

1

The right corner of voxel 3 is located at $x = 45$ mm and $y = 45$ mm, while the origin ($x = 0$ mm, $y = 0$ mm) is centered at the middle of voxel 5. Assuming the voxels are all the same size, we can see easily see that the distance to the boundary of the image is given by $s_{boundary} = 1.5s_{voxel}$, where $s_{boundary} = 45$ mm and s_{voxel} is the length of a side of a voxel. Then with some simple arithmetic, we can see that $s_{voxel} = 30$ mm. From that fact, we can easily generate the coordinates for each voxel:

Voxel 1: $x = -30$ mm, $y = 30$ mm

Voxel 2: $x = 0$ mm, $y = 30$ mm

Voxel 3: $x = 30$ mm, $y = 30$ mm

Voxel 4: $x = -30$ mm, $y = 0$ mm

Voxel 5: $x = 0$ mm, $y = 0$ mm

Voxel 6: $x = 30$ mm, $y = 0$ mm

Voxel 7: $x = -30$ mm, $y = -30$ mm

Voxel 8: $x = 0$ mm, $y = -30$ mm

Voxel 9: $x = 30$ mm, $y = -30$ mm

2

At $t = 0.3$ ms, an RF pulse has been applied, but no gradients have been applied yet. Since all voxels are on resonance, they experience the same phase. A 90° RF pulse in the $+x$ will tip the magnetization vectors into the $+y'$ direction. This means that each magnetization vector has a phase of 90° .

3

At $t = 0.6$ ms, a gradient in x direction at 29.359 mT/m was already applied for 0.1 ms. Using the equation:

$$\tilde{\phi} = 2\pi\gamma G_x x t$$

We can solve for the phase of the magnetization at each voxel. Note that since voxels (1, 4, 7), (2, 5, 8), and (3, 6, 9) respectively have the same x position, they will have the same phase.

Voxels 1, 4, 7

$$\begin{aligned}\tilde{\phi} &= 2\pi \left(42.6 * 10^6 \frac{Hz}{T} \right) \left(29.359 * 10^{-3} \frac{T}{m} \right) (-30 * 10^{-3} m) (0.1 * 10^{-3} s) \\ &= -23.5750 \text{ radians} = -1350.75^\circ \approx 89^\circ\end{aligned}$$

Adding the existing 90° phase, and the total phase at these voxels is 179° .

Voxels 2, 5, 8

$$\begin{aligned}\tilde{\phi} &= 2\pi \left(42.6 * 10^6 \frac{Hz}{T} \right) \left(29.359 * 10^{-3} \frac{T}{m} \right) (0 * 10^{-3} m) (0.1 * 10^{-3} s) \\ &= 0 \text{ radians} = 0^\circ\end{aligned}$$

Adding the existing 90° phase, and the total phase at these voxels is 90° .

Voxels 3, 6, 9

$$\begin{aligned}\tilde{\phi} &= 2\pi \left(42.6 * 10^6 \frac{Hz}{T} \right) \left(29.359 * 10^{-3} \frac{T}{m} \right) (30 * 10^{-3} m) (0.1 * 10^{-3} s) \\ &= 23.5750 \text{ radians} = 1350.75^\circ \approx 271^\circ\end{aligned}$$

Adding the existing 90° phase, and the total phase at these voxels is 361° or 1° .

4

At $t = 1.2$ ms, an additional gradient in the y direction at 19.572 mT/m was already applied for 0.3 ms. Using the equation:

$$\tilde{\phi} = 2\pi \gamma G_y y t$$

We can solve for the phase of the magnetization at each voxel. Note that since voxels (1, 2, 3), (4, 5, 6), and (7, 8, 9) respectively have the same y position, they will have the same additional phase added. Then we need to add the phase from the y gradient to the phase from the x gradient to get the total phase of the magnetization at each voxel.

Voxels 1, 2, 3

$$\begin{aligned}\tilde{\phi} &= 2\pi \left(42.6 * 10^6 \frac{Hz}{T} \right) \left(19.572 * 10^{-3} \frac{T}{m} \right) (30 * 10^{-3} m) (0.3 * 10^{-3} s) \\ &= 47.15 \text{ radians} = 2701.41^\circ \approx 181^\circ\end{aligned}$$

Voxels 4, 5, 6

$$\begin{aligned}\tilde{\phi} &= 2\pi \left(42.6 * 10^6 \frac{Hz}{T} \right) \left(19.572 * 10^{-3} \frac{T}{m} \right) (0 * 10^{-3} m) (0.1 * 10^{-3} s) \\ &= 0 \text{ radians} = 0^\circ\end{aligned}$$

Voxels 7, 8, 9

$$\begin{aligned}\tilde{\phi} &= 2\pi \left(42.6 * 10^6 \frac{Hz}{T} \right) \left(19.572 * 10^{-3} \frac{T}{m} \right) (-30 * 10^{-3} m) (0.3 * 10^{-3} s) \\ &= -47.15 \text{ radians} = -2701.41^\circ \approx 179^\circ\end{aligned}$$

Now we add the phases from each gradient and the RF pulse to get the total phase at each voxel at $t = 1.2$ ms:

$$\text{Voxel 1: } 179^\circ + 181^\circ = 360^\circ$$

$$\text{Voxel 2: } 90^\circ + 181^\circ = 271^\circ$$

$$\text{Voxel 3: } 1^\circ + 181^\circ = 182^\circ$$

$$\text{Voxel 4: } 179^\circ + 0^\circ = 179^\circ$$

$$\text{Voxel 5: } 90^\circ + 0^\circ = 90^\circ$$

$$\text{Voxel 6: } 1^\circ + 0^\circ = 1^\circ$$

$$\text{Voxel 7: } 179^\circ + 179^\circ = 358^\circ$$

$$\text{Voxel 8: } 90^\circ + 179^\circ = 269^\circ$$

$$\text{Voxel 9: } 1^\circ + 179^\circ = 180^\circ$$

Their phases are all unique!