1. Please see file named NCweights.m for the code I used to generate the following results:

```
>> [w,x_nodes] = NCweights(0,'open',[0,1])

w =

     1


x_nodes =

    0.5000

>> [w,x_nodes] = NCweights(1,'open',[0,1])

w =

    0.5000    0.5000


x_nodes =

    0.3333    0.6667

>> [w,x_nodes] = NCweights(2,'open',[0,1])

w =

    0.6667   -0.3333    0.6667


x_nodes =

    0.2500    0.5000    0.7500

>> [w,x_nodes] = NCweights(3,'open',[0,1])

w =

    0.4583    0.0417    0.0417    0.4583


x_nodes =
```

```
    0.2000    0.4000    0.6000    0.8000

>> [w,x_nodes] = NCweights(1,'closed',[0,1])

w =

    0.5000    0.5000


x_nodes =

     0     1

>> [w,x_nodes] = NCweights(2,'closed',[0,1])

w =

    0.1667    0.6667    0.1667


x_nodes =

         0    0.5000    1.0000

>> [w,x_nodes] = NCweights(3,'closed',[0,1])

w =

    0.1250    0.3750    0.3750    0.1250


x_nodes =

         0    0.3333    0.6667    1.0000

>> [w,x_nodes] = NCweights(4,'closed',[0,1])

w =
    0.0778    0.3556    0.1333    0.3556    0.0778


x_nodes =

         0    0.2500    0.5000    0.7500    1.0000
```

2.

Homework 5

2. Closed Newton-Cotes formulas:

Trapezoidal Rule: $n=1$

$$\int_{x_0}^{x_1} f(x)\,dx = \frac{h}{2}[f(x_0)+f(x_1)]$$

$x_0 = 1.8$ $x_1 = 2.6$ $h = \frac{2.6-1.8}{1} = 0.8$

$$\int_{1.8}^{2.6} f(x)\,dx = \frac{0.8}{2}[f(1.8)+f(2.6)]$$
$$= 0.4[3.12014 + 10.46675] = 5.434756$$

Simpson's Rule: $n=2$

$$\int_{x_0}^{x_2} f(x)\,dx = \frac{h}{3}[f(x_0)+4f(x_1)+f(x_2)]$$

$x_0 = 1.8$ $x_2 = 2.6$ $h = \frac{2.6-1.8}{2} = 0.4$ $x_1 = 1.8+0.4 = 2.2$

$$\int_{1.8}^{2.6} f(x)\,dx = \frac{0.4}{3}[f(1.8)+4f(2.2)+f(2.6)]$$
$$= \frac{0.4}{3}[3.12014+24.16964+10.46675] = 5.034204$$

The Rule for: $n=4$

$$\int_{x_0}^{x_4} f(x)\,dx = \frac{2h}{45}[7f(x_0)+32f(x_1)+12f(x_2)+32f(x_3)+7f(x_4)]$$

$x_0 = 1.8$ $x_4 = 2.6$ $h = \frac{2.6-1.8}{4} = 0.2$

$x_1 = 1.8+0.2 = 2.0$ $x_2 = 2.0+0.2 = 2.2$ $x_3 = 2.2+0.2 = 2.4$

$$\int_{1.8}^{2.6} f(x)\,dx = \frac{2(0.2)}{45}[7f(1.8)+32f(2)+12f(2.2)+32f(2.4)+7f(2.6)]$$
$$= \frac{2(0.2)}{45}[21.84098 + 141.62208 + 72.50892 + 256.96448 + 73.26725]$$
$$= 5.03292$$

Open Newton-Cotes formulas:

Midpoint Rule: $n=0$

$$\int_{x_{-1}}^{x_1} f(x)\,dx = 2hf(x_0)$$

$x_{-1} = 1.8$ $x_1 = 2.6$ $h = \frac{2.6-1.8}{0+2} = 0.4$ $x_0 = 1.8+0.4 = 2.2$

$$\int_{1.8}^{2.6} f(x)\,dx = 2(0.4)(f(2.2)) = (0.8)(6.04241) = 4.833928$$

The Rule for: $n=2$

$$\int_{x_{-1}}^{x_3} f(x)\,dx = \frac{4h}{3}[2f(x_0)-f(x_1)+2f(x_2)]$$

$x_{-1} = 1.8$ $x_3 = 2.6$ $h = \frac{2.6-1.8}{2+2} = 0.2$

$x_0 = 1.8+0.2 = 2.0$ $x_1 = 2.0+0.2 = 2.2$ $x_2 = 2.2+0.2 = 2.4$

$$\int_{1.8}^{2.6} f(x)\,dx = \frac{4(0.2)}{3}[2f(2.0)-f(2.2)+2f(2.4)]$$
$$= \frac{4(0.2)}{3}[8.85138-6.04241+16.06028]$$
$$= 5.0318$$

3.

3. $\int_0^2 x^2 e^{-x^2} dx$ $\quad f(x) = x^2 e^{-x^2}$ $\quad [0, 2]$

a) $\int_{a=x_0}^{b=x_n} f(x)dx = \frac{h}{2}[f(a) + 2\sum_{i=1}^{n-1} f(x_i) + f(b)]$

$a = 0 \quad b = 2 \quad 0.25 = \frac{2-0}{n} \quad n = 8$

$\int_0^2 f(x)dx = \frac{0.25}{2}[f(0) + 2[f(0.25) + f(0.5) + f(0.75) + f(1) + f(1.25) + f(1.50) + f(1.75)] + f(2)]$

$= 0.421582$

b) $\int_a^b f(x)dx = \frac{h}{3}[f(a) + 2\sum_{i=1}^{\frac{n}{2}-1} f(x_{2i}) + 4\sum_{i=1}^{n/2} f(x_{2i-1}) + f(b)]$

$a = 0 \quad b = 2 \quad 0.25 = \frac{2-0}{n} \quad n = 8$

$\int_0^2 f(x)dx = \frac{0.25}{3}[f(0) + 2(f(x_2) + f(x_4) + f(x_6)) + 4(f(x_1) + f(x_3) + f(x_5) + f(x_7)) + f(2)]$

$= \frac{0.25}{3}[f(0) + 2(f(0.5) + f(1) + f(1.50)) + 4(f(0.25) + f(0.75) + f(1.25) + f(1.75)) + f(2)]$

$= 0.422716$

c) $\int_a^b f(x)dx = 2h\sum_{j=0}^{n/2} f(x_{2j})$

$a = 0 \quad b = 2 \quad 0.25 = \frac{2-0}{n+2} \quad n+2 = 8 \quad n = 6$

$\int_0^2 f(x)dx = 2(0.25)[f(x_0) + f(x_2) + f(x_4) + f(x_6) + f(2)]$

$= 2(0.25)[f(0.25) + f(0.75) + f(1.25) + f(1.75)]$

$= 0.424984$

Absolute error:

3 a) $|0.422725 - 0.421582| = 0.001143$

b) $|0.422725 - 0.422716| = 0.000009$

c) $|0.422725 - 0.424984| = 0.002259$