Anisotropic Poisson Problem

An anisotropic Poisson problem in a two-dimensional domain Ω is given by the strong form

$$\begin{aligned} -\nabla \cdot (\boldsymbol{A}\nabla m) + m = f \qquad & \text{in } \Omega, \\ \boldsymbol{A}\nabla m \cdot \boldsymbol{n} = 0 \qquad & \text{on } \partial\Omega, \end{aligned}$$

where the conductivity tensor $\boldsymbol{A}(\boldsymbol{x}) \in \mathbb{R}^{2\times 2}$ is assumed to be symmetric and positive definite for all $\boldsymbol{x}$, $f(\boldsymbol{x})$ is a given distributed source, and $\boldsymbol{n}$ is the unit outward normal vector to $\partial\Omega$.

Solve BVP problem

Choose Ω to be a unit square $[-1, 1]^2$ and take the source terms to be

$$f = \exp(-100(x^2 + y^2)).$$

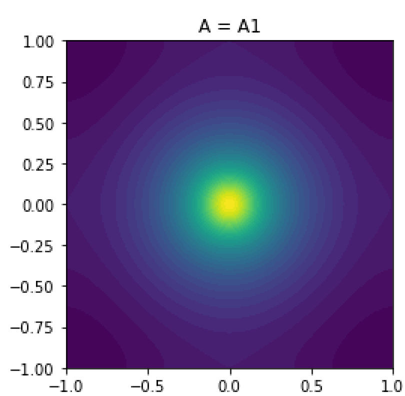
Use conductivity tensors $A(x)$ given by

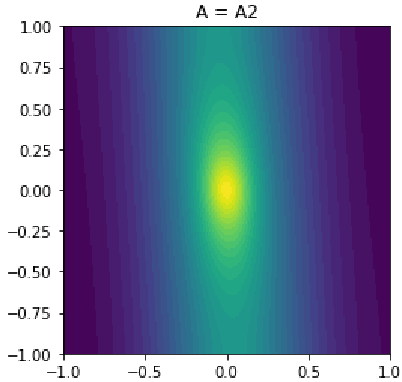
$$A_1 = \begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix} \text{ and } A_2 = \begin{pmatrix} 1 & -0.5 \\ -0.5 & 10 \end{pmatrix}$$

Part b

In [1]:
```
import matplotlib.pyplot as plt
%matplotlib inline

import dolfin as dl

import math
import numpy as np
import logging

logging.getLogger('FFC').setLevel(logging.WARNING)
logging.getLogger('UFL').setLevel(logging.WARNING)
dl.set_log_active(False)


mesh = dl.RectangleMesh(dl.Point(-1,-1), dl.Point(1,1), 128, 128)


V = dl.FunctionSpace(mesh, "Lagrange", 2)
print ("Number of unknowns = ", V.dim() )

f = dl.Expression("exp(-100*(x[0]*x[0] + x[1]*x[1]))", degree = 5)

A1 = dl.Constant(((10., 0.),(0., 10.0)))
A2 = dl.Constant(((1., -.5),(-0.5, 10.0)))

m_hat = dl.TrialFunction(V)
m_tilde = dl.TestFunction(V)

a1 = dl.inner(A1*dl.grad(m_hat), dl.grad(m_tilde))*dl.dx + dl.inner(m_hat, m_tilde)*dl.dx
a2 = dl.inner(A2*dl.grad(m_hat), dl.grad(m_tilde))*dl.dx + dl.inner(m_hat, m_tilde)*dl.dx

b = dl.inner(f,m_tilde)*dl.dx

m1 = dl.Function(V)
dl.solve(a1 == b, m1)

m2 = dl.Function(V)
dl.solve(a2 == b, m2)

dl.plot(m1, title="A = A1")
plt.show()
dl.plot(m2, title="A = A2")
plt.show()
```

```
Number of unknowns =  66049

--- Instant: compiling ---
```





Part c

When the diffusion matrix of the system, $A(x) = A_1$, the solution m seems to be an independent gaussian distribution. We comment on the fact that the diffusion matrix is a diagonal matrix with equal values on the diagonal. When $A(x) = A_2$, the solution m is "stretched" and seems to yield a dependent gaussian distribution. We note that the off-diagonals are not 0, and the diagonal values are not equal. It seems as though the diffusion matrix, $A(x)$ is acting as a covariance matrix for the displayed gaussian distribution in the image.