MODULE 1A: D.C CIRCUITS

1.1 Introduction

In practice, the electrical circuits may consist of one or more sources of energy and number of electrical parameters, connected in different ways. The different electrical parameters or elements are resistors, capacitors and inductors. The combination of such elements along with various sources of energy gives rise to complicated electrical circuits, generally referred as networks. The terms circuit and network are used synonymously in the electrical literature. The d.c. circuits consist of only resistances and d.c. sources of energy. And the circuit analysis means to find a current through or voltage across any branch of the circuit.

1.2 Network Terminology

In this section, we shall define some of the basic terms which are commonly associated with a network

1.2.1 Network

Any arrangement of the various electrical energy sources along with the different circuit elements is called an **electrical network**. Such a network is shown in the Fig. 1.1.

1.2.2 Network Element

Any individual circuit element with two terminals which can be connected to other circuit element is called a **network element.**

Network elements can be either active elements or passive elements. **Active elements** are the elements which supply power or energy to the network. Voltage source and current source are the examples of active elements. **Passive elements** are the elements which either store energy or dissipate energy in the form of heat. Resistor, inductor and capacitor are the three basic passive elements. Inductors and capacitors can store energy and resistors dissipate energy in the form of heat.

1.2.3 Branch

A part of the network which connects the various points of the network with one another is called a **branch**. In the Fig. 1.1, AB, BC, CD, DA, DE, CF and EF are the various branches. A branch may consist more than one element.

1.2.4 Junction Point

A point where three or more branches meet is called a **junction point**. Point D and C are the junction points in the network shown in the Fig. 1.1.

1.2.5 Node

A point at which two or more elements are joined together is called **node**. The junction points are also the nodes of the network. In the network shown in the Fig. 1.1, A, B, C, D, E and F are the nodes of the network.

1.2.6 Mesh (or Loop)

Mesh (or Loop) is a set of branches forming a closed path in a network in such a way that if one branch is removed then remaining branches do not form a closed path. A loop also can be defined as a closed path which originates from a particular node, terminating at the same node, travelling through various other nodes, without travelling through any node twice, In the Fig. 1.1 paths A-B-C-D-A, A-B-C-F-ED-A, D-C-F-E-D etc. are the loops of the network.

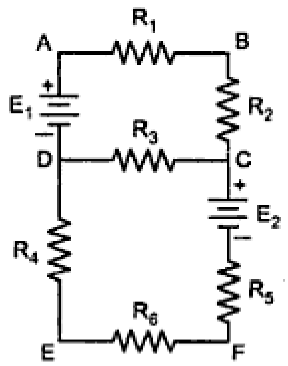


Fig 1.1 An electrical network

1.3 Energy Sources

There are basically two types of energy sources; voltage source and current source. These are classified as i) Ideal source and ii) Practical source.

Let us see the difference between ideal and practical sources.

1.3.1 Voltage Source

Ideal voltage source is defined as the energy source which gives constant voltage across its terminals irrespective of the current drawn through its terminals. The symbol for ideal voltage source is shown in the Fig. 1.2 (a). This is connected to the load as shown In Fig. 1.2 (b). At any time the value of voltage at load terminals remains same. This is indicated by V-I characteristics shown in the Fig. 1.2 (c).

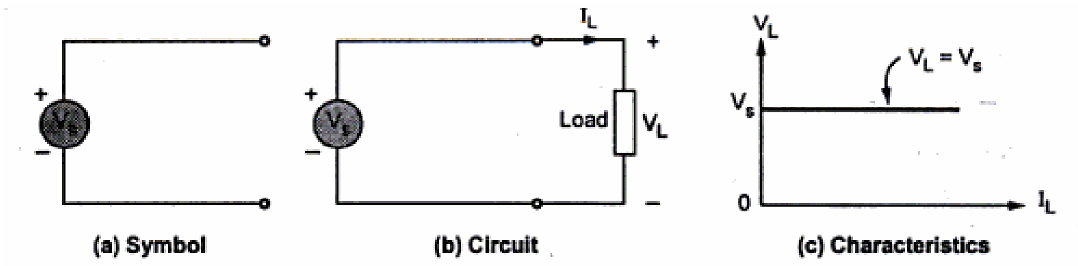


Fig 1.2 Ideal Voltage source

1.3.2 Practical voltage source:

But practically, every voltage source has small internal resistance shown in series with voltage source and is represented by R_{se} as shown in the Fig. 1.3.

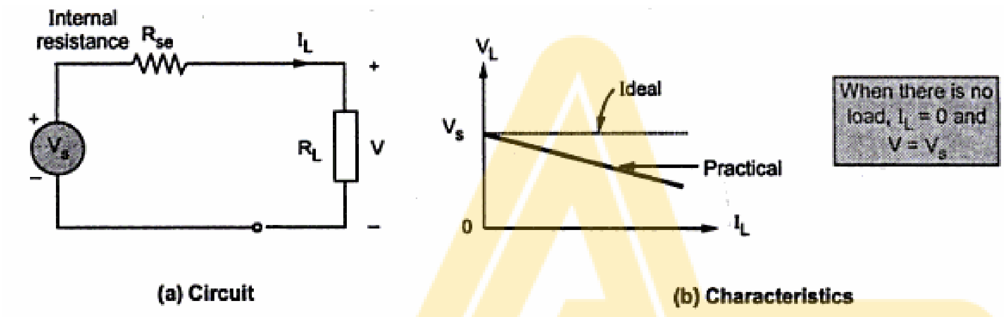


Fig 1.3 Practical Voltage Source

Because of the R_{se}, voltage across terminals decreases slightly with increase in current and it is given by expression.

$$\boxed{V_L = -(R_{se})\, I_L + V_S = V_S - I_L R_{se}}$$

1.3.2 Current Source

Ideal current source is the source which gives constant current at its terminals irrespective of the voltage appearing across its terminals. The symbol for ideal current source is shown in the Fig. 1.4 (a). This is connected to the load as shown In the Fig. 1.4 (b). At any time, the value of the current flowing through load I_L, is same i.e. is irrespective of voltage appearing across its terminals. This is explained by V-I characteristics shown in the Fig. 1.4(c).

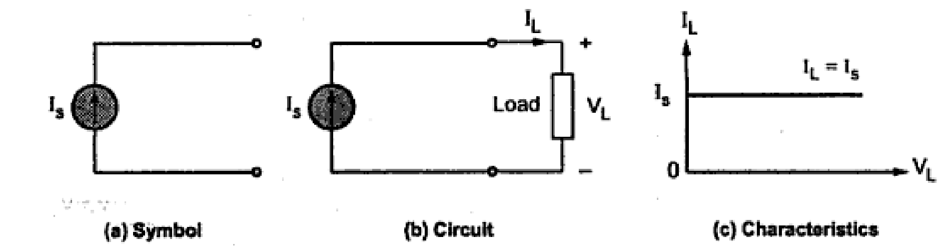


Fig 1.4 Ideal current source

But practically, every current source has high internal resistance, shown in parallel with current source and it is represented by R_{sh}. This is shown in the Fig. 1.5.

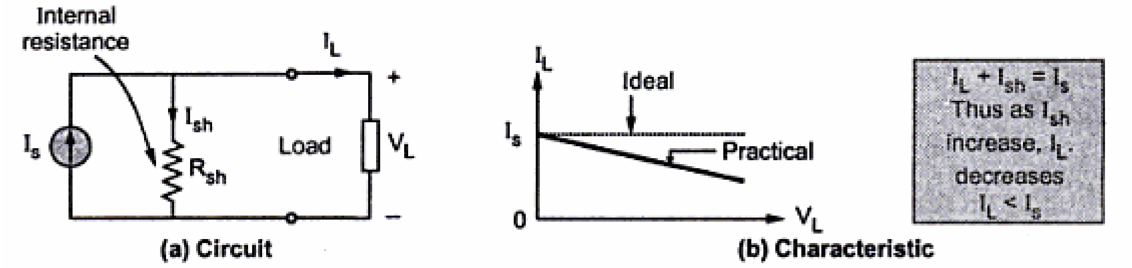


Fig 1.5 Practical Current source

1.4 Ohm's Law

This law gives relationship between the potential differences (V), the current (I) and the resistance (R) of a d.c. circuit. Dr. Ohm in 1827 discovered a law called Ohm's Law. It states,

Ohm's Law: *The current flowing through the electric circuit is directly proportional to the potential difference across the circuit and inversely proportional to the resistance of the circuit, provided the temperature remains constant.*

$$\boxed{I \propto \frac{V}{R}}$$

Where I is the current flowing in amperes, the V is the voltage applied and R is the resistance of the conductor, as shown In the Fig. 1.6,

Now $I = \frac{V}{R}$

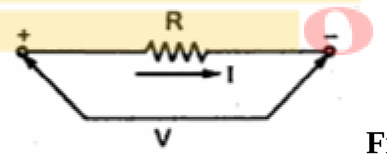


Fig 1.6 ohm`s law

The unit of potential difference is defined in such a way that the constant of proportionality is unity.

Ohm's Law is,	$I = \frac{V}{R}$	amperes
	$V = IR$	volts
	$\frac{V}{I} =$ constant $= R$	ohms

The Ohm's law can be defined as,

The ratio of potential difference (V) between any two points of a conductor to the current (I) flowing between them is constant, provided that the temperature of the conductor remains constant.

Key Point: *Ohm's Law can be applied either to the entire circuit or to the part of a circuit. If it is applied to entire circuit, the voltage across the entire circuit and resistance of the entire circuit should be taken into account. If the Ohm's Law is applied to the part of a circuit, then the resistance of that part and potential across that part should be used.*

1.4.1 Limitations of Ohm's Law

The limitations of the Ohms law are,

1. It is not applicable to the nonlinear devices such as diodes, zener diodes, voltage regulators etc.
2. It does not hold good for non-metallic conductors such as silicon carbide. The Law for such conductors is given by,

$$V = k\, I^{m} \text{ where } k, m \text{ are constants}$$

1.5 Series Circuit

A series circuit is one in which several resistances are connected one after the other. Such connection is also called end to end connection or cascade connection. There is only one path for the flow of current.

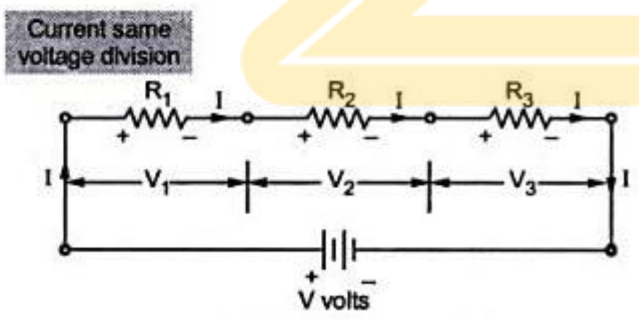


Fig 1.7 Series circuit

Consider the resistances shown in the Fig. 1.7

The resistance R1, R2 and R3 are said to be in series. The combination is connected across a source of voltage V volts. Naturally the current flowing through all of them is same indicated as I amperes. E.g. the chain of small lights, used for the decoration is good example of series combination.

Now let us study the voltage distribution.

Let V_1, V_2 and V_3 be the voltages across the terminals of resistances R_1, $R_2$and R_3 respectively.

Then, $V = V_1 + V_2 + V_3$

Now according to ohm`s law $V_1 = IR_1$, $V_2=IR_2$ $V_3 = IR_3$

Current through all of them is same as I

Therefore, $V = IR_1 + IR_2 + IR_3 = I (R_1 + R_2 + R_3)$

Applying ohm`s law to the overall circuit

$$V = I\,R_{eq}$$

Where R_{eq} = equivalent resistance of the circuit. By comparing of two equation

$$\mathbf{R_{eq} = R_1 + R_2 + R_3}$$

I.e. total or equivalent resistance of series circuit is the arithmetic sum of resistance connected in series.

1.5.1 Characteristics of Series Circuits

1. The same current flows through each resistance.
2. The supply voltage V is the sum of the individual voltage drops across the resistances.

$$V = V_1+V_2+\ldots\ldots\ldots +V_n$$

3. The equivalent resistance is equal to the sum of the individual resistances.
4. The equivalent resistance is the largest of all the individual resistances.

i.e $R > R_1$, $R>R_2$, $R>R_n$

1.6 Parallel Circuit

The parallel circuit is one in which several resistances are connected across one another in such a way that one terminal of each is connected to form a junction point while the remaining ends are also joined to form another junction point.

Consider a parallel circuit shown in the Fig. 1.8.

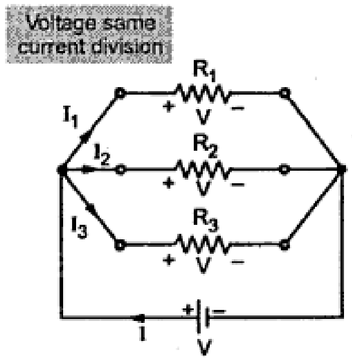


Fig 1.8 Parallel circuit

In the parallel connection shown, the three resistances R_1, R_2 and $R_3$are connected in parallel and combination is connected across a source of voltage 'V'.

In parallel circuit current passing through each resistance is different. Let total current drawn is say 'I' as shown. There are 3 paths for this current, one through R_1, second through R_2 and third through R_3. Depending upon the values of R_1 , R_2 and R_3 the appropriate fraction of total current passes through them. These individual currents are shown as I_1 , I_2 and I_3. While the voltage across the two ends of each resistances R_1 , R_2 and R_3 is the same and equals the supplyvoltage V.

Now let us study current distribution. Apply Ohm's law to each resistance.

$$V = I_1 R_1, \quad V = I_2 R_2 \ , \ V = I_3 R_3$$

$$I_1 = \frac{V}{R_1}, \quad I_2 = \frac{V}{R_2}, \quad I_3 = \frac{V}{R_3}$$

$$I = I_1 + I_2 + I_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$I = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

For the overall circuit if ohm`s is applied,

$$V = I\,R_{eq}$$

And $$I = \frac{V}{R_{eq}}$$

Where R_{eq} = equivalent resistance of the circuit. By comparing of two equation

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

In general if n resistors are connected in parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\ldots\ldots + \frac{1}{R_n}$$

Now if n=2, two resistance are in parallel then

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

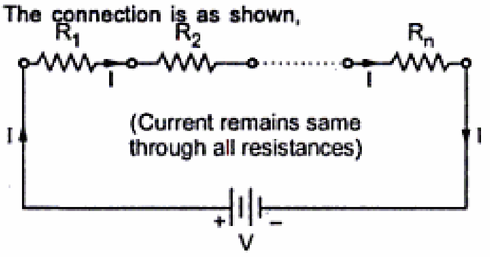
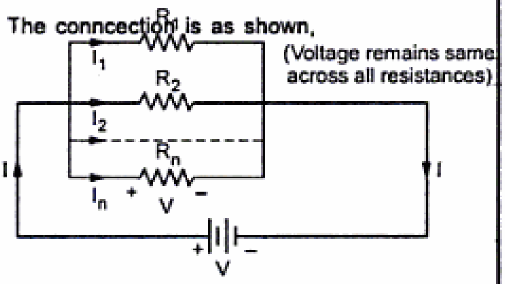
1.6.1 Characteristics of Parallel Circuits

1. The same potential difference gets across all the resistances in parallel.
2. The total current gets divided into the number of paths equal to the number of resistances in parallel. The total current is always sum of all the individual currents.

$$I = I_1 + I_2 + I_3 + \ldots\ldots + I_n$$

3. The reciprocal of the equivalent resistance of a parallel circuit is equal to the sum of the reciprocal of the individual resistances.
4. The equivalent resistance is the smallest of all the resistances.
   i.e $R < R_1$, $R < R_2$,……..$R<R_n$
5. The equivalent conductance is the arithmetic addition of the individual conductance.

1.7 Comparison of Series and Parallel Circuits

Sr. No.	Series Circuit	Parallel Circuit
1.	The connection is as shown, 	The conncection is as shown, 
2.	The same current flows through each resistance.	The same voltage exists across all the resistances in parallel.
3.	The voltage across each resistance is different.	The current through each resistance is different.
4.	The sum of the voltages across all the resistances is the supply voltage. $V = V_1 + V_2 + V_3 + ... + V_n$	The sum of the currents through all the resistances is the supply current. $I = I_1 + I_2 + ... I_n$
5.	The equivalent resistance is, $R_{eq} = R_1 + R_2 + ... + R_n$	The equivalent resistance is, $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_n}$
6.	The equivalent resistance is the largest than each of the resistances in series. $R_{eq} > R_1, R_{eq} > R_2 ...... R_{eq} > R_n$	The equivalent resistance is the smaller than the smallest of all the resistances in parallel.

1.8 Voltage Division in Series Circuit of Resistors

Consider a series circuit of two resistors R_1 and R_2 connected to source of V volts. As two resistors are connected in series, the current flowing through both the resistors is same, i.e. I. Then applying KVL, we get,

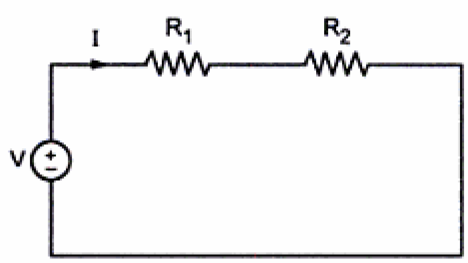


Fig 1.9

$$V = IR_1 + IR_2$$

$$I = \frac{V}{R_1 + R_2}$$

Total voltage applied is equal to the sum of voltage drops V_{R_1} and V_{R_2} across R_1 and R_2 respectively.

Therefore, $V_{R_1} = IR_1$

$$\boxed{V_{R1} = \frac{V}{R_1 + R_2} \cdot R_1 = \left[\frac{R_1}{R_1 + R_2}\right] V}$$

$V_{R_2} = IR_2$

$$\boxed{V_{R2} = \frac{V}{R_1 + R_2} \cdot R_2 = \left[\frac{R_2}{R_1 + R_2}\right] V}$$

So in general, voltage drop across any resistors or combination of resistors in a series circuit is equal to the ratio of that resistance value, to the total resistance multiplied by the source voltage

1.9 Current Division in Parallel Circuit of Resistors

Consider a parallel circuit of two resistors R_1 and R_2 connected across a source of V volts. Current through R_1 is I_1 and R_2 is I_2, while total current drawn from source is I_T.

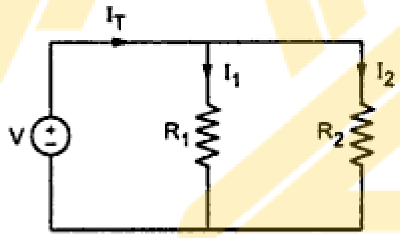


Fig 1.10

$I_T = I_1 + I_2$

$I_1 = \frac{V}{R_1}$, $I_2 = \frac{V}{R_2}$,

$V = I_1R_1 = I_2R_2$

$I_1 = I_2 \left(\frac{R_2}{R_1}\right)$

Substituting the value of I_1 in I_T

$$I_T = I_2 \left(\frac{R_2}{R_1}\right) + I_2 = I_2 \left(\frac{R_2}{R_1} + 1\right) = I_2 \left(\frac{R_1 + R_2}{R_1}\right)$$

$$\mathbf{I_2} = \left[\frac{\mathbf{R_1}}{\mathbf{R_1+R_2}}\right]\mathbf{I_T}$$

Now, $$I_1 = I_T - I_2 = I_T - \left[\frac{R_1}{R_1+R_2}\right] I_T$$

$$I_1 = \left[\frac{R_1+R_2-R_1}{R_1+R_2}\right] I_T$$

$$\mathbf{I_1} = \left[\frac{\mathbf{R_2}}{\mathbf{R_1+R_2}}\right]\mathbf{I_T}$$

In general, the current in any branch is equal to the ratio of opposite branch to the total resistance value, multiplied by the total current in the circuit.

1.10 Kirchhoff`s Laws

In 1847, a German Physicist, Kirchhoff, formulated two fundamental laws of electricity. These laws are of tremendous importance from network simplification point of view.

1.10.1 Kirchhoff's Current Law (KCL)

Consider a junction point in a complex network as shown in the Fig. 1.11.

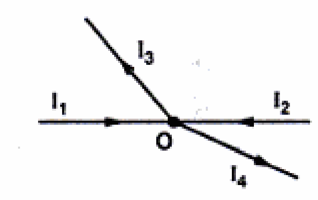


Fig 1.11 Junction point

The total current flowing towards a junction point is equal to the total current flowing away from that junction point.

Another way to state the law is,

The algebraic sum of all the current meeting at a junction point is always zero.

The word algebraic means considering the signs of various currents.

$$\sum I \text{ at junction pont} = 0$$

Sign convention: Currents flowing towards a junction point are assumed to be positive while currents flowing away from a junction point assumed to be negative.

Referring to figure 1.11, I_1 and I_2 are positive and I_3 and I_4 are negative

Applying KCL, $\sum$ I at junction pont =0

$$I_1 + I_2 - I_3 - I_4 = 0$$

Or $$I_1 + I_2 = I_3 + I_4$$

1.10.2 Kirchhoff's Voltage Law (KVL)

"In any network, the algebraic sum of the voltage drops across the circuit elements of any closed path (or loop or mesh) is equal to the algebraic sum of the e.m.fs in the path"

In other words, "***the algebraic sum of all the branch voltages, around any closed path or closed loop is always zero***."

Around a closed loop $\sum V = 0$

The law states that if one starts at a certain point of a closed path and goes on tracing and noting all the potential changes (either drops or rises), in any one particular direction, till the starting point is reached again, he must be at the same potential with which he started tracing a dosed path.

Sum of all the potential rises must be equal to sum of all the potential drops while tracing any dosed path of the circuit. The total change in potential along a closed path is always zero.

1.10.3 Sign Conventions to be Followed while Applying KVL

When current flows through a resistance, the voltage drop occurs across the resistance. The polarity of this voltage drop always depends on direction of the current. The current always flows from higher potential to lower potential.

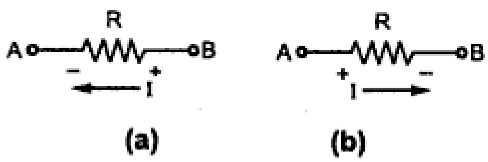


Fig 1.12

In the Fig. 1.12 (a), current I is flowing from right to left, hence point B is at higher potential than point A, as shown.

In the Fig. 1.12 (b), current I is flowing from left to right, hence point A is at higher potential than point B, as shown.

Once all such polarities are marked in the given circuit, we can apply KVL to any closed path in the circuit.

Now while tracing a closed path, if we go from -ve marked terminal to + ve marked terminal, that voltage must be taken as positive. This is called **potential rise.**

For example, if the branch AB is traced from A to B then the drop across it must be considered as rise and must be taken as + IR while writing the equations.

While tracing a closed path, if we go from +ve marked terminal to - ve marked terminal, that voltage must be taken as negative. This is called **potential drop**.

For example, in the Fig. 1.12 (a) only, if the branch is traced from B to A then it should be taken as negative, as - IR while writing the equations.

Similarly in the Fig. 1.12 (b), if branch is traced from A to B then there is a voltage drop and term must be written negative as - IR while writing the equation. If the branch is traced from B to A, it becomes a rise in voltage and term must be written positive as + IR while writing the equation.

1.10.4 Application of KVL to a Closed Path

Consider a closed path of a complex network with various branch currents assumed as shown in the Fig. 1.13 (a).

As the loop is assumed to be a part of complex network the branch currents are assumed to be different from each other.

Due to these currents the various voltage drops taken place across various resistances are marked as shown in the Fig. 1.13 (b).

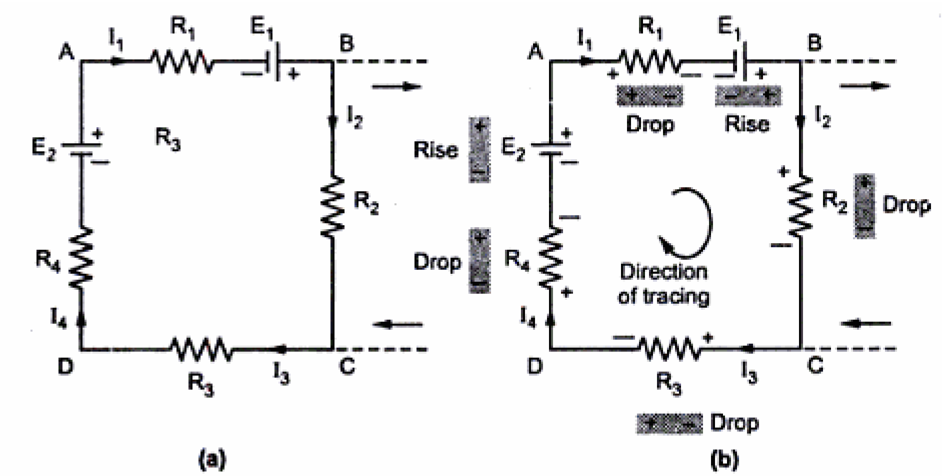


Fig 1.13

The polarity of voltage drop along the current direction is to be marked as positive (+) to negative (-).

Let us trace this closed path in clockwise direction i.e. A-B-C-D-A.

Across R_1 there is voltage drop I_1 R_1 and as getting traced from +ve to -ve, it is drop and must be taken as negative while applying KVL

Battery E_1 is getting traced from negative to positive i.e. it is a rise hence must be considered as positive.

Across R_2 there is a voltage drop I_2 R_2 and as getting traced from +ve to -ve, it is drop and must be taken negative.

Across R_3 there is a drop I_3 R_3 and as getting traced from +ve to -ve, it is drop and must be taken as negative.

Across R_4 there is drop I_3 R_3 and as getting traced from +ve to -ve, It is drop must be taken as negative.

Battery E_2 is getting traced from -ve to +ve, it is rise and must be taken as positive

:. We can write an equation by using KVL around this closed path as,

$$-I_1 R_1 + E_1 - I_2 R_2 - I_3 R_3 - I_4 R_4 + E_2 = 0$$

$$\text{i.e.} \qquad E_1 + E_2 = I_1 R_1 + I_2 R_2 + I_3 R_3 + I_4 R_4$$

If we trace the closed loop in opposite direction i.e. along A-D-C-B-A and follow the same sign convention, the resulting equation will be same as what we have obtained above.

1.10.5 Steps to Apply Kirchhoff's Laws to Get Network Equations

The steps are stated based on the branch current method.

Step 1: Draw the circuit diagram from the given information and insert all the values of sources with appropriate polarities and all the resistances.

Step 2: Mark all the branch currents with some assumed directions using KCL at various nodes and junction points. Kept the number of unknown currents minimum as far as possible to limit the mathematical calculations required to solve them later on.

Assumed directions may be wrong; in such case answer of such current will be mathematically negative which indicates the correct direction of the current. A particular current leaving a particular source has some magnitude, and then same magnitude of current should enter that source after travelling through various branches of the network.

Step 3: Mark all the polarities of voltage drops and rises as per directions of the assumed branch currents flowing through various branch resistances of the network. This is necessary for application of KVL to various closed loops.

Step 4: Apply KVL to different closed paths in the network and obtain the corresponding equations. Each equation must contain some clement which is not considered in any previous equation.

1.11 Electrical Work

- In an electrical circuit, movement of electrons i.e. transfer of charge is an electric current. The electric work done when there is a transfer of charge. The unit of such work is Joule.
- One joule of electrical work done is that work done in moving a charge of 1 coulomb through a potential difference of 1 volt.
- So if V is the potential difference in volts and Q is the charge in coulombs then we can write

Electrical work W= V * Q J

But $I = \frac{Q}{t}$

W = V I t J

1.12 Electrical Power

- The rate at which electrical work is done in an electric circuit is called an electrical power.

$$\text{Electrical Power} = \frac{\text{electrical work}}{\text{time}} = \frac{W}{t} = \frac{V\,I\,t}{t} = V\,I \;\; \text{J/sec}$$

- Thus the power consumed in the electric circuit is 1 watt if the potential difference of 1 volt applied across the circuit causes 1 ampere current to flow through it.
- According to Ohms law, V=IR or $I = \frac{V}{R}$
- Using this power can be expressed as

$$P = V\,I = I^2R = \frac{V^2}{R}$$

1.13 Electrical Energy

- An electrical energy is the total amount of electrical work done in electric circuit

  Electrical Energy E = Power * Time = V I t

- The unit of energy is joules or watt-sec.

MODULE 1b: A.C. FUNDAMENTALS

3.1 INTRODUCTION

90% of the electrical energy used nowadays is a.c. in nature. Electrical supply used for commercial purpose is alternating. The D.C. supply has constant magnitude with respect to time. Fig. 3.1 shows the graph of such current with respect to time

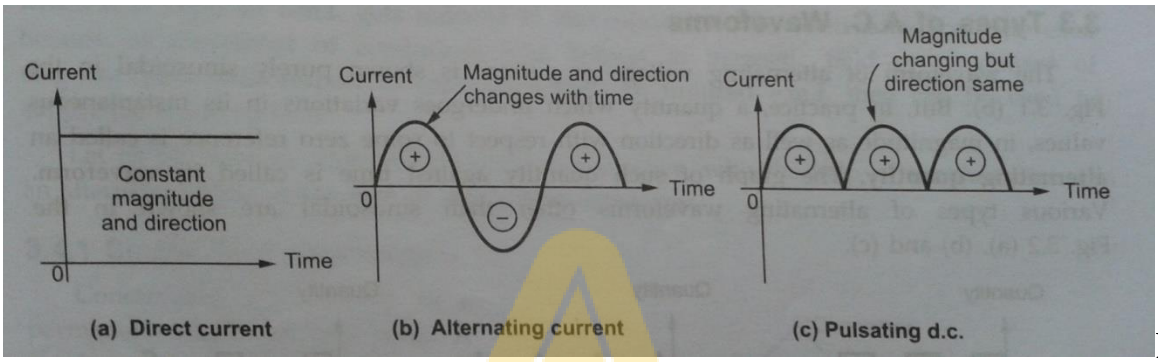


Fig 3.1

Such change in magnitude and direction is measured in terms of cycle. Each cycle of a.c. consists of two half cycles' namely positive cycle and negative cycle. Current increases in magnitude, in one particular direction, attains maximum and start decreasing, passing through zero it increases in opposite direction and behaves similarly.

In practice some waveform are available in which magnitude changes but its direction remains same as positive or negative. Such waveform is called pulsating d.c. The waveform obtained as output of full wave rectifier is an example of pulsating d.c.

3.2 ADVANTAGES OF A.C.

1. The voltages in a.c. system can be raised or lowered with the help of a device called transformer. In d.c. system, raising and lowering of voltages is not so easy.
2. As the voltages can be raised, electrical transmission at high voltages is possible. Now, higher the voltage, lesser is the current flowing through transmission line. Less the current, lesser are the copper losses and lesser is the conducting material required. This makes a.c. transmission always economical and efficient.
3. It is possible to build up high a.c. voltage; high speed a.c. generators of large capacity. The construction and cost of such generators are very low.
4. A.C. electrical motors are simple in construction, are cheaper and require less attention from maintenance point of view.
5. Whenever it is necessary, a.c. supply can be easily converted to obtain D.C. supply.

3.3 ADVANTAGES OF PURELY SINUSOIDAL WAVEFORM

1. Mathematically, it is very easy to write the equation for purely sinusoidal waveform.
2. Any other type of waveform can be resolved into a series of sine or cosine waves of fundamental and higher frequencies, sum of all these waves gives the original waveform. Hence it is always better to have sinusoidal waveform as the standard waveform.
3. The sine and cosine waves are the only waves which can pass through linear circuits containing resistance, inductance and capacitance without distortion. In case of other waveforms, there is a possibility of distortion when it passes through linear circuits.
4. The integration and derivatives of a sinusoidal function is again a sinusoidal function. This makes the analysis of linear electrical network with sinusoidal inputs, very easy.

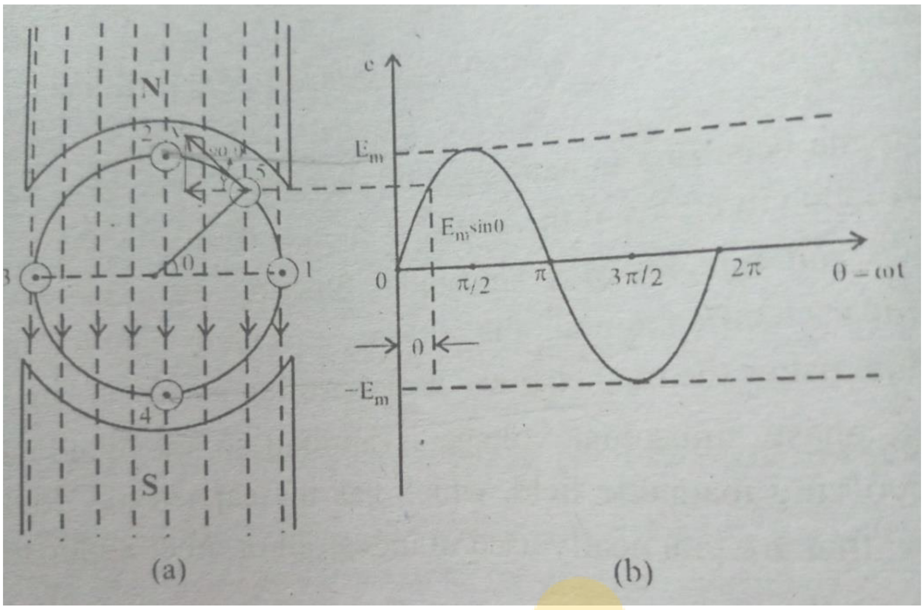
3.4 GENERATION OF SINUSOIDAL A.C. VOLTAGE

The machines which are used to generate electrical voltages are called generators. The generators which generate purely sinusoidal a.c. voltages are called alternators.

The basic principle of an alternator is the principle of electromagnetic induction. The sine wave is generated according to Faraday's law of electromagnetic induction. It says that whenever there is a relative motion between the conductor and the magnetic field in which it is kept, an e.m.f. gets induced in the conductor. The relative motion may exist because of movement of conductors with respect to magnetic field or movement of magnetic field with respect to conductor. Such an induced e.m.f. then can be used to supply the electrical load.

Let us see how an alternator produces a sine wave, with the help of simplest form of an alternator called single turn or single loop alternator.

Construction: Consider a conductor of length 'l' which is placed perpendicular to the lines of magnetic flux density 'B' wb/m^2 produced by two poles N and S as shown in figure below.



Let the conductor rotate in a circular path with an angular velocity ***ω*** or linear velocity v from position1. When it starts rotating from position1 i.e. when the angular displacement Ө=0, it moves parallel to the lines of flux and hence cuts no flux, so emf induced is zero.

At position 2, Ө=90^o conductor moves perpendicular to the lines of flux, cuts maximum flux and emf induced is E_m. Again conductor rotates through another 90^o and occupies position 3, Ө= 180^o or Π radians, it moves parallel to the lines of flux hence cuts no flux and emf induced is zero.

Now the conductor will be moved by another 90^o and occupies position 4, Ө= 270^o or 3Π/2 radians, it moves perpendicular to the lines of flux, cuts max flux and hence emf induced is max.

Again the conductor will be moved by another 90^o and occupies position 1, Ө= 360^o or 2Π radians, emf induced is zero.

When the conductor is rotating from position 1 to 2 and 2 to 3 i.e. from Ө=0 to Ө=Π (0 to 180^o) it is rotating under the influence of north pole and the direction of induced emf is taken as positive. Similarly when the conductor rotates from position 3 to 4 and 4 to 1 i.e. from Ө= Π to Ө= 2Π (180^o to 360^o) it is under the influence of south pole and hence the direction of the induced emf is taken as negative.

Consider the conductor has moved through an angle Ө and occupied position 5 as shown in figure.

The component of velocity v perpendicular to lines of flux is v cos(90-Ө)= v sinӨ

Therefore emf induced at position 5 is given by,

$$e = B\, l\, v \sin\Theta = E_m \sin\Theta$$

$$e = E_m \sin\boldsymbol{\omega} t = E_m \sin(2\Pi f)t$$

where $B\, l\, v = E_m$ = maximum value of the emf induced. Similar equation can be written for current also , $i = I_m \sin(2\Pi f)t$ or $I_m \sin\Theta$.

3.5 STANDARD TERMINOLOGY RELATED TO ALTERNATING QUANTITY

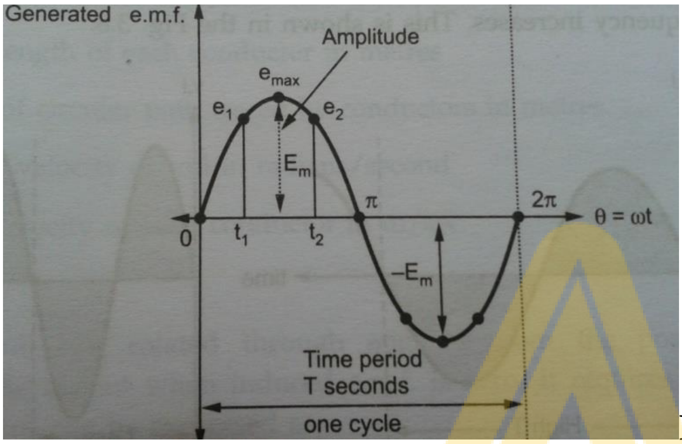


Fig 3.2 Waveform of an alternating e.m.f

3.5.1 Instantaneous value

The value of an alternating quantity at a particular instant is known as **instantaneous value**.

E.g. e_1 and e_2 are the instantaneous value of an alternating emf at the instant t_1 and t_2 respectively.

3.5.2 Waveform

The graph of instantaneous values of an alternating quantity plotted against time is called its **waveform.**

3.5.3 Cycle

Each repetition of a set of positive and negative instantaneous values of an alternating quantity is called a **cycle**.

3.5.4 Time Period (T)

Time taken by an alternating quantity to complete its one cycle is known as its **time period** denoted by T seconds. After every T seconds, the cycle of an alternating quantity repeats.

3.5.5 Frequency (f)

The number of cycles completed by an alternating quantity per second is known as its frequency. It is denoted by f and is measured in cycles/second which is known as hertz, denoted as Hz.

$$\mathbf{f = \frac{1}{T}\ Hz}$$

3.5.6 Amplitude

The maximum value attained by an alternating quantity during positive or negative half cycle is called its amplitude. It is denoted as E_m or I_m.

3.5.7 Angular Frequency (ω)

It is the frequency expressed in electrical radians per second. As one cycle of an alternating quantity corresponds to 2π radians, the angular frequency can be expressed as (2π * cycles/sec). its unit is radians/sec.

$$\boldsymbol{\omega = 2\pi f}\ \textbf{radians/sec}$$

3.6 EFFECTIVE VALUE OR R.M.S VALUE

*The **effective value or r.m.s. value** of an alternating current is given by that steady current which, when flowing through a given circuit for a given time produces the same amount of heat as produced by the alternating current, which when flowing through the same circuit for the same time.*

Consider sinusoidally varying alternating current and square of this current as shown in figure 3.3

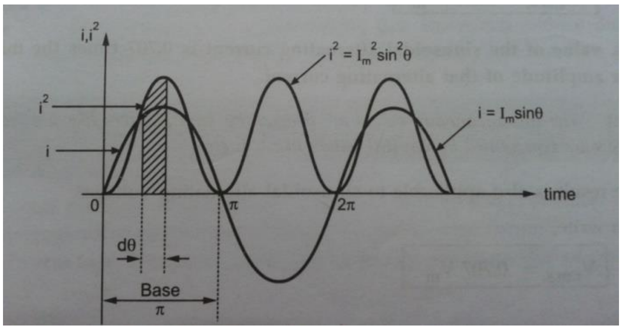


Fig 3.3 Waveform of current and square of the current

The current $i = I_m \sin\theta$ while

Square of the current $i^2 = I_m^2 \sin^2\theta$

Area of curve over half a cycle can be calculated by considering an interval dθ as shown.

Area of square curve over half cycle= $\int_0^{\pi} i^2\, d\theta$ and length of the base is π

Therefore Average value of square of the current over half cycle

$$= \frac{\text{area of curve over half cycle}}{\text{length of base over half cycle}} = \frac{\int_0^{\pi} i^2\, d\theta}{\pi}$$

$$= \frac{1}{\pi}\int_0^{\pi} i^2\, d\theta = \frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2\theta\, d\theta = \frac{I_m^2}{\pi}\int_0^{\pi} \frac{1-\cos 2\theta}{2} d\theta$$

$$= \frac{I_m^2}{\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_o^{\pi} = \frac{I_m^2}{2\pi}[\pi]$$

$$= \frac{I_m^2}{2}$$

Hence, root mean square value i.e. r.m.s. value can be calculated as

$$I_{r.m.s} = \sqrt{\text{mean or average value of current}} = \sqrt{\frac{I_m^2}{2}}$$

$$= \frac{I_m}{\sqrt{2}}$$

$$\mathbf{I_{r.m.s} = 0.707 I_m}$$

3.7 AVERAGE VALUE

The **average value** of an alternating quantity is defined as that value which is obtained by averaging all the instantaneous values over a period of half cycle.

For a symmetrical a.c., the average value over a complete cycle is zero as both positive and negative half cycles are identical. Hence, the average value is defined for half cycle only.

Average value can also be expressed by that steady current which transfers across any circuit, that same amount of charge as is transferred by that alternating current during the same time.

For an unsymmetrical a.c., the average value must be obtained for one complete cycle but for symmetrical a.c. like sinusoidal; it is to be obtained for half cycle.

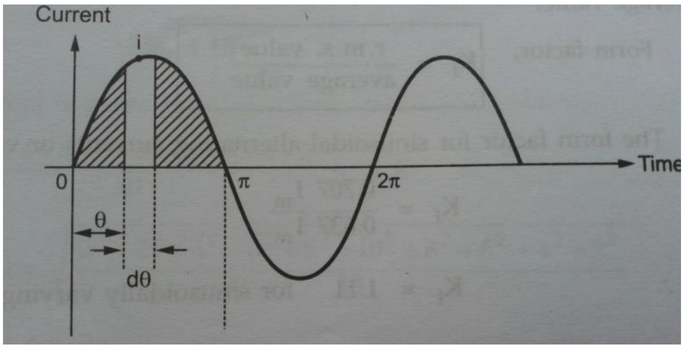


Fig 3.4 Average value of an alternating current

Consider sinusoidally varying current, $I = I_m \sin\theta$

Consider the elementary interval of instant dθ as shown in figure. The average instantaneous value of current in this interval say I.

The average value can be obtained by taking ratio of area under curve over half cycle to length of the base for half cycle.

$$I_{av} = \frac{\text{area under curve for half cycle}}{\text{length of base over half cycle}}$$

$$= \frac{\int_0^{\pi} i\, d\theta}{\pi}$$

$$= \frac{1}{\pi}\int_0^{\pi} i\, d\theta = \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta\ d\theta = \frac{I_m}{\pi}\int_0^{\pi} \sin\theta\ d\theta$$

$$= \frac{I_m}{\pi}[-\cos\theta]_0^{\pi} = \frac{I_m}{\pi}[-\cos\pi + \cos\theta]$$

$$= \frac{I_m}{\pi}[2]$$

$$= \frac{2I_m}{\pi}$$

$$\mathbf{I_{av} = 0.637 I_m}$$

3.8 FORM FACTOR (K_f)

The form factor of an alternating quantity is defined as the ratio of r.m.s. value to the average value.

$$K_f = \frac{\textbf{r.m.s value}}{\textbf{average value}}$$

The form factor for sinusoidal alternating currents or voltages can be obtained as,

$$K_f = \frac{0.707\, I_m}{0.637\, I_m}$$

K_f **= 1.11** for sinusoidally varying quantity

3.9 CREST OR PEAK FACTOR(K_P)

The peak value of an alternating quantity is defined as the ratio of maximum value to the r.m.s. value.

$$K_P = \frac{\textbf{maximum value}}{\textbf{r.m.s. value}}$$

The peak factor for sinusoidal alternating currents or voltages can be obtained as

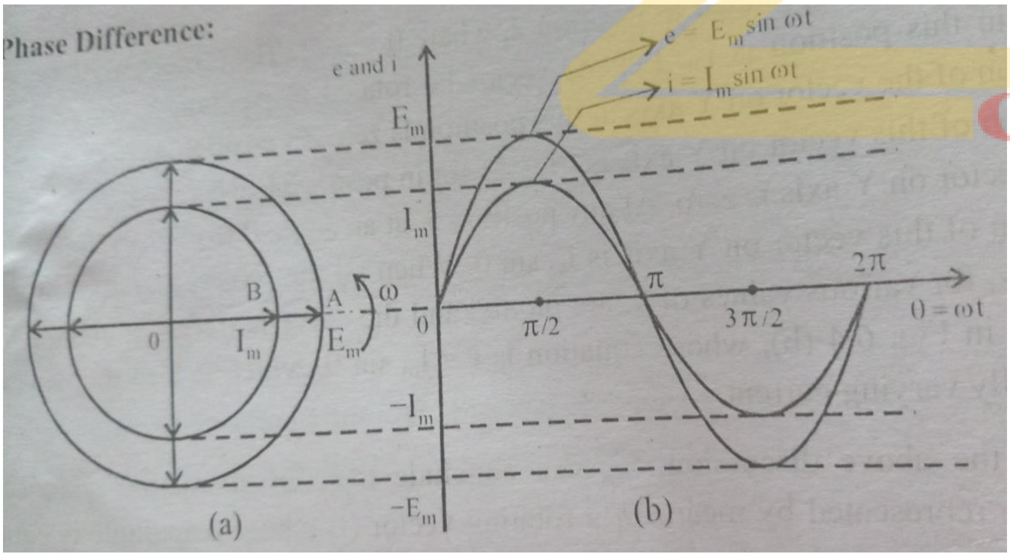
$$K_P = \frac{I_m}{0.707\, I_m} = 1.414$$

3.9 PHASE OF AN ALTERNATING QUANTITY

Phase of an alternating quantity is the angle through which the rotating vector representing the alternating quantity has rotated through from the reference axis.

Phase difference between the two alternating quantities is the angle difference between the two rotating vectors representing the two alternating quantities.

Case 1:



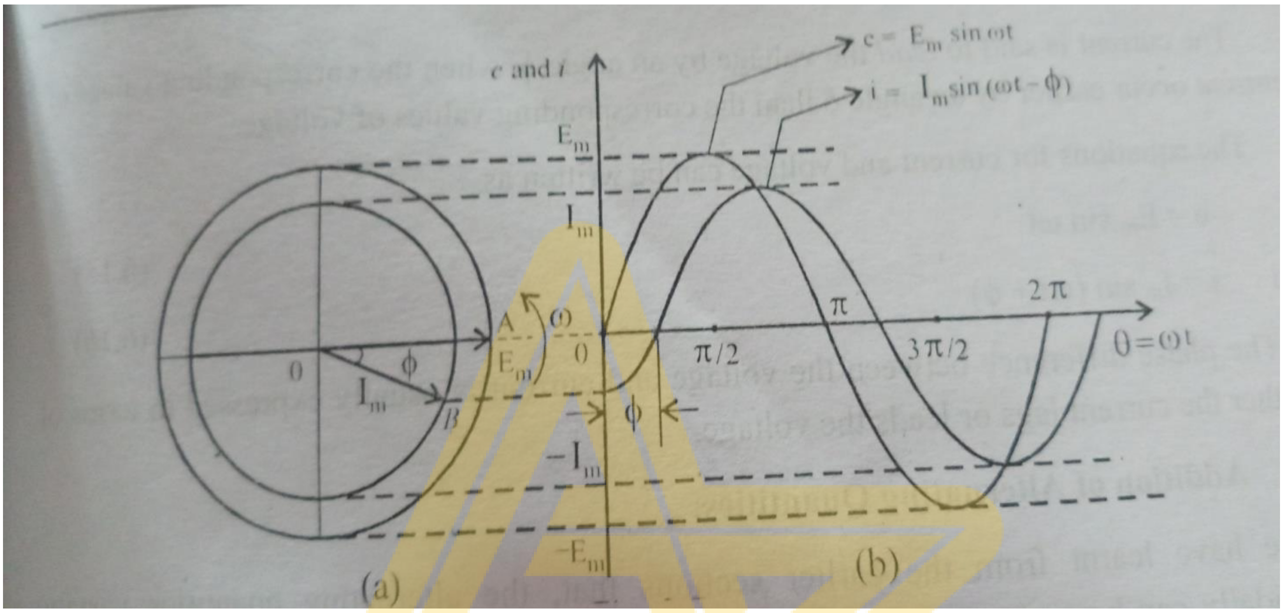
The rotating vector OA represents the alternating voltage and OB represents the alternating current. Both of them rotate together with an angular velocity $\boldsymbol{\omega}$ and phase difference is zero.

Two quantities are said to be in phase with each other when their corresponding values occur at the same time.

The equations for voltage and current are $e = E_m \sin\omega t$

$$i = I_m \sin\omega t$$

<u>Case 2</u>: Lagging



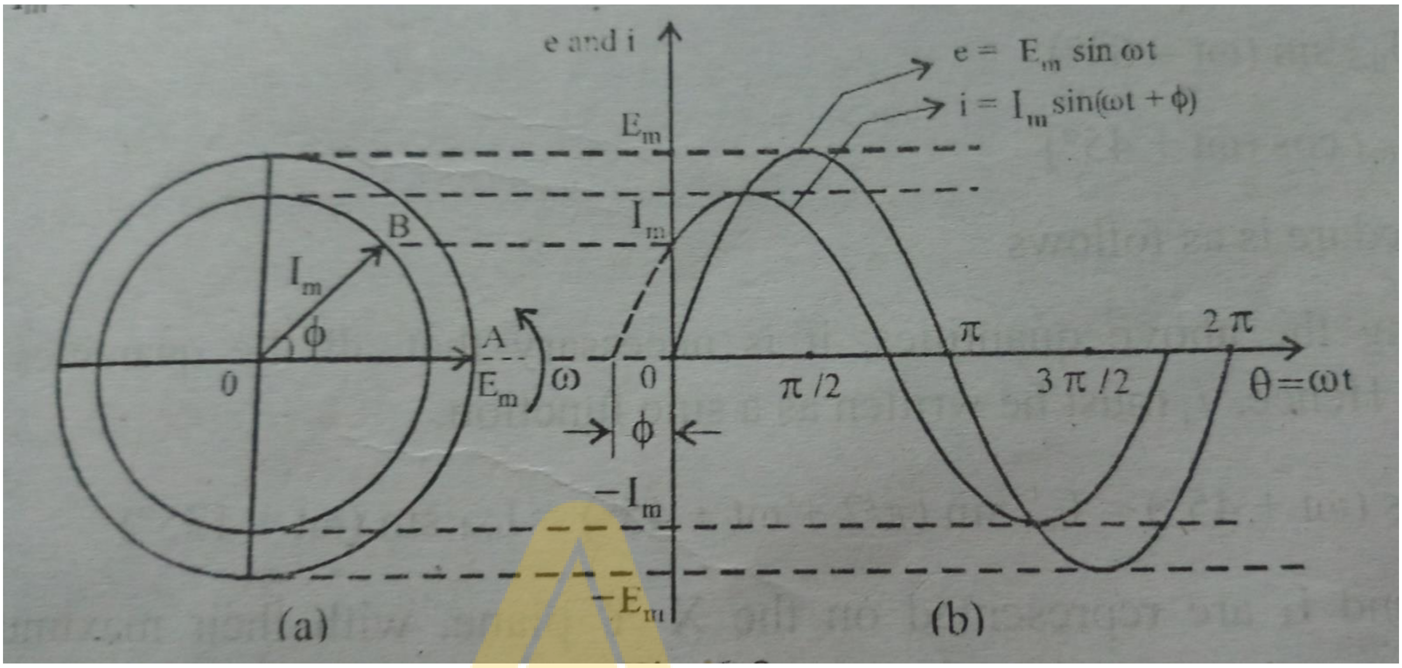
Vectors OA and OB rotate with an angle difference of ϕ, the current vector always lags the voltage vector by an angle ϕ.

Equations for voltage and current are

$e = E_m \sin\omega t$

$i = I_m \sin(\omega t - \phi)$

Case 3: Leading



Vectors OA and OB rotate with an angle difference of ϕ, current vector leads the voltage vector by an angle ϕ.

Equations for current and voltage are, $e = E_m \sin\boldsymbol{\omega}t$

$$i = I_m \sin(\boldsymbol{\omega}t + \phi)$$