Lecture 6 & 7
Reactive Elements in Power Electronic Systems

Winter 2023-24

Design of Inductor

- The inductor consists of a magnetic circuit and an electrical circuit.
- The design requires,
  - The size of wire to be used for the electric circuit, to carry the rated current safely.
  - The size and shape of magnetic core to be used such that
    - The peak flux is carried safely by the core without saturation.
    - The required size of the conductors are safely accommodated in the core.
  - The number of turns of the electric circuit to obtain the desired inductance.

Design of Inductor

- Material constraints
- Any given wire (conducting material) can only carry a certain maximum current per unit cross section of the wire size.
- When this limit is exceeded, the wire will overheat from the heat generated (I^2R) and melt or deteriorate.
- The safe current density for the conducting material is denoted by J A/m^2.

Design of Inductor

- Material constraints
- Any magnetic material can only carry a certain maximum flux density.
- When this limit is exceeded, the material saturates and the relative permeability drops substantially.
- This maximum allowable flux density for the magnetic material is denoted by B_m T.

Table 7.2 | Saturation flux density of common core materials

Core Type	B_{sat} (T)
Ferrite	0.3
Si steel	1.2 (CRGO), 1 (CRNGO)
Powdered iron	1.6
Amorphous glass	1.6
Mu metal	1

Notes: CRGO – cold rolled grain oriented; CNRGO – cold rolled non-grain oriented.

Design of Inductor

- Design Relationships
- In order to design an inductor of L Henry, capable of carrying an rms current of I_{rms} and peak current of I_p
- Let the wire size be a_w m^2.
- $$a_w = \frac{I_{rms}}{J} \cdots\cdots (1)$$
- Let the peak flux density in the core of area (A_C) be B_m on account of the peak current I_p in the inductor.
- $LI_p = N\phi_p = NB_mA_C \cdots\cdots (2)$

Design of Inductor

- Design Relationships
- The winding of the inductor is accommodated in the window of the core.
- Let the window area (A_W) be filled by conductors to a fraction of k_w.
- $k_w A_W = N a_w = N \frac{I_{rms}}{J} \cdots\cdots (3)$
- Cross multiplying equations (2) and (3), we get
- $L I_p N \frac{I_{rms}}{J} = N B_m A_C k_w A_W \cdots\cdots (4)$
- $L I_p I_{rms} = k_w J B_m A_C A_W \quad \cdots\cdots (5)$

Design of Inductor

- Design Relationships
- $LI_pI_{rms} = k_wJB_mA_CA_W \quad \cdots\cdots (5)$
- The above equation may be interpreted as a relationship between
  - the energy handling capacity ($0.5LI^2$) of the inductor to the size of the core (A_CA_W), the material properties (B_m, J), and our manufacturing skill (k_w).

Design of Inductor

- k_w depends on how well the winding can be accommodated in the window of the core.
- k_w is usually 0.3 to 0.5.
- B_m is the maximum unsaturated flux is about 1 T for iron and 0.2 T for ferrites.
- J is the maximum allowable current density for the conductor.
- For copper conductors J is between 2.0×10^6 A/m^2 to 5.0×10^6 A/m^2.

Design steps

- Input L I_p, I_{rms}, Core Tables, Wire Tables, J, B_m, k_w

1. Compute $A_C A_W = \dfrac{L I_p I_{rms}}{k_w J B_m}$
2. Select a core from core tables with the required $A_C A_W$.
3. For the selected core, find A_C, and A_W.
4. Compute $N = \dfrac{L I_p}{A_C B_m}$, select nearest whole number of N*.

Design steps

5. Compute, $a_w = \frac{I_{rms}}{J}$, select nearest whole number of wire gauge, $a_w{}^*$ from wire table.
6. Compute the required air gap in the core, $l_g = \frac{\mu_0 N^* I_p}{B_m}$
7. Check the assumptions:
   - Core reluctance << Air gap reluctance;

     $$R_c << R_g \; ; \frac{l}{\mu_r} \;<<\; l_g$$

   - No fringing:

     $$l_g \;<<\; \sqrt{A_C}$$

An air gap is generally introduced for inductive applications wherein the core needs to store energy. If an air gap of l_g is introduced in the magnetic path, then the reluctance of the magnetic path is given as

$$\Re = \frac{l_m}{\mu_o \mu_r A_c} + \frac{l_g}{\mu_o A_c} = \frac{1}{\mu_o A_c}\left(\frac{l_m}{\mu_r} + l_g\right) \tag{7.19}$$

The relative permeability μ_r is very large as compared to l_m. Therefore, $l_m / \mu_r \ll l_g$. The reluctance given in Eq. (7.19) can be written as

$$\Re \approx \frac{l_g}{\mu_o A_c} \tag{7.20}$$

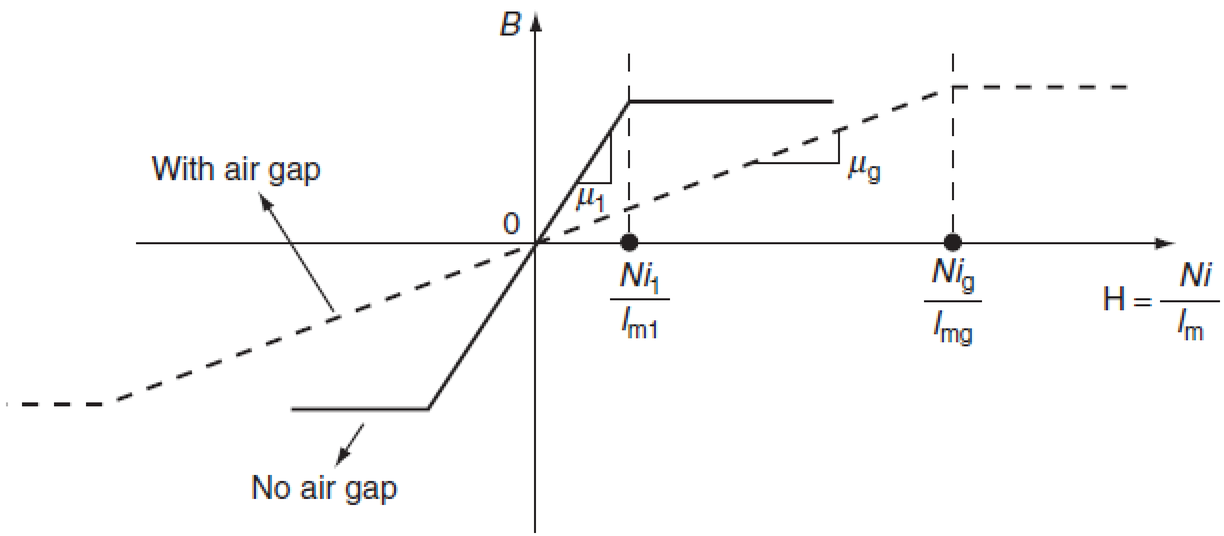


Figure 7.5 | Permeability change on introduction of air gap.

Design steps

8. Recalculate $J^* = \dfrac{I_{rms}}{a_w}$
9. Recalculate $k_w^* = \dfrac{N^* a_w^*}{A_W}$
10. Compute from the geometry of the core, mean length per turn and the length of the winding.
11. From wire tables, find the resistance of winding.

Table 2.1: Wire Table

Nominal Diameter	Wire Size SWG	Outer Diameter	Resistance Ohm/km	Area mm^2
0.025	50	0.036	34026	0.000506
0.030	49	0.041	23629	0.000729
0.041	48	0.051	13291	0.001297
0.051	47	0.064	8507	0.002027
0.061	46	0.074	5907	0.002919
0.071	45	0.086	4340	0.003973
0.081	44	0.097	3323	0.005189
0.091	43	0.109	2626	0.006567
0.102	42	0.119	2127	0.008107
0.112	41	0.132	1758	0.009810
0.122	40	0.142	1477	0.011675
0.132	39	0.152	1258	0.013701
0.152	38	0.175	945.2	0.018242
0.173	37	0.198	735.9	0.02343
0.193	36	0.218	589.1	0.02927
0.213	35	0.241	482.2	0.03575
0.234	34	0.264	402.0	0.04289
0.254	33	0.287	340.3	0.05067
0.274	32	0.307	291.7	0.05910
0.295	31	0.330	252.9	0.06818
0.315	30	0.351	221.3	0.07791
0.345	29	0.384	183.97	0.09372
0.376	28	0.417	155.34	0.1110
0.417	27	0.462	126.51	0.1363
0.457	26	0.505	105.02	0.1642
0.508	25	0.561	85.07	0.2027
0.559	24	0.612	70.30	0.2452
0.610	23	0.665	59.07	0.2919
0.711	22	0.770	43.40	0.3973
0.813	21	0.874	33.23	0.5189
0.914	20	0.978	26.26	0.6567
1.016	19	1.082	21.27	0.8107
1.219	18	1.293	17.768	1.167
1.422	17	1.501	10.850	1.589
1.626	16	1.709	8.307	2.075
1.829	15	1.920	6.564	2.627
2.032	14	2.129	5.317	3.243
2.337	13	2.441	4.020	4.289
2.642	12	2.756	3.146	5.480
2.946	11	3.068	2.529	6.818
3.251	10	3.383	2.077	8.302
3.658	9	3.800	1.640	10.51
4.064	8	4.219	1.329	12.97

Table 2.2: Design of Transformers and Inductors

Laminations: GKW | Core Section: Square
Flux Density: 1T for Inductor | 1.2T for Transformer
Current Density: J = 2.5 A/mm^2 | Window Space Factor: 0.3
Transformer Design: $N = 3754 V_{rms}/A_C$ | a_w in $mm^2 = I_{rms}/J$
Inductor Design: $N = 10^6 L I_{peak}/A_C$ | $l_g = 4\pi 10^4 N I_{peak}$ mm

Core Type No.	A_C mm^2	A_W mm^2	A_P mm^4	VA[@] As a Xformer	Energy[#] As an Inductor
L202	12.3	27.7	33.7	0.03	0.13
L164	23	53.3	1,227.6	0.12	0.46
L109	41	81.3	3329	0.33	1.25
12AX	90.3	210.9	19,033	1.9	7.1
T17	161.3	122.2	19,716	1.97	7.4
INT41	169	168	28,392	2.8	10.6
17A	204.5	1519	31,070	3.1	11.7
12A	252.8	188	47,533	4.7	17.8
10A	252.8	443.2	1,12,052	11.2	42
T 1	278.9	656.7	1,83,138	18.3	68.7
T 74	306.3	227.9	69,806	7	26.2
T 23	364.8	271.7	99,118	9.9	37.2
T 2	364.8	1,092.5	39,862	39.8	149.5
T 30	400	300	1,20,000	12	45
T 45	492.8	369.6	1,82,168	18.2	68.3
T 31	492.8	369.6	1,82,168	18.2	117
T 15	645.2	483.9	3,12,173	31.2	159
T 14	645.2	656.7	4,23,657	42.3	173
T.33	784	588	4,60,992	46.1	287
T.3	1011.2	756.8	7,65,346	76.5	595
T.16	1451.6	1,092.5	15,85,913	158.4	691
T 5	1451.6	1,269.8	18,43,312	184.1	1,054
T 6	1451.6	1,935.5	28,09,562	280.7	720
INT 120	1,600	1,200	19,20,000	191.8	1,873
T 43	2,580.6	1,935.5	49,94,777	499	4,824
T 8	2,580.6	4,984.9	1,28,64,258	1,285	3,645
INT 180	3,600	2700	97,20,000	971	15,452
8 A	5,806.4	7,096.8	4,12,06,911	4,117	10,854
8 B	5,806.4	4,984.9	2,89,44,581	2,892	21,699
8 C	5,806.4	9,965.7	5,78,65,181	5,781	44,953
T 100	10,322.6	11,612.9	11,98,74,000	1,1975	555
4 AX	566.4	2,612.2	14,79,626	147.8	4,28,500
35 A	1,451.6	7,871.8	1,14,26,755	1,142	
43 TP	645.2	2,903.2	18,73,041	281	
8 TP	1,451.6	7,871.8	1,14,26,755	1,583	
100 TP	2,580.6	15,483.8	3,99,58,217	5,988	
@ Transformer Primary $V_{rms} I_{rms}$; Frequency (for Transformer) f=50 Hz					
# Energy Capacity of the Inductor = $L I_{peak} I_{rms}/2$					

Problems

The following design refers to a 2 mH inductor suitable for dc application with a maximum current of 0.5 A.

Core: 26x19; $A_W = 40mm^2$; $A_C = 90mm^2$; N = 37 turns; $a_w = 0.29mm^2$ (23 SWG);

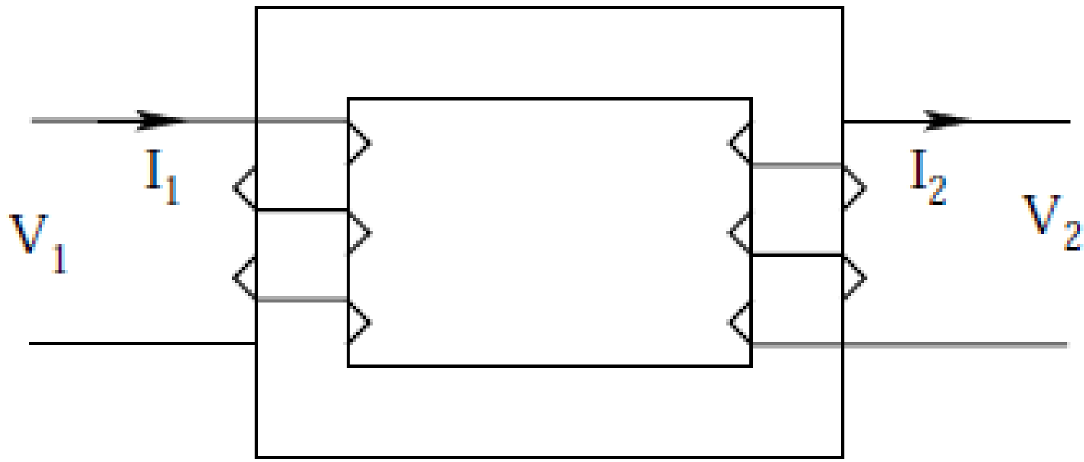
Evaluate the above design (i.e. peak flux density, peak current density, window space factor, and inductance value) for airgap values of 0.08mm and 1mm.

Design of Transformer

- Unlike the inductor, the transformer does not store energy.
- The transformer consists of more than one winding.
- Also, in order to keep the magnetization current low, the transformer does not have air gap in its magnetizing circuit.

Design of Transformer

- Consider a transformer with a single primary and single secondary as shown in Fig.



Design of Transformer

- Let the specifications be
  - Primary: V_1 volt; I_1 ampere;
  - Secondary: V_2 volt; I_2 ampere;
  - VA Rating: $V_1 I_1 = V_2 I_2$;
  - Frequency: f Hz
- For square wave of operation, the voltage of the transformer is
- $V_1 = 4fB_mA_CN_1$; $V_2 = 4fB_mA_CN_2$

Design of Transformer

- The window for the transformer accommodates both the primary and the secondary.
- With the same notation as for inductors,
- $k_w A_W = \frac{N_1 I_1 + N_2 I_2}{J}$
- From the above equations,
- $V_1 I_1 + V_2 I_2 = 4 f B_m A_C (N_1 I_1 + N_2 I_2)$
- $V_1 I_1 + V_2 I_2 = 4 k_w f J B_m A_C A_W$
- $VA = 2 k_w f J B_m A_C A_W$
- $A_C A_W = \frac{VA}{2 k_w f J B_m}$
- The above equation relates the area product ($A_C A_W$) required for a transformer to handle a given VA rating.

Design Steps

- For a given specification of VA, V_1, V2, J, B_m, k_w, and f, it is desired to design a suitable transformer.
- The design requires
  - Size of wire and number of turns to be used for primary and secondary windings.
  - Core to be used.
  - Resistance of the winding.
  - Magnetizing inductance of the transformer.

Design Steps

1. Compute the Area product ($A_C A_W$) of the desired core. ($A_C A_W = \frac{VA}{2k_w f J B_m}$)
2. Select the smallest core from the core tables having an area product higher than obtained in step (1).
3. Find the core area (AC) and window area (AW) of the selected core.
4. Compute the number of turns

$$N_1 = \frac{V_1}{4 f B_m A_C} \; ; \; N_2 = \frac{V_2}{4 f B_m A_C}$$

Design Steps

5. Select the nearest higher whole number to that obtained in step (4), for the primary and secondary turns.

6. Compute the wire size for secondary and primary.

$$a_{w1} = \frac{I_1}{J} \; ; \; a_{w2} = \frac{I_2}{J}$$

7. Select from the wire tables the desired wire size.

8. Compute the length of secondary and primary turns, from the mean length per turn of the core tables.

Design Steps

9. Find from the wire tables, the primary and secondary resistance.
10. Compute from the core details, the reluctance of the core.

$$R = \frac{l_c}{A_C \mu_o \mu_r}$$

11. Compute the magnetizing inductance.

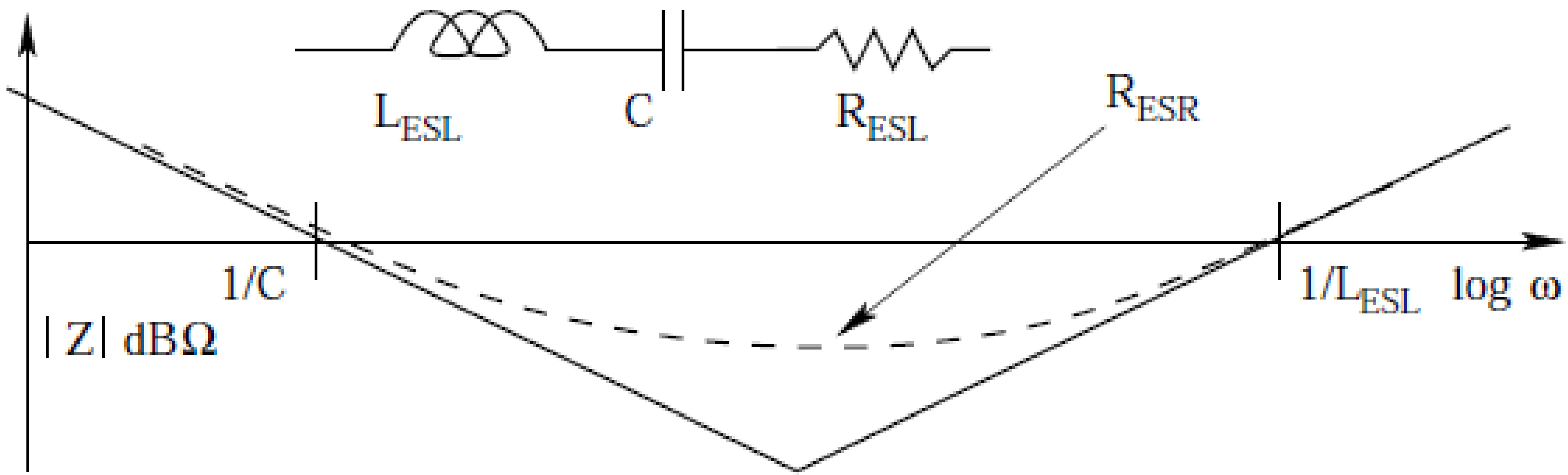
$$L_m = \frac{N^2}{R}$$

Capacitors for PE Application

- The capacitors in PES are required to handle large power.
- As a result they must be capable of carrying large current without overheating.
- To satisfy the demands in PES, the capacitors must be very close to their ideal characteristics namely low equivalent series resistance (ESR) and low equivalent series inductance (ESL).
- Low ESR will ensure low losses in the capacitor.
- Low ESL will ensure that the capacitor can be used in a large range of operating frequency.

Capacitors for PE Application

➢ Figure shows the impedance of a capacitor as a function of frequency.



Capacitors for PE Application

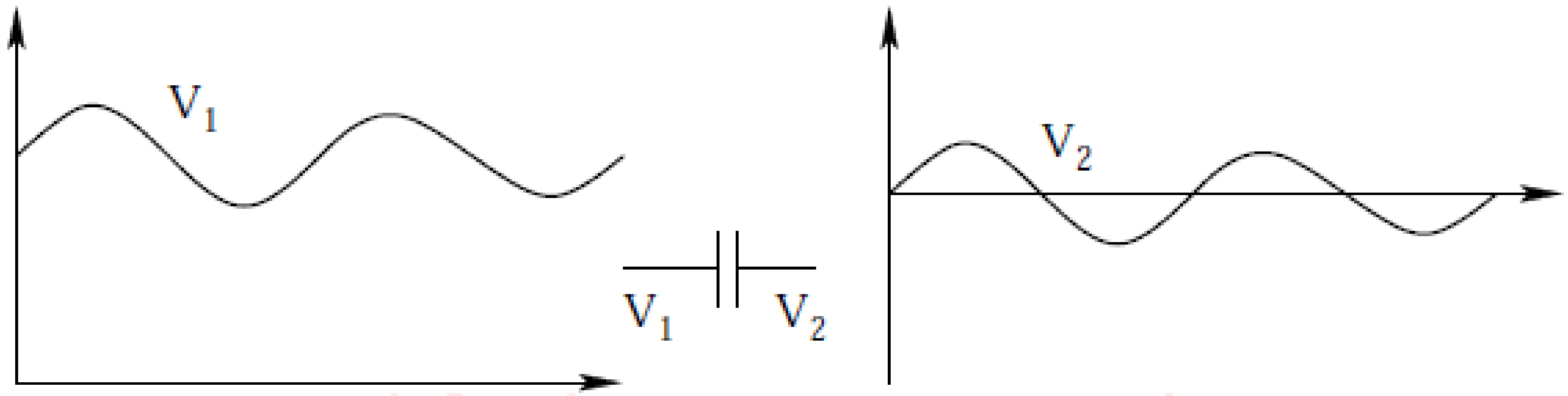
- A real capacitor is close to the ideal at lower frequencies.
- At higher frequencies, the ESR and the ESL of the real capacitor make it deviate from the ideal characteristics.
- For PES applications, it is necessary that the ESR and ESL of the capacitor are low.

Types of Capacitors

- Coupling Capacitors
- Coupling capacitors are used to transfer ac voltages between two circuits at different average potentials.
- Such capacitors are employed mostly in control circuits to couple ac signals from one circuit to another with differing dc potentials.
- The current carried by such a capacitor is comparatively low.
- The important feature of such capacitors is
  - High insulation resistance

Types of Capacitors

- Coupling Capacitors



Types of Capacitors

- Power capacitors (low frequency)
- These are used in PES mainly to improve power factor.
- They are generally used at low frequencies (predominantly 50/60 Hz).
- They compensate the reactive power demanded by the load so that the power handling portion of the PES are not called upon to supply the reactive power.
- Further, they also bypass harmonics generated in the PES.
- In such applications the voltage is predominantly sinusoidal; the current may be rich in harmonics.
- The important features of these capacitors are
  - Capability to handle high reactive power.
  - Capability to handle high harmonic current.

Types of Capacitors

- Power capacitors (high frequency)
- These are used for the same applications as the low frequency power capacitors but at higher frequencies (up to 20 kHz).
- Further they are also capable of carrying surge current resulting from switching.
- Such applications arise when capacitor banks are switched on and off to cater to conditions of varying load (typical in induction heating applications).
- The main features of these capacitors are
  - Capability to handle large reactive power.
  - Capability to operate at higher frequency.
  - Capability to handle switching surge currents.

Types of Capacitors

- Filter capacitors
- These capacitors are forward filtering capacitors to smooth out the variable source voltage applied to the load or reverse filtering capacitor to smooth out the variable load current from reaching the source.
- They are called upon to handle large periodic currents.
- The important features of these capacitors are
  - High capacitors value.
  - High rms current rating.

Types of Capacitors

- Pulse capacitors
- Pulse capacitors are used to provide very high surge currents to loads.
- They will be charged over a relatively long period and discharged in a very short period.
- Typical applications are precision welding, electronic photoflash, electronic ignition etc.
- The required features for these applications are
  - Large energy storage capacity.
  - Large peak current handling capacity.
  - Low ESL.

Types of Capacitors

- Damping capacitors
- Damping or snubber capacitors are used in parallel with power switching devices to suppress undesired voltage stresses on the device.
- The rms current in the capacitor will be high.
- The desired features are
  - High rms current capacity.
  - Low ESL.

Types of Capacitors

- Commutation capacitors
- These capacitors are employed in the commutation circuits of SCRs for forced turn-off of the device.
- They are subjected to very high reactive power and peak currents.
- The commutation process is quite short and so these capacitors must have purely capacitive reactance even at high operating frequency.
- The desired features are
  - High peak current capacity.
  - Low ESL.

Types of Capacitors

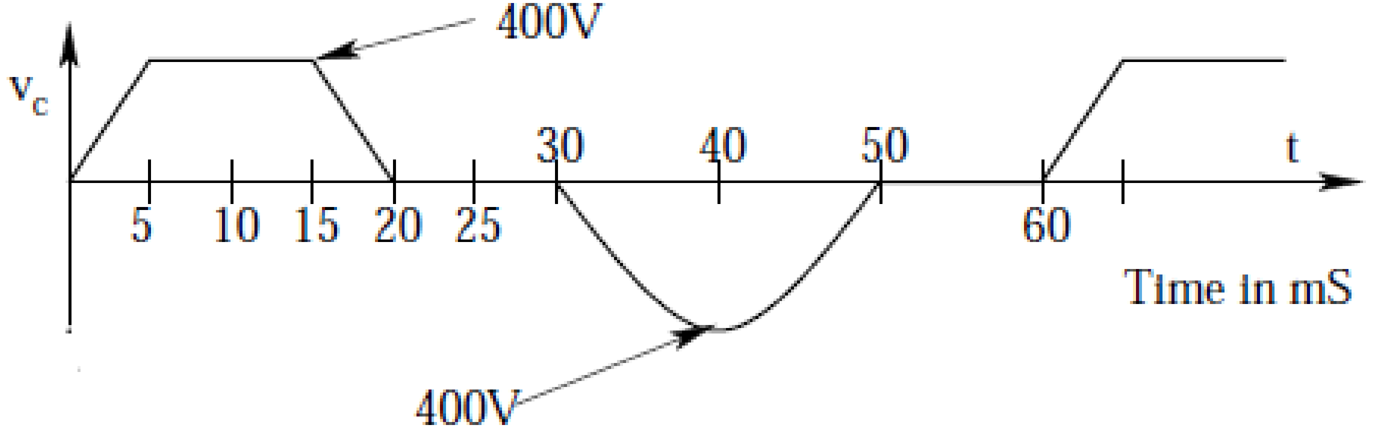
- Resonant capacitors
- Resonant capacitors are used in circuits in combination with inductors and are subjected to sinusoidal voltages and current.
- The operating frequency is high.
- The stability of the capacitor is important.
- The desirable features are
  - Stability of capacitance.
  - Low ESR.

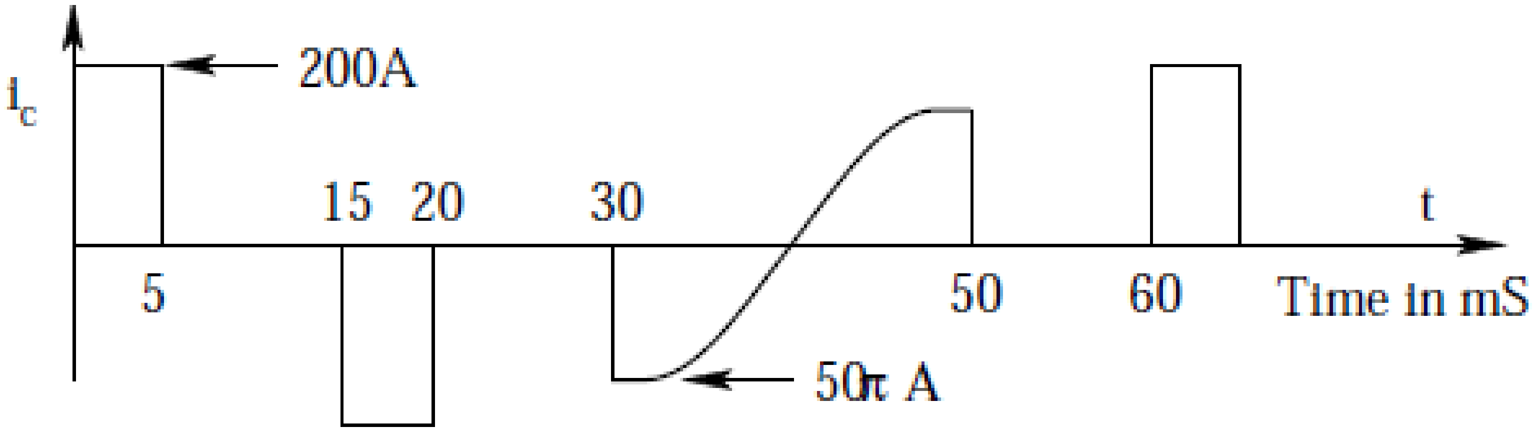
1. Figure 1 shows the voltage across a capacitor used for a power electronic application. The capacitance value is $2.5\mu F$. The capacitor has an equivalent resistance (ESR) of 10 mΩ. The dielectric of the capacitor has a thermal resistance of $0.2°C/W$ to the ambient.

   (A) Sketch the current waveform through the capacitor for one cycle.

   (B) Evaluate the losses in the capacitor.

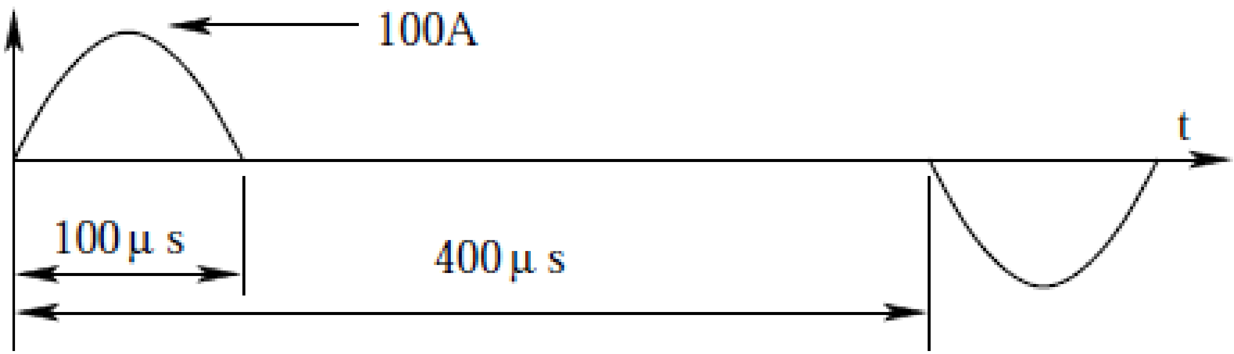
   (C) Evaluate the temperature rise in the dielectric of the capacitor.





I^2_{rms}	10779	ESR	0.01Ω
I_{rms}	103.8A	R_{th}	$0.2°C/W$
Loss	107.8W	Temperature Rise	$21.6°C$

5. The approximate wave shape of a capacitor current in a commutation circuit is shown in Fig. 5. The capacitor has an ESR of 20 mΩ. Evaluate the power dissipation in the capacitor.



4. The current through and the voltage across a power device is shown in Fig. 4. Evaluate the average current and the rms current rating of the device. Evaluate the conduction loss in the device.