Line Frequency Controlled Rectifiers

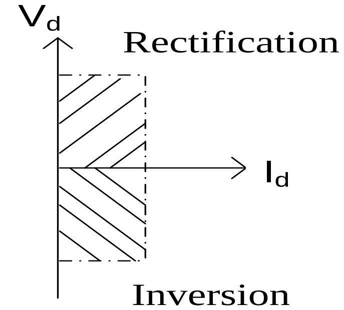
- Controlled dc output voltage

  → varies from +ve maximum to –ve maximum.

- Controllable devices like thyristors, IGBT BJT are used.

- Commutation of the devices depends on the ac side line frequency.

- Dc side current is always +ve
- Dc side voltage is +ve or -ve.



- +ve voltage and +ve current at dc side indicates rectification mode. Power flows from ac side to dc side.
  - operating points lie in the first quadrant of V_d-I_d plane.

- -ve voltage and +ve current at dc side indicates inversion mode. Power flow is from dc side to ac side.
  - operating points lie in the fourth quadrant of V_d-I_d plane.

Single phase half wave controlledrectifier

- **R load**

0 → α

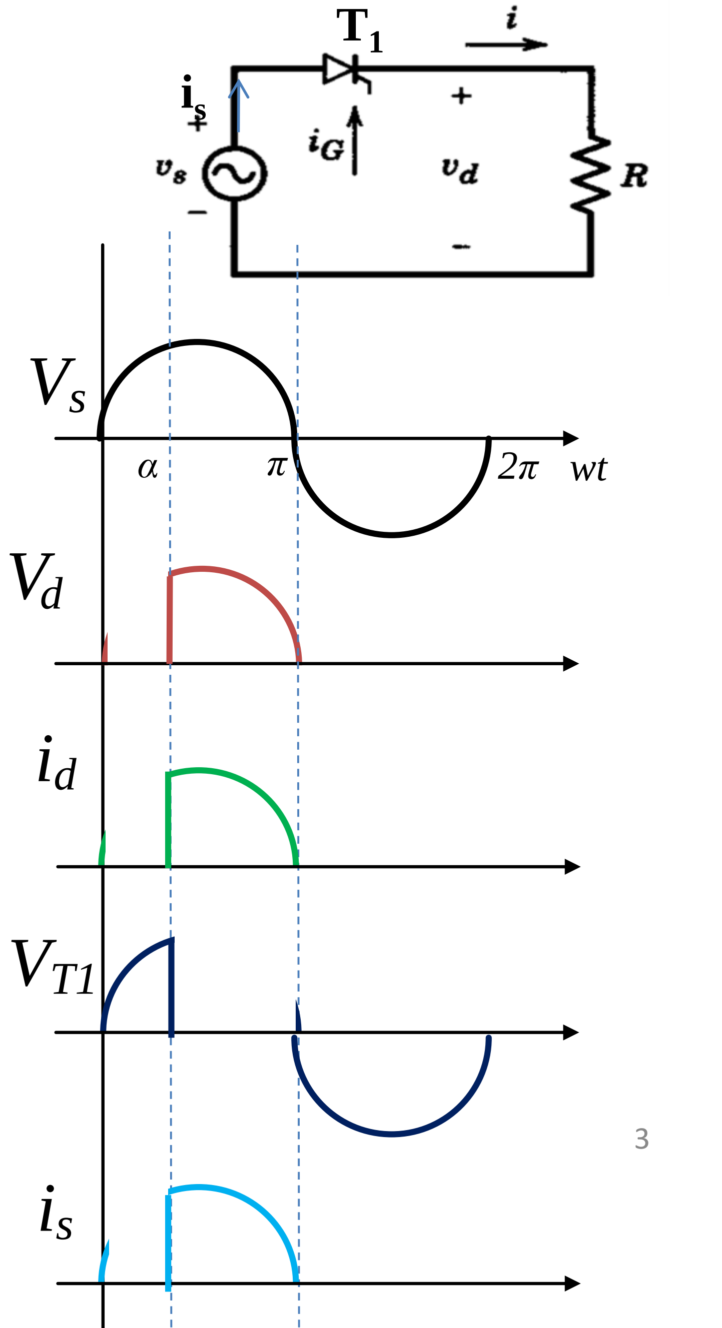
T_1 is off.

$i = 0. \quad v_d = 0. \quad i_s = 0 \quad v_{T1} = v_s$

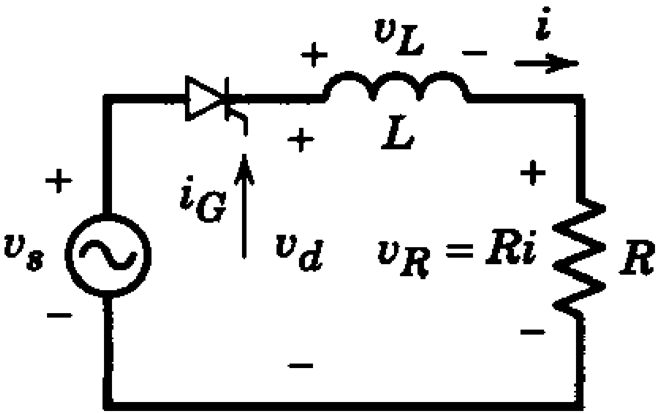
- At $\omega t = \alpha$, T_1 is triggered on
- **α → π**
- T_1 is on.
- $v_d = v_s \quad i = v_d/R = v_s/R = \dfrac{\sqrt{2}v_s}{R}\sin\omega t$
- $i_s = i$
- At $\omega t = \pi$, $i = 0$ $\quad T_1$ becomes off
- **π → 2π**

T_1 is off.

$i = 0. \quad v_d = 0. \quad i_s = 0 \quad v_{T1} = v_s$



R-L load



<u>$0 \rightarrow \alpha$</u>

Switching device is off.

$i = 0 \qquad V_d = 0$

$V_L = 0$, $V_R = 0$

$V_T = v_s$

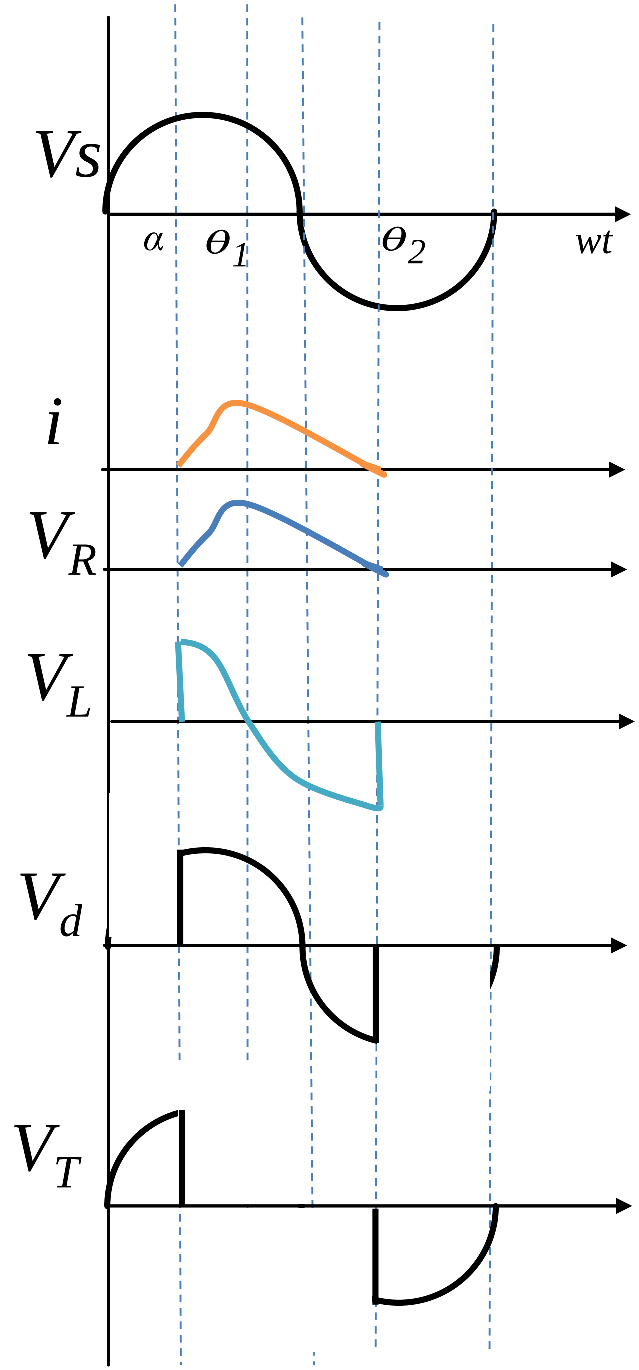
<u>$\alpha \rightarrow \theta_2$</u>

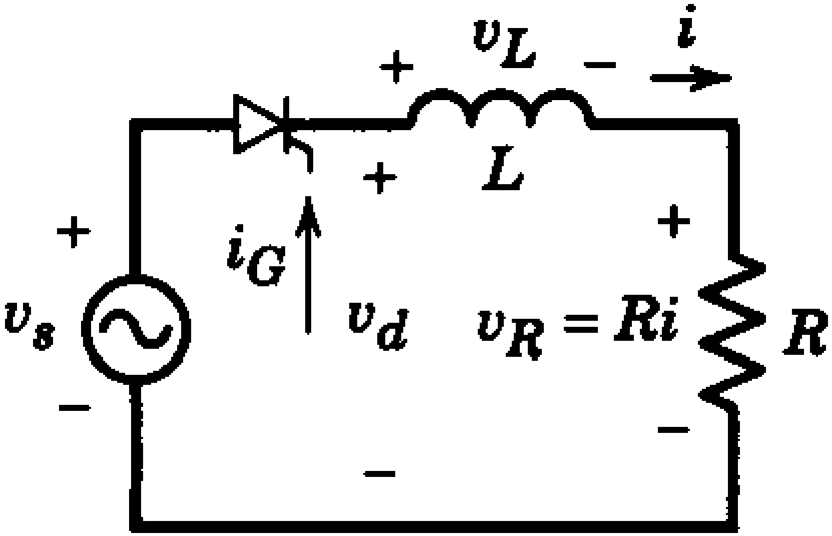
Thyristor is on.

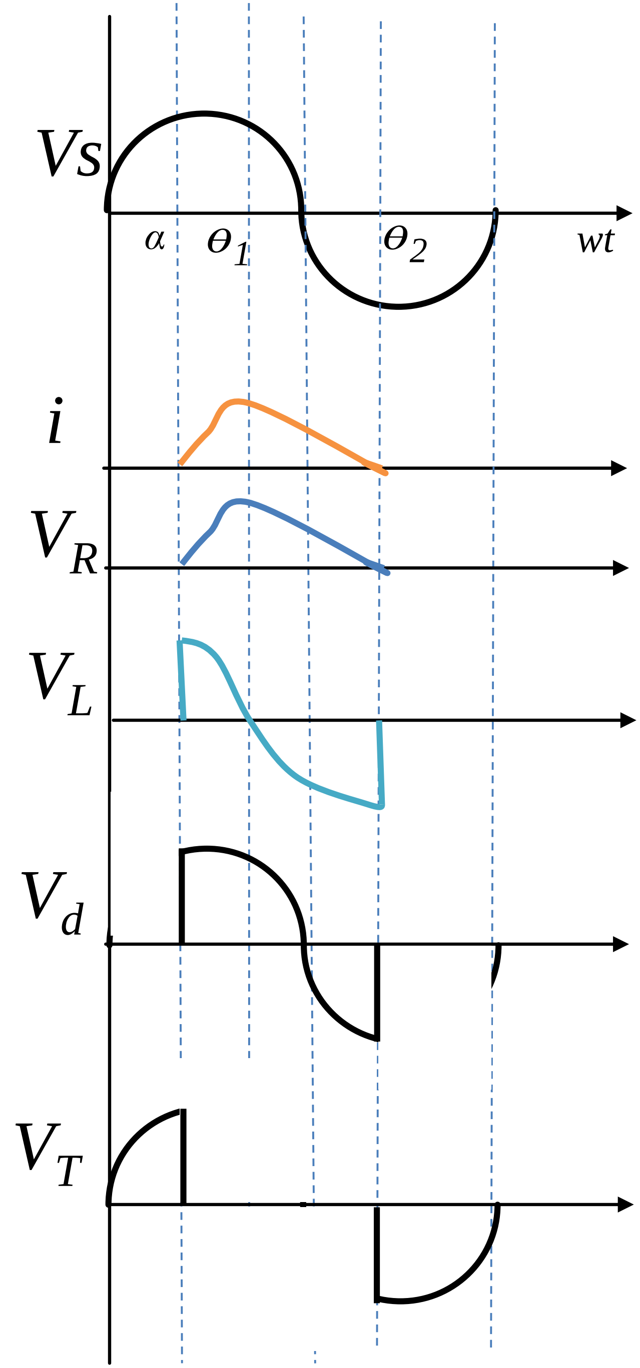
$V_d = Ri(t) + L\dfrac{di(t)}{dt} = v_s = \sqrt{2}v_s \sin \omega t$

$v_R(t) = Ri(t)$

$v_L(t) = v_s - v_R(t)$







At ωt= θ_1

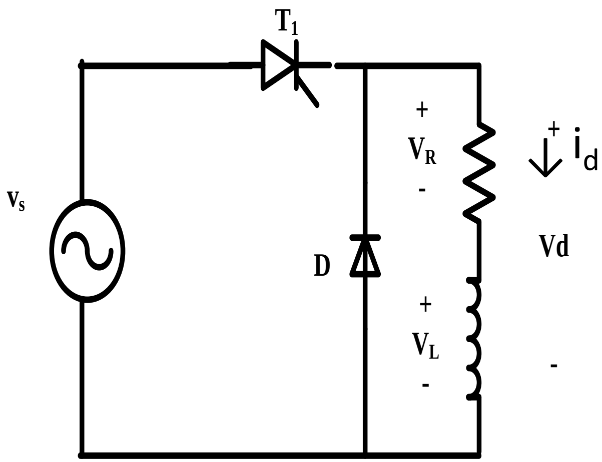
di/dt = 0, v_L = 0,

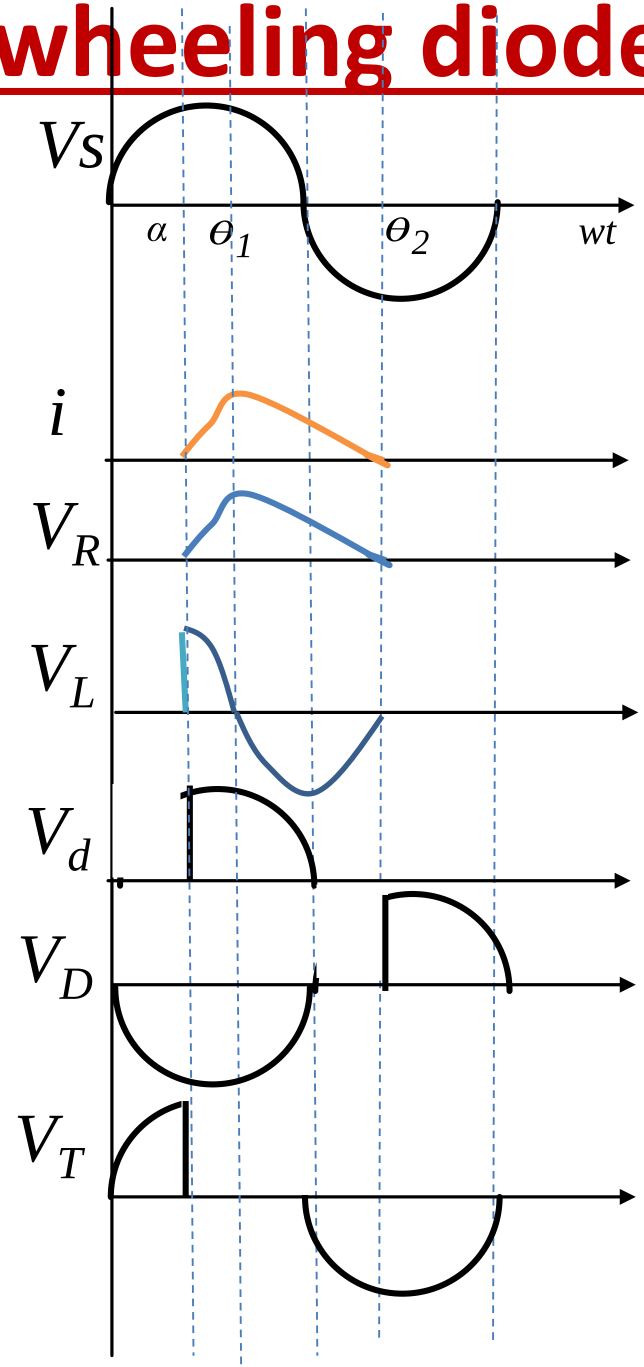
$v_R = v_s$

Area A1 = Area A2

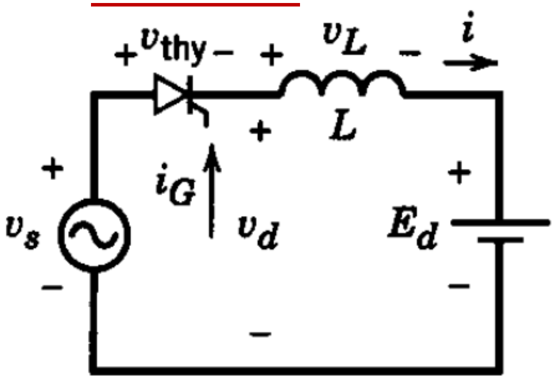
Energy stored in the inductor during α to θ_1 is sent back to ac source during θ_1 to θ_2

R-L load with a freewheeling diode





- **L-E load**



$0 \rightarrow \theta$

T is reverse biased.

$i = 0$

$V_L = 0$

$V_d = E_d$

$V_T = v_s - E_d$

$\theta \rightarrow \alpha$

T is forward biased and off.

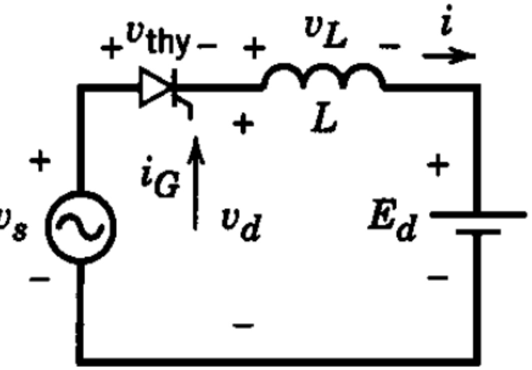
$i = 0$

$V_d = E_d$

$V_L = 0$

$V_T = v_s - E_d$

- **L-E load**



At α Thyristor is forward biased and triggered.

α →β

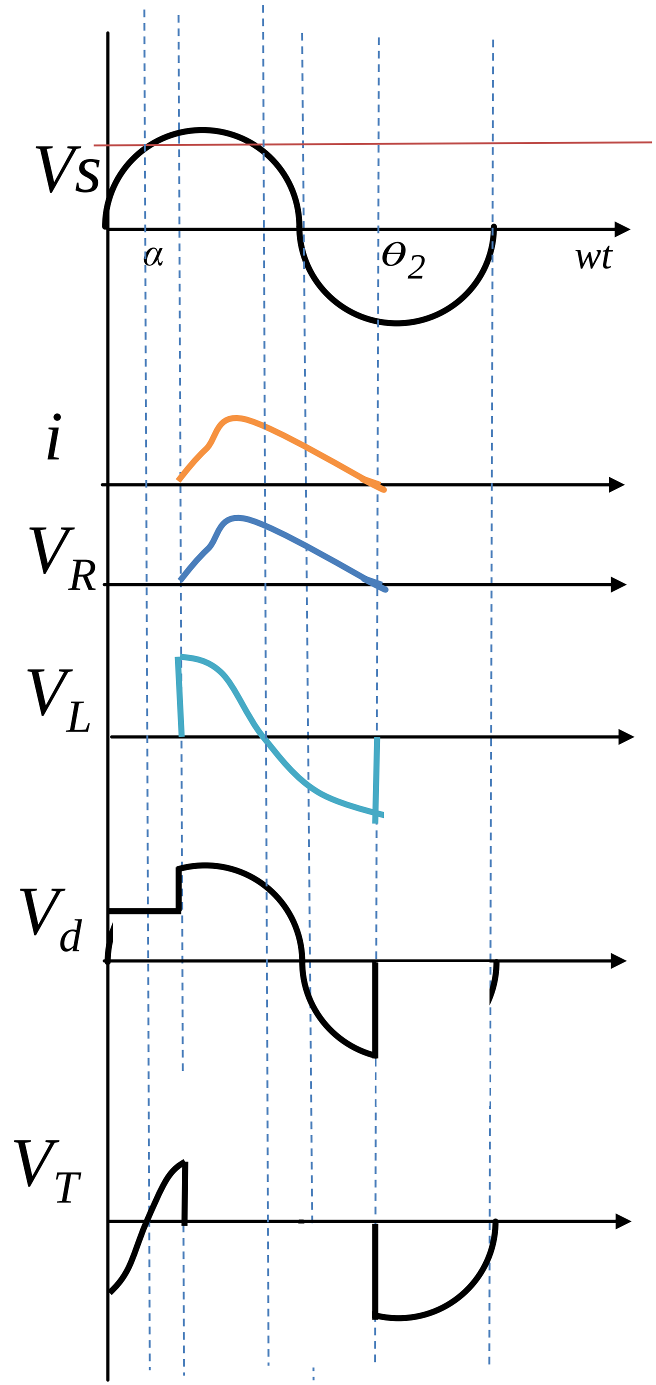
Thyristor is conducting.

$V_d = v_s$

$V_L = L\, di/dt = v_s - E_d$

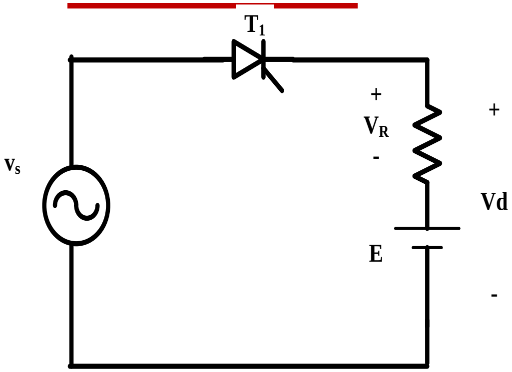
$$i_d = \frac{1}{\omega L} \int (v_s - E_d)\, d\omega t$$

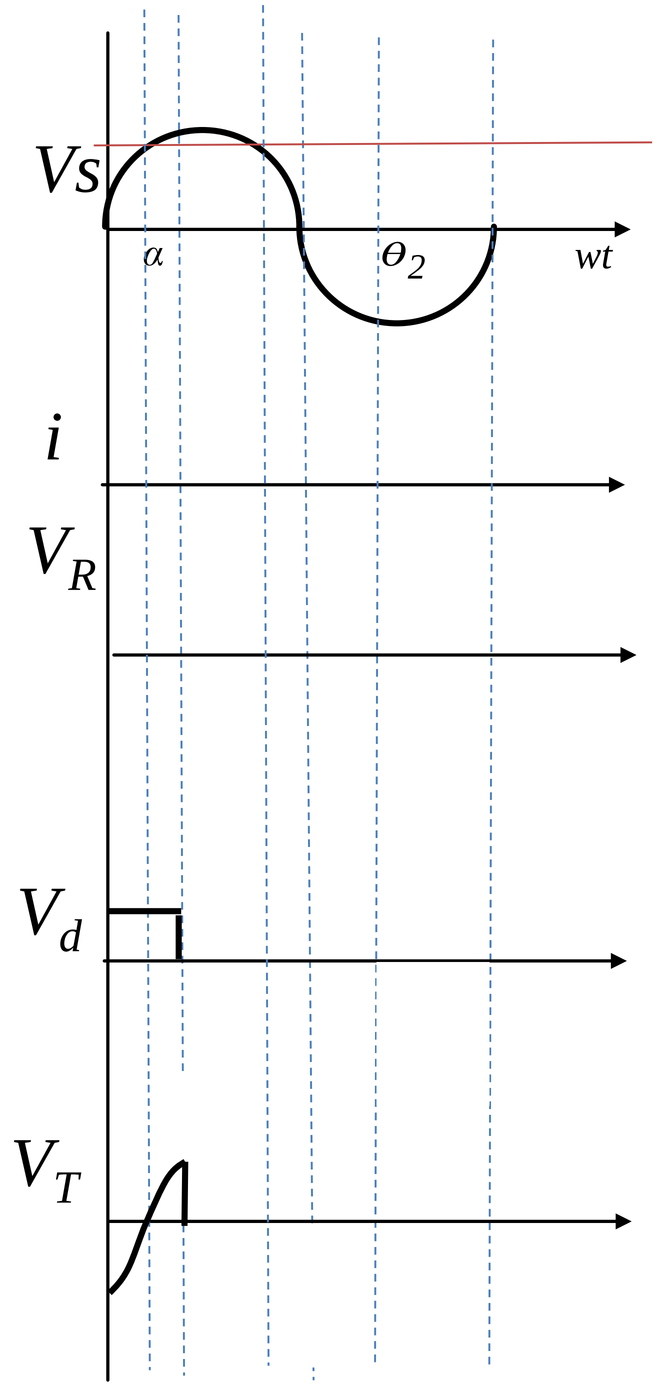
$V_T = 0$



- At β, $i = 0$, T becomes off.
- <u>$\beta \rightarrow 2\pi$</u>
- Thyristor is off
- $i = 0$
- $V_L = 0$
- $V_d = E_d$
- $V_T = v_s - E_d$

• R-E load





0 → θ

Thyristor is reverse biased.

$$i_d = 0$$

$$v_d = E_d$$

$$v_R = 0$$

$$v_T = v_s - E_d$$

θ → α

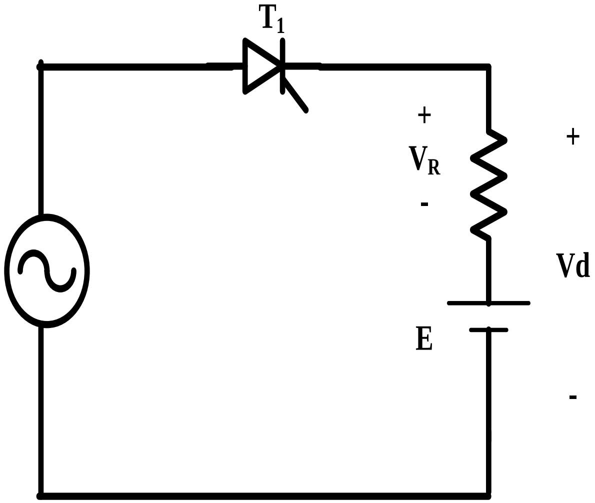
Thyristor is forward biased and in the off state.

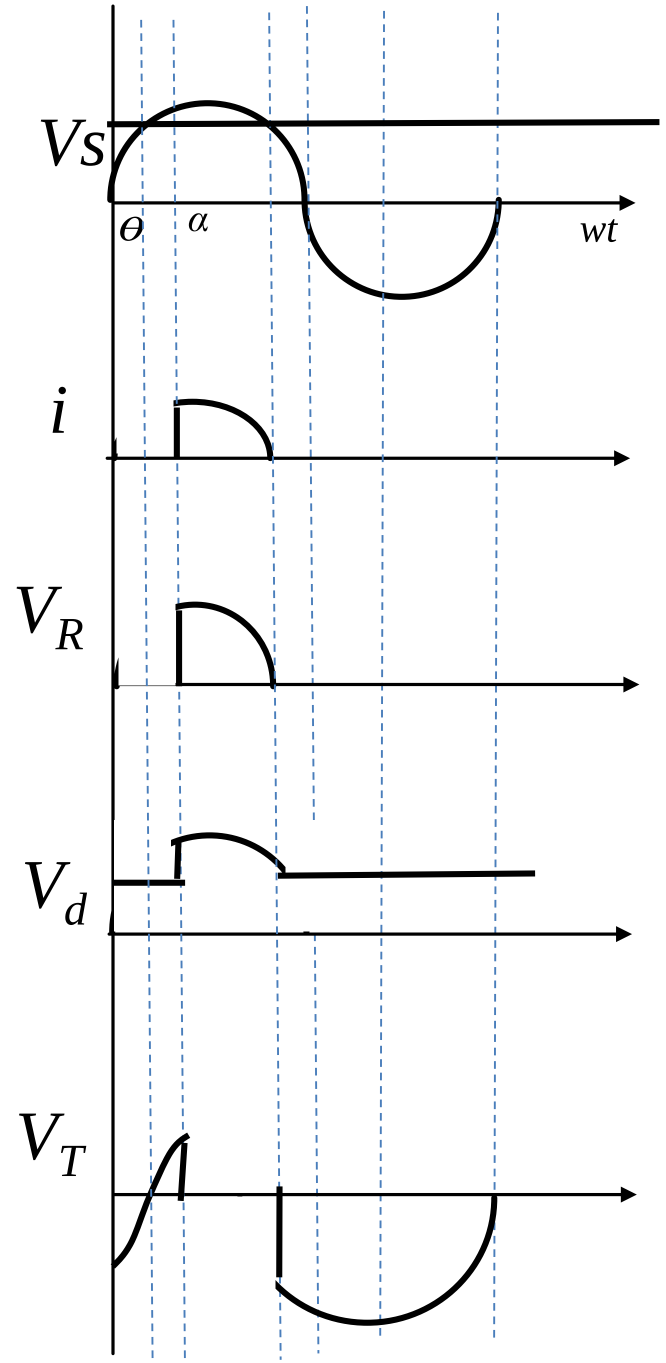
$$v_d = E_d$$

$$v_R = 0$$

$$i_d = 0$$

$$v_T = v_s - E$$





$\alpha \rightarrow \pi - \theta$

Thyristor is forward biased and in the on state.

$V_d = v_s$

$v_R = Ri = v_s - E_d$

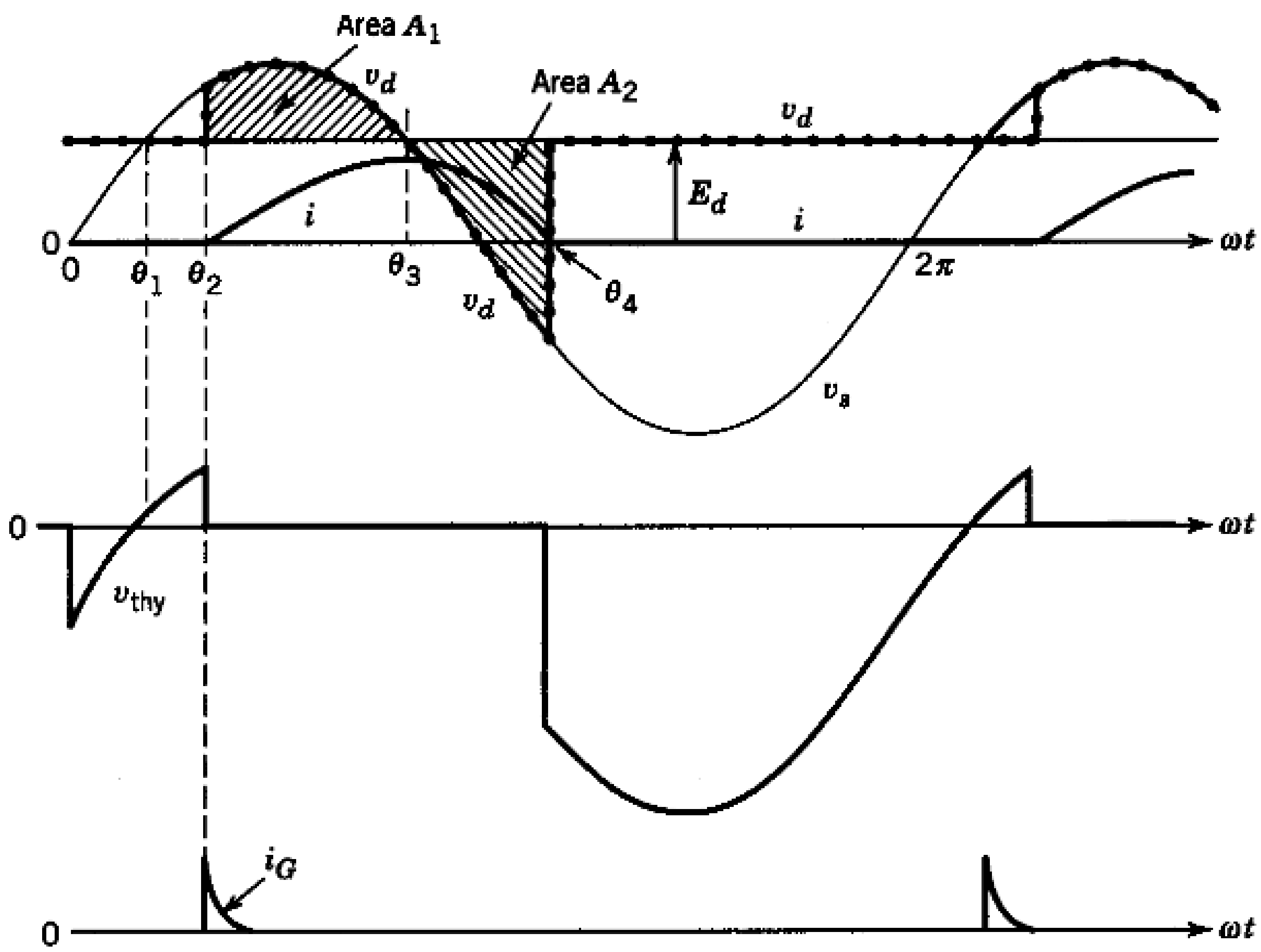
$i = (v_s - E_d)/R$

$V_T = 0$

- **At Π- θ,**
- $i_d = 0$
- **Thyristor becomes off**

- **Π- θ $\rightarrow$ 2Π**
- $v_d = E_d$
- $v_R = 0$
- $v_T = v_s - E_d$

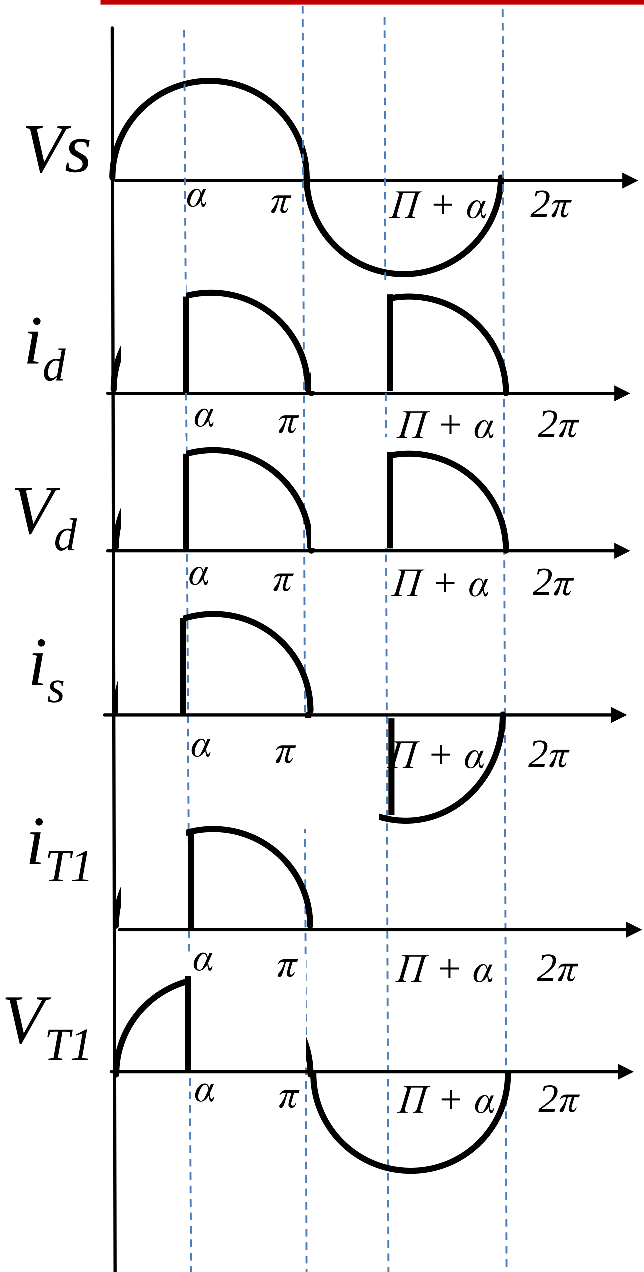
Waveform

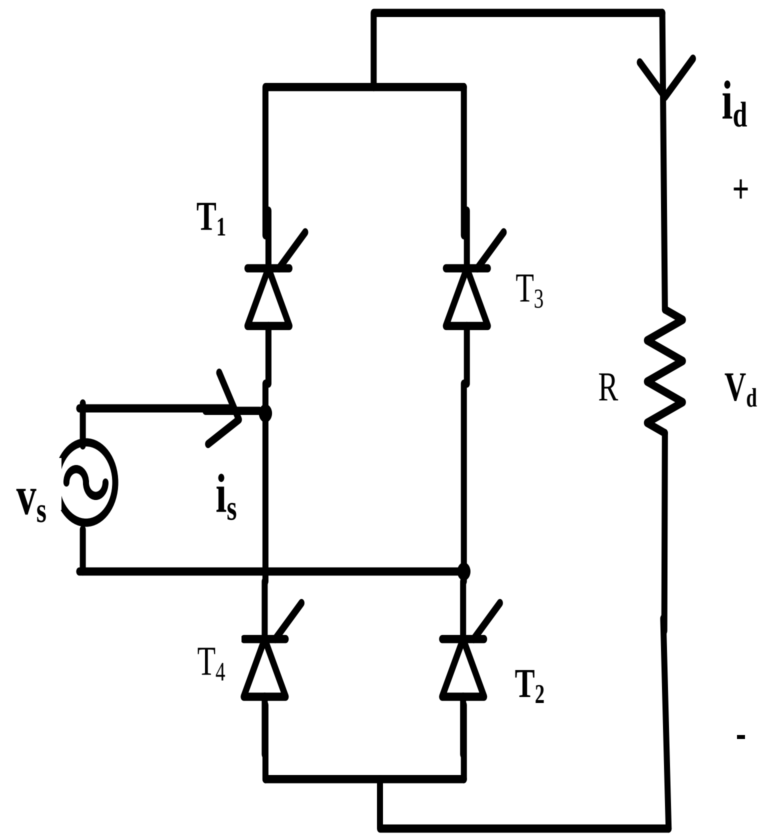


Area A_1
v_d
Area A_2
v_d
E_d
i
i
0
0
θ_1
θ_2
θ_3
2π
ωt
v_d
θ_4
v_s
0
ωt
v_{thy}
i_G
0
ωt

Single phase Fully Controlled bridge rectifier

- R Load





R – L Load -Discontinuous current mode

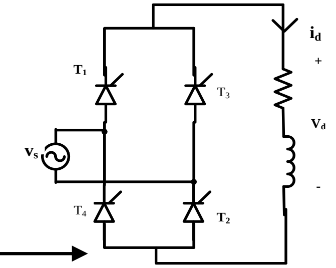
T_1 and T_2 conduct

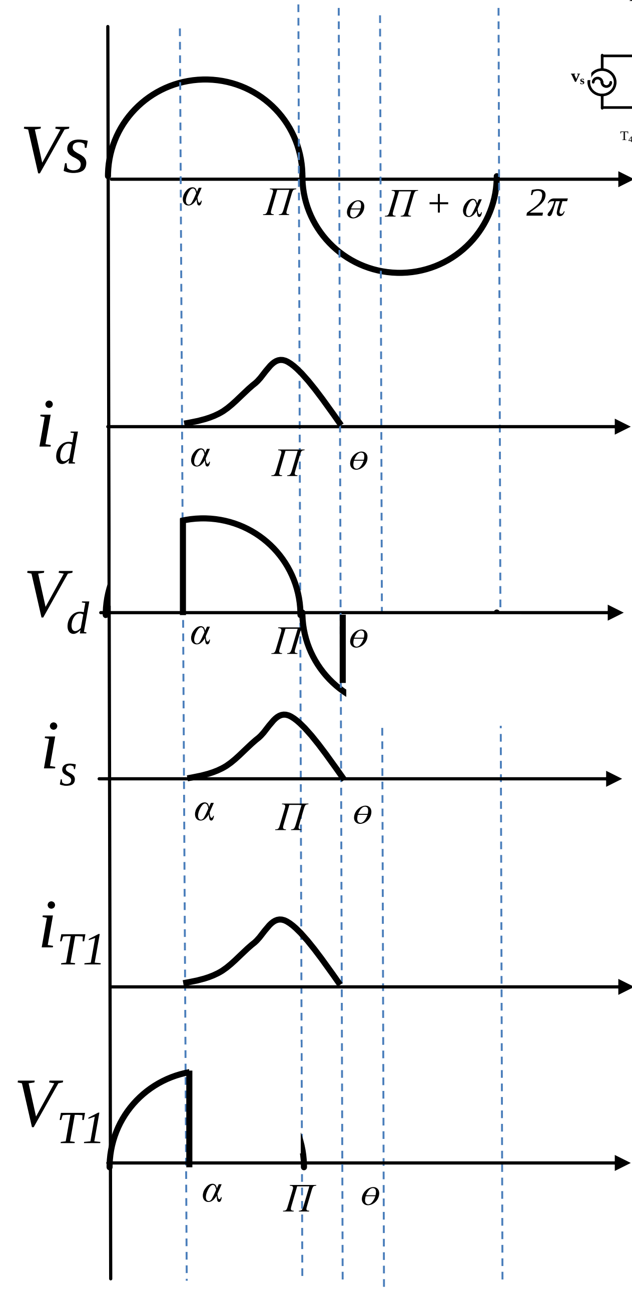
- $v_d = \underline{\alpha \rightarrow \theta_1} = v_s = \sqrt{2}v_s \sin \omega t$
- $i_d = \quad Ri_d(t) + L\dfrac{di_d(t)}{dt}$
- $i_s = i_d$
- $v_{T1} = 0, \quad v_{T2} = 0$
- $v_{T3} = -v_s$
- At $\omega t = \theta_1$, $i_d = 0$
- $i_{T1} = i_{T2} = 0$
- Both T_1 and T_2 go to the off state

- $\underline{\theta_1 \rightarrow \pi + \alpha}$

T_1 T_2 T_3 and T_4 remain in the off state

$i_s = i_d = 0$, $v_d = 0$,

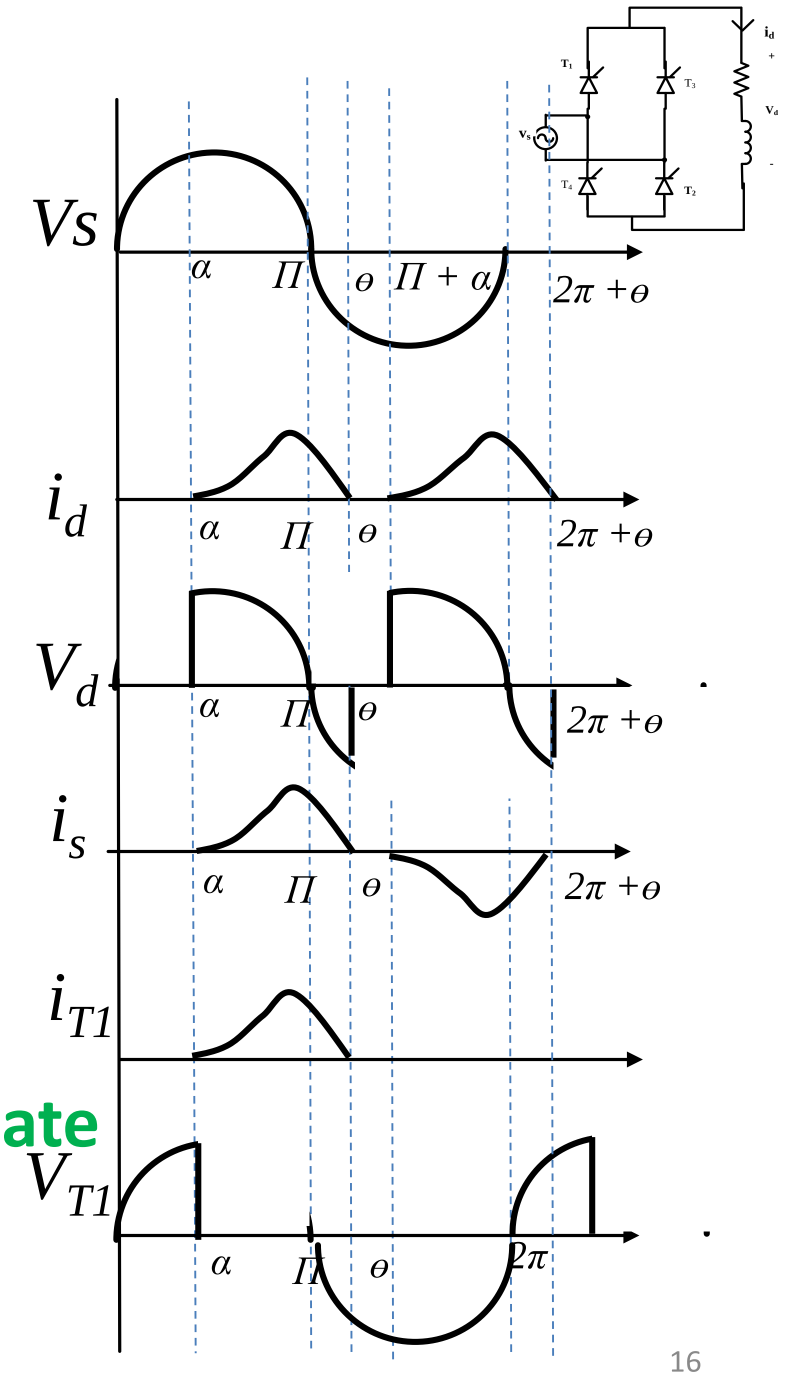




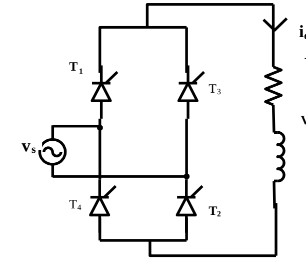
- $\pi+\alpha \rightarrow \pi+\theta_1$ T_3 and T_4 conduct
- $v_d = Ri_d(t) + L\dfrac{di_d(t)}{dt} = -v_s = -\sqrt{2}v_s \sin \omega t$
- i_d =
- $i_s = -i_d$
- $v_{T1} = v_s = v_{T2}$
- At $\omega t = \pi + \theta_1$, $i_d = 0$
- $i_{T3} = i_{T4} = 0$
- Both T_3 and T_4 go to the off state
- $\pi+\theta_1 \rightarrow 2\pi+\alpha$

T_1 T_2 T_3 and T_4 remain in the off state

$i_s = i_d = 0$, $v_d = 0$,



R – L Load, Continuous Current Mode



For $\omega t < \alpha$, T_3 & T_4 were conducting

At $\omega t = \alpha$, $i_d = I_1$ $(\neq 0)$

T_1 and T_2 are turned on

$\alpha \rightarrow \Pi + \alpha$

T_1 and T_2 conduct.

T_3 and T_4 are in the off state

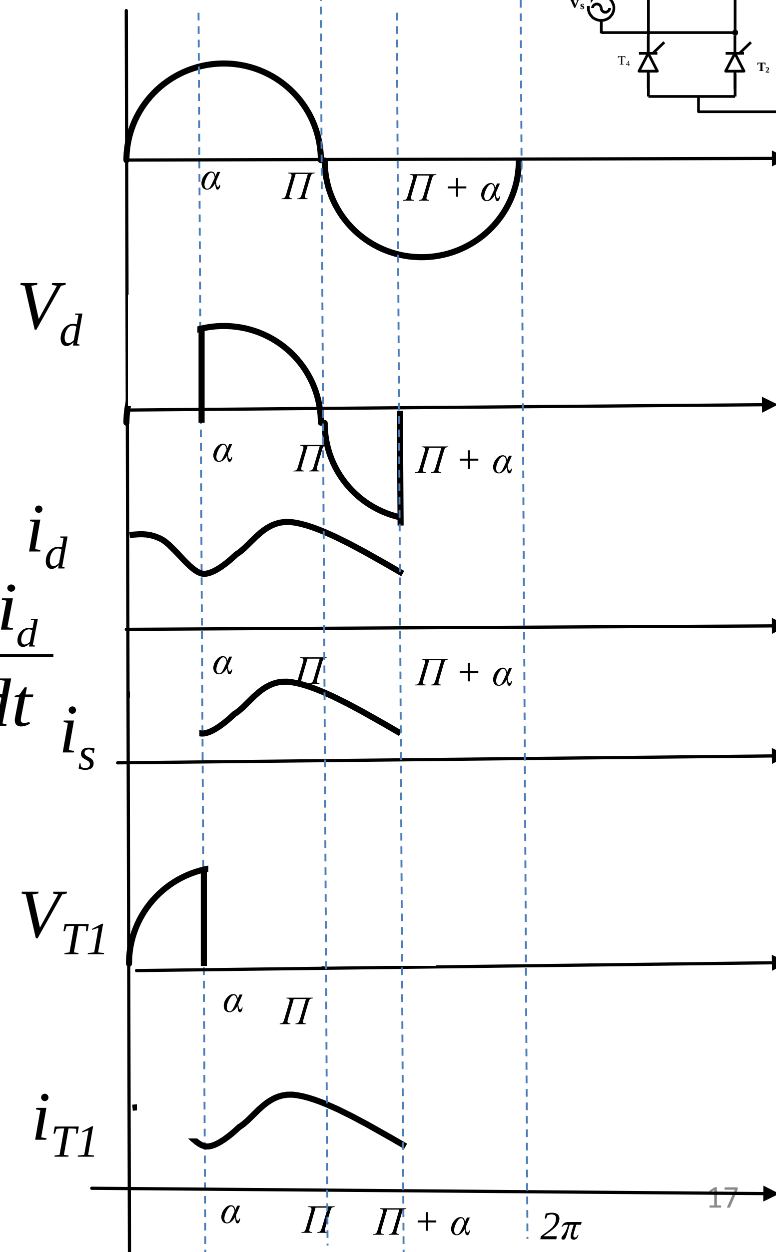
$$V_d = V_s = \sqrt{2}V_s \sin \omega t = Ri_d + L\frac{di_d}{dt}$$

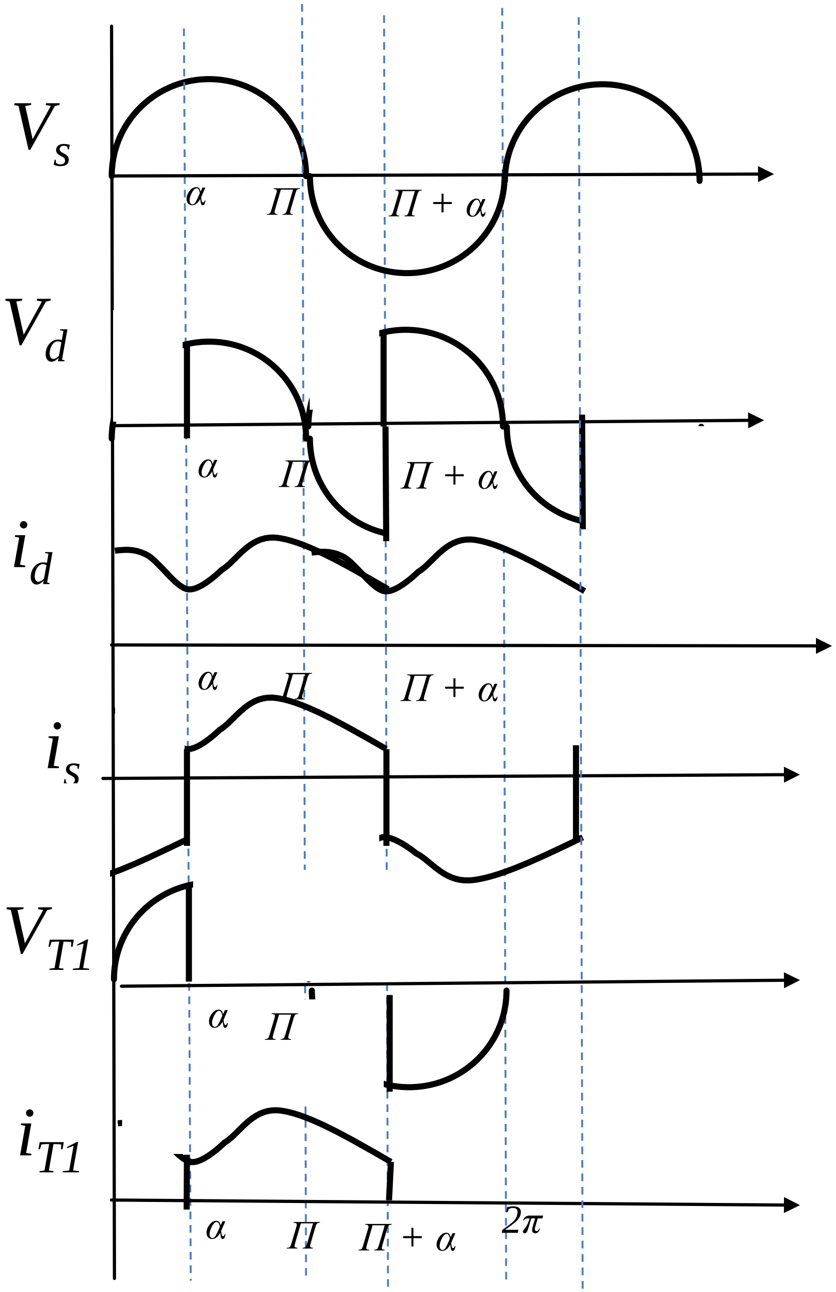
$$i_s = i_d$$

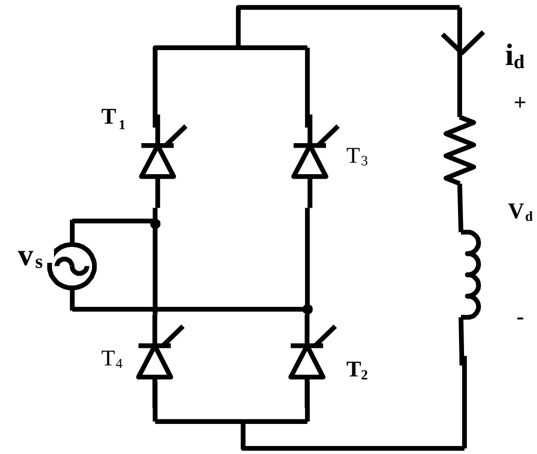
$$V_{T3} = -v_s = V_{T4}$$

$$At\ \omega t = \alpha,\ i_d = +I_1$$

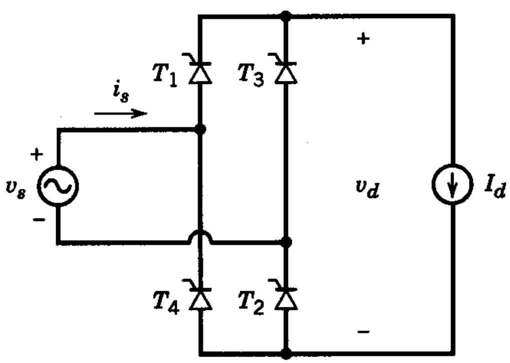
$$RI_1 + L\left.\frac{di_d}{dt}\right|_{\omega t=\alpha} = \sqrt{2}V_s \sin \alpha$$

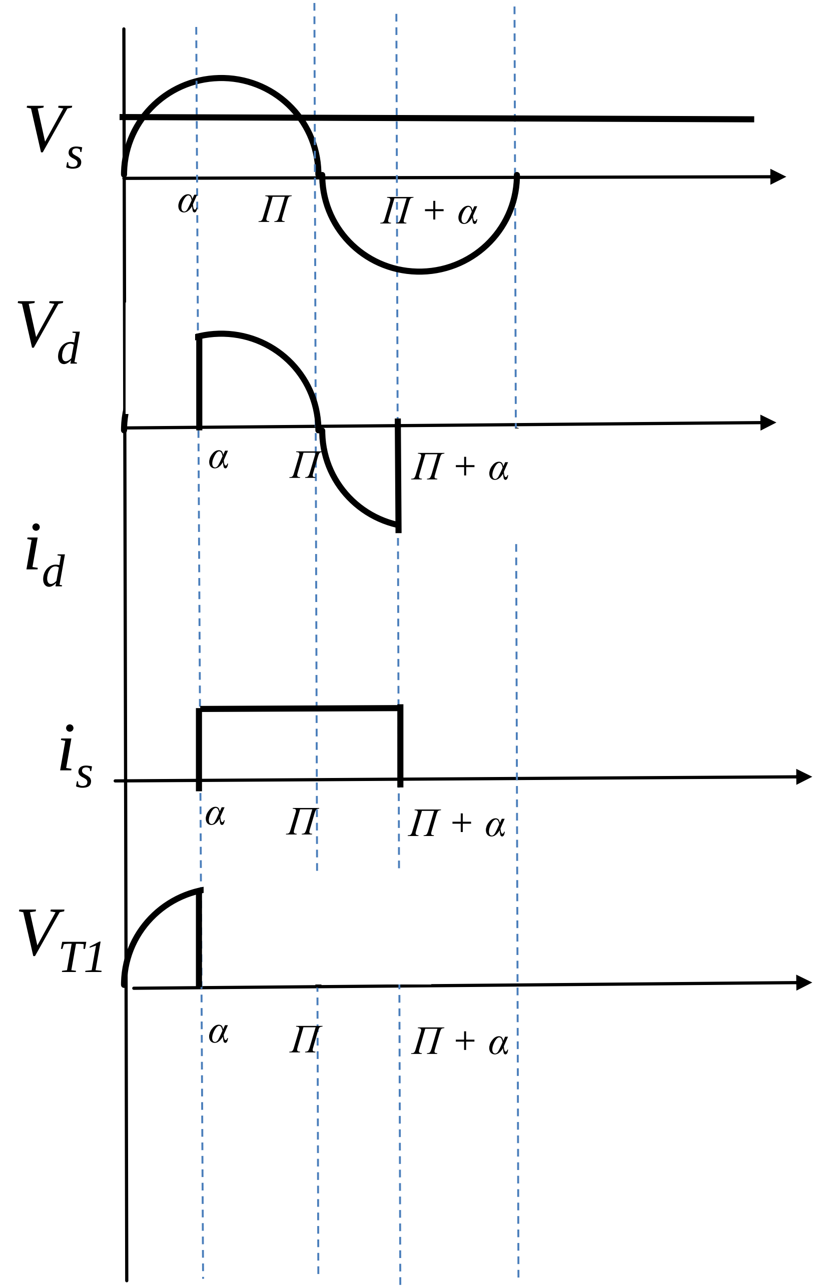



V_s
α
Π
Π + α
V_d
α
Π
Π + α
i_d
α
Π
Π + α
i_s
V_{T1}
α
Π
i_{T1}
α
Π
Π + α
2π

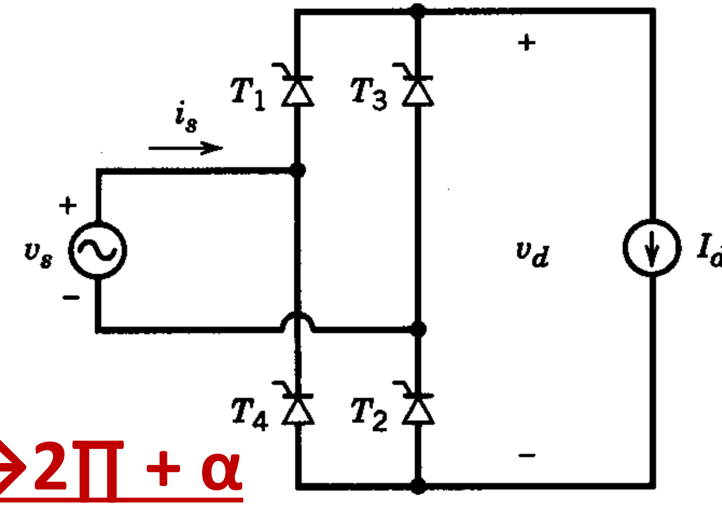

i_d
+
T_1
T_3
V_d
V_s
-
T_4
T_2

- At $\Pi + \alpha$, T_3 and T_4 are turned on
- T_1 and T_2 are line commutated
- $\Pi + \alpha \rightarrow 2\Pi + \alpha$
- T_3 and T_4 conduct.
- $V_d = -V_s$
- $i_s = -i_d$
- $V_{T1} = v_s = V_{T2}$





R – L Load, constant load current I_d



$\alpha \rightarrow \Pi + \alpha$

T_1 and T_2 conduct,
T_3 and T_4 are line commutated

$V_d = V_s$

$i_s = I_d$

$V_{T3} = - v_s = V_{T4}$

- $\Pi + \alpha \rightarrow 2\Pi + \alpha$
- T_3 and T_4 conduct,
- T_1 and T_2 are line commutated
- $V_d =- V_s$
- $i_s = -I_d$
- $V_{T1} = v_s = V_{T2}$

$\Pi + \alpha \rightarrow 2\Pi + \alpha$

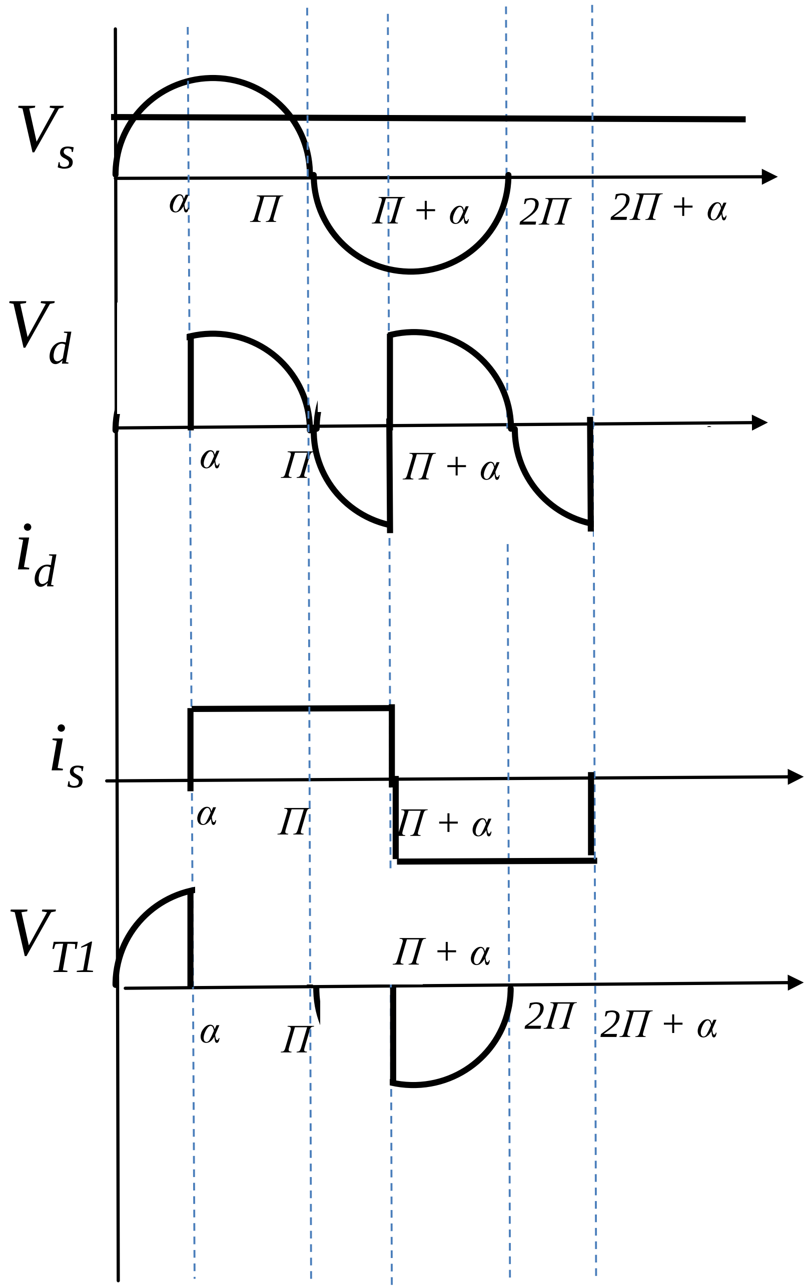
T_3 and T_4 conduct,

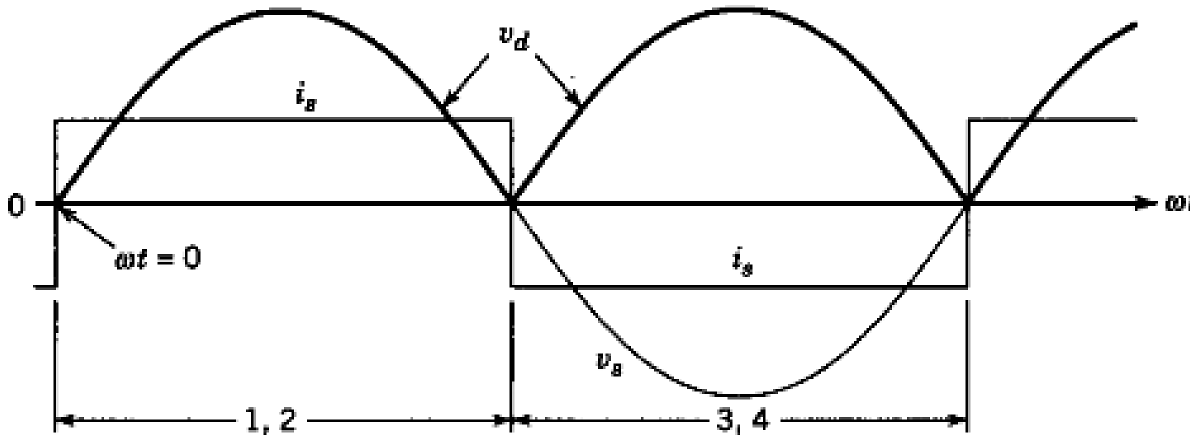
T_1 and T_2 are line commutated

$V_d = -V_s$

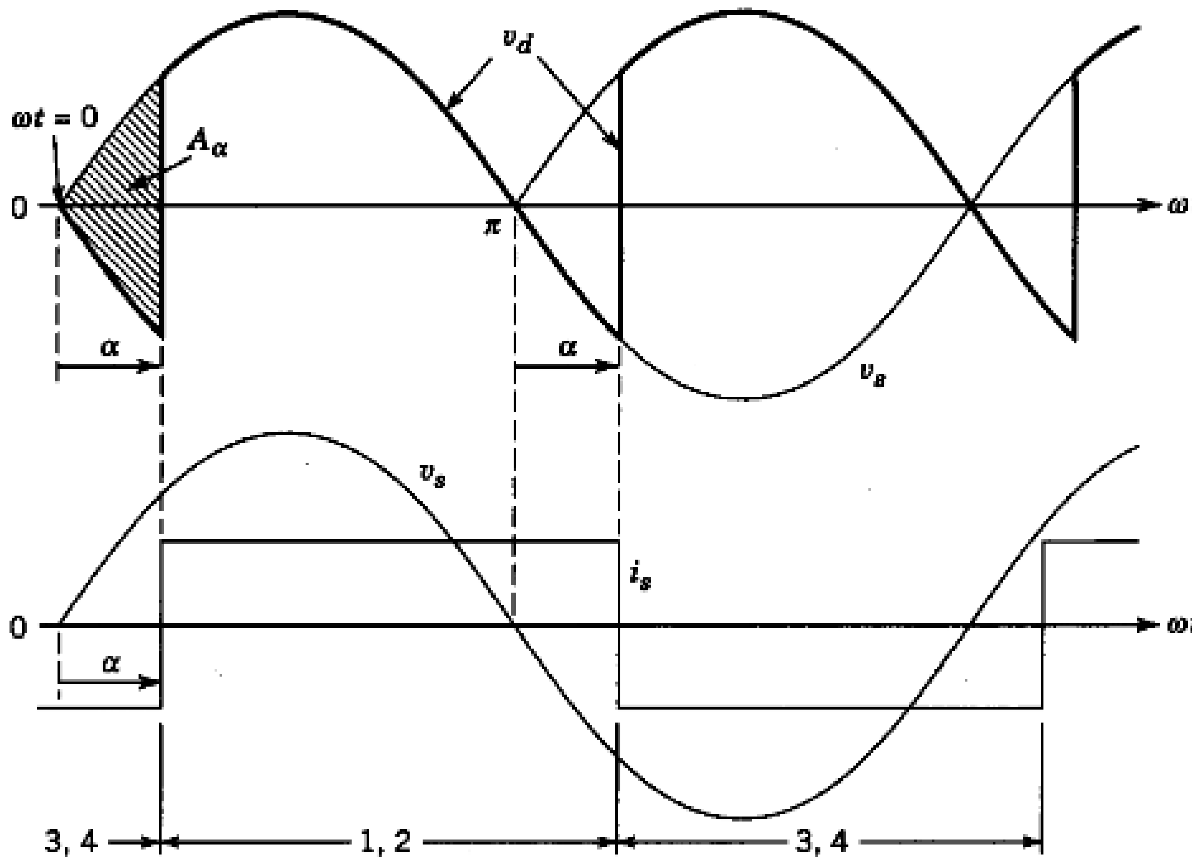
$i_s = -I_d$

$V_{T1} = v_s = V_{T2}$





(a) $\alpha = 0$



Performance parameters

- DC Side
- **Average load voltage**

$$V_{davg} = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha}(\sqrt{2}V_s \sin\omega t)d\omega t$$

$$= \frac{2\sqrt{2}V_s \cos\alpha}{\pi} = 0.9V_s \cos\alpha$$

V_{dRMS} $= V_s$

- Average Power through the converter =
- $P=V_{davg} I_d = 0.9V_sI_d\cos\alpha$
- Ripple frequency = twice the line frequency
- Voltage Ripple Factor K_V =

$$i_s(t) = a_0 + \sum_{1,2,3,..}^{\infty} a_n \cos n\omega t + b_n \sin n\omega t \qquad a_0 = 0$$

$$i_s(t) = \sum_{1}^{\infty} c_n \sin(n\omega t + \phi_n) \qquad c_n = \sqrt{a_n{}^2 + b_n{}^2} \qquad \phi_n = \tan^{-1}\frac{a_n}{b_n}$$

$$i_{s1}(t) = c_1 \sin(\omega t + \phi_1) \qquad c_1 = \sqrt{a_1{}^2 + b_1{}^2}$$

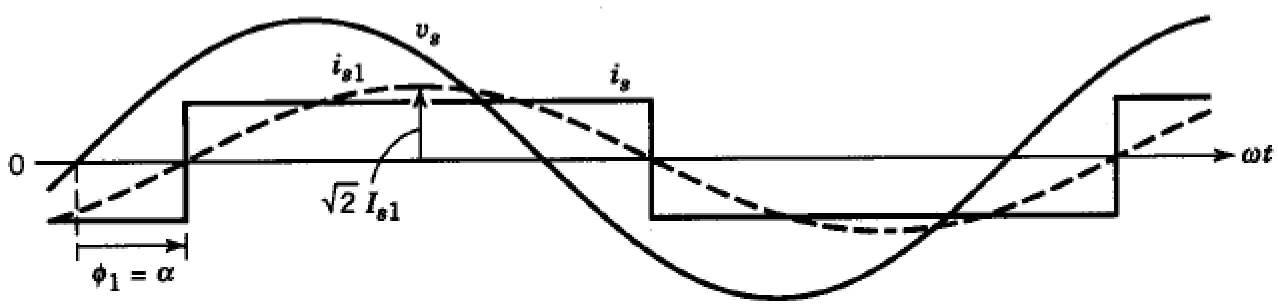
$$a_1 = \frac{1}{\pi}\left[\int_{\alpha}^{\pi+\alpha} I_d \cos\omega t \;\, \mathrm{d}\omega t + \int_{\pi+\alpha}^{2\pi+\alpha} -I_d \cos\omega t \;\, \mathrm{d}\omega t\right] = \quad -\frac{4I_d}{\pi}\sin\alpha$$

$$b_1 = \frac{1}{\pi}\left[\int_{\alpha}^{\pi+\alpha} I_d \sin\omega t \;\, \mathrm{d}\omega t + \int_{\pi+\alpha}^{2\pi+\alpha} -I_d \sin\omega t \;\, \mathrm{d}\omega t\right] = \quad = \frac{4I_d}{\pi}\cos\alpha$$

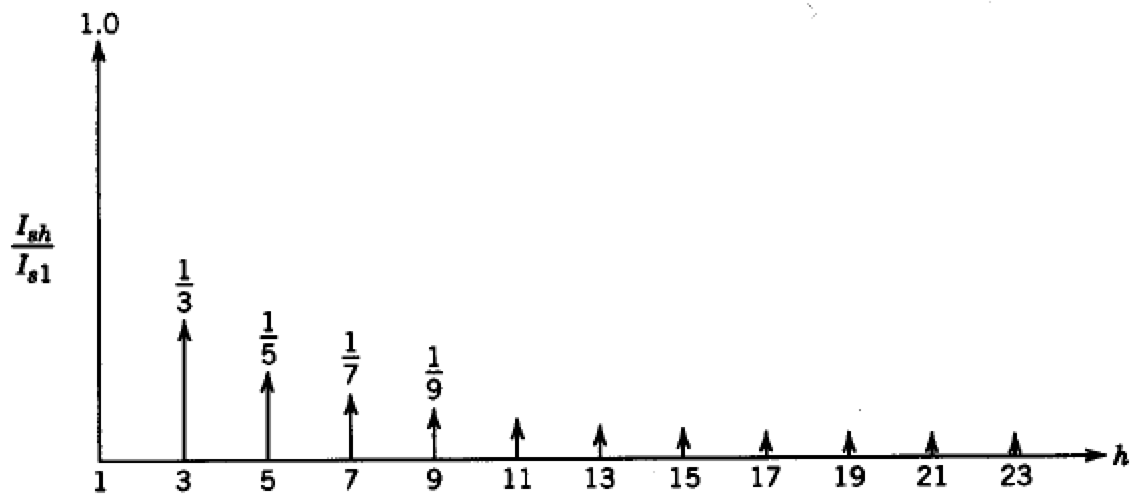
$$c_1 = \frac{4I_d}{\pi} \qquad \phi_1 = \tan^{-1}\frac{a_1}{b_1} = -\alpha \qquad i_{s1} = \frac{4I_d}{\pi}\sin(\omega t - \alpha)$$

- RMS value of i_{s1} $\quad I_{s1} = \dfrac{2\sqrt{2}I_d}{\pi}$
- RMS value of h^{th} harmonic i_{sh} $\quad I_{sh} = \dfrac{I_{s1}}{h}$
- $h = 3, \; 5, \; 7, \ldots\ldots$
- RMS value of total source current i_s
- $I_s = I_d$

$$I_{sh} = \begin{cases} 0 \ for \ even \ h \\ \dfrac{I_{s1}}{h} \ for \ odd \ h \end{cases}$$



(a)



Harmonic spectrum

- **Total harmonic distortion THD =**
- $$\sqrt{\frac{I_s{}^2 - I_{s1}{}^2}{I_{s1}{}^2}} = \quad 48.43 \quad \%$$

- Displacement factor
- DF = cos(-α) = cos α

- **Ac side Power factor =**

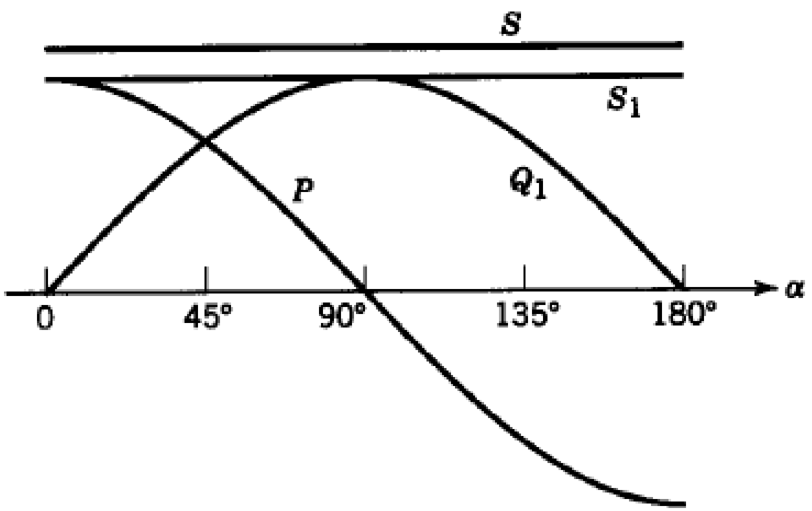
$$\frac{V_s I_{s1} \cos\phi_1}{V_s I_s} = \quad \frac{I_{s1} \cos\phi_1}{I_s} = \quad \frac{2\sqrt{2}}{\pi}\cos\alpha$$

- Power at the source side

Total apparent Power $S = V_s I_s$

Active Power P

$$P = V_s I_{s1} \cos \varphi 1$$



- Fundamental frequency current results in fundamental reactive volt amperes
  - $Q_1 = V_s\, I_{s1} \sin\phi_1 = V_s\, I_{s1} \sin\alpha$

- Fundamental frequency apparent power
- $S_1 = V_s\, I_{s1} \quad = \sqrt{P^2 + Q_1^{\,2}}$

$V_{davg\alpha}/V_{davg0}$

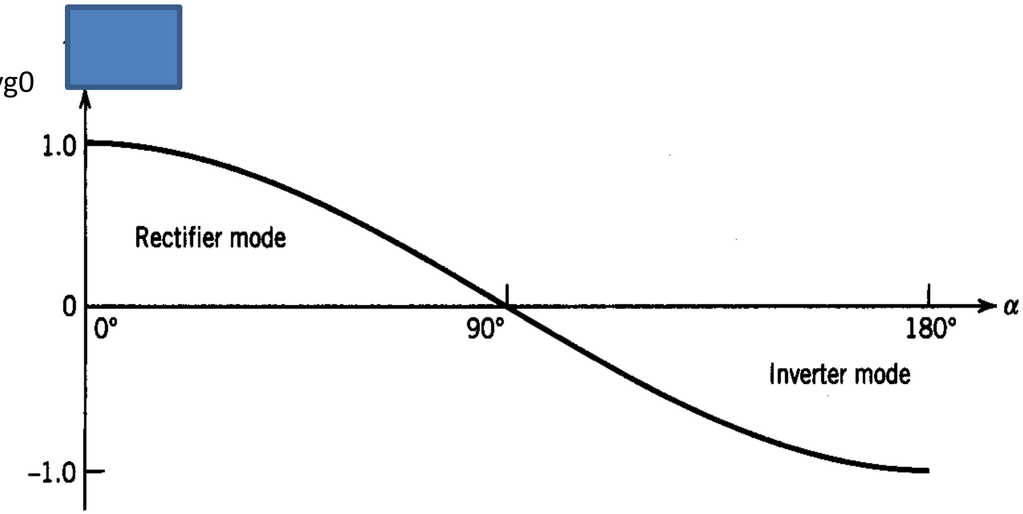
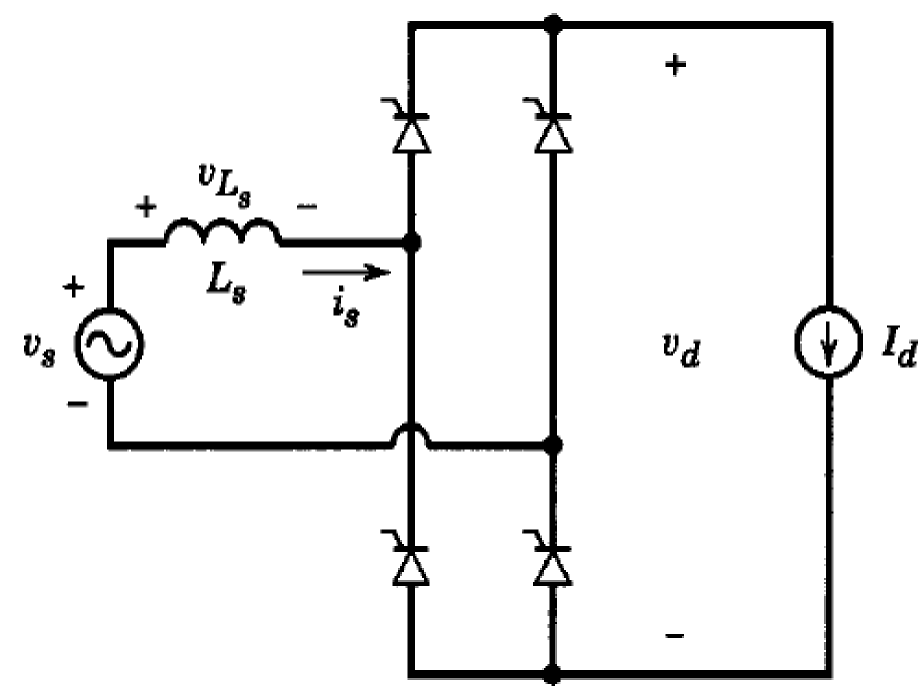


Figure 6-7 Normalized V_d as a function of α.

Effect of source inductance

- The current commutation takes a finite time called μ or commutation interval

constant load current I_d



$0 \rightarrow \alpha$

T_3 and T_4 are on

T_1 and T_2 are off

$v_d = -v_s$

$i_d = I_d$

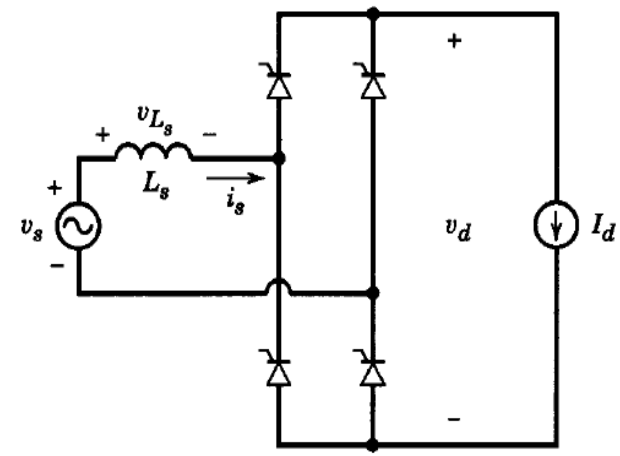
$i_s = -I_d$

$V_{T3} = 0 = V_{T4}$

$V_{T1} = v_s = V_{T2}$

At α, T_1 and T_2 are turned on

- $\alpha \rightarrow \alpha + \mu$
- i_{T3} and i_{T4} decrease from $I_d \rightarrow 0$



i_{T1} and i_{T2} rise from $0 \rightarrow I_d$

T_1, T_2, T_3, T_4 are on

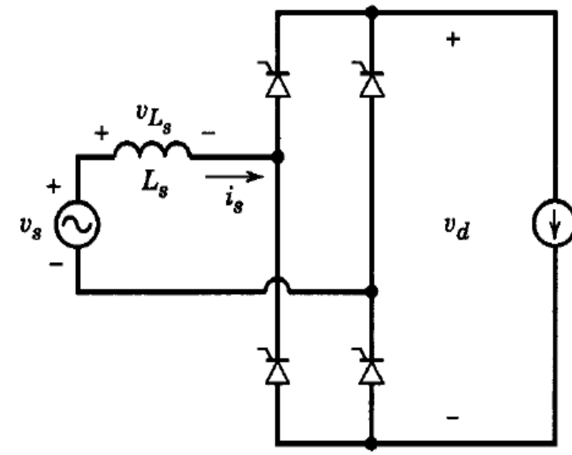
$V_d = 0, \quad i_d = I_d$

i_s varies from $-I_d$ to $\rightarrow I_d$.

$v_{Ls} = v_s$

At $(\alpha + \mu)$, $i_{T3} = 0 = i_{T4}$ T_3 and T_4 are turned off naturally

- $\underline{\alpha + \mu \rightarrow \Pi + \alpha}$



T_1 and T_2 are on

T_3 and T_4 are off

$V_d = V_s$

$i_d = I_d$

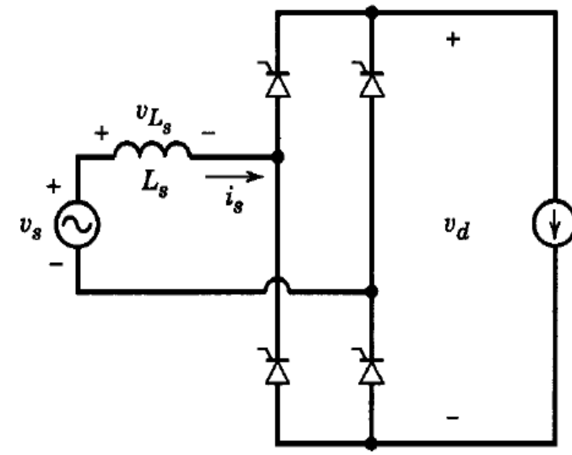
$i_s = I_d$

$V_{T1} = 0 = V_{T2}$

$V_{T3} = -v_s = V_{T4}$

At $\underline{\Pi + \alpha}$, T_3 and T_4 are turned on

- $\Pi + \alpha \rightarrow \Pi + \alpha + \mu$
- i_{T1}, i_{T2} decreases from $I_d \rightarrow 0$



i_{T3}, i_{T4} rises from $0 \rightarrow I_d$

T_1, T_2, T_3, T_4 conduct

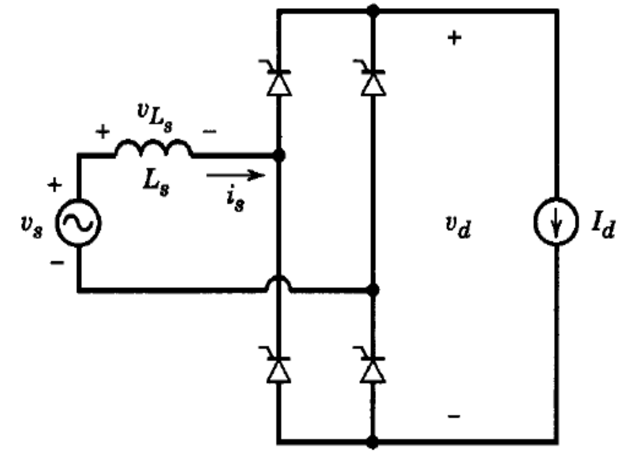
$V_d = 0$, $i_d = I_d$

i_s varies from $+I_d$ to $\rightarrow -I_d$.

$v_{Ls} = v_s$

At $(\Pi + \alpha + \mu)$, $i_{T1} = 0 = i_{T2}$ T_1 and T_2 are turned off

- <u>$\Pi + \alpha + \mu \rightarrow 2\Pi + \alpha$</u>
- T_3 and T_4 are on
- T_1 and T_2 are off



- $V_d = - V_s$
- $i_d = I_d$
- $i_s = - I_d$
- $V_{T1} = v_s = V_{T2}$
- $V_{T3} = 0 = V_{T4}$
- $v_{Ls} = 0$

- **During the commutation period μ, T_1, T_2, T_3 and T_4 are on**

$$V_{LS} = V_S$$

$$\sqrt{2}V_s \sin \omega t = L_s \frac{di_s(t)}{dt} = \omega L_s \frac{di_s(t)}{d\omega t}$$

$$\frac{\sqrt{2}V_s}{\omega L_s} \sin \omega t \; d\omega t = di_s(t)$$

$$\int_{\alpha}^{\alpha+\mu} \frac{\sqrt{2}V_s}{\omega L_s} \sin \omega t d\omega t = \int_{-I_d}^{+I_d} di_s(t)$$

$$\frac{\sqrt{2}V_s}{\omega L_s}\left[\cos \alpha - \cos(\alpha + \mu)\right] = 2I_d$$

$$\cos(\alpha + \mu) = \cos \alpha - \frac{2\omega L_s I_d}{\sqrt{2}V_s}$$

- Average o/p voltage = $V_{davg} =$

$$\frac{1}{\pi}\int_{\alpha+\mu}^{\pi+\alpha}\sqrt{2}V_s \sin\omega t\, d\omega t =$$

$$= \frac{2\sqrt{2}V_s}{\pi}\cos\alpha - \frac{2\omega L_s I_d}{\pi}$$