Lectures 12 and 13

We started formal logic which concerns *statements.* A statement is either TRUE (1) or FALSE (0).

Statements can be formed from $\wedge$ (AND), $\vee$ (OR) and $\neg$ (NOT).

We stated the distributive laws:

$$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r) \qquad\qquad p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$$

and Demorgan's laws:

$$\neg(p \vee q) = (\neg p) \wedge (\neg q) \qquad\qquad \neg(p \wedge q) = (\neg p) \vee (\neg q)$$

We discussed logical implication: $p \Rightarrow q$, IF p THEN q, or p IMPLIES q.

$$(p \Rightarrow q) = (q \vee \neg p)$$

Exercises for Lectures 12 and 13

1. Decide whether each of the following is a statement.
   (a) Decide whether each of the following is a statement.
   (b) π is not a number.
   (c) Black is a color.
   (d) $5 - 4 > 10$
   (e) Black is a mood.
   (f) Every right handed person in this class will die.
   (g) Every real number is countable.
   (h) $x^2 + 1 = 0$ has no solutions.
   (i) Margarine tastes better than butter.
   (j) Not all laws are just.
   (k) None of these are statements.

2. Suppose p is TRUE and q is FALSE, and r is a statement. Label each of the following as true or false, or undecidable:
   ____ $(p \wedge q \wedge r)$
   ____ $(p \vee q \vee r)$
   ____ $p \wedge \neg(q \vee \neg q)$.
   ____ $p \wedge (p \vee q) \wedge (p \vee q \vee r)$.
   ____ $p \vee (p \wedge q) \vee (p \wedge q \wedge r)$.
   ____ $p \vee \neg((p \wedge q) \vee \neg(p \wedge q \wedge r))$.

3. Suppose p is TRUE and q is FALSE, and r is a statement. Label each of the following as true or false, or undecidable:

   ____ $(p \wedge q) \Rightarrow r$

   ____ $(p \vee q) \Rightarrow r$

   ____ $p \Rightarrow \neg p$

   ____ $\neg p \Rightarrow p$

   ____ $\neg p \Rightarrow (q \wedge r)$

   ____ $\neg p \Rightarrow \neg(q \vee r)$

   ____ $(p \Rightarrow q) \Rightarrow (q \Rightarrow p)$

4. Suppose $p \wedge (q \vee (p \wedge q))$ is TRUE. What can you conclude about the truth of p and q?

5. Use the double implication method to show that

$$p \vee (q \wedge r) \Longleftrightarrow (p \vee q) \wedge (p \vee r).$$

6. Use the double implication method to show that

$$p \wedge (q \vee r) \Longleftrightarrow (p \wedge q) \vee (p \wedge r).$$

7. Use the double implication method to show that

$$\neg(q \vee r) \Longleftrightarrow (\neg p \wedge \neg q).$$

8. Use the double implication method to show that

$$\neg(q \wedge r) \Longleftrightarrow (\neg p \vee \neg q).$$

9. Show that $[(p \Rightarrow q) \Rightarrow r] \Longrightarrow [p \Rightarrow (q \Rightarrow r)]$.

10. Show that $[(p \Rightarrow q) \Rightarrow r] \Longrightarrow [p \Rightarrow (q \Rightarrow r)]$.

11. Show that $[(p \wedge \neg r) \vee (q \wedge \neg p) \vee (r \vee \neg q)] \Longleftrightarrow [(p \wedge \neg q) \vee (q \wedge \neg r) \vee (r \vee \neg p)]$.