第一周 第二讲习题

from Problem Set 1.4

8 This matrix A has 3 independent columns. So C has the same 3 columns as A. What is the 3 by 3 matrix R so that $A = CR$? What is different about $B = CR$?

Upper triangular $A = \begin{bmatrix} 2 & 2 & 2 \\ 0 & 4 & 4 \\ 0 & 0 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 2 & 2 \\ 0 & 0 & 4 \\ 0 & 0 & 6 \end{bmatrix}$

12 Factor these matrices into $A = CR = (m \text{ by } r)\,(r \text{ by } n)$: all ranks equal to r.

$$A_1 = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \end{bmatrix} \quad A_2 = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 0 & 1 & 3 & 5 \end{bmatrix} \quad A_3 = \begin{bmatrix} 2 & 1 & 3 \\ 6 & 3 & 9 \end{bmatrix} \quad A_4 = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 2 & 2 & 0 \end{bmatrix}$$

19 Test the column-row matrix multiplication in equation (16) to find AB and BA:

$$AB = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad BA = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\boldsymbol{AB} = \begin{bmatrix} | & & | \\ \boldsymbol{a}_1 & \cdots & \boldsymbol{a}_n \\ | & & | \end{bmatrix} \begin{bmatrix} — & \boldsymbol{b}_1^* & — \\ & \vdots & \\ — & \boldsymbol{b}_n^* & — \end{bmatrix} = \boldsymbol{a}_1\boldsymbol{b}_1^* + \boldsymbol{a}_2\boldsymbol{b}_2^* + \cdots + \boldsymbol{a}_n\boldsymbol{b}_n^*. \tag{16}$$

columns $\boldsymbol{a}_k$ **rows $\boldsymbol{b}_k^*$** **Add columns $\boldsymbol{a}_k$ times rows $\boldsymbol{b}_k^*$**

from Problem Set 2.2

35 This matrix has a remarkable inverse. Find A^{-1} by elimination on $[A \;\; I]$. Extend to a 5 by 5 "alternating matrix" and guess its inverse; then multiply to confirm.

Invert $A = \begin{bmatrix} 1 & -1 & 1 & -1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ and solve $A\boldsymbol{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$.