Derivation of EPI Distortion

This derivation is based off of the Jezzard and Balaban 1995 paper.

We first start with part of Equation 2 from the paper, the below equation represents the phase term for the exponential term of an EPI experiment:

$$2\pi\gamma\left(\int_0^t G_r(t)xdt + \int_0^t G_{pe}(t)ydt + \int_0^t \Delta B_0(x,y,z)dt\right)$$

The paper then defines $\Delta\phi_r$ and $\Delta\phi_{pe}$ to be the distance between each k-space point. For the readout, this implies the time difference between two adjacent points in the readout. For the phase-encode, this will be the time between the same x-points between the current and next readout lines.

First, we consider $\Delta\phi_r$. Between adjacent points of the readout, the time is given by the dwell time. So for some time in the readout t^*, we have its adjacent point $t^* + t_{dwell}$, where t_{dwell} is the dwell time. We note that G_{pe} is 0 during this time, so that entire term goes to 0. This gives us:

$$\begin{aligned}
\Delta\phi_r &= 2\pi\gamma\left(\int_0^{t^*+t_{dwell}} G_r(t)xdt + \int_0^{t^*+t_{dwell}} \Delta B_0(x,y,z)dt\right) - 2\pi\gamma\left(\int_0^{t^*} G_r(t)xdt + \int_0^{t^*} \Delta B_0(x,y,z)dt\right) \\
&= 2\pi\gamma\left((t^*+t_{dwell})G_r(t)x + (t^*+t_{dwell})\Delta B_0(x,y,z)\right) - 2\pi\gamma\left(t^* G_r(t)x + t^*\Delta B_0(x,y,z)\right) \\
&= 2\pi\gamma\left(t_{dwell}G_r(t)x + t_{dwell}\Delta B_0(x,y,z)\right)
\end{aligned}$$

Now we consider $\Delta\phi_{pe}$. The time between readout lines is given by the dwell time multiplied by twice the number of points in the readout axis of the FOV (which the paper defines as N), this is the time from the start of the current readout line to the end time of the next readout line. Adjacent points in k_y are structured this way, due to the "zig-zag"-like trajectory of an EPI experiment. So the time difference between readout lines is $2Nt_{dwell}$. However, the paper also considers the ramp time of the phase-encoding gradients, so this time is augmented by τ_{ramp}. So the total time between points is $2Nt_{dwell} + 2\tau_{ramp}$. We also note that since the k_x doesn't change between these two samples, it is easy to see that the readout Gradient will sum to 0 during this time, so we can ignore the G_r term. So for some time, t^* corresponding to the beginning of a readout line, and the time corresponding to the end of the next readout line, $t^* + 2Nt_{dwell} + 2\tau_{ramp}$, we have:

$$\begin{aligned}
\Delta\phi_{pe} = {} & 2\pi\gamma\left(\int_0^{t^*+2Nt_{dwell}+2\tau_{ramp}} G_{pe}(t)ydt + \int_0^{t^*+2Nt_{dwell}+2\tau_{ramp}} \Delta B_0(x,y,z)dt\right) - \\
& 2\pi\gamma\left(\int_0^{t^*} G_{pe}(t)ydt + \int_0^{t^*} \Delta B_0(x,y,z)dt\right) \\
& G_{pe} \text{ is only active for } \tau_{ramp} \text{ for each readout line.} \\
& = 2\pi\gamma\left(2\tau_{ramp}G_{pe}(t)y + (t^* + 2Nt_{dwell} + 2\tau_{ramp})\Delta B_0(x,y,z)\right) - \\
& 2\pi\gamma\left(\tau_{ramp}G_{pe}(t)ydt + t^*\Delta B_0(x,y,z)dt\right) \\
& = 2\pi\gamma\left(\tau_{ramp}G_{pe}(t)y + (2Nt_{dwell} + 2\tau_{ramp})\Delta B_0(x,y,z)\right)
\end{aligned}$$