Ma2201/CS2022 **Discrete Mathematics** A Term, MMXVII

Quiz 0110

Print Name: ____________________

Sign: ____________________

1. (**4 points**) Suppose $\{p_n \mid n \in \mathbb{N}\}$ is a set of statements and suppose that $p_n \Rightarrow p_{n+5}$ is true for all $n \geq 0$.

Suppose also that p_{10} is true and p_{23} is false.

Label each of the following as 'T' if it must be true, 'F' if it must be false, and '?' if it's truth cannot be determined from the given information.

♣ *From base case p_{10} and inductive step $p_n \Rightarrow p_{n+5}$ we know that p_k is true if $k \in \{10, 15, 20, 25, 30, 25, 40, 45, \ldots\}$.*

We also know from p_{23} being false that p_3, p_8, p_{13}, and p_{18} are all false, otherwise they would be a base case for an induction showing p_{23} is true.

About no other p_i can we conclude anything.

Thus p_{12345} is true, so the first question is T. The second is '?' since we only know that some of them are definitely true and know none of them to be definitely false. The third 'T' since all the terms in the and statement are true by induction. The fourth is 'T' since we know p_{18} is false, hence the implication is true. ♣

________ $p_{12345} \vee p_{54321}$.

________ $\displaystyle\bigwedge_{k=24}^{\infty} p_k$.

________ $\displaystyle\bigwedge_{k=1}^{\infty} p_{100k}$.

________ $p_{18} \Rightarrow p_{19}$.

2. (**6 points**) Prove by induction that for all $n \geq 1$ that

$$1 + 2(1 + 3^1 + 3^2 + \cdots + 3^n) = 3^{n+1}.$$

♣ *Proof by Induction: Base case: $n = 1$. The left side is $1 + 2(1 + 3^1) = 1 + 2(1 + 3) = 1 + 2 \cdot 4 = 1 + 8 = 9$. The right side is $3^{1+1} = 3^2 = 9$, so they are equal.*

Inductive Step: Suppose $1 + 2(1 + 3^1 + 3^2 + \cdots + 3^n) = 3^{n+1}$ for some particular n, and we compute $1 + 2(1 + 3^1 + 3^2 + \cdots + 3^{n+1})$:

$$\begin{aligned}
1 + 2(1 + 3^1 + 3^2 + \cdots + 3^{n+1}) &= 1 + 2(1 + 3^1 + 3^2 + \cdots + 3^n + 3^{n+1}) \\
&= [1 + 2(1 + 3^1 + 3^2 + \cdots + 3^n)] + 2 \cdot 3^{n+1}) \\
&= 3^{n+1} + 2 \cdot 3^{n+1}) \quad \textit{by the inductive hypothesis} \\
&= 3^{n+1}(1 + 2) = 3^{n+1}(3) = 3^{n+2} = 3^{(n+1)+1}
\end{aligned}$$

as required, so the statement is true for $n + 1$, and the inductive step is proved.

Since the base case is true for $n = 1$ and the inductive step is true for all n, the statement is true for all n by induction. ♣