Homework 3

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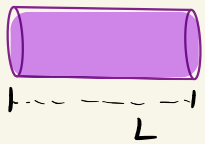
11/14/2023

Problem 1

¡Please Read!

Due to my bathroom ceiling burning (ask Jack for pics), I was given an extension until 11:59 on Monday, 11/20/2023 !



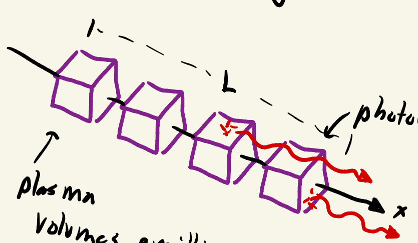
n, T constant

$$j(E) = n^2 T^{-1/2} \exp\left(-E/T\right)$$

1.1 Normally, $j(E)$ has units of power per unit frequency per unit volume. The 1-D case we are looking at, however, makes $j(E)$ have units power per unit frequency per unit length. (Assume $j(E) \equiv \iint_{z\,y} j_v(E)\,dz\,dy$, where $j_v(E)$ is the per volume version). So, integrating $j(E)$ over length gives units of power per unit frequency, which for light is the same as power per unit photon energy because $E = h\nu$. Power per unit energy is the units of the emission $I(E)$. So:

$$I(E) = \int_0^L j(E)\,dx$$

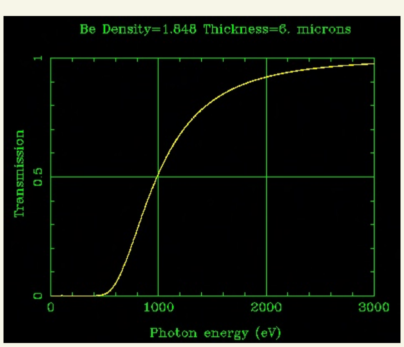
Basically, by neglecting absorption, any photon born along L will follow the isotropic local emissivity $j(E)$ and pass through the other radiating volumes to the detector.

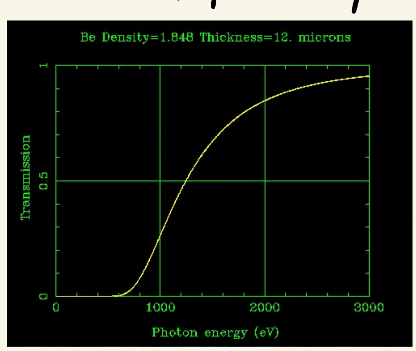


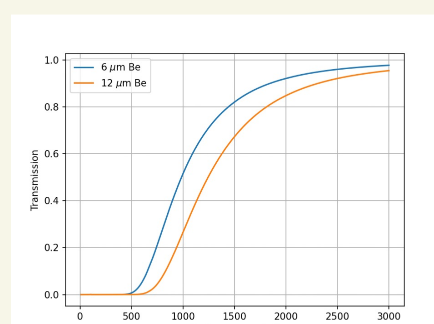
1.2 Grabbing filter tables for:

- 6 μm Be
- 12 μm Be

for photon energies from 10 to 3000 eV with 1,000 linearly spaced data points.







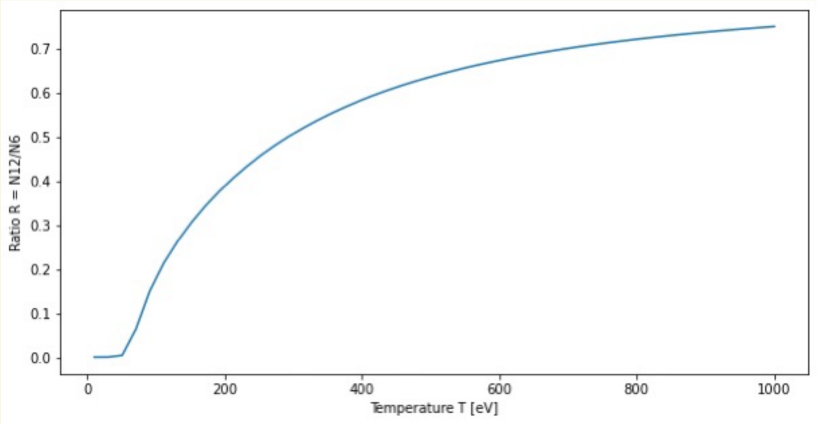
I can also calculate $j(E)$ and $I(E) = \int_0^L j(E)\,dx = j(E)L$ because n, T const.

Let $n = 1\times10^{22}\ m^{-3}$ for now, so I can write $J(E,T)$. Let $L = 1$ m for now.

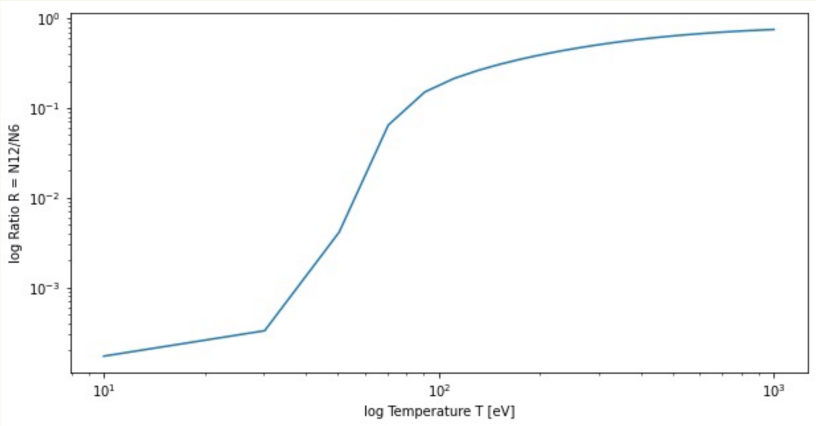
$\boxed{1.3}$ $$N_i(T) = \int j(E,T)\,W_i(E)\,dE$$

I want to calculate this for both filters, and over the temp range $T = 10 - 1000$ eV. I will write a python function which reads in T and exicutes the above integral. I do this for $n = 1\times10^{22}$. This shouldnt matter as it cancels.

Here is my ratio plot.



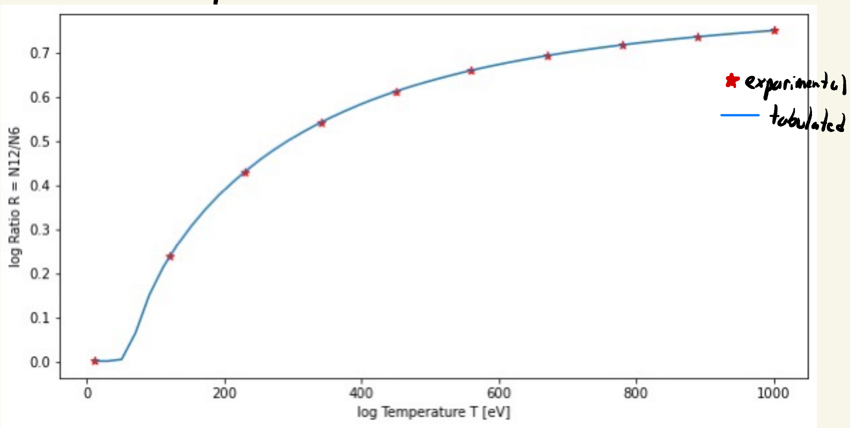
The paper points out that when the ratio is relativly flat, small changes in the measured ratio R gives rise to huge changes in the predicted temperature T. So, my plot is flat below ~30 eV, and is again flat around > 500 eV. On a log-log plot, a clear bend-over happens around 100 eV, so I'd expect the best readily distiguishable temp range to be between 30 eV and ~100 eV. Its not so bad up to 500 eV.



$\boxed{1.4}$ For this step, I will look at the total emission through the two filters W_1 and W_2 for 10 temperature evenly spaced between 10 eV and 1,000 eV and plot thier ratio against the plot I made in 1.3.

$$I_6 = \int J(E,T)\, W_6(E)\, dE \qquad T = 1 \text{ to } 1000 \text{ in 10 steps.}$$
$$I_{12} = \int J(E,T)\, W_{12}(E)\, dE$$

$R_{exp} = \frac{I_{12}}{I_6}$. Plotting as red * versus the solid blue predicted curve from the previous step.

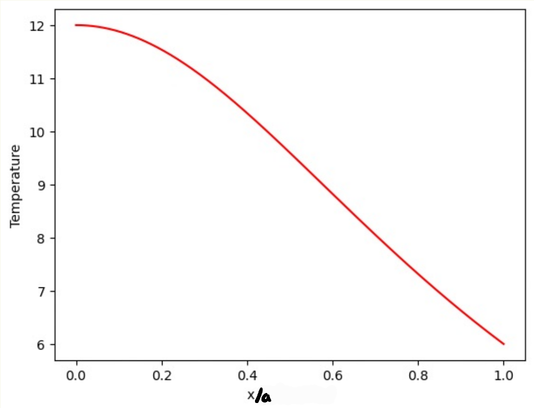


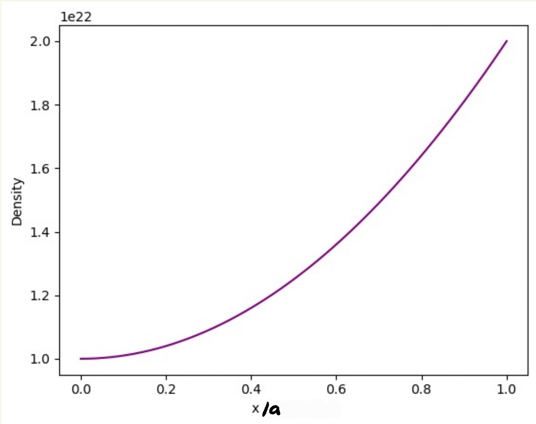
Question 2

$\boxed{2.1}$ generate $n = n_0\left(1 + \frac{x^2}{a^2}\right)$ and $T = \frac{T_0}{1 + x^2/a^2}$, $n(x>a) = 0$, $T(x>a) = 0$

Approximates a chord through the center of a spherically symmetric isobaric $p = nT = p_0$ plasma.

Here is a plot of them: $T_0 = 12$ (keV), $n_0 = 1 \times 10^{22}\ m^{-3}$



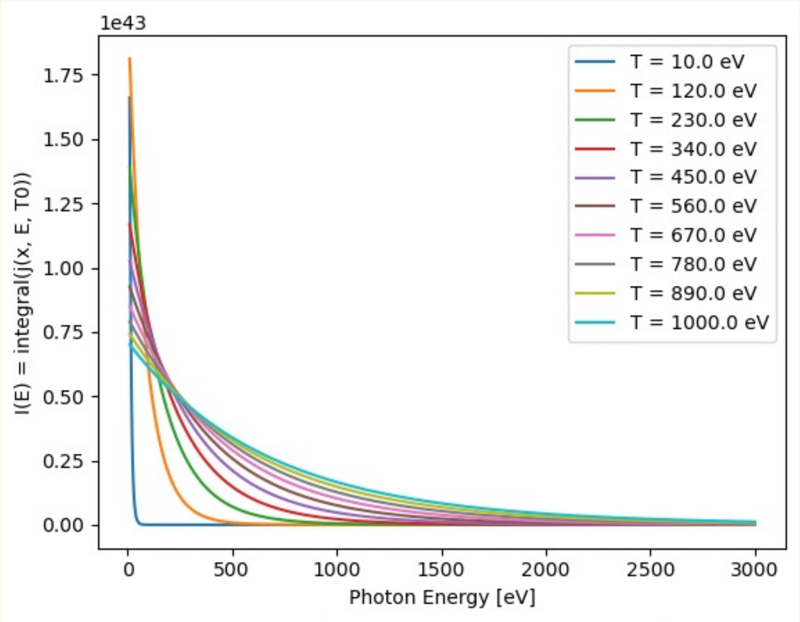


$\boxed{2.2}$ $I(E) = \int j(x, E)\,dx$ where

$$j(x, E) = n^2_{(x)} T_{(x)}^{-1/2} \exp\left(-E/T_{(x)}\right)$$

I will calulate $I(E)$ for the density and temp profiles given in 2.1, but where I vary the T_0 from 10 eV too 1000 eV in 10 evenly spaced grid points:

Here is my plot of $I(E)$ for 10 different temperatures, shown in the figure label. We can see that increasing the temperature makes $I(E)$ decrease more slowly with energy E. This makes sense.



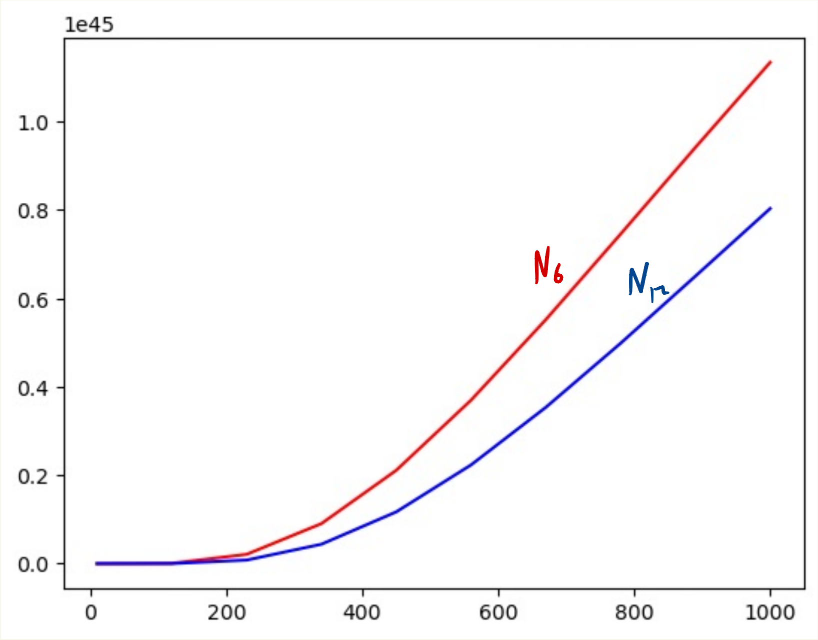
$\boxed{2.3}$ Now I take the $I(E)$ I plotted above, and perform the integral over energy times $W_i(E)$ to get the total filtered emission:

$$N_6 = \int I(E)\, W_6(E)\, dE$$

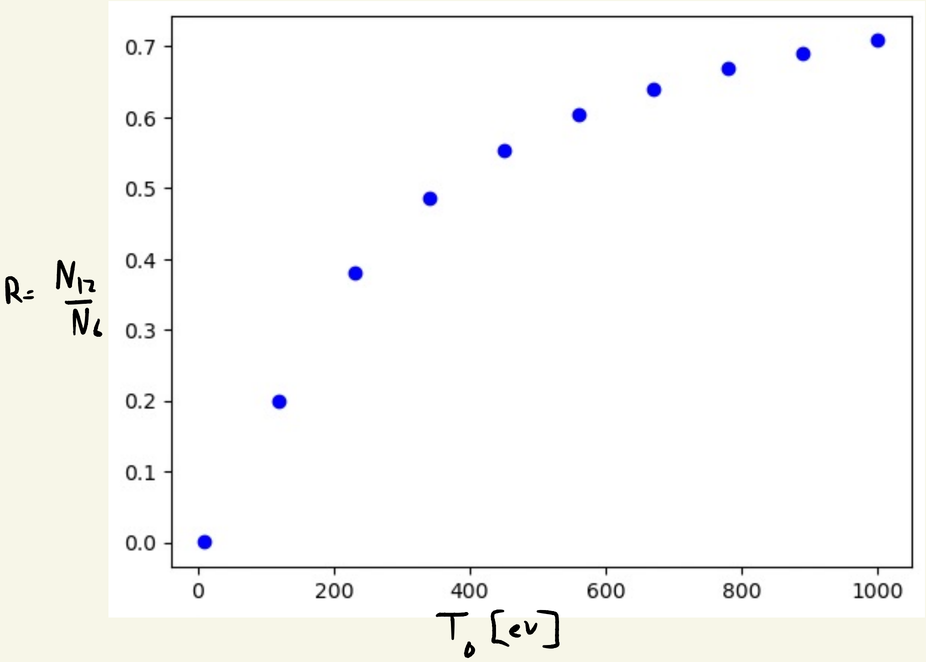
$$N_{12} = \int I(E)\, W_{12}(E)\, dE$$

I can do this at the ten temperatures I used for T_0 (in the legend of my previous figure).

The result of doing this is the following $N_i(T_0)$ plot for $i = 6$ um, 12 um:



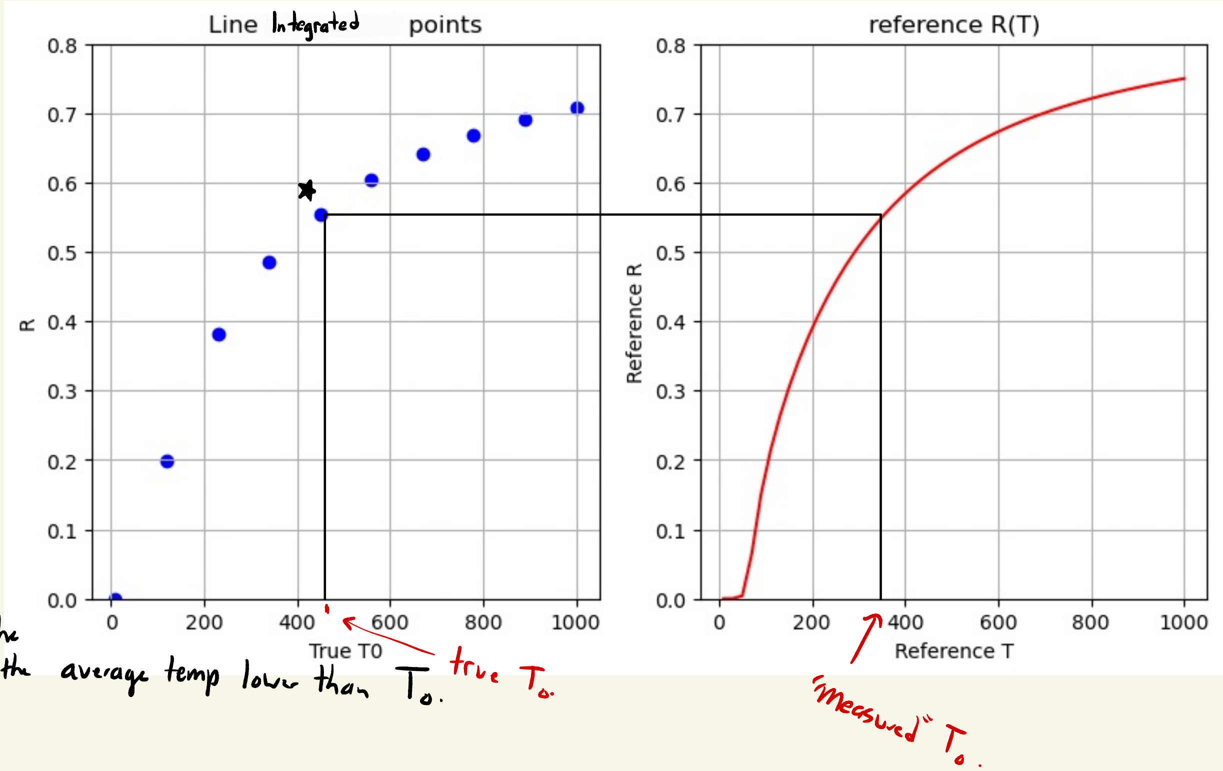
Thier ratio $R = N_{12}/N_6$ is shown below plotted over T_0:



I can use this plot side by side with my old plot of $R(T)$ to see if I can estimate T_0 from only knowing the measured R:

Lets for example look at the blue point labeled "★". I can draw a horizontal line to the red curve and extract the T:

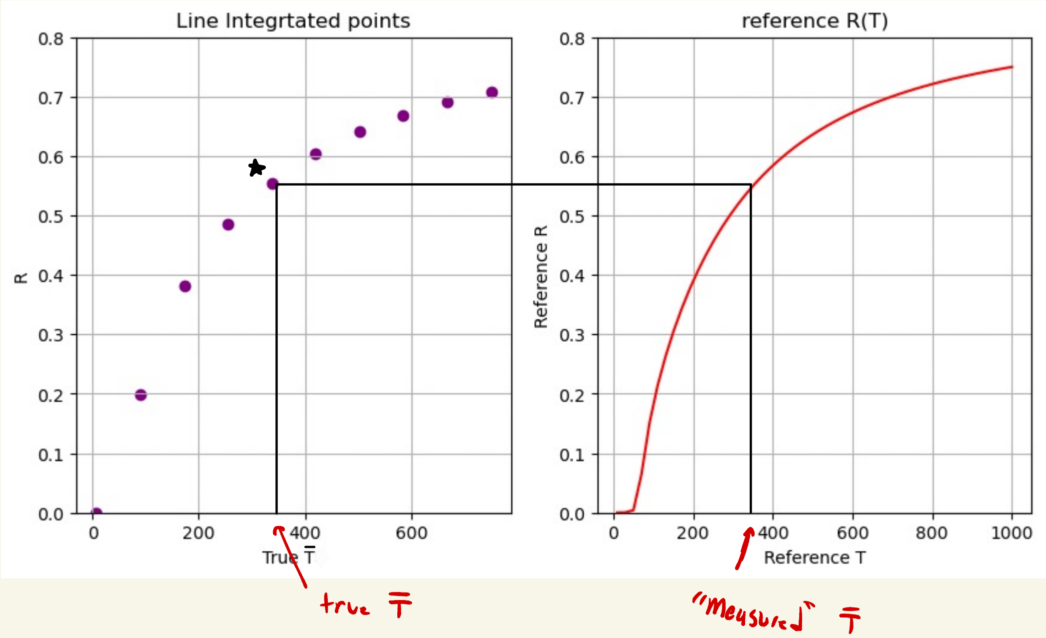
I read about 350 eV. but, T_0 that the ★ point actually corisponds to is 450 ish. So, I am underestimating the peak T_0. This makes sense because the line-integration makes the average temp lower than T_0.



$\boxed{2.4}$

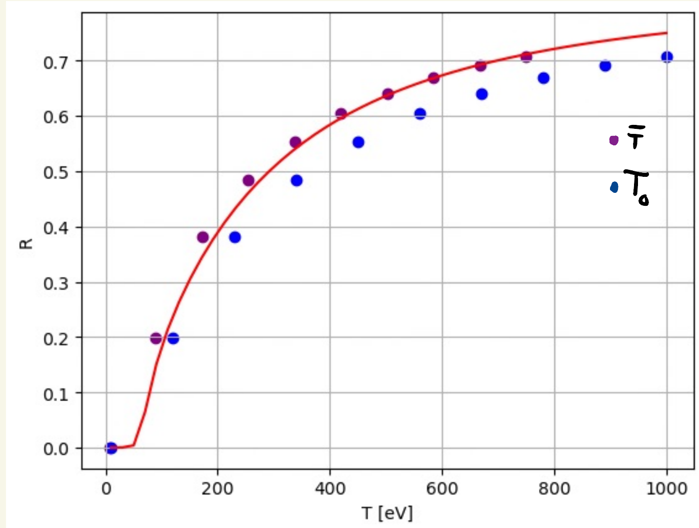
If I re-plot $R(T)$ over $\bar{T} = \dfrac{\int n(x)\,T(x)\,dx}{\int n(x)\,dx}$ I am now plotting over the density-weighted average temperature:

With this change, I have a prediction that is much closer to the actual $\bar{T}$ value. The same star point ★ has a true $\bar{T}$ of ~360, and the "measured" $\bar{T}$ is very nearly the same.



An easy way to see the improvment is

The horizontal displacment between the red curve and the $\bar{T}$ purple points is much smaller than the same displacment for the blue T_0 points.

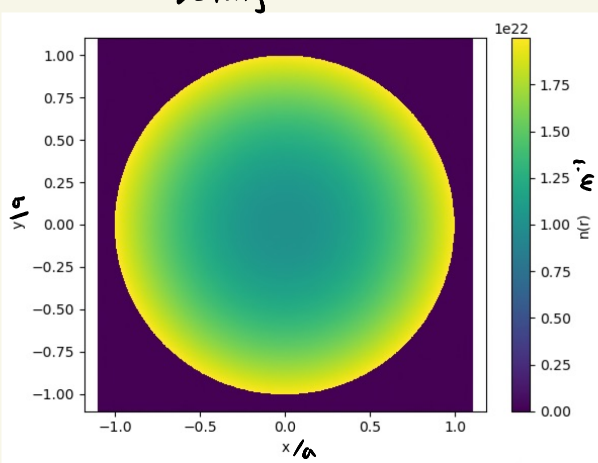


Problem 3

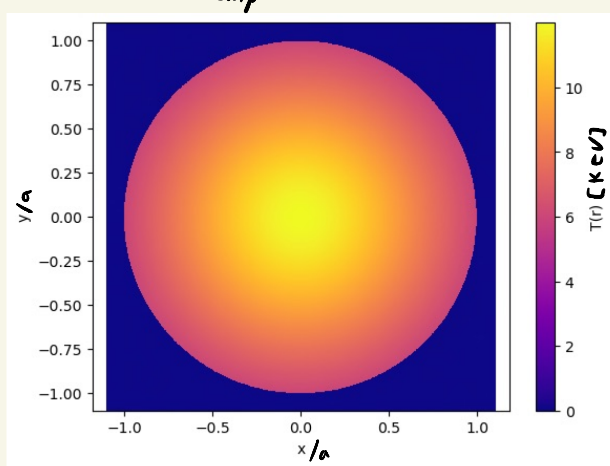
3.1 With $a=1$, $T_0 = 12$ keV, $n_0 = 1\times10^{22}$, here are my 2D plots:

$n(r) = n_0(1+r^2/a^2)$ $\quad T = \frac{T_0}{1+r^2/a^2}$, 0 outside, $r>a$.

Density



Temp

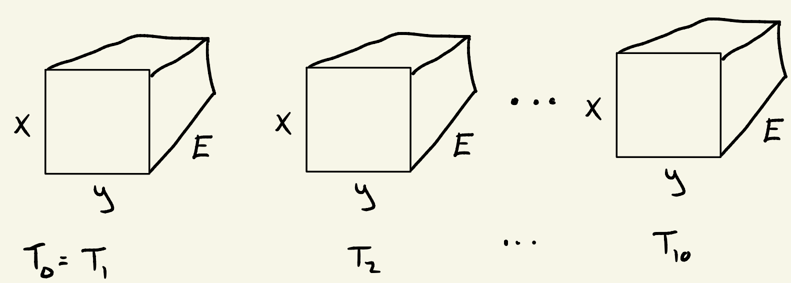


3.2

Evaluate $j(x,y,E) = n(x,y)^2\, T(x,y)^{-1/2} \exp\left(-E/T(x,y)\right)$

I will do this using my 2D $n(x,y)$ and $T(x,y)$ matricies I have. The data structure for storing values of $j(x,y,E)$ for varying T_0 between 10 and 1000 eV in ten spaces:

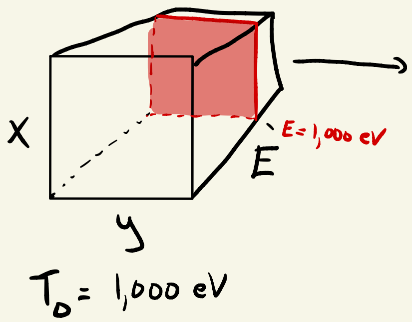
So index by T_0 to pull out one of the x,y,E j grids.

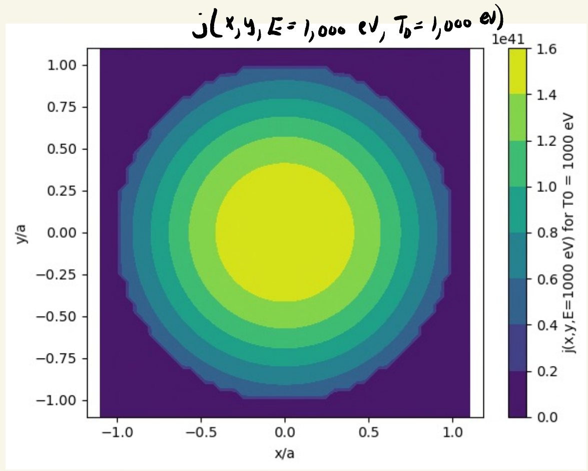


Here is an example of one xy slice at $E = 1{,}000$ eV for $T_0 = 1{,}000$ eV:

$$T(x,y) = \frac{T_0}{1 + r^2/a^2}$$

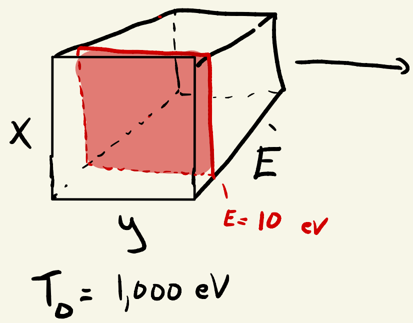
example $j(x, y, E = 1{,}000 \text{ eV}, T_0 = 1{,}000 \text{ eV})$

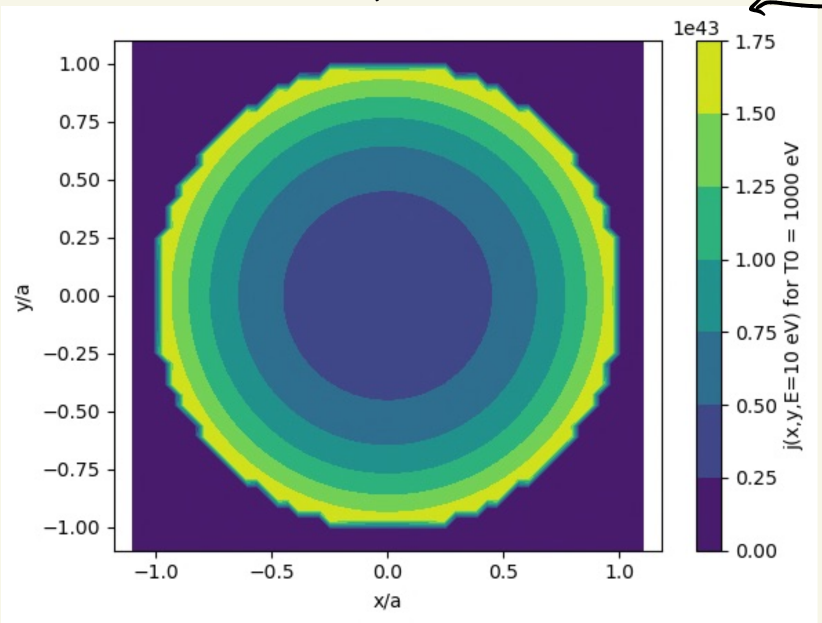




Heres another slice at $E = 10$ eV. Notice the hole in the low density region.

$j(x, y, E = 10 \text{ eV}, T_0 = 1{,}000 \text{ eV})$

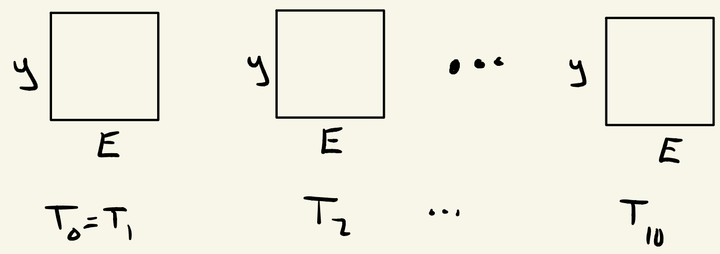




2 orders higher than $E = 1{,}000$ eV.

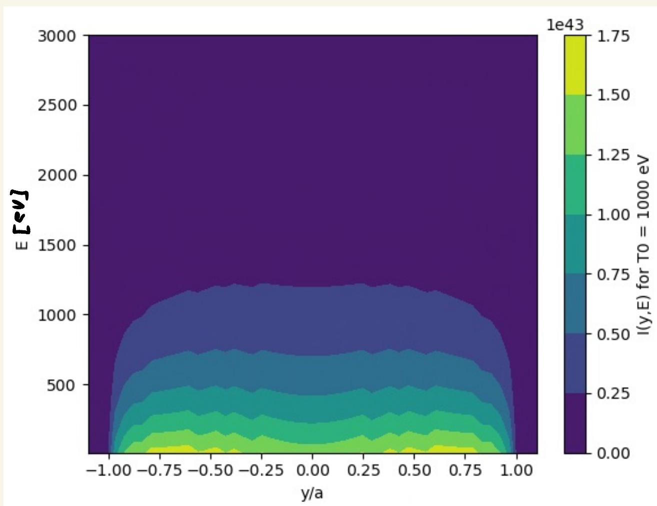
Next, I integrate along x to generate $I(y,E) = \int j(x,y,E)\,dx$

The new data structure looks like:



Here is an example $I(y,E)$ plot for $T_0 = 1{,}000$ eV:

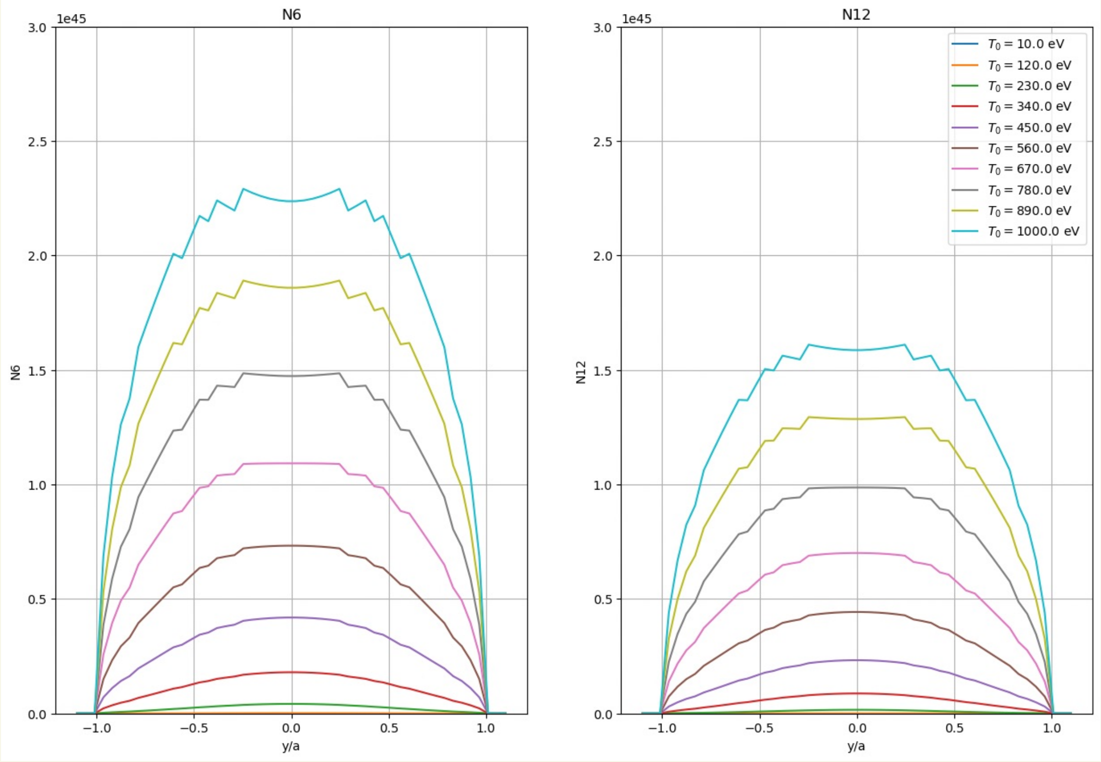
You can clearly see how when integrated over x, the majority of photons are arriving at low E and in two bumps where the density and temp are both largish. In the center, density is lower so emission is less.



I have a plot like this for the 10 temperatures I have $T_0 = 10 \ldots 1{,}000$ eV in 10 even steps.

$\boxed{3.3}$ Now, I will integrate over E to get $N_i(y) = \int I(y,E)\,W_i(E)\,dE$ For $i = 6, 12$ um. This is a straightforward task because I can just integrate along y in my existing data structure $* W(E)$. The results are plotted on the next page for the 10 temps I looked at for N_6 and N_{12}.

Here are the plots of $N_6(y)$ and $N_{12}(y)$ for the 10 different temperatures. Note that as T_0 increases, the total filtered emission increases. The 12 μm filter also attenuates more light than the 6 μm filter.



$\boxed{3.4}$ Now I will try my luck at Abel inverting $N_6(y)$ and $N_{12}(y)$. The Abel inverted profiles will be called $\tilde{N}_6(r)$ and $\tilde{N}_{12}(r)$.

These two quantities are related by the following equation:

$$\tilde{N}_i(r) = -\frac{1}{\pi}\int_r^a \frac{dN_i}{dy}\frac{dy}{(y^2-r^2)^{1/2}}$$

I can calculate this numerically by first calculating $\frac{dN_i}{dy}$ numerically, and then by creating an array of r's I want to know about. Then, I do a numerical integral from $y=r$ to $y=a$. Some sudo code for python:

rarray = linspace$(0, a, 1000)$

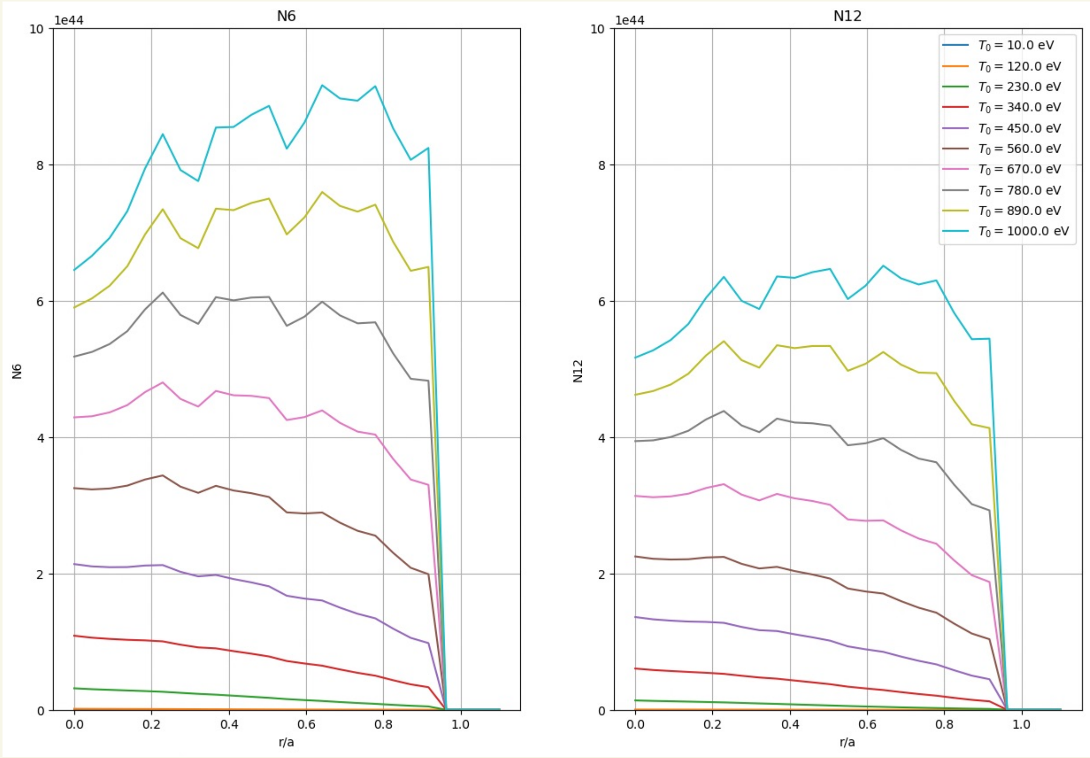
for i in range(rarray.shape[0]):

r = rarray[i]

$\tilde{N}_i[i]$ = np.trapz$\left(-\frac{1}{\pi}\frac{dN_i[r:a]}{dy}\frac{1}{(\text{yarray}[r:a]^2 - r^2)^{1/2}}, \text{yarray}\right)$

Note the array slicing is taking care of the integral bounds.

Here is a plot of $\tilde{N}_6(r)$ and $\tilde{N}_{12}(r)$ for my 10 different temperatures after carrying out my manual Abel inversion:



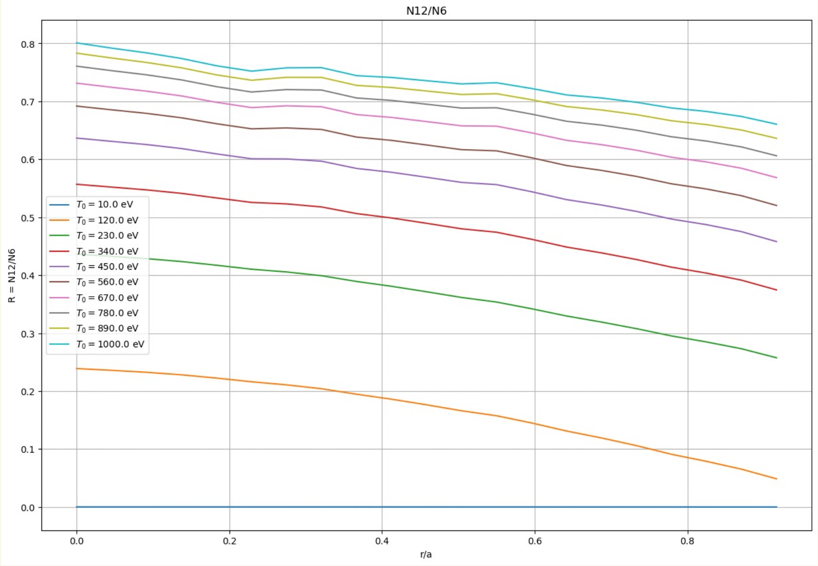
$\boxed{3.5}$

I can now take the ratio of $\tilde{N}_{12}/\tilde{N}_6\,(r) = R(r)$. Then, I can pass this to my known $R(T)$ from part 1.3 but invert it:

$$T(R(r)) \text{ is given by } \text{scipy.interp1D}(R_{array}, T_{array})\left(\frac{\tilde{N}_{12}(r)}{\tilde{N}_6(r)}\right)$$

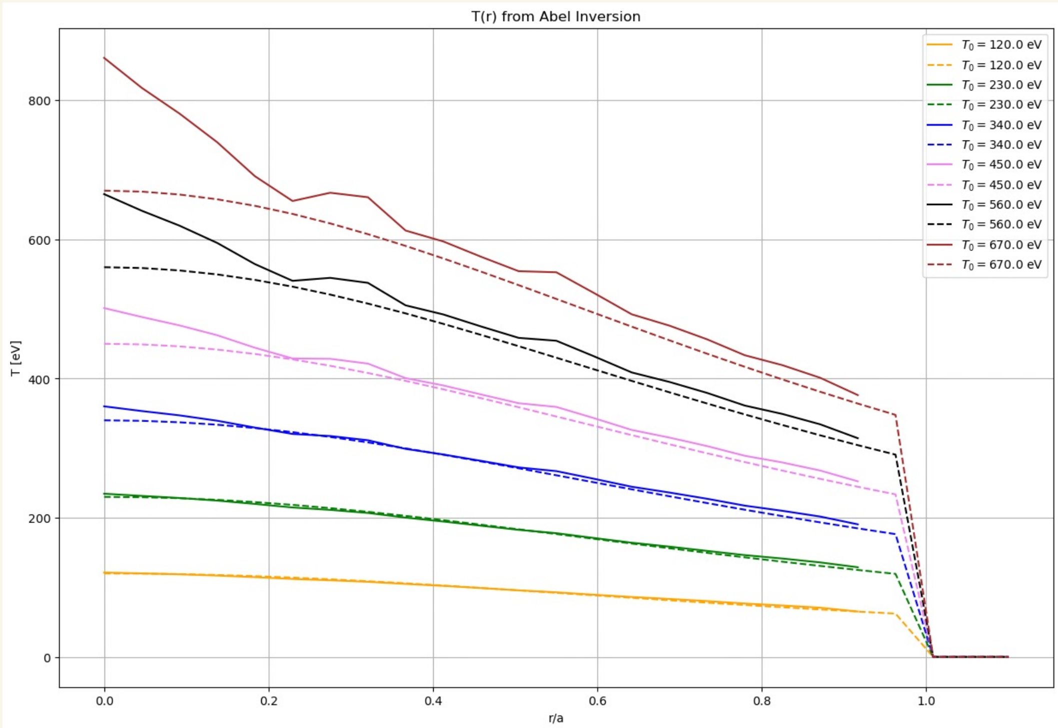
plotting $R(r)$ for my 10 evenly spaced temperatures:

As we can see, it looks like the $T_0 = 10$ eV case is too low to be distiguishable. This was predicted in 1.3.



Now I am ready to interpolate my result from 1.3 and plot $T(R(r))$:

Also plotted in dashed are the true $T(r) = \frac{T_0}{1+r^2/a^2}$ profiles I'm trying to reconstruct. Note the Abel inversion did a fantastic job. Not shown are $T_0 = 10$ eV, $T_0 = 780, 890, 1000$ eV, which are all outside the range of where I said in 1.3 I could map R to T using my 6 nm and 12 nm filters.



3.6

I felt that the derivatives $\frac{dN_6}{dy}$ and $\frac{dN_{12}}{dy}$ felt a little choppy, and the fact that the temperature I measured is effectively line-integrated over x. I was thinking I would need to compare to some density weighted x-average temperature as we did in 2.4 rather than simply the $T(r) = \frac{T_0}{1+r^2/a^2}$ temp profile. However, as can be seen above, my Abel inversion reproduced the $T(r)$ profiles for several different T_0 temperatures very well. My qualms are soothed. $\therefore$