Induction Motors

Problems

Example 5.1

(i) Find the efficiency of an induction motor operating at full load. The machine details are given in the following:

2000 hp, 2300 V, 3 phase, Star connected, 4 pole, 60 Hz, Full load slip = 0.03746

$$R_s = 0.02\ \Omega\ ;\ R_r = 0.12\ \Omega;\ R_c = 451.2\ \Omega;\ X_m = 50\ \Omega;\ X_{ls} = X_{lr} = 0.32\ \Omega$$

(ii) The line power factor needs to be improved to unity by installing capacitors at the input terminals of the induction motor. Calculate the per-phase capacitance required to obtain a line power factor of unity.

Solution **(i)** The equivalent circuit of the induction motor is utilized to solve the problem. First, calculate the equivalent input impedance of the induction motor. This is achieved by finding the equivalent impedance between the magnetizing and core-loss resistance, and then the combined impedance of this with the rotor impedance is found, which, when added to the stator impedance, gives the equivalent impedance of the induction machine.

$$\text{Magnetizing-branch equivalent impedance, } Z_o = \frac{jX_m R_c}{R_c + jX_m} = 5.47 + j49.39\ \Omega$$

$$\text{Rotor impedance, } Z_r = \frac{R_r}{s} + jX_{lr} = \frac{0.12}{0.03447} + j0.32\ \Omega$$

Equivalent rotor and magnetizing impedance, $Z_{eq} = \frac{Z_o Z_r}{Z_o + Z_r} = 3.13 + j0.51\ \Omega$

Motor equivalent impedance, $Z_{im} = R_s + jX_{ls} + Z_{eq} = 3.15 + j0.83\ \Omega$

Phase input voltage, $V_{as} = \frac{V_{ll}}{\sqrt{3}} = \frac{2300}{\sqrt{3}} = 1327.9\ \text{V}$

Stator current, $I_s = \frac{V_{as}}{Z_{im}} = 394.2 - j104.1\ \text{A}$

Rotor current, $I_r = \frac{Z_{eq}}{Z_{im}} I_s = 393.87 - j78.07\ \text{A}$

No-load current, $I_o = I_s - I_r = 0.33 - j26.03\ \text{A}$

Core-loss current, $I_c = \frac{Z_o}{R_c} I_o = 2.85 - j0.28\ \text{A}$

Rotor angular speed,

$$\omega_m = (1 - s)\frac{\omega_s}{P/2} = (1 - s)\frac{2\pi f_s}{P/2} = 0.03746\,\frac{2 * \pi * 60}{4/2} = 181.43 \text{ rad/sec (mech)}$$

$$\text{Air-gap torque, } T_e = 3\,\frac{P}{2}\,|I_r|^2\,\frac{R_r}{s\omega_s} = 8220.1 \text{ N·m}$$

Mechanical power, $P = \omega_m T_e = 1491.2$ kW
Shaft power output, $P_m = P_s - P_m = 1491.2 - 0 = 1491.2$ kW
Stator resistive losses, $P_{sc} = 3|I_s|^2 R_s = 3 * 407.74^2 * 0.2 = 9.975$ kW
Rotor resistive losses, $P_{rc} = 3|I_r|^2 R_r = 3 * 401.53^2 * 0.12 = 58.04$ kW
Core losses, $P_{co} = 3|I_c|^2 R_c * 2.865^2 * 451.2 = 11.11$ (kW)
Input power, $P_i = P_m + P_{sce} + P_{rc} + P_{co} = 1491.2 + 9.975 + 58.04 + 11.11 = 1570.5$ kW

$$\%\text{ Efficiency, } \eta = \frac{P_s}{P_i}\,100 = \frac{1491.2}{1570.5}\,100 = 94.96\%$$

(ii) The principle of power-factor improvement with capacitor installation at the machine stator terminals is based on the capacitor's drawing a leading reactive current from the supply to cancel the lagging reactive current drawn by the induction machine. In order for the line power factor to be unity, the reactive component of the line current must be zero. The reactive line current is the sum of the capacitor and induction machine reactive currents.

Therefore, the capacitive reactive current (I_{cap}) has to be equal in magnitude but opposite in direction to the machine lagging reactive current, but the machine reactive current is the imaginary part of the stator current and is given by

$$I_{cap} + \text{imag}(I_s) = 0$$

$$\text{Hence, } I_{cap} = -\,\text{imag}(I_s) = -(-j104.1) = j104.1 \text{ A}$$

This current is controlled by the capacitors installed at the input, and therefore the capacitor required is

$$C = \frac{I_{cap}}{j\omega_s V_{as}} = \frac{j104.1}{j377 * 1327.9} = 20.792\ \mu\text{F}$$

Example 5.2

The no-load and locked-rotor test results for a three-phase, star-connected, 60-Hz, 2000-hp induction machine with a stator phase resistance of 0.02 Ω are as follows:

Test	Input line to line voltage, V	Line current, A	Three-phase input power, kW
No load	2300	26.55	11.617
Locked rotor	462.68	407.75	319.22

Find the machine equivalent-circuit parameters.

Solution **(i)** From the no-load test results:

Input power, $P_1 = 11.617/3 = 3.872$ kW/phase

Power factor, $\cos\phi_o = \dfrac{P_1}{V_{as}I_o} = \dfrac{11.617 * 10^3}{\sqrt{3} * 2300 * 26.55} = 0.1098$

Magnetizing current, $I_m = I_o \sin\phi_o = 26.39$ (A)

Core-loss branch current, $I_c = I_o \cos\phi_o = 2.9216$ (A)

Neglecting stator impedance, the following are calculated:

Magnetizing inductance, $L_m = \dfrac{V_{as}}{2\pi f_s I_m} = \dfrac{2300/\sqrt{3}}{2\pi * 60 * 26.39} = 0.1335$ H

Core-loss resistance, $R_c = \dfrac{V_{as}}{I_c} = \dfrac{2300/\sqrt{3}}{2.916} = 455.37\ (\Omega)$

(ii) From the locked-rotor test results:

$$\text{Power factor, } \cos\phi_{sc} = \frac{P_{sc}}{V_{sc} I_{sc}} = \frac{(319.22/3) * 10^3}{(462.68/\sqrt{3}) * 407.75} = 0.2137$$

$$\text{Phase angle, } \phi_{sc} = 1.355 \text{ rad}$$

$$\text{Locked-rotor impedance, } Z_{sc} = \frac{V_{sc}}{I_{sc}} = \frac{462.68/\sqrt{3}}{407.75} = \frac{267.13}{407.75} = 0.6551\Omega$$

$$\text{Stator-referred rotor resistance per phase, } R_r = Z_{sc}\cos\phi_{sc} - R_s = 0.12\ \Omega$$

$$\text{Total leakage reactance, } X_{eq} = Z_{sc}\sin\phi_{sc} = 0.64\ \Omega$$

The per-phase stator and rotor leakage reactances are half of the total leakage reactance; hence,

$$X_{ls} = X_{lr} = 0.32\ \Omega$$

and the leakage inductances are

$$L_{ls} = L_{lr} = 0.8488 \text{ mH}$$

(iii) Further refinement of the equivalent-circuit parameters: The stator impedance was neglected in the calculation of magnetizing inductance and core-loss resistance because, at that time, the stator leakage inductance was not available. Now, having obtained it from the locked rotor test, we can use the no-load equivalent circuit to recalculate the magnetizing inductance and core-loss resistance; their final values are obtained as 0.1326 H and 451.2 Ω, respectively.

Example

A 460-V, 25-hp, 60 Hz, four-pole, Y-connected induction motor has the following impedances in ohms per phase referred to the stator circuit:

$R_1 = 0.641\Omega \quad R_2 = 0.332\Omega$

$X_1 = 1.106\ \Omega \quad X_2 = 0.464\ \Omega \quad X_M = 26.3\ \Omega$

The total rotational losses are 1100 W and are assumed to be constant. The core loss is lumped in with the rotational losses. For a rotor slip of 2.2 percent at the rated voltage and rated frequency, find the motor's

1. Speed
2. Stator current
3. Power factor
4. P_{conv} and P_{out}
5. τ_{ind} and τ_{load}
6. Efficiency

Solution

1.
$$n_{sync} = \frac{120 f_e}{P} = \frac{120 \times 60}{4} = 1800 \text{ rpm}$$

$$n_m = (1-s)n_{sync} = (1-0.022)\times 1800 = 1760 \text{ rpm}$$

2.
$$Z_2 = \frac{R_2}{s} + jX_2 = \frac{0.332}{0.022} + j0.464$$

$$= 15.09 + j0.464 = 15.1\angle 1.76^\circ\ \Omega$$

$$Z_f = \frac{1}{1/jX_M + 1/Z_2} = \frac{1}{-j0.038 + 0.0662\angle -1.76^\circ}$$

$$= \frac{1}{0.0773\angle -31.1^\circ} = 12.94\angle 31.1^\circ\ \Omega$$

Solution

$$Z_{tot} = Z_{stat} + Z_f$$
$$= 0.641 + j1.106 + 12.94\angle 31.1^\circ\ \Omega$$
$$= 11.72 + j7.79 = 14.07\angle 33.6^\circ\ \Omega$$

$$I_1 = \frac{V_\phi}{Z_{tot}} = \frac{\frac{460\angle 0^\circ}{\sqrt{3}}}{14.07\angle 33.6^\circ} = 18.88\angle -33.6^\circ\ \text{A}$$

$$PF = \cos 33.6^\circ = 0.833 \quad \text{lagging}$$

3. $$P_{in} = \sqrt{3} V_L I_L \cos\theta = \sqrt{3} \times 460 \times 18.88 \times 0.833 = 12530\ \text{W}$$

4. $$P_{SCL} = 3 I_1^2 R_1 = 3(18.88)^2 \times 0.641 = 685\ \text{W}$$

$$P_{AG} = P_{in} - P_{SCL} = 12530 - 685 = 11845\ \text{W}$$

Solution

$$P_{conv} = (1-s)P_{AG} = (1-0.022)(11845) = 11585 \text{ W}$$

$$P_{out} = P_{conv} - P_{F\&W} = 11585 - 1100 = 10485 \text{ W}$$

$$= \frac{10485}{746} = 14.1 \text{ hp}$$

5. $$\tau_{ind} = \frac{P_{AG}}{\omega_{sync}} = \frac{11845}{2\pi \times 1800/60} = 62.8 \text{ N.m}$$

$$\tau_{load} = \frac{P_{out}}{\omega_m} = \frac{10485}{2\pi \times 1760/60} = 56.9 \text{ N.m}$$

6. $$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{10485}{12530} \times 100 = 83.7\%$$