Transfer Functions and Bode Plots in the Laplace Domain

Assignment 0

December 4, 2025

Instructions

For each transfer function in **Part A** and for the numerical example in **Part B**, you must:

1. Identify **poles** and **zeros**.
2. Sketch the **asymptotic Bode magnitude** and **phase** plots by hand.
3. Generate Bode magnitude and phase plots using **MATLAB**.
4. Insert into your solution document:
   - A clear **photo/scan** of your hand-sketched asymptotic Bode plots.
   - A **screenshot** of the corresponding MatLab Bode plots.

Optional reference videos (YouTube):

- Control System Lectures – Bode Plots, Introduction (Brian Douglas)
- Bode Plot: First-Order Transfer Function (KK Awasthi EduHub)
- Spring-Mass-Damper System, 1DOF – Transfer Function Derivation

1 Part A: Transfer Functions and Bode Plots

Use the following four transfer functions:

$$G_1(s) = \frac{10}{s+10} \quad \text{(simple first-order low-pass)},$$

$$G_2(s) = \frac{s-2}{s+10} \quad \text{(first-order with RHP zero)},$$

$$G_3(s) = \frac{100}{s^2+10s+100} \quad \text{(second-order underdamped)},$$

$$G_4(s) = \frac{0.1s+1}{0.01s+1} \quad \text{(lead-type compensator)}.$$

1.1 Problem A.1: $G_1(s) = \dfrac{10}{s+10}$

1. Find the pole and DC gain:

$$\text{Pole: } s = \underline{\hspace{3cm}}, \quad G_1(0) = \underline{\hspace{3cm}}$$

2. Sketch the **asymptotic Bode magnitude** and **phase** plots for $\omega \in [0.1, 100]$ rad/s.
3. Attach:

(a) Hand-sketched asymptotic Bode plots (magnitude and phase).
(b) Screenshot of MATLAB/Octave/Python Bode plots for $G_1(s)$.

1.2 Problem A.2: $G_2(s) = \dfrac{s-2}{s+10}$

1. Find the zero, pole, and DC gain:

   Zero: $s = \underline{\hspace{3cm}}$, Pole: $s = \underline{\hspace{3cm}}$, $G_2(0) = \underline{\hspace{3cm}}$

2. Sketch the **asymptotic Bode magnitude** and **phase** plots for $\omega \in [0.1, 100]$ rad/s.
3. Attach:

(a) Hand-sketched asymptotic Bode plots (magnitude and phase).
(b) Screenshot of MATLAB/Octave/Python Bode plots for $G_2(s)$.

4. **Very short question:** This system has a **right-half-plane zero**. One sentence: how does that usually affect the phase (compared to a left-half-plane zero)?

1.3 Problem A.3: $G_3(s) = \dfrac{100}{s^2+10s+100}$

1. Find the poles:

$$s_{1,2} = \underline{\hspace{6cm}}$$

2. Sketch the **asymptotic Bode magnitude** and **phase** plots for $\omega \in [0.1, 100]$ rad/s.
3. Attach:

(a) Hand-sketched asymptotic Bode plots (magnitude and phase).
(b) Screenshot of MATLAB/Octave/Python Bode plots for $G_3(s)$.

1.4 Problem A.4: $G_4(s) = \dfrac{0.1s+1}{0.01s+1}$

1. Find the zero and pole:

$$\text{Zero at } s = \underline{\hspace{3cm}}, \quad \text{Pole at } s = \underline{\hspace{3cm}}$$

2. Sketch the **asymptotic Bode magnitude** and **phase** plots for $\omega \in [0.1, 100]$ rad/s.

3. Attach:

(a) Hand-sketched asymptotic Bode plots (magnitude and phase).
(b) Screenshot of MATLAB/Octave/Python Bode plots for $G_4(s)$.

4. **Very short question:** Around the frequency between the zero and pole, does $G_4(s)$ tend to add **positive** phase (phase lead) or **negative** phase (phase lag)?

2 Part B: Mass–Spring–Damper Transfer Function

Consider the mechanical system shown below. A mass m is connected to a fixed wall via a spring (stiffness c) and a damper (damping coefficient d). An external force $F(t)$ is applied to the mass, producing displacement $x(t)$.

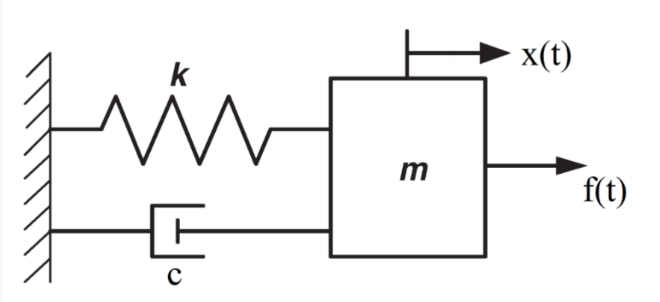

Figure 1: Mass–spring–damper system with input force $F(t)$ and output displacement $x(t)$.

B.1 Derive the Differential Equation and Transfer Function

1. Using Newton's second law and the diagram, write the differential equation relating $x(t)$ and $F(t)$:

   ____

2. Assume zero initial conditions and apply the Laplace transform to obtain an equation in $X(s)$ and $F(s)$:

   ____

3. Derive the transfer function:

$$G(s) = \frac{X(s)}{F(s)} = \underline{\hspace{6cm}}$$

B.2 Numerical Example and Bode Plots

For:

$$m = 1 \text{ kg}, \quad d = 4 \text{ N·s/m}, \quad c = 16 \text{ N/m},$$

1. Write the numerical transfer function $G(s)$:

$$G(s) = \underline{\hspace{7cm}}$$

2. Find the poles:

$$s_{1,2} = \underline{\hspace{8cm}}$$

3. Sketch the **asymptotic Bode magnitude** and **phase** plots for $\omega \in [0.1, 100]$ rad/s.
4. Generate Bode plots in MATLAB/Octave/Python and attach:

(a) Hand-sketched asymptotic Bode plots (magnitude and phase).
(b) Screenshot of MATLAB/Octave/Python Bode plots for this numerical example.

Optional MATLAB/Octave Template

```
s = tf('s');    % define Laplace variable

G1 = 10/(s + 10);
figure; bode(G1, {0.1, 100}); grid on;

G2 = (s - 2)/(s + 10);
figure; bode(G2, {0.1, 100}); grid on;

G3 = 100/(s^2 + 10*s + 100);
figure; bode(G3, {0.1, 100}); grid on;

G4 = (0.1*s + 1)/(0.01*s + 1);
figure; bode(G4, {0.1, 100}); grid on;
```