Ma2201/CS2022 Quiz 0101

Discrete Mathematics

A Term, MMXVII

Print Name: ____________________

Sign: ____________________

1. (**4 points**) Label each of the following sets as finite, countably infinite or uncountable.

________ $\mathcal{P}(\{x \in \mathbb{Q} \mid -1 \leq x \leq 1\})$.

♣ *The rationals between -1 and 1 are infinite, and a subset of $\mathbb{Q}$, so countably infinite. Hence their power set is uncountable.* ♣

________ $\mathbb{Z} \cup (\mathbb{Z} \times \mathbb{Q})$.

♣ *Countably infinite: The sets $\mathbb{Z}$ and $(\mathbb{Z} \times \mathbb{Q})$ are both countably infinite, so their union in as well.* ♣

________ $(\mathbb{R} \cap \mathbb{Z}) \times (\mathbb{R} \cap \mathbb{Q})$.

♣ *Countably Infinite: $(\mathbb{R} \cap \mathbb{Z}) = \mathbb{Z}$ and $(\mathbb{R} \cap \mathbb{Q}) = \mathbb{Q}$ which are both countable, and the product of two countably infinite sets is countably infinite.* ♣

________ $\{x \in \mathbb{R} \mid 1.001 \leq x \leq 1.002\}$.

♣ *Uncountable: The interval is only 10^{-3} wide, but multiplying by 1000 is one-to-one and onto, and we proved that the real numbers in an interval of width 1 is uncountable.* ♣

2. (**4 points**) Let p and q be statements. Label each of the following as TRUE if it must be true. Otherwise FALSE.

____ $p \Rightarrow (q \vee \neg q)$.

♣ *TRUE: Since $q \vee \neg q$ is always true, consequence is true, so implication is true.* ♣

____ $(p \Rightarrow q) = \neg(q \Rightarrow p)$. E

♣ *FALSE: If p and q are both true, then both $p \Rightarrow q$ and $q \Rightarrow p$ are true, so equating the first with the negation of the second is not always true so FALSE.* ♣

____ $(p \Rightarrow q) = \neg(p \wedge \neg q)$.

♣ *TRUE: The implication is by definition $q \vee \neg p$. The right hand side, by Demorgan's law is $\neg(p \wedge \neg q) = \neg p \vee q$.* ♣

____ $(p \vee q) \Rightarrow p$.

♣ *FALSE: Consider if p is false and q is true.* ♣

3. (**2 points**) Define a function $f : \mathbb{N} \to \mathcal{P}(\mathbb{N})$ such that the set $C_f = \{n \in \mathbb{N} \mid n \notin f(n)\}$ is the set of even numbers.

♣ *We can define any function we please provided that each even numbers are sent to a set not containing themselves, and the opposite for the odd numbers. So, for example define $f(n) = \emptyset$ for n even and $f(n) = \{0, 1, 2, 3, \ldots, n\}$ for n odd.*

[Note: C_f is used to show f cannot be onto, and indeed, none of the functions $f(n) = C_f$, since $f(n)$ is always a finite set, and the even numbers is an infinite set.] ♣