Biological Neural Computation

Homework problem set 1

Spring 2019

Data Assigned: 01/16/2019
Data Due: 02/06/2019

<u>**General Guidelines:**</u> The homework solutions should include figures that clearly capture the result. The figures have to be labeled, well explained and the results must be clearly discussed. When appropriate, it is recommended that you use the ***Hypothesis – Rationale – Experiments/data – Analysis – Results – Discussion/Conclusions – Limitation(s)*** framework to discuss your work.

The first sheet of the homework must certify that this is completely your work and list the students/people you have consulted or received help from (with your signature and date of submission). All online references used must be listed in the reference section at the end of the homework. It is sufficient to include Matlab code only in the online submission.

All identification information should be self-contained in the second page of the homework. No other place should this information be available to facilitate blind review.

Good luck,
Barani Raman

Problem 1. (a) Implement the HH model in Matlab. The equations describing the model are:

$$I = C_M \frac{dV}{dt} + \bar{g}_K n^4 (V - V_K) + \bar{g}_{Na} m^3 h (V - V_{Na}) + \bar{g}_l (V - V_l),$$

where
$$dn/dt = \alpha_n (1 - n) - \beta_n n,$$
$$dm/dt = \alpha_m (1 - m) - \beta_m m,$$
$$dh/dt = \alpha_h (1 - h) - \beta_h h,$$

$$\alpha_n = 0.01 \, (V + 10) \Big/ \left(\exp \frac{V + 10}{10} - 1 \right),$$
$$\beta_n = 0.125 \exp (V/80),$$
$$\alpha_m = 0.1 \, (V + 25) \Big/ \left(\exp \frac{V + 25}{10} - 1 \right),$$
$$\beta_m = 4 \exp (V/18),$$
$$\alpha_h = 0.07 \exp (V/20),$$
$$\beta_h = 1 \Big/ \left(\exp \frac{V + 30}{10} + 1 \right).$$

Note: V in these equations represents displacement voltage with respect to the resting membrane potential and not the actual membrane potential. Hence you will have to substitute -65-V in your implementation.

Implement using Euler's integration technique. Use the following values for the constants:

C = 1 microF/cm^2, Leak reversal potential = -61 mV, K reversal potential = -77mV, Na reversal potential = 55 mv, Leakage conductance (g_L = 0.3 mS/cm^2), K conductance (gK = 36 mS/cm^2, N conductance (gNa) = 120 mS/cm^2.

Initial conditions V = -65 mV.

Plot the following figures to validate your model:
  (i)   Evolution of the HH variables m,n,h following an action potential.
  (ii)   Relative evolution of sodium and potassium conductances
  (iii) Relative evolution of capacitive, leak, sodium and potassium currents
  (iv)   Demonstrate the threshold and rebound spiking behavior of the neuron

Additional problem only for BME 572
  (v)   Replace the exponential relation between α_n and V with a quadratic fit and check whether the model is still valid

Additional problem only for BME 572

(b) Implement the reduced version of the HH model, where

$$Original \quad C\dot{V} \;=\; I \;-\; \overbrace{g_K n^4 (V - E_K)}^{I_K} \;-\; \overbrace{g_{Na} m^3 h (V - E_{Na})}^{I_{Na}} \;-\; \overbrace{g_L (V - E_L)}^{I_L}$$

$$Reduced\, C\dot{V} \;=\; I - \overbrace{g_K n^4 (V - E_K)}^{I_K} - \overbrace{g_{Na} m_\infty^3 (V)(0.89 - 1.1n)(V - E_{Na})}^{instantaneous\ I_{Na}} - \overbrace{g_L (V - E_L)}^{I_L}$$

Compare the response of the original and reduced model to injected current pulses of the same amplitude.

[30 pts]

Note: For those enrolled in L41 5657 this problem will carry [40 pts].

Problem 2. (a) Implement a simple integrate and fire neuron (IAF) in Matlab by adding a threshold (Vthr) to the membrane patch. Whenever the voltage crosses the threshold, set the voltage on that time step to +70 mV to represent the generation of an action potential, and then reset the voltage to 0 mV on the next time step to represent the repolarization phase of the action potential.

$$I(t) = \frac{V(t)}{R} + C\frac{dV(t)}{dt}$$

$$V(t) = \begin{cases} V(t-1) + \dfrac{dv(t)}{dt} \bullet dt & V(t) < V_{THR} \\[2em] \begin{aligned} V(t) &= 70mv \\ V(t+1) &= 0 \end{aligned} & V(t) \geq V_{THR} \end{cases}$$

Using the following parameters: R=10 MOhm, C=1 nF, Vthr=5 mV, Vspk=70 mV, and a time step (dt) of 1 ms, simulate the response of the integrate-and-fire model using Euler integration. Generate a representative plot of the membrane voltage as a function of time for a step current injection with I=1 nA for 10 <= t < 60 ms and I=0 otherwise.

Now examine the response of your IAF model to sinusoidal stimulation. Use a sinusoidal current injection with a peak amplitude of 1 nA. Generate a plot of "spike count vs. stimulus frequency," where "spike count" is the total number of spikes generated during the 1 second stimulus interval (it would be sufficient to characterize the model for the following input frequencies: 1, 2, 5, 10, 20, 50, 100 Hertz. Remember to run the simulation for 1s. Show the input, the spiking pattern generated and the relationship between spike count vs. stimulus frequency).
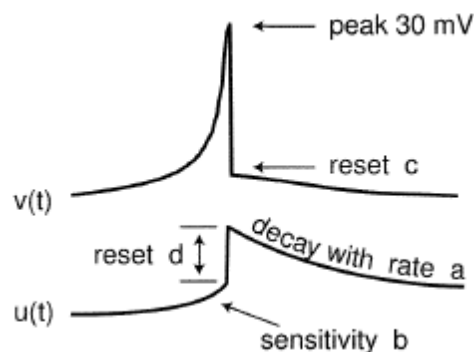
[25 pts]

Additional problems only for BME 572
(b) Implement the same problem as Problem 4(a) but using the following model of a neuron:

$$v' = 0.04v^2 + 5v + 140 - u + I$$
$$u' = a(bv - u)$$

if v = 30 mV,
then $v \leftarrow c$, $u \leftarrow u + d$

peak 30 mV

reset c

v(t)

reset d ↕   decay with rate a

u(t)

sensitivity b

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Initialize the variables in the following fashion:
a = 0.02, b=0.2, c=-65, d=8,v = -65, u = b*v;

For those who are interested more about the model, refer this paper:
E. M. Izhikevich, Simple model of spiking neurons, IEEE Transactions on Neural Networks, Vol. 14, no. 6, pp. 1569-1572, 2003.

(c) Construct a two-neuron oscillator using reciprocal inhibition. The neurons will be modeled again as leaky integrate-and-fire units but now with an adaptive threshold mechanism that generates firing-rate adaptation and post-inhibitory rebound.
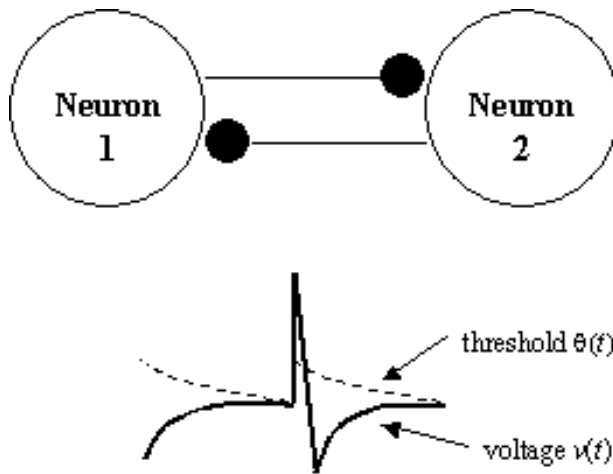
The update equations for each individual neuron are:

$$C\frac{dv}{dt} = \frac{-v}{R} - g_{syn}(v - E_{syn}) + I_{inject}$$ voltage update

$$\frac{d\theta}{dt} = \frac{-\theta + v}{\tau_{thresh}}$$ threshold update

$$\frac{dz}{dt} = \frac{-z}{\tau_{syn}} + \frac{g_{peak}}{(\tau_{syn}/e)}u(t)$$ conductance update I

$$\frac{dg}{dt} = \frac{-g}{\tau_{syn}} + z(t)$$ conductance update II

Remember that the input u(t) comes from spike activity of the pre-synaptic unit. The parameter values for the model are:

$C = 1$ membrane capacitance (nF)
$R = 10$ membrane resistance (MΩ)
$V_{rest} = 0$ resting membrane potential (mV)
$V_{spk} = 70$ action potential amplitude (mV)
$\tau_{thresh} = 50$ threshold time constant (ms)
$E_{inh} = -15$ synaptic reversal potential (mV)
$\tau_{syn} = 15$ synaptic time constant (ms)
$g_{peak} = 0.1$ peak synaptic conductance (μS)
$T_{max} = 1500$ total simulation time (ms)
$\Delta t = 1$ integration time step (ms)

Esyn = Einh = -15 mV and e is exponent of 1 = 2.7183.

Both neurons should receive constant current injection. To break the symmetry of the model, inject slightly more current into neuron 1 than neuron 2. Specifically, inject 1.1 nA into neuron 1 and 0.9 nA into neuron 2.

When the neuron fires an action potential, reset the membrane voltage to Einh on the next time step. [Note: This is related to the adaptive threshold level, which can fall below zero in this model, but not below Einh. We need to reset the membrane voltage to a value that is below the threshold level, hence we choose Einh as the reset value.]

[15 + 20 = 35 pts]

Problem 3.

For this problem, you will implement a linear-nonlinear model (LN model) of a neuron. The following two files are provided: (i) 'Spikes.txt' – contains when spikes happened in a neuron during the experiment, and (ii) 'Stimulus.txt' – has information about when the stimulus was provided. There are five trials in this experiment and the duration of each trial is 20 sec. The same stimulus was provided for all trials. (See figures below for additional details).
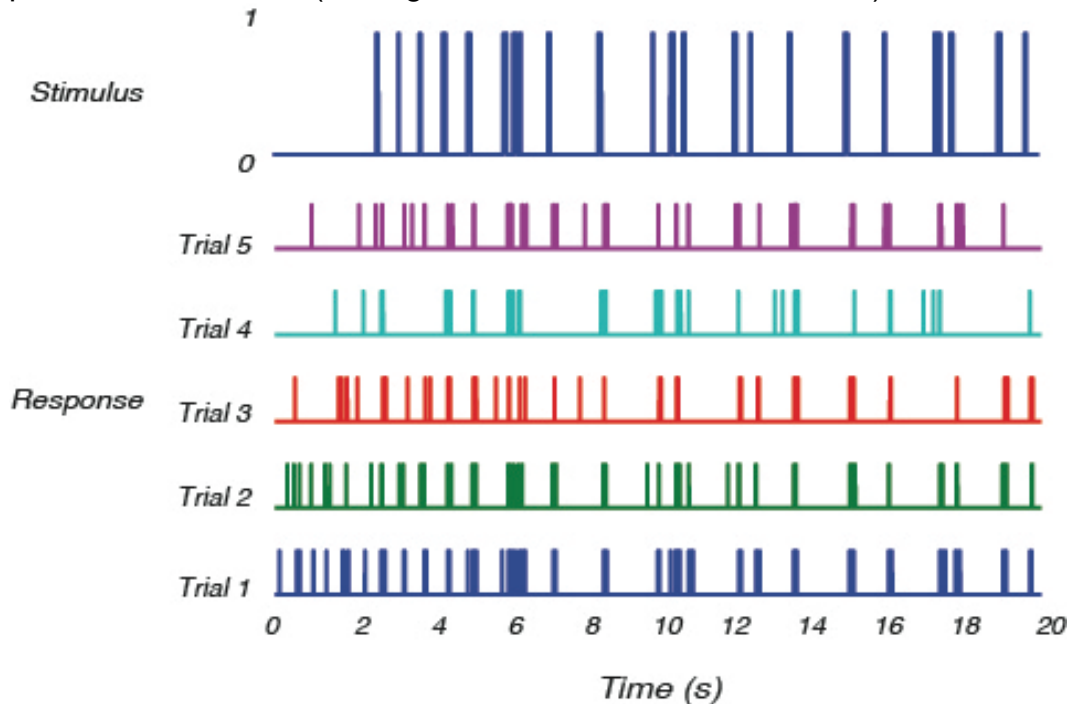
You are expected to implement a Linear-Non-linear filter as described in the following paper: *Geffen, MN, Broome, BM, Laurent, G and Meister, M. (2009). "Neural encoding of rapidly fluctuating odors." Neuron 61(4): 570-586*

A useful strategy to construct the linear filter is as follows:

$SW = R$

$Find \quad W \; that \quad \min imizes \quad (R - SW)^2$

where,

S is stimulus during the previous 2 s (binned in 100 ms time bins)
R is the firing rate response in the current time bin (again 100ms time bin)
W is the linear filter

You will move the stimulus window in 100ms steps to obtain a matrix S (first row in S: 0-2s; second row in S: 0.1-2.1s… and so on) and corresponding response matrix R. Then, you should build a LN model using the first four trials and test your model using the fifth trial.

[20 pts]

Note: For L41 5657 majors, building the linear part would be sufficient. This problem will carry **[35 pts]**.

Problem 4. In the last problem, you created a linear or a linear-nonlinear model of neuron that predicted the firing rate of the neuron given a stimulus. Here, you will extend this model to predict the spike trains. Given the predicted firing rate over time, you are expected to create a spike train using inhomogeneous Poisson process. To do this, you will start with a homogeneous Poisson process with a $\lambda_{max} = \max(\lambda(t))$. Here, $\lambda(t)$ is the predicted firing rate of the neuron. Then, you will thin the homogeneous Poisson process by accepting a spike by generating an uniform random number U[0,1] (use rand in matlab) and if U[0,1] $\leq \dfrac{\lambda(t)}{\lambda_{max}}$. Plot the spike trains, ISI distributions, compute the Fano Factor, and Coefficient of Variation of the ISIs.

[15 pts]

Note: This problem is exclusively for BME 572 students only.