

In this document I show how to obtain the maximum likelihood estimates of  $\beta_0$  and  $\beta_1$  using both the `optim` function and the `glm` function with the argument `family` set to `binomial`. The dataset `wcgsdata` is also available on Learn and it has several (exposure) variables. For now, we will only use the weight and CHD variables.

```
require(readxl)
data_wchs <- read_excel("wcgsdata.xls")
names(data_wchs)

## [1] "Id"      "Age0"    "Height0" "Weight0" "Sbp0"    "Dbp0"    "Chol0"
## [8] "Behpat0" "Ncigs0"  "Dibpat0" "Chd69"   "Typechd" "Time169" "Arcus0"

head(data_wchs)

## # A tibble: 6 x 14
##   Id Age0 Height0 Weight0 Sbp0 Dbp0 Chol0 Behpat0 Ncigs0 Dibpat0 Chd69
##   <dbl> <dbl>   <dbl>   <dbl> <dbl> <dbl> <chr>   <dbl>   <dbl>   <dbl> <dbl>
## 1 2001  49      73      150   110   76 225      2      25      1      0
## 2 2002  42      70      160   154   84 177      2      20      1      0
## 3 2003  42      69      160   110   78 181      3       0      0      0
## 4 2004  41      68      152   124   78 132      4      20      0      0
## 5 2005  59      70      150   144   86 255      3      20      0      1
## 6 2006  44      72      204   150   90 182      4       0      0      0
## # ... with 3 more variables: Typechd <dbl>, Time169 <dbl>, Arcus0 <chr>

#Log likelihood function logistic regression model (slide 12)
log_like <- function(param, d, x){
  beta0 <- param[1]
  beta1 <- param[2]
  sum((d*(beta0+beta1*x)) - log(1+exp(beta0+beta1*x)))
}

res_optim <- optim(c(0, 0), log_like,
                  d = data_wchs$Chd69, x = data_wchs$Weight0,
                  control = list(fnscale = -1), hessian = TRUE)

res_optim

## $par
## [1] -4.21609890 0.01043202
##
## $value
## [1] -884.4688
##
## $counts
## function gradient
##      101      NA
##
## $convergence
## [1] 0
##
## $message
## NULL
##
## $hessian
##           [,1]      [,2]
## [1,]    -235.0617 -40999.69
```

```
## [2,] -40999.6914 -7268930.98
#Inverse of the observed Fisher information matrix
# (the diagonal extract the standard errors of beta0 and beta1)
sqrt(diag(solve(-1*res_optim$hessian)))

## [1] 0.512516688 0.002914498
#Results using the glm function
res_glm <- glm(Chd69 ~ Weight0, family = "binomial",
               data = data_wchs)
out_glm <- summary(res_glm)
out_glm

##
## Call:
## glm(formula = Chd69 ~ Weight0, family = "binomial", data = data_wchs)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -0.7283  -0.4292  -0.3982  -0.3693   2.4146
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept) -4.21471    0.51206  -8.231  < 2e-16 ***
## Weight0      0.01042    0.00292   3.570 0.000356 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##      Null deviance: 1781.2  on 3153  degrees of freedom
## Residual deviance: 1768.9  on 3152  degrees of freedom
## AIC: 1772.9
##
## Number of Fisher Scoring iterations: 5

exp(res_glm$coefficients)

## (Intercept)      Weight0
## 0.01477667  1.01047871

exp(confint.default(res_glm, level = 0.95))

##              2.5 %      97.5 %
## (Intercept) 0.00541636 0.04031303
## Weight0      1.00471301 1.01627750

alpha <- 0.05
exp(out_glm$coefficients[2, 1] - qnorm(1-(alpha/2))* out_glm$coefficients[2, 2])

## [1] 1.004713

exp(out_glm$coefficients[2, 1] + qnorm(1-(alpha/2))* out_glm$coefficients[2, 2])

## [1] 1.016278
```