Summary for lectures 3 and 4

We are studying sets.

We defined sets, and the notation for sets, and gave several examples.

We discussed the notion of a set being well-defined.

Introduced the notation $\in$ and $\subseteq$.

We computed the number of subsets of a finite set.

We discussed the empty set $\emptyset$ and the power set of A, $\mathcal{P}(A)$.

We defined the bit vector, and showed how to order the subsets of A.

We defined intersection, $\cap$, and union, $\cup$, and complement $(--)^c$ and their relationship.

Exercises for Lectures 3 and 4

1. Discuss whether or not each of these determines a well-defined set:
   (a) The set of people in the United States.
   (b) The set of people in the United States.
   (c) The set of current members of congress.
   (d) The set students in this class.
   (e) the set of bicycles on the WPI campus.
   (f) the set of cars registered in Massachusetts
2. Discuss whether or not each of these determines a well-defined set:
   (a) The set of numbers numbers which are the sum of two prime numbers.
   (b) The set of integers which are larger than any prime number.
   (c) The set of rational numbers which are prime.
   (d) The set of natural numbers which are the lengths of words in Shakespeare's "Hamlet".
3. List all the elements of each of the following sets:
   (a) $\{n \in \mathbb{N} \mid 5 < n^2 < 100, n = 3k, k \in \mathbb{N}\}$
   (b) $\{n^2 + m^2 \mid m \in \mathbb{N}, n \in \mathbb{N}, n + m < 10\}$
   (c) $\{n + m \mid n \in \mathbb{Z}, m \in \mathbb{N}, n^2 + m^2 \leq 5\}$
   (d) $\{n + m \mid n \in \mathbb{Z}, m \in \mathbb{Z}, n^2 + m^2 \leq 5\}$
   (e) $\{n + m \mid n \in \mathbb{N}, m \in \mathbb{N}, n^2 + m^2 \leq 5\}$
4. Let $A = \{a, b, c, d, \ldots, x, y, z\}$ be the set of lower case letters in the alphabet.
   (a) How many sets are subsets of A?

(b) How many subsets of A are also subsets of a subset of A?

(c) How many subsets of A contain both the elements f and s?

(d) What is the set of subsets of A such that a is it's first element?

(e) How many subsets of A do not contain a vowel? ($\{a, e, i, o, u\}$)

5. Let $B = \{b, o, d\}$, $F = \{a, b, c, d\}$, and $S = \{c, d, o, i\}$. Give each of the following sets:

   (a) $B \cup F$.

   (b) $B \cap F$.

   (c) $(B \cap F) \cup (S \cap F)$.

   (d) $(B \cup F) \cap (S \cup F)$.

   (e) $(B \cup F) \cap (S \cup F) \cap (B \cup S)$.

   (f) $B \cap F \cap S$.

6. Let $B = \{b, o, d\}$, $F = \{a, b, c, d\}$, and $S = \{c, d, o, i\}$. Give each of the following sets:

   (a) $\mathcal{P}(B) \cup \mathcal{P}(F)$.

   (b) $\mathcal{P}(B \cup F)$.

   (c) $\mathcal{P}(B) \cap \mathcal{P}(F)$.

   (d) $\mathcal{P}(B \cap F)$.

   (e) $\mathcal{P}(B) \cap \mathcal{P}(F) \cap \mathcal{P}(F)$.

   (f) $\mathcal{P}(B) \cup \mathcal{P}(F) \cup \mathcal{P}(F)$.

7. List the elements of the set $\mathcal{P}(\mathcal{P}(\mathcal{P}(\emptyset)))$. (There are only four.)

8. Show using the double inclusion method that $(X \cap Y) \cup Z = (X \cup Z) \cap (Y \cup Z)$.

9. Show using the double inclusion method that $(X \cup Y) \cap Z = (X \cap Z) \cup (Y \cap Z)$.

10. Show using the double inclusion method that $(X \cup Y)^c = X^c \cap Y^c$.

11. Show using the double inclusion method that $(X \cap Y)^c = X^c \cup Y^c$.