Homework 3, Due: Thursday, 1/31

This assignment is due on **Thursday, January 31**, by 11:59 PM. Your assignment should be well-organized, typed (or neatly written and scanned) and saved as a .pdf for submission on Canvas. You must show all of your work to receive full credit. For problems requiring the use of MATLAB code, remember to include your output in the main body of your solution .pdf and to also submit your .m-files on Canvas as a part of your completed assignment. Your code should be appropriately commented to receive full credit.

Problems

$\boxed{1}$ (4 points) Use the LU factorization

$$\mathsf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & -1 \\ 0 & -2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$

to solve the linear system $\mathsf{A}\mathbf{x} = \mathbf{b}$, where

$$\mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}.$$

$\boxed{2}$ Consider the matrix

$$\mathsf{A} = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 2 & 3 \\ 2 & -1 & 4 \end{bmatrix}$$

(a) (4 points) Analytically (i.e., by hand) compute the matrix factorization of A into the form $\mathsf{A} = \mathsf{P}^\mathsf{T}\mathsf{L}\mathsf{U}$ by finding the permutation matrix P, lower-triangular matrix L, and upper-triangular matrix U. Use Doolittle's form of the LU factorization (which is the version we discussed in class).

(b) (4 points) Use MATLAB's built-in `lu` function to numerically compute the factorization from part (a). Your input should be the matrix A, and your command window output should be the resulting matrices P, L, and U. Refer to `help lu` to determine the syntax that you will need to use in order to display the necessary output matrices. Do you get the same matrices P, L, and U as in part (a)? Explain.

(c) (4 points) **BONUS:** Write your own MATLAB **function** implementing Doolittle's LU factorization method. Name your function `myLU.m`. Your function should input the matrix A and output the matrices P, L, and U that define the factorization $\mathsf{A} = \mathsf{P}^\mathsf{T}\mathsf{L}\mathsf{U}$. (Note that if no row exchanges are necessary, $\mathsf{P} = \mathsf{I}$.) Demonstrate that your function works by testing it on the matrix A given in this problem, as well as another matrix of your choice.

$\boxed{3}$ (6 points) Prove that if $||\cdot||$ is a vector norm on $\mathbb{R}^n$, then

$$||\mathsf{A}|| = \max_{||\mathbf{x}||=1} ||\mathsf{A}\mathbf{x}||$$

is a matrix norm.

$\boxed{4}$ Consider the matrix

$$\mathsf{A} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

(a) (4 points) Compute the eigenvalues and associated eigenvectors of A.

(b) (2 points) Find the spectral radius $\rho(\mathsf{A})$.

(c) (2 points) Find the ℓ_2 norm of A.

$\boxed{5}$ (4 points) Show that if A is symmetric, then $||\mathsf{A}||_2 = \rho(\mathsf{A})$.

Note: For any of the above problems for which you use MATLAB to help you solve, you must submit your code/.m-files as part of your work. Your code must run in order to receive full credit. If you include any plots, make sure that each has a title, axis labels, and readable font size, and include the final version of your plots as well as the code used to generate them.