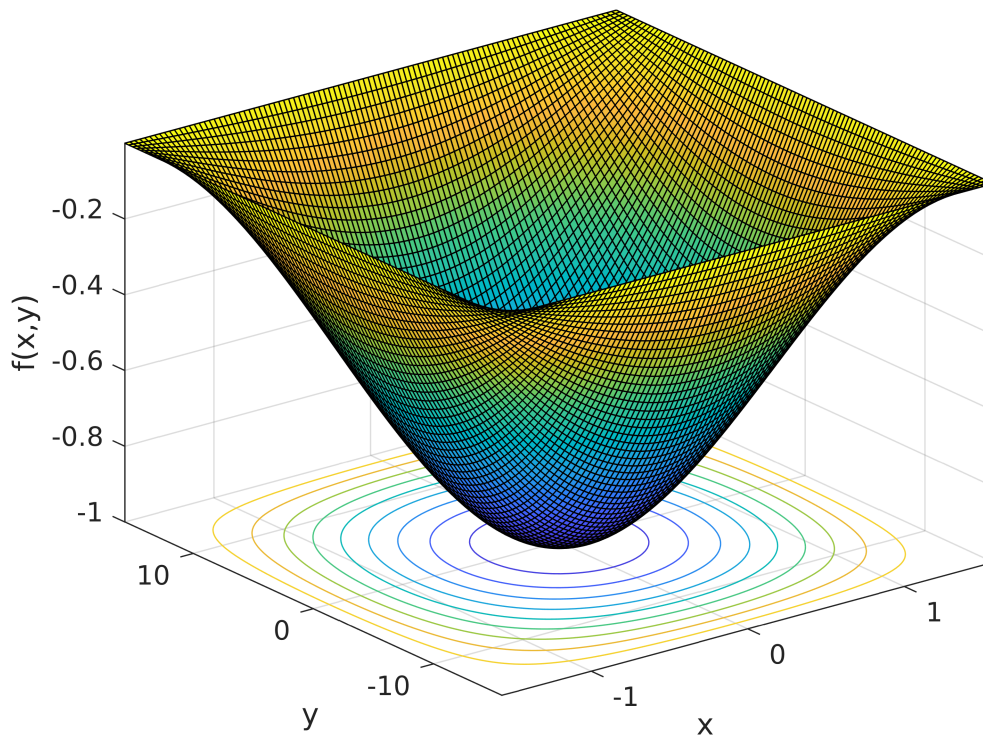


## Problem 1

```
clc;  
clear;  
close all;
```

### Part A,B,C - See Written Portion

```
% define the function to minimize  
f = @(x,y) -cos(x).*cos(y/10);  
  
% define the domain  
x = linspace(-pi/2,pi/2,100);  
y = linspace(-10*pi/2,10*pi/2,100);  
[xx,yy] = meshgrid(x,y);  
  
% plot the function  
figure;  
surfc(xx,yy,f(xx,yy));  
xlabel('x'); ylabel('y'); zlabel('f(x,y)');
```



## Part D and E

### Some Random Points

```
descent_methods(0.25,0.25);
```

Gradient Descent w/ exact line search optimum:  $x = 0$ ,  $y = 9.9573e-05$  @ iteration 779  
Gradient Descent w/o line search optimum:  $x = 0$ ,  $y = 9.9521e-05$  @ iteration 779  
Newton Method w/ exact line search optimum:  $x = 7.043e-09$ ,  $y = -6.906e-07$  @ iteration 2  
Newton Method w/o line search optimum:  $x = 7.0587e-08$ ,  $y = 5.1186e-07$  @ iteration 2

```
descent_methods(-0.5,0.5);
```

Gradient Descent w/ exact line search optimum:  $x = 0$ ,  $y = 9.9653e-05$  @ iteration 848  
Gradient Descent w/o line search optimum:  $x = 0$ ,  $y = 9.9597e-05$  @ iteration 848  
Newton Method w/ exact line search optimum:  $x = 4.387e-09$ ,  $y = -9.0439e-08$  @ iteration 3  
Newton Method w/o line search optimum:  $x = 9.8726e-14$ ,  $y = -9.8144e-13$  @ iteration 3

```
descent_methods(-0.75,-0.75);
```

Gradient Descent w/ exact line search optimum:  $x = 0$ ,  $y = -9.9174e-05$  @ iteration 889  
Gradient Descent w/o line search optimum:  $x = 0$ ,  $y = -9.9116e-05$  @ iteration 889  
Newton Method w/ exact line search optimum:  $x = 1.4239e-08$ ,  $y = 2.1381e-08$  @ iteration 3  
Newton Method w/o line search optimum:  $x = 3.3371e-08$ ,  $y = 3.3275e-07$  @ iteration 3

```
descent_methods(1,-1);
```

Gradient Descent w/ exact line search optimum:  $x = 0$ ,  $y = -9.904e-05$  @ iteration 918  
Gradient Descent w/o line search optimum:  $x = 0$ ,  $y = -9.9979e-05$  @ iteration 917  
Newton Method w/ exact line search optimum:  $x = -7.7678e-09$ ,  $y = 1.7162e-08$  @ iteration 3  
Newton Method w/o line search optimum:  $x = -6.9086e-21$ ,  $y = 6.9245e-20$  @ iteration 5

### In Positive-Definite Interval

```
descent_methods(pi/4,pi/2);
```

Gradient Descent w/ exact line search optimum:  $x = 0$ ,  $y = 9.9922e-05$  @ iteration 962  
Gradient Descent w/o line search optimum:  $x = 0$ ,  $y = 9.9858e-05$  @ iteration 962  
Newton Method w/ exact line search optimum:  $x = -5.3525e-09$ ,  $y = -1.6621e-08$  @ iteration 3  
Newton Method w/o line search optimum:  $x = 1.0299e-16$ ,  $y = 1.0299e-15$  @ iteration 4

```
descent_methods(pi/4,-pi/3);
```

Gradient Descent w/ exact line search optimum:  $x = 0$ ,  $y = -9.9459e-05$  @ iteration 922  
Gradient Descent w/o line search optimum:  $x = 0$ ,  $y = -9.9398e-05$  @ iteration 922  
Newton Method w/ exact line search optimum:  $x = -8.5819e-09$ ,  $y = 1.7903e-08$  @ iteration 3  
Newton Method w/o line search optimum:  $x = 1.5861e-19$ ,  $y = -1.5873e-18$  @ iteration 4

```
descent_methods(-pi/5,-pi/3);
```

Gradient Descent w/ exact line search optimum:  $x = 0$ ,  $y = -9.9355e-05$  @ iteration 922

Gradient Descent w/o line search optimum:  $x = 0$ ,  $y = -9.9294e-05$  @ iteration 922  
Newton Method w/ exact line search optimum:  $x = 7.0872e-09$ ,  $y = 1.5508e-08$  @ iteration 3  
Newton Method w/o line search optimum:  $x = 6.5793e-10$ ,  $y = 6.5789e-09$  @ iteration 3

```
descent_methods(-pi/5,pi/2);
```

Gradient Descent w/ exact line search optimum:  $x = 0$ ,  $y = 9.9817e-05$  @ iteration 962  
Gradient Descent w/o line search optimum:  $x = 0$ ,  $y = 9.9753e-05$  @ iteration 962  
Newton Method w/ exact line search optimum:  $x = 4.9803e-09$ ,  $y = -1.6208e-08$  @ iteration 3  
Newton Method w/o line search optimum:  $x = 6.3011e-09$ ,  $y = -6.3011e-08$  @ iteration 3

## Not in Positive-Definite Interval

```
descent_methods(-pi/2,pi/2);
```

Gradient Descent w/ exact line search optimum:  $x = 0$ ,  $y = 9.9791e-05$  @ iteration 963  
Gradient Descent w/o line search optimum:  $x = 0$ ,  $y = 9.9727e-05$  @ iteration 963  
Newton Method w/ exact line search optimum:  $x = 9.7454e-09$ ,  $y = 439.823$  @ iteration 18  
Newton Method w/o line search optimum:  $x = -1.5708$ ,  $y = 109.9557$  @ iteration 5

```
descent_methods(pi/2,3*pi);
```

Gradient Descent w/ exact line search optimum:  $x = 0$ ,  $y = 0.00044786$  @ iteration 1000  
Gradient Descent w/o line search optimum:  $x = 0$ ,  $y = 0.00044756$  @ iteration 1000  
Newton Method w/ exact line search optimum:  $x = 1.5708$ ,  $y = 9.8821$  @ iteration 1000  
Newton Method w/o line search optimum:  $x = 1.5708$ ,  $y = 15.708$  @ iteration 3

## In Positive-Definite Interval

### (showing restricted radius of convergence for non-line search)

```
descent_methods(pi/3,pi);
```

Gradient Descent w/ exact line search optimum:  $x = 0$ ,  $y = 0.00013759$  @ iteration 1000  
Gradient Descent w/o line search optimum:  $x = 0$ ,  $y = 0.0001375$  @ iteration 1000  
Newton Method w/ exact line search optimum:  $x = -8.9179e-09$ ,  $y = -5.8983e-08$  @ iteration 4  
Newton Method w/o line search optimum:  $x = -1.5708$ ,  $y = -15.708$  @ iteration 3

```
descent_methods(pi/4,2*pi);
```

Gradient Descent w/ exact line search optimum:  $x = 0$ ,  $y = 0.00028177$  @ iteration 1000  
Gradient Descent w/o line search optimum:  $x = 0$ ,  $y = 0.00028158$  @ iteration 1000  
Newton Method w/ exact line search optimum:  $x = -8.7033e-09$ ,  $y = -8.2597e-08$  @ iteration 3  
Newton Method w/o line search optimum:  $x = -4.7124$ ,  $y = -47.1239$  @ iteration 5

## Part F

We note that the rate of convergence of Newton's method is much faster than the method of steepest descent. However, steepest descent converges to the minimum of the function from any initial point, while Newton's method can only converge from where the initial point is in the positive-definite interval of the function. We also note that the line search does not affect the convergence of steepest descent by much.

However, for Newton's method, the radius of convergence is much smaller without doing the line search. This can be seen by choices of initial points inside the positive-definite interval where Newton's method only converges to the optimum with a line search.

```
function [c,v_sd_ne,v_sd_e,v_nm_ne,v_nm_e] = descent_methods(x0,y0)
% descent_methods - function defining descent methods to run on data
% runs steepest descent and newton method w/ and w/o line search

% define the function to minimize
f = @(x,y) -cos(x).*cos(y/10);

% define search direction for steepest descent (from 1C)
pk_sd = @(x,y) [-sin(x).*cos(y/10); -0.1*cos(x).*sin(y/10)];

% define search direction for newton method (from 1C)
pk_nm = @(x,y) [(-0.5*sin(2*x).*(cos(y/10).^2 + sin(y/10).^2))./...
    (cos(x-y/10).*cos(x+y/10));...
    (-5*sin(y/5).*(sin(x).^2 + cos(x).^2))./...
    (cos(x-y/10).*cos(x+y/10))];

% run line search algorithms
% set initial vectors
v_sd_ne = [x0;y0];
v_sd_e = [x0;y0];
v_nm_ne = [x0;y0];
v_nm_e = [x0;y0];
% set alpha values for no line search (this probably can be omitted...)
a_sd_ne = 1;
a_nm_ne = 1;

% save iteration # on convergence when norm of descent vector < 10^-6
lim = 1e-6;
c = [0,0,0,0];

% descent loop
for n=1:1000
    % steepest descent with no line search
    pk = pk_sd(v_sd_ne(1),v_sd_ne(2));
    if (norm(pk) > lim)
        v_sd_ne = v_sd_ne + a_sd_ne*pk;
        c(1) = c(1) + 1;
    end

    % steepest descent with exact line search
    x = v_sd_e(1); y = v_sd_e(2);
    pk = pk_sd(x,y);
    if (norm(pk) > lim)
        fes = @(a) f(x+a*pk(1),y+a*pk(2));
        a_sd_e = fminbnd(fes,1e-10,1);
        v_sd_e = v_sd_e + a_sd_e*pk;
        c(2) = c(2) + 1;
    end
end
```

```

end

% newton method with no line search
pk = pk_nm(v_nm_ne(1),v_nm_ne(2));
if (norm(pk) > lim)
    v_nm_ne = v_nm_ne + a_nm_ne*pk;
    c(3) = c(3) + 1;
end

% newton method with exact line search
x = v_nm_e(1); y = v_nm_e(2);
pk = pk_nm(x,y);
if (norm(pk) > lim)
    fes = @(a) f(x+a*pk(1),y+a*pk(2));
    a_nm_e = fminbnd(fes,1e-10,1);
    v_nm_e = v_nm_e + a_nm_e*pk;
    c(4) = c(4) + 1;
end
end
% display results
disp(['Gradient Descent w/ exact line search optimum: x = ',...
    num2str(v_sd_e(1)), ', y = ',...
    num2str(v_sd_e(2)), ' @ iteration ', num2str(c(2))]);
disp(['Gradient Descent w/o line search optimum: x = ',...
    num2str(v_sd_ne(1)), ', y = ',...
    num2str(v_sd_ne(2)), ' @ iteration ', num2str(c(1))]);
disp(['Newton Method w/ exact line search optimum: x = ',...
    num2str(v_nm_e(1)), ', y = ',...
    num2str(v_nm_e(2)), ' @ iteration ', num2str(c(4))]);
disp(['Newton Method w/o line search optimum: x = ',...
    num2str(v_nm_ne(1)), ', y = ',...
    num2str(v_nm_ne(2)), ' @ iteration ', num2str(c(3))]);
end

```