Assignment #3- Example of the GARCH(1,1) Model Using Contemporary Market Data

MSDS 492- Analysis of Financial Markets

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AI used in this report for custom coding, background search, and wording clarification. The work is my own.

Contents

Executive Summary

This study examines the volatility dynamics of a diverse set of financial assets which include U.S. equities (SPY SPDR S&P 500 ETF Trust), energy equities (PSX Phillips 66; VLO Valero Energy Corporation; MPC Marathon Petroleum Corporation), commodities (CL=F WTI Crude Oil Futures (NYMEX); GLD SPDR Gold Shares ETF), and the EUR/USD exchange rate(FX Spot). The study reviews exploratory data analysis and univariate GARCH(1,1) modeling for each of the assets. The analysis follows the standard conditional variance framework popularized in the empirical finance literature and closely parallels the illustrative GARCH(1,1) example presented in Section 9.4 of Taylor (2005), which emphasizes volatility clustering, high persistence, and the economic interpretability of conditional variance dynamics. Daily log returns are analyzed to characterize distributional

properties, cross-asset relationships, and the persistence of volatility shocks across asset classes.

The exploratory analysis verifies the key stylized facts emphasized by Taylor (2005) that returns are approximately mean-zero, exhibit excess kurtosis, and display weak linear autocorrelation but strong autocorrelation in squared and absolute returns. These features are especially pronounced in energy-related assets, consistent with Taylor's discussion of commodities and financial assets exhibiting heterogeneous volatility regimes. Cross-asset correlations align with economic intuition and mirror the comparative examples in the text, with strong comovement among energy equities, moderate correlation with the broad equity market, and near-zero correlation between EUR/USD returns and other asset classes.

Univariate GARCH(1,1) models provide a reasonable and effective representation of conditional variance dynamics across all assets. This aligns closely with the behavior illustrated in Taylor's Section 9.4 example on the US Dollar to Deutsch Mark exchange rate. Estimated models display high volatility persistence, with the sum of ARCH and GARCH coefficients close to unity, implying slow mean reversion and extended volatility half-lives which is an outcome explicitly highlighted by Taylor as typical in empirical financial data. Conditional volatility estimates track realized volatility measures closely, and diagnostic tests indicate minimal remaining autocorrelation in squared standardized residuals, suggesting that second-moment dependence is largely absorbed by the model. Multi-step volatility forecasts converge smoothly toward asset-specific long term variance levels, replicating the gradual forecast mean reversion emphasized in Taylor's discussion. Energy-related assets converge to substantially higher long-run volatility levels than equities and foreign exchange, illustrating how the same GARCH structure yields economically distinct risk profiles across markets.

Overall, the results reinforce Taylor's conclusion that the GARCH(1,1) model, despite its simplicity, captures the dominant features of financial market volatility and provides a robust baseline for volatility forecasting and risk assessment. At the same time, differences across asset classes point to the practical value of extensions discussed in the literature such as heavy-tailed errors, asymmetric models, or multivariate approaches when modeling tail risk or volatility spillovers beyond a univariate framework.

Literature Review

Recent empirical research on financial volatility forecasting continues to treat GARCH-type models as a central benchmark while evaluating their performance relative to more flexible alternatives. A common theme across this literature is the persistence of volatility and the trade-off between complexity, interpretability, and forecasting accuracy. Rather than replacing GARCH outright, most recent studies assess how far its limitations can be mitigated through extensions or hybrid approaches.

Persistence and Model Complexity in Applied GARCH Studies

Several recent peer-reviewed studies reaffirm that the standard GARCH(1,1) model remains competitive for modeling volatility across major asset classes. Using global equity indices, Marisetty (2024) shows that GARCH(1,1) consistently balances statistical significance and predictive stability, even when higher-order GARCH specifications marginally improve in-sample fit. Similar conclusions emerge in applied energy and commodity contexts, where GARCH models capture strong volatility persistence but struggle during abrupt regime changes (Fałdziński, Fiszeder, and Orzeszko 2021). These studies highlight the principal achievement of the GARCH framework that it provides a stable and interpretable representation of time-varying risk using a minimal number of parameters.

The drawback emphasized in these reviews is that volatility persistence may be overstated when structural breaks or regime shifts are ignored. Recent evidence suggests that neglecting such features can weaken forecast accuracy, particularly during periods of macroeconomic stress (Chung, Espinoza, and Quispe 2025). Nonetheless, GARCH models remain widely used because their simplicity facilitates cross-asset comparisons and practical implementation.

GARCH Extensions and Hybrid Models

A second group of recent studies explores extensions designed to address well-known limitations of standard GARCH models. Realized-GARCH approaches integrate high-frequency or realized volatility measures to improve forecast precision and reduce measurement error. Fang and Han (2025) demonstrate that incorporating realized volatility components enhances forecasting performance for major equity indices and improves option pricing outcomes. The advantage of this approach lies in its ability to capture multi-scale volatility dynamics, though it requires richer data and introduces additional modeling complexity.

Hybrid models combining GARCH with machine learning techniques represent another active research direction. Recent comparisons show that neural networks and support

vector regression models often outperform GARCH in medium- and long-horizon forecasts, particularly in markets characterized by nonlinear dynamics (Sun and Yu 2020; Chung 2024). However, these models typically sacrifice interpretability and may exhibit instability across different market conditions. As a result, several authors argue for hybrid frameworks in which GARCH provides a structural backbone, while machine learning components capture residual nonlinearities.

GARCH versus Stochastic Volatility Frameworks

A further strand of recent literature directly compares GARCH models with stochastic volatility (SV) models. Pang and Zhao (2025) find that autoregressive stochastic volatility models outperform GARCH(1,1) in forecasting equity volatility under the physical measure, particularly during stress periods, although the differences narrow for certain derivative pricing applications. These findings suggest that treating volatility as a latent stochastic process may offer statistical advantages, but at the cost of substantially more complex estimation and reduced transparency. Consequently, GARCH models remain attractive in applied settings where interpretability and computational efficiency are important.

Positioning of the Present Study

Relative to the recent literature, the present study aligns with applied research that treats GARCH(1,1) as a robust baseline rather than an obsolete methodology. Unlike studies focused on methodological innovation or forecast competition, this investigation emphasizes cross-asset comparability and economic interpretation by applying a unified GARCH framework across equities, energy equities, commodities, and foreign exchange. In doing so, it echoes recent findings that simple GARCH models remain informative when the objective is to characterize volatility persistence and long-run risk levels, rather than to optimize short-horizon forecast accuracy.

The contribution of this work is primarily consolidative in that it synthesizes established volatility modeling techniques with contemporary empirical insights, demonstrating that classic GARCH results documented in earlier literature remain relevant in modern, multi-asset contexts, even as more sophisticated alternatives continue to emerge.

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Introduction- Explanation of Analysis Conducted

Volatility plays a central role in financial decision-making, influencing risk management, portfolio allocation, derivative pricing, and capital planning across a wide range of organizations. Unlike returns, which are notoriously difficult to predict, volatility exhibits strong persistence and clustering, making it both economically meaningful and statistically tractable. Despite the growth of increasingly sophisticated forecasting methods,

organizations still rely heavily on relatively simple econometric models, especially GARCH-type models, to measure and forecast time-varying risk. This persistence raises an important question: to what extent do simple, well-established volatility models remain adequate for contemporary, multi-asset risk analysis?

The core problem addressed in this study is the measurement and comparison of volatility dynamics across diverse asset classes using a unified modeling framework. While extensive research documents volatility behavior within individual markets, fewer studies systematically compare how volatility persistence, shock transmission, and long-run risk differ across equities, energy equities, commodities, and foreign exchange when modeled consistently.

Research Questions and Hypotheses

This study is guided by the following research questions-

- Do different asset classes exhibit materially different volatility persistence and mean-reversion behavior when modeled using a common GARCH(1,1) specification?
- To what extent does the GARCH framework model adequately capture realized volatility dynamics across equities, commodities, and foreign exchange?
- How do cross-asset differences in volatility dynamics inform comparative risk assessment and forecasting?

Correspondingly, the study tests the following hypotheses-

- Volatility persistence is high across all asset classes but varies systematically by market type, with energy-related assets exhibiting longer volatility half-lives than equities and foreign exchange.
- A standard GARCH(1,1) model captures the dominant second-moment dynamics of returns across asset classes, despite differences in return distributions and market structure.
- Cross-asset comparisons based on a unified volatility model yield economically meaningful distinctions that are relevant for organizational risk management.

Nature of the Problem and Relation to Other Research

The problem examined in this report is fundamentally a risk comparability problem rather than a pure forecasting competition. It relates closely to broader challenges in financial economics concerning volatility persistence, regime dependence, and tail risk, but differs in emphasis from studies that prioritize marginal forecast accuracy or methodological novelty. Whereas much of the recent literature evaluates increasingly complex models against GARCH benchmarks, this study focuses on understanding what information a simple, transparent model can reliably deliver across diverse markets.

In this sense, the investigation complements existing research on volatility modeling by emphasizing synthesis and interpretability rather than replacement. It builds on well-established stylized facts while responding to modern portfolio contexts in which organizations simultaneously manage exposures to equities, commodities, and currencies.

Importance to Organizations

For financial institutions, energy firms, asset managers, and corporate risk functions, volatility is not an abstract statistical concept but a key operational input. Volatility estimates influence hedge ratios, risk limits, capital buffers, and stress-testing outcomes. Inconsistent or incomparable volatility measures across asset classes can lead to distorted risk assessments, inefficient hedging strategies, and suboptimal investment decisions.

By providing a systematic, cross-asset analysis of volatility dynamics within a unified GARCH framework, this study offers organizations a clearer basis for comparing risk across markets. The results help clarify when simple models remain sufficient for strategic decision-making and when more complex approaches may be warranted. In doing so, the research supports more transparent, consistent, and economically grounded risk management practices.

Theoretical Framework

This study is grounded in the theory of conditional heteroskedasticity in financial time series, which holds that the variance of asset returns is not constant over time but evolves as a predictable function of past information. Unlike classical models that assume homoskedastic disturbances, modern financial theory recognizes volatility as a dynamic state variable that reflects changing market conditions, information arrival, and risk

perceptions. Within this framework, volatility is both economically meaningful and statistically modellable, providing a foundation for risk measurement and forecasting.

Volatility as a Conditional Process

The theoretical starting point is the distinction between unconditional variance and conditional variance. While unconditional variance summarizes long-run risk, conditional variance reflects time-varying uncertainty given the information set available at a particular point in time. Financial markets are characterized by periods of calm and turbulence, implying that conditional variance clusters over time. This phenomenon that is commonly referred to as volatility clustering, suggests that large shocks tend to be followed by further large shocks, regardless of sign, while small shocks tend to be followed by small shocks. There is also a trend where high volatility will gradually reduce towards its base level and low volatility will potentially increase to a similar base level.

In this framework, volatility is treated as a latent process that can be inferred from observed returns. Theoretical models therefore focus on specifying how current volatility depends on past shocks and past volatility, rather than attempting to predict returns directly. This perspective justifies the emphasis on second-moment dynamics rather than conditional mean behavior.

GARCH Models and Volatility Persistence

The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model provides a formal representation of conditional variance dynamics. In its standard GARCH(1,1) form, conditional variance is modeled as a weighted combination of a long-run variance level, the squared innovation from the previous period, and the previous period's conditional variance. The theoretical implication is that volatility responds both to new information (news effects) and to existing volatility conditions (persistence effects).

A key concept within this framework is volatility persistence, typically measured by the sum of the ARCH and GARCH coefficients. When persistence is high, shocks to volatility decay slowly, leading to extended periods of elevated or depressed risk. Theoretical and empirical work suggests that such persistence is a fundamental property of financial markets, reflecting gradual information diffusion, behavioral effects, and institutional frictions. Mean reversion in volatility, while present, operates over relatively long horizons.

Cross-Asset Heterogeneity in Volatility Dynamics

Although volatility clustering and persistence are universal features of financial returns, the theoretical framework allows for heterogeneity across asset classes. Differences in market microstructure, supply-demand dynamics, and exposure to macroeconomic shocks imply that the parameters governing volatility dynamics may differ systematically across equities,

commodities, and foreign exchange. For example, commodity markets may exhibit stronger volatility responses to supply shocks, while foreign exchange markets may display lower overall volatility due to higher liquidity and continuous trading.

The framework adopted in this study treats asset-specific volatility processes as realizations of a common structural form, enabling direct comparison of persistence, shock sensitivity, and long-run variance across markets. This approach emphasizes comparability and interpretability rather than asset-specific model customization.

Relationship to Risk Measurement and Organizational Decision-Making

From an organizational perspective, the theoretical framework links conditional volatility directly to risk exposure and decision-making. Conditional variance serves as a key input into value-at-risk calculations, stress testing, hedging strategies, and capital allocation decisions. The assumption that conditional volatility is predictable underpins many risk management systems used by financial institutions, energy firms, and corporate treasury functions.

Importantly, the framework recognizes a trade-off between model complexity and usability. While more elaborate models may capture additional features of volatility dynamics, simple and transparent models often provide sufficient information for strategic decisions, particularly when the objective is comparative risk assessment across assets rather than marginal forecast improvements.

Conceptual Positioning of the Present Study

Within this theoretical framework, the present study adopts the GARCH(1,1) model as a baseline representation of conditional volatility dynamics. The focus is not on proposing a new volatility model, but on using an established theoretical structure to examine how volatility behaves across different asset classes under a unified specification. By doing so, the study leverages core theoretical insights volatility clustering, persistence, and mean reversion to address a practical problem of cross-asset risk comparability.

This framework provides a coherent bridge between financial theory, empirical modeling, and organizational risk management, ensuring that the analysis remains both theoretically grounded and practically relevant.

Data

This study employs daily financial market data for a set of representative assets spanning equities, energy equities, commodities, and foreign exchange. The objective of the data selection is to cover the requirements of the programing assignment. Thes assets

represent coverage of major asset classes while maintaining consistency in frequency and availability across markets, thereby enabling meaningful cross-asset comparison of volatility dynamics.

Asset Selection

The dataset includes the following instruments-

- U.S. equity market: S&P 500 exchange-traded fund (SPY- ETF), serving as a proxy for broad equity market risk.
- Energy equities: Phillips 66 (PSX), Valero Energy Corporation (VLO), and Marathon Petroleum Corporation (MPC), representing downstream energy firms with exposure to commodity price fluctuations and refining margins.
- Commodities: West Texas Intermediate crude oil futures (CL=F) and gold (GLD-ETF), capturing energy and precious metal markets with distinct supply-demand and hedging characteristics.
- Foreign exchange: EUR/USD spot exchange rate, representing a highly liquid currency market with continuous trading and comparatively low volatility.

These assets were selected to reflect differences in market structure, liquidity, and exposure to macroeconomic shocks, while remaining sufficiently liquid to support reliable volatility estimation.

Time Period and Frequency

All series are observed at a daily frequency, which is standard in empirical volatility modeling and consistent with the assumptions underlying GARCH-type models. The sample spans January 2019 through mid-February 2026, covering multiple market regimes, including periods of relative stability and episodes of elevated financial stress. This time span encompasses major macroeconomic and market disruptions as well as subsequent normalization phases, allowing the analysis to capture volatility clustering and persistence under varying conditions.

Data Construction

For each asset, daily adjusted closing prices are used to account for corporate actions such as dividends and stock splits where applicable. Returns are computed as daily logarithmic returns, defined as the first difference of the natural logarithm of prices. Log returns are preferred because they are time-additive and approximately symmetric for small price changes, making them suitable for econometric modeling of conditional variance.

In addition to returns, several auxiliary series are constructed for descriptive and diagnostic purposes, including squared returns and rolling volatility measures. These transformations are used to assess volatility clustering and to compare conditional volatility estimates with realized volatility proxies.

Data Quality and Preprocessing

Prior to analysis, all series are inspected for missing values, outliers, and inconsistencies. Days with no trading activity are handled in a manner consistent with market conventions for each asset class. Summary statistics and distributional diagnostics are computed to verify key stylized facts of financial returns, including excess kurtosis and weak serial correlation in raw returns.

Stationarity of the return series is assumed based on standard theoretical results and confirmed empirically through exploratory analysis. As the focus of the study is on second-moment dynamics, no detrending or additional filtering of returns is applied.

Suitability for Volatility Analysis

The selected dataset is well suited for the objectives of this study for three reasons. First, the daily frequency provides sufficient granularity to capture volatility clustering without introducing excessive microstructure noise. Second, the extended sample from 2019 to early 2026 allows the analysis to examine volatility behavior across diverse market regimes using a consistent framework. Third, the inclusion of multiple asset classes enables direct comparison of volatility persistence and long-run risk under a unified modeling specification.

Together, these features support robust estimation of conditional volatility models and facilitate economically meaningful interpretation of cross-asset risk dynamics.

Methodology

This study adopts a quantitative, model-based decision analytics approach to examine and compare volatility dynamics across multiple asset classes. The methodology is designed to balance theoretical rigor, empirical transparency, and practical relevance for organizational risk assessment.

Research Objective and Analytical Approach

The primary objective of this research is to analyze and compare conditional volatility behavior across heterogeneous financial assets using a unified modeling framework. Rather than optimizing short-horizon forecast accuracy for a single market, the study

focuses on understanding volatility persistence, shock sensitivity, and long-run risk levels in a manner that supports cross-asset comparability and organizational decision-making.

To achieve this objective, the study employs univariate GARCH-type volatility models, with particular emphasis on the standard GARCH(1,1) specification. This is the model selected in the programing assignment and it represents a widely accepted baseline in both academic research and applied risk management, allowing the results to be interpreted relative to established theory and practice.

Proposed Methodology

The empirical methodology proceeds in four stages:

1. Exploratory data analysis, including summary statistics, distributional diagnostics, and autocorrelation analysis of returns and transformed returns.

2. Model specification and estimation, using a consistent GARCH(1,1) framework applied separately to each asset.

3. Model diagnostics and validation, assessing whether the fitted models adequately capture conditional variance dynamics.

4. Volatility inference and comparison, focusing on persistence measures, implied half-lives, and long-run variance estimates across asset classes.

This staged approach ensures that modeling choices are informed by data characteristics and that results are both statistically and economically interpretable.

Model Formulation Background

Let r_t denote the daily log return of an asset at time t, expressed as:

$$r_t = \mu + \varepsilon_t, \ \varepsilon_t = \sigma_t z_t,$$

where μ is the conditional mean, σ_t^2 is the conditional variance, and z_t is an independent and identically distributed innovation with zero mean and unit variance.

The conditional variance follows a GARCH(1,1) process:

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2,$$

where $\omega > 0$, $\alpha \geq 0$, and $\beta \geq 0$. The parameters α and β capture the short-run response to new shocks and the persistence of volatility, respectively. Stationarity requires $\alpha + \beta < 1$, implying mean-reverting volatility.

The same model structure is applied uniformly across all assets to ensure comparability of estimated parameters and derived volatility measures.

Modeling Assumptions

The methodology rests on several standard assumptions:

- Conditional heteroskedasticity: Volatility is time-varying and predictable based on past information.

- Weakly stationary returns: Log return series are stationary in mean, allowing focus on second-moment dynamics.

- Symmetric shock response: The baseline GARCH(1,1) model assumes that positive and negative shocks of equal magnitude have the same impact on volatility.

These assumptions are consistent with the theoretical framework and are evaluated empirically through diagnostic testing.

Parameter Estimation Using Real Data

Model parameters are estimated using maximum likelihood estimation (MLE) applied to the observed daily return series for each asset. Estimation is conducted separately for each asset, allowing parameters to reflect asset-specific volatility dynamics while maintaining a common structural form.

Parameter estimates are generated directly from the data rather than imposed or calibrated externally. Robust numerical optimization techniques are used to ensure convergence, and estimated parameters are examined for economic plausibility and adherence to theoretical constraints, such as positivity and stationarity.

Model Diagnostic and Evaluation Measures

Model performance is evaluated using a combination of statistical diagnostics and economic criteria rather than a single forecast-error metric. Key evaluation components include:

- Parameter interpretation: Magnitude and significance of estimated volatility parameters, particularly total persistence ($\alpha + \beta$).

- Residual diagnostics: Time-series plots of standardized residuals and autocorrelation diagnostics of squared standardized residuals, with confidence bands used to assess remaining dependence.

- Volatility comparison: Visual comparison of model-implied conditional volatility with realized volatility proxies, measured as rolling-window realized volatility derived from observed returns.

- Forecast behavior: Stability and convergence of multi-step volatility forecasts toward long-run variance levels.

These diagnostics emphasize whether the model captures key features of volatility dynamics, such as clustering and persistence, rather than whether it marginally outperforms alternative models in short-term predictive accuracy.

Volatility Measures and Forecasting

Conditional volatility estimates are converted to annualized percentage units to facilitate interpretation and cross-asset comparison. Long-run variance is computed as $\omega/(1 - \alpha - \beta)$ when stationarity conditions hold, and volatility persistence is summarized using implied half-life measures.

Forward-looking volatility dynamics are examined using closed-form multi-step forecasts implied by the GARCH(1,1) structure. These forecasts illustrate the expected decay of volatility shocks over time and provide an intuitive link between estimated parameters and future risk levels.

Cross-asset validation is achieved by applying the same estimation, diagnostic, and reporting procedures to all assets. Differences in persistence, volatility levels, and adjustment speeds across asset classes form the basis for comparative interpretation.

The methodology intentionally prioritizes interpretability, transparency, and comparability over methodological novelty. While more complex models may capture asymmetries, tail behavior, or structural breaks, the GARCH(1,1) framework provides a clear and theoretically grounded baseline. Known limitations such as symmetric shock response and sensitivity to structural change are acknowledged and inform interpretation rather than undermining the validity of the analysis.

Results

This section presents and interprets the empirical findings from the univariate GARCH(1,1) analysis within the context of the study's decision-analytic objectives. To provide a clear and representative illustration of the modeling results, the analysis first focuses on West Texas Intermediate crude oil futures (CL=F) as a detailed case study. Complete estimation results, diagnostic plots, and volatility forecasts for all assets included in the study are

provided in the accompanying code-output PDF. Following the discussion of the crude oil example, results are then compared across the diverse assets to highlight differences and commonalities in volatility persistence, long-run variance, and risk dynamics, thereby informing cross-market volatility assessment.

Verification of Stylized Facts of Financial Markets

The following plots provide illustrative evidence that the assets examined in this study exhibit the well-documented stylized facts of financial return series and are therefore appropriate candidates for GARCH(1,1) modeling. As emphasized by Taylor (2005), these stylized facts include returns that are approximately mean-zero, display excess kurtosis (fat tails), and exhibit weak linear autocorrelation, alongside pronounced and persistent autocorrelation in squared and absolute returns. Together, these characteristics motivate modeling approaches that focus on conditional variance dynamics.

The discussion below focuses on West Texas Intermediate crude oil front-month futures (CL=F) as a representative example; however, the remaining assets analyzed in the study exhibit qualitatively similar patterns. In particular, the extreme volatility shocks observed in March 2020 during the onset of the COVID-19 crisis provide a clear illustration of volatility clustering and regime shifts in crude oil markets. These episodes underscore the sensitivity of energy markets to macroeconomic disruptions and highlight the persistence of volatility following large shocks. This background analysis is presented briefly to establish the empirical foundation for the subsequent GARCH modeling.

This plot presents the price trends for all assets included in the study to provide contextual background. In this plot, CL=F is shown as the green series, illustrating both the pronounced price movements during periods of market stress and the broader volatility

environment within which the return-based analysis is conducted.

Cross-Ticker • Adjusted/Close Prices (Raw Levels)
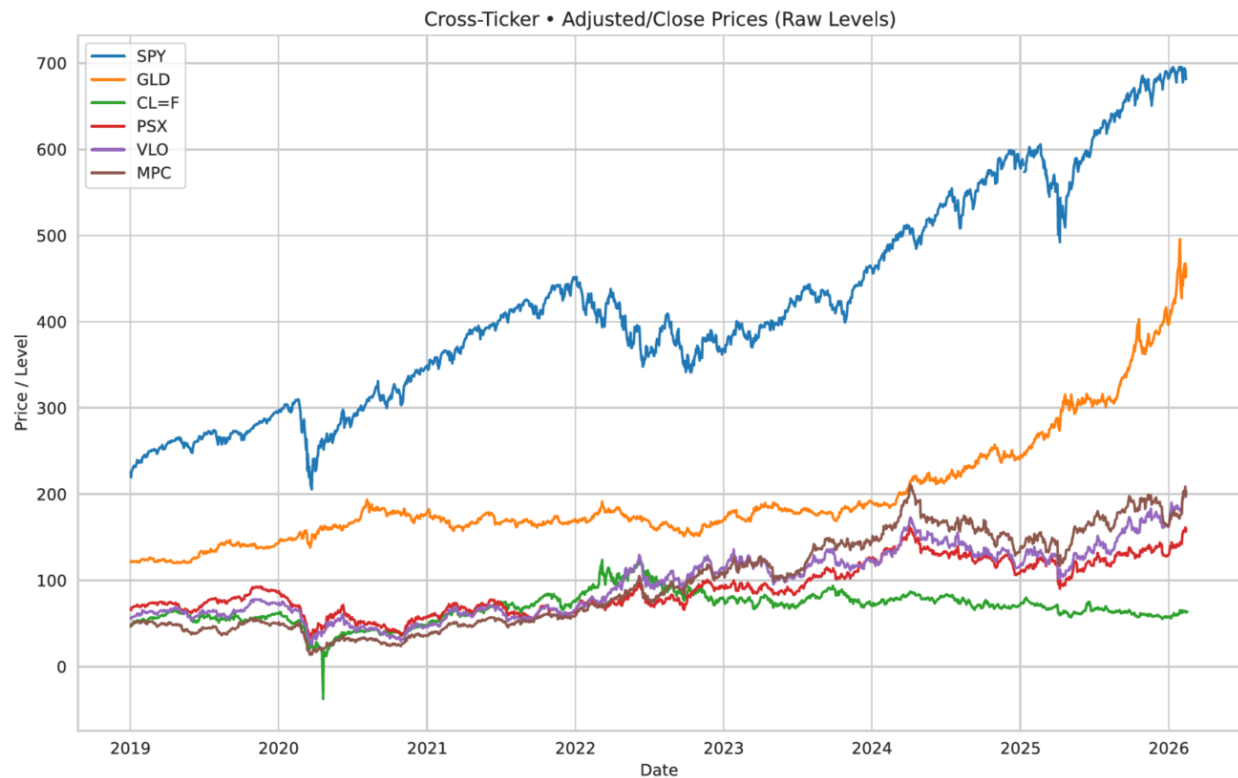
Return distribution

The return distribution plots indicate that daily returns are centered close to zero and exhibit pronounced excess kurtosis, characterized by heavier-than-normal tails relative to the Gaussian distribution. This behavior reflects the presence of large, infrequent shocks and is a well-established feature of financial return data. The combination of near-zero mean returns and fat-tailed distributions underscores the limited predictability of returns

themselves while highlighting the importance of modeling high time-varying volatility.

CL=F • Returns • Distribution
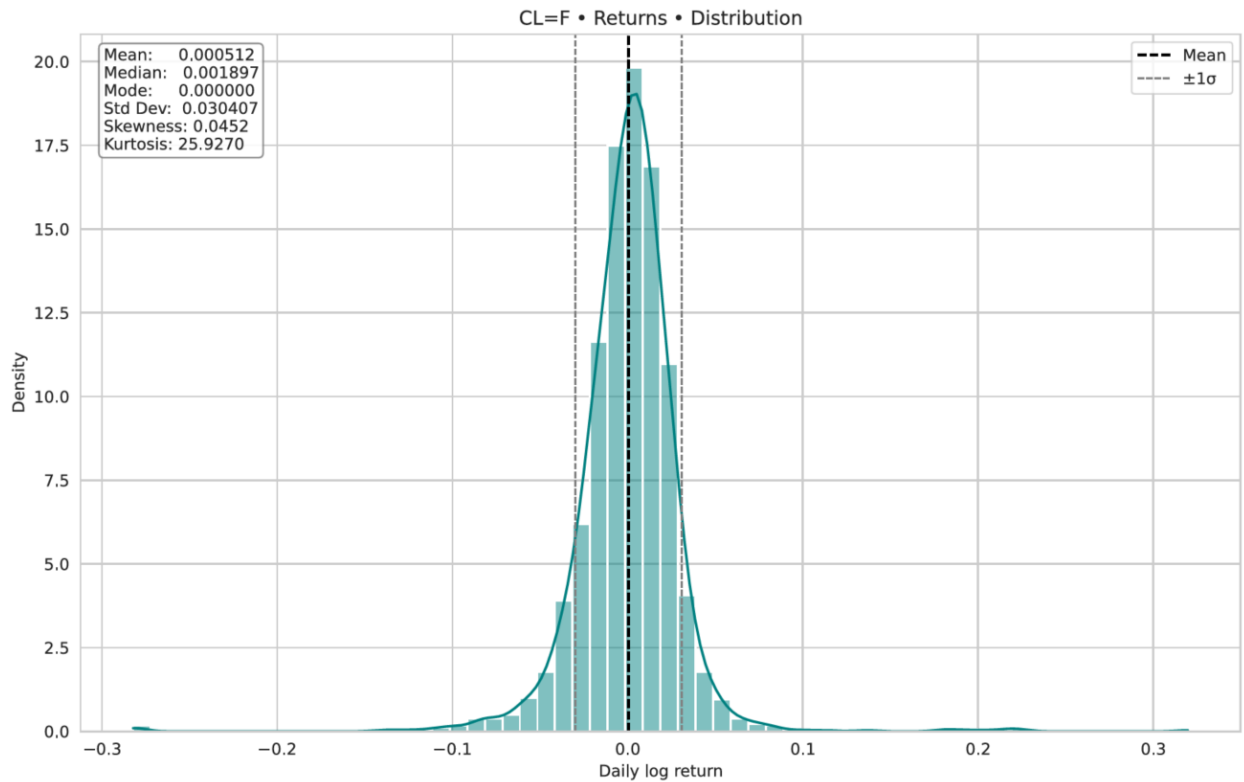
The Q–Q plot demonstrates clear departures from normality, particularly in the tails of the return distribution, indicating the presence of extreme outcomes and fat-tailed behavior that is characteristic of financial returns. The extreme returns are close to ±30% for single

day.

CL=F • Returns • Q–Q Plot vs Normal
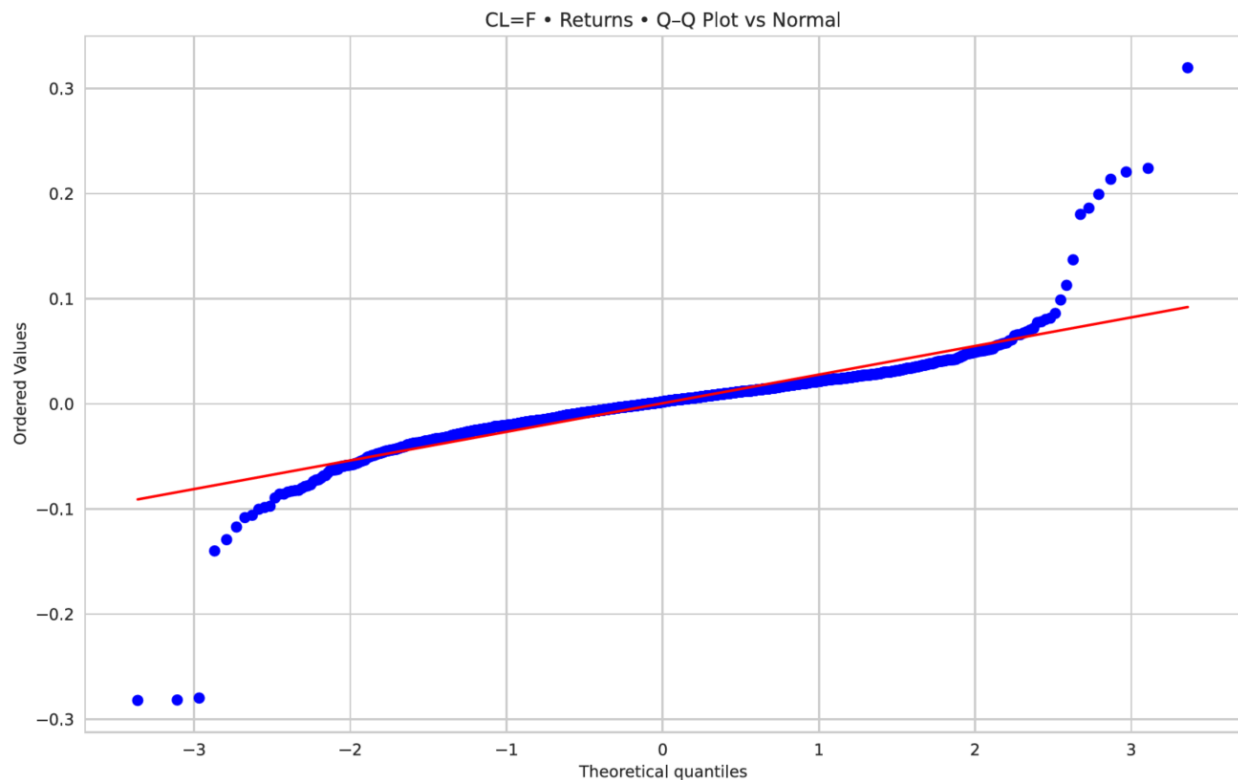
The plot below illustrates the low autocorrelation of lagged daily returns, indicating minimal predictability of raw returns from one day to the next. Autocorrelation estimates are generally small in magnitude and are not consistently above conventional statistical significance thresholds. While isolated lags may occasionally appear statistically significant, the associated correlation values are economically negligible. As a result, lagged returns provide little meaningful information for predicting subsequent return realizations. This finding is consistent with standard empirical evidence that linear dependence in raw financial returns is weak, reinforcing the focus on volatility dynamics
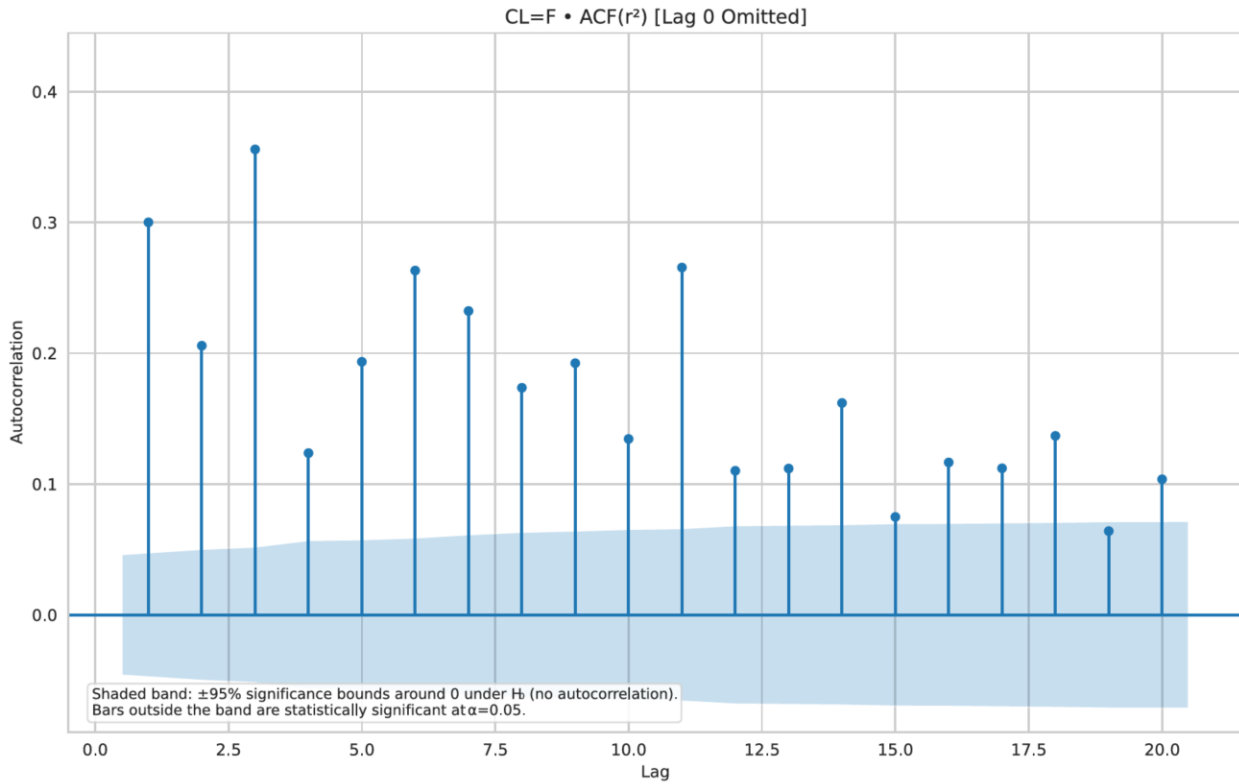
rather than return predictability.

CL=F • ACF • Returns
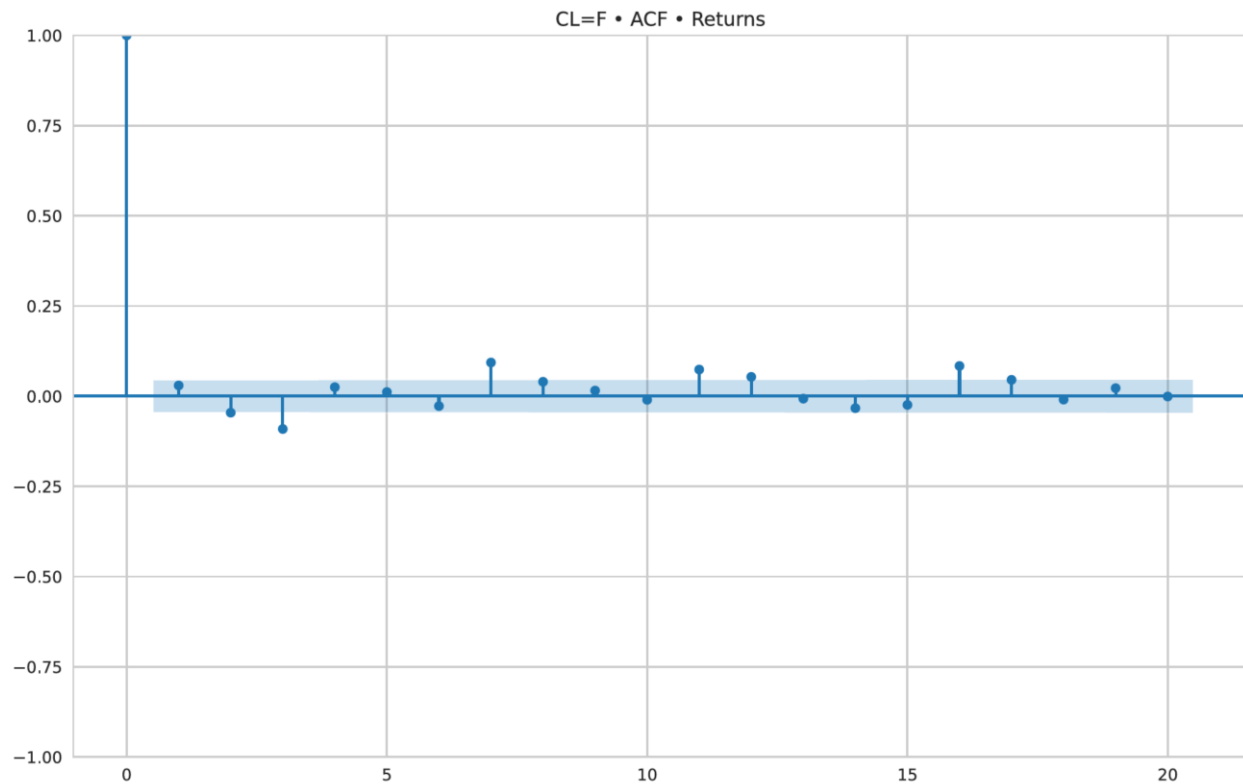
In contrast to raw returns, substantially higher autocorrelations are observed in the absolute and squared return series. These autocorrelations are consistently above conventional statistical significance thresholds and are markedly larger in magnitude than those observed for untransformed returns. Moreover, the dependence in these transformed returns persists across multiple lags, remaining elevated for several trading days before gradually decaying over time. This pattern reflects volatility clustering, whereby periods of high (or low) volatility tend to be followed by similar conditions.

The presence of strong and persistent autocorrelation in absolute and squared returns provides the empirical foundation for GARCH-type models. Because these transformations capture second-moment dynamics rather than mean behavior, their serial dependence indicates that volatility is predictable even when returns themselves are not. As a result, GARCH models can exploit this structure to generate informative forecasts of future

volatility despite the limited predictability of raw returns.

CL=F • ACF(|r|) [Lag 0 Omitted]

Shaded band: ±95% significance bounds around 0 under H_0 (no autocorrelation).
Bars outside the band are statistically significant at $\alpha=0.05$.

CL=F • ACF(r²) [Lag 0 Omitted]

Shaded band: ±95% significance bounds around 0 under H_0 (no autocorrelation).
Bars outside the band are statistically significant at $\alpha=0.05$.
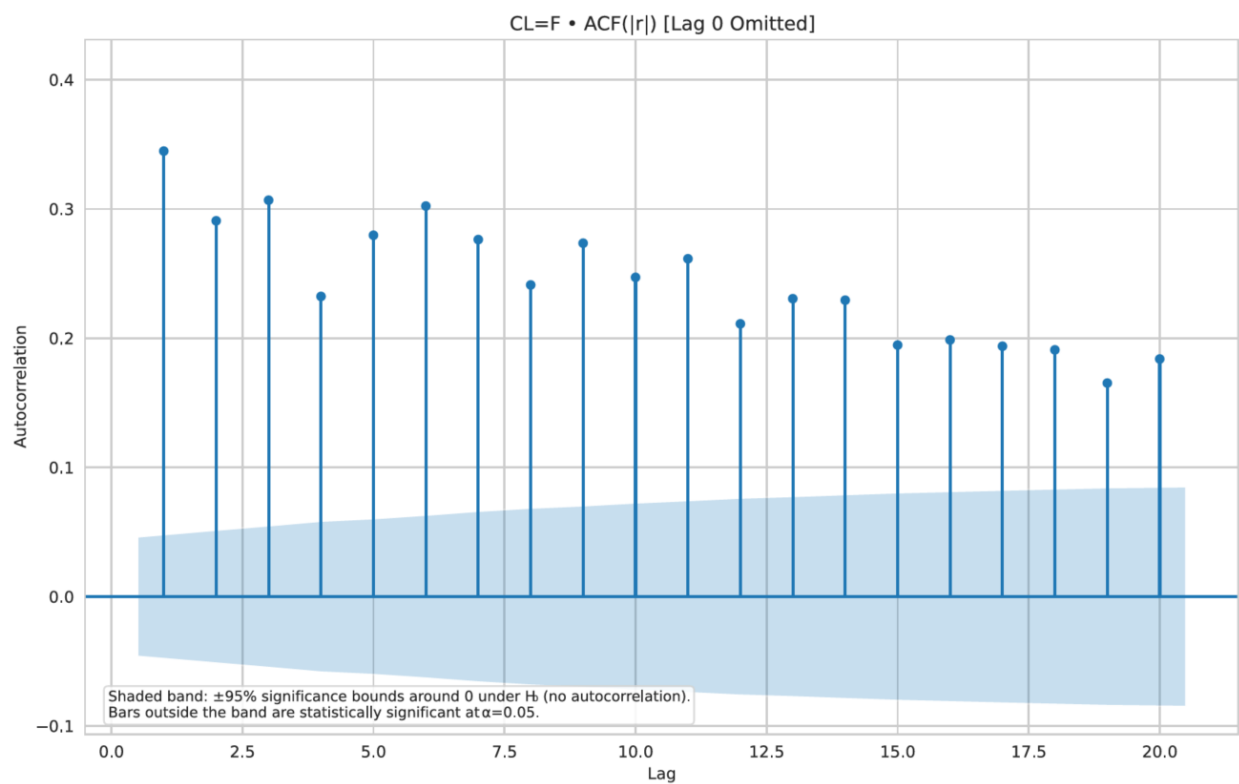
GARCH(1,1) modeling results

The analysis continues by presenting the GARCH(1,1) modeling results for West Texas Intermediate crude oil futures (CL=F) as a representative example. Detailed estimation results, diagnostic plots, and volatility forecasts for all assets included in the study are provided in the accompanying code-output PDF. Following the discussion of the crude oil results, the analysis compares and contrasts key model outcomes across assets to inform cross-market differences in volatility persistence, long-run risk levels, and conditional variance dynamics.

Modeling Setup and Distributional Assumptions

A Gaussian innovation distribution is adopted for the GARCH(1,1) specification to maintain consistency with the illustrative example presented in Section 9.4 of Taylor (2005), which examines the U.S. dollar–Deutsch mark exchange rate. Although empirical return distributions are known to deviate from normality, the Gaussian assumption provides a simpler and transparent baseline for examining conditional volatility dynamics. This choice allows the analysis to focus on volatility persistence and mean-reversion behavior under a standard specification, while acknowledging that extensions incorporating heavy-tailed or asymmetric innovations may offer improvements in modeling tail risk.

Report Settings

Metric	Value
Model	GARCH(1,1) with mean=constant; ML estimation
Innovation Distribution	Gaussian
Forecast Horizon (h)	90
Volatility Units	Annualized % (daily σ * $\sqrt{252}$ * 100)
Realized Vol Window	100 trading days (annualized %)
ACF Shaded Band	95% bounds around 0 under H_0 (no autocorrelation)
Assets Included	CL=F, EUR_USD, GLD, MPC, PSX, SPY, VLO

GARCH(1,1) Modeling Variable Estimates

The table below reports estimated parameters and diagnostic metrics from the GARCH(1,1) model, summarizing both the underlying volatility dynamics and overall model fit.

- The estimated conditional mean is close to zero, indicating that average daily returns are small relative to volatility and that risk dynamics are driven primarily by second-moment behavior rather than predictable mean movements.
- The variance intercept, ω, represents the baseline contribution to conditional variance and feeds directly into the implied long-run variance of the process.

- The ARCH parameter, α, measures the short-run impact of new shocks, capturing how strongly large return innovations immediately increase volatility.
- The GARCH parameter, β, reflects volatility persistence through its dependence on past conditional variance.
- The sum $\alpha + \beta$ is very close to unity, indicating strong volatility clustering and slow mean reversion—features that are widely documented in financial return series. High persistence implies a long volatility half-life, meaning that shocks to volatility decay gradually rather than dissipating quickly.
- The model-implied long-run (unconditional) variance, given by $\omega/(1 - \alpha - \beta)$, represents the risk level toward which conditional volatility converges over time.
- Model fit is assessed using the log-likelihood, which measures how well the model explains the observed return distribution under maximum likelihood estimation, as well as the Akaike and Bayesian information criteria (AIC and BIC), which balance goodness of fit against model complexity.
    - Lower AIC and BIC values indicate a more efficient fit relative to alternative specifications estimated on the same data. Taken together, these results suggest that the GARCH(1,1) model captures economically meaningful volatility persistence while providing a statistically coherent description of conditional variance dynamics.

CL=F • GARCH(1,1) Estimates

Metric	Value	Explanation
μ (mean)	0.000697	Estimated average daily return (constant mean in the return equation).
ω	1.848354e-05	Variance intercept; baseline level feeding the long-run variance.
α	0.100000	Shock (ARCH) effect; how strongly yesterday's squared residual increases today's variance.
β	0.880000	Persistence (GARCH) effect; how strongly yesterday's variance carries into today.
ν (Student-t df)	—	Not estimated under Gaussian innovations.
α+β (persistence)	0.980000	Total variance persistence; closer to 1 implies slower mean reversion and stronger volatility clustering.
σ∞² = ω/(1−α−β)	0.000924	Long-run (unconditional) variance implied by the model, assuming α+β < 1.
Half-life (days)	34.309283	Approx. days for a volatility shock to decay by 50% (based on α+β).
log-likelihood	4180.956160	Model fit objective value under maximum likelihood; higher is better (within same data/model).
AIC	-8353.912319	Akaike Information Criterion (penalized fit); lower is better for comparing models on the same data.
BIC	-8331.952435	Bayesian Information Criterion (stronger penalty than AIC); lower is better for comparing models on the same data.

Plot of Annualized Conditional Volatility

The chart below illustrates that CL=F experienced an extreme volatility shock in early 2020 associated with the onset of the COVID-19 crisis. During this period, GARCH-estimated conditional volatility increased sharply (over 100% annualized when crude oil prices went negative), while the 100-day realized volatility rose more gradually, reflecting the smoothing effect inherent in a long rolling window. Following the crisis, both volatility measures

declined, with conditional volatility reverting more rapidly and realized volatility adjusting with a noticeable lag.

From 2021 onward, the two series track each other closely. Conditional volatility exhibits short-lived spikes in response to new information, while realized volatility captures broader volatility regimes over longer horizons. This behavior highlights classic volatility clustering in crude oil markets and demonstrates the responsiveness of the GARCH model to shocks, in contrast to realized volatility measures that lag due to their averaging nature.
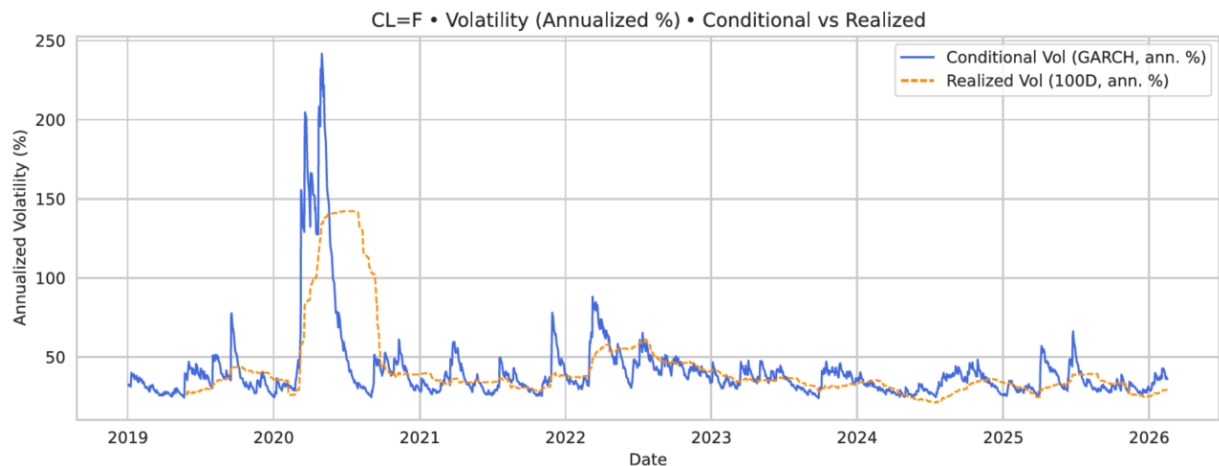

Plot of Conditional Volatility Compared to Daily Returns.

The plot below illustrates that daily percentage returns for CL=F are highly volatile and noisy, characterized by frequent sharp positive and negative swings. In contrast, the GARCH-estimated conditional volatility captures the underlying risk regime by rising sharply during periods of market stress and gradually declining as conditions stabilize. This behavior is most evident in early 2020 during the onset of the COVID-19 crisis. Additional volatility responses are visible around the start of the Russia–Ukraine war in 2022 and following tariff-related announcements in spring 2025, each reflecting significant shocks to global energy markets. The extreme return realizations observed during these episodes coincide with pronounced increases in conditional volatility, underscoring the magnitude of these disturbances to crude oil markets.

Following major shock periods, conditional volatility steadily reverts toward lower levels even as daily returns continue to fluctuate from day to day. This pattern illustrates classic volatility clustering: large return movements tend to be followed by sustained periods of elevated volatility rather than persistent directional trends in returns. Overall, the figure highlights the distinction between erratic daily returns and conditional volatility, which provides a smoother, structural measure of prevailing market uncertainty.

This behavior closely parallels the illustrative results presented in Taylor (2005, Section 9.4). As in Taylor's example, large changes in daily returns are followed by sharp increases in estimated volatility, with elevated volatility persisting for some time before gradually decaying. These dynamics demonstrate the GARCH model's ability to capture the time-varying response of volatility to market shocks.

CL=F • Daily % Returns vs Conditional Volatility (Annualized %)
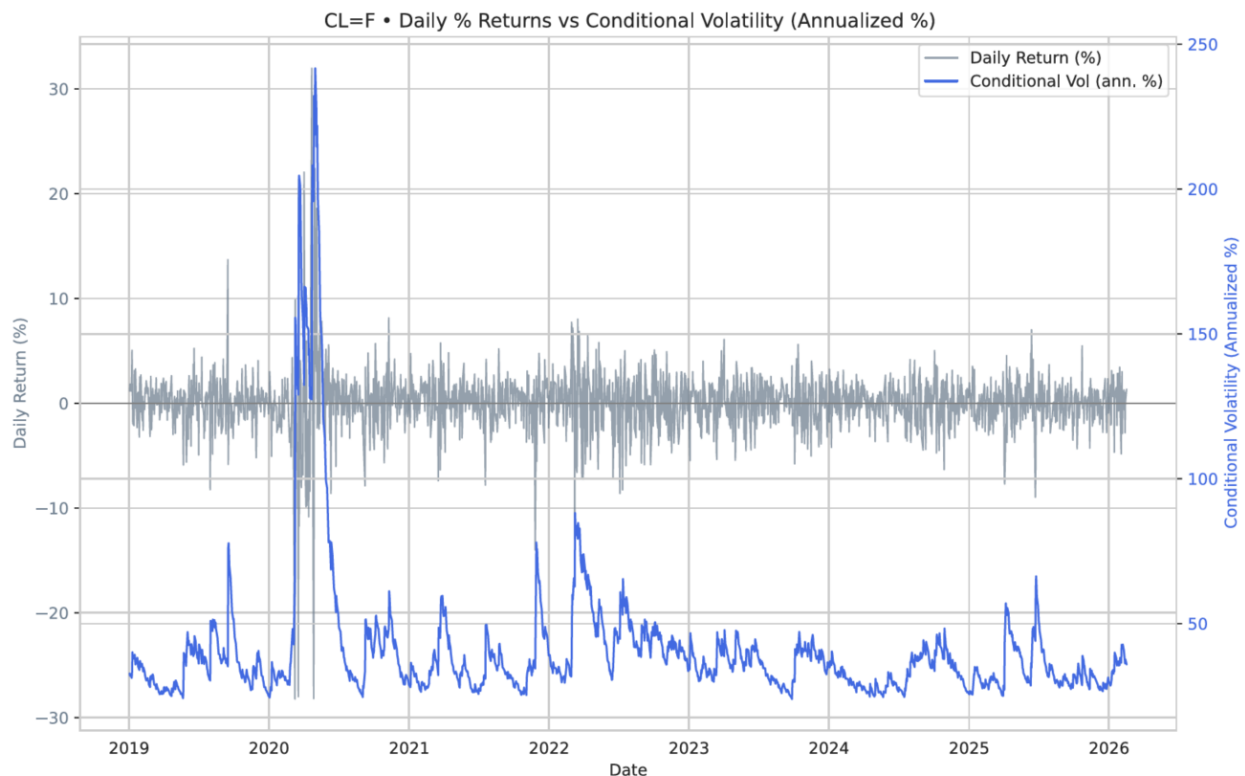
Review of Standardized Residuals

Standardized residual diagnostics are used to assess whether the GARCH(1,1) model has adequately captured serial dependence and volatility clustering in the return series. Standardized residuals are defined as the model residuals scaled by the fitted conditional volatility; if the model is well specified, these residuals should behave like white noise. The time-series plot of standardized residuals therefore provides a visual check for remaining structure, while the autocorrelation function (ACF) of squared standardized residuals formally tests for residual dependence in the magnitude of shocks. In a correctly specified GARCH model, squared standardized residuals should exhibit no statistically significant autocorrelation beyond lag zero, indicating that conditional variance dynamics have been successfully absorbed by the model.

For the crude oil (CL=F) example, the extreme return realizations observed during the COVID-19 crisis may exceed the capacity of a basic symmetric GARCH(1,1) specification to fully capture all second-moment dynamics. Consistent with this interpretation, the ACF of squared standardized residuals shows slight statistical significance at the one-day lag. However, the magnitude of this autocorrelation is very small, suggesting limited economic significance and indicating that most volatility clustering has been successfully modeled. Beyond this isolated case, no systematic autocorrelation is observed in the standardized residual diagnostics.

The EUR/USD results are also included for reference and exhibit no statistically significant autocorrelation in squared standardized residuals. Taken together, these findings indicate that the standard GARCH(1,1) specification performs reasonably well across assets, capturing the dominant features of conditional variance dynamics while leaving only minimal residual dependence during periods of extreme market stress.

CL=F • Standardized Residuals

CL=F • ACF(Standardized Residuals²) [Lag 0 Omitted]
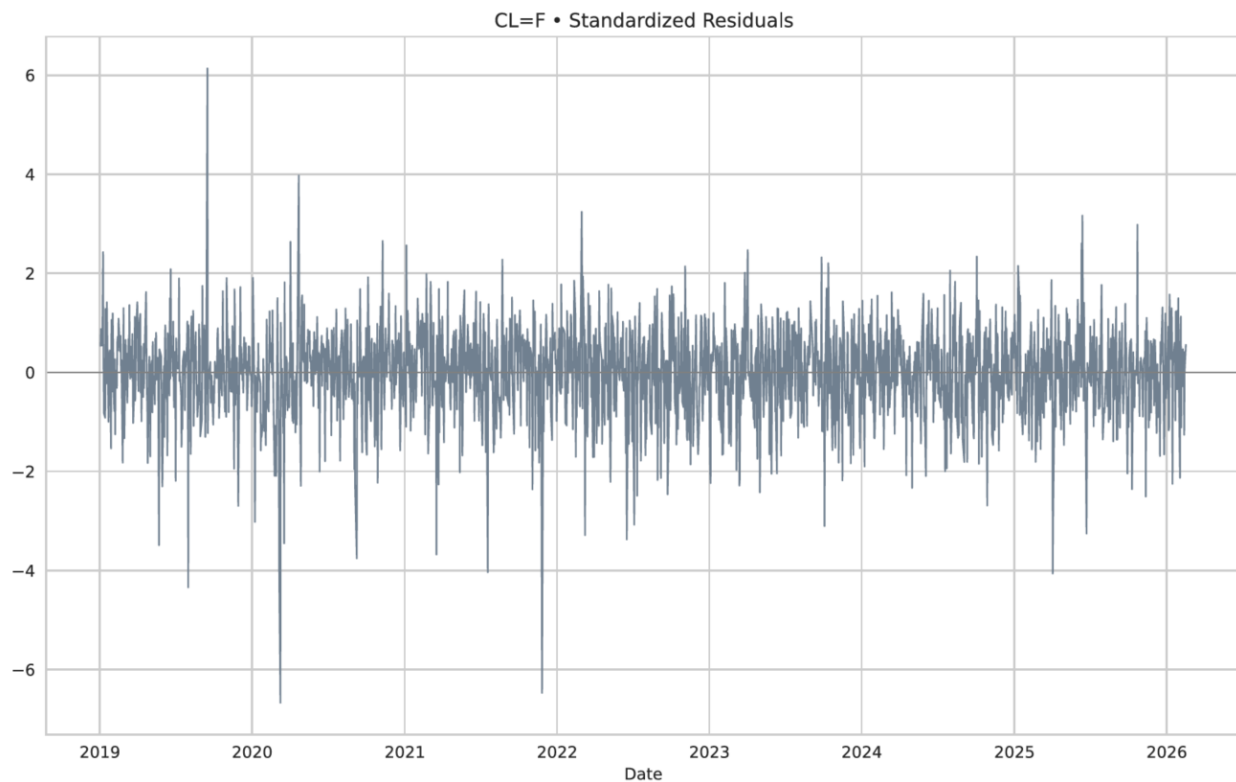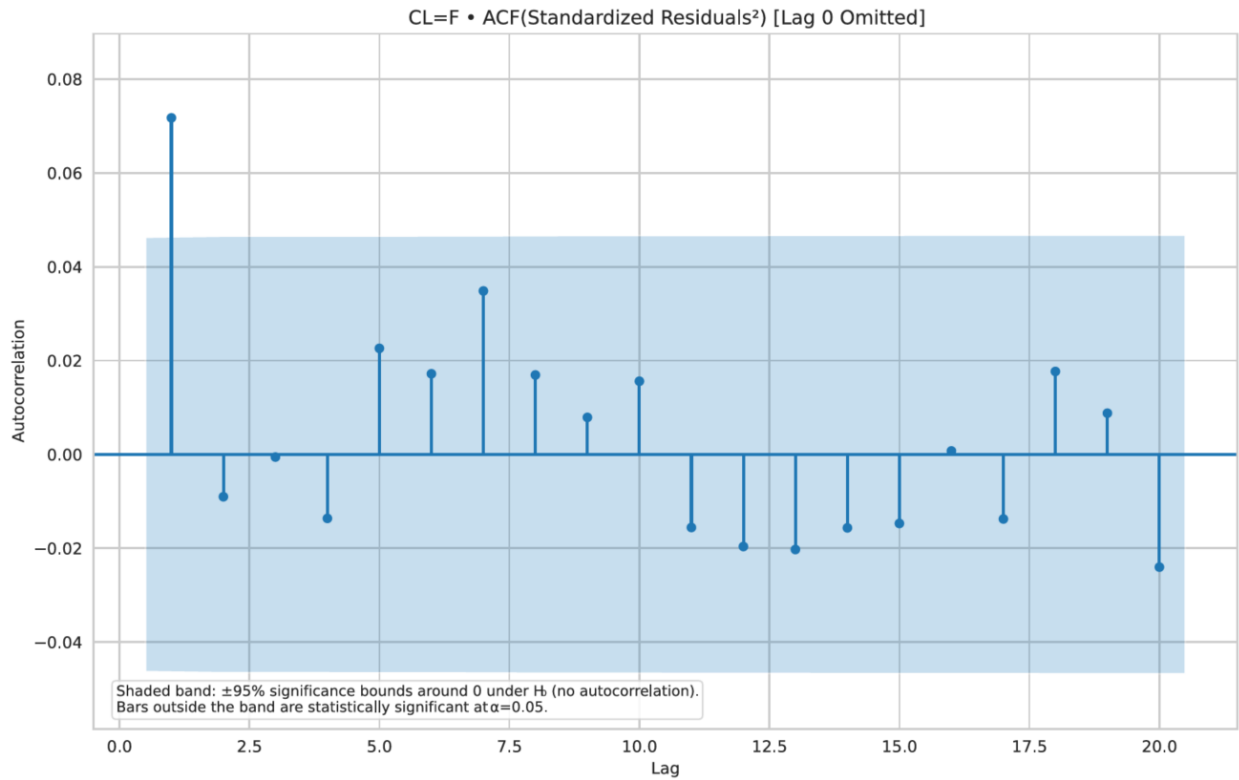
EUR-USD exchange showing no statistically significant autocorrelation of standardized residuals.

EUR_USD • Standardized Residuals

EUR_USD • ACF(Standardized Residuals²) [Lag 0 Omitted]

Shaded band: ±95% significance bounds around 0 under H_0 (no autocorrelation).
Bars outside the band are statistically significant at $\alpha = 0.05$.
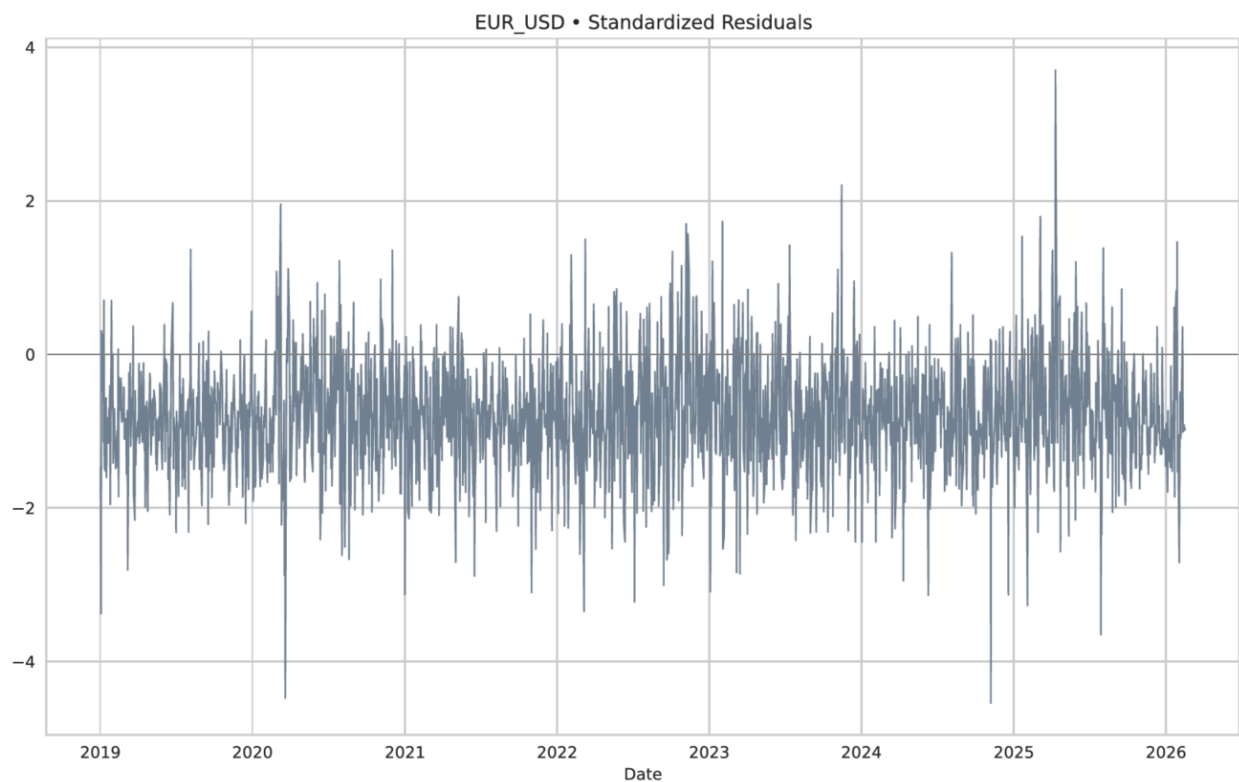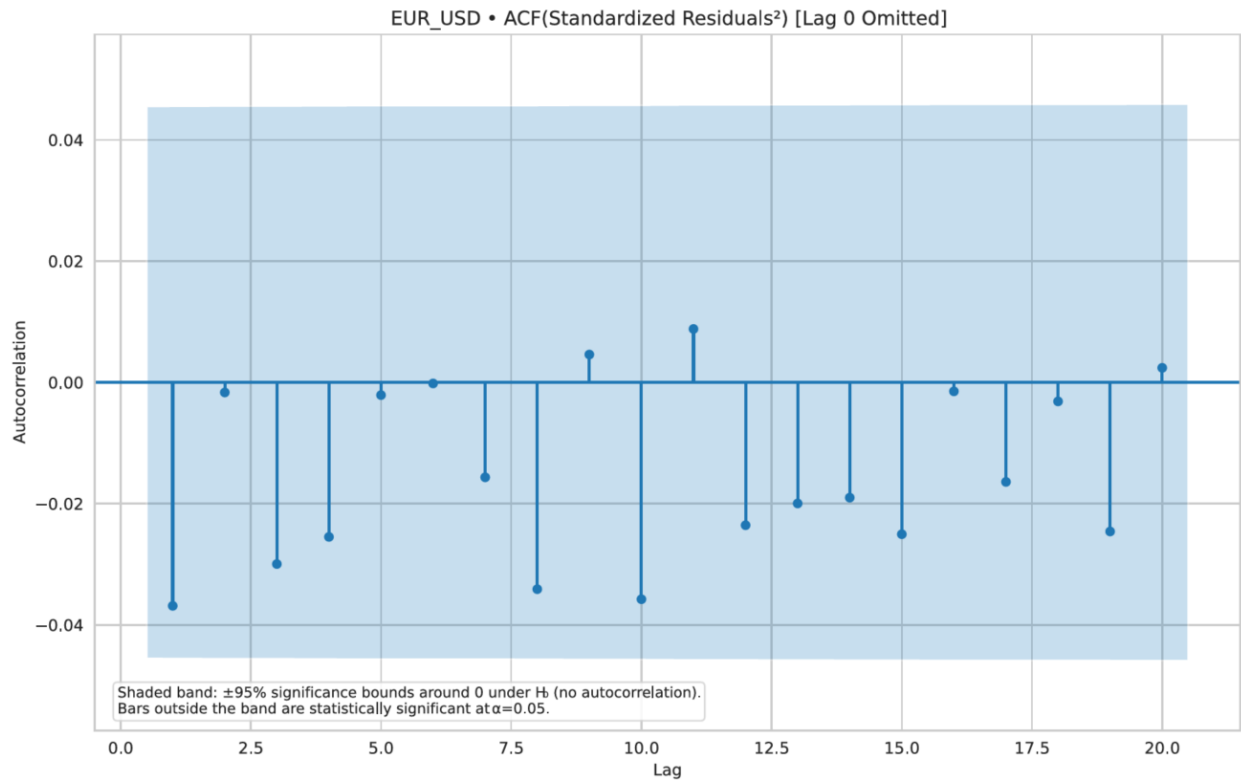
Volatility Forecast

The figure below presents the h-step volatility forecast for CL=F derived from the GARCH(1,1) model. The forecast exhibits a smooth, upward-sloping trajectory that gradually converges toward the model-implied long-run volatility level, reflecting the high persistence characteristic of GARCH processes. Beginning near the most recent conditional volatility estimate, the forecast increases steadily as the horizon extends, illustrating how short-term uncertainty propagates forward before stabilizing at the unconditional variance. This behavior indicates that even when current volatility is relatively low, the model anticipates a gradual increase in risk over longer horizons, consistent with slow mean reversion and persistent volatility dynamics.

Recent conditional volatility estimates also show a modest upward movement from previously low levels, indicating a mild increase in near-term risk. For crude oil markets, annualized volatility typically ranges between approximately 30% and 60%, as reflected in both the conditional and realized volatility series. The forecasted volatility path aligns closely with these historical ranges and with theoretical expectations for a highly persistent GARCH(1,1) process, reinforcing the model's consistency with observed long-term volatility behavior in crude oil markets.

CL=F • h-Step Volatility Forecast (Annualized %, Closed-Form)
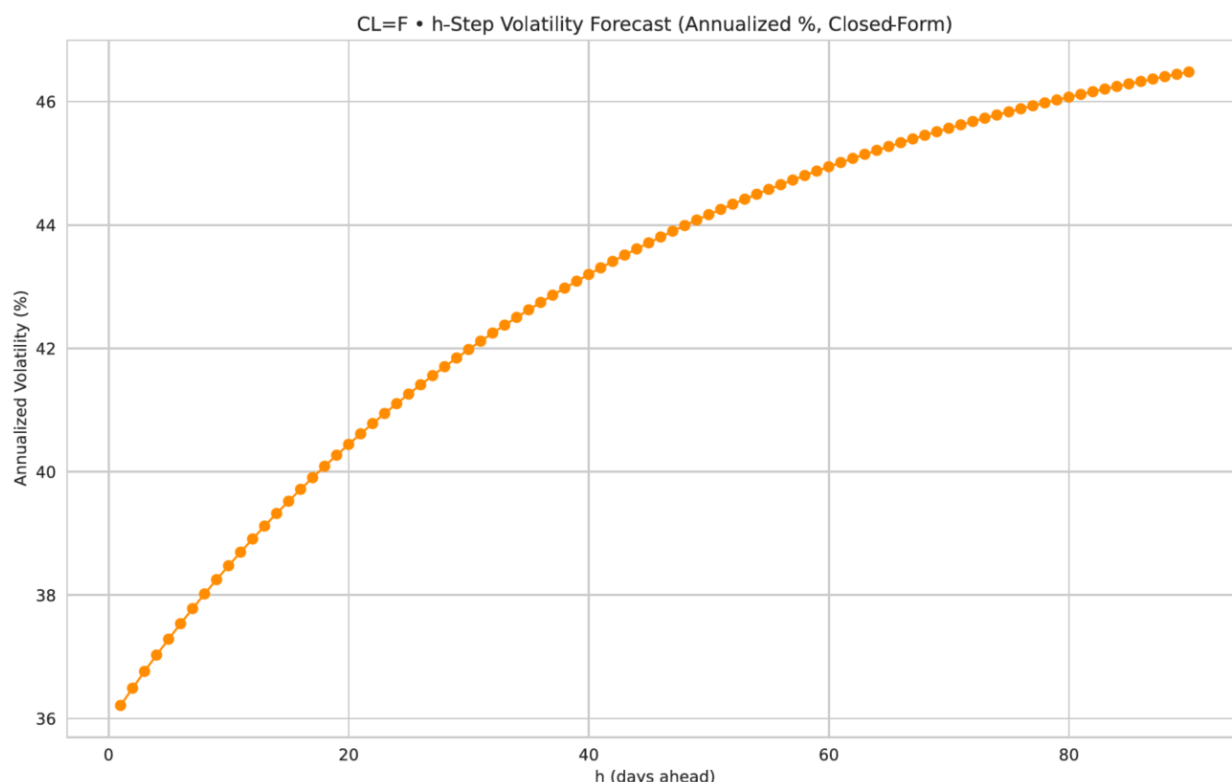
Review of Modeling Results Across Assets

Table 1 summarizes the key variance-equation parameter estimates from the GARCH(1,1) models for each asset. Across all series, volatility persistence is very high, with estimated values of $\alpha + \beta$ clustered around 0.98. This indicates that shocks to volatility decay slowly over time, a defining characteristic of financial return series. The ARCH parameters (α) vary modestly across assets, reflecting differences in sensitivity to recent return shocks, while the GARCH parameters (β) capture the degree of longer-run volatility carryover. Together, these parameters describe how quickly volatility responds to new information and how persistent those effects remain.

While most assets exhibit persistence levels close to 0.98, Phillips 66 (PSX) stands out with a higher persistence estimate of approximately 0.987. This implies that volatility shocks for PSX decay more slowly and that volatility reverts to its long-run mean at a slower rate relative to the other assets. Overall, however, the results indicate that all assets display strong volatility clustering, with only modest variation in the speed at which volatility responds to and recovers from market shocks.

The concentration of persistence estimates around $\alpha + \beta \approx 0.98$ is not indicative of a coding or estimation issue. Rather, it is a well-documented outcome when applying GARCH(1,1) models to liquid financial assets such as equities, commodities, foreign exchange, and broad market indices. These markets share common volatility dynamics

characterized by extended periods of relative calm punctuated by sharp volatility bursts. Because GARCH persistence is estimated directly from the data via maximum likelihood, assets with similar volatility clustering patterns give similar persistence parameters. In this study, CL=F, EUR/USD, GLD, MPC, SPY, and VLO all exhibit volatility behavior consistent with this pattern, while PSX displays a slightly higher persistence level, indicating that the model is capable of distinguishing meaningful cross-asset differences when they exist. The similarity of persistence estimates therefore reflects genuine commonality in underlying return dynamics rather than any deficiency in the modeling framework.

Across assets, the relative balance between the ARCH and GARCH components varies systematically. Energy-related assets exhibit larger short-run shock responses, while foreign exchange volatility is dominated by persistence rather than immediate news effects, consistent with differences in market structure and information flow.

Although volatility persistence ($\alpha + \beta$) is broadly similar across assets, the variance intercept (ω) differs meaningfully and reflects important cross-asset differences in baseline risk. Energy-related assets such as crude oil (CL=F) and refining equities (MPC, VLO, PSX) exhibit larger variance intercepts, indicating higher underlying volatility even in the absence of recent shocks, while EUR/USD displays a much smaller intercept consistent with the structurally lower volatility of highly liquid foreign exchange markets. As a result, assets may share comparable persistence and mean-reversion dynamics yet differ substantially in their long-run risk levels, underscoring the importance of distinguishing volatility magnitude from volatility persistence in cross-asset risk assessment.

Table 1. GARCH(1,1) Parameter Estimates (by Asset)

These are the key estimated parameters in the variance equation
$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2$.

Asset	Mean (μ)	ω (variance intercept)	α (ARCH)	β (GARCH)	α+β (persistence)
CL=F	0.000697	1.848354e-05	0.100000	0.880000	0.980000
EUR/USD	0.004518	4.106655e-07	0.050000	0.930000	0.980000
GLD	0.000384	2.245379e-06	0.100000	0.880000	0.980000
MPC	0.001235	1.534174e-05	0.100000	0.880000	0.980000

PSX	0.000796	8.117445e-06	0.065584	0.921553	0.987137
SPY	0.000901	3.085275e-06	0.100000	0.880000	0.980000
VLO	0.001064	1.565674e-05	0.050000	0.930000	0.980000

Table 2 illustrates how similar volatility persistence across most assets translates into closely aligned shock half-lives and long-run variance dynamics. For assets with persistence estimates $\alpha + \beta$ near 0.98, volatility shocks are estimated to take approximately 34 trading days to decay by half. These results indicate that volatility is highly persistent across markets and that mean reversion operates slowly but consistently, leaving risk levels elevated for several weeks following large return shocks.

Phillips 66 (PSX) emerges as a clear outlier in this comparison. Its higher persistence estimate of approximately 0.987 implies a substantially longer volatility half-life of roughly 53.5 trading days. As a result, volatility shocks for PSX dissipate more gradually, and elevated risk conditions persist for a longer period relative to the other assets examined. This finding highlights meaningful cross-asset heterogeneity in the speed of volatility mean reversion, even within a common GARCH(1,1) framework.

Overall, the table demonstrates that while unconditional (long-run) variance levels differ across assets, the dominant driver of comparative volatility persistence is the persistence parameter $\alpha + \beta$. In this respect, PSX stands out as the asset for which volatility takes the longest to return toward its long-run equilibrium, reinforcing its distinct risk profile within the sample.

Table 2. Long-Run Variance and Shock Half-Life (Comparative Risk Persistence)

The report provides the implied **unconditional (long-run) variance** and the **half-life** (in trading days) implied by persistence.

Asset	α+β (persistence)	Long-run variance σ_∞^2	Half-life (days)
CL=F	0.980000	0.000924	34.309283
EUR/USD	0.980000	2.053328e-05	34.309636
GLD	0.980000	0.000112	34.309546
MPC	0.980000	0.000767	34.309594

PSX	0.987137	0.000631	53.539909
SPY	0.980000	0.000154	34.309690
VLO	0.980000	0.000783	34.309220

Table 3 reports model-fit statistics for the GARCH(1,1) specifications across all assets and indicates consistently strong performance. Each series yields large log-likelihood values and highly negative Akaike and Bayesian information criteria (AIC and BIC), suggesting a good fit under the chosen model structure. These outcomes are characteristic of a well-behaved GARCH model when applied to liquid financial return series.

Importantly, differences in the magnitudes of the log-likelihood, AIC, and BIC across assets such as the substantially larger log-likelihood and more negative information criteria observed for EUR/USD relative to CL=F or VLO primarily reflect differences in return scale rather than meaningful variation in model quality. As a result, direct cross-asset comparisons of these raw fit statistics are not informative and do not indicate relative superiority or inferiority of the model across markets.

Overall, the fit statistics provide no evidence of misspecification or instability that would warrant altering the GARCH(1,1) structure for any of the assets examined. Taken together, these results suggest that the models perform consistently well across asset classes, and no modifications are required based solely on the reported fit measures.

Table 3. Model Fit Statistics (Within-Asset Comparability)

These statistics are useful for **within-asset** model comparison (same data/model family), and provide a compact sense of fit quality under the report's specification.

Asset	Log-Likelihood	AIC	BIC
CL=F	4180.956160	-8353.912319	-8331.952435
EUR/USD	6722.373639	-13436.75	-13414.65
GLD	5756.585331	-11505.17	-11483.21
MPC	4203.115547	-8398.231094	-8376.273445
PSX	4332.290886	-8656.581773	-8634.624125

| SPY | 5747.942045 | -11487.88 | -11465.93 |
| VLO | 4083.009895 | -8158.019791 | -8136.062143 |

Other interesting findings

Conditional and Realized Volatility: EUR/USD Exchange Rate

The chart below presents the conditional and realized volatility estimates for the EUR/USD exchange rate. For EUR/USD, the relatively large and persistent gap between conditional and realized volatility is a well-known feature of foreign exchange markets rather than an indication of model misspecification. FX returns are typically smooth and low-variance due to high liquidity, continuous trading, and strong policy anchoring. As a result, long-window realized volatility measures remain low and adjust only gradually over time.

In contrast, the GARCH-estimated conditional volatility responds immediately to even small day-to-day return innovations, producing a more jagged and consistently higher volatility series. This divergence reflects the fundamental difference between forward-looking, model-based conditional volatility and backward-looking realized volatility measures that rely on rolling averages. In highly liquid currency pairs such as EUR/USD, this dynamic leads to conditional volatility that captures short-term risk fluctuations while realized volatility reflects broader, more stable volatility regimes.

Overall, the observed pattern is consistent with the structural characteristics of currency markets and highlights the appropriateness of the GARCH framework for modeling time-varying volatility in FX returns. The behavior observed in the chart therefore reflects intrinsic features of EUR/USD volatility dynamics rather than any limitation of the modeling approach.

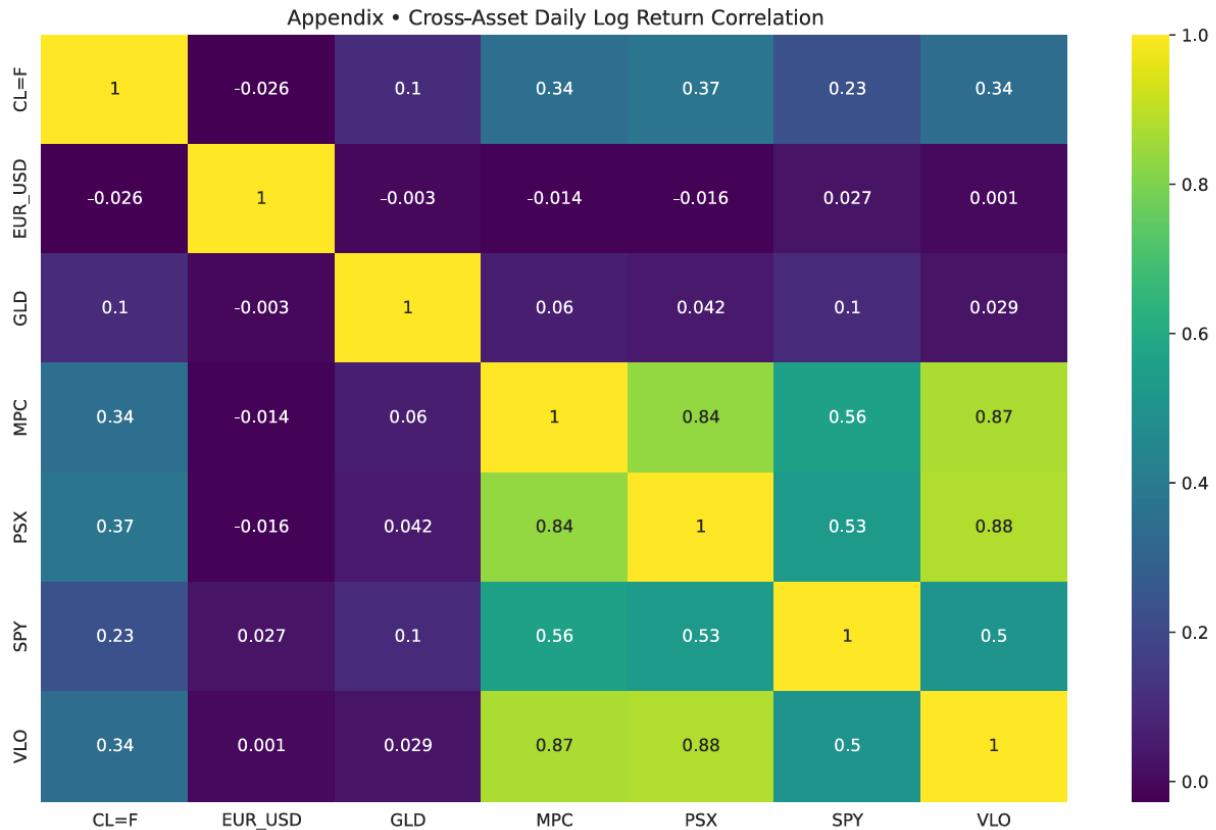EUR_USD • Volatility (Annualized %) • Conditional vs Realized
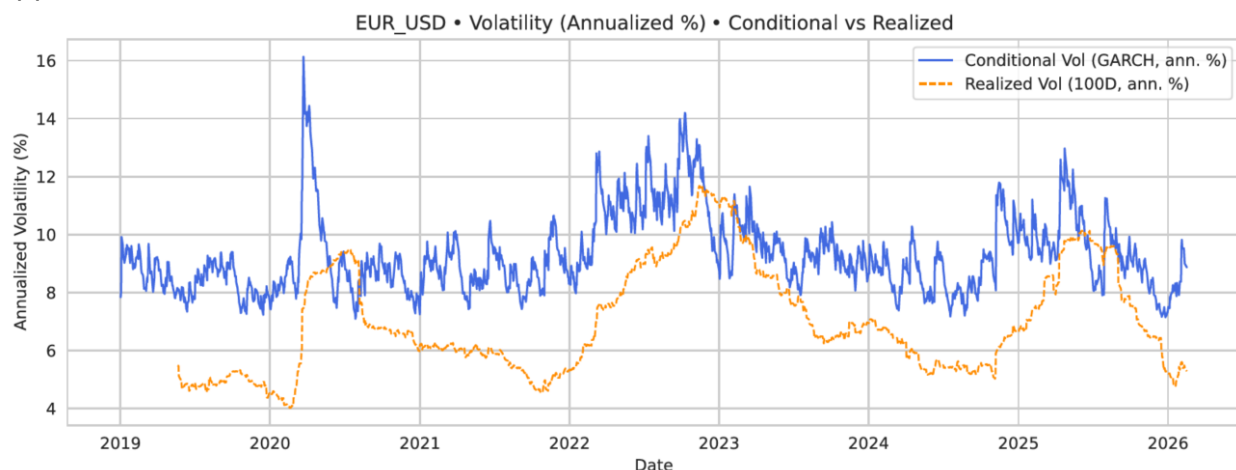
Cross-Asset Return Correlations

The correlation heat map reveals a strong clustering of return correlations among U.S. refining equities (Phillips 66 (PSX), Valero Energy Corporation (VLO), Marathon Petroleum Corporation (MPC)) with pairwise correlations in the range of approximately 0.84 to 0.88. This tight co-movement reflects their shared exposure to refining margins, crude oil price dynamics, and common macro-energy drivers. The S&P 500 proxy (SPY) also exhibits moderate positive correlations with these refiners (approximately 0.50 to 0.56), indicating that broader equity-market factors play a meaningful role in shaping their returns.

In contrast, gold (GLD) and the EUR/USD exchange rate display weak correlations with both the energy equities and the broader equity market, with coefficients close to zero. This pattern reinforces their roles as alternative or diversifying assets within the sample. Crude oil futures (CL=F) occupy an intermediate position: returns show a modest positive correlation with refining equities (approximately 0.34 to 0.37), consistent with operational and economic linkages between crude prices and refining activity. However, these correlations are notably weaker than those observed among equity-to-equity pairs, reflecting differing sensitivities to factors such as crack spreads, inventory dynamics, and downstream margins rather than crude price movements alone.

Overall, the correlation structure highlights clear segmentation across asset classes. Energy equities move closely together and exhibit meaningful co-movement with the broader equity market, while commodities and foreign exchange behave more independently. This structure underscores the potential diversification benefits of commodities and FX relative to equities and complements the cross-asset volatility results discussed earlier.

Appendix • Cross-Asset Daily Log Return Correlation

	CL=F	EUR_USD	GLD	MPC	PSX	SPY	VLO
CL=F	1	−0.026	0.1	0.34	0.37	0.23	0.34
EUR_USD	−0.026	1	−0.003	−0.014	−0.016	0.027	0.001
GLD	0.1	−0.003	1	0.06	0.042	0.1	0.029
MPC	0.34	−0.014	0.06	1	0.84	0.56	0.87
PSX	0.37	−0.016	0.042	0.84	1	0.53	0.88
SPY	0.23	0.027	0.1	0.56	0.53	1	0.5
VLO	0.34	0.001	0.029	0.87	0.88	0.5	1

Conclusions

This study examined volatility dynamics across a diverse set of financial assets including broad equities, energy equities, commodities, and foreign exchange by using a unified univariate GARCH(1,1) framework. The objective was not to compete among alternative volatility models, but to evaluate whether a simple and transparent conditional variance specification can provide economically meaningful and comparable risk insights across heterogeneous markets. The results strongly support this objective.

Across all assets analyzed, daily returns exhibit the well-documented stylized facts of financial markets: approximately zero mean, pronounced excess kurtosis, weak linear autocorrelation, and strong dependence in the magnitude of returns. Autocorrelation

diagnostics confirm that while raw returns are largely unpredictable, transformations such as squared and absolute returns display persistent serial dependence. These features provide the empirical foundation for GARCH-type modeling and are consistently observed across equities, commodities, and foreign exchange in this sample.

Univariate GARCH(1,1) models estimated under Gaussian distributions capture the dominant features of conditional variance dynamics for every asset considered. Estimated persistence parameters ($\alpha+\beta$) cluster near unity, indicating strong volatility clustering and slow mean reversion which is an outcome that aligns closely with classical empirical results documented in the literature and the illustrative example in Taylor (2005). The implied half-lives of volatility shocks are on the order of several weeks for most assets, reinforcing the interpretation of volatility as a highly persistent state variable rather than a transitory disturbance.

Despite similarities in persistence, the models reveal economically meaningful cross-asset differences in long-run volatility levels. Energy-related assets, particularly refining equities and crude oil, converge to substantially higher unconditional volatility than broad equities and foreign exchange. This distinction highlights that comparable persistence does not imply comparable risk levels and underscores the importance of separating volatility persistence from volatility magnitude in cross-asset risk assessment.

Diagnostic checks indicate that the fitted GARCH models adequately absorb second-moment dependence. Standardized residuals exhibit no systematic structure, and the autocorrelation of squared standardized residuals shows little evidence of remaining volatility clustering. While isolated lag-level significance appears occasionally these effects are small in magnitude and do not suggest material misspecification. Conditional volatility estimates track realized volatility closely, with GARCH responding rapidly to new information and realized volatility adjusting more gradually due to its rolling-window construction.

Multi-step volatility forecasts derived from the GARCH solution exhibit smooth convergence toward asset-specific long-run variance levels. This behavior provides an intuitive illustration of volatility mean reversion and links estimated parameters directly to forward-looking risk expectations. Importantly, these forecasts are stable and interpretable, reinforcing the suitability of the model for strategic risk assessment rather than short-horizon trading applications.

Overall, the findings reaffirm that the standard GARCH(1,1) model remains a robust and informative baseline for modeling financial market volatility. While more sophisticated approaches may offer improvements in tail modeling, asymmetry, or short-term forecast

accuracy, the simplicity and transparency of the GARCH framework make it particularly well suited for cross-asset comparison and organizational risk analysis. Differences in volatility persistence, long-run risk levels, and cross-asset dependence structures emerge clearly even under this simpler model setup, underscoring the continued practical relevance of classical volatility models in contemporary financial contexts.

Keywords

GARCH(1,1); Conditional Volatility; Volatility Clustering; Financial Time Series; Risk Measurement; Volatility Persistence; Energy Equities; Commodities; Foreign Exchange; Univariate Volatility Modeling; Realized Volatility; Forecasting Risk

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