

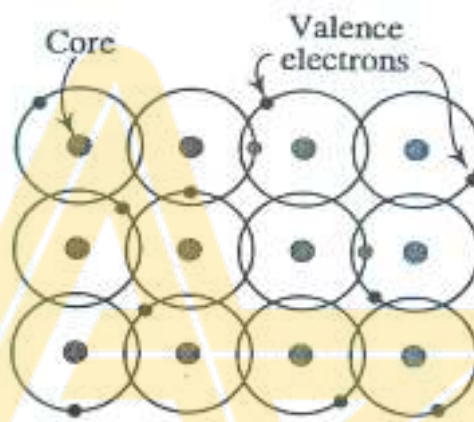
## Module – V

# Material Science

### Free-electron concept

A metal is made up of atoms and these atoms consist of valence electrons, which are responsible for electrical conduction in the bulk state of the metal. **For e.g.:** A copper atom consists of 29 electrons out of which 28 electrons are bound in the 3 shells forms the core and one electron remaining in the fourth shell is the free valence electron. As a whole an atom is neutral.

When a large number of copper atoms join to form a metal the boundaries of the neighboring atoms slightly overlap on each other. Due to this, the valence electrons find continuity from atom to atom and can move easily throughout the body of the metal, but are restricted within the boundaries of the solid.



Since each atom contributes equal number of electron, there will be a very large number of electrons which are free in a metal. Such electrons are called as **free electrons** and they account for the bulk properties of the metal such as electrical conductivity, thermal conductivity etc. They are also called conduction electrons.

The disconnection of the valence electrons results in a virtual loss of a negative charge for that atom so that it becomes a positive ion. The array of such ions forms a three dimensional structure called **lattice**.

Under thermal equilibrium condition, the free electrons in a solid are in a state of random motion. At temperature T, they possess an average kinetic energy given by

$$\frac{1}{2}mv_{th}^2 = \frac{3}{2}kT$$

Where  $v_{th}$  is the mean **thermal velocity**.

### **Basic Terminologies**

#### **1. Bound Electrons:**

All the valence electrons in an isolated atom are bound to their parent nuclei are called as bound electrons.

## 2. Free electrons:

Electrons which moves freely or randomly in all directions in the absence of external field in a metal.

## 3. Drift Velocity

The average velocity acquired by the free electron in a particular direction during the presence of electric field

## 4. Electric Field (E):

The electric field E of a conductor having uniform cross section is defined as the potential drop (V) per unit length (L).

$$\text{i.e., } E = V / L \quad \text{V/m}$$

**5. Current density (J):** It is defined as the current per unit area of cross section of an imaginary plane held normal to the direction of the flow of current in a current carrying conductor.

$$J = I / A \quad \text{Am}^{-2}$$

**6. Relaxation time ( $\tau_r$ )** The average time in which the drift velocity of an electron decays to  $1/e$  times of its initial velocity is known as relaxation time (where  $e = 2.71$ ). It is of the order  $10^{-14}$  sec

**7. Mean Collision time ( $\tau$ ):** The average time taken by electrons between two successive collisions during their random motion is called mean collision time, it is denoted by  $\tau$

**8. Mean free path ( $\lambda$ ):** The average distance traveled by electrons between two successive collisions during their random motion is called mean free path, it is denoted by  $\lambda$

## Classical Free electron Theory (Drude – Lorentz Theory):

This theory was developed by Drude and Lorentz during 1900. According to this theory, a metal consists of electrons which are free to move about in the crystal like molecules of a gas in a container. Mutual repulsion between electrons is ignored and hence potential energy is taken as zero. Therefore the total energy of the electron is equal to its kinetic energy.

## **Assumptions of Classical free electron theory:**

1. Atoms in a metal are considered to be made up of ion cores which are nuclei surrounded by inner electrons excluding the valence electrons and the valence electrons which are free to move anywhere inside the sample.
2. The effect of electric potential due to the positive ion cores on the electrons is considered to be constant and hence neglected.
3. The electrostatic force of repulsion between the electrons and force of attraction between the electron and the ion core is neglected.
4. The electrons in the metal are considered equivalent to molecules in a gas and the gas laws are made applicable to the electrons in a metal. Accordingly, the

distribution of energy and velocity is assumed to follow Maxwell-Boltzmann statistics.

$$\frac{3}{2}kT = \frac{1}{2}mv^2$$

### **Expression for Drift Velocity:**

$$v_d = \frac{eE}{m} \tau$$

### **EXPRESSION FOR CURRENT IN A CONDUCTOR**

**Current flowing through a conductor is given by**

$$I = nev_d A$$

**A** - area of cross section of a conductor,

**n**- number of free electrons per unit volume in the metal

**I** - current flowing through the conductor

**e**- is the charge of electron; **v<sub>d</sub>** - drift velocity of the electrons

### **ELECTRICAL CONDUCTIVITY**

The current density per unit electric field is called as electrical conductivity and is given by

$$\sigma = \frac{ne^2 \tau}{m}$$

n - the number of electrons/unit volume; e- charge of electron

**τ**- mean collision time or relaxation time; m – mass of the electrons

### **MOBILITY OF ELECTRONS**

The **mobility of electrons** is defined as the magnitude of the drift velocity acquired by the electrons in a unit electric field.

$$\therefore \mu = \frac{v_d}{E} = \frac{e\tau}{m}$$

### **RELATION BETWEEN CONDUCTIVITY AND MOBILITY**

$$\sigma = (ne^2 \tau / m)$$

$$\sigma = (ne)(e \tau / m)$$

$$\sigma = ne\mu$$

## **Failures of Classical Free electron Theory:**

Even though the theory is successful in explaining some experimental facts such as electrical & thermal conductivity in metal, it failed to explain other experimental facts such as

### **1. Specific Heat:**

At constant volume, the specific heat,  $C_v = \frac{3}{2}R$ . But experimentally it was found that the contribution to the specific heat of a metal by its conduction electrons was  $C_v = 10^{-4}RT$ . Thus the value of  $C_v$  as per prediction is far higher than the experimentally observed value and also the theory predicts that the specific heat does not depend on temperature.

### **2. Temperature Dependence of electrical conductivity:**

Experimentally it was found that

$$\sigma \propto \frac{1}{T} \rightarrow (1)$$

But according to the assumption of classical free electron theory,

$$\frac{3}{2}kT = \frac{1}{2}mv_{th}^2 \quad \text{or} \quad v_{th} = \sqrt{\frac{3kT}{m}}$$

i.e.  $v_{th} \propto \sqrt{T}$

But  $\tau \propto \frac{1}{v_{th}}$

$$\therefore \tau \propto \frac{1}{\sqrt{T}} \quad \text{But} \quad \sigma = \frac{ne^2\tau}{m}$$

$$\therefore \sigma \propto \tau$$

Or  $\sigma \propto \frac{1}{\sqrt{T}} \rightarrow (2)$

From (1) and (2) it is clear that the prediction of classical free electron theory is not agreeing with the experimental observations.

### **3. Dependence of Electrical conductivity on Electron Concentration:**

As per the classical free electron theory, the electrical conductivity  $\sigma$  is given

by 
$$\sigma = \frac{ne^2\tau}{m}$$

where  $n$  is the electron concentration,  $\therefore \sigma \propto n$

Let us consider the case of copper and aluminum. The electron concentration in Al is  $18.06 \times 10^{28}/\text{m}^3$  whereas in Cu it is  $8.45 \times 10^{28}/\text{m}^3$ . But the conductivity of Cu is  $5.88 \times 10^7 \Omega^{-1}\text{m}$  and that of Al is  $3.65 \times 10^7 \Omega^{-1}\text{m}$ . Hence the theory failed to account for the exact dependence of conductivity on electron concentration.

## QUANTUM FREE ELECTRON THEORY

In 1928, Arnold Sommerfeld succeeded in overcoming many of the drawbacks of the classical free electron theory by using Pauli's exclusion principle and is known as quantum free electron theory.

The main **assumptions** of quantum free electron theory are

1. The energy values of the conduction electrons are quantized.
2. The distribution of electrons in the various allowed levels occur as per Pauli's exclusion principle.
3. The electrons travel in a constant potential inside the metal but stay confined within its boundaries.
4. The attraction between the electrons and the lattice ions and the repulsion between the electrons are ignored.

### Energy bands in solids

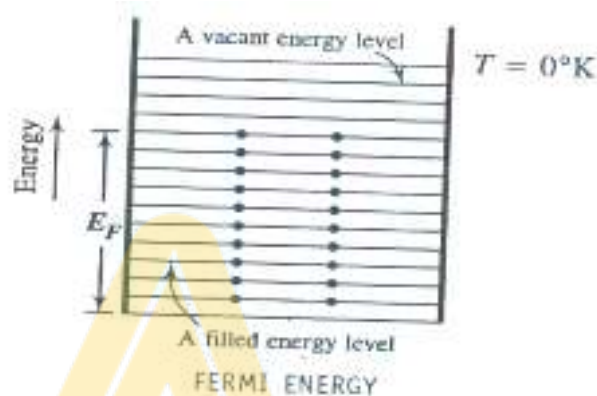
- The discrete energy level of an atom becomes bands during the formation of solid due to mutual influence of constituent atoms
- Each band consists of a large number of energy levels which correspond to a range of energy values.
- The bands are separated by certain gaps called **forbidden bands** or **energy band gaps**.
- The electron can occupy only those energy levels which lie within an energy band.
- Because of quantization rules there is a limit on the number of electrons which can fill an energy band.
- If an energy band contains the maximum quota of electrons, it is said to be filled band
- An empty band or partially filled band provides energy levels to which electrons can be energized from lower levels or bands.
- In every energy band diagram, there will be a band called valence band below which all the bands are occupied and above which all the bands are empty.
- The empty band which is immediately above the valence is called conduction band.
- The gap between the valence band the conduction band is called the **forbidden band or energy gap**.

### FERMI ENERGY:

There are 'N' allowed energy levels for free electrons in a metal which are separated by energy differences that are the characteristics of the material and are quantized. As per Pauli's exclusion principle, each allowed energy level can accommodate a maximum of two electrons with opposite spin. The filling up of the electrons in the energy levels should be undertaken from the lowest energy level. So, in each energy level there are two electrons one with spin up and other with spin down. However there are still higher energy levels left vacant.

“The energy of the highest occupied energy level at zero degree absolute is called **Fermi energy ( $E_F$ )** and the energy level is referred as **Fermi level**”

At absolute zero temperature i.e. when the metal is not under the influence of any electrical field or thermal energy, no electrons will be present above the Fermi level. They are completely empty and those levels below Fermi level are completely filled.



### **Fermi – Dirac statistics:**

In a metal there are large number of free electrons and each of them possesses an energy corresponding to the energy state of the valence band. Under thermal equilibrium the free electrons acquire energy obeying statistical rule known as Fermi – Dirac statistics. The electrons are identical particles and indistinguishable particles. Fermi – Dirac statistics permits the evaluation of probability of finding electrons occupying energy levels in a certain energy range. This evaluation is done through a function called Fermi factor.

### **Fermi Factor:**

At temperature above absolute zero, the material will be receiving thermal energy from the surroundings. So the electrons just below the Fermi level absorb the thermal energy and will occupy the higher empty levels. The electrons occupying energy levels far below the Fermi level cannot absorb this energy because there are no unoccupied higher energy levels into which the electrons can come into, when their energies increase by small amount.

Though such excitations seem to be random the resulting distribution of electrons in various energy levels after excitation will be systematic. The distribution is governed by a statistical function when the system is in thermal equilibrium (steady state).

The probability ' $f(E)$ ' that a given energy state with energy ' $E$ ' is occupied at a steady temperature is given by

$$f(E) = \frac{1}{e^{\frac{E-E_F}{kT}} + 1}$$

Here  $f(E)$  is called Fermi factor.

“**Fermi factor** is defined as the probability of occupation of a given energy state for a material in thermal equilibrium”.

### **Dependence of Fermi factor on temperature and effect on occupancy of energy levels:**

The probability ‘ $f(E)$ ’ that a given energy state with energy ‘ $E$ ’ is occupied at a steady temperature is given by

$$f(E) = \frac{1}{e^{\frac{E-E_F}{kT}} + 1}$$

The different cases of distribution are as follows;

#### **CASE1: Probability of occupation for $E < E_F$ at $T = 0K$**

When  $T = 0K$  and  $E < E_F$ , we have

$$f(E) = \frac{1}{e^{-\infty} + 1} = \frac{1}{0 + 1} = 1$$

$$\therefore f(E) = 1, \text{ for } E < E_F$$

It means that the energy level is certainly occupied and  $E < E_F$  applies to all the energy levels below ‘ $E_F$ ’.

#### **CASE2: Probability of occupation for $E > E_F$ at $T = 0K$**

When  $T = 0K$  and  $E > E_F$ , we have

$$f(E) = \frac{1}{e^{+\infty} + 1} = \frac{1}{\infty + 1} = 0$$

$$\therefore f(E) = 0, \text{ for } E > E_F$$

∴ At  $T = 0K$ , all the energy levels above the Fermi level are unoccupied.

Thus at  $T = 0K$ , the variation of  $f(E)$  for different energy values, becomes a step function as shown in the figure below

#### **CASE 3: Probability of occupation at ordinary temperature:**

At ordinary temperatures  $f(E)$  remains 1 for  $E \ll E_F$ , and starts decreasing from 1 as ‘ $E$ ’ becomes closer to  $E_F$  as shown in fig.

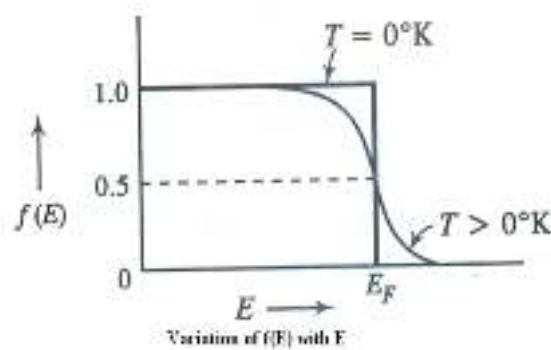
At  $E = E_F$  we have,

$$f(E) = \frac{1}{e^0 + 1} = \frac{1}{1 + 1} = \frac{1}{2}$$

∴ The value of  $f(E)$  becomes  $\frac{1}{2}$  at  $E = E_F$



Thus Fermi energy is the most probable energy or the average energy of the electrons which undergo transitions across the Fermi level at temperature above zero degree absolute.



### **DENSITY OF STATES:**

In a solid material, the permitted energy levels are in terms of bands. Each band is spread over an energy range of few eVs. And the number of energy levels in each band is extremely large. Because of such distribution, the energy values appear to be virtually continuous over the band. A closer look reveals that the energy levels are not evenly distributed in the band. i.e. the density of energy levels in the band varies with the energy. This is realized through a function known as **density of states function** denoted as  **$g(E)$** .

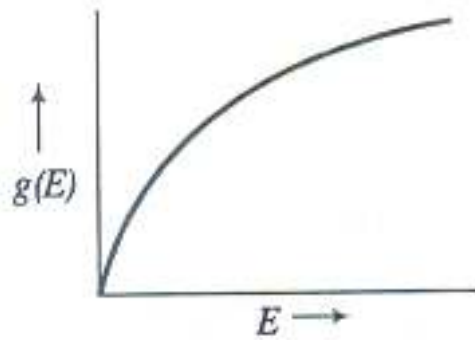


**“It is the number of allowed energy levels per unit energy interval in the band associated with the material per unit volume”.**

The number of states in an energy range  $E$  and  $E+dE$  for the electrons in a 3 – dimensional solid of unit volume or Density of states in the energy interval  $dE$  at ‘ $E$ ’ is given by

$$g(E)dE = \left[ \frac{8\sqrt{2}\pi m^{3/2}}{h^3} \right] E^{1/2} dE$$





### FERMI ENERGY AT 0K:

Let  $g(E)$  be the density of states function and  $f(E)$  be the probability of occupation of an energy level 'E' at temperature 'T'. Then the number of electrons per unit volume possessing the energy in the range E to E+dE is given by

$$N(E)dE = g(E)dE \times f(E) \text{ ----- (1)}$$

Let 'n' be the number electrons per unit volume of the material occupying the energy levels from  $E = 0$  to Fermi level at 0K ( $E_{F0}$ ) and is evaluated as

$$n = \int_0^{E_{F0}} N(E)dE$$

$$n = \int_0^{E_{F0}} g(E)dE \times f(E)$$

But,  $f(E) = 1$  at  $T = 0K$ .

Therefore, 
$$n = \int_0^{E_{F0}} g(E)dE \times 1$$

Substituting for  $g(E)dE$  we get

$$n = \int_0^{E_{F0}} \frac{8\sqrt{2}\pi m^{3/2}}{h^3} E^{1/2} dE$$

$$n = \frac{8\sqrt{2}\pi m^{3/2}}{h^3} \int_0^{E_{F0}} E^{1/2} dE$$

$$n = \frac{8\sqrt{2}\pi m^{3/2}}{h^3} \times \left[ \frac{2}{3} E^{3/2} \right]_0^{E_{F0}}$$

$$n = \frac{8\sqrt{2}\pi m^{3/2}}{h^3} \times \frac{2}{3} (E_{F0})^{3/2}$$

Simplifying,

$$(E_{F0})^{3/2} = \frac{h^3}{(8m)^{3/2}} \left( \frac{3n}{\pi} \right)$$

$$(E_{F0}) = \frac{h^2}{(8m)} \left( \frac{3n}{\pi} \right)^{2/3} \text{ is the Fermi energy at 0K.}$$

Where 'n' is the no of electrons per unit volume 'm' is the mass of the electron.

### **Fermi energy at T > 0°K:**

The Fermi energy 'E<sub>F</sub>' at any temperature 'T' is given by

$$E_F = E_{F0} \left[ 1 - \frac{\pi^2}{12} \left( \frac{kT}{E_{F0}} \right)^2 \right]$$

Since the 2<sup>nd</sup> term is very small except for very high temperatures, we can write

$$E_F = E_{F0}$$

### **Fermi Velocity (v<sub>F</sub>):**

"The velocity of the electrons which occupy Fermi level (or possess Fermi energy) is called Fermi Velocity."

$$\therefore E_F = \frac{1}{2} m v_F^2$$

$$\text{Or } v_F = \left( \frac{2E_F}{m} \right)^{1/2}$$

### **Fermi temperature (T<sub>F</sub>):**

"Fermi temperature (T<sub>F</sub>) is the temperature at which the average thermal energy of the free electrons in a solid becomes equal to the Fermi energy at 0°K.

i.e. at T = T<sub>F</sub>, kT<sub>F</sub> = E<sub>F0</sub>

But  $E_{F0} \approx E_F$

$$\therefore kT_F = E_F$$

$$\text{Or } T_F = \frac{E_F}{k}$$

### **Expression for electrical resistivity and conductivity based on quantum free electron theory**

According to Sommerfeld, the free electrons obey Fermi – Dirac statistics. Hence the expression for electrical conductivity can be written as

$$\sigma = \frac{ne^2 \lambda}{m^* v_F} ; \text{Where } m^* \text{ is the effective mass of electrons.}$$

Also, the expression for resistivity is given by

$$\rho = \frac{m^* v_F}{ne^2 \lambda}$$

### **Merits of quantum free electron theory:**

The quantum free electron theory is successful in explaining the drawbacks of classical free electron theory:

#### **1. Specific heat capacity:**

As per classical free electron theory all the conduction electrons are capable of absorbing the heat energy which results in a large value of specific heat ( $C_v = \frac{3}{2}R$ ). But according to quantum free electron theory only those electrons which are close to  $E_F$  are capable of absorbing the heat energy to get excited to higher energy levels and hence the specific heat value becomes very small.

It was shown that

$$C_v = \left( \frac{2k}{E_F} \right) RT$$

Taking the typical value of  $E_F = 5\text{eV}$ , we get

$$\left( \frac{2k}{E_F} \right) \approx 10^{-4}$$

$$\therefore C_v = 10^{-4} RT$$

This is experimental value.

#### **2. Temperature Dependence of electrical conductivity:**

According to classical theory

$$\sigma \propto \frac{1}{\sqrt{T}}$$

According to Quantum free electron theory, the electrical conductivity is given by

$$\sigma = \frac{ne^2 \lambda}{m^* v_F} \rightarrow (1)$$

We have ' $E_F$ ' temperature independent.  $\therefore$  ' $v_F$ ' also temperature independent. But ' $\lambda$ ' is temperature dependent. The nature of dependence of ' $\lambda$ ' and ' $T$ ' can be analyzed as follows.

During the conduction electrons scattered by the vibration of lattice ions and the displacement of ions takes place equally in all directions. If 'r' is the amplitude larger area of cross section should scatter more efficiently then the means free path of the electron will reduce.

$$\therefore \lambda \propto \frac{1}{\pi r^2}$$

But  $\pi r^2 \propto T$

$$\therefore \lambda \propto \frac{1}{T}$$

Comparing this with equation (1) we have

$$\sigma \propto \lambda \propto \frac{1}{T}$$

$$\therefore \sigma \propto \frac{1}{T}$$

### **3. Electrical conductivity and electron concentration:**

We have equation for the electrical conductivity

$$\sigma = \frac{ne^2 \lambda}{m^* v_F}$$

⇒ The value of 'σ' depends on both 'n' and ' $\lambda/v_F$ '.

If we compare the cases of copper and aluminium, the value of 'n' for aluminium is 2.13 times higher than that of copper. But the value of ' $\lambda/v_F$ ' for copper is about 3.73 times higher than that of 'Al'. Thus the conductivity of copper exceeds that of 'Al'. Hence the dependence of electrical conductivity on electron concentration is correctly explained by the quantum free electron theory.

### **Important Questions:**

1. What are the assumptions of quantum free electron theory?
2. Discuss the merits of quantum free electron theory.
3. Explain the failures of classical free electron theory.
4. Define Fermi factor. Explain the dependence of Fermi factor on temperature and energy.
5. Define Fermi energy. Obtain an expression for Fermi energy at 0K.

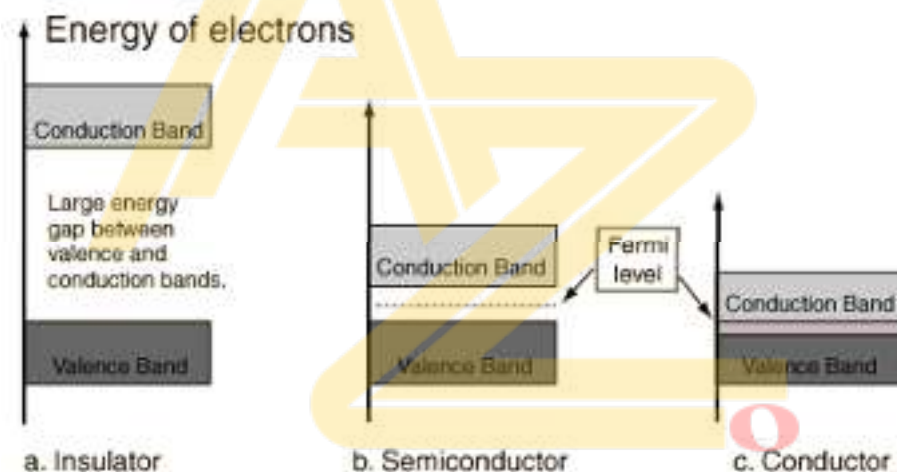
## PHYSICS OF SEMICONDUCTORS

Conductivity of solids can be explained on the basis of band theory. The important energy bands in solids are

1. **Valence Band:** The electron in the outer most orbit of an atom is known as valence electron. The range of energies possessed by valence electrons is known as valence band.. The valence band may be completely or partially filled.

2. **Conduction Band:** The range of energies possessed by conduction electrons is known as conduction band. In some metals, the valence electron may be loosely packed and may get detached to become free electron. These free electrons are responsible for conduction of current in a conductor and are hence called conduction electrons.

**Forbidden energy band/gap:** The separation between bottom of conduction band and top of valence band in the energy level diagram of solids is known as forbidden energy band. There are no electrons in the forbidden energy gap.



### **SIGNIFICANCE OF BAND GAP $E_g$**

The band gap energy  $E_g$  is minimum amount of energy required for breaking a covalent bond and to excite an electron from valence band to conduction band. The energy required to break a covalent bond in Ge is 0.72eV and for Si it is 1.12eV at 300K.

### **Classification of semiconductors:**

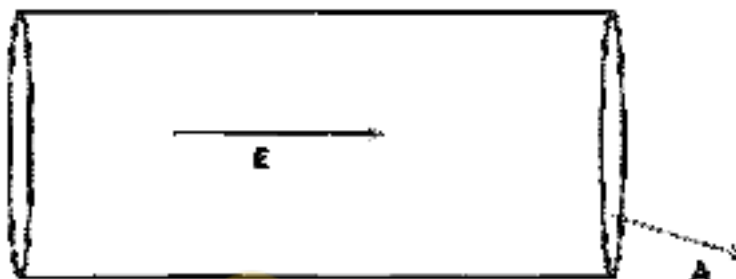
The materials having moderate electrical conductivity are called as semiconductors. These materials have a completely filled VB and completely filled CB at 0K. Semiconductors are classified into two types on the basis of concentrations of electrons and holes in the material.

#### **1. Intrinsic or pure semiconductor**

## 2. Extrinsic or doped semiconductor

### **Expression for electrical conductivity of a semiconductor:**

When an electric field is applied to a semiconductor, there will be movement of electrons in conduction band (C.B) and holes in valence band (V.B). Hence there will be two currents, an electron current in C.B and hole current in V.B. As these currents are constituted by oppositely charged carriers, the net current will be sum of the two currents.



Consider a semiconductor of area cross section 'A'. Let an electric field of 'E' is applied to the conductor. Let  $v_e$  and  $v_h$  be the drift velocities of electrons and holes respectively due to applied field E.

The current due to electron is

$$I_e = n_e e A v_e \rightarrow (1)$$

The current due to holes is

$$I_h = n_h e A v_h \rightarrow (2)$$

Here and  $n_e$  &  $n_h$  are the number of electrons and holes per unit volume.

The total current is given by

$$I = I_e + I_h = n_e e A v_e + n_h e A v_h$$

The total current is the sum of the current due electrons and holes. The total current density is given by

$$J = n_e e v_e + n_h e v_h \rightarrow (3)$$

By definition, mobility of charge carriers is the drift velocity per unit electric field.

Therefore the drift velocity of electrons is

$$v_e = \mu_e E$$

and the drift velocity of holes is

$$v_h = \mu_h E$$

where  $\mu_e$  &  $\mu_h$  are the motilities of electrons and holes respectively.

Substituting for  $v_e$  &  $v_h$  in equation (3) we get

$$J = n_e e \mu_e E + n_h e \mu_h E \rightarrow (4)$$

We have Ohm's law  $J = \sigma E \rightarrow (5)$

Where  $\sigma$  is conductivity, which is defined as the current density per unit applied electric field.

Comparing equation (4) and (5), we get

$$\sigma = n_e e \mu_e + n_h e \mu_h \rightarrow (6)$$

This equation gives total conductivity of a semiconductor.

**For intrinsic semiconductors,**  $n_e = n_h = n_i$ ;  $n_i$  is the intrinsic carrier concentration. Hence

$$\sigma_i = n_i e (\mu_e + \mu_h)$$

### **Fermi Level in semiconductors:**

In semiconductors, at Temperature,  $T = 0\text{K}$  the conduction band (CB) is completely empty and the valence band (VB) is completely filled. But, at ordinary temperatures such as room temperatures, electrons get excited from the top of the valence band to conduction band and occupy energy levels at the bottom of the conduction band. As CB is a higher energy band, the electrons undergo de-excitation to VB. This process of excitation and de-excitation continues and the electrons involved become conduction electrons. This means that the conduction electrons are distributed between the energy levels in the bottom of the CB and top of the VB. The average energy of all such energy levels is called Fermi level of the semiconductor. In the case of pure or intrinsic semiconductors, the Fermi level corresponds to level in the forbidden gap exactly in between CB and VB. Fermi level also signifies the average energy of the conduction electrons.

### **Electron and hole concentration:**

The number of electrons per unit volume in the conduction band of a semiconductor is known as electron concentration. Similarly, the number of holes per unit volume in the valence band of a semiconductor is known as hole concentration.

The electron concentration  $n_e$  is given by

$$n_e = \frac{4\sqrt{2}}{h^3} (\pi m_e^* kT)^{3/2} e^{\left(\frac{E_F - E_g}{kT}\right)}$$

The hole concentration  $n_h$  are given by



$$n_h = \frac{4\sqrt{2}}{h^3} (\pi m_h^* kT)^{3/2} e^{-\left(\frac{E_F}{kT}\right)}$$

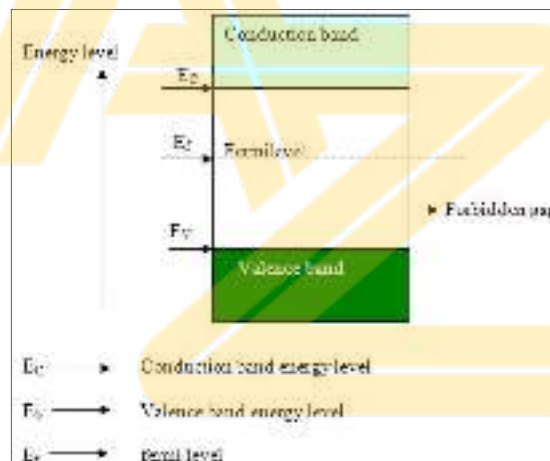
In the above equations  $E_F$  is the Fermi energy of the semiconductor,  $E_g$  is the energy gap of the semiconductor,  $m_e^*$  and  $m_h^*$  are the effective masses of electrons and holes,  $T$  is the absolute temperature.

### **RELATION BETWEEN FERMI-LEVEL & ENERGY GAP IN INTRINSIC SEMICONDUCTOR**

In a pure semiconductor, the electron in the conduction band cluster very close to the bottom of C.B, and we assume that electrons are located right at the bottom of the edge of C.B, as shown in the figure. Similarly we assume that the holes are at the top edge of V.B.

The electron concentration in the C.B is given by

$$n_e = \frac{4\sqrt{2}}{h^3} (\pi m_e^* kT)^{3/2} e^{\left(\frac{E_F - E_g}{kT}\right)}$$



The hole concentration in V.B is given by

$$n_h = \frac{4\sqrt{2}}{h^3} (\pi m_h^* kT)^{3/2} e^{-\left(\frac{E_F}{kT}\right)}$$

In an intrinsic semiconductor, the electron and hole concentrations are equal,

i.e. 
$$n_e = n_h$$

$$\left(m_e^*\right)^{3/2} e^{\left(\frac{E_F - E_g}{kT}\right)} = \left(m_h^*\right)^{3/2} e^{-\left(\frac{E_F}{kT}\right)}$$

$$e^{\frac{2E_F - E_g}{kT}} = \left(\frac{m_h^*}{m_e^*}\right)^{3/2}$$

Taking natural logarithm on both sides we get,

$$\frac{2E_F - E_g}{kT} = \frac{3}{2} \ln\left(\frac{m_h^*}{m_e^*}\right)$$

i.e.  $2E_F = E_g + \frac{3kT}{2} \ln\left(\frac{m_h^*}{m_e^*}\right)$

Under practical conditions,  $m_e^*$  and  $m_h^*$  are equal and hence second term on the right hand side of the above equation reduces to zero.

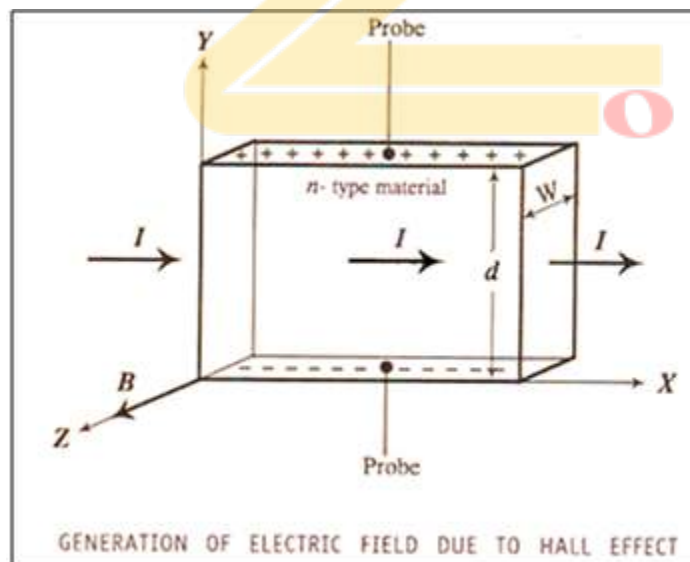
Therefore,  $E_F = \frac{E_g}{2}$

This means that in an intrinsic semiconductor, the Fermi level lies exactly midway between the valance band and conduction band.

## Hall Effect

If a material carrying current is placed in a transverse magnetic field, an electric field is produced in the material in a direction perpendicular to both the current and magnetic field. This phenomenon is called **Hall Effect**. The electric field generated is called **Hall field** and corresponding voltage is **Hall voltage**.

### Theory:



Consider a rectangle slab of a semiconductor material in which a current 'I' is flowing in positive x-direction. Let the semiconductor material be of n type, which means that the charge carriers are electrons.

Let a magnetic field, **B** is applied along Z-direction as shown in the figure. Under the influence of a magnetic field, the electrons experience the Lorentz-force,  $F_L$  given by

$$F_L = -Bev \rightarrow (1); \text{ -ve is due to electron charge}$$

Where 'e' is the magnitude of charge on the electron and 'v' is drift velocity of the electrons.

Applying the Flemings left -hand rule, the force **F<sub>L</sub>** is acting on the electron along the negative y-direction. The electrons are therefore deflected downwards. As a result the density of electrons increases in lower end of the material, due to which its bottom edge becomes negatively charged. On the other hand the loss of electrons from the upper end causes the top edge of the material to be positively charged. Hence a potential difference **V<sub>H</sub>** called **Hall voltage** appears between the upper and lower surface of the semiconductor material which establishes an electric field **E<sub>H</sub>**, called **Hall field** across the conductor in the negative y- direction.

The field **E<sub>H</sub>**, exerts an upward force **F<sub>H</sub>** on the electrons given by,

$$F_H = -eE_H \rightarrow (2)$$

Now as the deflection of electrons continues in the downward direction due to Lorentz force **F<sub>L</sub>**, the Hall field increases. As a result the force **F<sub>H</sub>** which acts on the electron in upward direction also increases till it becomes equal to **F<sub>L</sub>**. Thus at equilibrium,

$$\begin{aligned} F_L &= F_H \\ \text{i.e. } -Bev &= -eE_H \\ \Rightarrow E_H &= Bv \rightarrow (3) \end{aligned}$$

The current density,

$$\begin{aligned} J &= \frac{I}{A} = \frac{nevA}{A} \\ J &= nev \text{ or } v = \frac{J}{ne} \end{aligned}$$

Substituting for v, in equation (3) we get

$$E_H = B \frac{J}{ne} \text{ ----- (4)}$$

$$\text{The Hall voltage, } V_H = E_H d = \frac{BJd}{ne} \text{ ----- (5)}$$

where **n** is charge carrier concentration.

Let 'w' be the thickness of the material in z-direction, then the area of cross section normal to direction of current I is **A = wd**.

$$\begin{aligned} \therefore V_H &= \frac{BId}{ne(wd)} = \frac{BI}{new} \\ V_H &= \frac{BI}{\rho_{ch} w} \end{aligned}$$

Where  $\rho_{ch}$  is the charge density which is a constant for a given material.

Hence

$$V_H = \frac{R_H BI}{w}$$

where  $R_H$  is a constant called Hall coefficient,  $R_H = \frac{1}{\rho_{ch}} = \frac{1}{ne}$

Thus by measuring  $V_H$ ,  $I$  &  $w$  and by knowing  $B$ , the charge density  $\rho_{ch}$  can be determined using which Hall coefficient can be evaluated.

### **Importance of Hall Effect:**

The importance of Hall Effect in the field of semiconductor is that it helps to determine

- 1) The type of semiconductor.
- 2) The sign of majority charge carriers.
- 3) The majority charge carrier concentration.
- 4) The mobility of majority charge carriers.
- 5) The mean drift velocity of majority charge carriers.

### **Important Questions:**

1. Derive an expression for the electrical conductivity of intrinsic semiconductors.
2. Obtain the relation between Fermi energy and energy gap in the case of intrinsic semiconductors.
3. What is Hall Effect? Obtain an expression for hall voltage in terms of hall coefficient.

# DIELECTRIC PROPERTIES OF MATERIALS

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## 1. What are dielectrics?

A dielectric is an electrically non-conducting material which provides electrical insulation between two media (conductors) which are at different potentials, and also serves as an electrical charge storage aid under certain circumstances.

**E.g.:** glass, porcelain, wood, rubber or waxed paper.

## 2. What is polarization in dielectrics?

The Process of formation of dipoles or alignment of already existing dipoles on the application of an electric field on the dielectric material is called polarization.

## 3. What are polar and non-polar dielectric materials?

A dielectric material doesn't possess any free electrons. All the electrons are bound very strongly to the respective nuclei of the parent molecules. Each molecule (or atom) in the normal state consists of equal amount of positive and negative charges. All the positive charges are concentrated in the nuclei, which are surrounded by electron clouds in which all the negative charges are distributed.

In the molecules (or atoms) of some dielectric materials, the effective center of the negative charge distribution coincides with the effective center of the positive charges thus neutralizing each other's effect. Such materials are called non-polar dielectrics.

There are some dielectric materials like water, in which the effective centers of the negative and positive charges in the molecules do not coincide with each other even in the absence of any external field. Each molecule (or atom), behaves as though it consists of a pair of negative and positive charges separated by a small constant distance. Such a pair is referred to as a permanent dipole and the materials comprising of such dipoles are called polar dielectrics.

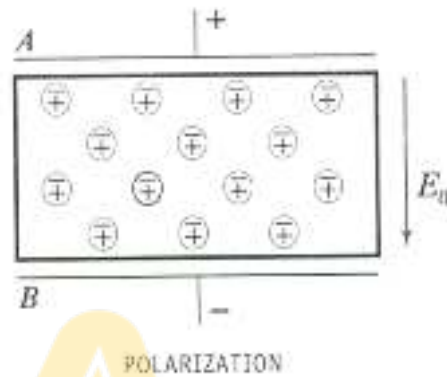
## 4. What is polarization? Explain polarization process in case of polar and non-polar dielectrics?

Process of formation of dipoles or alignment of already existing dipoles on the application of an electric field on the dielectric material is called **polarization**.

### (i) Polarization in the case of non-polar dielectric:

Consider a rectangular slab of a non-polar dielectric material. Let the slab be placed between two electrodes A & B, between which a uniform electric field  $E_0$  is set up.

Under the influence of the electric field, the positive and negative charges of the various molecules in it experience pulling forces in opposite directions. As a result, the effective centers of positive and negative charges get separated till the restoring forces balance the forces due to the applied field. Due to the separation between the two charge centers, each molecule develops a dipole moment in the direction of the electric field. Such dipoles are produced throughout the solid.



Now, inside the body of the dielectric slab, though the polarization charges appear in every molecule, opposite charges in the neighboring molecules neutralize each other. This kind of cancellation takes place throughout the body of the material since there are equal amount of negative and positive charges. However, the charges which are at the end surfaces of the slab do not find the opposite kind of charges for cancellation as is evident from the figure. Thus the effect of the applied field is to cause the appearance of net opposing charges at the end faces of the slab.

#### (ii) Case of polar dielectric:

In the case of a polar dielectric material, there are permanent dipoles oriented randomly. Under the influence of the applied electric field, the molecular dipoles experience a torque which tends to align their dipole moments along the direction of the field. This tendency is opposed by the thermal agitation inside the material. Between these two opposing events, some alignment is achieved under sufficiently strong fields. Once the alignment is established, the surface charges appear at the ends of the slab in a way similar to the case of nonpolar dielectric materials.

### 5. Explain various types of polarization processes?

There are three different mechanisms through which electrical polarization can occur in dielectric materials when they are subjected to an external electric field. Accordingly there are three main types of polarizations.

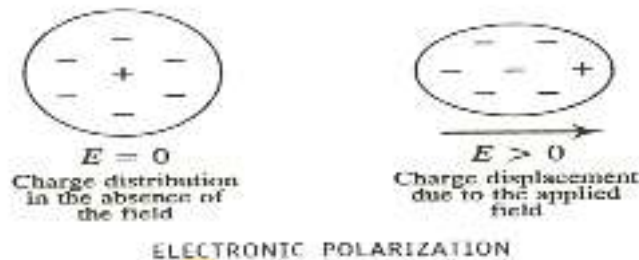
They are,

- 1) Electronic polarization,

- 2) Ionic polarization
- 3) Orientational polarization

### 1. Electronic polarization:

The electronic polarization occurs due to displacement of the positive and negative charges in a dielectric material owing to the application of an external electric field. The separation created between the charges leads to development of a dipole moment. This process occurs throughout the material. Thus the material as a whole will be polarized.

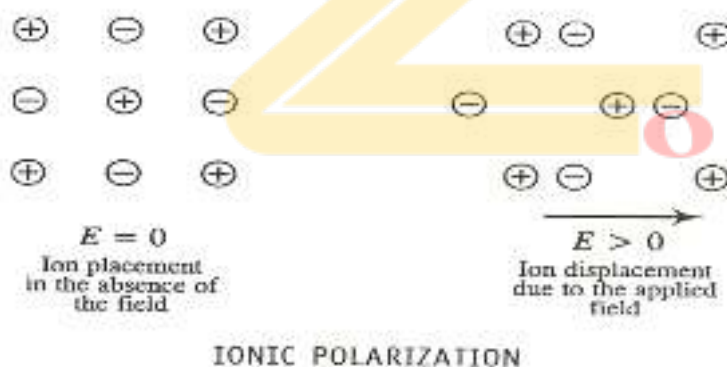


The electronic polarizability,  $\alpha_e$  for a rare gas atom is given by,

$$\alpha_e = \frac{\epsilon_0(\epsilon_r - 1)}{N} ; \text{ where } N \text{ is the number of atoms/unit volume.}$$

### 2. Ionic Polarization:

Ionic polarization occurs only in those dielectric materials which possess ionic bonds such as in NaCl. When ionic solids are subjected to an external electric field, the adjacent ions of opposite sign undergo displacement. The displacement causes an increase or decrease in the distance of separation between the atoms depending upon the location of the ion pair in the lattice.



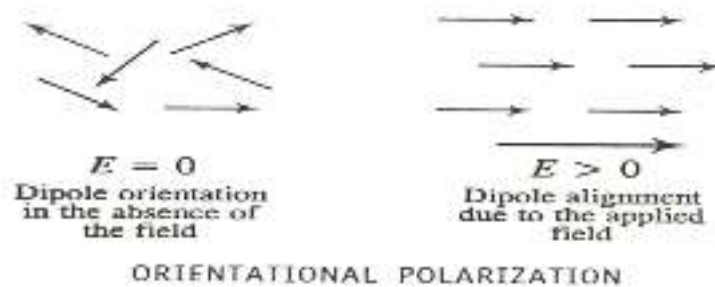
This results in a net dipole moment due to the shifting of electron clouds relative to the nuclei.

### 3. Orientational Polarization:

Orientational polarization occurs in those dielectric materials which possess molecules with permanent dipole moment (i.e., in polar dielectrics). The orientation of these molecules will be random normally due to thermal agitation. Because of randomness in orientation, the material has net zero dipole moment. But under the influence of an applied



electric field, each of the dipoles undergo rotation so as to reorient along the direction of the field because of which, the overall cancellation of dipole moments due to randomness does not hold good any more (Fig. 7). Thus the material itself develops electrical polarization.

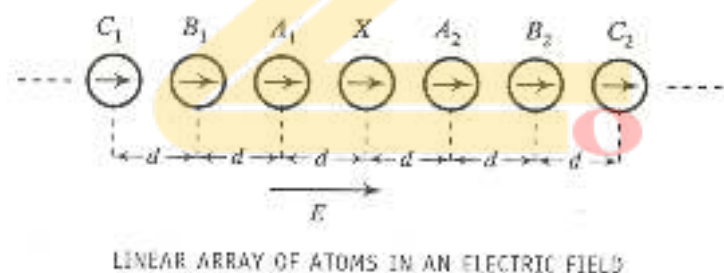


The orientational polarization is strongly temperature dependent and decreases with increase of temperature.

## 6. What is internal field? Write an expression for the internal field in case of solid and liquid dielectrics?

**"The internal field, or the local field,** is the electric field that acts at the site of any given atom of a solid or a liquid dielectric subjected to an external electric field. It is defined as **"the resultant of the applied field and the field due to all the surrounding dipoles"**.

Consider a dielectric material either solid or liquid kept in an external uniform Electric field of strength  $E$ . In the material let us consider an array of equidistant atomic dipoles arranged parallel to the direction of the field as shown in fig.



Let the interatomic distance be 'd' and dipole moment of each of atomic dipole be  $\mu$ . Then the internal field at the site of an atom is given by

$$E_i = E + \frac{1.2\mu}{\pi\epsilon_0 d^3}$$

If  $\alpha_e$  is the electronic polarizability of the atoms, we can write,

$$\mu = \alpha_e E_i$$

Hence 
$$E_i = E + \frac{1.2\alpha_e E_i}{\pi\epsilon_0 d^3}$$

Or

$$E_i = \frac{E}{1 - \frac{1.2\alpha_e}{\pi\epsilon_0 d^3}}$$

In 3-dimensional cases, the general equation for internal field is expressed as,

$$E_i = E + \frac{\gamma P}{\epsilon_0}$$

where, P is the dipole moment/unit volume for the material, and  $\gamma$  is the proportionality constant called internal field constant.

In the 3-dimensional case, if it is cubic lattice, then it can be shown that  $\gamma = 1/3$  in which event, the internal field is named Lorentz field given by,

$$E_{Lorentz} = E + \frac{P}{3\epsilon_0}$$

Above equation is known as Lorentz relation.

## 7. Derive Clausius – Mossotti relation?

Consider an elemental solid dielectric material of dielectric constant  $\epsilon_r$

If N is the number of atoms/unit volume of the material,  $\mu$  is the atomic dipole moment, then we have,

$$\text{Dipole moment/unit volume} = N\mu \quad \text{----- (1)}$$

Here the field experienced by the atoms is the internal field  $E_i$ . Hence, if  $\alpha_e$  is the electronic polarizability of the atoms, we can write the equation for  $\mu$  as,

$$\mu = \alpha_e E_i \quad \text{---- (2)}$$

$$\text{Therefore dipole moment/unit volume} = N\alpha_e E_i \quad \text{----- (3)}$$

But dipole moment/ unit volume is Polarization P

$$\text{Therefore} \quad P = N\alpha_e E_i \quad \text{----- (4)}$$

$$\text{or} \quad E_i = \frac{P}{N\alpha_e} \quad \text{----- (5)}$$

$$\text{But} \quad P = \epsilon_0 (\epsilon_r - 1)E$$

$$\text{or} \quad E = \frac{P}{\epsilon_0 (\epsilon_r - 1)} \quad \text{----- (6)}$$

For 3-D lattice, internal field is given by

$$E_i = E + \frac{\gamma P}{\epsilon_0}$$

Substituting for  $E_i$  and E from Eqs (5) and (6) we have

$$\frac{P}{N\alpha_e} = \frac{P}{\epsilon_0 (\epsilon_r - 1)} + \frac{\gamma P}{\epsilon_0} \quad \text{----- (7)}$$

Considering the internal field in the material to be Lorentz field, we have  $\gamma = 1/3$

$$\frac{1}{N\alpha_e} = \frac{1}{\epsilon_0(\epsilon_r - 1)} + \frac{1}{3\epsilon_0}$$

$$\frac{1}{N\alpha_e} = \frac{3 + (\epsilon_r - 1)}{3\epsilon_0(\epsilon_r - 1)}$$

$$\frac{3\epsilon_0}{N\alpha_e} = \frac{\epsilon_r + 2}{\epsilon_r - 1}$$

By rearranging, we have

$$\frac{N\alpha_e}{3\epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2} \text{----- (8)}$$

Eq (8) is called Clausius - Mossotti equation, and it holds good for crystals of high degree of symmetry. The equation provides a relation between  $\epsilon_r$  the dielectric constant which is a macroscopically measurable quantity, and  $\alpha_e$  the electronic polarizability which is an atomic property i.e., microscopic.

### **8. Discuss solid, liquid and gaseous dielectrics with one example for each.**

Dielectric materials are used in various electrical machines and appliances are for their trouble free working and to enhance the life time and reliability of the devices. The dielectric materials are broadly classified into solid, liquid and gaseous dielectrics.

#### **i) Solid dielectrics:**

Solid dielectric materials are available in a number of varieties and have number of applications such as jacketing materials, moulding materials and filling materials. Ceramics, glass (toughened glass), fiber glass reinforced plastics and epoxy – resins are used as mechanically rigid form of insulation (Moulding materials). Dielectrics such as polyethylene (PE), polyvinylchloride (PVC), natural and synthetic (ethylene propylene) rubber etc. are used on the conductor in power cables as jacketing materials for insulations. Polyethylene and paper are used in capacitors and transformers. Mica and fiberglass based polyethylene tapes are used in electrical machines.

#### **ii) Liquid dielectrics:**

Liquid dielectrics perform a number of functions simultaneously such as Providing insulation between various parts carrying voltage, cooling action by convection of heat generated during operation of an electrical device, filling of the voids or air gaps etc. These are mainly used in transformers, switches, circuit breakers etc. Examples for liquid dielectrics are silicon fluids, transformer oil, viscous Vaseline, poly isobutylene, purified water, liquid nitrogen etc.

#### **iii) Gaseous dielectrics:**

A dielectric gas is an insulating material in gaseous state which can prevent electrical discharge. They acts as good insulators and work well as heat transferring media.

Examples of dielectric gases are Ammonia, Air, Carbon dioxide, sulfur hexafluoride (SF<sub>6</sub>), Carbon Monoxide, Nitrogen, hydrogen etc. Generally, air provides insulation between the overhead transmission power lines and the earth.

### 9. Explain the application of dielectrics in transformers.

A transformer consists of two insulated conducting coils wound on an insulated core. In case of high voltage transformers, further insulation is provided between the individual windings of the coils and also between the core and the coils. This insulation is provided by using mica, paper or cloth. To fill the air gaps paper used is impregnated with varnish or wax.

In high voltage transformers, the ionization of the air in air gaps leads to excessive heating which damages the insulation. This effect is known as **corona**. Mica is used to guard against corona. In very high voltage transformers, a kind of mineral oil known as **transformer oil** is used which helps not only in guarding against corona but also to keep the transformer cool.

### Important points:

1. **Dielectric constant ( $\epsilon_r$  or  $K$ ):** The ratio of permittivity of a medium to permittivity of free space is known as dielectric constant.

2. **Electric Dipole:** Two equal and opposite charges separated by small distance constitutes an electric dipole.

3. **Dipole moment ( $\mu$ ):** The product of magnitude of the charge ( $q$ ) constituting the dipole and distance ( $d$ ) of separation between the charges is called dipole moment.

$$\mu = qd \text{ (Cm)}$$

4. **Polarization ( $P$ ):** The dipole moment acquired by a dielectric material per unit volume is called polarization.

$$P = \epsilon_0 \chi E \text{ where } \chi \text{ is electric susceptibility of the material.}$$

5. **Relation between Polarization and dielectric constant:**

$$P = \epsilon_0(\epsilon_r - 1)E \text{ where } E \text{ is the applied electric field.}$$

6. **Polarizability( $\alpha$ ):**

It is defined as the dipole moment per unit electric field.

$$\alpha = \frac{\mu}{E}$$