MODULE 2A: SINGLE PHASE AC CIRCUITS

SINGLE PHASE A.C. CIRCUITS

The resistance, inductance and capacitance are three basic elements of any electrical network. In order to analyse any electric circuit, it is necessary to understand the following three cases,

1. A.C. through pure resistive circuit.
2. A.C. through pure inductive circuit.
3. A.C. through pure capacitive circuit.

In each case, it is assumed that a purely sinusoidal alternating voltage given by the equation $v = V_m \sin\omega t$ is applied to the circuit. The equation of the current, power and phase shift is developed in each case. The voltage applied having zero phase angle is assumed reference while plotting the phasor diagram in each case

A.C. THROUGH PURE RESISTANCE

Consider a simple circuit consisting of a pure resistance 'R' ohm connected across a voltage $v = V_m \sin\omega t$.

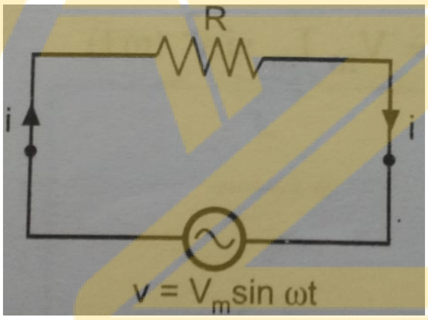


Fig 3.5 Pure resistive circuit

According to ohms law, we can find the equation for the current i as

$$i= \frac{v}{R} = \frac{V_m \sin\omega t}{R} = \left(\frac{V_m}{R}\right) \sin\omega t$$

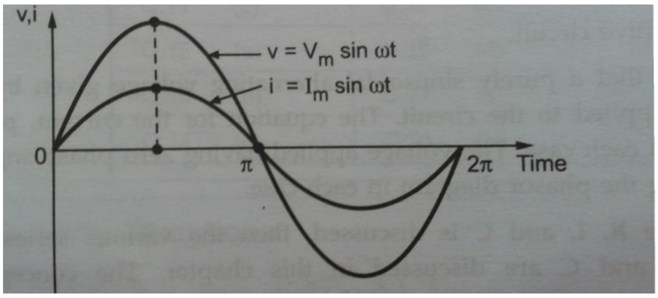
This is the equation giving instantaneous value of the current.
Comparing this with standard equation,

$$i = I_m \sin(\omega t + \emptyset)$$

$$I_m = \frac{V_m}{R} \text{ and } \emptyset=0$$

So, maximum value of alternating current, i is $I_m = \frac{V_m}{R}$ while, as Ø=0, it indicates that it is in phase with the voltage applied. There is no phase difference between the two. The current is going to achieve its maximum and zero whenever voltage is going to achieve its maximum and zero values.

The waveform of voltage and current and the corresponding phasor diagram is shown in fig 3.6



Power

The instantaneous power in a.c. circuits can be obtained by taking product of the instantaneous values of current and voltage.

$P = v*i = V_m \sin \omega t * I_m \sin \omega t = V_m\ I_m\ \sin^2 \omega t$

$= \frac{V_m\ I_m}{2}(1 - \cos 2\omega t)$

$= \frac{V_m\ I_m}{2} - \frac{V_m\ I_m}{2} \cos 2\omega t$

From the above equation, it is clear that the instantaneous power consist of two components,

1. Constant power component $\left(\frac{V_m\ I_m}{2}\right)$
2. Fluctuating component $\left[\frac{V_m\ I_m}{2} \cos 2\omega t\right]$ having frequency, double the frequency of applied voltage.

Now, the average value of the fluctuating component of double frequency is zero, over one complete cycle. So, average power consumption over one cycle is equal to the constant power component i.e. $\left(\frac{V_m\ I_m}{2}\right)$

$$P_{av} = \frac{V_m\ I_m}{2} = \frac{V_m}{\sqrt{2}} * \frac{I_m}{\sqrt{2}}$$

$$\mathbf{P_{av} = V_{rms} * I_{rms}\ watts}$$

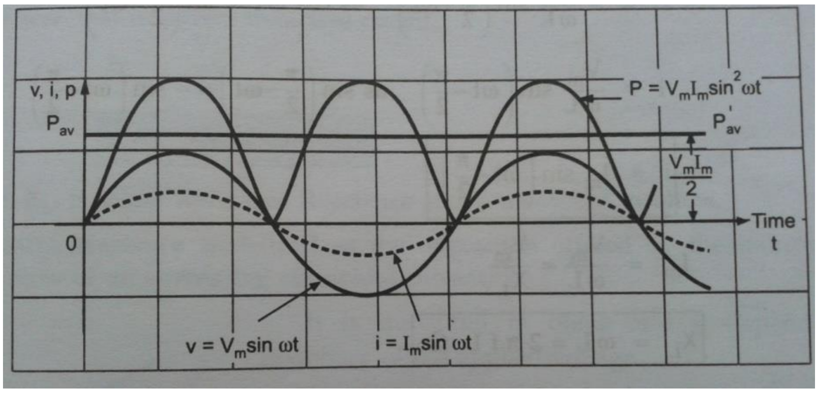


Fig 3.7 v,i, p for purely resistive circuit

A.C. THROUGH PURE INDUCTANCE

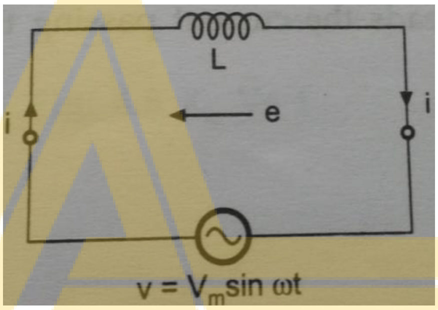


Fig 3.8 purely inductive circuit

Consider a simple circuit consisting of a pure inductance of L henries, connected across a voltage given by the equation,$v = V_m \sin \omega t$.

Pure inductance has zero ohmic resistance. Its internal resistance is zero. The coil has pure inductance of L henries.

When alternating quantity i flows through inductance 'L', it sets up an alternating magnetic field around the inductance. This changing flux links the coil and due to self inductance, emf gets induced in the coil. This emf opposes the applied voltage.

The self induced emf in the coil is given by, $e = -L\frac{di}{dt}$

At all instant, the applied voltage v is equal and opposite to the self induced emf

$$v = -e = -\left(-L\frac{di}{dt}\right)$$

$$v = L\frac{di}{dt}$$

$$V_m \sin \omega t = L\frac{di}{dt}$$

$$di = \frac{V_m}{L}\sin \omega t$$

$$i = \int di = \int \frac{V_m}{L}\sin \omega t = \frac{V_m}{L}\left(\frac{-\cos \omega t}{\omega}\right)$$

$$= \quad -\frac{V_m}{\omega L}\sin\left(\frac{\pi}{2}-\omega t\right) \qquad \text{as } \cos\omega t = \sin\left(\frac{\pi}{2}-\omega t\right)$$

$$i = \quad \frac{V_m}{\omega L}\sin\left(\omega t-\frac{\pi}{2}\right)$$

$$\mathbf{i = \quad I_m \sin\left(\omega t-\frac{\pi}{2}\right)}$$

where $I_m = \frac{V_m}{\omega L} = \frac{V_m}{X_L}$

where $X_L = \omega L = 2\pi f L\ \Omega$

The term, X_L is called **Inductive Reactance** and is measured in **ohms**

The above equation clearly shows that the current is purely sinusoidal and having phase angle of $-\frac{\pi}{2}$ radians i.e. -90°. This means that **the current lags voltage applied by 90°.**

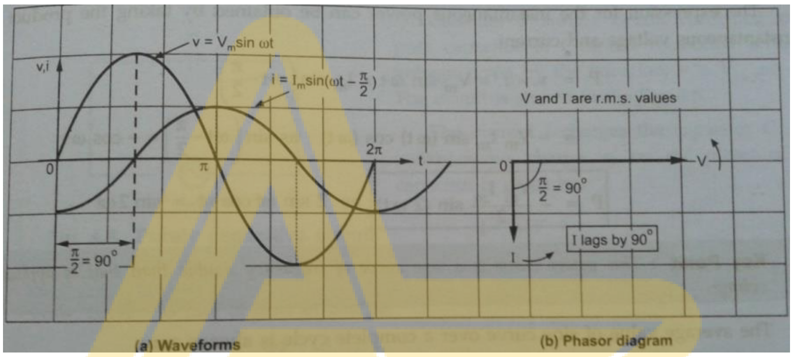


Fig 3.9

Power

The instantaneous power in a.c. circuits can be obtained by taking product of the instantaneous values of current and voltage.

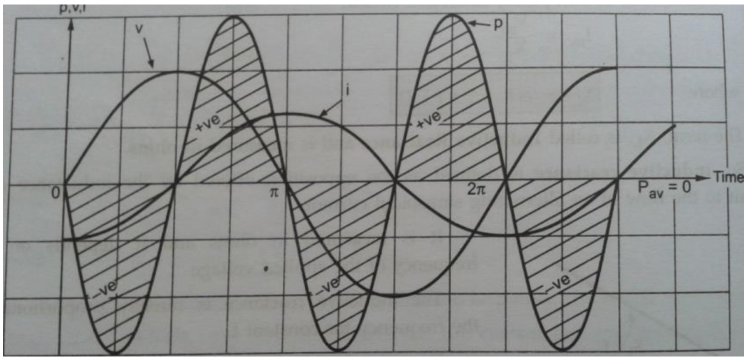
$$P = v*i = V_m \sin\omega t * I_m \sin(\omega t - \frac{\pi}{2})$$

$$= -V_m I_m \sin(\omega t)\cos(\omega t)$$

$$\mathbf{P \quad = -\frac{V_m I_m}{2}\sin(2\omega t)}$$

The average value of sine curve over a complete cycle is always zero

$$P = \int_0^{2\pi}\frac{V_m I_m}{2}\sin(2\omega t)\ d(\omega t) = 0$$



It can observed from it that when power curve is positive, energy gets stored in the magnetic field established due to increasing current while during negative power curve, this power is returned back to the supply.

The areas of positive loop and negative loop are exactly same and hence, average power consumption is zero.

A.C. THROUGH PURE CAPACITANCE

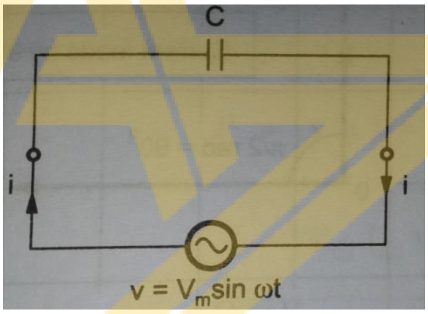


Fig 3.11

Consider a simple circuit consisting of a pure capacitor of C-farads, connected across a voltage given by the equation $v = V_m \sin \omega t$.

The current i charges the capacitor C. The instantaneous charge 'q' on the plates of the capacitor is given by

$$q = C\,v$$

Therefore $\quad q = C\,V_m \sin \omega t$

Now, current is rate of flow of charge

$$i = \frac{dq}{dt} = \frac{d}{dt}(CV_m \sin \omega t)$$

$$i = CV_m \frac{d}{dt}(\sin \omega t) = CV_m \omega \cos(\omega t)$$

$$i = \frac{V_m}{\frac{1}{\omega C}} \sin(\omega t + \frac{\pi}{2})$$

$$\mathbf{i = I_m \sin(\omega t + \frac{\pi}{2})}$$

where $I_m = \frac{V_m}{X_c}$ where $X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}\ \Omega$

The term X_c is called **capacitive reactance** and measured in **ohms.**

The above current equation clearly shows that the current is purely sinusoidal and having phase angle of $+\frac{\pi}{2}$ radians i.e. +90°.

This means **current leads voltage applied by 90°.** The positive sign indicates leading nature of the current.

Fig 3.12 shows waveform of voltage and current and the corresponding phasor diagram. The current waveform starts earlier by 90° in comparison with voltage waveform. When voltage is zero, the current has positive maximum value.

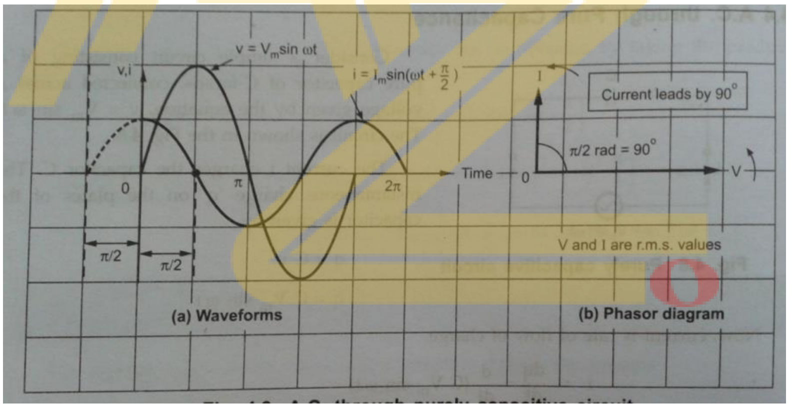


Fig 3.12

POWER

The instantaneous power in a.c. circuits can be obtained by taking product of the instantaneous values of current and voltage.

$$P = v * i = V_m \sin \omega t * I_m \sin(\omega t + \frac{\pi}{2})$$

$= V_m I_m \sin(\omega t)\cos(\omega t)$

$$\mathbf{P} = \frac{\mathbf{V_m I_m}}{\mathbf{2}} \mathbf{\sin(2\omega t)}$$

The average value of sine curve over a complete cycle is always zero

$$P = \int_0^{2\pi} \frac{V_m I_m}{2} \sin(2\omega t)\ d(\omega t) = 0$$

Fig 3.14 shows waveforms of current, voltage and power. It can observed from the figure that when power curve is positive, in practice, an electrostatic energy gets stored in the capacitor during its charging while the negative power curve represents that the energy stored is returned back to the supply during its discharging. The areas of positive and negative loops are exactly the same and hence, average power consumption is zero.

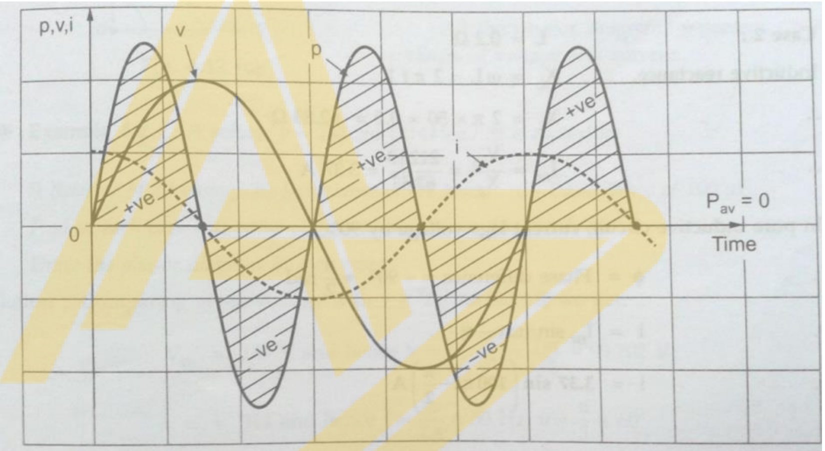
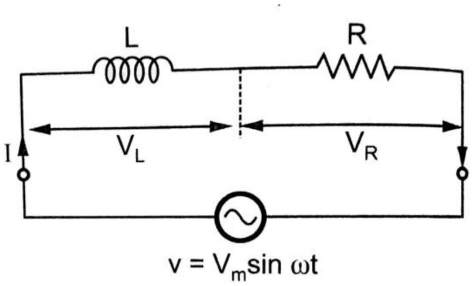


Fig 3.14

3.14 A.C. THROUGH SERIES R-L CIRCUIT

Consider a circuit consisting of pure resistance R ohms connected in series with a pure inductance of L henries. The series combination is connected across a.c. supply given by $v = V_m \sin \omega t$.



Circuit draws a current I then there are two voltage drops,

a) Drop across pure resistance, $V_R = I R$

b) Drop across pure inductance, $V_L = I\,X_L$ where $X_L = 2\,\pi\,f\,L$

I= r.m.s. value of current drawn

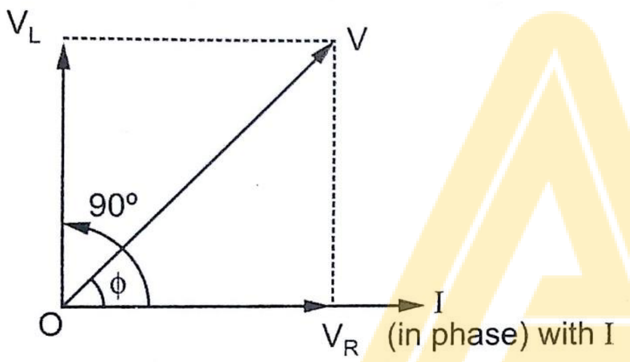
V_R, V_L= r.m.s. values of voltage drops

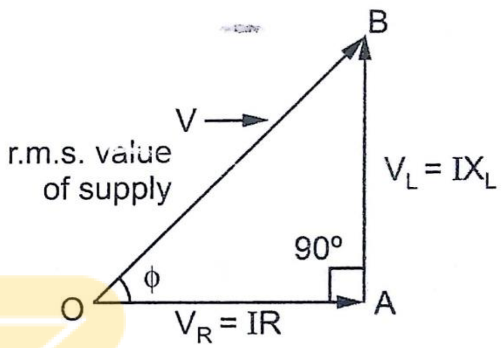
The Kirchhoff's law can be applied to the a.c. circuit but only the point to remember is the addition of voltages should be vector addition.

Therefore $\overline{V} = \overline{V_R} + \overline{V_L}$

Therefore $\overline{V} = \overline{IR} + \overline{IX_L}$

Let us draw the phasor diagram for the above case





From the voltage triangle

$V= \sqrt{(V_R)^2 + (V_L)^2} = \sqrt{(IR)^2 + (IX_L)^2} = I\sqrt{(R)^2 + (X_L)^2}$

V=IZ

Where $Z= \sqrt{(R)^2 + (X_L)^2}$

The impedance Z is measured in ohms.

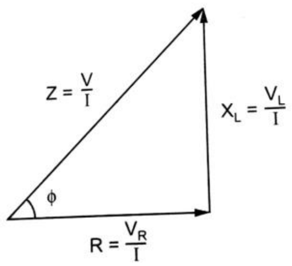
IMPEDANCE

Impedance is defined as the opposition of circuit to the flow of alternating current. It is denoted by Z and its unit is ohms.

For the R-L series circuit, it can observed from the phasor diagram that the current lags behind the applied voltage by an angle Ø. From the voltage triangle, we can write

$\text{Tan}\,Ø = \frac{V_L}{V_R} = \frac{X_L}{R}$, $\cos Ø = \frac{V_R}{V} = \frac{R}{Z}$, $\text{Sin}\,Ø = \frac{V_L}{V} = \frac{X_L}{Z}$

If all the sides of the voltage triangle are divided by current, we get a triangle called impedance triangle.



From the impedance triangle, we can see that the X component of impedance is R and is given by R= Zcosφ

And Y component of impedance is X_L and is given by X_L=Zsinφ

In rectangular form the impedance is denoted as

$$Z= R+jX_L$$

While in the polar form, it is denoted as

$$Z= |Z|\angle\varphi$$

Where $|Z| = \sqrt{R^2 + {X_L}^2}$ and $\varphi= tan^{-1}\frac{X_L}{R}$

POWER AND POWER TRIANGLE

The expression for the current in a series R-L circuit is

$$i = I_m \sin(\omega t - \phi) \text{ as current lags voltage.}$$

The power is product of instantaneous values of voltage and current,

$$\therefore \quad P = v \times i = V_m \sin \omega t \times I_m \sin(\omega t - \phi) = V_m I_m [\sin(\omega t) \cdot \sin(\omega t - \phi)]$$

$$= V_m I_m \left[\frac{\cos(\phi) - \cos(2\omega t - \phi)}{2}\right] = \frac{V_m I_m}{2} \cos\phi - \frac{V_m I_m}{2} \cos(2\omega t - \phi)$$

Now, the second term is cosine term whose average value over a cycle is zero. Hence, average power consumed is,

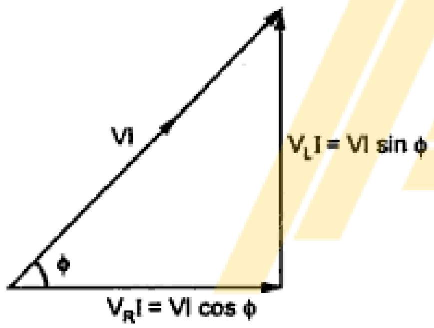
$$P_{av} = \frac{V_m I_m}{2} \cos\phi = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos\phi$$

$$\therefore \quad \boxed{P = V I \cos\phi \quad \text{watts}} \quad \text{where V and I are r.m.s. values}$$

If we multiply voltage equation by current I, we get the power equation.

$$\overline{VI} = \overline{V_R I} + \overline{V_L I}$$

$$\therefore \quad \overline{VI} = \overline{V \cos\phi I} + \overline{V \sin\phi I}$$



From this equation, power triangle can be obtained as shown in the Fig.

So, three sides of this triangle are,

1) VI, 2) VI $\cos\phi$ 3) VI $\sin\phi$

These three terms can be defined as below.

APPARENT POWER

It is defined as the product of r.m.s. value of voltage (V) and current (I). It is denoted by S.

$$\boxed{S = V I \quad \text{VA}}$$

It is measured in unit volt-amp (VA) or kilo volt-amp (kVA).

REAL OR TRUE POWER (P)

It is defined as the product of the applied voltage and the active component of the current.

It is real component of the apparent power. It is measured in unit watts (W) or kilowatts (kW).

$$\boxed{P = V I \cos\phi \quad \text{watts}}$$

Reactive Power (Q)

It is defined as product of the applied voltage arid the reactive component of the current.

It is also defined as imaginary component of the apparent power. It is represented by ‘Q’ and it is measured in unit volt-amp reactive (VAR) or kilovolt-amp reactive (kVAR).

$$\boxed{Q = VI \sin\phi \quad \text{VAR}}$$

POWER FACTOR (COSΦ)

It is defined as factor by which the apparent power must be multiplied in order to obtain the true power.

It is the ratio of true power to apparent power.

$$\boxed{\text{Power factor} = \frac{\text{True Power}}{\text{Apparent Power}} = \frac{VI\cos\phi}{VI} = \cos\phi}$$

The numerical value of cosine of the phase angle between the applied voltage and the current drawn from the supply voltage gives the power factor. It cannot be greater than 1.

It is also defined as the ratio of resistance to the Impedance.

$$\boxed{\cos\phi = \frac{R}{Z}}$$

If current lags voltage power factor is said to be lagging. If current leads voltage power factor is said to be leading.

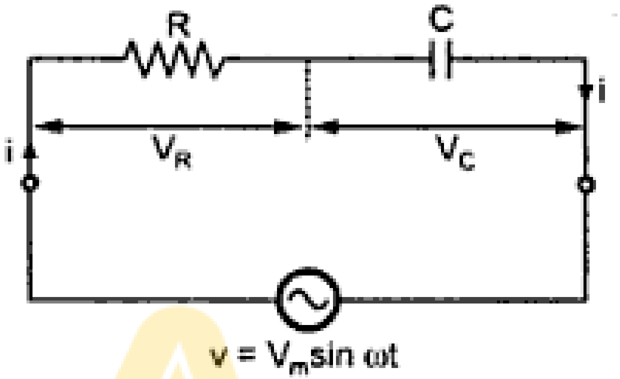
So, for pure inductance, the power factor is cos (90°) i.e. zero lagging while for pure capacitance, the power factor is cos (90°) i.e. zero but leading. For purely resistive circuit voltage and current are in phase i.e. φ= 0. Therefore, power factor is cos (0°) = 1. Such circuit is called unity power factor circuit.

$$\boxed{\text{Power factor} = \cos\phi}$$

φ is the angle between supply voltage and current.

A.C. THROUGH SERIES R-C CIRCUIT

Consider a circuit consisting of pure resistance R-ohms and connected in series with a pure capacitor of C-farads as shown in the Fig. The series combination is connected across ac. supply given by $v = V_m \sin \omega t$.



Circuit draws a current I, then there are two voltage drops,

a) Drop across pure resistance, $V_R = I\,R$
b) Drop across pure inductance, $V_C = I\,X_c$ where $X_c = \frac{1}{2\pi f C}$

I= r.m.s. value of current drawn

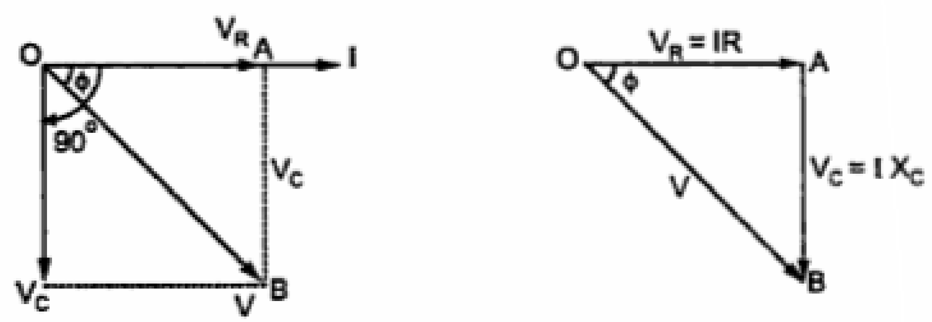
V_R, V_c= r.m.s. values of voltage drops

The Kirchhoff's voltage law can be applied to get,

$$\mathbf{V} = \overline{V_R} + \overline{V_C}$$

$$\overline{V} = \overline{IR} + \overline{IX_C}$$

Let us draw the phasor diagram. Current I is taken as reference as it is common to both the elements.



From the voltage triangles,

$$V = \sqrt{(V_R)^2 + (V_C)^2} = \sqrt{(IR)^2 + (I X_C)^2} = I \sqrt{(R)^2 + (X_C)^2}$$

$\therefore$ $$V = I Z$$

Where $\boxed{Z = \sqrt{(R)^2 + (X_C)^2}}$ is the impedance of the circuit.

Impedance

Similar to R-L series circuit, in this case also, the impedance is nothing but opposition to the flow of alternating current. It is measured in ohms given by Z = $\sqrt{R^2 + {X_c}^2}$ where $X_c = \frac{1}{2\,\pi\, f\, C}\,\Omega$ called capacitive reactance.

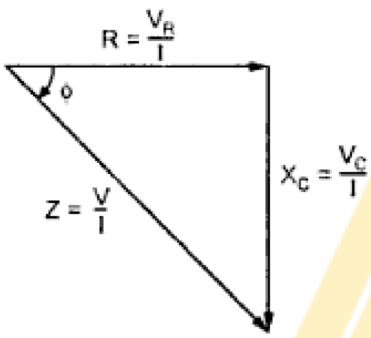
In R-C series circuit, current leads voltage by angle ϕ or supply voltage V lags current I by angle ϕ as shown in the phasor diagram in Fig. 7.20.

From voltage triangle, we can write,

$$\tan\phi = \frac{V_C}{V_R} = \frac{X_C}{R}, \quad \cos\phi = \frac{V_R}{V} = \frac{R}{Z}, \quad \sin\phi = \frac{V_C}{V} = \frac{X_C}{Z}$$

If all the sides of the voltage triangle are divided by the current, we get a triangle called **impedance triangle.**

Two sides of the triangle are 'R' and 'X_C' and the third side is impedance 'Z'.



Impedance triangle

The X component of impedance is R and is given by

$$R = Z\cos\phi$$

and Y component of impedance is X_C and is given by

$$X_C = Z\sin\phi$$

But, as direction of the X_C is the negative Y direction, the rectangular form of the impedance is denoted as,

$$\boxed{Z = R - jX_C \ \Omega}$$

While in polar form, it is denoted as,

$$\boxed{Z = |Z| \angle -\phi \ \Omega}$$

$$Z = R - jX_C = |Z| \angle -\phi$$

where $|Z| = \sqrt{R^2 + X_C^2}$, $\phi = \tan^{-1}\left[\frac{-X_C}{R}\right]$

Power and Power Triangle

The current leads voltage by angle ϕ, hence its expression is,

$$i = I_m \sin(\omega t + \phi) \text{ as current leads voltage}$$

The power is the product of instantaneous values of voltage and current.

$$\therefore \quad P = v \times i = V_m \sin\omega t \times I_m \sin(\omega t + \phi)$$

$$= V_m I_m [\sin(\omega t) \cdot \sin(\omega t + \phi)]$$

$$= V_m I_m \left[\frac{\cos(-\phi) - \cos(2\omega t + \phi)}{2}\right]$$

$$= \frac{V_m I_m \cos\phi}{2} - \frac{V_m I_m}{2} \cos(2\omega t + \phi) \qquad \text{as } \cos(-\phi) = \cos\phi$$

Now, second term is cosine term whose average value over a cycle is zero. Hence, average power consumed by the circuit is,

$$P_{av} = \frac{V_m I_m}{2} \cos\phi = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos\phi$$

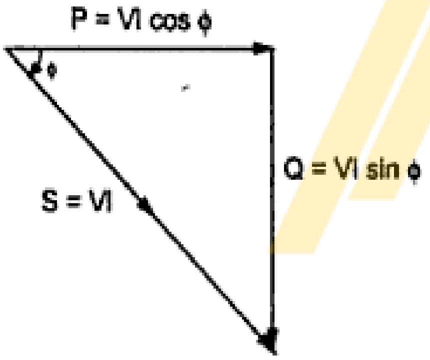
$\therefore$ $\boxed{P = V\,I \cos\phi \quad \text{watts}}$ where V and I are r.m.s. values

If we multiply voltage equation by current I, we get the power equation,

$$\overline{VI} = \overline{V_R I} + \overline{V_C I}$$

$$\therefore \quad \overline{VI} = \overline{VI\cos\phi} + \overline{VI\sin\phi}$$

Hence, the power triangle can be shown as in the Fig.



Thus, the various powers are,

Apparent power,	$S = V\,I$ VA
True or average power,	$P = V\,I\cos\phi$ W
Reactive power,	$Q = V\,I\sin\phi$ VAR

Remember that, X_L term appears positive in Z.

$$Z = R + j X_L = |Z| \angle \phi \qquad \phi \text{ is positive for inductive Z}$$

While X_C term appears negative in Z.

$$Z = R - j X_C = |Z| \angle -\phi \qquad \phi \text{ is negative for capacitive Z}$$

For any single phase a.c. circuit, the average power is given by,

$$P = V\,I\cos\phi \quad \text{watts}$$

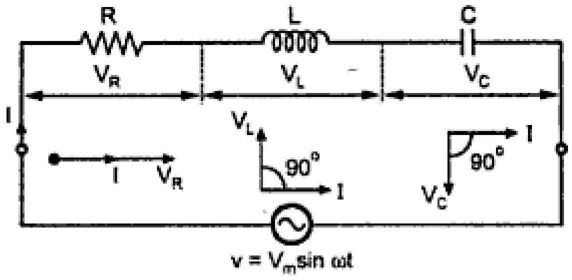
Where V, I are r.m.s. values

$\cos\phi$ = Power factor of circuit

$\cos\phi$ is **lagging** for inductive circuit and $\cos\phi$ is **leading for capacitive** circuit.

A.C. THROUGH SERIES R-L-C CIRCUIT

Consider a circuit consisting of resistance R ohms pure Inductance L henries and capacitance C farads connected in series with each other across a.c. supply. The circuit s shown below.



The a.c. supply is given by, $v = V_m \sin \omega t$

The circuit draws a current I. Due to current I, there are different voltage drops across R, L and C which is given by.

a) Drop across resistance R is $V_R = I R$

b) Drop across inductance L is $V_L = I X_L$

c) Drop across capacitance C is $V_C = I X_C$

The values of I, V_R, V_L and V_C are r.m.s. values

The characteristics of three drops are,

a) V_R is in phase with current I.

b) V_L leads current I by 90°.

c) V_C lags current I by 90°.

According to Kirchhoff's laws, we can write,

$$\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_C \quad \text{... Phasor addition}$$

Let us see the phasor diagram. Current I is taken as reference as it is common to all the elements.

Following are the steps to draw the phasor diagram :

1) Take current as reference.
2) V_R is in phase with I.
3) V_L leads current I by 90°.
4) V_C lags current I by 90°.
5) Obtain the resultant of V_L and V_C. Both V_L and V_C are in phase opposition (180° out of phase).
6) Add that with V_R by law of parallelogram to get the supply voltage.

The phasor diagram depends on the conditions of the magnitudes of V_L and V_C which ultimately depends on the values of X_L and X_C. Let us consider the different cases.

1 $X_L > X_C$

When $X_L > X_C$, obviously, I X_L i.e. V_L is greater than I X_C i.e. V_C. So, resultant of V_L and V_C will be directed towards V_L i.e. leading current I. Current I will lag the resultant of V_L and V_C i.e. $(V_L - V_C)$.

The circuit is said to be **inductive** in nature. The phasor sum of V_R and $(V_L - V_C)$ gives the resultant supply voltage, V. This is shown in the Fig.

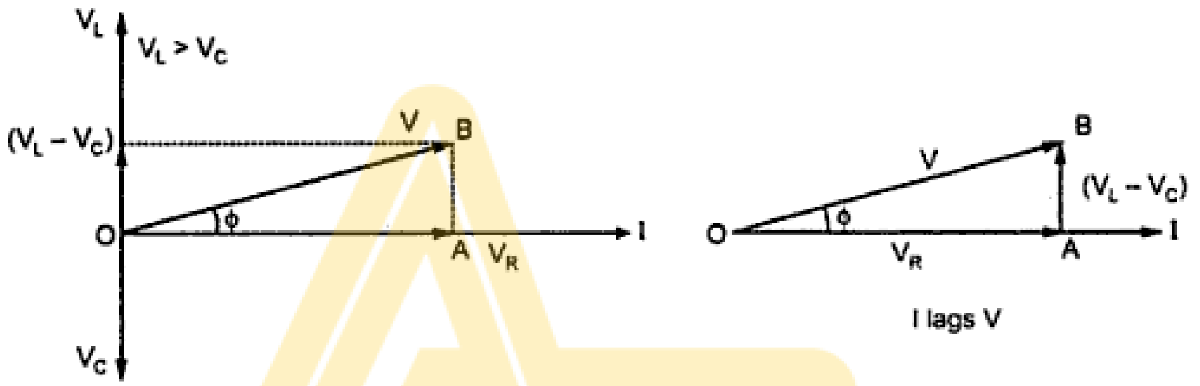


Fig. Phasor diagram and voltage triangle for $X_L > X_C$

From the voltage triangle, $V = \sqrt{(V_R)^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$

$$= I\sqrt{(R)^2 + (X_L - X_C)^2}$$

$$\therefore \quad V = IZ$$

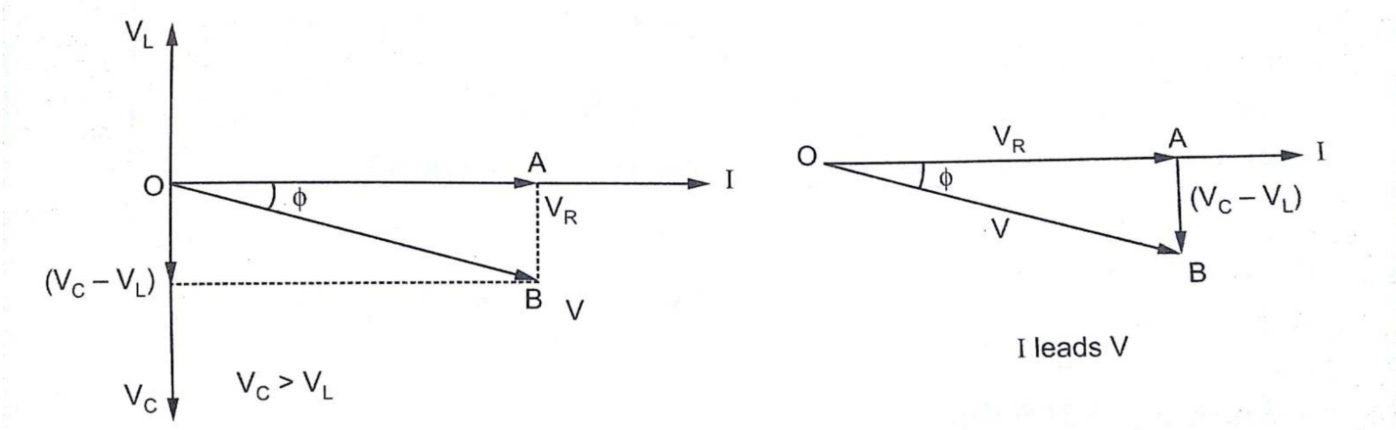
Where $Z = \sqrt{R^2 + (X_L - X_C)^2}$

So, if $v = V_m \sin \omega t$, then $i = I_m \sin(\omega t - \varphi)$ as current lags voltage by angle φ

2. $\mathbf{X_L < X_C}$

When $\mathbf{X_L < X_C}$, obviously, IX_L i.e V_L is less than IX_C i.e V_C. So the resultant of V_L and V_C will be directed towards V_C. Current I will lead $(V_C - V_L)$.

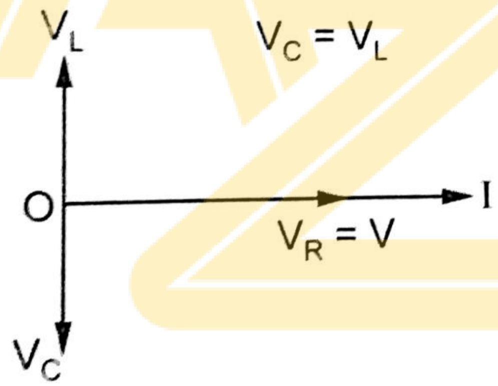
The current is said to be capacitive in nature. The phasor sum of V_R and $(V_C - V_L)$ gives the resultant supply voltage V. This is shown in the fig below.



3. $X_L = X_C$

When $\mathbf{X_L = X_C}$, obviously $\mathbf{V_L = V_C}$. So V_L and V_C will cancel each other and their resultant is zero.

So, $\mathbf{V_R = V}$ in such case and overall circuit is purely resistive in nature. The phasor diagram is shown in fig.



From the phasor diagram V= $\mathbf{V_R}$ **= IR= IZ** where Z=R

The circuit is purely resistive with unity power factor.

Impedance

In general, for RLC series Circuit impedance is given by,

$$Z = R + jX$$

Where $X = X_L - X_C$ = total reactance of circuit

If $X_L > X_C$, X is positive and circuit is inductive.

If $X_L < X_C$, X is negative and circuit is capacitive.

If $X_L = X_C$, X is zero and circuit is purely resistive.

$$\tan\phi = \left[\frac{X_L - X_C}{R}\right],\ \cos\phi = \frac{R}{Z} \text{ and } Z = \sqrt{R^2 + (X_L - X_C)^2}$$

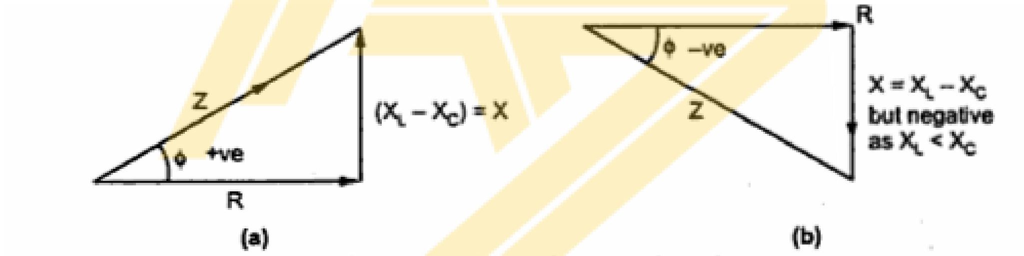
Impedance triangle

The impedance is expressed as,

$$Z = R + jX \quad \text{where} \quad X = X_L - X_C$$

For $X_L > X_C$, φ is positive and impedance triangle is as shown in fig (a)

For $X_L < X_C$, φ is negative and the impedance triangle is as shown in fig (b)



In both cases R= Z cosφ and X= Z sinφ

POWER

The average power consumed by the circuit is,

P_{av} = Average power consumed by R+ Average power consumed by L + Average power consumed by C

But, pure L and C never consume any power.

∴ P_{av} = Power taken by R $= I^2 R = I\,(I\,R) = I\,V_R$

But, $V_R = V\cos\phi$ in both the cases

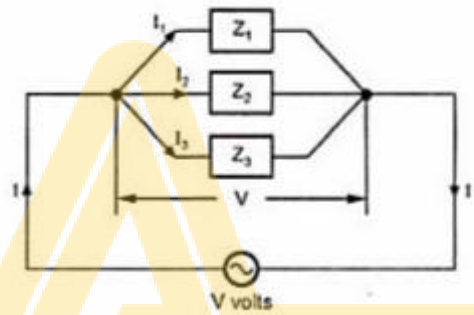
∴ **P = VIcosφ W**

Thus for any condition, in general power can be expressed as **P = VI cosφ**

A.C. Parallel Circuit

A parallel circuit is one in which two or more impedance are connected in parallel across the supply voltage. Each impedance may be a separate series circuit. Each impedance is called branch of the parallel circuit.

The Fig. shows a parallel circuit consisting of three impedances connected in parallel across an ac. supply of V volts.



The current taken by each impedance is different.

Applying Kirchhoff's law, $\bar{I} = \bar{I_1} + \bar{I_2} + \bar{I_3}$ ... (Phasor addition)

$$\therefore \quad \frac{\bar{V}}{\bar{Z}} = \frac{\bar{V}}{\bar{Z_1}} + \frac{\bar{V}}{\bar{Z_2}} + \frac{\bar{V}}{\bar{Z_3}}$$

$$\therefore \quad \frac{1}{\bar{Z}} = \frac{1}{\bar{Z_1}} + \frac{1}{\bar{Z_2}} + \frac{1}{\bar{Z_3}}$$

Where Z is called **equivalent impedance.** This result is applicable for 'n' such impedances connected in parallel.

TWO IMPEDANCE CONNECTED IN PARALLEL

If there are two impedances connected in parallel and if I_T is the total current, then current division rule can be applied to find individual branch currents.

$$\boxed{\begin{aligned} \bar{I}_1 &= \bar{I}_T \times \frac{\bar{Z_2}}{\bar{Z_1} + \bar{Z_2}} \\ \bar{I}_2 &= \bar{I}_T \times \frac{\bar{Z_1}}{\bar{Z_1} + \bar{Z_2}} \end{aligned}}$$

Following are the steps to solve parallel a.c. circuit :

1) The currents in the individual branches are to be calculated by using the relation

$$\bar{I}_1 = \frac{\bar{V}}{\bar{Z}_1}, \quad \bar{I}_2 = \frac{\bar{V}}{\bar{Z}_2}, \ldots\ldots, \quad \bar{I}_n = \frac{\bar{V}}{\bar{Z}_n}$$

While the individual phase angles can be calculated by the relation,

$$\tan\phi_1 = \frac{X_1}{R_1}, \tan\phi_2 = \frac{X_2}{R_2}, \ldots, \tan\phi_n = \frac{X_n}{R_n}$$

2) Voltage must be taken as reference phasor as it is common to all branches.
3) Represent all the currents on the phasor diagram and add them graphically or mathematically by expressing them in rectangular form. This is the resultant current drawn from the supply.
4) The phase angle of resultant current I is power factor angle. Cosine of this angle is the power factor of the circuit..

CONCEPT OF ADMITTANCE

Admittance is defined as the reciprocal of the impedance. It is denoted by Y and is measured in unit siemens or mho.

Now, current equation for the circuit shown in the Fig. is,

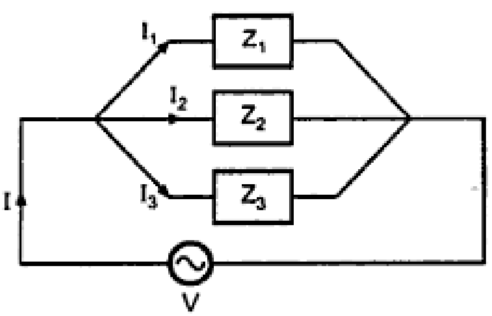
$$\bar{I} = \bar{I}_1 + \bar{I}_2 + \bar{I}_3$$

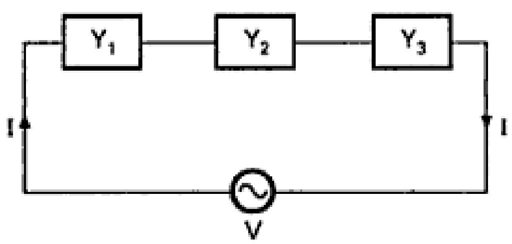
$$\bar{I} = \bar{V} \times \left(\frac{1}{\bar{Z}_1}\right) + \bar{V} \times \left(\frac{1}{\bar{Z}_2}\right) + \bar{V} \times \left(\frac{1}{\bar{Z}_3}\right)$$

$$\bar{V}\bar{Y} = \bar{V}\bar{Y}_1 + \bar{V}\bar{Y}_2 + \bar{V}\bar{Y}_3$$

$$\therefore \quad \bar{Y} = \bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3$$

where Y is the admittance of the total circuit. The three impedances connected in parallel can be replaced by an equivalent circuit, where three admittances are connected in series, as shown in the Fig. 7.41.





Components of admittance

Consider an impedance given as,

$$Z = R \pm jX$$

Positive sign for inductive and negative for capacitive circuit.

Admittance $$Y = \frac{1}{Z} = \frac{1}{R \pm jX}$$

Rationalising the above expression,

$$Y = \frac{R \mp jX}{(R \pm jX)(R \mp jX)} = \frac{R \mp jX}{R^2 + X^2}$$

$$= \left(\frac{R}{R^2 + X^2}\right) \mp j\left(\frac{X}{R^2 + X^2}\right) = \frac{R}{Z^2} \mp j\frac{X}{Z^2}$$

$\therefore$ $$\boxed{Y = G \mp jB}$$

In the above expression, G = Conductance = $\frac{R}{Z^2}$

and B = Susceptance = $\frac{X}{Z^2}$

Conductance (G)

It is defined as the ratio of the resistance to the square of the Impedance. It is measured in the unit siemens.

Susceptance (B)

It is defined as the ratio of the reactance to the square of the impedance. It is measured in the unit siemens.

The susceptance is said to be inductive (B_L) if its sign is negative. The susceptance is said to be capacitive (B_C) if its sign is positive.

Note: The sign convention for the reactance and the susceptance are opposite to each other.

MODULE 2B: THREE PHASE CIRCUIT

Introduction

- We have seen that a single phase a.c. voltage can be generated by rotating a turn made up of two conductors, in a magnetic field. Such an a.c. producing machine is called single turn alternator.
- But voltage produced by such a single turn is very less and not enough to supply practical loads.
- Hence number of turns are connected in series to form one winding in a practical alternator, such a winding is called armature winding.
- The sum of the voltages induced in all the turns is now available as a single phase a.c. voltage, which is sufficient to drive the practical loads.
- But in practice there are certain loads which require polyphase supply.
- Phase means branch, circuit or winding while poly means many. So such applications need a supply having many a.c. voltages present in it simultaneously. Such a system is called polyphase system.

- To develop polyphase system, the armature winding in an alternator is divided into number of phases required.
- In each section, a separate a.c. voltage gets induced. So there are many independent ac. voltages present equal to number of phases of armature winding.
- The various phases of armature winding are arranged in such a manner that the magnitudes and frequencies of all these voltages is same but they have definite phase difference with respect to each other.
- The phase difference depends on number of phases in which armature is divided.
- For example, if armature is divided into three coils then three separate a.c. voltages will be available having same magnitude and frequency but they will have a phase difference of 360°/3 = 120° with respect to each other.
- All three voltages with a phase difference of 120° are available to supply a three phase load.
- Such a supply system is called three phase system. Similarly by dividing armature into various numbers of phases, a 2 phase, 6 phase supply system also can be obtained.
- A phase difference between such voltages Is 360°/n where n is number of phases.

Advantages of Three Phase System

In the three phase system, the alternator armature has three windings and it produces three independent alternating voltages. The magnitude and frequency of all of them is equal but they have a phase difference of 120° between each other. Such a three phase system has following advantages over single phase system.

1. The output of three phase machine is always greater than single phase machine of same size, approximately 1.5 times. So for a given size and voltage a three phase alternator occupies less space and has less cost too than single phase having same rating.
2. For a transmission and distribution, three phase system needs less copper or less conducting material than single phase system for given volt amperes and voltage rating so transmission becomes very much economical.
3. It is possible to produce rotating magnetic field with stationary coils by using three phase system. Hence three phase motors are self starting.
4. In single phase system, the instantaneous power is a function of time and hence fluctuates w.r.t. time. This fluctuating power causes considerable vibrations in single phase motors. Hence performance of single phase motors is poor. While instantaneous power in symmetrical three phase system is constant.
5. Three phase system give steady output.
6. Single phase supply can be obtained from three phase but three phase cannot be obtained from single phase.
7. Power factor of single phase motors is poor than three phase motors of same rating.
8. For converting machines like rectifiers, the D.C. output voltage becomes smoother if number of phases is increased.

But it is found that optimum number of phases required to get all above said advantages is three. Any further increase in number of phases cause a lot of complications. Hence three phase system is accepted as standard system throughout the world.

Generation of Three Phase Voltage System

It is already discussed that alternator consisting of one group of coils on armature produces one alternating voltage. But if armature coils are divided into three groups such that they are displaced by the angle 120° from each other, three separate alternating voltages get developed.

- Consider armature of alternator divided into three groups as shown in the Fig. 4.1. The coils are named as R1- R2, Y1 -Y2 and B1- B2 and mounted on same shaft.

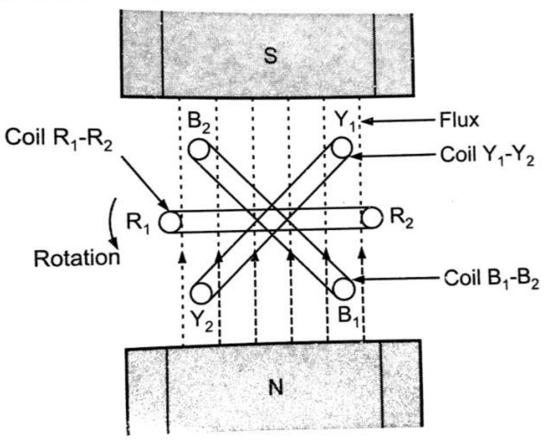


Fig 4.1

- The ends of each coil are brought out through the slip ring and brush arrangement to collect the Induced e.m.f.
- Let e_R, e_Y and e_B be the three independent voltages in coil R_1R_2, Y_1Y_2 and B_1B_2 respectively.
- All are alternating voltages having same magnitude and frequency as they are rotated at uniform speed.
- All of them will be displaced by one another by 120°.
- Suppose e_R is assumed to be reference and is zero for the instant shown in fig 4.2.

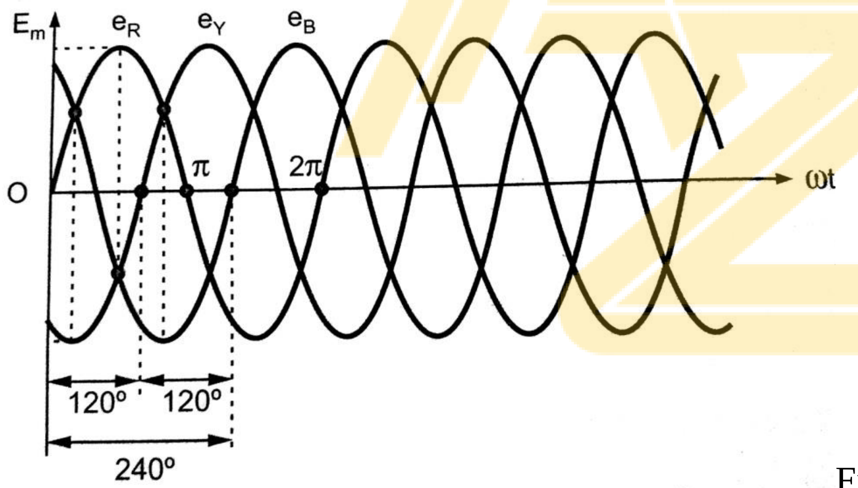


Fig 4.2

- At the same instant e_Y will be displaced by 120° from e_R and will follow e_R while e_B will be displaced by 120° from e_Y and will follow e_Y.
- All coils together represent three phase supply system.
- The equation of the induced e.m.f are

$$e_R = E_m \sin(\omega t)$$
$$e_Y = E_m \sin(\omega t - 120°)$$
$$e_B = E_m \sin(\omega t - 240°)$$

- The phasor diagram is shown in fig 4.3

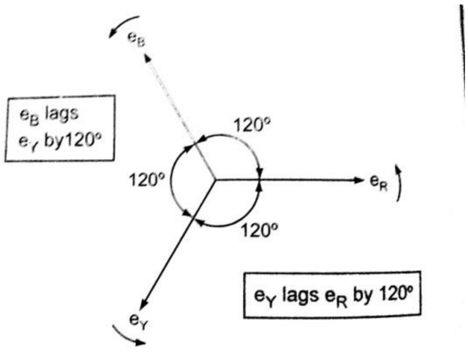


Fig 4.3

- If the three voltages are added vectorially, it can be observed that the sum of these three voltages at any instant is zero.

Mathematically this can be shown as :

$e_R + e_Y + e_B$

$= E_m \sin \omega t + E_m \sin(\omega t - 120°) + E_m \sin(\omega t + 120°)$

$= E_m [\sin \omega t + \sin \omega t \cos 120° - \cos \omega t \sin 120° + \sin \omega t \cos 120° + \cos \omega t \sin 120°]$

$= E_m [\sin \omega t + 2 \sin \omega t \cos 120°] = E_m \left[\sin \omega t + 2 \sin \omega t \left(\frac{-1}{2}\right)\right] = 0$

$\therefore \quad \boxed{\bar{e}_R + \bar{e}_Y + \bar{e}_B = 0}$

Phase Sequence

The sequence in which the voltages in three phases reach their maximum positive values is called phase-sequence. Generally the phase sequence is R-Y-B.

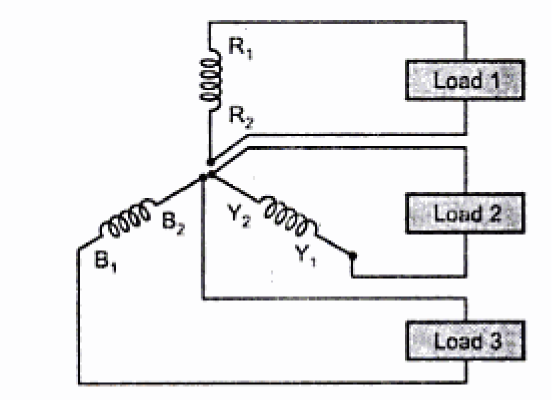
The significance of the phase sequence of the three phase supply is:

1. When the 3 phase supply of a particular sequence is given to a static three phase load, certain current flows through the line and phase of the load. If the phase sequence is changed, then both magnitude and phase of the currents flowing in the lines and the phase of the load will change.
2. If the load is a three phase induction motor, when the sequence of the supply is changed, not only the magnitude and phase of the line current and phase current change, but the direction of rotation of motor also changes.

Three Phase Supply Connections

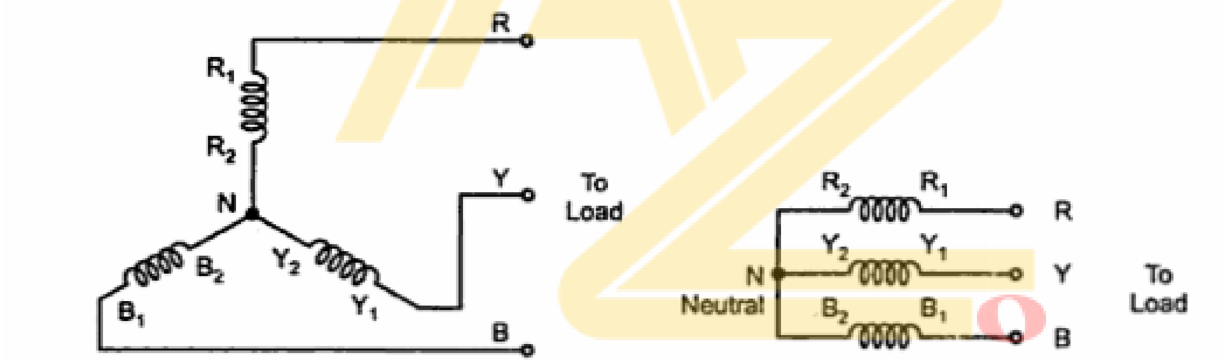
In single phase system, two wires are sufficient for transmitting voltage to the load i.e. phase and neutral. But in case of three phase system, two ends of each phase i.e. R1-R2, Y1-Y2 and B1-B2 are available to supply voltage to the load. If all six terminals are used independently to supply voltage to load as shown in the Fig, then total six wires will be required and it will be very much costly.

To reduce the cost by reducing the number of windings, the three windings are interconnected in a particular fashion. This gives different three phase connections.



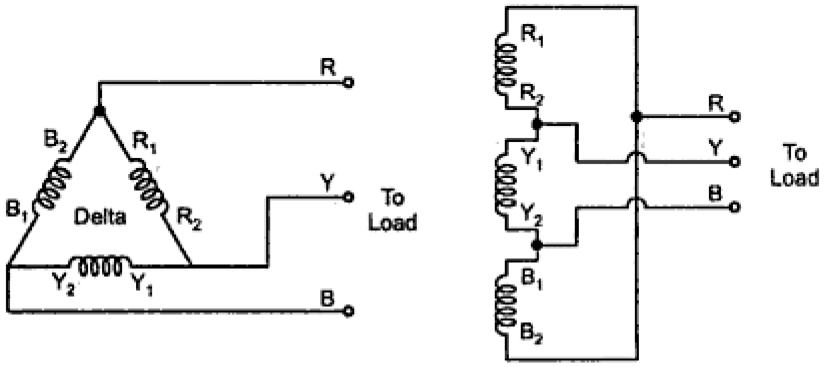
Star Connection

The star connection is formed by connecting starting or terminating ends of all the three windings together. The ends R1 -Y1 -B1 are connected or ends R2 -Y2 -B2 are connected together. This common point is called Neutral Point. The remaining three ends are brought out for connection purpose. These ends are generally referred as R-Y-B, to which load is to be connected.



Delta Connection

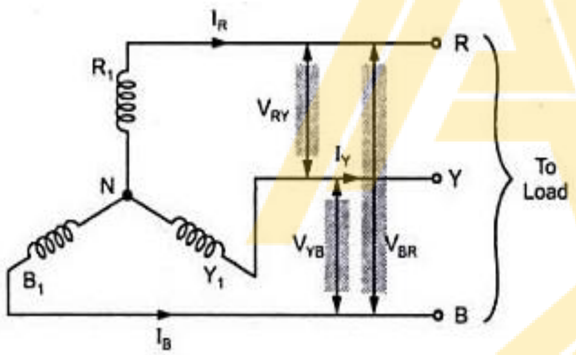
The delta is formed by connecting one end of winding to starting end of other and connections are continued to form a dosed loop. The supply terminals are taken out from the three junction points. The delta connection is shown in the Fig.

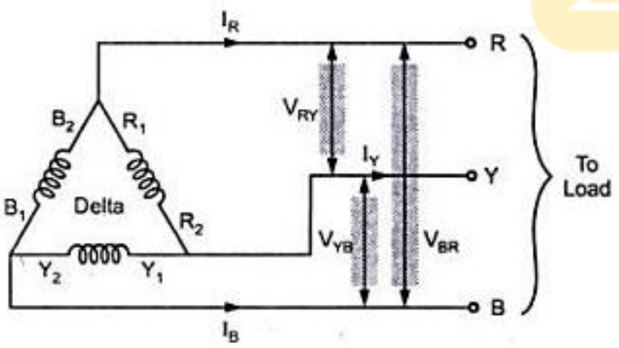


Concept of Line Voltages and Line Currents

The potential difference between any two lines of supply is called line voltage and current passing through any line is called line current.

Consider a star connected system as shown in the Fig.





Line voltages are denoted by V_L. These are V_{RY}, V_{YB} and V_{BR}. Line currents are denoted by I_L. These are I_R, I_Y and I_B

Similarly for delta connected system we can show the Line voltages and line currents as in the Fig.

Line voltages V_L are V_{RY}, V_{YB} and V_{BR}.

While Line currents I_L are I_R, I_Y and I_B.

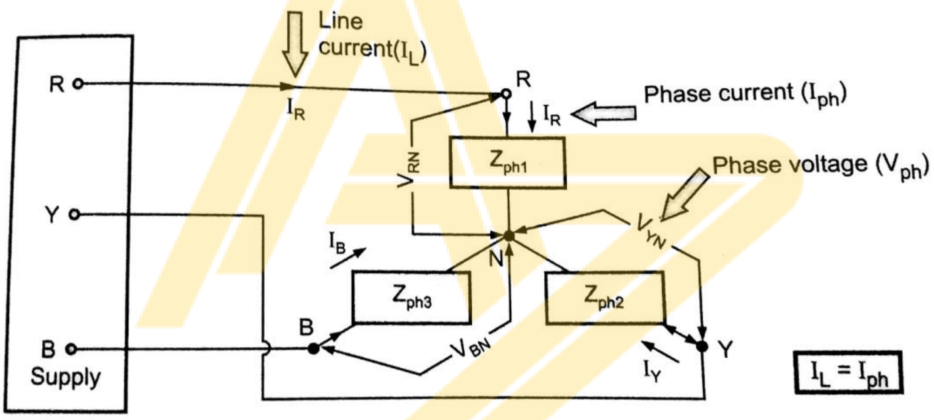
Concept of Phase Voltages and Phase Currents

Now to define the phase voltages and phase currents let us see the connections of the three phase load to the supply lines. Generally Red, Yellow and Blue coloured wires are used to differentiate three phases and hence the names given to three phases are R, Y and B.

The load can be connected in two ways, i) Star connection, ii) Delta connection

The three phase load is nothing but three different impedances connected together in star or delta fashion

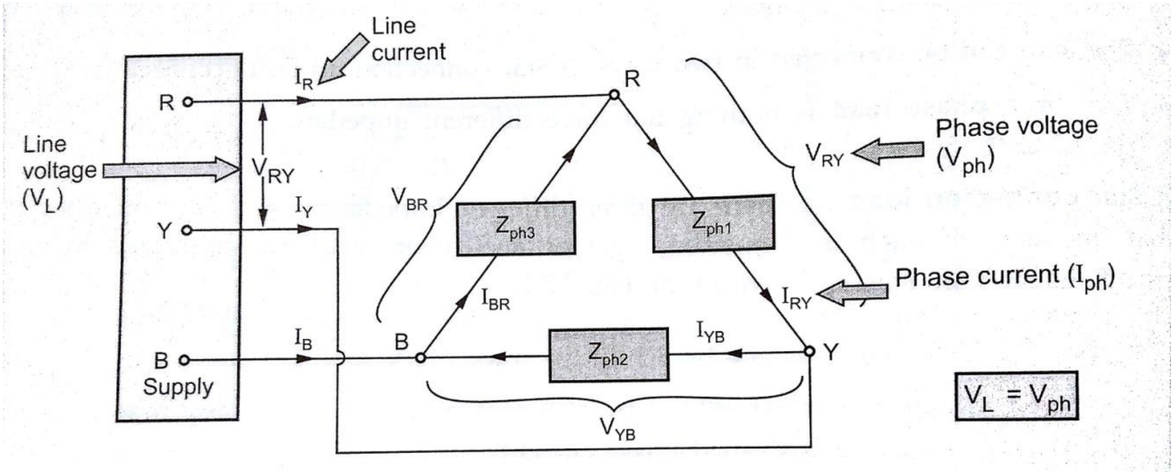
1. **Star connected load**: There are three different impedances and are connected such that one end of each is connected together and other three are connected to supply terminals R-Y-B. This is shown in the Fig.



- In the diagram shown V_{RN}, V_{YN} and V_{BN} are the phase voltages while I_R, I_Y and I_B are phase currents.
- The phase voltages are denoted as V_{ph} while the phase currents are denoted as I_{ph}.
- Generally suffix N is not indicated for phase voltages in star connected load. So, $V_{ph}= V_R= V_Y=V_B$.
- It can be seen from the diagram that $I_{ph}=I_R=I_Y=I_B$.
- But same are the currents flowing in the three lines also. Thus we can conclude that for star connection

$$\mathbf{I_{ph}=I_L}$$

2. **Delta Connection:** If the three impedances are connected such that the starting end of one is connected to the terminating end of other, to form a closed loop it is called delta connection of the load. The junction points are connected to supply terminals R-Y-B.



- The currents I_{RY}, I_{YB} and I_{BR} flowing through the various branches of the load are phase currents. The line currents are I_R, I_Y and I_B flowing through supply lines. Thus in delta connection of the load, line and phase currents are different.
- The voltages across $Z_{ph1} = V_{RY}$, across $Z_{ph2} = V_{YB}$ and across $Z_{ph3} = V_{BR}$ and all are phase voltages.

$$V_{ph} = V_{RY} = V_{YB} = V_{BR}$$

- But as per definition of line voltages, same are the voltage across the supply line also. Thus it can be concluded that in delta connection line voltage is equal to phase voltage.

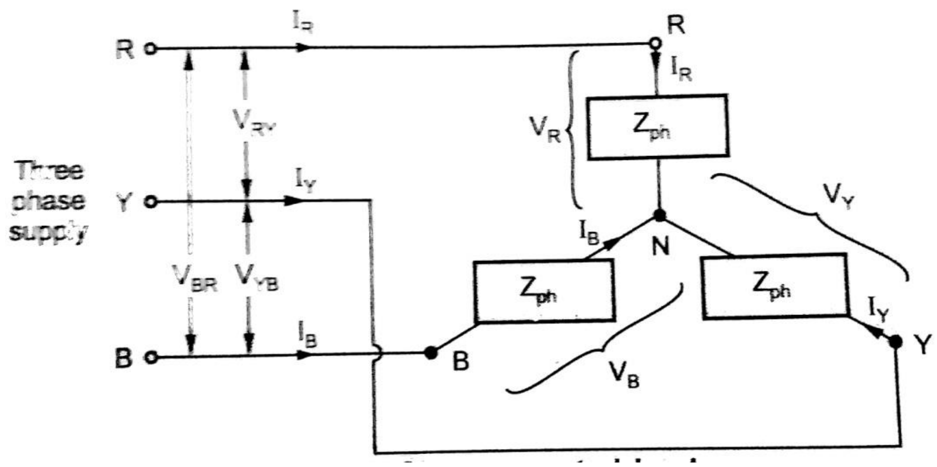
$$\mathbf{V_{ph} = V_L}$$

Balanced Load

- The load is said to be balanced when magnitude of all the impedances are equal and phase angle of all of them are equal and of same nature either all inductive or all capacitive or all resistive.
- In such cases all phase voltages have equal magnitude and are displaced from each other by 120° while all phase currents also have equal magnitude and are displaced from each other by 120°.
- The same is true for all the line voltages and line currents.
- The load is said to be unbalanced when magnitude of all the impedances are unequal and phase angle of all of them are unequal. In such cases all phase voltages have unequal magnitude and are not displaced from each other by 120°.

Relation for star connected load

Consider a balanced star connected load as shown in fig.



Line voltages $V_L = V_{RY} = V_{YB} = V_{BR}$.

While Line currents $I_L = I_R = I_Y = I_B$.

Phase voltages $V_{ph} = V_R = V_Y = V_B$.

Phase currents $I_{ph} = I_R = I_Y = I_B$

As seen earlier $\mathbf{I_{ph} = I_L}$

To derive relation between V_{ph} and V_L, consider the voltage V_{RY} . we can write

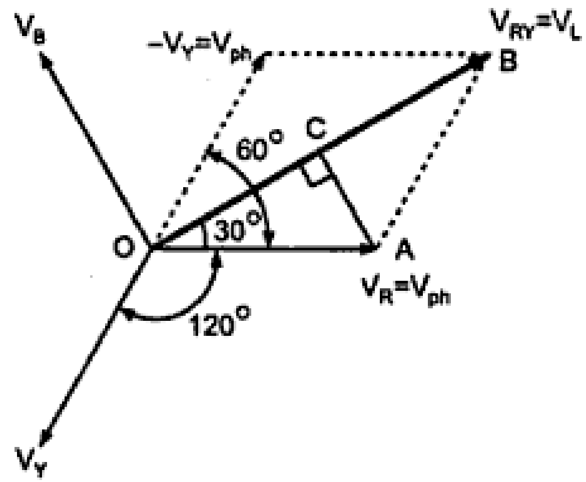
$$\bar{V}_{RY} = \bar{V}_{RN} + \bar{V}_{NY}$$

But $\bar{V}_{NY} = -\bar{V}_{YN}$

Hence $\bar{V}_{RY} = \bar{V}_R - \bar{V}_Y$ ...(1)

Similarly, $\bar{V}_{YB} = \bar{V}_{YN} + \bar{V}_{NB} = \bar{V}_{YN} - \bar{V}_{BN} = \bar{V}_Y - \bar{V}_B$ ...(2)

and $\bar{V}_{BR} = \bar{V}_B - \bar{V}_R$ ...(3)



The three phase voltages are displaced by 120° from each other. The phasor diagram to get V_{RY} is shown in the above. The V_Y is reversed to get $-V_Y$ and then it is added to V_R to get V_{RY}.

The perpendicular is drawn from point A on vector OB representing V_L. In triangle OAB, the sides OA and AB are same as phase voltages. Hence OB bisects angle between V_R and $-V_Y$.

$$\therefore \angle BOA = 30°$$

And perpendicular AC bisects the vector OB.

$$OC = CB = \frac{V_L}{2}$$

From triangle OAB, $\cos 30° = \frac{OC}{OA} = \frac{(V_{RY}/2)}{V_R}$

$$\therefore \quad \frac{\sqrt{3}}{2} = \frac{(V_L/2)}{V_{ph}}$$

$$\therefore \quad V_L = \sqrt{3}\ V_{ph} \text{ for star connection}$$

Thus line voltage is √3 times the phase voltage in star connection.

Now lagging or leading nature of the current depends on per phase Impedance. If Z_{ph} is inductive i.e. R+j X_L then current I_{ph} lags V_{ph} by angle φ where φ is $\tan^{-1}\frac{X_L}{R}$. If Z_{ph} is capacitive i.e. R-j X_C then I_{ph} leads V_{ph} by angle φ. If Z_{ph} is resistive i.e. R+j 0 then I_{ph} is in phase with V_{ph}.

And $$\boxed{|Z_{ph}| = \frac{|V_{ph}|}{|I_{ph}|}}$$

$$\boxed{\phi = V_{ph}\hat{}I_{ph} \neq V_L\hat{}I_L}$$

Power : The power consumed in each phase is single phase power given by,

$$\boxed{P_{ph} = V_{ph} I_{ph} \cos\phi}$$

For balanced load, all phase powers are equal. Hence total three phase power consumed is,

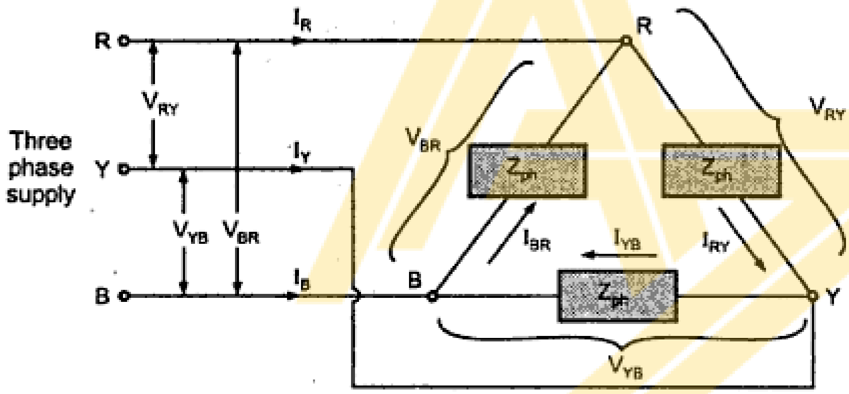
$$P = 3P_{ph} = 3 V_{ph} I_{ph} \cos\phi = 3 \frac{V_L}{\sqrt{3}} I_L \cos\phi$$

$$\therefore \quad \boxed{P = \sqrt{3}\, V_L I_L \cos\phi}$$

For star connection, to draw phasor diagram, use

$$\bar{V}_{RY} = \bar{V}_R - \bar{V}_Y, \quad \bar{V}_{YB} = \bar{V}_Y - \bar{V}_B \quad \text{and} \quad \bar{V}_{BR} = \bar{V}_B - \bar{V}_R$$

4.9 Relation for Delta Connected Load



Line voltages $V_L = V_{RY} = V_{YB} = V_{BR}$

Line currents $I_L = I_R = I_Y = I_B$

Phase voltages $V_{ph} = V_{RY} = V_{YB} = V_{BR}$

Phase currents $I_{ph} = I_{RY} = I_{YB} = I_{BR}$

As seen earlier, $V_{ph} = V_L$ for delta connected load. To derive the relation between I_L and I_{ph}, apply the KCL at the node R of the load shown in the Fig.

$$\sum I_{entering} = \sum I_{leaving} \text{ at node R}$$

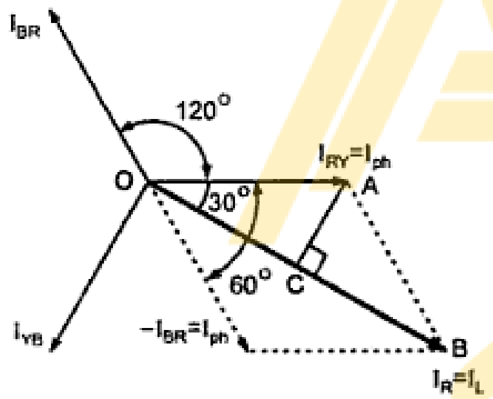
$$\therefore \quad \bar{I}_R + \bar{I}_{BR} = \bar{I}_{RY}$$

$$\therefore \quad \bar{I}_R = \bar{I}_{RY} - \bar{I}_{BR} \qquad \text{...(1)}$$

Applying KCL at node Y and B, we can write equations for line currents I_Y and I_B as,

$$\bar{I}_Y = \bar{I}_{YB} - \bar{I}_{RY} \qquad \text{...(2)}$$

$$\bar{I}_B = \bar{I}_{BR} - \bar{I}_{YB} \qquad \text{...(3)}$$



The phasor diagram to obtain line current I_R by carrying out vector subtraction of phase currents I_{RY} and I_{YB} is shown in the Fig.

The three phase currents are displaced from each other by 120°.

I_{BR} is reversed to get $-I_{BR}$ and then added to I_{RY} to get I_R.

The perpendicular AC drawn on vector OB, bisects the vector OB which represents I_L. Similarly OB bisects angle between $-I_{YB}$ and I_{RY} which is 60°

$$\therefore \quad \angle BOA = 30° \quad \text{and} \quad OC = CB = \frac{I_L}{2}$$

From triangle OAB,

$$\cos 30° = \frac{OC}{OA} = \frac{I_R/2}{I_{RY}}$$

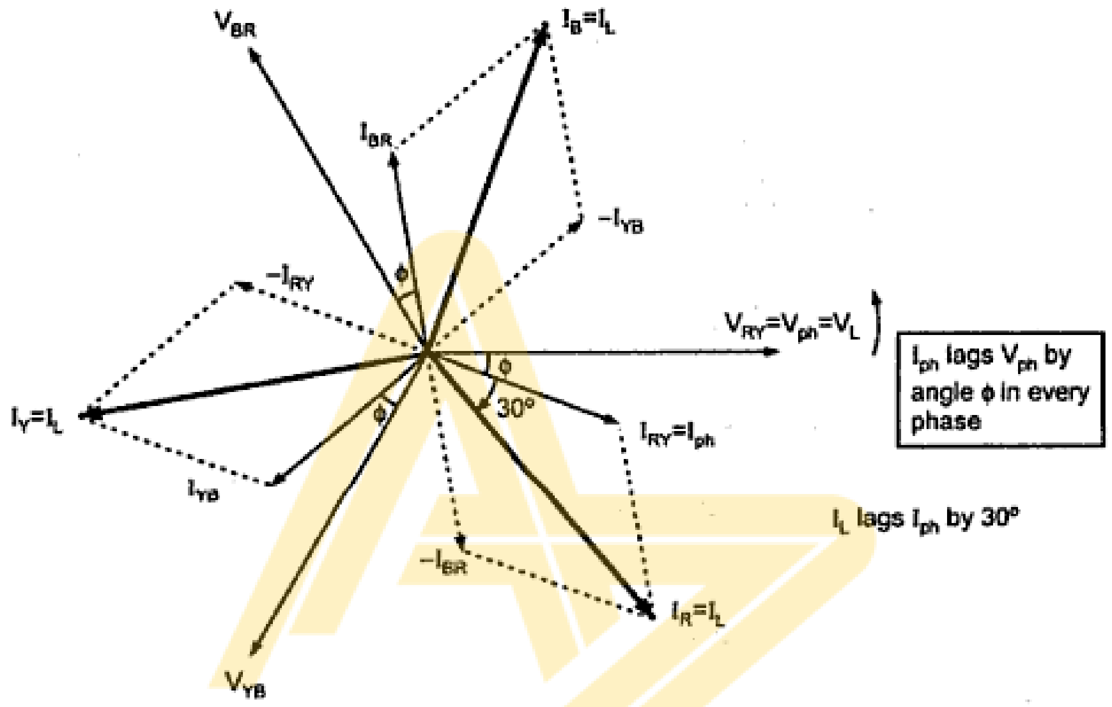
$$\therefore \quad \frac{\sqrt{3}}{2} = \frac{I_L/2}{I_{ph}}$$

$$\therefore \quad \boxed{I_L = \sqrt{3}\ I_{ph}} \qquad \text{... for delta connection}$$

Again Z_{ph} decides whether I_{ph} has to lag, lead or remain in phase with V_{ph}. **Angle between V_{ph} and I_{ph} is ϕ.**

Thus for delta connection, to draw phasor diagram, use $\bar{I}_R = \bar{I}_{RY} - \bar{I}_{BR}$, $\bar{I}_Y = \bar{I}_{YB} - \bar{I}_{RY}$ and $\bar{I}_B = \bar{I}_{BR} - \bar{I}_{YB}$

The complete phasor diagram for cos ϕ lagging power factor load is shown in the Fig.



$$Z_{ph} = R_{ph} + j\,X_{Lph} = |Z_{ph}| \angle \phi\,\Omega$$

Power : Power consumed in each phase is single phase power given by,

$$\boxed{P_{ph} = V_{ph}\, I_{ph} \cos\phi}$$

Total power $\quad P = 3P_{ph} = 3V_{ph} I_{ph} \cos\phi = 3V_L \frac{I_L}{\sqrt{3}} \cos\phi$

$\therefore \quad P = \sqrt{3}\, V_L I_L \cos\phi$

Power Triangle for Three Phase Load

Total apparent power $S = 3 \times$ Apparent power per phase

$\therefore$ $S = 3\ V_{ph}I_{ph} = 3\ \frac{V_L}{\sqrt{3}} I_L = 3\ V_L \frac{I_L}{\sqrt{3}}$

$\therefore$ $\boxed{S = \sqrt{3}\, V_L I_L}$ volt-amperes (VA) or kVA

Total active power $P = \sqrt{3}\, V_L I_L \cos\phi$ watts (W) or kW

Total reactive power $Q = \sqrt{3}\, V_L I_L \sin\phi$ reactive volt amperes (VAR) or kVAR

Hence power triangle is as shown in the Fig.

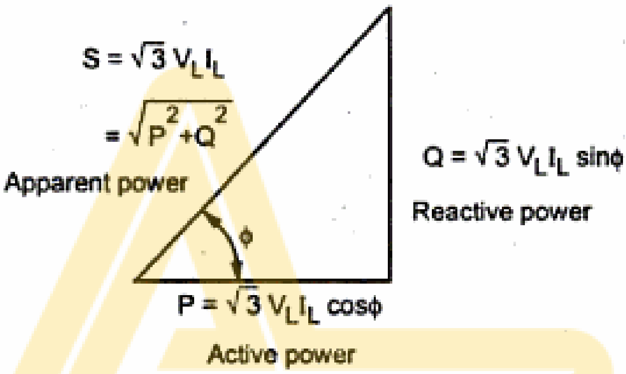
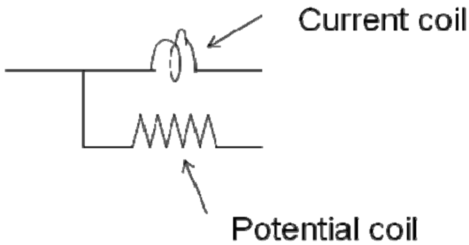


Fig. Power triangle

Measurement of power and power factor by two wattmeter method

The power in a three phase circuit can be measured by connecting two wattmeters in any of the two phases of the three phase circuit. A wattmeter consists of a current coil and a potential coil as shown in the figure.



The wattmeter is connected in the circuit in such a way that the current coil is in series and carries the load current and the potential coil is connected in parallel across the load voltage. The wattmeter reading will then he equal to the product of the current carried by the current coil, the voltage across the potential coil and the cosine of the angle between the voltage and current.

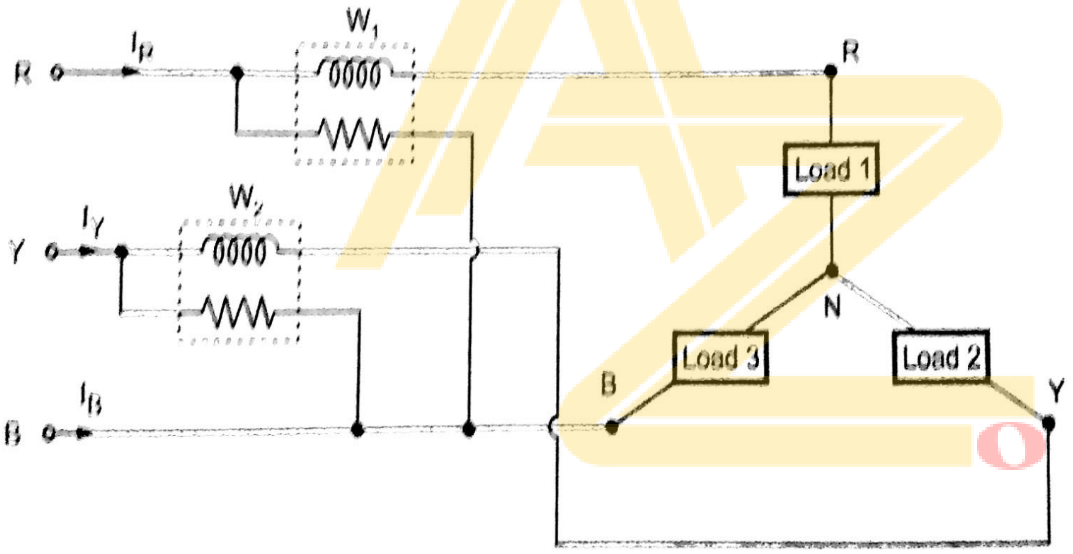
Balanced star connected load

- The current coils of the two wattmeters are connected in any two lines while the voltage coil of each wattmeter is connected between its own current coil terminal and the line without the current coil.
- For example, the current coils are inserted in the lines R and Y then the voltage coils are connected between R-B for one wattmeter and Y-B for other wattmeter.
- It can be shown that when two wattmeters are connected in this way, the algebraic sum of the two wattmeter readings gives the total power dissipated in the three phase circuit.
- If W_1 and W_2 are the two wattmeter readings then total power

$$\mathbf{W = W_1 + W_2 = three\ phase\ power = \sqrt{3}\, V_L I_L \cos\varphi}$$

PROOF:

- Consider star connected load and two wattmeter connected as shown in fig.



- Let us consider the RMS values of current and voltage to prove that sum of two wattmeter gives the total power consumed by the three phase load.

$$W_1 = I_R X\, V_{RB} X \cos\angle I_R \& V_{RB}$$

$$W_2 = I_Y X\, V_{YB} X \cos\angle I_Y \& V_{YB}$$

- To find angle between (I_R and V_{RB}) and (I_Y and V_{YB}) phasor diagram is drawn. (Assuming power factor to be lagging)

$$\overline{V_{RB}} = \overline{V_R} - \overline{V_b}$$

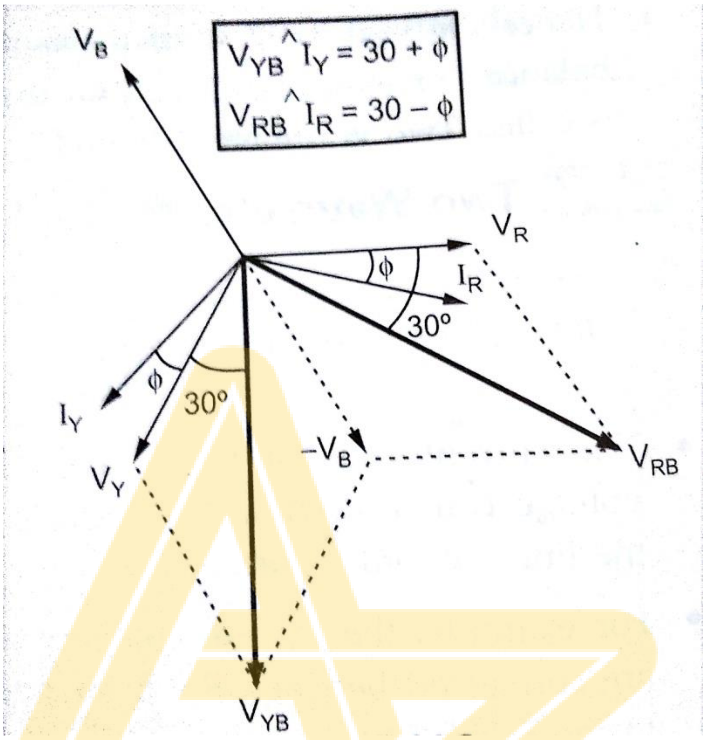
And $\overline{V_{YB}} = \overline{V_Y} - \overline{V_B}$

Angle between V_R and $I_R = \varphi$

Angle between V_Y and $I_Y = \varphi$

$V_{ph} = V_R = V_Y = V_B$ and $V_{RB} = V_{YB} = V_L$

$I_R = I_Y = I_L = I_{PH}$(star)



From the vector diagram

Angle between V_{RB} and $I_R = 30° - \varphi$

Angle between V_{YB} and $I_Y = 30° + \varphi$

$\therefore \quad W_1 = I_R V_{RB} \cos(30° - \varphi)$ i.e. $W_1 = I_L V_L \cos(30° - \varphi)$

$W_2 = I_Y V_{YB} \cos(30° + \varphi)$ i.e. $W_2 = I_L V_L \cos(30° + \varphi)$

$\therefore \quad W_1 + W_2 = I_L V_L [\cos(30° - \varphi) + \cos(30° + \varphi)]$

$= I_L V_L [\cos 30 \cos\varphi + \sin 30 \sin\varphi + \cos 30 \cos\varphi - \sin 30 \sin\varphi]$

$= 2 I_L V_L \cos 30 \cos\varphi$

$= 2 I_L V_L \frac{\sqrt{3}}{2} \cos\varphi$

$\mathbf{W_1 + W_2 = \sqrt{3}\, I_L V_L \cos\varphi}$ **= Total 3 phase power**

Power Factor Calculation by Two Wattmeter Method

- In case of balanced load, the p.f. can be calculated from W_1 and W_2 readings.
- For balanced lagging p.f.

$$W_1 = I_L V_L \cos(30° - \varphi)$$

$$W_2 = I_L V_L \cos(30° + \varphi)$$

$$\mathbf{W_1 + W_2 = \sqrt{3}\, I_L V_L \cos\varphi} \text{ ---------------------------------(i)}$$

$$W_1 - W_2 = I_L V_L [\cos(30° - \varphi) - \cos(30° + \varphi)]$$
$$W_1 - W_2 = I_L V_L [\cos 30 \cos\varphi + \sin 30 \sin\varphi - \cos 30 \cos\varphi + \sin 30 \sin\varphi]$$
$$W_1 - W_2 = 2 I_L V_L \sin 30 \sin\varphi$$
$$\mathbf{W_1 - W_2 = I_L V_L \sin\varphi} \text{ --------------------------------------(ii)}$$

- Taking ratio of equation (ii) and (i)

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{I_L V_L \sin\varphi}{\sqrt{3} I_L V_L \cos\varphi} = \frac{\tan\varphi}{\sqrt{3}}$$

$$\text{Tan } \varphi = \frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}$$

$$\varphi = \tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}\right]$$

$$\textbf{Power factor } \mathbf{\cos\varphi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}\right]\right\}}$$

Effect of Power Factor on Wattmeter Readings

- For balanced lagging p.f.

$$W_1 = I_L V_L \cos(30° - \varphi)$$

$$W_2 = I_L V_L \cos(30° + \varphi)$$

- Consider different cases

Case (i) cosφ = 0 i.e. φ=90°

$$\therefore W_1 = I_L V_L \cos(30° - 90°) = 1/2 I_L V_L$$

$$W_2 = I_L V_L \cos(30° + 90°) = -1/2 I_L V_L$$

i.e. $\mathbf{W_1 + W_2 = 0}$

$\mathbf{|W_1| = |W_2|}$ **but** $\mathbf{W_2 = -W_1}$

Case (ii) cos φ = 0.5 i.e. φ= 60°

$W_1 = I_L V_L \cos(30° - 60°) = \sqrt{3}/2 I_L V_L$

$W_2 = I_L V_L \cos(30° + 60°) = 0$

$\mathbf{W_1 + W_2 = W_1}$ **= Total Power**

- One wattmeter shows zero reading

Case (iii) cosφ =1 i.e φ = 0

$W_1 = I_L V_L \cos(30° - 0°) = \sqrt{3}/2 I_L V_L$

$W_2 = I_L V_L \cos(30° + 0°) = \sqrt{3}/2 I_L V_L$

- Both wattmeter read equal and positive