

Instructions!!

- ① Keep a copy and a pen
- ② Be interactive

Bit Manipulation

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What matters the most in CP?

- faster coding time
- faster run-time

Bit Manipulation

It is a very important topic to handle questions related to binary.

Number System

Decimal $(0 - 9) \rightarrow 10$

Binary $(0, 1) \rightarrow 2$

Octal $(0 - 7) \rightarrow 8$

Hexa Decimal $(0 - 9, A - F) \rightarrow 16$

Basics of bit manipulation

① Decimal to Binary

2	1 0 2	
2	5 1	0
2	2 5	1
2	1 2	1
2	6	0
2	3	0

$$(102)_{10} = (1100110)_2$$

↓
Decimal
Binary

Code it up!

② Binary to Decimal

$$(1101101)_2 = 126$$

Code it up!

similarly other conversions can be done like octal - decimal

Conversion

Octal - binary	— octal - decimal
decimal - binary	— decimal - octal

Hexadecimal - octal - Hexa-decimal
decimal - octal

binary — hexadecinal —

Simple shortcut

Base 1

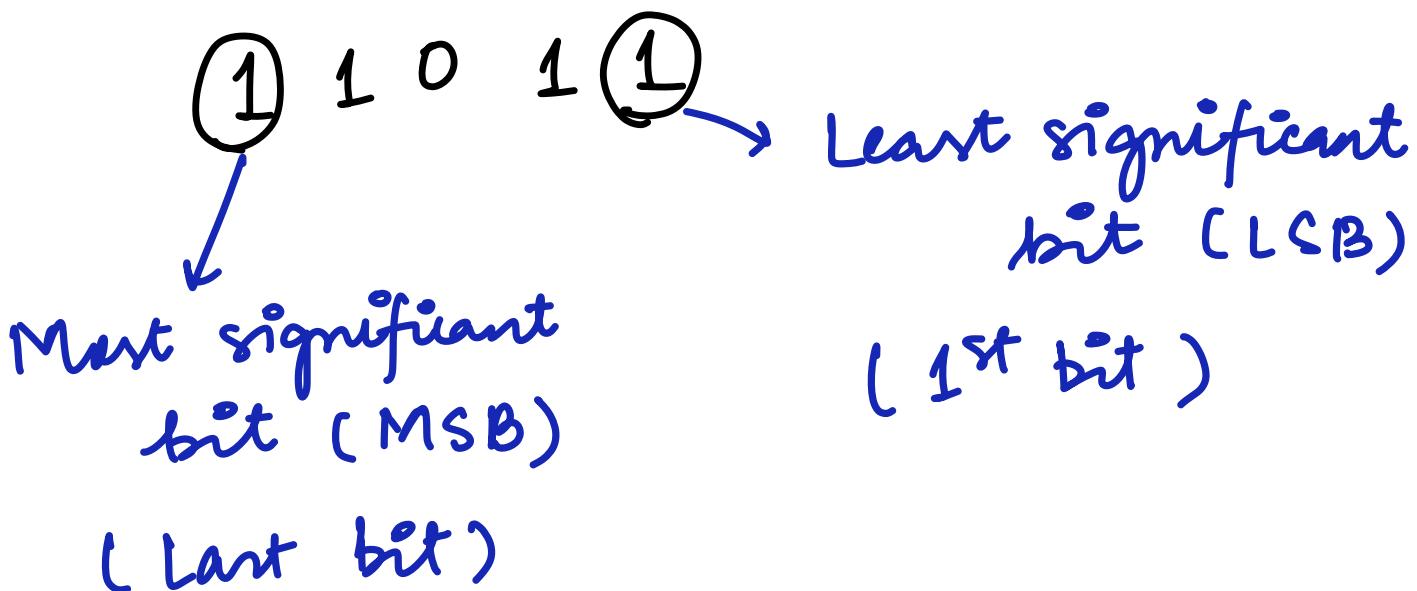
Multiply

Decimal

Base 2

divide

Now we are only concerned with binary numbers.



Bitwise operators

① OR operator (`|`)

Bitwise OR gives 1 if either of the bits is set.

② AND operator (`&`)

Bitwise AND gives 1 if both the bits are set.

③ XOR operator (`^`)

even 1's \rightarrow 0

$\begin{array}{|c|c|c|} \hline 1 & 0 & 1 \\ \hline 1 & 1 & 0 \\ \hline \end{array} \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 0 & 1 & 1 \\ \hline \end{array}$

odd 1's \rightarrow 1

0 1 1 0

④ Bitwise Left shift operator \ll

\rightarrow shift all the bits by one position
to the left

$\begin{array}{r} \uparrow \uparrow \uparrow \uparrow \\ 0 1 1 0 1 \\ 1 1 0 1 0 \end{array}$

Equivalent to
multiply the
number by 2

$a \ll b$

$$\rightarrow \boxed{a \ll b} = a * 2^b$$

5, $5 \ll 1 \rightarrow 10$ 5×2^1

$5 \ll 2 \rightarrow 20$ 5×2^2

$5 \ll 3 \rightarrow 40$ 5×2^3

$\boxed{a \ll b}$
 $\boxed{a >> b}$

⑤ Bitwise Right shift operator

shift all bits one position to the
right

$\begin{array}{r} \uparrow \uparrow \uparrow \uparrow \uparrow \\ 0 1 1 0 1 \\ \boxed{0} 0 1 1 0 \end{array}$

Equivalent to
divide the
number by 2

$5 \rightarrow \boxed{1} 0 1^2 \rightarrow 2$

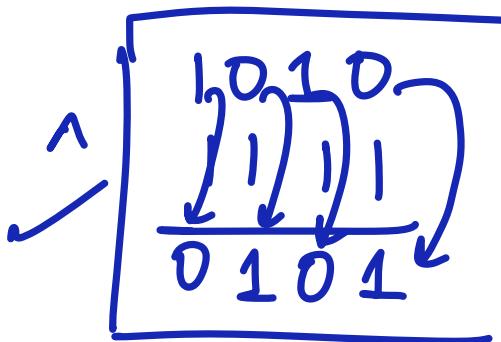
$5 >> 1 \rightarrow \boxed{0 1 0}$

$$\rightarrow \boxed{a >> b} = \frac{a}{2^b}$$

In bit manipulation we will only be working with integers and not decimal point numbers.

$$5 \gg 2 \boxed{001}$$

int - 32 bits ✓
 long long - 64 bits



Properties of bitwise operators

① OR, AND, XOR are all commutative and associative

$$\begin{array}{r} 1010 \\ ^\wedge \\ 0000 \\ \hline 1010 \\ 0 \end{array}$$

② $A \wedge A = 0$ → useful to find a single number

③ $A \wedge 0 = A$ ✓ (XOR with 0 gives same element)

④ $A \wedge 1 = A'$ (XOR with 1 toggles the bit) $1 \rightarrow 0$
 $0 \rightarrow 1$

⑤ $a \wedge b \leq \underline{\min(a, b)}$ ✓ ✓
 As we keep doing and the value

Keeps decreasing $a \& b \& c \leq \min(a, b, c)$

✓

⑥

$$\boxed{a \mid b} \geq \boxed{\max(a, b)}$$

As we keep doing or the value
keeps increasing

⑦

cyclic XOR

$$A \wedge B = C$$

$$A \wedge C = B$$

$$B \wedge C = A$$

$$\begin{array}{r} 10 \\ b \\ \hline 1110 \\ 14 \\ \hline 010, 4 \rightarrow 100 \\ 3 \rightarrow 01 \\ 5 \rightarrow 10 \\ 7 \rightarrow 11 \end{array}$$

⑧

for a number to be odd the lsb is always one, How to check?

$$n \% 2 == 0$$

if $\text{num} \not\equiv 1 \pmod{2} \quad \{ \begin{cases} \text{even} \\ \text{odd} \end{cases}$

contr^t "number is odd" is end

{ (Better than remainder check \rightarrow faster)

⑨

How to check if power of 2 or
not?

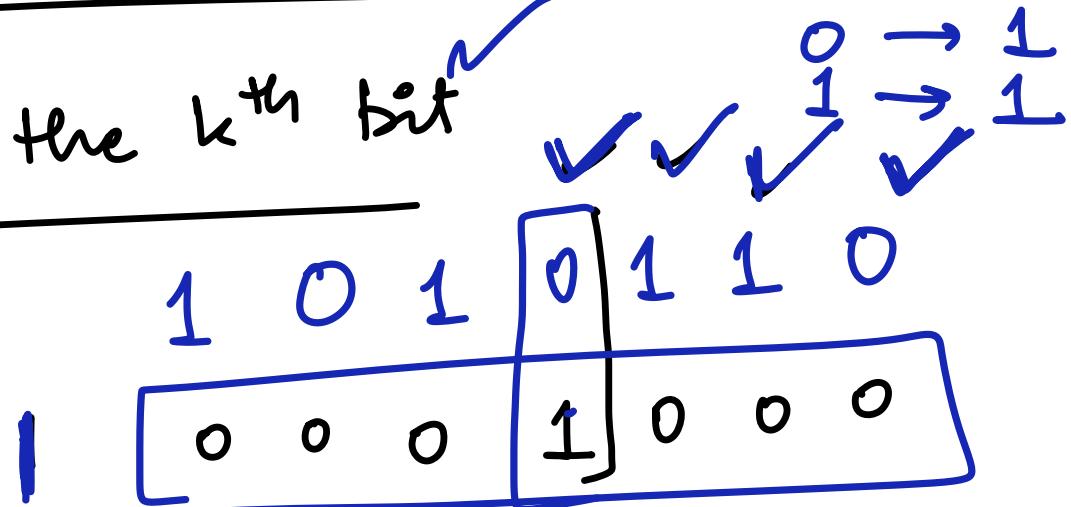
$A \& A - 1 = 0$ then Yes
else No

How?

$$\begin{array}{r} 1000 \rightarrow 8 \\ 0111 \rightarrow 7 \\ \hline 0000 \end{array}$$

Tricks and Tips using Bit Manipulation

① Set the k^{th} bit



$$\begin{array}{l} 011 = 1 \\ 11 = 1 \end{array}$$

$$1011110$$

0 based $\leftarrow A | (1 \ll k)$

$$A | ((1 \ll k) - 1)$$

$$A | (1 \ll k - 1)$$

1 based

②

Check if the k^{th} bit is set or not?

$$\begin{array}{r}
 0001010 \\
 \oplus 0000001 \\
 \hline
 0001000
 \end{array}$$

$$\begin{array}{r}
 1010110 \\
 \oplus 0000000 \\
 \hline
 1000100
 \end{array}$$

$$A \& (1 \ll k - 1) != 0$$

$$\rightarrow A \& (1 \ll k) != 0$$

$$\rightarrow (A >> k) \& 1 != 0$$

↳ then set

↳ else not set

③

Toggle the k^{th} bit

$$1010110$$

$$\begin{array}{r} ^\wedge \\ 0001000 \end{array}$$

$$A \wedge (1 \ll k - 1)$$