad ♣$$

[1] $\{\emptyset\}$ is not a subset of C since $\emptyset$ is not an *element* of C.