Homework 4, Due: Wednesday, 11/28

This assignment is due on **Wednesday, November 28**, by 11:59 PM. Your assignment should be well-organized, typed (or neatly written and scanned) and saved as a .pdf for submission on Canvas. You must show all of your work to receive full credit. For problems requiring the use of MATLAB code, remember to also submit your .m-files on Canvas as a part of your completed assignment. Your code should be appropriately commented to receive full credit.

Problems

1 (8 points) In a circuit with impressed voltage $\varepsilon(t)$ and inductance L, Kirchhoff's first law gives the relationship

$$\varepsilon(t) = L\frac{di}{dt} + Ri$$

where R is the resistance in the circuit and i is the current. Suppose we measure the current for several values of t and obtain

t	1.00	1.01	1.02	1.03	1.04
i	3.01	3.12	3.14	3.18	3.24

where t is measured in seconds, i is in amperes, the inductance L is a constant 0.98 henries, and the resistance is 0.142 ohms. Approximate the voltage $\varepsilon(t)$ when $t = 1.00$, 1.01, 1.02, 1.03, and 1.04.

2 Consider the three-point endpoint formula

$$f'(x_0) = \frac{1}{2h}\Big[-3f(x_0) + 4f(x_0+h) - f(x_0+2h)\Big] + \frac{h^2}{3}f^{(3)}(\xi_0)$$

for approximating $f'(x_0)$, where ξ_0 lies between x_0 and $x_0 + 2h$.

(a) (6 points) Use Taylor expansion and the function values at x_0, x_0+h, and x_0+2h to derive the three-point endpoint formula.

(b) (12 points) Write a MATLAB code that implements the three-point endpoint formula for approximating the first derivative of the function

$$f(x) = e^{5x}$$

at $x_0 = 0.25$ for a given step size h. Starting with $h = 0.1$, test how decreasing h (by order of magnitude, e.g. $h = 0.01$, $h = 0.001$, ..., $h = 10^{-12}$) affects your finite difference approximation. For each h, compare your finite difference approximation to the true value of $f'(0.25)$ and report the absolute error. Make a plot showing the absolute error vs. h using the `loglog` command. Does decreasing h systematically decrease the error? Explain.

3 (6 points) Apply Richardson's extrapolation process to determine $N_3(h)$, an approximation to $f'(x_0)$, for the following function $f(x)$ and step size h.

$$f(x) = x + e^x, \qquad x_0 = 0, \qquad h = 0.4$$

Note: For any of the above problems for which you use MATLAB to help you solve, you must submit your code/.m-files as part of your work. Your code must run in order to receive full credit. If you include any plots, make sure that each has a title, axis labels, and readable font size, and include the final version of your plots as well as the code used to generate them.