ELEMENTS OF CIVIL ENGINEERING AND MECHANICS

MODULE-2

EQUILIBRIUM OF FORCES

Equilibrium

When a stationary body is subjected to external forces and if the body remains in the state of rest under the action of forces, it is said to be in equilibrium

Equilibrant

A force which nullifies the effect of forces on a body is called equilibrant of forces. Sometimes the resultant of the force system is not equal to zero, it means that the body is not in the state of equilibrium. The external force which is required to keeps the body in equilibrium s known as equilibrant.

Principle of Equilibrium

According to this principle "**A body is said to be in equilibrium if the algebraic sum of all forces acting on the body is zero ad also if the algebraic sum of moments of forces about any fixed point is zero,**

I.e. $\Sigma F=0$, $\Sigma F_x=0$, $\Sigma F_y=0$ and $\Sigma M=0$

A body is said to be in equilibrium if there is no translation and rotation of the body under the application of external forces.

Conditions of equilibrium for different force systems

1. Coplanar concurrent force system- $\Sigma F_x=0$, $\Sigma F_y=0$

2. Coplanar Non-concurrent force system-- $\Sigma F_x=0$, $\Sigma F_y=0$,$\Sigma M=0$

3. Coplanar parallel force system-$\Sigma F=0$ $\Sigma M=0$

4. Three coplanar concurrent force-Lamis theorem

5. Coplanar Collinear forces- $\Sigma F=0$

6. Non-coplanar force system- $\Sigma F_x=0$, $\Sigma F_y=0$, $\Sigma F_z=0$ $\Sigma M=0$

LAMI'S THEOREM

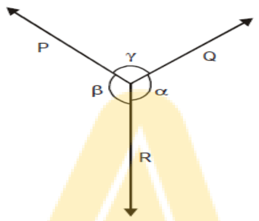
Statement:

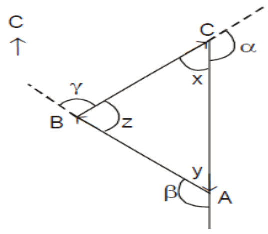
"If three force acting on a particle are in equilibrium, then each force is directly proportional to the sine of the angle between the other two forces".

Explanation:

Let P, Q and R three forces acting at a point keeping it in equilibrium, If α, β and γ are the angles opposite to each of them respectively.

$$\frac{P}{sin\alpha} = \frac{Q}{sin\beta} = \frac{R}{sin\gamma}$$





This law is a direct consequence of the triangle law. Since the forces in equilibrium, they can be represented by the sides of the triangle *ABC* taken in order. A general property of any triangle is that each side is proportional to the sine of the angle opposite to it. Thus in the triangle *ABC* drawn with sides parallel to the forces *P*, *Q* and *R*.

$$\frac{AB}{sinx} = \frac{BC}{siny} = \frac{AC}{sinz}$$

Here x, y and z are the angle if the triangle ABC. But by the triangle law of forces, the sides of the triangle are proportional to the respective force. From the above figure

$$\sin x = \sin(180-\alpha) = \sin\alpha$$

$$\sin y = \sin(180-\beta) = \sin\beta$$

$$\sin z = \sin(180-\gamma) = \sin\gamma$$

Hence,

$$\frac{P}{sin\alpha} = \frac{Q}{sin\beta} = \frac{R}{sin\gamma} = constant$$

Thus, if three forces acting on a particle are in equilibrium, each force is proportional to the sine of the angle between the other two forces.

Significance of Lamis theorem:

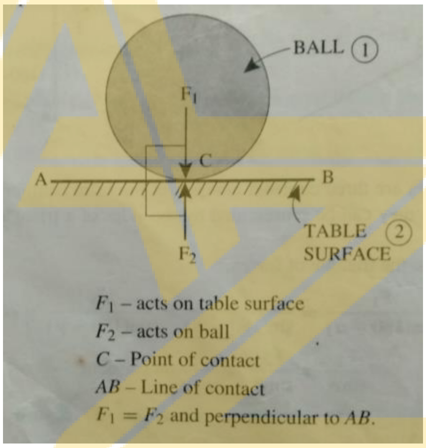
- It is used to find out the unknown forces where three concurrent and coplanar force acting on a body and the body should be in the state of equilibrium.

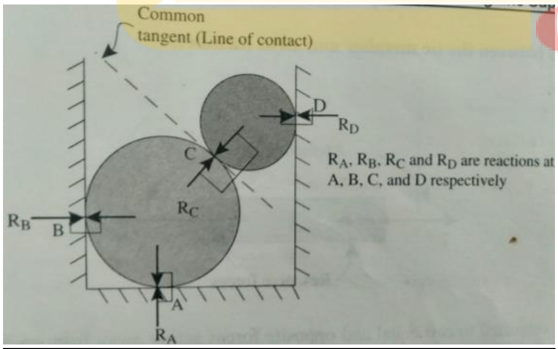
Note: Derivation of Lamis theorem refer class notes

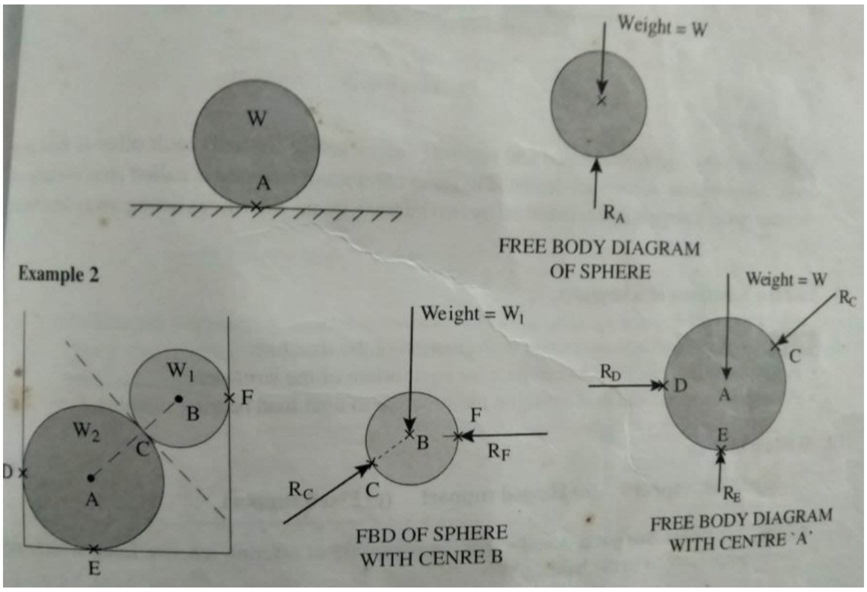
Triangle law:

"If three coplanar concurrent forces can be represented by the sides of a triangle both in magnitude and direction, taken in order, then the forces will be in equilibrium".

FREE BODY DIAGRAM (FBD): Free body diagram is nothing but a sketch which shows the various forces acting on a body. The forces acting on a body could be in form of weight, reactive forces contact forces etc. an example for Free Body Diagram is shown below.







<u>Significance of FBD:</u>

- It is an important step in solving the problems in mechanics since it helps to visualize all the forces acting on a single point.
- It is also useful in problems involving equilibrium of forces.

(Numerical Problems on coplanar concurrent force system and Lami's theorem)-Refer class notes

General procedure to solve problems:

1. Draw the FBD diagram for the given system of forces/system of objects inn equilibrium. Consider all forces acting on the system including support reactions.
2. If the forces are concurrent use $\Sigma F_x=0$, $\Sigma F_y=0$
3. If the foes are not concurrent use $\Sigma F_x=0$, $\Sigma F_y=0$ and $\Sigma M=0$
4. For three concurrent forces in equilibrium use Lamis theorem.

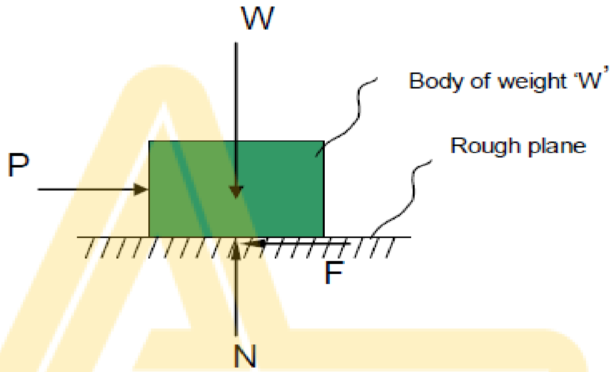
$$\frac{P}{sin\alpha} = \frac{Q}{sin\beta} = \frac{R}{sin\gamma} = constant$$

5. For an object hanging freely from the cable the tension in the cable will be equal to the weight of the object.
6. For equilibrium of a system of more than one objects draw free body diagram of all objects. Start solving with that F.B.D which has minimum number of unknowns

FRICTION

Whenever a body moves or tends to move over another surface or body, a force which opposes the motion of the body is developed tangentially **at the surface of contact**, such on opposing force developed is called friction or frictional resistance**.** The frictional resistance is developed due to the interlocking of the surface irregularities at the contact surface b/w two bodies.

Consider a body weighing W resting on a rough plane & subjected to a force P to displace the body.



Where,

P = Applied force.

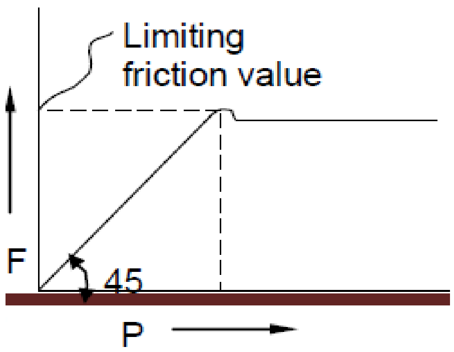
N = Normal reaction from rough surface.

F = Frictional resistance.

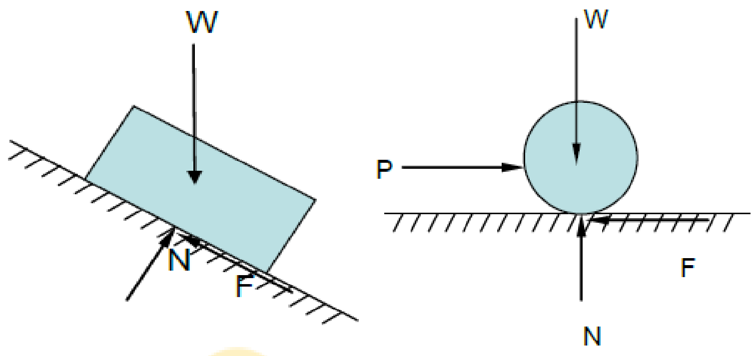
W = Weight of the body.

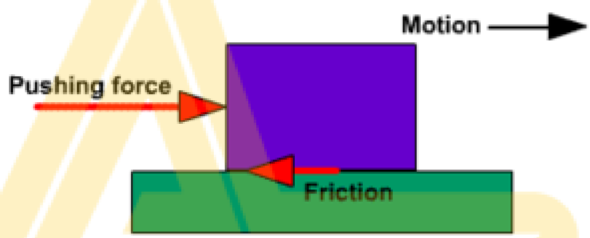
The body can start moving or slide over the plane if the **force P overcomes the frictional force.**

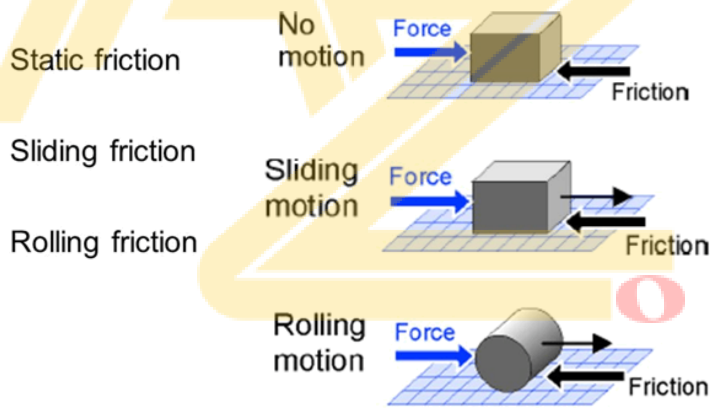
The frictional resistance developed is proportional to the magnitude of the applied force which is responsible for causing motion up to a certain limit.



From the above graph we see that as P increases, F also increases. However F cannot increase beyond a certain limit. Beyond this limit (Limiting friction value) the frictional resistance becomes constant for any value of applied force. If the magnitude of the applied force is less than the limiting friction value, the body remains at rest or in equilibrium. If the magnitude of the applied force is greater than the limiting friction value the body starts moving over the surface.







TYPES OF FRICTION

Based on the surface of contact frictional force is broadly classified as Dry Friction and Fluid Friction.

1) Dry Friction: it is the frictional force developed between two bodies when they are sliding one over the other. This is also called as Coloumb's Dry Friction.

Dry Friction further divided into two types:

(a) Solid/ Sliding Friction: It is the friction developed between two bodies having relative motion, as they are sliding one over the other. This friction force is greater to rolling friction in magnitude.

(b) Rolling friction: It is the friction developed between the two bodies when they are rolling one over the other the magnitude of rolling friction is always less than sliding friction

.Eg:Riding a bicycle

2) Fluid Friction: It is the frictional force developed between bodes because of the fluid introduced between them.

Fluid Friction further divided into two types:

(a) Viscous friction: The friction action on the body when the contact surface are completely separated by lubricant is called Viscous or Film Friction.

(b) Non-Viscous friction: The friction acting on the body when contact surface are lubricated with extremely thin layer of lubricant is called Non-Viscous Friction.

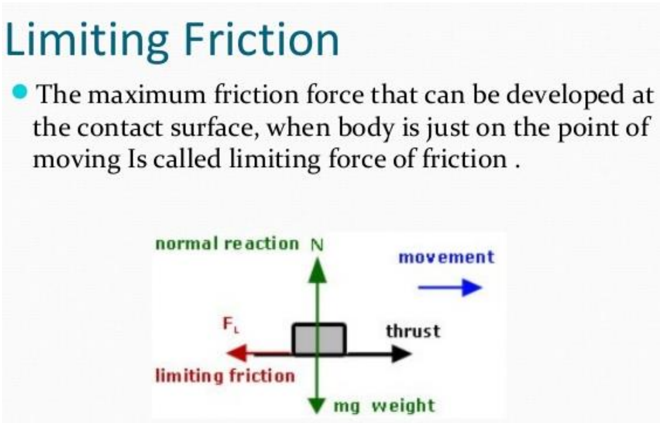
3) Static friction: It is the friction experienced between two bodies when both bodies are at rest.

4) Limiting Friction: the friction acting on a body which is just on the point or verge of sliding is called Limiting Friction.

5) Dynamical Friction: The friction acting on a body which is actually in motion is called Dynamical Friction or Kinetic Friction. Eg: Slipping on wet floor.

LIMITING FRICTION

The self-adjusting opposite and resisting friction F which opposes the sliding motion of one body over another, has a limiting value and if the applied force exceeds this values, the body begins to move. This limiting values of the force is called limiting friction and it this stage the body is in limiting equilibrium and just on the verge of motion.



<u>CO-EFFICIENT OF FRICTION (μ)</u>

It is the constant ratio which the limiting friction F bears to the normal reaction N called as co-efficient of friction.

It is defined by the relationship, $\mu = \frac{F}{N}$

Where,

μ – Represents co-efficient of friction

F – Represents frictional resistance

N – Represents normal reaction.

<u>LAWS OF STATIC FRICTION [LAWS OF DRY FRICTION (COLOUMB'S LAWS)]</u>

The frictional resistance developed between bodies having dry surfaces of contact obey certain laws called laws of static/dry friction.

They are as follows:

1) The frictional resistance depends upon the roughness or smoothness of the surface.

2) Frictional resistance acts in a direction opposite to the motion of the body.

3) The frictional resistance is independent of the area of contact between the two bodies.

4) The ratio of the limiting friction value (F) to the normal reaction (N) is a constant is called co-efficient of friction (μ).

$$\mu = \frac{F}{N}$$

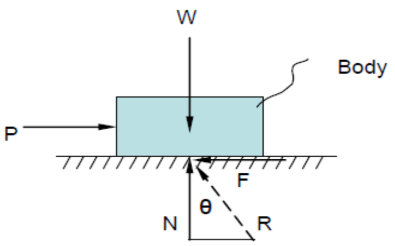
5) The magnitude of the frictional resistance developed is exactly equal to the applied force till limiting friction value is reached or where the bodies is about to move.

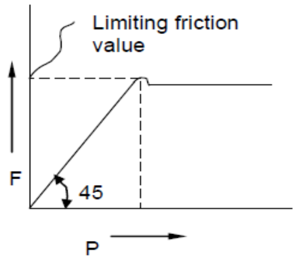
Explain the following terms

1. Angle of friction.
2. Angle of repose.
3. Cone of friction

1) ANGLE OF FRICTION (ϕ)

<u>Definition:</u> It is the angle made by the resultant of friction force (F) and normal reaction (N) force.





Consider a body weighing W placed on a horizontal plane. Let P be an applied force required to just move the body such that, frictional resistance reaches limiting friction value. Let R be resultant of F & N. Let θ be the angle made by the resultant with the direction of N. Such an angle θ is called the Angle of friction.

As P increases, F also increases and correspondingly θ increases. However, F cannot increase beyond the limiting friction value and as such θ can attain a maximum value only.

Let $\theta_{max} = \alpha$

Where α represents angle of limiting friction

$$\tan \theta_{max} = \tan\alpha = \frac{F}{N}$$

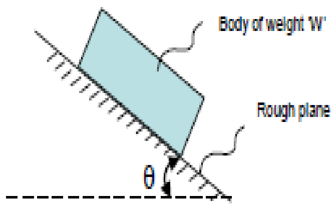
$$\text{But } \frac{F}{N} = \mu$$

Therefore $\mu = \tan\alpha$

i.e., co-efficient of friction is equal to the tangent of the angle of limiting friction.

2) ANGLE OF REPOSE (θ)

<u>Definition:</u> The maximum inclination of the plane with the horizontal, on which a body free from external forces can rest without sliding is called angle of repose.



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Let us consider a body of weight W which is placed on an inclined plane as shown in below figure. The object is just at the point of sliding down the plane when the angle of inclination is θ. The various forces acting on the object are self-weight, normal reaction and frictional force.

Let $\theta max = \Phi$

Where Φ = angle of repose

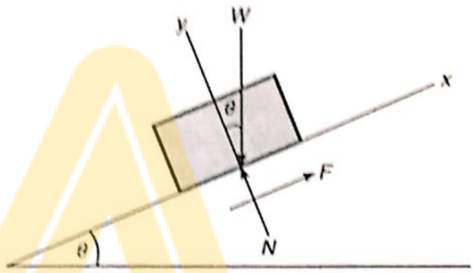


Figure 8.4 Angle of repose (θ).

Applying the conditions of equilibrium,

$$\Sigma F_x = 0; \ \Sigma F_y = 0$$

Resolving forces along the x-axis,

$$-F + W \sin \theta = 0$$

or
$$F = W \sin \theta \tag{8.2}$$

Resolving forces along the y-axis,

$$N - W \cos \theta = 0$$

or
$$N = W \cos \theta \tag{8.3}$$

We know that

$$\mu = \frac{F}{N}$$

⇒
$$\mu = \frac{W \sin \theta}{W \cos \theta} = \tan \theta \tag{8.4}$$

or
$$\tan \phi = \tan \theta$$

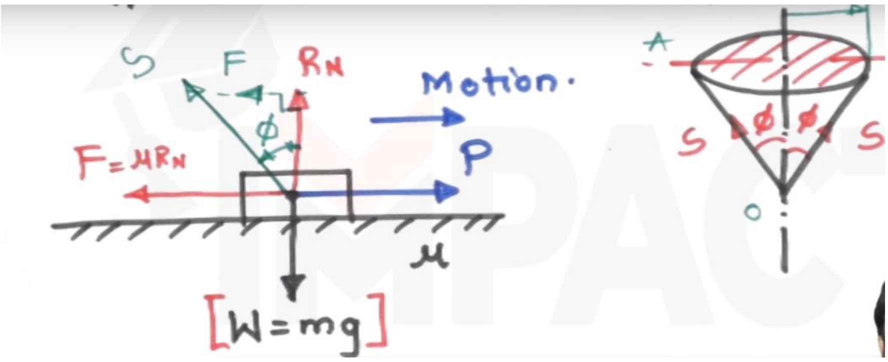
or
$$\phi = \theta$$

It is evident from Eqs. (8.1) and (8.4) that

Angle of friction = Angle of repose

3) CONE OF FRICTION

It is an imaginary cone of revolution formed by the resultant reaction at the contact point. If the resultant reaction is rotated about normal reaction force, it will form a right circular cone of angle 2Φ known as cone of friction.



Consider a body weighting W resting on a rough horizontal surface. Let P be a force required to just move the body such that frictional resistance reaches limiting value. Let R be the resultant of F & N making an angel with the direction of N.

If the direction of P is changed the direction of F changes and accordingly R also changes its direction. If P is rotated through 360^{o}, R also rotates through 360^{o} and generates an imaginary cone called cone of friction.

Note: In this discussion, all the surface that been consider are rough surfaces, such that, when the body tends to move frictional resistance opposing the motion comes into picture tangentially at the surface of contact in all the examples, the body considered is at the verge of moving such that frictional resistance reaches limiting value. We can consider the body to be at rest or in equilibrium & we can still apply conditions of equilibrium on the body to calculate unknown force.

(Numerical Problems on single and two blocks on inclined planes/friction)-Refer class notes

Tips to solve the problems

I. Draw a free body diagram
II. Draw the reference axis. We have to choose the reference axis in such a way that one of the axis must be long the direction of motion.

The following forces should be considered while drawing the free body diagram

i) Self-
yweight always acts **vertically downwards**
ii) External or internal forces
iii) In a rough surface –**a)Frictional force** is always opposite to the direction of motion at the contact surface and is parallel to the contact surface
b) Normal reaction is always perpendicular to the contact surface

iv) Write the algebraic sum of the forces along the X-axis i.e. $\Sigma F_x=0$

iv) Write the algebraic sum of the forces along the Y-axis i.e. $\Sigma F_y=0$