Pset #2

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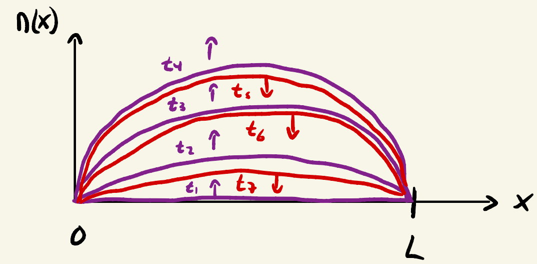
Problem 1

1.1 My phantom density profile is $n(x,t) = n_o \sin\left(\frac{\pi t}{t_p}\right)\sin\left(\frac{\pi x}{L}\right)$

where t_p is the length of the plasma liftime, and L is the length of the plasma in the cord direction:

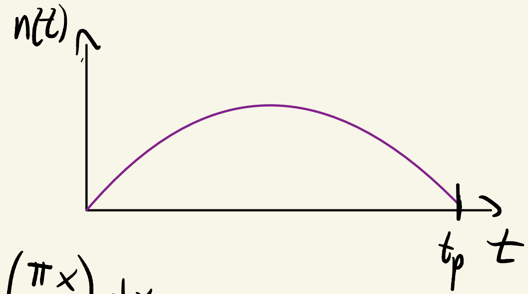
The plot at the right shows what is going on with this. The plasma is plotted verus x for different times. Purple is the plasma growing in time, and red is it shrinking in time.



I will pick: $n_o = 4 \times 10^{20}\ m^{-3}$

$L = 0.1\ m$

$t_p = 100\ ms$



$$\int n_e(x,t)\,dl = \int_0^L n_o \sin\left(\frac{\pi t}{t_p}\right)\sin\left(\frac{\pi x}{L}\right)dx$$

$$= \boxed{\langle n_e L\rangle = n_o\left[\frac{2L}{\pi}\sin\left(\frac{\pi t}{t_p}\right)\right]}$$

1.2 Q: Calculate the signal from a homodyne Mach-Zehnder Interferometer. Note where phase ambiguity arises.

To start, the max system density is $n_o = 4 \times 10^{20}\ m^{-3}$. Lets pick a frequency high enough that it is well above the critical density:

$$n_c = \frac{\omega^2 m \varepsilon_o}{e^2},$$

$$n_c \gg n_o$$

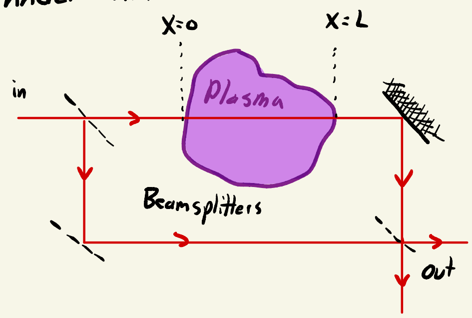
$$n_o \ll \frac{\omega^2 m \varepsilon_o}{e^2}$$

$$2\pi f = \omega \gg \sqrt{\frac{n_o e^2}{\varepsilon_o m_e}} = 1128 \times 10^9\ rad/s$$

$$f \gg \frac{1}{2\pi}\sqrt{\frac{n_o e^2}{\varepsilon_o m_e}} = 179.57\ GHz.$$

So, a red laser at 650 nm, or 4.61×10^{14} Hz, is well above the critical 28.39 GHz frequency and is fit to be a probe of this plasma.

The Mach-Zehnder interferometer is shown below:



The two output paths are evenly mixed between the plasma leg and the bypass leg, so it will be and interference of the two. The plasma refraction index is

$$N^2 = 1 - \frac{\omega_p^2}{\omega^2} = 1 - \frac{n_e}{n_c} \quad \text{where} \quad \omega_p^2 = \frac{n_e e^2}{\varepsilon_0 m}$$

$$\omega^2 = \frac{n_c e^2}{\varepsilon_0 m}$$

$$N = \frac{kc}{\omega}$$

$$k = \frac{\omega}{c} N$$

From Ian's book, $\Delta\phi = \int (k_p - k_o) dl = \int (N-1) \frac{\omega}{c} dl$

where k_p plasma is the wave vector when plasma is present, $k_p = \begin{cases} k_o & \text{, not in plasma} \\ \frac{\omega}{c}\left(1 - \frac{n_e}{n_c}\right) & \text{in plasma:} \end{cases}$

and k_o is the free space $k_o = \omega/c$

$$\Delta\phi = \int_{\text{not in plasma}} (k_o - k_o) dl + \int_{\text{in plasma}} \frac{\omega}{c}\left[\left(1 - \frac{n_e}{n_c}\right) - 1\right] dl$$

$$\Delta\phi = \int_{\text{in plasma}} (N-1) \frac{\omega}{c} dl \text{ , as above.}$$

From my chosen red laser frequency, $n_c = \frac{(2\pi f_{red})^2 m_e \varepsilon_0}{e^2} = 2.6 \times 10^{27} \, m^{-3} \ggg n_0$

So, I can definitly approximate N as:

$$N = \sqrt{1 - n_e/n_c} \cong 1 - \frac{1}{2} \frac{n_e}{n_c}$$

So

$$\Delta\phi = \int_0^L (N-1) \frac{\omega}{c} dl \cong \int_0^L \left(-\frac{1}{2} \frac{n_e}{n_c}\right) \frac{\omega}{c} dx$$

$$\Delta\phi = -\frac{\omega}{2n_c c}\int_0^L n_e \, dx$$ plugging in my expression for the integral

Note: $\alpha \equiv \frac{L\omega n_o}{\pi n_c c}$ prefactor

$$\Delta\phi = -\frac{\omega}{2n_c c} n_o \left[\frac{2L}{\pi}\sin\left(\frac{\pi t}{t_p}\right)\right]$$

We imediatly run into an issue. The prefactor $-\frac{\omega n_o}{2n_c c}\frac{2L}{\pi} < 1 \approx 0.046$ radians.

This is much to small to measure properly. So, I am changing the 650 nm red laser to a CH_3OH $\lambda = 119\ \mu m$ far infrared laser. this gives:

$\omega = 2\pi f = 2\pi \frac{c}{\lambda} = 1.58 \times 10^{13}$ rad/s, 2.52×10^{12} Hz laser.

$$n_c = \frac{(2\pi f_{CH_3OH})^2 m_e \varepsilon_0}{e^2} = 7.876 \times 10^{22}\ m^{-3}.$$ This still satisfies $n_c \gg n_o$. (by over 100x).

The prefactor is now 8.5314 radians. Now we are cooking.

$$\Delta\phi = -\alpha \sin\left(\frac{\pi t}{t_p}\right) \cong -8.5 \sin\left(\frac{\pi t}{t_p}\right)$$, so $\Delta\phi$ will go up from 0 to 8.5 radians, and back to zero over the t_p plasma liftime.

The detector can only measure intensity, not $\Delta\phi$. The intensity of

$$E_t = E_1 e^{i(\omega t)} + E_2 e^{i(\omega t + \Delta\phi)}$$

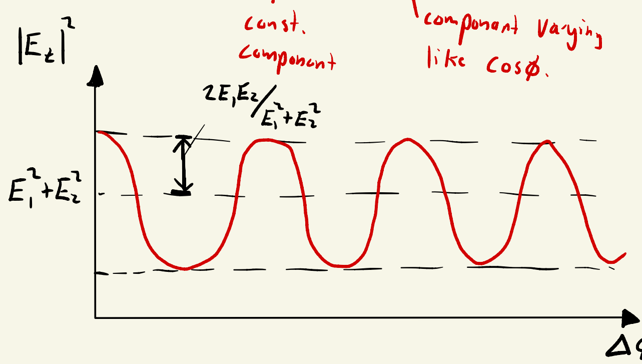
$$E_t = \left[E_1 + E_2 e^{i\Delta\phi}\right] e^{i\omega t}$$

The power detected is proportional to $|E_t|^2$:

$$|E_t|^2 = (E_1^2 + E_2^2)\left(1 + \frac{2E_1E_2}{E_1^2+E_2^2}\cos(\Delta\phi)\right)$$

const. component

component varying like $\cos\phi$.



Varyation in output power based on the phase difference between two signals.

For simplicity, lets take $E_1 = E_2 = 1$

Then

$$|E_t|^2 = 2\left(1 + \cos(\Delta\phi)\right)$$

plugging in $\Delta\phi$ to find $|E_t|^2$ as a function of t:

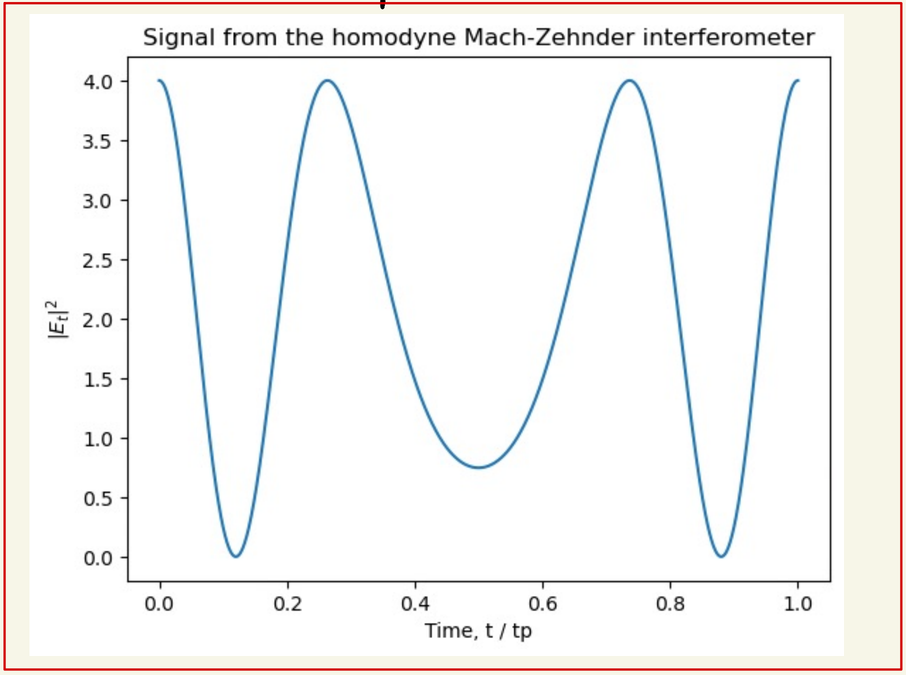
(cos is even)

$$|E_t|^2(t) = 2 + 2\cos\left[-\alpha \sin\left(\frac{\pi t}{t_p}\right)\right] = 2 + 2\cos\left(\alpha \sin\left(\frac{\pi t}{t_p}\right)\right)$$

where again,

$$\alpha \equiv \frac{L\omega n_0}{\pi n_c c} \quad \text{prefactor}$$

I will plot $|E_t|^2$ now as a function of time from $t=0$ to $t=t_p$:



There is phase ambiguity when ever $\frac{d|E_t|^2}{d\phi} = 0$, or when $|E_t|^2$ does not change with ϕ.

$$0 = \sin(\Delta\phi) \qquad \Delta\phi = -\alpha \sin\left(\frac{\pi t}{t_p}\right)$$

which happends when $\Delta\phi = 0, \pi, 2\pi, 3\pi, \dots n\pi \dots$

for my case, $|\Delta\phi| \leq \alpha = 8.53 = 2.71\pi$, so $n = 0, 1, 2, 1, 0$

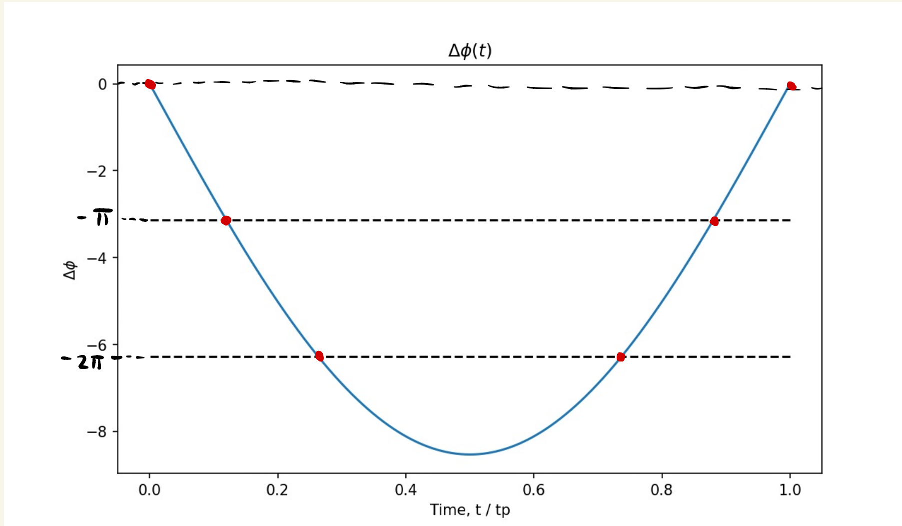
$$\frac{n\pi}{-\alpha} = \sin\left(\frac{\pi t}{t_p}\right), \text{ the ambiguous } t/t_p \text{ are}$$

$$\frac{t_{amb}}{t_p} = \frac{1}{\pi}\arcsin\left(\frac{n\pi}{-\alpha}\right)$$

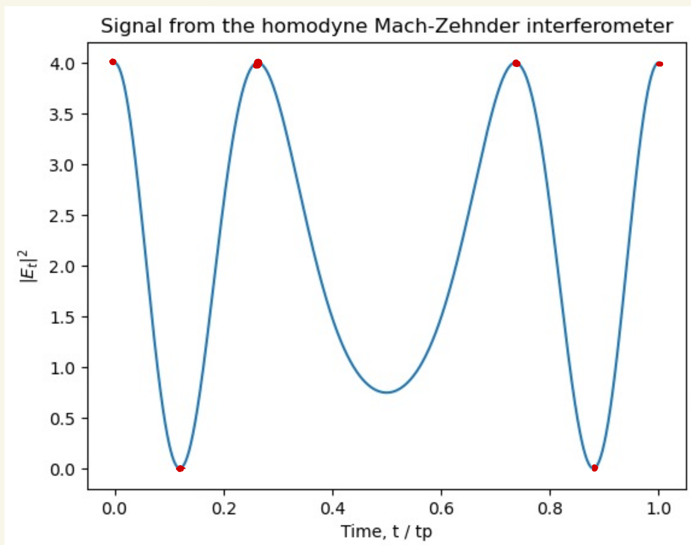
plotting $\Delta\phi(t)$ helps to see when $\Delta\phi = n\pi$:

The red dots are where $\Delta\phi = n\pi$, So at these times $|E_t|^2$ is ambigous. this happens for

$\frac{t}{t_p}$ = 0, 0.1194, 0.263, 0.732, 0.879, 1

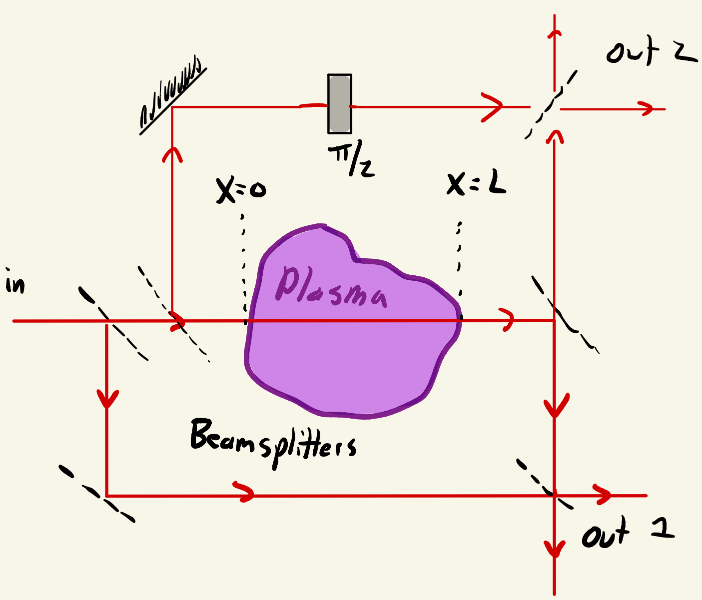


plotting these on the intensity graph shows where the ambigous points are.



$\boxed{1.3}$ I am interpreting this as the following settup:

The wording of the problem makes it sound like we measure out 1: ref leg + plasma leg and out 2: $\pi/2$ leg and plasma leg. Then we "combine this information" which I take to mean out1 + out2.



So, our output now looks like:

$$E_t = E_1 e^{i\omega t} + E_2 e^{i(\omega t + \Delta\phi)} + E_3 e^{i(\omega t + \pi/2)}$$

$$E_t = \left(E_1 + E_2 e^{i\Delta\phi} + E_3 e^{i\pi/2} \right) e^{i\omega t}$$

$$|E_t|^2 = E_1^2 + 2E_1E_2\cos(\Delta\phi) + E_2^2\left[\sin^2(\Delta\phi) + \cos^2(\Delta\phi)\right] + 2E_2E_3\sin(\Delta\phi) + E_3^2$$

$$|E_t|^2 = E_1^2 + E_2^2 + E_3^2 + 2E_1E_2\cos(\Delta\phi) + 2E_2E_3\sin(\Delta\phi)$$

$$|E_t|^2 = \left[E_1^2 + E_2^2 + E_3^2\right]\left(1 + \frac{2E_1E_2}{\left[E_1^2+E_2^2+E_3^2\right]}\cos(\Delta\phi) + \frac{2E_2E_3}{\left[E_1^2+E_2^2+E_3^2\right]}\sin(\Delta\phi) \right)$$

Now we can see where $|E_t|^2$ does not change with phase:

$$\frac{d|E_t|^2}{d\phi} = -2E_1E_2\sin(\Delta\phi) + 2E_2E_3\cos(\Delta\phi)$$

Set this equal to Zero:

$$0 = -2E_1E_2\sin(\Delta\phi) + 2E_2E_3\cos(\Delta\phi)$$

$$0 = -E_1\sin(\Delta\phi) + E_3\cos(\Delta\phi)$$

$$E_1\sin(\Delta\phi) = E_3\cos(\Delta\phi)$$

$$\tan(\Delta\phi) = \frac{E_3}{E_1}$$

So, $\frac{d|E_t|^2}{d\Delta\phi} = 0$ when $\tan(\Delta\phi) = \frac{E_3}{E_1}$, so this is when $|E_t|^2$ looses sensitivity to changes in phase

In my example, let $E_1 = E_2 = E_3 = 1$, and $\tan(\Delta\phi) = 1$ is the condition of ambiguity. This occurs whenever $\Delta\phi = \frac{\pi}{4} + n\pi, n = 0,1,2,3.$

$\boxed{1.4}$ Now consider a triature system where the beam is split into 3 beams, 0, $\pi/3$, $2\pi/3$ radians out of phase:

$$E_t = E_1 e^{i\omega t} + E_2 e^{i(\omega t + \Delta\phi)} + E_3 e^{i(\omega t + \pi/3)} + E_4 e^{i(\omega t + 2\pi/3)}$$

$$E_t = \left(E_1 + E_2 e^{i\Delta\phi} + E_3 e^{i\pi/3} + E_4 e^{i2\pi/3} \right) e^{i\omega t}$$

putting this into wolfram:

$$|E_t|^2 = E_1^2 + 2E_1E_2\cos(\Delta\phi) + E_1E_3 - E_1E_4 + E_2^2 + \sqrt{3}E_2E_3\sin(\Delta\phi)$$
$$+ E_2E_3\cos(\Delta\phi) + \sqrt{3}E_2E_4\sin(\Delta\phi) - E_2E_4\cos(\Delta\phi) + E_3^2 + E_3E_4 + E_4^2$$

$$|E_t|^2 = \sum_{i=1}^{4} E_i^2 + E_1E_3 - E_1E_4 + E_3E_4 + 2E_1E_2\cos(\Delta\phi) + \sqrt{3}E_2E_3\sin(\Delta\phi)$$
$$+ E_2E_3\cos(\Delta\phi) + \sqrt{3}E_2E_4\sin(\Delta\phi) - E_2E_4\cos(\Delta\phi)$$

$$\frac{d|E_t|^2}{d\phi} = -2E_1E_2\sin(\Delta\phi) + \sqrt{3}E_2E_3\cos(\Delta\phi)$$
$$- E_2E_3\sin(\Delta\phi) + \sqrt{3}E_2E_4\cos(\Delta\phi) + E_2E_4\sin(\Delta\phi)$$

set this equal to zero, divid by E_2:

$$0 = -2E_1\sin(\Delta\phi) + \sqrt{3}E_3\cos(\Delta\phi) - E_3\sin(\Delta\phi) + \sqrt{3}E_4\cos(\Delta\phi) + E_4\sin(\Delta\phi)$$

$$0 = \sin(\Delta\phi)\left[E_4 - E_3 - 2E_1\right] + \cos(\Delta\phi)\left[\sqrt{3}E_3 + \sqrt{3}E_4\right]$$

$$\boxed{\tan(\Delta\phi) = \frac{-\sqrt{3}\left[E_3 + E_4\right]}{\left[E_4 - E_3 - 2E_1\right]}} \text{, the phase change is ambigous here.}$$

in my case, let $E_1 = E_2 = E_3 = E_4 = 1$

$$\tan(\Delta\phi) = \frac{-\sqrt{3}\,2}{1-1-2} = \sqrt{3}$$

So anywhere $\tan(\Delta\phi) = \sqrt{3}$, the phase is ambigous.

$\boxed{\text{I should not keep going}}$ by adding more phases, $|E_t|^2(\Delta\phi) = |E_t|^2(\Delta\phi + 2\pi)$, and there are $\Delta\phi$ where $|E_t|^2$ does not change with respect to changes in $\Delta\phi$. So, I have not fixed my problem.

$\boxed{1.5}$ Now consider a temporally heterodyne interferometer, where the probe and reference beams oscillate at ω_1 and ω_2:

$$E_t = E_1 e^{i\omega_1 t} + E_2 e^{i\omega_2 t + \Delta\phi}$$

$$|E_t|^2 = E_1^2 + E_2^2 + 2E_1E_2\cos\left((\omega_2-\omega_1)t + \Delta\phi\right)$$

This is a beat fiequency pattern.

$$|E_t|^2 = E_1^2 + E_2^2 + 2E_1E_2\cos\left(\left(\omega_2-\omega_1 + \frac{\partial\phi}{\partial t}\right)t\right)$$

So, as long as the phase change $\frac{\partial\phi}{\partial t}$ is much slower than $\omega_2-\omega_1$, I should be able to determine the phase change by counting intensity peaks in $|E_t|^2$, where T_m is the measured time seperation between peaks. Then, $(\omega_2-\omega_1) + \frac{\partial\phi}{\partial t} = \frac{2\pi}{T_m}$

and $$\frac{\partial\phi}{\partial t} = \frac{2\pi}{T_m} - (\omega_2-\omega_1)$$

an estimate for the phase change is

$$\partial\phi \approx \frac{\partial\phi}{\partial t}T_m = -(\omega_2-\omega_1)T_m + 2\pi$$

Thus, I can build a time array where $t_{i+1} = t_i + T_{mi}$, $t_0 = 0$.

Then, the corrisponding $\Delta\phi$ array is

$$\Delta\phi_{i+1} = \Delta\phi_i + \partial\phi_i = \Delta\phi_i \left(\omega_2-\omega_1\right)T_{mi} - 2\pi \,, \Delta\phi_0 = 0.$$

Then, once I have my $\Delta\phi$ array, I can back out a line integrated density array:

$$\Delta\phi_i = -\frac{\omega_2}{2n_c c}\left[\int_0^L n_e\, dx\right]_i$$

$$\frac{-2n_c c\,\Delta\phi_i}{\omega_2} = \left[\int_0^L n_e dx\right]_i = \langle n_e L\rangle_i$$

then I can plot the $\langle n_e L\rangle_i$ array versus the t_i time array. This should match the true line integrated density: $\langle n_e L\rangle = n_0\left[\frac{2L}{\pi}\sin\left(\frac{\pi t}{t_p}\right)\right]$

In my case, the plasma changes from 0 to its peak and back to 0 over $t_p = 100\times10^{-3}$ seconds $= 0.1$ seconds. I will pick:

plasma leg: $\omega_2 = 2\pi\left(2.52\times10^{12}\right)$

free space leg: $\omega_1 = 2\pi\left(2.52\times10^{12} - 1\times10^{4}\right)$

So that $\omega_2 - \omega_1 = 2\pi\left(1\times10^4\right) \gg \frac{\partial\phi}{\partial t} \approx \frac{\alpha}{t_p} = \frac{8.5\text{ rad}}{0.1\text{ secods}} = 85\text{ rad/s}$

First, I will generate the plot of

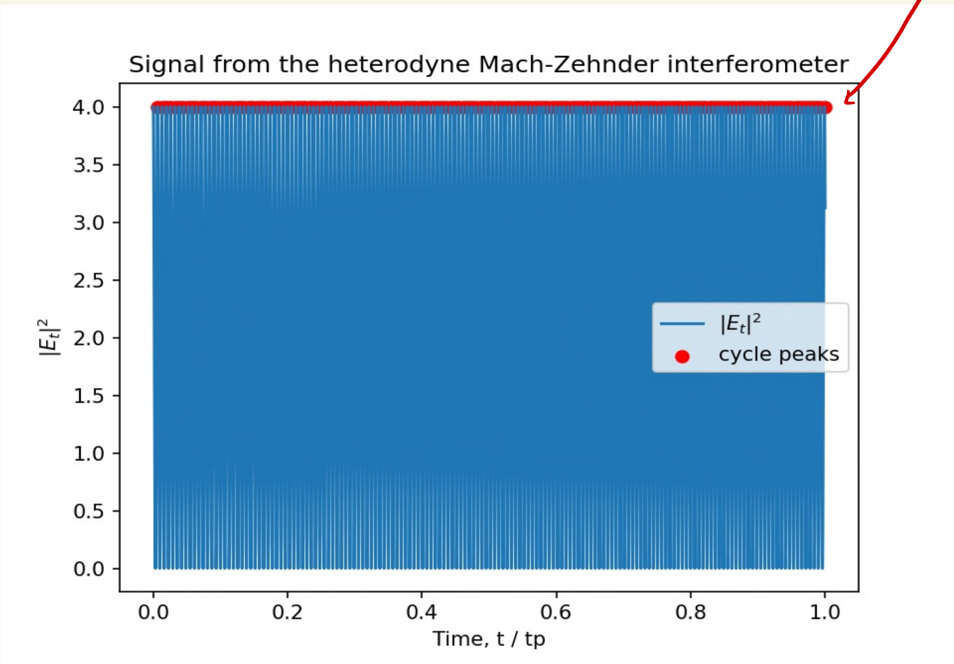
$$|E_t|^2 = E_1^2 + E_2^2 + 2E_1E_2\cos\left((\omega_2-\omega_1)t + \Delta\phi(t)\right)$$ versus time from

0 to t_p to get a feel for whats going on. let $E_1 = E_2 = 1$, and recall

$$\Delta\phi(t) = -\frac{\omega}{2n_c c} n_0\left[\frac{2L}{\pi}\sin\left(\frac{\pi t}{t_p}\right)\right] = -\alpha\sin\left(\frac{\pi t}{t_p}\right).$$

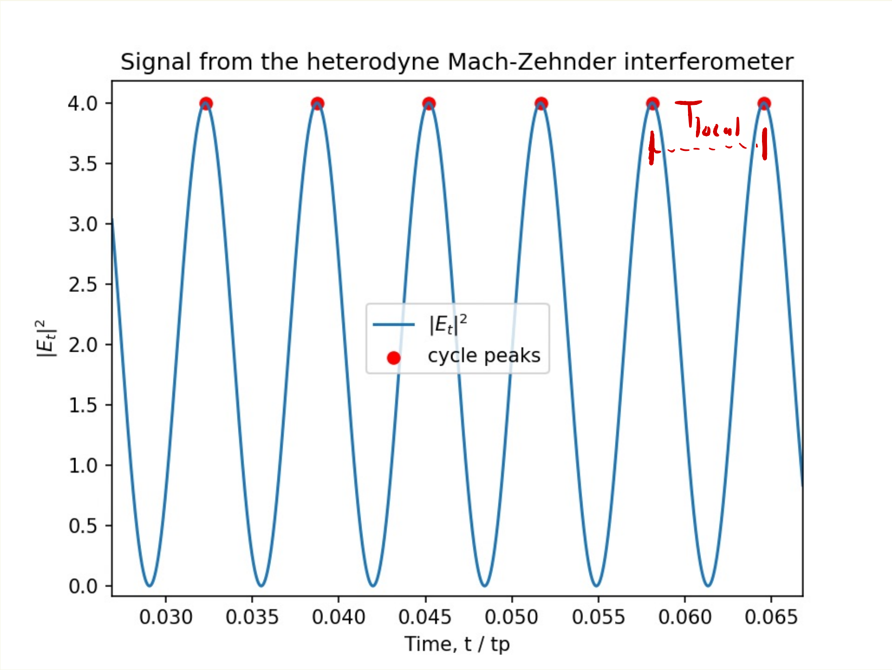
Intensity plot $|E_t|^2$:

red dots indicate cycle peaks. The time delta between them is the local period.



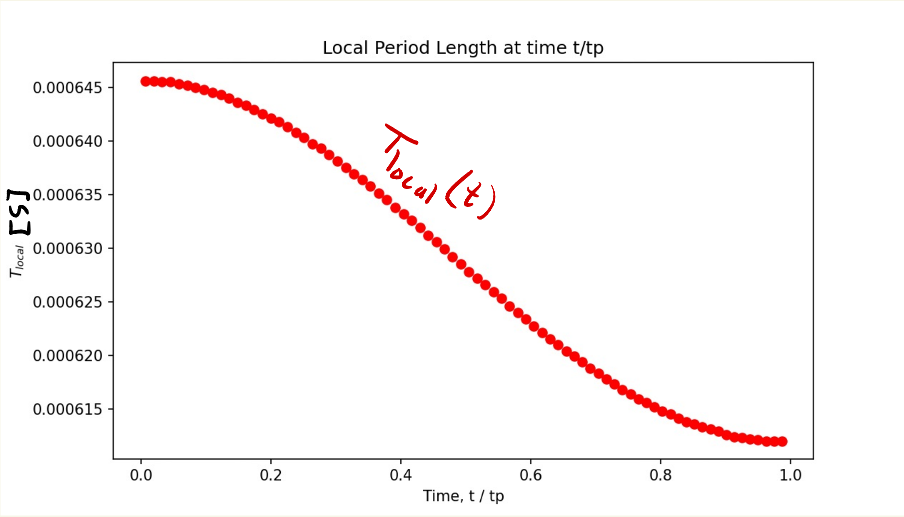
Note: if I also measured the difference between troughs and zeros, I'd get better local T_m resolution

Same plot, zoomed in:



T_{local}, the local period, changes.

plot of T_{local} changing from $t/t_p=0$ to $t/t_p=1$.

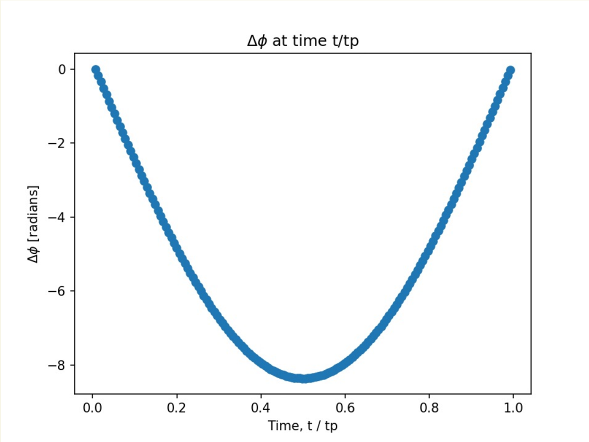


Now, I know

$$\Delta\phi_{i+1} = \Delta\phi_i + (\omega_2-\omega_1)T_{mi} - 2\pi \ , \Delta\phi_0 = 0 .$$

($\Delta\phi$ is the accumulated $\Delta\phi$ over $x=0,L$ at time t, not a difference in ϕ over time, $\partial\phi$ in time ∂t)

building this array from my known T_{mi} array (plotted befor) I have.

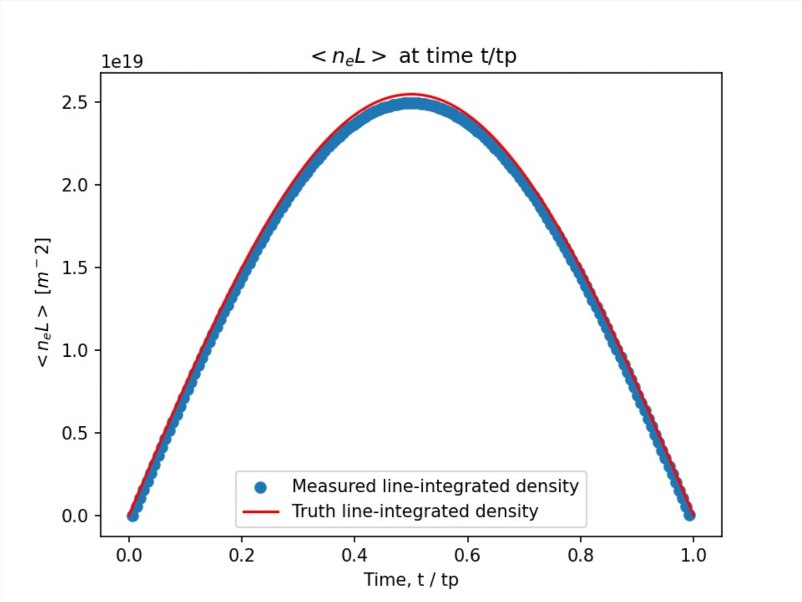


Whew, this thing is looking sinusoidal! Now lets get

$$\frac{-2n_c c\,\Delta\phi_i}{\omega_2} = \left[\int_0^L n_e\,dx\right]_i = \langle n_e L\rangle_i$$

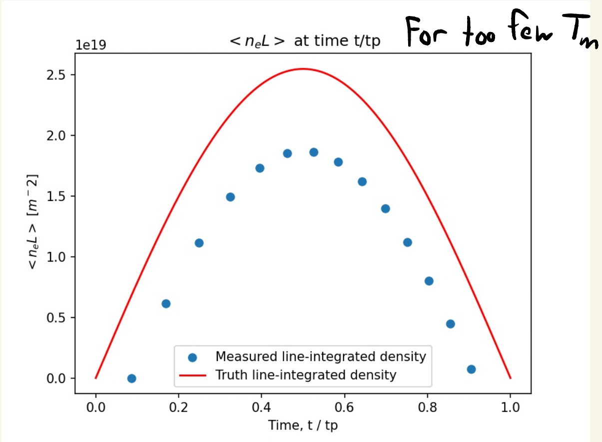
This can be compared to the true value: $\langle n_e L\rangle = n_0\left[\frac{2L}{\pi}\sin\left(\frac{\pi t}{t_p}\right)\right]$;

Yess this looks great! The measured line-integrated density comes extreemly close to the truth profile.



Playing with the beat frequency $\omega_2 - \omega_1$:

When $\omega_2 - \omega_1$ gets too low, approaching $\frac{\partial \phi}{\partial t} \simeq \frac{\alpha}{t_p} = 85$ rad/s, the local measurments of T_m, the local period, are too crude. For example, even at $\omega_2 - \omega_1 = 1{,}000$, information is lost:

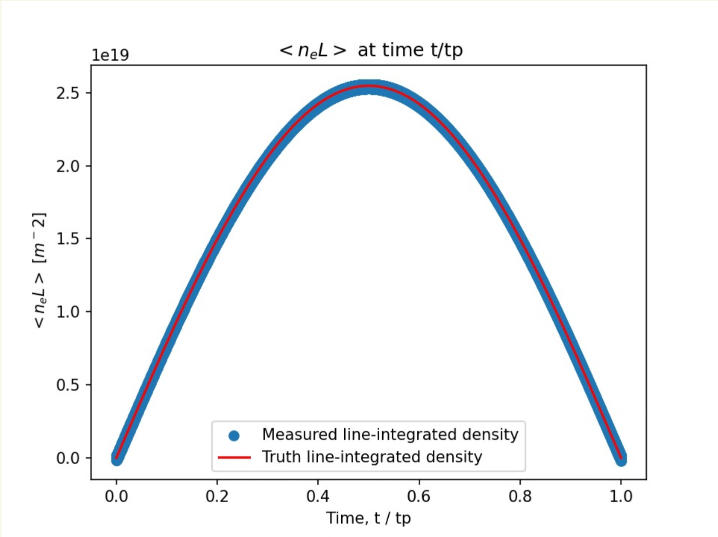


For too few T_m measurments

One could imagine also measuring differences between pulse troughs and zeros to get more resolution, which would give you more measurments of T_m for the same data. (I am only using distances between peaks, see cycle peaks plot)

When $\omega_2 - \omega_1 \simeq \frac{\alpha}{t_p}$, all things break down: there are not enough peaks and troughs to measure T_m at all, let alone locally.

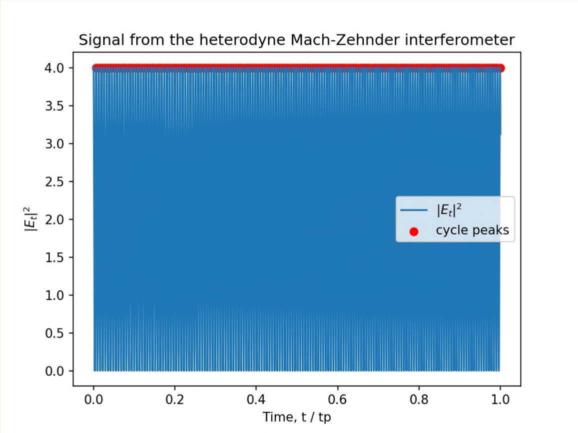
At high frequency, $\omega_2 - \omega_1 = 1 \times 10^7$, for example, I even more closely approximate:



(note before the top was slightly not approximated correctly)

$\boxed{1.6}$

The heterodyne system has many performance advantages. In addition, it looked like as long as $\omega_2 - \omega_1$ kept going up, your data got nicer. Wahoo! Not so fast! Notice this figure:



Finding the peaks meant the blue curve, oscillating at roughly $\omega_2 - \omega_1$, needs to be digitized, or at the very least the red dot peaks. The larger the $\omega_2 - \omega_1$, the more beefy and robust digitizer that is required. Using a heterodyne system at all is pricy because you need good frequency stability between ω_1 and ω_2. One channel is required. The $\pi/2$ and $0, \pi/3, 2\pi/3$ homodyne systems require extra channels which adds to cost. A simple homodyne might work if $\Delta\phi$ is expected to remain between 0 and π for frequencies used and densities expected.

Problem 2

2.1 For my phanton density profile, I will use a similar theme to what I had befor:

$$n_e(x,y,z) = n_0 \sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{\pi y}{W}\right)\sin\left(\frac{\pi z}{H}\right)$$

So I can debug easier, $L \neq W \neq H$: $L = 0.1$ m

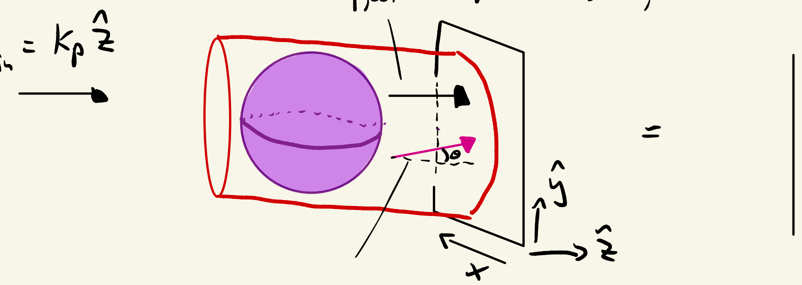
$W = 0.2$ m

$H = 0.5$ m

This function goes to zero at the edges.

2.2 what I want to plot is the intensity as a function of x and y for the situation shown below: $\vec{k}_{p,out} \approx (k_{pin} + \Delta\phi)\hat{z}$, when $\Delta\phi = f(x,y)$ due to plasma variation

$\vec{k}_{p,in} = k_p \hat{z}$



$\vec{k}_{ref} = k_p \sin(\theta)\hat{y} + k_p\cos(\theta)\hat{z}$, when the referen $\vec{k}_{ref}$ is tilted from the z-axis by angle θ in the y-z plane.

The intensity $I(x,y) \propto |E_t|^2(x,y)$.

The total $\vec{E}$ leaving the plasma, incident on the detector is given by

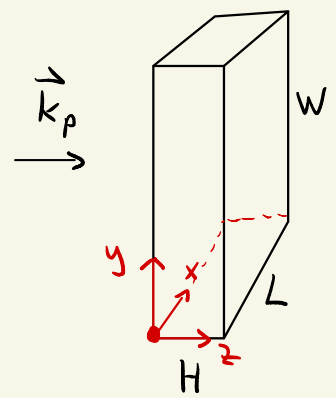
$$E_t = E_1 e^{i(\vec{k}_{p,out}\cdot\vec{r} - \omega t)} + E_2 e^{i(\vec{k}_{ref}\cdot\vec{r} - \omega t)}$$

$$E_t = \left[E_1 e^{i([k_p + \Delta\phi(x,y)]z)} + E_2 e^{i(k_p\cos(\theta)z + k_p\sin(\theta)y)}\right] e^{-i\omega t}$$

The intensity is perportional to the square magnitude of E_t:

$$|E_t|^2 = E_1^2 + E_2^2 + 2E_1E_2\cos\left[-k_p y\sin(\theta) - k_p z\cos(\theta) + z(k_p + \Delta\phi_p(x,y))\right]$$

Thus, I need to determine $\Delta\phi(x,y)$. Let the plasma itself be a rectangle of dimensions $L \times W \times H$, as follows:



$$n_e(x,y,z) = n_o \sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{\pi y}{W}\right)\sin\left(\frac{\pi z}{H}\right)$$

So, the line-integrated density as a function of x,y at $z = H$ is

$$\langle n_e H\rangle(x,y,z=H) = n_o \int_0^H \sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{\pi y}{W}\right)\sin\left(\frac{\pi z}{H}\right) dz$$

$$\langle n_e H\rangle(x,y,z=H) = \frac{2Hn_o}{\pi}\sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{\pi y}{W}\right)$$

again, $N \simeq 1 - \frac{1}{2} n_e/n_c$ when $n_e \ll n_c$.

$$\Delta\phi = \frac{\omega}{c}\int_0^H \left[\left(1-n_e/n_c\right)^{1/2} - 1\right] dz \simeq \frac{-\omega}{2cn_c}\int_0^H n_e(x,y)\,dz = \frac{-\omega}{2cn_c}\langle n_e H\rangle(x,y)$$

plugging in $\langle n_e H\rangle$ gives

$$\Delta\phi(x,y) = \frac{-\omega}{2cn_c}\frac{2Hn_o}{\pi}\sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{\pi y}{W}\right) \; *$$

and plugging this into:

$$|E_t|^2 = E_1^2 + E_2^2 + 2E_1E_2\cos\left[-k_p y\sin(\theta) - k_p z\cos(\theta) + z\left(k_p + \Delta\phi_p(x,y)\right)\right] \; **$$

I now know $|E_t|^2(x,y)$ as a function of θ, the probe offset angle, and of probe frequency $\omega = k_p c$. I will choose $\omega = 2\pi\left(2.52\times10^{12}\right)$, so

$$k_p = \frac{\omega}{c} = 52{,}778.7 \text{ m}^{-1}$$

Then I will write code that makes an x by y mesh from $0 \leq x \leq L$ and $0 \leq y \leq W$, calculate $\Delta\phi(x,y)$ using equation *, and then use equation ** to get $|E_t|^2(x,y)$, at $z = H$ (assume the camera is somehow right up against the plasma at location $z = H$). Let $E_1 = E_2 = 1$ for simplicity.

$$|E_t|^2 = 2 + 2\cos\left[-k_p y \sin(\theta) - k_p H\cos(\theta) + \underset{z=H}{H}\left(k_p + \Delta\phi_p(x,y)\right)\right]$$

Here is my code, where W_CH is the angular frequency of the $f = 2.52 \times 10^{12}$ Hz laser, and c = light speed. nc_CH is the critical density $\frac{(2\pi f_{CH})^2 m_e \varepsilon_0}{e^2}$

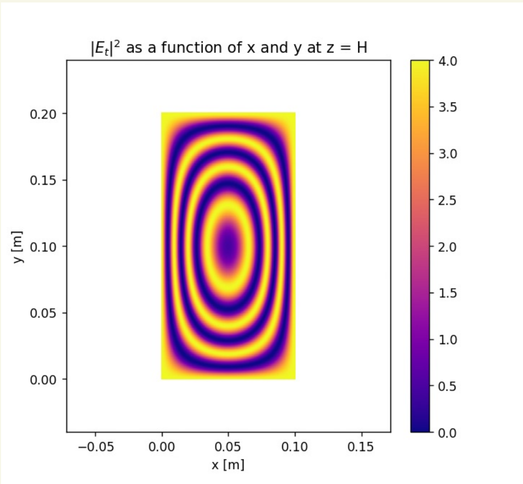
```
In [26]:
wp = W_CH
kp = wp / c
L = 0.1
W = 0.2
H = 0.5
xarray = np.linspace(0, L, 1000)
yarray = np.linspace(0, W, 1000)
Xmesh, Ymesh = np.meshgrid(xarray, yarray)
print(kp)

52778.75658030852

In [27]:
def get_Esq(x, y, z, theta):
    delphi = -W_CH*H*n0*np.sin(pi*x/L)*np.sin(pi*y/W)/(c*nc_CH*pi)
    #return 2 + 2*np.cos( z*(kp*(1 - np.cos(theta)) + delphi) )
    return 2 + 2*np.cos(-kp*y*np.sin(theta) -zkp*np.cos(theta) + z*(kp+delphi))
def get_Esq_no_plasma(x, y, z, theta):
    delphi = 0*x
    #return 2 + 2*np.cos( z*(kp*(1 - np.cos(theta)) + delphi) )
    return 2 + 2*np.cos(-kp*y*np.sin(theta) -zkp*np.cos(theta) + z*(kp+delphi))

In [28]:
theta = 1*(pi/180)
Esq_xy = get_Esq(Xmesh, Ymesh, z=H, theta=theta)
Esq_xy_no_plasma = get_Esq_no_plasma(Xmesh, Ymesh, z=H, theta=theta)

In [29]:
plt.pcolor(Xmesh, Ymesh, Esq_xy)
plt.axis('equal')
plt.colorbar()
plt.xlabel('x [m]')
plt.ylabel('y [m]')
plt.title(r'$|E_t|^2$ as a function of x and y at z = H')
```

2.3

First, I will show the homodyne result and the result if no plasma is present for $\theta = 1°$.

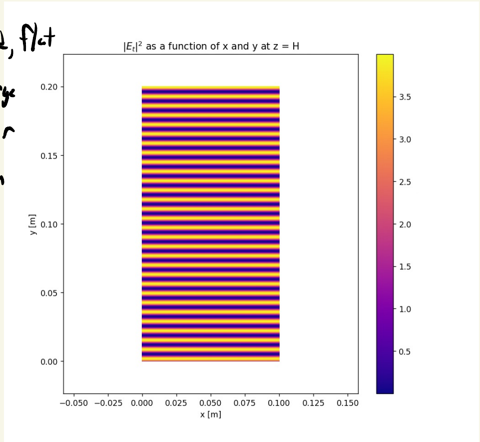
homodye result:

· closed fringes



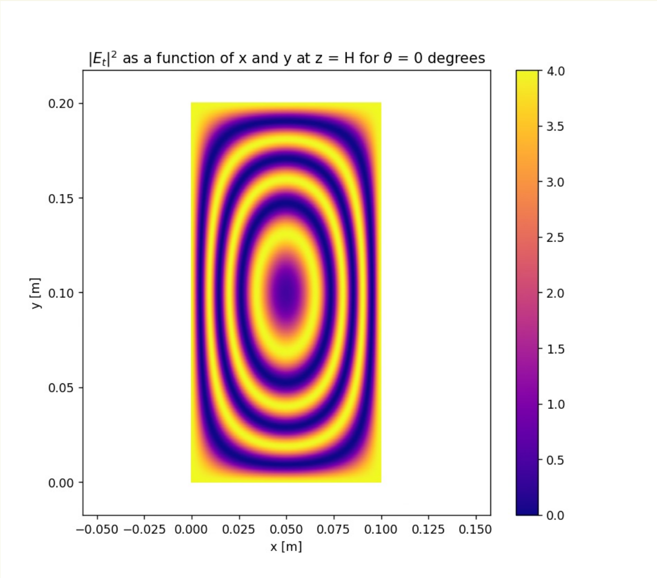
heterodye, $\theta = 1°$, no plasma

As expected, flat striped fringe pattern for no plasma case.



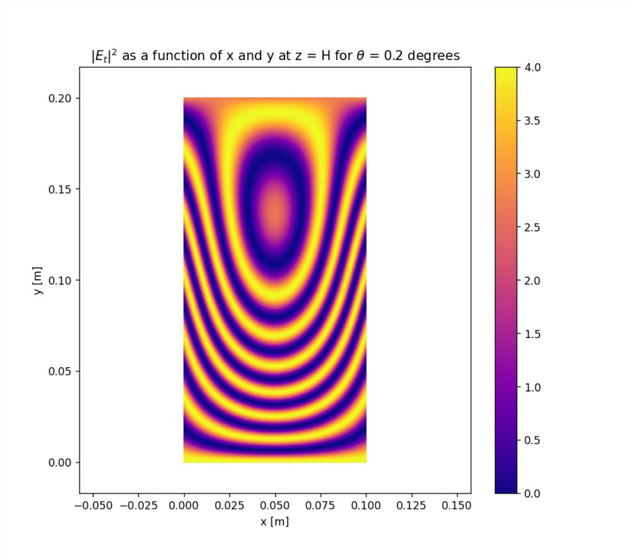
Heterodyne: Now I will start increasing the reference angle from 0° to 1° in steps of 0.2°

0°:



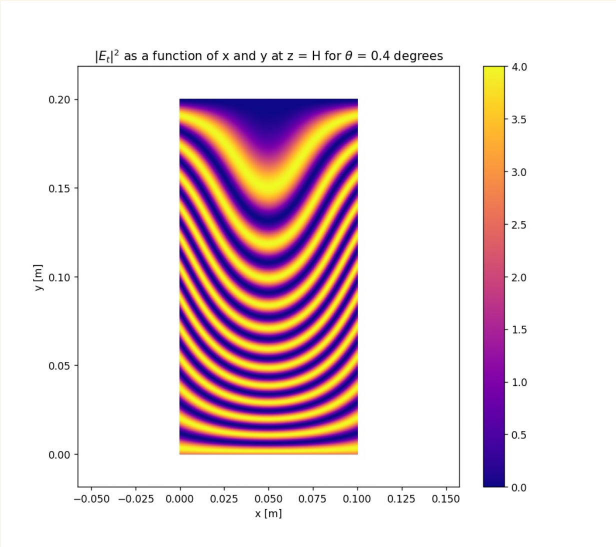
This is equivalent to the homodyne case.

$\theta = 0.2°$:



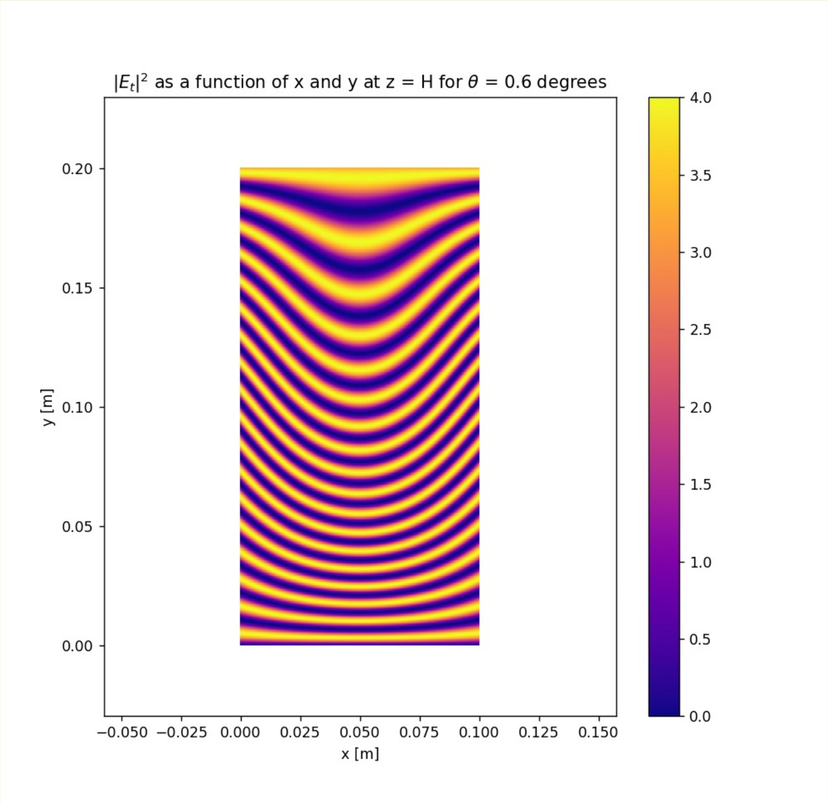
The bottom half looks to have unambiguous phase, but the top still has closed interference fringes

$\theta = 0.4°$:

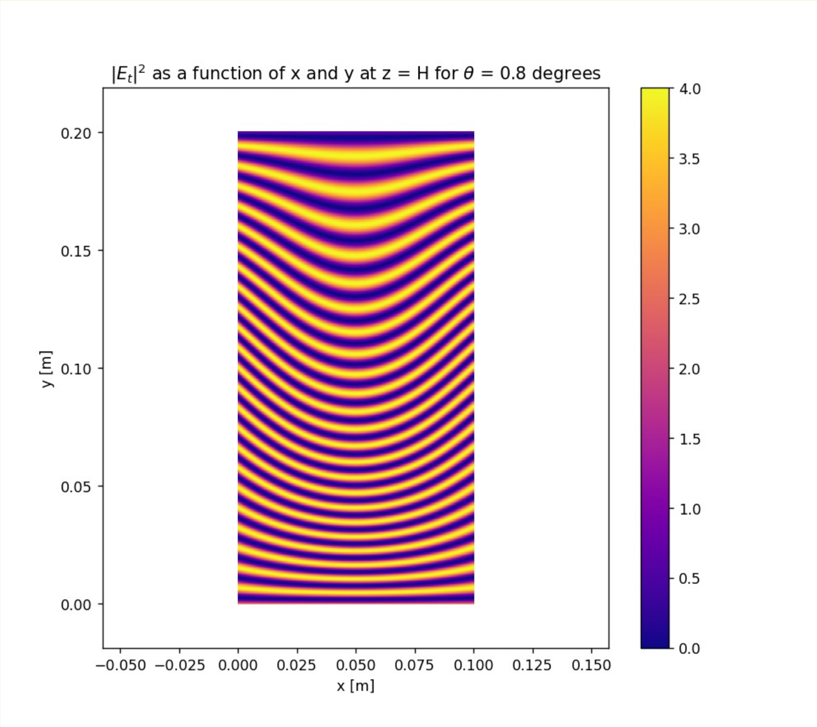


looks like there are finally no closed interference fringes.

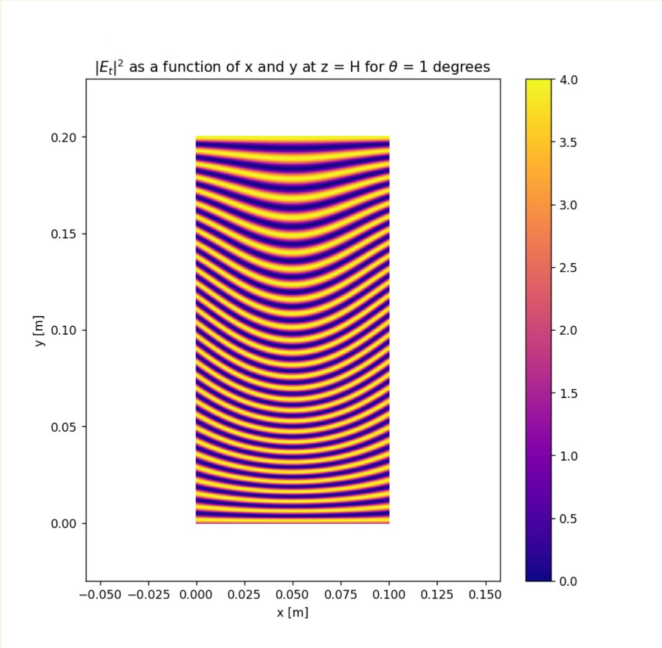
$\theta = 0.6°$



$\theta = 0.8°$:



1°



$\boxed{2.4}$

The angle required to remove ambiguity can be determined by looking at the expressions for intensity and phase shift due to plasma:

$$|E_t|^2 = 2 + 2\cos\left[\underbrace{-k_p y \sin(\theta) - k_p H\cos(\theta) + H\left(k_p + \Delta\phi_p(x,y)\right)}_{\equiv \psi,\ \text{total phase}}\right]$$

$$\Delta\phi(x,y) = \frac{-\omega}{2cn_c}\frac{2Hn_0}{\pi}\sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{\pi y}{W}\right) = \frac{-\omega}{2cn_c}\langle n_e H\rangle(x,y)$$

As θ increases from 0, there is a critical point where the closed fringe patterns vanish. θ increasing corresponds to the $k_p y \sin(\theta)$ term growing. The closed fringes vanish when the full phase term ψ is monotomically decreasing in the y direction:

$$\frac{\partial}{\partial y}\Big[\overbrace{-k_p y\sin(\theta) - k_p H\cos(\theta) + H\left(k_p + \Delta\phi_p(x,y)\right)}^{\psi}\Big] < 0 \quad \text{at all } (x,y)$$

or

$$-k_p\sin(\theta) + H\frac{\partial\Delta\phi_p}{\partial y} < 0 \quad \text{at all } (x,y)$$

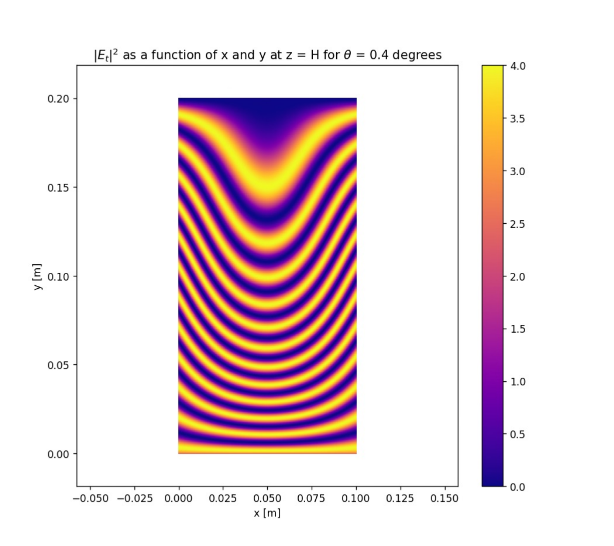
or $$\boxed{\frac{\partial(\Delta\phi)}{\partial y} < k_p\sin(\theta) \quad \text{at all } (x,y)}$$

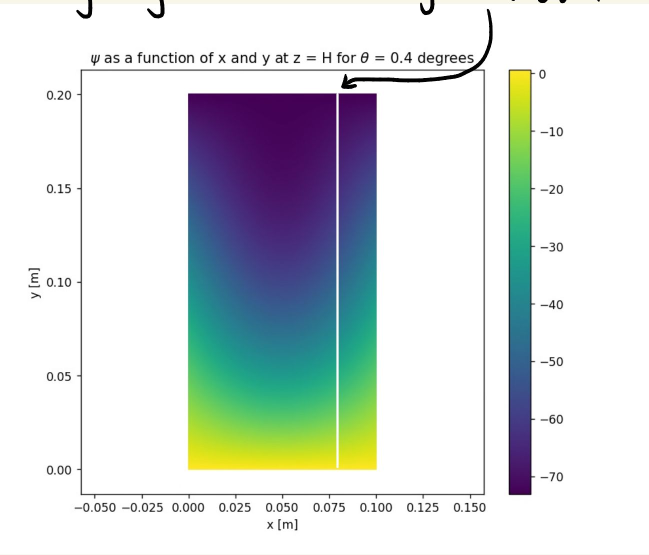
Thus, increasing θ helps.

I will show some visual examples of ψ when there are and aren't fringes, and one can easily see when fringes vanishing correspond to $\frac{\partial \psi}{\partial y} < 0$ at all x, y:

For $\theta = 0.4^0$, there are no closed fringes.

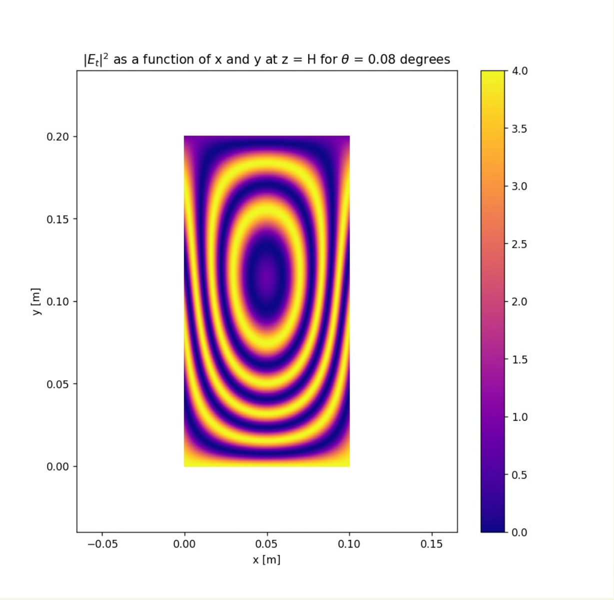
The total phase ψ decreases monotomically along any vertical line you draw.

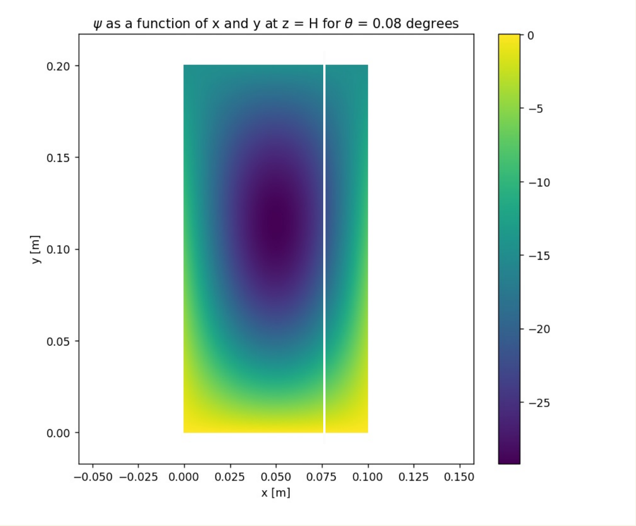




For $\theta = 0.08^0$, there are closed fringes

Now note that the condition $\frac{\partial \psi}{\partial y} < 0$ is not satisfied everywhere (at the top).



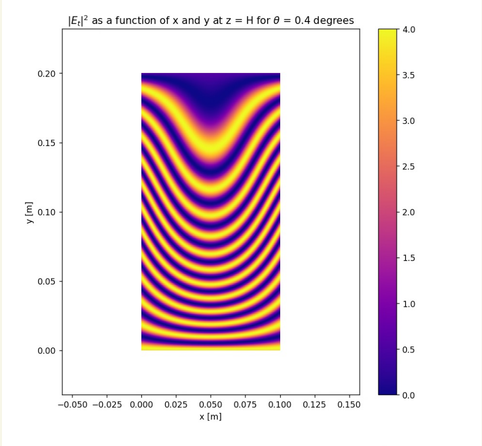


2.5

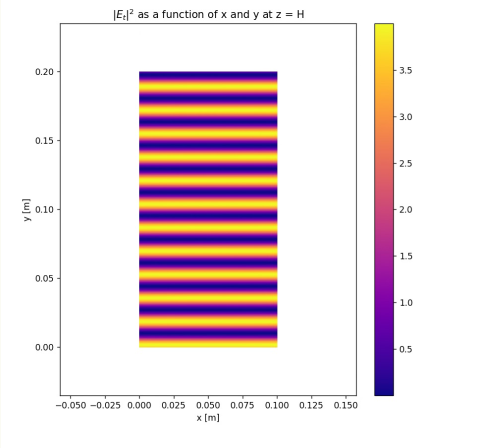
In order to infer the line-integrated electron density $\langle n_e L \rangle(x,y)$ from the the interferogram, I would plot the interferogram both when there is plasma and when there isn't, and at any point x,y, I identify on the no-plasma plot, I would count how many fringes the line has moved on the corrisponding line in the with-plasma plot. An example is in order:

For the $\theta = 0.4°$ case:

with plasma

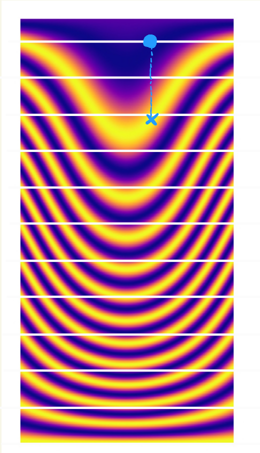


no plasma

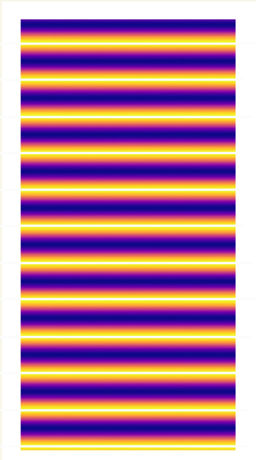


Lets put them side-by-side now:

plasma



no plasma



I have added white lines where the no-plasma fringes would be. Look at the blue dot. Its fringe has moved forward about two fringes to the blue x. Call this F=2. The line integrated density at the blue dot (an x,y point) is

$$F = 4.5\times10^{-16}\,\lambda\,\langle n_e L\rangle$$

$$\langle n_e L\rangle(x,y) = \frac{2.22\times10^{15}}{\lambda}F \quad [m^{-2}]$$

where my $\lambda = \frac{2\pi}{k_p} = 0.000119$ m. Thus, if I have a map of $F(x,y)$, I can quickly use to get my line-integrated density x-y map.