MODULE 5: KINEMATICS AND KINETICS

1. Define displacement, distance travelled, velocity, average velocity, acceleration. Mention their SI units

Displacement: It is the change in position of body with respect to time from an arbitrary fixed point. Displacement is a vector quantity

SI unit: meter **Dimension:** L

Distance travelled: Distance is a scalar quantity that refers to "how much ground an object has covered" during its motion.

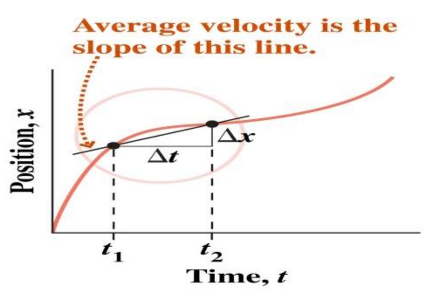
SI unit: meter **Dimension:** L

Velocity: The **velocity** of an object is the rate of change of its position with respect to a frame of reference, and is a function of time. **Velocity** is equivalent to a specification of an object's speed and direction of motion

Other units: mph, ft/s

In SI base units: m/s **Dimension:** $L\,T^{-1}$

Average Velocity: The average velocity of an object is its total displacement divided by the total time taken. In other words, it is the rate at which an object changes its position from one place to another. Average velocity is a Vector quantity. The SI unit is meters per second.

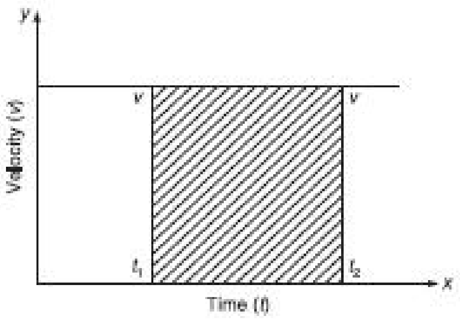


Acceleration: Acceleration is the rate of change of velocity of an object with respect to time. An object's acceleration is the net result of all forces acting on the object, as described by Newton's Second Law. The SI unit for acceleration is metre per second squared.

SI unit: m/s^2 **Dimension:** $L\,T^{-2}$

2. Derive the equations of motion

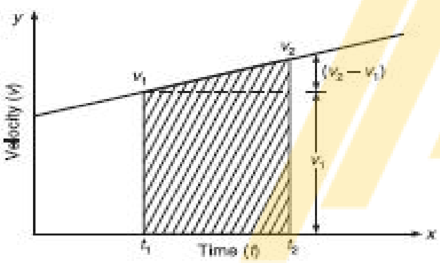
Velocity–time graph:



The area under V–T graph will produce the distance traveled by the body/particle from time *t1 to t2*

s = v × (t2 – t1) = v t……………….(1)

This is applicable only when the velocity is uniform.



The slope of the line is gives acceleration

$$a = \frac{(v_2 - v_1)}{(t_2 - t_1)}$$

$$(v_2 - v_1) = a(t_2 - t_1)$$

$$v_2 = v_1 + a(t_2 - t_1)$$

$$v = u + at \qquad (1)$$

where v = final velocity, u = initial velocity and $t = (t_2 - t_1)$.

As seen from earlier graph, the total distance traveled is given by the area under the curve and hence the area is given as

$$s = v_1 \times t + \frac{1}{2} \times (v_2 - v_1)\, t$$

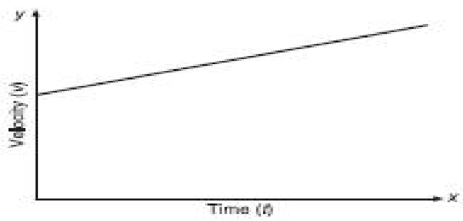
$$\text{But acceleration} = a = \frac{(v_2 - v_1)}{t}$$

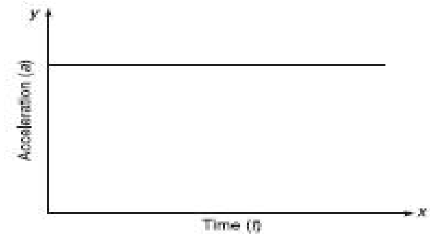
Substituting, we get

$$s = v_1 \times t + \frac{1}{2} \times a\, t^2 \ \text{ or } \ ut + \frac{1}{2} \times at^2 \qquad (2)$$

where u is the initial velocity or velocity at time t_1.

Acceleration–time graph:





The acceleration is constant with respect to time t. The same can be connected to velocity–time graph (Figure 11.6), wherein the velocity variation is constant, and the variation of acceleration to be uniform.

Using Equation (1) and (2), to get an equation without time *t, we substitute for t from Equation.1 in* Equation. 2, we get

$$s = u\frac{(v-u)}{a} + \frac{1}{2}\times a\left[\frac{(v-u)}{a}\right]^2 = \frac{u(v-u)}{a} + \frac{(v-u)^2}{2a}$$

$$2as = 2uv - 2u^2 + v^2 + u^2 - 2uv = v^2 - u^2$$

$$v^2 - u^2 = 2as$$

3. Define a) Rectilinear Motion b) Curvilinear Motion c) Projectile Motion d) Trajectory

- **Rectilinear motion**

When a particle or a body moves along a straight line path, then it is called linear motion or rectilinear motion.

- **Curvilinear motion**

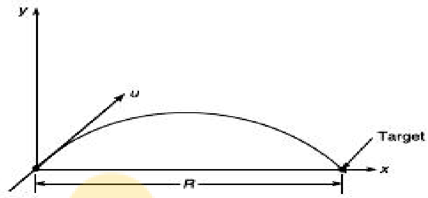
When a moving particle describes a path other than a straight line is said be a particle in curvilinear motion.

- **Projectile motion**

Whenever a particle is projected upwards with some inclination to the horizontal (but not vertical), it travels in the air and traces a Parabolic path and falls on the ground point (target) other than the point of projection

- **Trajectory**

Whenever a particle is projected upwards with some inclination to the horizontal (but not vertical), it travels in the air and traces a parabolic path and falls on the ground point (target) other than the point of projection. The particle itself is called projectile and the path traced by the projectile is called trajectory



4. What is super elevation? Why it is necessary?

Super Elevation:

If the road surface is horizontal, then the centrifugal force acting radially outwards will draw the vehicle away from the center effecting lateral slipping of the vehicle. If this is to be prevented, then the frictional resistance offered at the wheel and road has to be balance this centrifugal force. To prevent lateral slipping of the vehicle due to the centrifugal force, the road edge away from the center (outer edge) will be slightly raised above the inner edge by a vertical height *e*. Consider vehicle of mass *m* negotiate a circular curve of radius *r* with a velocity *v*. Let the angle with which the road surface is inclined at an angle θ with respect to horizontal surface.

For dynamic equilibrium along the road surface,

$$mg\sin\theta - \left(\frac{mv^2}{r}\right)\cos\theta$$

$$\tan\theta = \frac{\left(\frac{mv^2}{r}\right)}{mg} = \frac{v^2}{gr}$$

Where θ is the banking angle known as super elevation, for highways, the super elevation is provided for average speeds of the vehicles.

Terms used in projectile

1. Velocity of projection (u): It is the velocity with which projectile is projected in the upward direction with some inclination to the horizontal.

2. Angle of projection (α): It is the angle with which the projectile is projected with respect to horizontal.

3. Time of flight (T): It is the total time required for the projectile to travel from the point of projection to the point of target.

4. Horizontal range (R): It is the horizontal distance between the point of projection and target point.

5. Vertical height (h): It is the vertical distance/height reached by the projectile from the point of projection.

Time of Flight:

Let T be the time of flight. We know that the vertical ordinate at any point on the path of projectile after a time T is given by

$$y = (u \sin \alpha)\, t - \frac{1}{2} g t^2$$

When the projectile hits the ground, say at B: $y = 0$ at $t = T$

Above equation becomes

$$0 = (u \sin \alpha)\, t - \frac{1}{2} g t^2$$

$$(u \sin \alpha) = \frac{1}{2} g t$$

$$T = \frac{2u \sin \alpha}{g}$$

Horizontal range of the projectile: During the time of flight, the horizontal component of velocity of projectile = $u \cos \alpha$

{Horizontal distance of the projectile} = R = {Horizontal component of velocity of projection} {Time of flight} = $u \cos \alpha \times T$

$$R = u \cos \alpha \times \frac{2u \sin \alpha}{g} = \frac{u^2 \sin (2\alpha)}{g}$$

$\sin (2\alpha)$ will be maximum only when $\sin 2\alpha = 1$

$$\sin 2\alpha = \sin 90 \text{ or } \alpha = 45°$$

Hence maximum horizontal range is given by

$$R_{max} = \frac{u^2 \sin 90}{g} = \frac{u^2}{g}$$

Maximum height attained by the projectile: When the projectile reaches its maximum height, vertical component of velocity of projection becomes zero.

$$v^2 - u^2 = 2gs$$

$$0 - u^2 \sin^2 \alpha = -2g h_{max}$$

$$h_{max} = \frac{u^2 \sin^2 \alpha}{2g}$$

Time required to reach the maximum height is given by

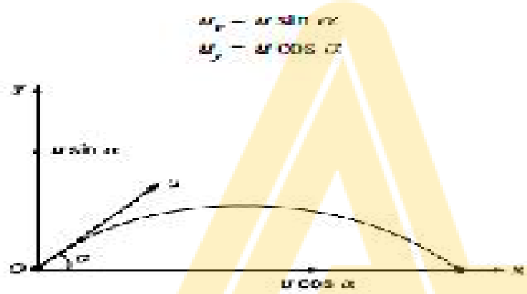
$$v = u + at$$

$$0 = u \sin \alpha - gt$$

then $$t = \frac{u \sin \alpha}{g}$$

Motion of projectile:

Let a particle be projected upward from a point *O at an angle with* horizontal with an initial velocity of *u m/s*



The vertical component of velocity is always affected by acceleration due to gravity. The particle will reach the maximum height when vertical component becomes zero. The horizontal component of velocity will remains constant since there is no effect of acceleration due to gravity. The combined effect of horizontal and vertical components of velocity will move the particle along some path in air and then fall on the ground other than the point of projection

Equation for the path of projectile (Trajectory equation): Let a particle is projected at a certain angle from point *O*. The particle will move along a certain path *OPA* in the air and will fall down at *A*.

Let u = velocity of projection

α = angle of projection

After 't' seconds, let a particle reach any point 'P' with x and y as coordinates as shown in Figure 11.40.

We know that, horizontal component of velocity of projection = $u \cos \alpha$

Vertical component of velocity of projection = $u \sin \alpha$

Therefore, $$x = u \cos \alpha t \quad (1)$$

$$y = u \sin \alpha t - \frac{1}{2} g t^2 \quad (2)$$

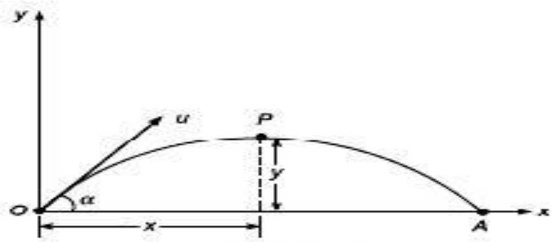


Figure 11.42

From Eq. (1)

$$t = \frac{x}{u\cos\alpha}$$

substitute in Eq. (2), we get

$$y = u\sin\alpha\left(\frac{x}{u\cos\alpha}\right) - \frac{1}{2}g\left(\frac{x}{u\cos\alpha}\right)^2$$

or
$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$$