

CS 3133 Foundations of Computer Science
C term 2018

Solutions for the Midterm Exam

1. Give a regular expression that represents the set of strings over $\Sigma = \{a, b\}$ that do not contain the substring aa .

Solution:

$$b^*(ab^+)^*(\lambda \cup a)$$

(20 points)

2. Consider the following grammar G :

$$\begin{aligned} S &\rightarrow XYZ \\ X &\rightarrow Xa \mid Xb \mid a \\ Y &\rightarrow aY \mid bY \mid b \\ Z &\rightarrow aZ \mid bZ \mid a \end{aligned}$$

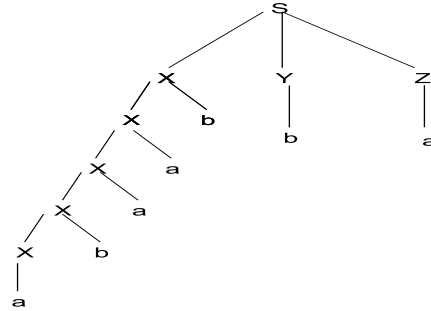
- (a) Give a leftmost derivation of $abaabba$.
(b) Build the derivation tree for the derivation in part (a).
(c) What is $L(G)$?

Solution:

- (a) The following is a leftmost derivation of $abaabba$:

$$\begin{aligned} S &\Rightarrow XYZ \\ &\Rightarrow XbYZ \\ &\Rightarrow XabYZ \\ &\Rightarrow XaabYZ \\ &\Rightarrow XbaabYZ \\ &\Rightarrow abaabYZ \\ &\Rightarrow abaabbZ \\ &\Rightarrow abaabba \end{aligned}$$

(b)



(c)

$$L(G) = a(a \cup b)^*b(a \cup b)^*a$$

(20 points)

3. Construct two regular grammars, one ambiguous and one unambiguous, that generate the language determined in the previous problem 2(c).

Solution:

Unambiguous regular grammar:

$$\begin{aligned} S &\rightarrow aA \\ A &\rightarrow aA \mid bB \\ B &\rightarrow aB \mid bB \mid a \end{aligned}$$

Ambiguous regular grammar:

$$\begin{aligned} S &\rightarrow aA \mid aC \\ A &\rightarrow aA \mid bA \mid bB \\ B &\rightarrow aB \mid bB \mid a \\ C &\rightarrow aC \mid bC \mid bB \end{aligned}$$

It is ambiguous because there are two different leftmost derivations for the string aba :

$$\begin{aligned} S &\Rightarrow aA \\ &\Rightarrow abB \\ &\Rightarrow aba \end{aligned}$$

and

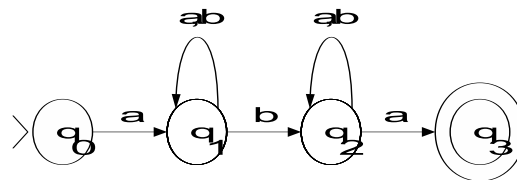
$$\begin{aligned} S &\Rightarrow aC \\ &\Rightarrow abB \\ &\Rightarrow aba \end{aligned}$$

(20 points)

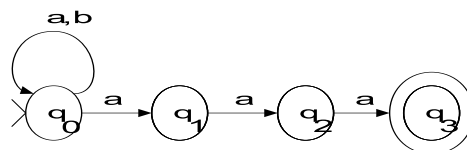
4. Design an NFA that accepts the language determined in problem 2(c).
(20 points)

Solution:

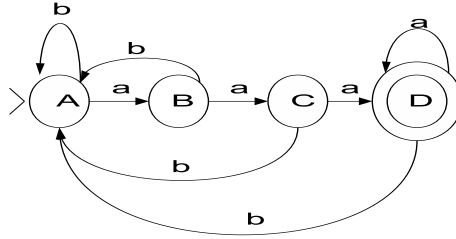
The state diagram of an NFA is



5. Construct the state diagram of a DFA equivalent to the following NFA by using the subset construction method. What is the language accepted by these machines? (20 points)



Solution:



Here the states correspond to the following subsets: $A = \{q_0\}$, $B = \{q_0, q_1\}$, $C = \{q_0, q_1, q_2\}$ and $D = \{q_0, q_1, q_2, q_3\}$. The language is $(a \cup b)^*aaa$.