राष्ट्रीय प्रौद्योगिकी संस्थान कालिकट
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EE6303D Dynamics of Electrical Machines (DEM)

Module 2

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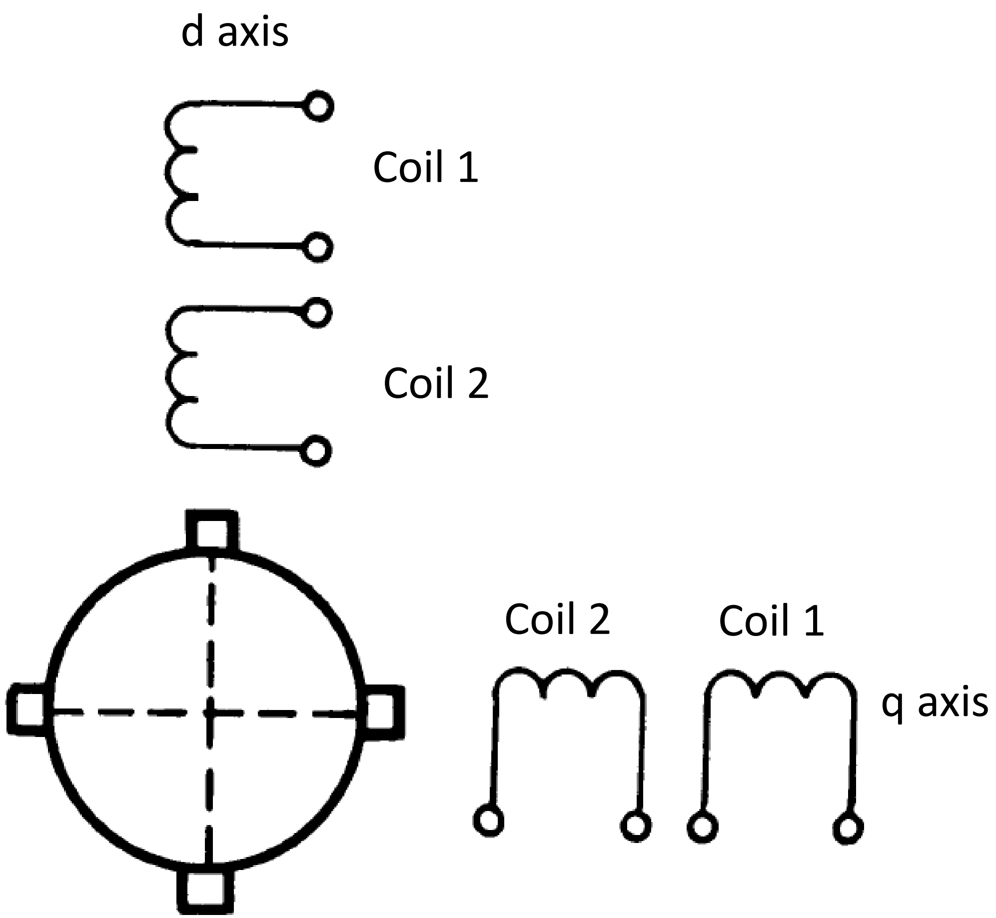
PRIMITIVE MACHINE EQUATIONS ALONG STATIONARY AXES

Rotor Voltages



$$V_{dr} = R_{dr} i_{dr} + \frac{d\psi_{dr}}{dt} + B_{qr} p\theta$$

$$V_{qr} = R_{qr} i_{qr} + \frac{d\psi_{qr}}{dt} - B_{dr} p\theta$$

<u>flux density terms</u>

$$B_{qr} = L'_{qr} i_{qr} + M'_{q1} i_{qs1} + M'_{q2} i_{qs2}$$

$$B_{dr} = L'_{dr} i_{dr} + M'_{d1} i_{ds1} + M'_{d2} i_{ds2}$$

$$V_{dr} = R_{dr} i_{dr} + L_{dr} p i_{dr} + M_{d1} p i_{ds1} + M_{d2} p i_{ds2} + L'_{qr} i_{qr} p\theta + M'_{q1} i_{qs1} p\theta + M'_{q2} i_{qs2} p\theta$$

$$V_{qr} = R_{qr} i_{qr} + L_{qr} p i_{q2} + M_{q1} p i_{qs1} + M_{q2} p i_{qs2} - L'_{dr} i_{dr} p\theta - M'_{d1} i_{ds1} p\theta - M'_{d2} i_{ds2} p\theta$$

PRIMITIVE MACHINE EQUATIONS ALONG STATIONARY AXES

$$[\mathbf{V}] = [\mathbf{Z}][\mathbf{i}]$$

$$[\mathbf{V}] = [\mathbf{R}][\mathbf{i}] + [\mathbf{L}]p[\mathbf{i}] + [\mathbf{G}][\mathbf{i}]p\theta$$

$$\mathbf{V} = \mathbf{R}\mathbf{i} + \mathbf{L}p\mathbf{i} + \mathbf{G}\mathbf{i}p\theta$$

R =

	ds1	ds2	dr	qr	qs1	qs2
ds1	R_{ds1}					
ds2		R_{ds2}				
dr			R_r			
qr				R_r		
qs1					R_{qs1}	
qs2						R_{qs2}

L =

	ds1	ds2	dr	qr	qs1	qs2
ds1	L_{ds1}	M_{d12}	M_{d1}			
ds2	M_{d12}	L_{ds2}	M_{d2}			
dr	M_{d1}	M_{d2}	L_{dr}			
qr				L_{qr}	M_{q1}	M_{q2}
qs1				M_{q1}	L_{qs1}	M_{q12}
qs2				M_{q2}	M_{q12}	L_{qs2}

G =

	ds1	ds2	dr	qr	qs1	qs2
ds1						
ds2						
dr				L'_{qr}	M'_{q1}	M'_{q2}
qr	$-M'_{d1}$	$-M'_{d2}$	$-L'_{dr}$			
qs1						
qs2						

$$\begin{bmatrix} V_{ds1} \\ V_{ds2} \\ V_{dr} \\ V_{qr} \\ V_{qs1} \\ V_{qs2} \end{bmatrix} = \begin{array}{cc} & \begin{array}{cccccc} \text{ds1} & \text{ds2} & \text{dr} & \text{qr} & \text{qs1} & \text{qs2} \end{array} \\ \begin{array}{c} \text{ds1} \\ \text{ds2} \\ \text{dr} \\ \text{qr} \\ \text{qs1} \\ \text{qs2} \end{array} & \begin{bmatrix} R_{ds1}+L_{ds1}p & M_{d12}p & M_{d1}p & & & \\ M_{d12}p & R_{ds2}+L_{ds2}p & M_{d2}p & & & \\ M_{d1}p & M_{d2}p & R_r+L_{dr}p & L'_{qr}p\theta & M'_{q1}p\theta & M'_{q2}p\theta \\ -M'_{d1}p\theta & -M'_{d2}p\theta & -L'_{dr}p\theta & R_r+L_{qr}p & M_{q1}p & M_{q2}p \\ & & & M_{q1}p & R_{qs}+L_{qs1}p & M_{q12}p \\ & & & M_{q2}p & M_{q12}p & R_{qs2}+L_{qs2}p \end{bmatrix} \end{array} \cdot \begin{bmatrix} i_{ds1} \\ i_{ds2} \\ i_{dr} \\ i_{qr} \\ i_{qs1} \\ i_{qs2} \end{bmatrix}$$

SQUIRREL CAGE INDUCTION MOTOR MODEL FROM PRIMITIVE MACHINE EQUATIONS

Step 1

Remove coil 2 and corresponding rows and column

Replace the operator p by $j\omega_1$

$p\theta = \omega_r$

$$\omega_r = \omega_1(1-s)$$

$$\begin{bmatrix} V_{ds1} \\ V_{dr} \\ V_{qr} \\ V_{qs1} \end{bmatrix} = \begin{array}{c|cccc} & ds1 & dr & qr & qs1 \\ \hline ds1 & R_{ds1}+j\omega_1 L_{ds1} & j\omega_1 M_{d1} & & \\ dr & j\omega_1 M_{d1} & R_r + j\omega_1 L_{dr} & \omega_1(1-s)M_{q1} & \omega_1(1-s)M_{q1} \\ qr & -\omega_1(1-s)M_{d1} & -\omega_1(1-s)L_{dr} & R_r + j\omega_1 L_{qr} & j\omega_1 M_{q1} \\ qs1 & & & j\omega_1 M_{q1} & R_{qs1}+j\omega_1 L_{qs1} \end{array} \begin{bmatrix} i_{ds1} \\ i_{dr} \\ i_{qr} \\ i_{qs1} \end{bmatrix}$$

SQUIRREL CAGE INDUCTION MOTOR MODEL FROM PRIMITIVE MACHINE EQUATIONS

Step 2

$$R_{ds1} = R_{qs1} = R_1, \qquad R_r = R_2$$

$$L_{ds1} = L_{qs1} = L_1, \qquad L_{dr} = L_{qr} = L_2$$

$$M_{d1} = M_{q1} = M$$

$$M'_{d1} = M_{d1} = M$$

$$M'_{q1} = M_{q1} = M$$

stator coils which are symmetrically distributed

air gap is uniform

flux wave is sinusoidally distributed in space and hence the coefficients of mutual inductance for transformer and generated voltages are the same

$$\begin{bmatrix} V_{ds1} \\ V_{dr} \\ V_{qr} \\ V_{qs1} \end{bmatrix} = \begin{array}{c|cccc} & ds1 & dr & qr & qs1 \\ \hline ds1 & R_1 + jX_1 & jX_m & & \\ dr & jX_m & R_2 + jX_2 & (1-s)X_2 & (1-s)X_m \\ qr & -(1-s)X_m & -(1-s)X_2 & R_2 + jX_2 & jX_m \\ qs1 & & & jX_m & R_1 + jX_1 \end{array} \begin{bmatrix} i_{ds1} \\ i_{dr} \\ i_{qr} \\ i_{qs1} \end{bmatrix}$$

SQUIRREL CAGE INDUCTION MOTOR MODEL FROM PRIMITIVE MACHINE EQUATIONS

Step 3

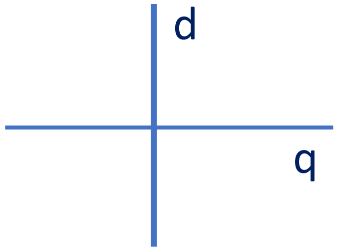
$$
\begin{bmatrix} V_1 \\ 0 \\ 0 \\ -jV_1 \end{bmatrix}
=
\begin{array}{c|cccc|}
 & \mathrm{ds1} & \mathrm{dr} & \mathrm{qr} & \mathrm{qs1} \\
\hline
\mathrm{ds1} & R_1 + jX_1 & jX_m & & \\
\mathrm{dr} & jX_m & R_2 + jX_2 & (1-s)X_2 & (1-s)X_m \\
\mathrm{qr} & -(1-s)X_m & -(1-s)X_2 & R_2 + jX_2 & jX_m \\
\mathrm{qs1} & & & jX_m & R_1 + jX_1 \\
\hline
\end{array}
\cdot
\begin{bmatrix} I_1 \\ I_2 \\ -jI_2 \\ -jI_1 \end{bmatrix}
$$

net m.m.f. they produce rotates at synchronous speed

During balanced operation, net MMF

$$N_1(I_1^2 \sin^2 \omega_1 t + I_1^2 \cos \omega_1 t)^{\frac{1}{2}}$$

rotor voltages are zero since the rotor coils in an induction motor are short-circuited



SQUIRREL CAGE INDUCTION MOTOR MODEL FROM PRIMITIVE MACHINE EQUATIONS

dr

$$0 = jX_m I_1 - jI_1(1-s)X_m + (R_2 + jX_2)I_2 - j(1-s)X_2 I_2$$

$$= jI_1 s X_m + (R_2 + jsX_2)I_2$$

$$= jI_1 X_m + \left[\frac{R_2}{s} + jX_2\right] I_2$$

qr

$$0 = jI_1 X_m + \left[\frac{R_2}{s} + jX_2\right] I_2$$

Per phase voltage equation

$$\begin{bmatrix} V_1 \\ 0 \end{bmatrix} = \begin{array}{c|cc} & \text{ds1} & \text{dr} \\ \hline \text{ds1} & R_1 + jX_1 & jX_m \\ \text{dr} & jX_m & \dfrac{R_2}{s} + jX_2 \end{array} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ 0 \end{bmatrix} = \begin{array}{c|cc} & \text{ds1} & \text{dr} \\ \hline \text{ds1} & R_1 + jX_1 & jX_m \\ \text{dr} & jX_m & \dfrac{(R_2 + jX_2) + R_2(1-s)}{s} \end{array} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

SQUIRREL CAGE INDUCTION MOTOR MODEL FROM PRIMITIVE MACHINE EQUATIONS

Per phase voltage equation

$$\begin{bmatrix} V_1 \\ 0 \end{bmatrix} = \begin{array}{c|cc} & \text{ds1} & \text{dr} \\ \hline \text{ds1} & R_1 + jX_1 & jX_m \\ \text{dr} & jX_m & \frac{R_2}{s} + jX_2 \end{array} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} V \\ 0 \end{bmatrix} = \begin{array}{c|cc} & \text{ds} & \text{dr} \\ \hline \text{ds} & R_{ds} + jX_{ds} & jX_m \\ \text{dr} & jsX_m & R_{dr} + jsX_{dr} \end{array} \cdot \begin{bmatrix} i_{ds} \\ i_{dr} \end{bmatrix}$$

$$\begin{bmatrix} i_{ds} \\ i_{dr} \end{bmatrix} = \frac{1}{D} \begin{array}{c|cc} & \text{ds} & \text{dr} \\ \hline \text{ds} & Z_{dr} & -jX_m \\ \text{dr} & -jsX_m & Z_{ds} \end{array} \cdot \begin{bmatrix} V \\ 0 \end{bmatrix}$$

D is the determinant $(Z_{ds} Z_{dr} + s X_m^2)$

$$i_{ds} = \frac{1}{D}(Z_{dr} V)$$

$$i_{dr} = jsX_m V/D$$

$$T = -\mathbf{i}^* \mathbf{G} \mathbf{i}$$

$$\mathbf{G} = \begin{array}{c|cccccc} & \text{ds1} & \text{ds2} & \text{dr} & \text{qr} & \text{qs1} & \text{qs2} \\ \hline \text{ds1} & & & & & & \\ \text{ds2} & & & & & & \\ \text{dr} & & & & L'_{qr} & M'_{q1} & M'_{q2} \\ \text{qr} & -M'_{d1} & -M'_{d2} & -L'_{dr} & & & \\ \text{qs1} & & & & & & \\ \text{qs2} & & & & & & \end{array}$$

Per phase torque

$$T_{ph} = -i_{ds} jX_m i_{dr}^* \quad \text{(synchronous W)}$$

Generated Shaft torque in Sync. Watts/phase = Per phase synchronous power - Per phase rotor copper loss

Per phase rotor copper loss= $i_{dr}^2 R_r$

$$= i_{dr}^2 R_r \left(\frac{1}{s} - 1\right)$$

Substitute for i_{ds} in terms of V
Substitute for V in terms of i_{dr}
$Z_{dr}=R_r$

SQUIRREL CAGE INDUCTION MOTOR MODEL FROM PRIMITIVE MACHINE EQUATIONS

Inference

$$T = -\mathbf{i}^* \mathbf{G} \mathbf{i}$$

$$= -i^*_{\mathrm{qr}} M_{\mathrm{d}} i_{\mathrm{ds}} + i^*_{\mathrm{dr}} M_{\mathrm{q}} i_{\mathrm{qs}}$$