Assignment. I. (MA6003D).

1) Let $V = \{(a_1, a_2) : a_1, a_2 \in \mathbb{R}\}$. For $(a_1, a_2), (b_1, b_2) \in V$ and $\alpha \in \mathbb{R}$.

define $$(a_1, a_2) + (b_1, b_2) = (a_1 + 2b_1, a_2 + 3b_2)$$

$$\alpha(a_1, a_2) = (\alpha a_1, \alpha a_2).$$

Is V a vector space over $\mathbb{R}$ with these operations?

2) Let $V = M_{n\times n}(\mathbb{R})$ be set of all $n\times n$ matrices whose entries from $\mathbb{R}$. Verify the following subsets $W \subseteq V$ are subspace or not?

a) $W = \{A \in M_{n\times n}(\mathbb{R}) \mid A = A^T\}$

b) $W = \{A \in M_{n\times n}(\mathbb{R}) \mid A = -A^T\}$

c) $W = \{A \in M_{n\times n}(\mathbb{R}) \mid \text{trace}(A) = 0\}$.

3) Let V be a vector space over $\mathbb{R}$ or $\mathbb{C}$. and u, v, w be distinct vectors in V.

Prove that

a) $\{u, v\}$ is linearly Independent $\Leftrightarrow$ $\{u+v, u-v\}$ is linearly Independent.

b) $\{u, v, w\}$ is linearly Independent $\Leftrightarrow$ $\{u+v, v+w, w+u\}$ is linearly Independent.

c) verify $\{(1,2,3,4), (0,5,-1,2), (1,0,2,3)\} \subseteq \mathbb{R}^4$ is L.I or not?

4). Let U and W be the subspaces of R^4 generated by

$(1,4,-1,3)$, $(1,5,0,5)$, $(3,10,-5,5)$ and

$(1,4,0,6)$, $(1,2,-1,5)$, $(2,2,-3,9)$

Find $\dim(U+V)$ and $\dim(U\cap W)$.

5) Let $T: R^3 \to R^4$ be the linear mapping defined by

$T(x,y,z) = (x+2y-z,\ y+z,\ x+y-2z,\ -x+6z)$

Find the basis and dimension of $R(T)$ and $N(T)$.

6) Let $T: R^3 \to R^2$ be linear map defined by

$T(x,y,z) = (2x+y-z,\ 3x-2y+4z)$

Find the matrix of T, relative to the bases

$\beta = \{(1,1,1), (1,1,0), (1,0,0)\}$, $\gamma = \{(1,3), (1,4)\}$.

7) Let $T: R^3 \to R^4$ be linear map defined by

$T(a,b,c) = (a+b,\ b+c,\ c+d,\ a+b+c+d)$.

Find $N(T)$ and rank T.

8) Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 1 \\ 0 & 2 & -1 \\ 0 & 6 & 0 \end{bmatrix}$ $T: R^3 \to R^4$

Find a linear Transformation with respect to the standard basis in R^3 and R^4.

9) Solve the system of linear equation.

$$x_1 + 2x_2 + 2x_3 = 2$$

$$x_1 + 8x_3 + 5x_4 = -6$$

$$x_1 + x_2 + 5x_3 + 5x_4 = 3$$

10) Find eigen value and eigen vector for

$$A = \begin{bmatrix} 0 & -2 & -3 \\ -1 & 1 & -1 \\ 2 & 2 & 5 \end{bmatrix}.$$