Ma2201/CS2022 Quiz 0100

Discrete Mathematics

A Term, MMXVII

Print Name: ___________________

Sign: ___________________

1. (**4 points**) Let $A = \{a, b, c\}$ be a set and let $R \subseteq \mathcal{P}(A) \times \mathcal{P}(A)$ defined by

$$R = \{(X, Y) \in \mathcal{P}(A) \times \mathcal{P}(A) \mid b \in X \cap Y\}$$

. Label the following TRUE or FALSE.

____ The relation R is transitive.

♣ *TRUE: If $(X, Y) \in R$ and $(Y, Z) \in R$, then $b \in X \cap Y$ and $b \in Y \cap Z$, so b is in all three sets and $b \in X$ and $b \in Z$, and $b \in X \cap Z$, so $(X, Z) \in R$.* ♣

____ The relation R is symmetric.

♣ *TRUE: If $(X, Y) \in R$ then $b \in X \cap Y = Y \cap X$ so $(Y, X) \in R$.* ♣

____ The relation R is reflexive.

♣ *FALSE: This requires every subset to be related to itself. But $b \notin \emptyset \cap \emptyset$ so $(\emptyset, \emptyset) \notin R$.*♣

____ *The equivalence class of $\emptyset$ is $\{\emptyset\}$.*

♣ FALSE: Since the relation is not an equivalence relation, not being symmetric, there are no equivalence classes. But even beyond that, we just noted that $\emptyset$ is not related to itself, and in fact, it is related to no elements. ♣

*2. (**4 points**) Let $A = \{a, b, c\}$ be a set and let the relation $S \subseteq \mathcal{P}(A) \times \mathcal{P}(A)$ defined by*

$$S = \{(X, Y) \in \mathcal{P}(A) \times \mathcal{P}(A) \mid \textit{There exists and onto function } X \to Y\}$$

. Label the following TRUE or FALSE

____ *The relation S is transitive.*

♣ TRUE: If $(X, Y) \in S$ and $(Y, Z) \in S$, then there exists $f : X \to Y$ and $g : Y \to Z$ both onto. and $g(f(x)$ is an onto function $gf : X \to Z$, so $(X, Z) \in S$ ♣.

____ *The relation S is symmetric.* ♣ FALSE: There is an onto function $\{a, b\} \to \{a\}$ sending both elements to a. But there is no onto function $\{a\} \to \{a, b\}$. ♣

____ *The relation S is reflexive.*

♣ TRUE: The identity function which maps every element to itself, is also an onto map from $X \to X$. ♣

____ *The equivalence class of $\{a, b, c\}$ has three elements.*

♣ FALSE: There are no equivalence classes, just as before. But besides that, $(\{a, b, c\}, X) \in S$ is true for 7 sets X, and only one of those $\{a, b, c\}$ itself is it true that $(X, \{a, b, c\} \in S$. ♣

*3. (**2 points**) Let $B = \{a, b, c, d\}$. Find a relation $T \subseteq B \times B$ which is both an equivalence relation and a functional relation.*

♣ For a relation be a function, each element of the first set, the domain, must be related to exactly one element in the second set, the target. For a relation to be an equivalence relation, each element must be related to itself. So the only relation that can satisfy both conditions is: $T = \{(a, a), (b, b), (c, c), (d, d)\}$. This is an equivalence relation, and there are four equivalence classes: $\{a\}$, $\{b\}$, $\{c\}$, and $\{d\}$. ♣