Lecture - 19

Induction Motors

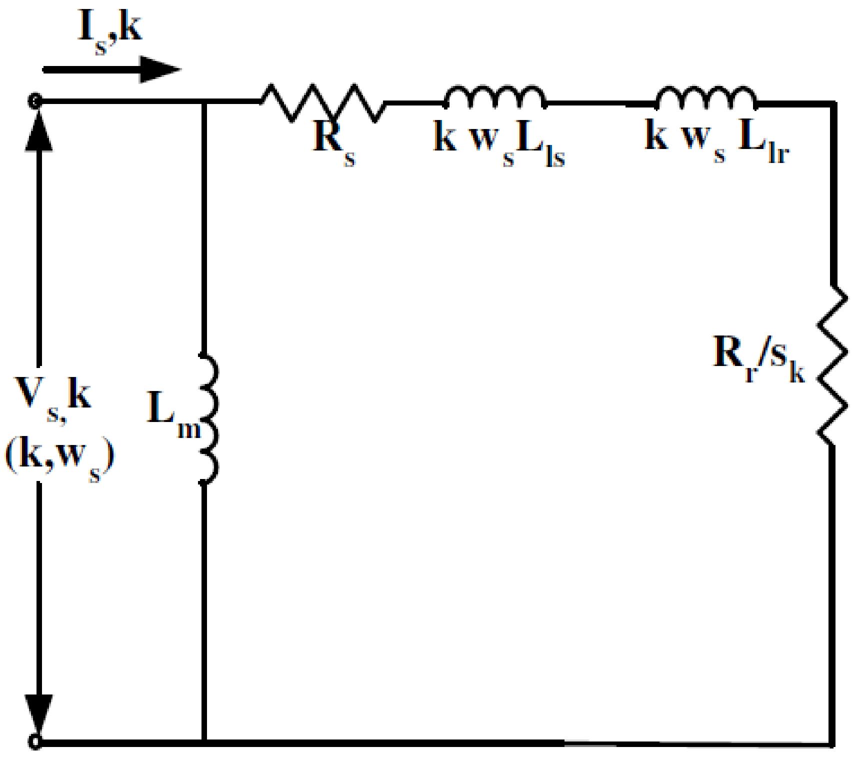
Operation from Non-Sinusoidal Sources

Operation from Non-Sinusoidal Sources

- When an induction motor is operated from an inverter, the applied voltages are not sinusoidal.
- They contain, besides the fundamental, odd harmonics.
- For example, in the six step inverter, harmonics of order 6m±1 are present.
- Therefore, it becomes necessary to study the response of the machine to these harmonic voltages.
- The equivalent circuit offers a convenient starting point for this study.

Operation from Non-Sinusoidal Sources

- The equivalent circuit of induction motor shown in Figure below is for a general harmonic of order k



- All reactances get multiplied by the harmonic order k.
- It is assumed that resistance values remain same, i.e. skin effect is neglected.

Operation from Non-Sinusoidal Sources

- The value of the slip s_k has to be interpreted properly.
- The slip is the ratio of (synchronous speed - rotor speed) to the synchronous speed.
- The rotor speed is the same, whatever be the harmonic considered.
- But the synchronous speed, i.e., the speed at which the stator mmf is rotating, depends on the harmonic being considered.
- In general, the mmf due to the k^{th} harmonic will rotate at k times the speed of the fundamental.
- However not all the harmonic mmfs rotate in the same direction as the fundamental.

Operation from Non-Sinusoidal Sources

- It has been pointed out that harmonics of the order 6m-1 (5,11,17,etc.) have a negative phase sequence and so the mmf produced by these harmonics will rotate in the opposite direction to the fundamental.
- Therefore, the slip for the 5th harmonic is given by

$$s_5 = \frac{-5w_s - w}{5w_s} = 1 + \frac{1}{5}\frac{w}{w_s} = 1 + \frac{1}{5}(1 - s_1)$$

- Since s_1 is very small, it can be approximately said that

$$s_5 = 1 + \frac{1}{5}$$

Similarly, $s_{11} = 1 + \frac{1}{11}$; $s_{17} = 1 + \frac{1}{17}$; etc.

Operation from Non-Sinusoidal Sources

- On the other hand, for a positive sequence harmonic such as 7th,

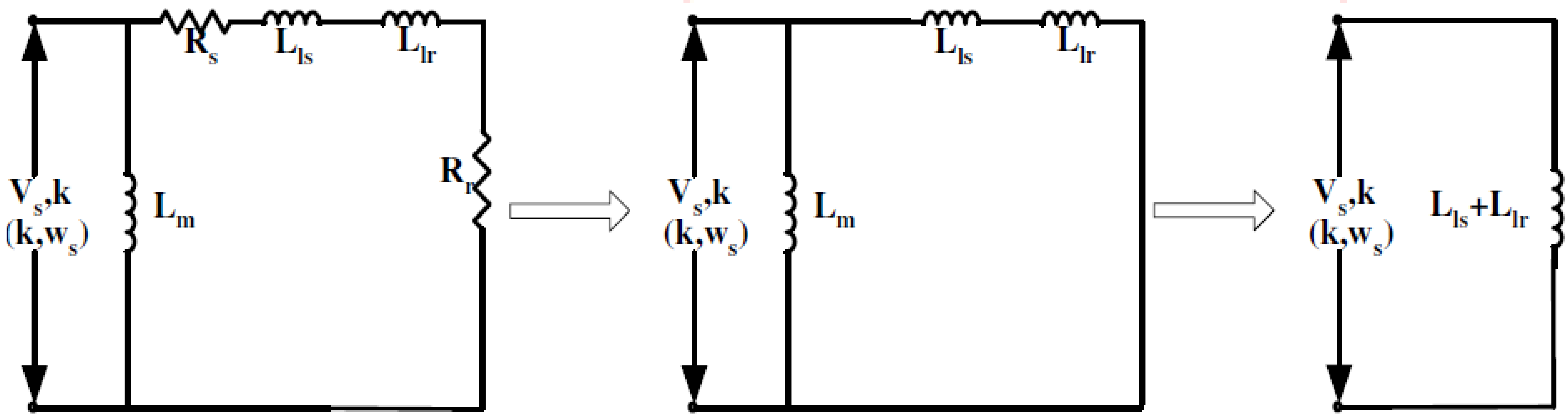
$$s_7 = \frac{7w_s - w}{7w_s} = 1 - \frac{1}{7}\frac{w}{w_s} = 1 - \frac{1}{7}(1 - s_1)$$

$$s_7 = 1 - \frac{1}{7}$$

similarly, $s_{13} = 1 - \frac{1}{13}; s_{19} = 1 - \frac{1}{19};$ etc.

Operation from Non-Sinusoidal Sources

- In general, therefore, the slip corresponding to the harmonics is very close to unity.
- Therefore the equivalent circuit can be successively simplified as follows.



Operation from Non-Sinusoidal Sources

- In the case of the six-step inverter, harmonics of the output voltage have amplitude which are inversely proportional to the harmonic order.
- If X_l is the leakage impedance of the machine at fundamental frequency, various harmonic currents can be written as

$$I_5 = \frac{V_1}{5 \times 5X_1} = \frac{\frac{V_1}{X_l}}{25} \qquad I_7 = \frac{\frac{V_1}{X_1}}{49}; \quad I_{11} = \frac{\frac{V_1}{X_1}}{121}, \quad etc.$$

- The total rms stator current is therefore given by

$$I_{rms}^2 = I_1^2 + (\frac{V_1}{X_1})^2 [\frac{1}{25^2} + \frac{1}{49^2} + \frac{1}{121^2} + ...]$$

$$= I_1^2 [1 + (\frac{V_1}{I_1 X_1})^2] [\frac{1}{25^2} + \frac{1}{49^2} + \frac{1}{121^2} + ...]$$

Operation from Non-Sinusoidal Sources

- Considering a machine with 10% leakage impedance, V_1/X_1 corresponds to 10 times rated current.
- Therefore the harmonic currents, expressed in p.u. with rated current as base, can be calculated as:

$$I_5 = \frac{10}{25} = 0.4p.u$$

$$I_7 = \frac{10}{49} = 0.2p.u$$

$$I_{11} = \frac{10}{121} = 0.083p.u$$

Operation from Non-Sinusoidal Sources

- Therefore the harmonic currents considerably increase the rms stator current and contribute to the increased copper losses in the motor.
- Further, the maximum instantaneous current handled by the inverter switches can also go up by a factor of 1.5 to 2 and the inverter devices have to be suitably rated.
- Thus some amount of derating for the whole inverter drive system becomes inevitable.
- A further important effect created by the current harmonics is that they produce a torque pulsations in the motor.

Effects of Harmonics on the Torque

- The basic mechanism of torque production in the motor is due to the interaction of the rotor current with the air gap flux.
- In a machine which is excited by purely sinusoidal voltages, only fundamental flux and rotor current are produced and these rotate in synchronism, i.e., a constant spatial angle is maintained between the flux and the current.
- Since torque is proportional to the magnitudes of the flux and the current and also the sine of the spatial angle between two, a steady torque is produced.

Effects of Harmonics on the Torque

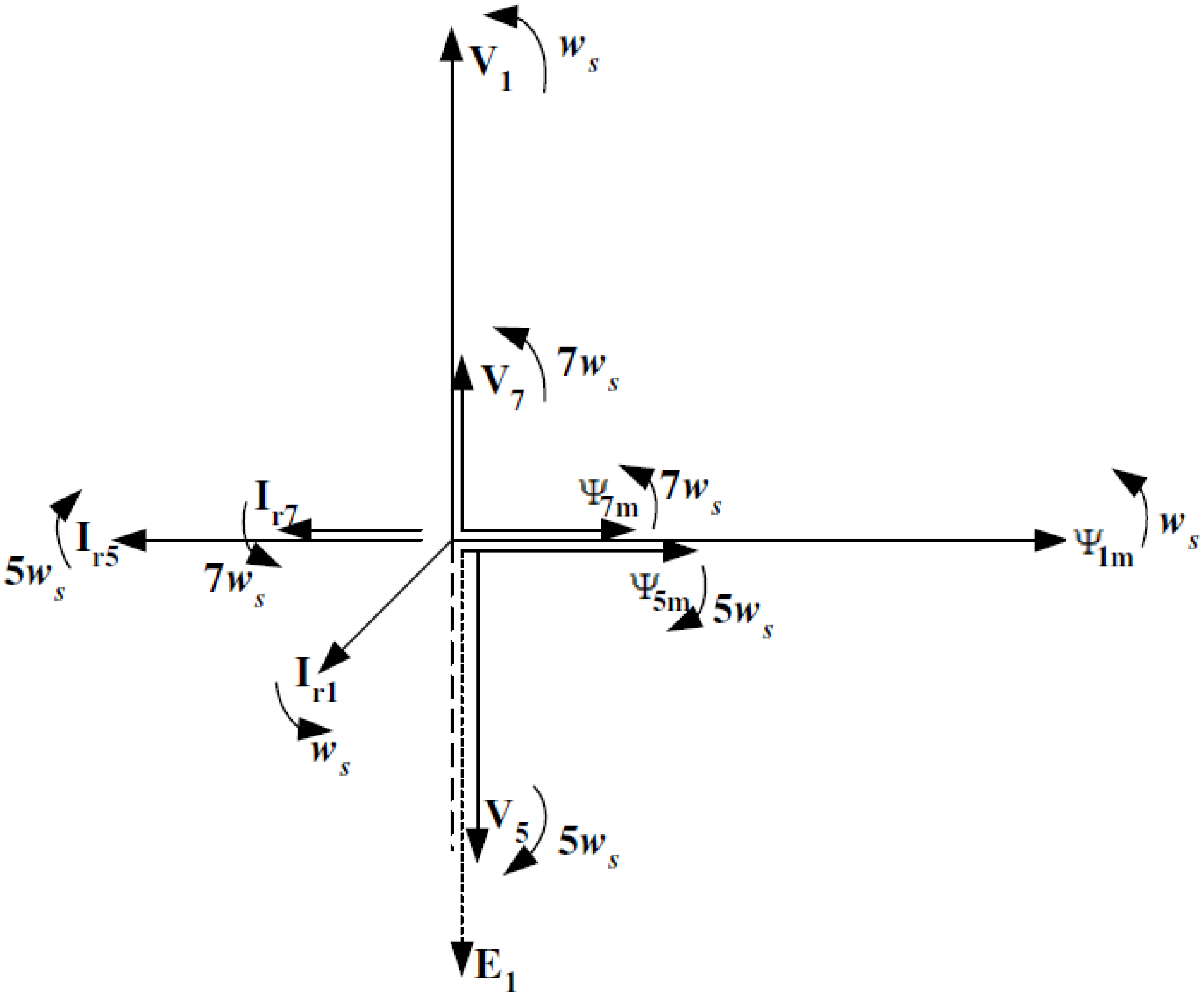
- In inverter fed machines, however, fluxes and currents at various harmonic frequencies are also produced.
- If the interaction between flux at one frequency and the current at same frequency is considered, steady torque is produced.
- In the case of positive sequence harmonics, this adds to the torque produced by the fundamental and in the case of negative sequence harmonics, it opposes the torque due to the fundamental.
- However, the contributions of the harmonics to the steady output torque of the motor are negligible in magnitude and it can be said that the useful output torque is only due to the fundamental.

Effects of Harmonics on the Torque

- When the interaction between flux at one frequency and rotor current at another frequency is considered, the two are in relative motion as they rotate at different speeds and possibly in the opposite directions.
- The torque produced by such interactions therefore pulsates with respect to time at the frequency of relative motion between the flux and current considered.

Effects of Harmonics on the Torque

- Such interactions can be understood by looking at the phasor diagram.



Effects of Harmonics on the Torque

- The phasor diagram has been drawn taking the time origin as the instant at which the fundamental flux ψ_{1m} has the peak value.
- Correspondingly, the fundamental applied voltage will be at its negative zero crossing and is therefore pointing upwards.
- Now, it has been mentioned that due to the symmetry of the inverter output voltage waveform, it can be expressed as a Fourier series containing only sine terms, i.e., the zero crossings of fundamental coincide with the zero crossings of the harmonics.

Effects of Harmonics on the Torque

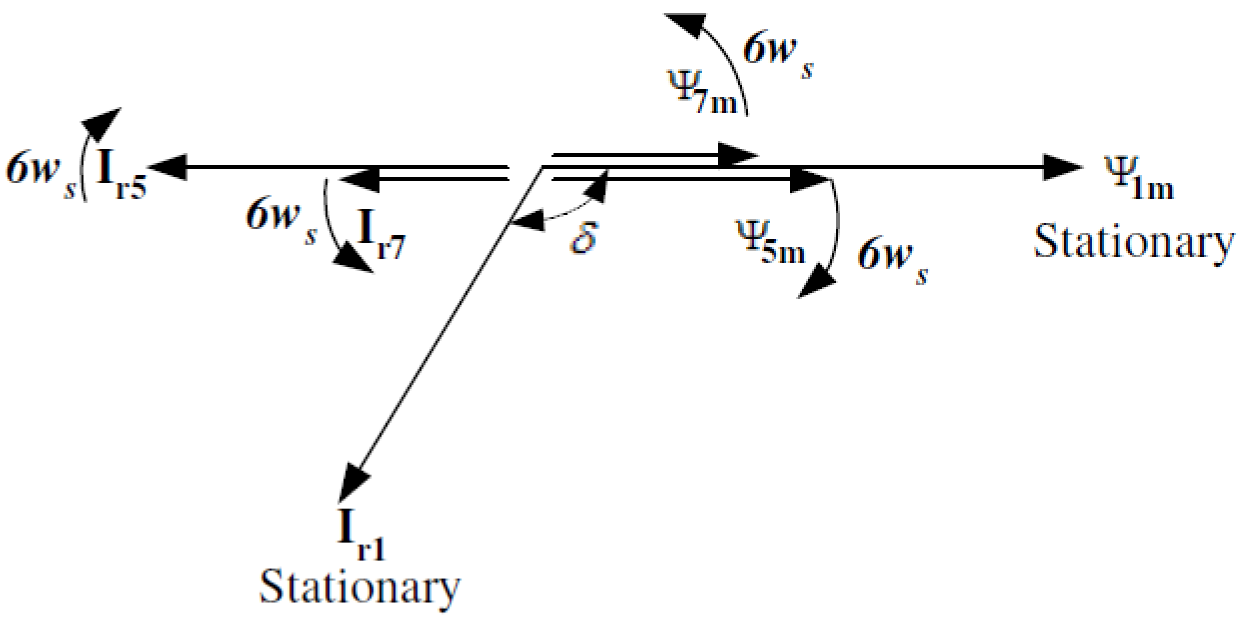
- Therefore the harmonic voltages V_5 and V_7 should also be at there negative zero crossings.
- However, V_5 is a negative sequence component and rotates in the clockwise direction.
- Therefore the phasor of the V_5 points downwards.
- Once the voltage phasors are drawn, corresponding flux phasors can be located at a lag angle of 90^o with respect to the voltage, taking the direction of rotation into account.

Effects of Harmonics on the Torque

- For each harmonic flux, the rotor induced emf lags by 90^o.
- Also at the harmonic frequencies, rotor leakage reactance dominates over the resistance and therefore the rotor can be taken to lag the induced voltage by 90^o.
- The currents I_{r5} and I_{r7} are drawn in phasor diagram taking into account the above considerations.

Effects of Harmonics on the Torque

- The phasor diagram can be given a clockwise rotation at a speed w_s, thereby making the fundamental quantities stationary.
- The resulting diagram is shown in Figure below



Effects of Harmonics on the Torque

- From the phasor diagram in previous slide, it is clear that I_{r5} and I_{r7} are rotating with respect to ψ_{1m} at 6 times the fundamental speed, but in the opposite direction.
- They will both produce torque components pulsating at 6 times the fundamental frequency w_s.
- These torque components can be expressed as

$$M_{d6,1} = k\,[\psi_{1m} I_{r5} sin(\pi + 6w_s t) + \psi_{1m} I_{r7} sin(\pi - 6w_s t)]$$

- Similarly the flux components ψ_{5m} and ψ_{7m} will interact with I_{r1} and produce 6th harmonic torque pulsation.
- These components can be expressed as

$$M_{d6,2} = k\,[\psi_{5m} I_{r1} sin(\delta - 6w_s t) + \psi_{7m} I_{r1} sin(\delta + 6w_s t)]$$

Effects of Harmonics on the Torque

- If δ is approximately taken as 90°, the total 6th harmonic torque pulsation due to all the four flux currents pairs is

$$M_{d6} = k\,[\psi_{1m}(I_{r5} - I_{r7})sin(6w_s t) + (\psi_{7m} + \psi_{5m})I_{r1}cos(6w_s t)]$$

- The harmonic fluxes ψ_{5m} and ψ_{7m} are generally very small and in any case the second term adds to the first in quadrature.
- Therefore the 6th harmonic torque pulsation can be expressed as

$$M_{d6} = k\,[\psi_{1m}(I_{r7} - I_{r5})sin(6w_s t)]$$

Effects of Harmonics on the Torque

- Thus the fundamental flux interacting the 7th and 5th harmonic currents produces 6th harmonic torque pulsation.
- Note that there is a cancelling effect between the contribution of the two currents.
- Similarly torque pulsation at the 12th; 18th; etc. harmonics are also produced, although the 6th harmonic pulsation is the predominant one.

Effects of Harmonics on the Torque

- The time variation of the instantaneous developed torque in the induction machine fed by a six step inverter therefore is as shown in Figure