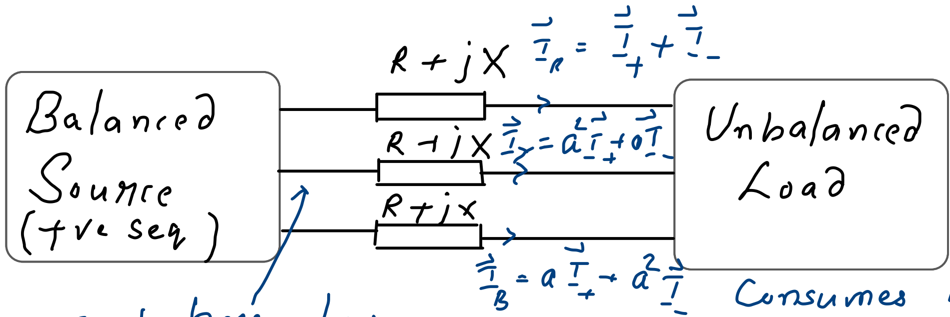
Symmetrical Components - Part 4

- Effect of Unbalanced Load on Power Transmission Efficiency



Current here has −ve Seq & +ve Seq
But only +ve Seq carries power

Consumes total 3φ active power of P

Power loss in R in first line

$$= |\vec{I}_R|^2 R = \vec{I}_R \vec{I}_R^{\,*} R$$

$$= (\vec{I}_+ + \vec{I}_-)(\vec{I}_+^{\,*} + \vec{I}_-^{\,*})\, R$$

$$= R\left[|\vec{I}_+|^2 + |\vec{I}_-|^2 + \vec{I}_- \vec{I}_+^{\,*} + \vec{I}_-^{\,*} \vec{I}_+\right]$$

[Note: Power loss in the 3 lines will be different when $\vec{I}_- \neq 0$]

Power loss in R in second line

$$= R\,(a^2 \vec{I}_+ + a\vec{I}_-)(a^2 \vec{I}_+ + a\vec{I}_-)^*$$

$$= R\,(a^2 \vec{I}_+ + a\vec{I}_-)(a\,\vec{I}_+^{\,*} + a^2 \vec{I}_-^{\,*})$$

Since $a^* = a^2$, $a^{2*} = a$

$$= R\left[|\vec{I}_+|^2 + |\vec{I}_-|^2 + a^2 \vec{I}_- \vec{I}_+^{\,*} + a\,\vec{I}_+ \vec{I}_-^{\,*}\right]$$

Since $a^3 = 1$, $a^4 = a$

|||ly power loss in R in third line

$$= R\left[|\vec{I}_+|^2 + |\vec{I}_-|^2 + a\,\vec{I}_- \vec{I}_+^{\,*} + a^2 \vec{I}_+ \vec{I}_-^{\,*}\right]$$

Total Power Loss

$$= 3R\left[|\vec{I}_+|^2 + |\vec{I}_-|^2\right]$$ Since $1 + a + a^2 = 0$

$\therefore$ Input Power $= P + 3R|\vec{I}_+|^2 + 3R|\vec{I}_-|^2$

Suppose load was balanced, consuming same power P. Then input power would have been $P + 3R|\vec{I}_+|^2$ only

Symmetrical Components - Part 4

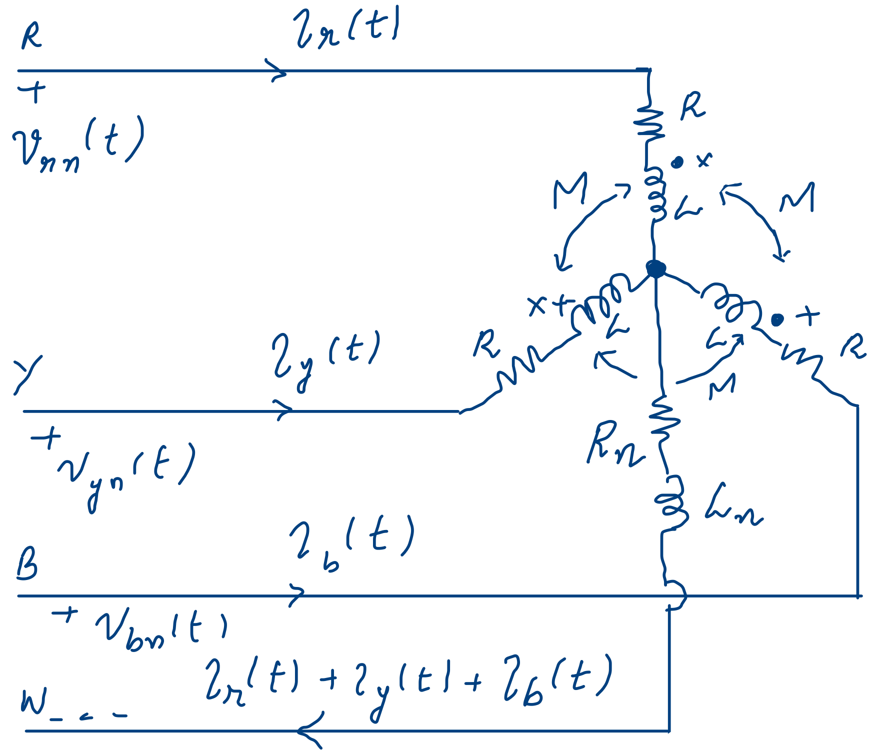
and value of $|\vec{I}_+|$ would have been less too (why?)

$\therefore$ for same amount of power delivered to load, the transmission efficiency will be lesser if load takes that power in an unbalanced manner.

Further, $|\vec{I}_+|$ will be a minimum when load power factor is unity.

$\therefore$ Maximum Power Transfer efficiency happens when load is balanced and is of unity power-factor.

- Sequence Impedances of a Balanced Impedance and Sequence Decoupling Property of Such Impedances.



$$v_{rn} = (R+R_n)i_r + (L+L_n)\frac{di_r}{dt} + R_n i_y + (M+L_n)\frac{di_y}{dt} + R_n i_b + (M+L_n)\frac{di_b}{dt}$$

$$v_{yn} = (R+R_n)i_y + (L+L_n)\frac{di_y}{dt} + R_n i_r + (M+L_n)\frac{di_r}{dt} + R_n i_b + (M+L_n)\frac{di_b}{dt}$$

$$v_{bn} = (R+R_n)i_b + (L+L_n)\frac{di_b}{dt} + R_n i_r + (M+L_n)\frac{di_r}{dt} + R_n i_y + (M+L_n)\frac{di_y}{dt}$$

Symmetrical Components - Part 4

For Sinusoidal Steady-State analysis, use $\frac{d}{dt} = j\omega$

$$\begin{bmatrix} \vec{V}_{RN} \\ \vec{V}_{YN} \\ \vec{V}_{BN} \end{bmatrix} = \begin{bmatrix} (R+R_n)+j\omega(L+L_n) & R_n+j\omega(M+L_n) & R_n+j\omega(M+L_n) \\ R_n+j\omega(M+L_n) & (R+R_n)+j\omega(L+L_n) & R_n+j\omega(M+L_n) \\ R_n+j\omega(M+L_n) & R_n+j\omega(M+L_n) & (R+R_n)+j\omega(L+L_n) \end{bmatrix} \begin{bmatrix} \vec{I}_R \\ \vec{I}_Y \\ \vec{I}_B \end{bmatrix}$$

$$\begin{bmatrix} \vec{V}_0 \\ \vec{V}_+ \\ \vec{V}_- \end{bmatrix} = \underbrace{A^{-1} \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix} A}_{\text{Similarity Transformation}} \begin{bmatrix} \vec{I}_0 \\ \vec{I}_+ \\ \vec{I}_- \end{bmatrix}$$

Neutral Shift $= \dfrac{V_0 \times 3Z_n}{Z_s + 2Z_m + 3Z_n}$

$$A = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix}, \quad A^{-1} = \frac{1}{3} \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{vmatrix}$$

Completing the matrix multiplication, and using $1+a+a^2=0$, $a^4=a$, $a^3=1$, we get

$$\begin{bmatrix} \vec{V}_0 \\ \vec{V}_+ \\ \vec{V}_- \end{bmatrix} = \begin{bmatrix} Z_s+2Z_m & 0 & 0 \\ 0 & Z_s-Z_m & 0 \\ 0 & 0 & Z_s-Z_m \end{bmatrix} \begin{bmatrix} \vec{I}_0 \\ \vec{I}_+ \\ \vec{I}_- \end{bmatrix}$$

Z_0 = the zero-seq impedance

$= Z_s + 2Z_m$

$= \underbrace{[R + j\omega(L+2M)]} + \underbrace{3[R_n + j\omega L_n]}_{\text{3 times neutral impedance}}$

Z_+ = the +ve seq impedance

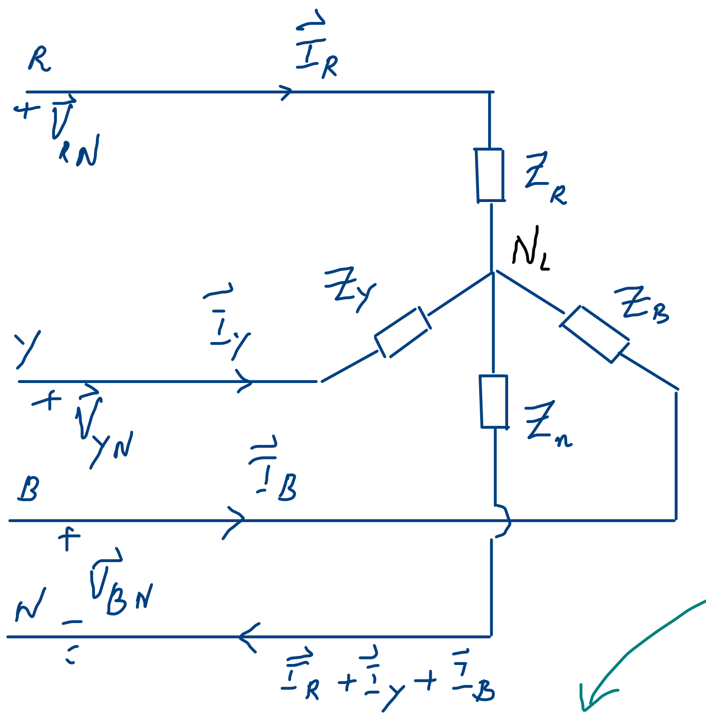
$= Z_s - Z_m = R + j\omega(L-M)$

Z_- = the −ve seq impedance

$= Z_s - Z_m = R + j\omega(L-M)$.

Symmetrical Components - Part 4

- Sequence Impedance Matrix of an Unbalanced Impedance and Sequence Coupling Property of Such Impedances



Z matrix

$$\begin{bmatrix} \vec{V}_{RN} \\ \vec{V}_{YN} \\ \vec{V}_{BN} \end{bmatrix} = \begin{bmatrix} Z_R + Z_N & Z_N & Z_N \\ Z_N & Z_Y + Z_N & Z_N \\ Z_N & Z_N & Z_B + Z_N \end{bmatrix} \begin{bmatrix} \vec{I}_R \\ \vec{I}_Y \\ \vec{I}_B \end{bmatrix}$$

$$\begin{bmatrix} \vec{V}_0 \\ \vec{V}_+ \\ \vec{V}_- \end{bmatrix} = \bar{A}^{-1} \, Z \, A \begin{bmatrix} \vec{I}_0 \\ \vec{I}_+ \\ \vec{I}_- \end{bmatrix}$$

$$\begin{bmatrix} \vec{V}_0 \\ \vec{V}_+ \\ \vec{V}_- \end{bmatrix} = \begin{array}{c|ccc} & 0 & + & - \\ \hline 0 & \frac{Z_R+Z_Y+Z_B}{3} + 3Z_N & \frac{Z_R + a^2 Z_Y + aZ_B}{3} & \frac{Z_R + aZ_Y + a^2 Z_B}{3} \\ + & \frac{Z_R + aZ_Y + a^2 Z_B}{3} & \frac{Z_R + Z_Y + Z_B}{3} & \frac{Z_R + a^2 Z_Y + aZ_B}{3} \\ - & \frac{Z_R + a^2 Z_Y + aZ_B}{3} & \frac{Z_R + aZ_Y + a^2 Z_B}{3} & \frac{Z_R + Z_Y + Z_B}{3} \end{array} \begin{bmatrix} \vec{I}_0 \\ \vec{I}_+ \\ \vec{I}_- \end{bmatrix}$$

Note the asymmetry

Some Effects of This Fully Populated Sequence Impedance Matrix

- Neutral Shift Calculation

Let the applied voltage have $\vec{V}_0, \vec{V}_+, \vec{V}_-$ as its symmetrical components. Then

$$\vec{V}_0 = \left(\frac{Z_R + Z_Y + Z_B}{3} + 3Z_n\right)\vec{I}_0 + \frac{Z_R + a^2 Z_Y + aZ_B}{3}\,\vec{I}_+ + \frac{Z_R + aZ_Y + a^2 Z_B}{3}\,\vec{I}_-$$

Symmetrical Components - Part 4

$$\therefore \vec{I}_0 = \frac{\vec{V}_0 - \dfrac{Z_R + a^2 Z_Y + a Z_b}{3}\vec{I}_+ - \dfrac{Z_R + a Z_Y + a^2 Z_b}{3}\vec{I}_-}{\dfrac{Z_R + Z_Y + Z_b}{3} + 3Z_n}$$

$$V_{N_L N} = 3\vec{I}_0 Z_n$$

$$= 3Z_n \frac{\vec{V}_0 - \dfrac{Z_R + a^2 Z_y + a Z_b}{3}\vec{I}_+ - \dfrac{Z_R + a Z_y + a^2 Z_b}{3}\vec{I}_-}{\dfrac{Z_R + Z_Y + Z_B}{3} + 3Z_n}$$

and $V_{N_L N} \rightarrow \vec{V}_0 - \dfrac{Z_R + a^2 Z_y + a Z_b}{3}\vec{I}_+ - \dfrac{Z_R + a Z_y + a^2 Z_B}{3}\vec{I}_-$

as $Z_n \rightarrow \infty$.

In a Balanced impedance neutral shift Voltage is $\vec{V}_0$ itself when neutral connection is open. But in an unbalanced load, neutral shift depends on +ve and −ve sequence currents too.

- Coupling between Sequence Components.
  +ve Seq voltage can lead to +, − and 0-Seq currents
  −ve Seq current can give +ve seq voltage drop
  0-Seq current can yield +ve & −ve Seq voltage drops etc.

  ∴ Given a Set $(\vec{V}_0, \vec{V}_+, \vec{V}_-)$ applied voltage, $\vec{I}_0$, $\vec{I}_+$ & $\vec{I}_-$ cannot be solved independent of each other. We canot prepare sequence equivalent circuits.

- An impedance matrix with equal diagonal terms and all off-diagonal terms equal gets DIAGONALISED when Similarity Transformation using Symmetrical Component Transformation Matrix. Impedance Matrix of an Unbalanced load DOES NOT.

An Useful Application of Sequence Coupling Property of Unbalanced Three Phase Impedance - LOAD BALANCING USING REACTIVE THREE-PHASE IMPEDANCE

Context : 3ϕ 3-Wire Balanced System delivering power to a 3ϕ Unbalanced Load. Transmission efficiency to be improved by cancelling $-$ve Sequence Current in connecting link to 0. Zero-Seq current does not flow due to 3-wire nature.

$$\begin{cases} \vec{V}_0 = 0 \\ \vec{V}_+ = \text{Rated} \\ \vec{V}_- = 0 \end{cases}$$

The equipment that does this is called "Load Balancing Eqpt" and when it contains only passive inductances and capacitances, it is called 'Passive Load Balancing Equipment'.

Why Reactive Impedance ?
— We do not want to lose power in the process of nullifying the $-$ve Seq current drawn by the load.

Question : Does there exist a 3ϕ unbalanced impedance that draws only $-$ve Sequence current from a $+$ve Sequence voltage source ?

Answer : Yes.
Such an impedance will have its neutral open (since we do not want zero-seq current) or it will be Δ connected. So $\vec{I}_0 = 0$. Then

$$\begin{bmatrix} \vec{V}_+ \\ 0 \end{bmatrix} = \begin{bmatrix} \dfrac{Z_R + Z_Y + Z_B}{3} & \dfrac{Z_R + a^2 Z_Y + a Z_B}{3} \\ \dfrac{Z_R + a Z_Y + a^2 Z_B}{3} & \dfrac{Z_R + Z_Y + Z_B}{3} \end{bmatrix} \begin{bmatrix} \vec{I}_+ \\ \vec{I}_- \end{bmatrix}$$

Symmetrical Components - Part 4

Suppose we make $Z_R + Z_Y + Z_B = 0$.
Note that this is possible iff all the three are pure reactances since we are not free to employ negative resistances. Then,

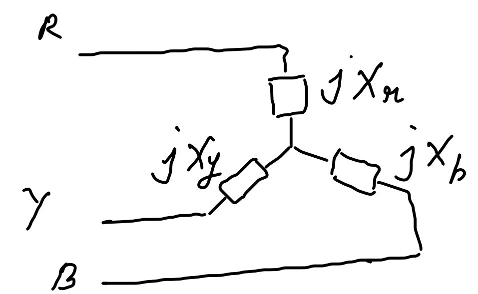
$$\begin{bmatrix} \vec{V}_+ \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & \dfrac{Z_R + a^2 Z_Y + a Z_B}{3} \\ \dfrac{Z_R + a Z_Y + a^2 Z_B}{3} & 0 \end{bmatrix} \begin{bmatrix} \vec{I}_+ \\ \vec{I}_- \end{bmatrix}$$

Solution is,

$$\vec{I}_- = \frac{3\vec{V}_+}{Z_R + a^2 Z_Y + a Z_B} \quad ; \quad \vec{I}_+ = 0$$

Now, suppose we make this $\vec{I}_-$ opposite to negative sequence current taken by unbalanced load, then, the combination of unbalanced load and this compensator will appear as a balanced load to the supply.

Let us call the desired -ve seq current in compensator as $\vec{I}_{-c}$.
Let the compensator be a pure reactive network as below.



Then $j(X_r + a^2 X_y + a X_b) = \dfrac{3\vec{V}_+}{\vec{I}_{-c}}$

known

known

Equating real parts & imaginary parts we get 2 eqns. And the third equation is $(X_r + X_y + X_b) = 0$
We can solve for X_r, X_y, X_b.

Symmetrical Components - Part 4

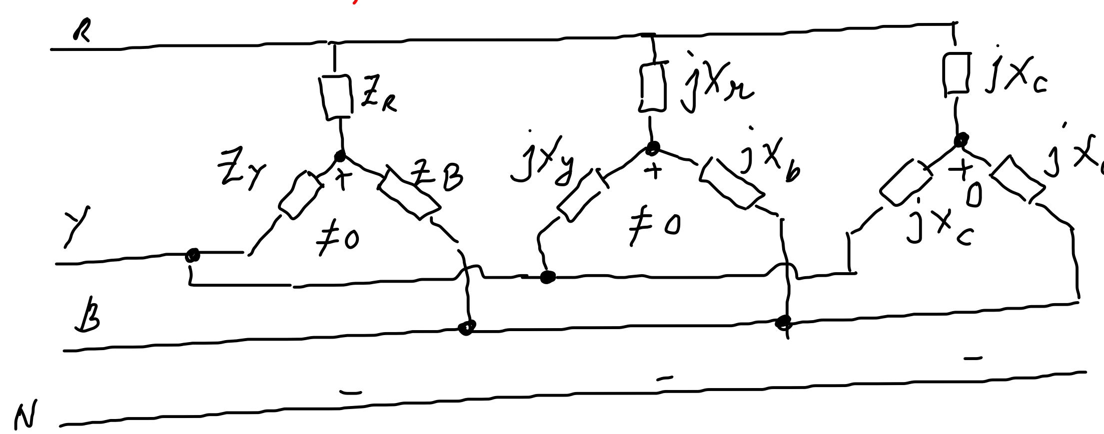
Now if we connect this network in $||^{le}$ with the unbalanced load, the resulting combination will appear as a balanced load to the source. But note the following important points.

- Now the line current is only $\vec{I}_+$ drawn by unbalanced load. It may be carrying reactive power along with active power. If we can nullify that reactive power also, we will get minimum transmission loss.
- The compensation network is an unbalanced Star impedance with neutral open. Neutral shift at the neutral of unbalanced load and at the neutral of compensator will be different. So they should not be connected together.

Let $\vec{I}_+ = I_a + j I_r$ with $\vec{V}_+$ as reference

Then, we can cancel jI_r by a 3ϕ Balanced reactive network with reactance value $= \dfrac{|\vec{V}_+|}{-jI_r} \triangleq jX_c$

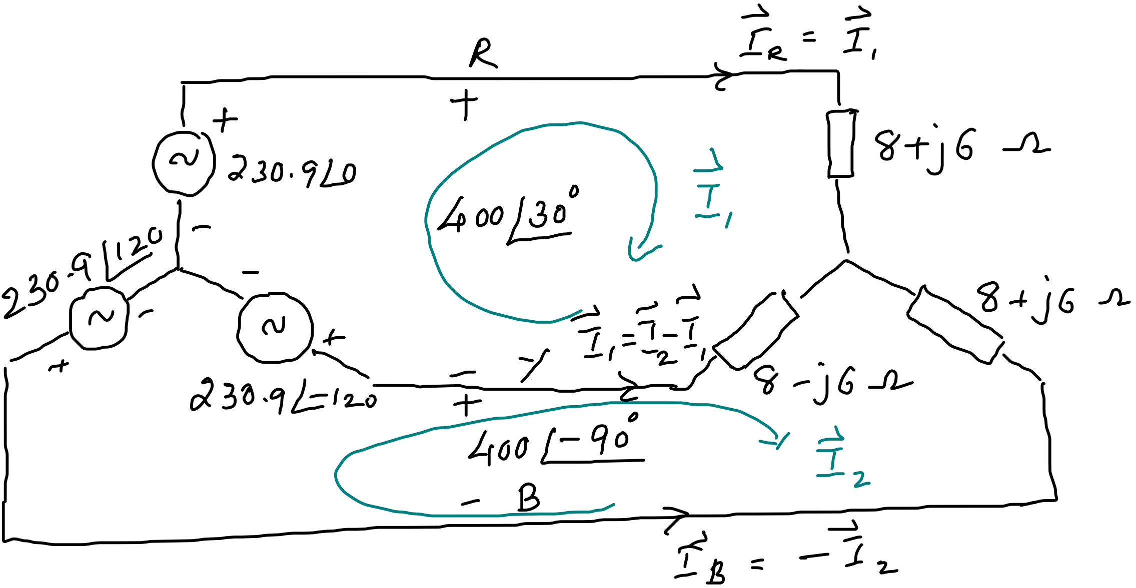
Being a balanced network, neutral shift at its neutral $= 0$ and so its neutral should not be tied to neutral of $-$ve sequence compensator



Second & Third Stars may be converted to Δ, the Δ's paralleled and converted back to Star to get a single Y network that does $-$ve Seq cancellation & reactive compensation.

Symmetrical Components - Part 4

An Example Problem Illustrating Load Compensation



Solving by Mesh Analysis.

$$\begin{bmatrix} 16 & -8+j6 \\ -8+j6 & 16 \end{bmatrix} \begin{bmatrix} \vec{I}_1 \\ \vec{I}_2 \end{bmatrix} = \begin{bmatrix} 400\angle 30^\circ \\ 400\angle -90^\circ \end{bmatrix}$$

Solving by Kramer's rule,

$\vec{I}_1 = 32.1\angle -22.83^\circ$ Arms

$\vec{I}_2 = 41.9\angle -109.23^\circ$ Arms

$\therefore\ \vec{I}_R = 32.1\angle -22.83^\circ,\ \vec{I}_Y = 51.16\angle 2.12^\circ,\ \vec{I}_B = 41.9\angle 70.77^\circ$

[Check: $\vec{I}_0 = (\vec{I}_R + \vec{I}_Y + \vec{I}_B)/3 = 0$]

$$\vec{I}_+ = \frac{\vec{I}_R + a\vec{I}_Y + a^2\vec{I}_B}{3} = 34.03 - j\,22.74 \text{ Arms}$$

active   Reactive

$$\vec{I}_- = \frac{\vec{I}_R + a^2\vec{I}_Y + a\vec{I}_B}{3} = -4.45 + j\,10.28 \text{ Arms}$$

$\therefore\ \vec{I}_{-c} = 4.45 - j\,10.28 = 11.2\angle -66.6^\circ$ Arms

$$x_n + a^2 x_y + a x_b = \frac{3\vec{V}_+}{j\,\vec{I}_{-c}} = \frac{3\times 230.9\angle 0}{1\angle 90 \times 11.2\angle -66.6}$$

$$= 61.85\angle -23.4^\circ$$

$$= 56.76 - j\,24.56$$

With $a^2 = 1\angle -120^\circ$ $a = 1\angle 120^\circ$

$[x_n - 0.5(x_y + x_b)] + j\,0.866(x_b - x_y) = 56.76 - j\,24.56$

$\therefore\ x_n - 0.5(x_y + x_b) = 56.76$ —— (1)

$x_y - x_b = 28.36$ —— (2)

Symmetrical Components - Part 4

and $X_r + X_y + X_b = 0$ —— (3)

Using (3) in (1), we get

$1.5\, X_r = 56.76\ \Omega$

$\Rightarrow X_r = 37.84\ \Omega$ (Inductive)

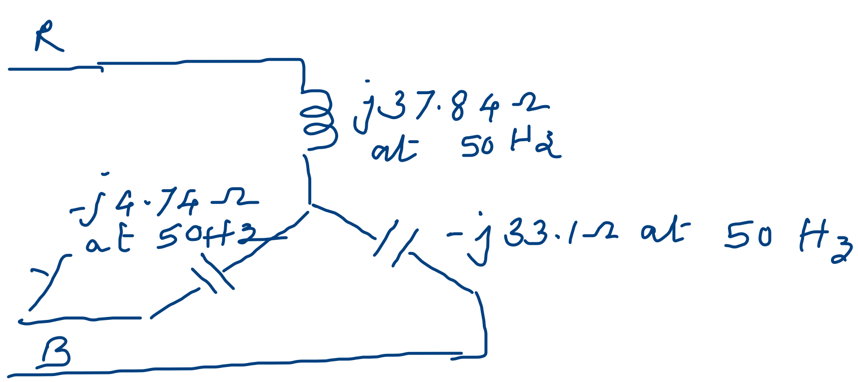
$\therefore X_y + X_b = -37.84\ \Omega$ —— (4)

$X_y - X_b = 28.36\ \Omega$ —— (2)

$2X_y = -9.48\ \Omega$

$\therefore X_y = -4.74\ \Omega$ (Capacitive)

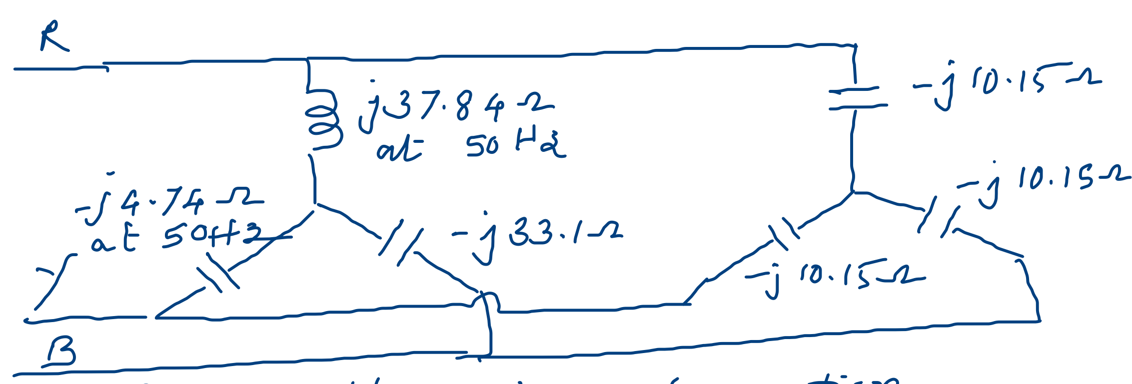
& $X_b = -33.1\ \Omega$ (Capacitive)



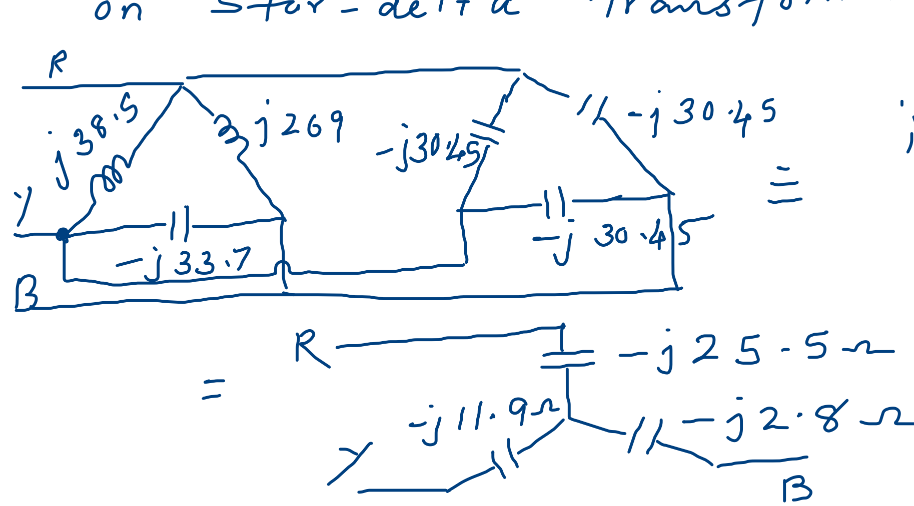
Reactive component in $\vec{I}_+ = -j22.74$.

$\therefore X_c = \dfrac{230.9}{-(-j22.74)} = -j10.15\ \Omega$ (Capacitive).

So, final Compensator is,



on Star-delta transformation,



← This is the final Compensator Ckt