राष्ट्रीय प्रौद्योगिकी संस्थान कालिकट
NATIONAL INSTITUTE OF TECHNOLOGY CALICUT

Monsoon 2023-2024

EE6302E Dynamics of Electrical Machines (DEM)

Module 1

References

1. *D P Sengupta & J.B. Lynn, Electrical Machine Dynamics, The Macmillan Press Ltd., 1980.*
2. *R Krishnan, Electric Motor Drives, Modeling, Analysis and Control, Pearson Education, 2001.*
3. *P.C. Kraus, Analysis of Electrical Machines, McGraw Hill Book Company,1987*

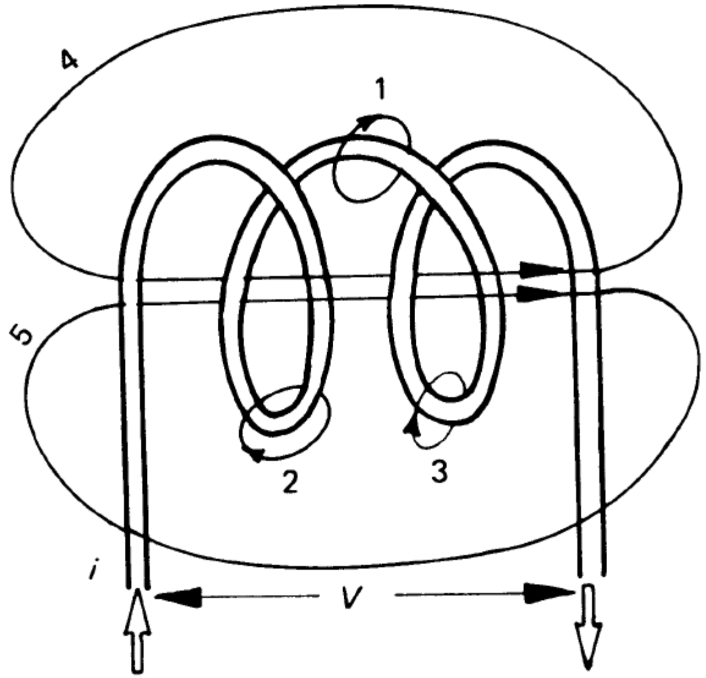
Relevance

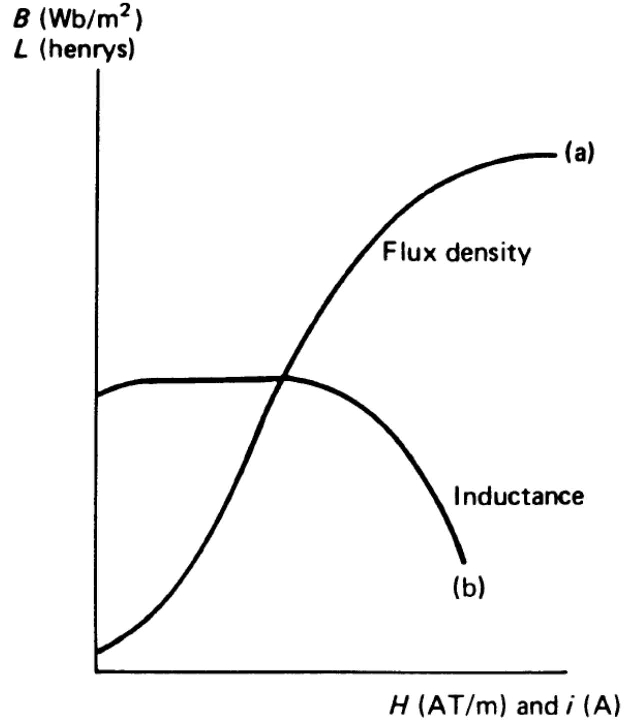
How to run any machine for variable speed application??

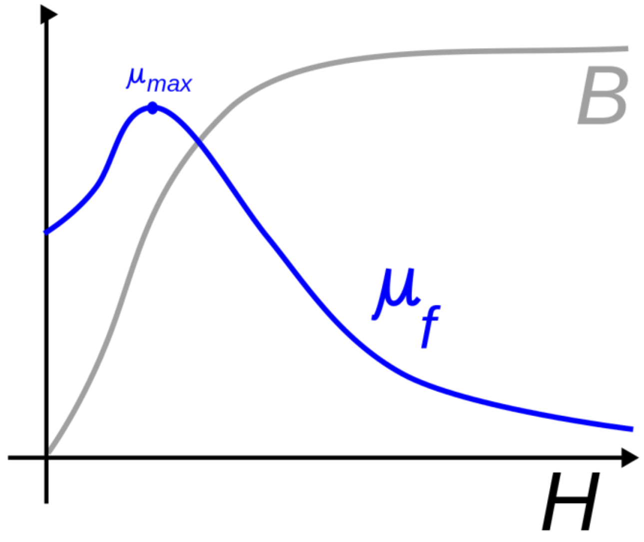
Dr. Gabriel Kron

Electrical machines, despite the differences in their construction and characteristics, have a lot in common. In earlier days, different types of machine were analysed in quite different ways, until in 1934 Dr. Gabriel Kron showed that all machines are basically the same, and can be analysed with the same set of generalised matrix equations. Kron's researches started a trend towards generalised machine concepts in the analysis of all types of machines

Fundamentals: Inductance

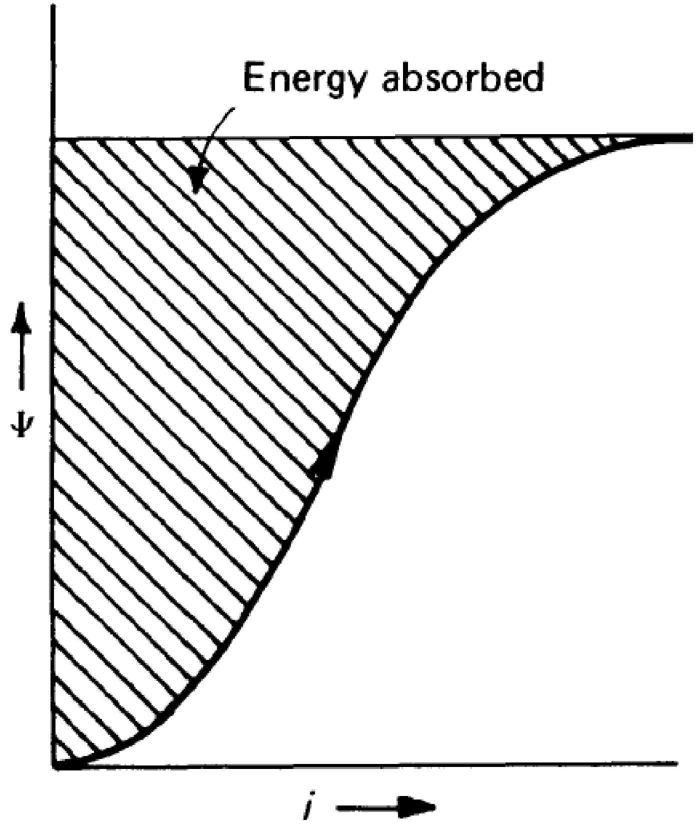


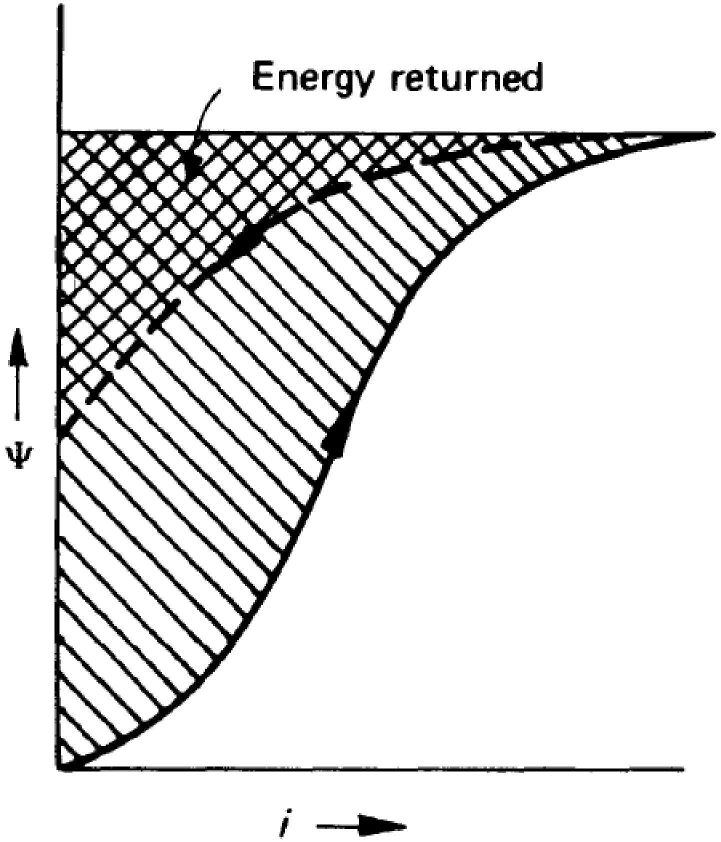


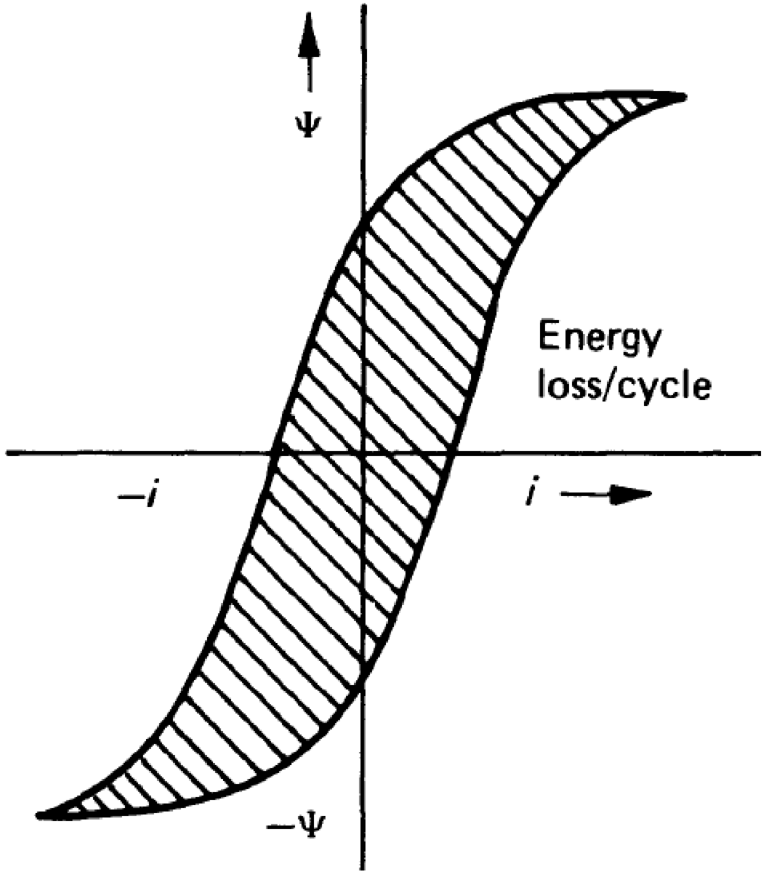


Permeability is the measure of magnetization that a material obtains in response to an applied magnetic field

Fundamentals: Inductance







$$E = \int_0^t V\, i\, \mathrm{d}t = \int_0^{\psi_1} i\, \mathrm{d}\psi$$

Fundamentals: Inductance

$$\Phi = Ni\mathscr{P} \quad \text{(Wb)}$$

$$\mathscr{P} = \mu A/l$$

$$\psi = N\Phi = N^2 i\mathscr{P}$$

Permeance is the property of allowing the passage of lines of magnetic flux

Faraday-Lenz law

$$e = -\frac{d\psi}{dt} = -\frac{d}{dt}(N^2\mathscr{P}i)$$

If the number of turns and the magnetic permeance do not change with time then

$$e = -N^2\mathscr{P}\frac{di}{dt} = -L\frac{di}{dt}$$

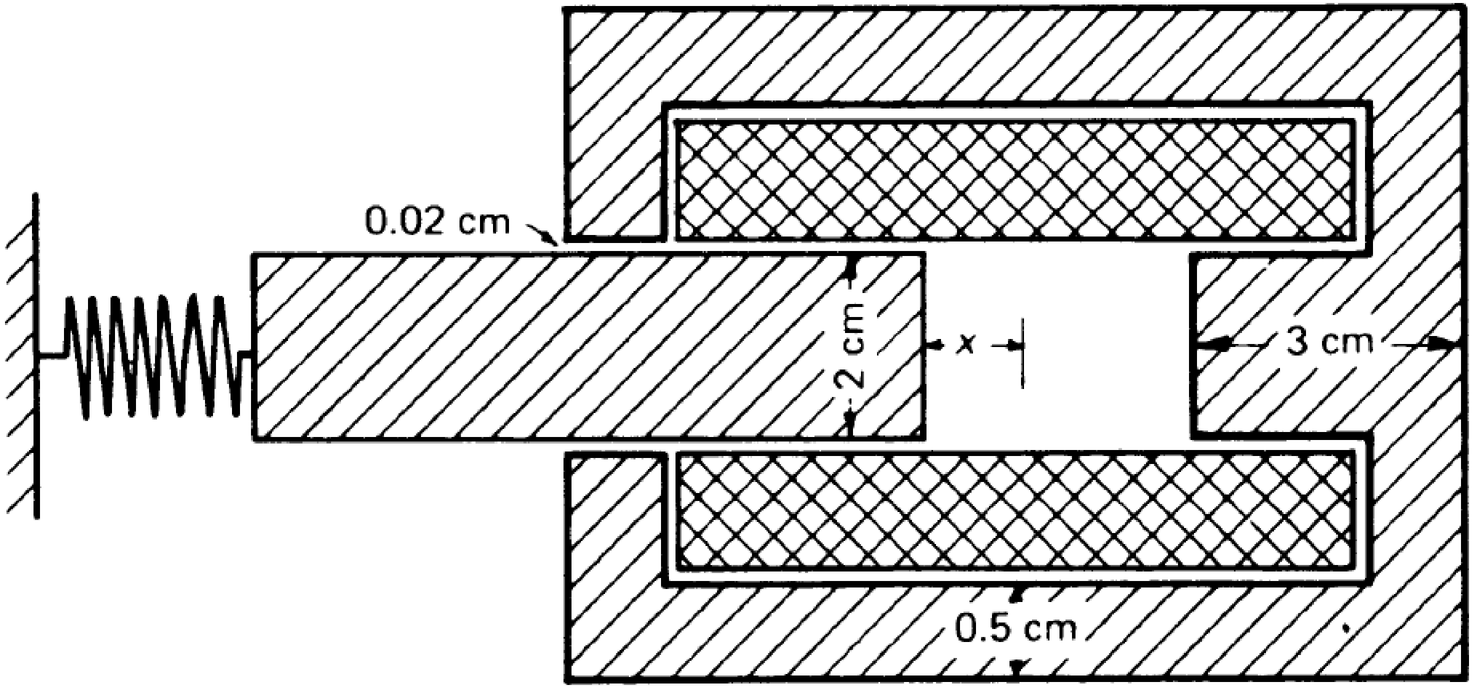
Fundamentals: Inductance

If the coils are in motion, this relation is no longer true and we have to write

$$
\begin{aligned}
e &= -\frac{\mathrm{d}(Li)}{\mathrm{d}t} \\
&= -L\frac{\mathrm{d}i}{\mathrm{d}t} - i\frac{\mathrm{d}L}{\mathrm{d}t} \\
&= -L\frac{\mathrm{d}i}{\mathrm{d}t} - i\frac{\partial L}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t}
\end{aligned}
$$

if L changes with x.

Electrodynamics



$$V = Ri + \frac{\mathrm{d}}{\mathrm{d}t}(L\,i)$$

$$= Ri + L\frac{\mathrm{d}i}{\mathrm{d}t} + i\frac{\partial L}{\partial x}\cdot\frac{\mathrm{d}x}{\mathrm{d}t}$$

$$Vi = Ri^2 + iL\frac{\mathrm{d}i}{\mathrm{d}t} + i^2\frac{\partial L}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t}$$

$$= Ri^2 + \frac{\mathrm{d}}{\mathrm{d}t}\left(\tfrac{1}{2}Li^2\right) + \tfrac{1}{2}i^2\frac{\partial L}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t}$$

$$P_e = \left(\tfrac{1}{2}i^2\frac{\partial L}{\partial x}\right)\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)$$

First term: power loss due to the resistance of the coil
Second term: rate of change of the stored magnetic energy (i.e. reactive power).
Third term: the power necessary to accelerate the plunger and overcome the tension of the spring. This term represents the mechanical power. Since it has electrical origin it may be called electromechanical power

Electrodynamics

electromechanical force $f_e = \frac{1}{2} i^2 \frac{\partial L}{\partial x}$

K is the stiffness of the spring

in which direction does the force act?

involves the square of the current, this implies that irrespective of the direction of the currents through the coil the force on the plunger acts only in one direction. Reversing the polarity of the d.c. source, or replacing it by an a.c. source will not alter the direction of force.

$$m \frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + Kx - \frac{1}{2} i^2 \frac{\partial L}{\partial x} = 0$$

(electrodynamic equation of the plunger)

Electrodynamics

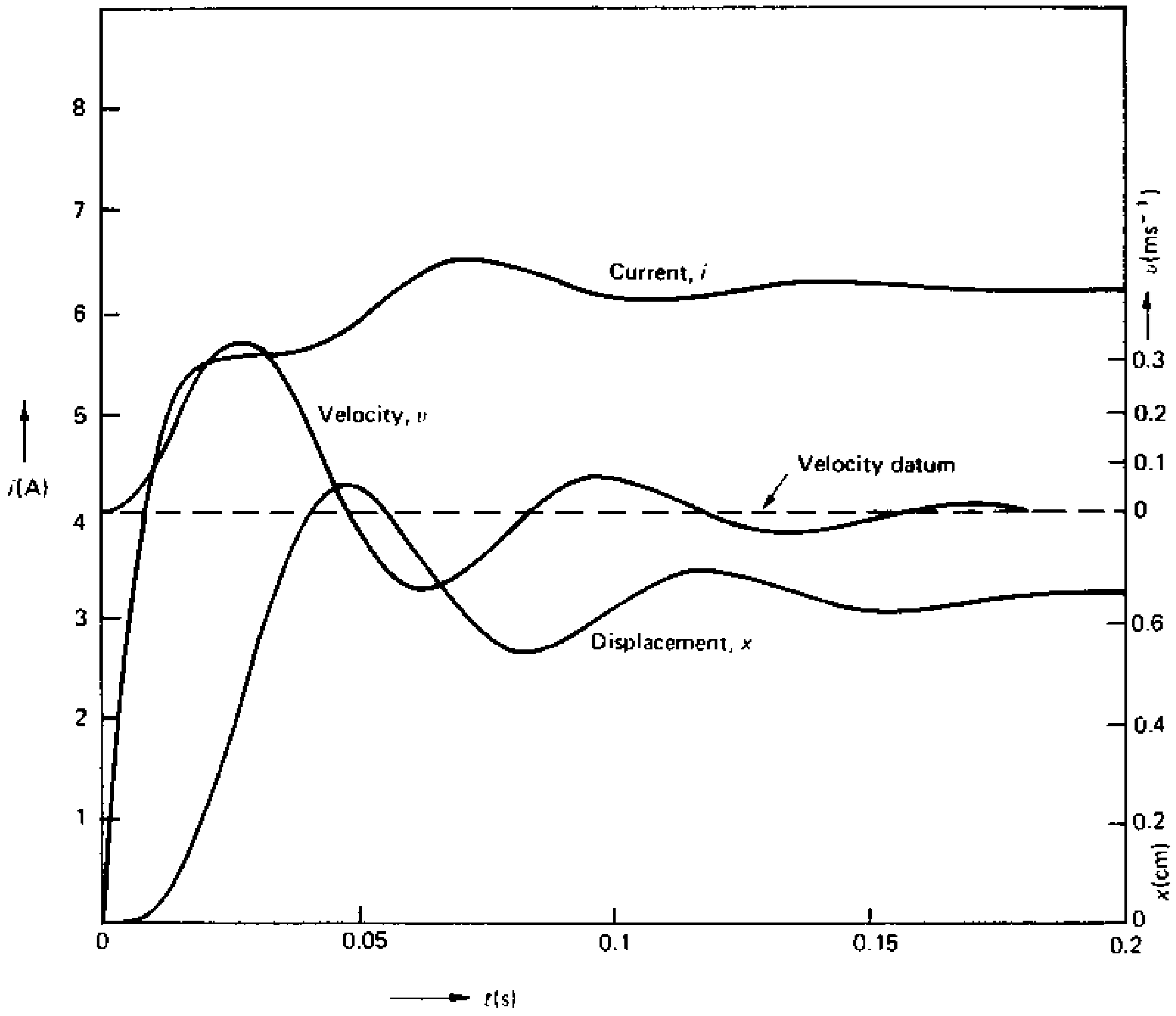
Clearly fe is positive if dL/dx is positive which implies that the plunger will move in such a direction that L increases with x or that the permeance increases with x, i.e., it will tend to move into the coil. This movement will be opposed by the spring and the acceleration will depend on the resultant of these opposing forces

If the plunger overshoots through the coil, the force f will reverse its direction and combine with the restoring force from the spring to pull the plunger back into the coil. It may be generally said that the electromechanical force will act in such a direction as to make the reluctance of the magnetic path a minimum

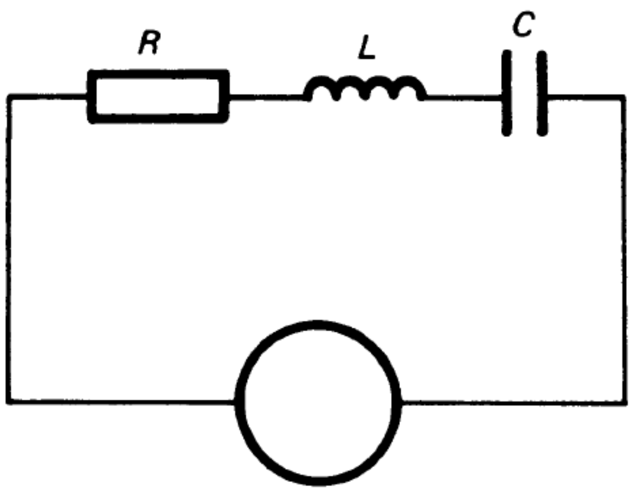
Tutorial

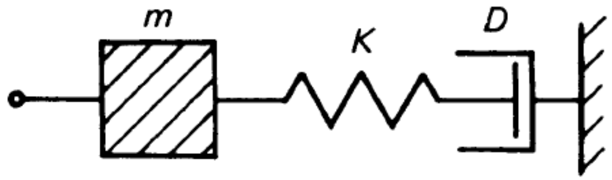
Compute the power loss, rate of change of stored magnetic energy and electromechanical power for a spring-plunger system with a dynamic as as shown in figure. Assume that the supply voltage is 10 V, R= 1.6 Ω, L as given below. Consider time as 0.05 s

$$L = \frac{0.012\,76}{1 - 32.33x}$$



Electrodynamics





$$\mathscr{E} = N^2 \mathscr{P} \int_0^i i\,\mathrm{d}i = \tfrac{1}{2} L i^2 \quad \text{(J)}$$

$$\mathscr{E}_{\mathrm{c}} = \tfrac{1}{2} C V^2 = \tfrac{1}{2} q^2 / C$$

$$\text{kinetic energy} = \tfrac{1}{2} m v^2 = \tfrac{1}{2} J \omega^2$$

$$\int_0^x F\,\mathrm{d}x = \int_0^x K x\,\mathrm{d}x = \tfrac{1}{2} K x^2$$

$$L\ddot{q} + R\dot{q} + q/C = V, \qquad \text{electrical network}$$

$$m\ddot{x} + D\dot{x} + Kx = F, \qquad \text{mechanical system}$$

the analogy in general form :
Lagrange's equation

Lagrangian Analysis (lossless system)

Joseph-Louis Lagrange

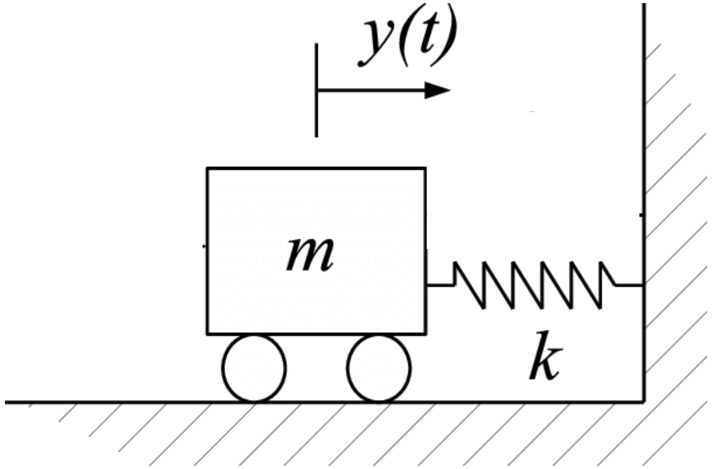
The Lagrangian

$$\mathscr{L} = \mathscr{T} - \mathscr{V}$$

Lagrangian equation in general form for a system with ‘n’ degrees of freedom

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathscr{L}}{\partial \dot{x}_n}\right) - \frac{\partial \mathscr{L}}{\partial x_n} = 0$$

Lagrangian Analysis (Lossless system)



kinetic energy of the mass

$$\mathcal{T} = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m(\dot{x})^2$$

potential energy stored in the spring

$$\mathcal{V} = \int_0^x F\,\mathrm{d}x = \int_0^x Kx\mathrm{d}x = Kx^2/2$$

$$\frac{\partial \mathcal{T}}{\partial \dot{x}} = \frac{\partial}{\partial \dot{x}}\left[\tfrac{1}{2}m(\dot{x})^2\right] = m\dot{x} = mv$$

$$\frac{\partial \mathcal{V}}{\partial x} = \frac{\partial}{\partial x}(\tfrac{1}{2}Kx^2) = Kx = F$$

$$\mathcal{L} = \mathcal{T} - \mathcal{V}$$

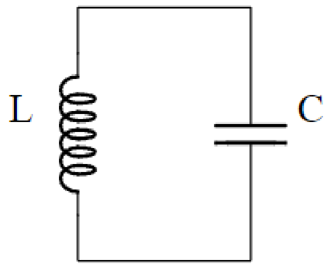
$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}_n}\right) - \frac{\partial \mathcal{L}}{\partial x_n} = 0$$

Newton's second law of motion

$$\frac{\mathrm{d}}{\mathrm{d}t}(mv) = -F$$

negative sign appears in this case because force is a restoring force

Lagrangian Analysis (Lossless system)



$$\frac{\partial \mathcal{T}}{\partial \dot{x}} = \frac{\partial}{\partial i}\left(\tfrac{1}{2}Li^2\right) = Li$$

$$\frac{\partial \mathcal{V}}{\partial q} = \frac{\partial}{\partial q}\left(\frac{1}{2}\frac{q^2}{C}\right) = \frac{q}{C}$$

$$\mathcal{L} = \mathcal{T} - \mathcal{V}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}_n}\right) - \frac{\partial \mathcal{L}}{\partial x_n} = 0$$

Substituting these values into Lagrange's equation

$$\frac{\mathrm{d}}{\mathrm{d}t}(Li) + \frac{q}{C} = 0$$

$$L\frac{\mathrm{d}i}{\mathrm{d}t} + \frac{q}{C} = 0$$

Lagrangian Analysis (Lossy system)

The Lagrangian

$$\mathscr{L} = \mathscr{T} - \mathscr{V}$$

In order to include the effects of loss, a velocity-dependent (or current-dependent) function, the Rayleigh dissipation function is adopted

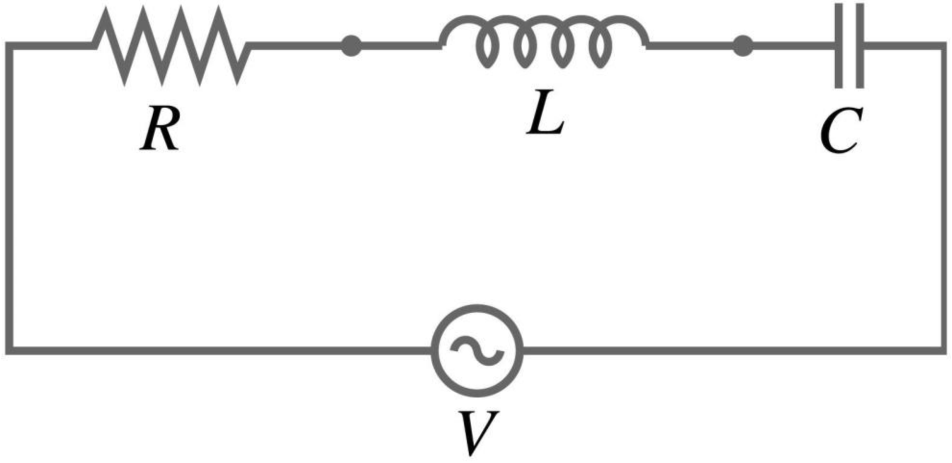
$$\frac{\partial \mathscr{F}}{\partial \dot{x}_n} = R_n \dot{x}_n \text{ represents force (or voltage)}$$

$$\mathscr{F} = \sum_{n=1}^{k} \tfrac{1}{2} R_n (\dot{x}_n)^2$$

Lagrangian equation in general form for a system with 'n' degrees of freedom

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathscr{L}}{\partial \dot{x}_n}\right) - \frac{\partial \mathscr{L}}{\partial x_n} + \frac{\partial \mathscr{F}}{\partial \dot{x}_n} = V_n$$

Lagrangian Analysis (Lossy system)



$$\frac{\partial \mathscr{T}}{\partial \dot{x}} = \frac{\partial}{\partial i}\left(\tfrac{1}{2}Li^2\right) = Li$$

$$\frac{\partial \mathscr{V}}{\partial q} = \frac{\partial}{\partial q}\left(\frac{1}{2}\frac{q^2}{C}\right) = \frac{q}{C}$$

$$\frac{\partial \mathscr{F}}{\partial \dot{x}_n} = R_n \dot{x}_n$$

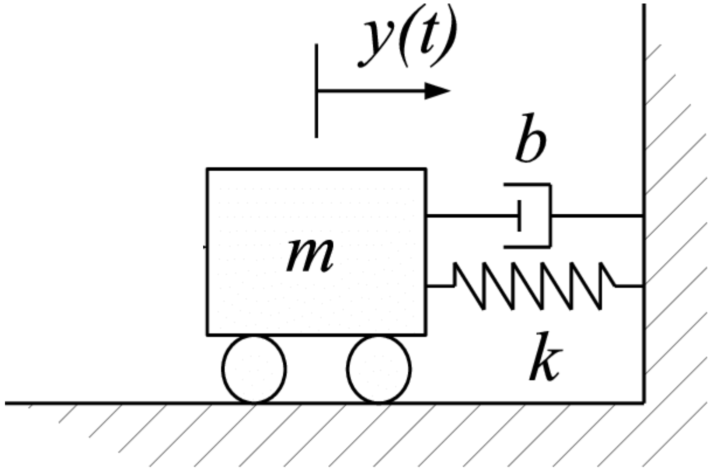
$$\mathscr{L} = \mathscr{T} - \mathscr{V}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathscr{L}}{\partial \dot{x}_n}\right) - \frac{\partial \mathscr{L}}{\partial x_n} + \frac{\partial \mathscr{F}}{\partial \dot{x}_n} = V_n$$

Substituting these values into Lagrange's equation

$$L\frac{\mathrm{d}^2 q}{\mathrm{d}t^2} + R\frac{\mathrm{d}q}{\mathrm{d}t} + \frac{q}{C} = V$$

Lagrangian Analysis (Lossy system)



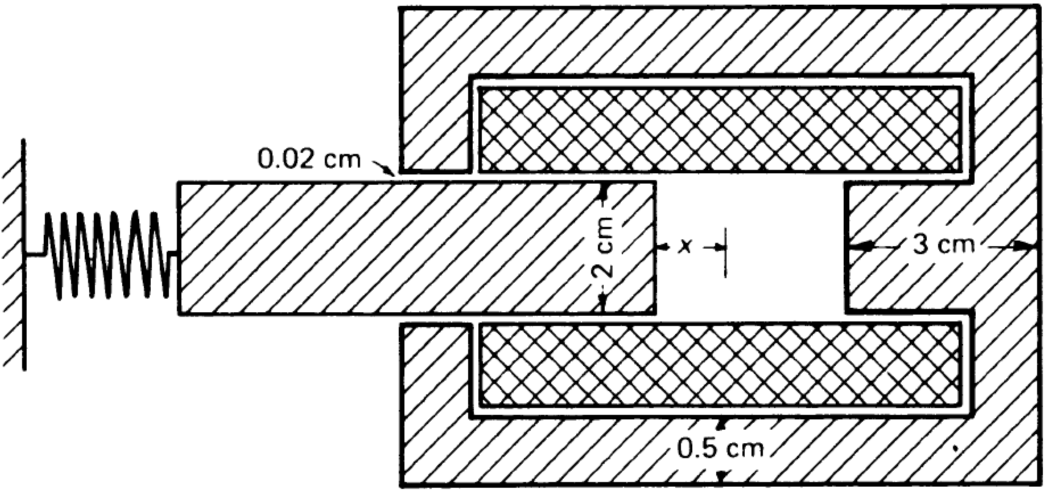
$$\mathscr{L} = \mathscr{T} - \mathscr{V}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathscr{L}}{\partial \dot{x}_n}\right) - \frac{\partial \mathscr{L}}{\partial x_n} + \frac{\partial \mathscr{F}}{\partial \dot{x}_n} = V_n$$

Substituting these values into Lagrange's equation

$$m\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + b\,\frac{\mathrm{d}x}{\mathrm{d}t} + Kx = F$$

Lagrangian Analysis (Lossy system)



Total kinetic energy	Total potential energy	Dissipation function
$\mathscr{T} = \frac{1}{2}Li^2 + \frac{1}{2}mv^2$	$\mathscr{V} = \frac{1}{2}Kx^2$	$\mathscr{F} = \frac{1}{2}Ri^2 + \frac{1}{2}Dv^2$
elect mech	mechanical	elect mech

$$\mathscr{L} = \mathscr{T} - \mathscr{V} = \frac{1}{2}L\dot{x}_1^2 + \frac{1}{2}m\dot{x}_2^2 - \frac{1}{2}Kx_2^2$$

$$\mathscr{F} = \frac{1}{2}R\dot{x}_1^2 + \frac{1}{2}D\dot{x}_2^2$$

$$\frac{\partial \mathscr{L}}{\partial \dot{x}_1} = L\dot{x}_1 \qquad \frac{\partial \mathscr{L}}{\partial x_1} = 0 \qquad \frac{\partial \mathscr{F}}{\partial \dot{x}_1} = R_1\dot{x}_1$$

$V_n = V_1 =$ voltage applied to the coil

$$L\frac{di}{dt} + Ri + i\frac{dL}{dt} = V_1$$

(Voltage equation of the coil)

$$\frac{\partial \mathscr{L}}{\partial \dot{x}_2} = m\dot{x}_2 \qquad \frac{\partial \mathscr{L}}{\partial x_2} = -Kx_2 + \frac{1}{2}\dot{x}_1^2\frac{\partial L}{\partial x_2}$$

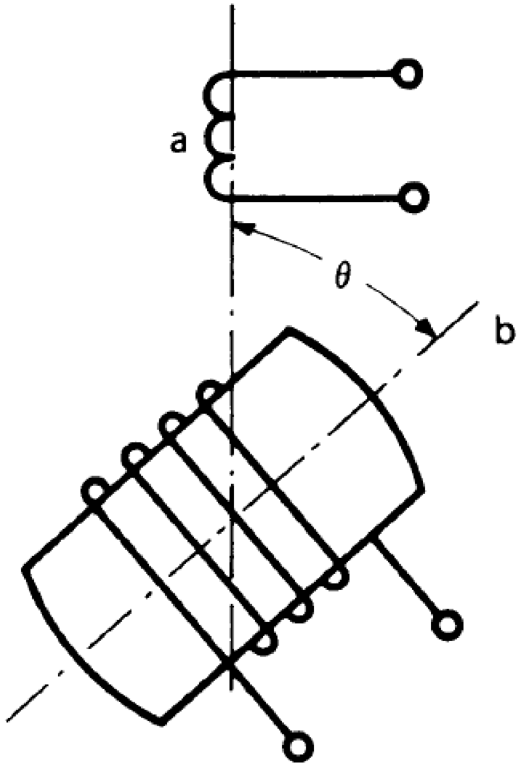
$$\frac{\partial \mathscr{F}}{\partial \dot{x}_2} = D\dot{x}_2 \qquad V_2 = \text{applied force} = F = 0$$

$$m\ddot{x}_2 + D\dot{x}_2 + Kx_2 - \frac{1}{2}i^2\frac{\partial L}{\partial x_2} = 0$$

(Dynamic equation of the plunger)

Lagrangian Analysis (Lossy system)

Doubly Excited Coil



x_1 represent the charge q_1 through coil 1
x_2 represent the charge q_2 through coil 2
x_3 represent the displacement θ of the rotor

Total kinetic energy

$$\mathscr{T} = \mathscr{T}_{\mathrm{e}} + \mathscr{T}_{\mathrm{m}} = \tfrac{1}{2}L_{11}i_1^2 + \tfrac{1}{2}L_{22}i_2^2 + i_1 i_2 M_{12} + \tfrac{1}{2}J\omega^2$$
$$= \tfrac{1}{2}L_{11}\dot{x}_1^2 + \tfrac{1}{2}L_{22}\dot{x}_2^2 + \dot{x}_1\dot{x}_2 M_{12} + \tfrac{1}{2}J\dot{x}_3^2$$

Total potential energy

$$\mathscr{V} = 0$$

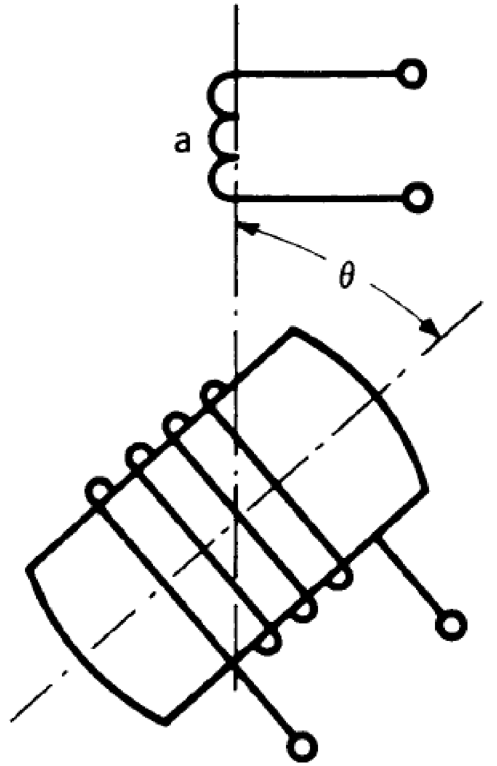
Dissipation function

$$\mathscr{F} = \tfrac{1}{2}R_1 i_1^2 + \tfrac{1}{2}R_2 i_2^2 + \tfrac{1}{2}R_{\mathrm{F}}\omega^2$$
$$= \tfrac{1}{2}R_1\dot{x}_1^2 + \tfrac{1}{2}R_2\dot{x}_2^2 + \tfrac{1}{2}R_{\mathrm{F}}\dot{x}_3^2$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathscr{L}}{\partial \dot{x}_n}\right) - \frac{\partial \mathscr{L}}{\partial x_n} + \frac{\partial \mathscr{F}}{\partial \dot{x}_n} = V_n$$

Lagrangian Analysis (Lossy system)

Doubly Excited Coil



x_1 represent the charge q_1 through coil 1
x_2 represent the charge q_2 through coil 2
x_3 represent the displacement θ of the rotor

$$\frac{\partial \mathscr{L}}{\partial \dot{x}_1} = \frac{\partial \mathscr{L}}{\partial i_1} = \frac{\partial \mathscr{T}}{\partial i_1} = L_{11} i_1 + M_{12} i_2$$

$$\frac{\partial \mathscr{L}}{\partial \dot{x}_2} = \frac{\partial \mathscr{L}}{\partial i_2} = \frac{\partial \mathscr{T}}{\partial i_2} = L_{22} i_2 + M_{12} i_1$$

$$\frac{\partial \mathscr{L}}{\partial \dot{x}_3} = \frac{\partial \mathscr{L}}{\partial \omega} = \frac{\partial \mathscr{T}}{\partial \omega} = J\omega$$

$$\frac{\partial \mathscr{L}}{\partial x_3} = \frac{\partial \mathscr{L}}{\partial \theta} = \tfrac{1}{2} i_1^2 \frac{\partial L_{11}}{\partial \theta} + \tfrac{1}{2} i_2^2 \frac{\partial L_{22}}{\partial \theta} + i_1 i_2 \frac{\partial M_{12}}{\partial \theta}$$

$$\frac{\partial \mathscr{F}}{\partial \dot{x}_1} = \frac{\partial \mathscr{F}}{\partial i_1} = R_1 i_1$$

$$\frac{\partial \mathscr{F}}{\partial \dot{x}_2} = \frac{\partial \mathscr{F}}{\partial i_2} = R_2 i_2$$

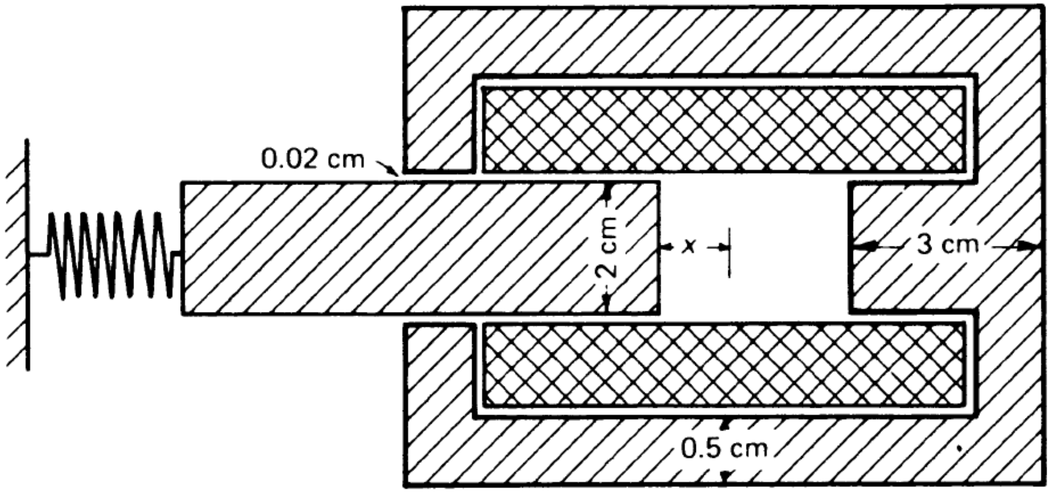
$$\frac{\partial \mathscr{F}}{\partial \dot{x}_3} = \frac{\partial \mathscr{F}}{\partial \omega} = R_F \omega$$

$$V_1 = R_1 i_1 + p[L_{11} i_1 + M_{12} i_2]$$

$$V_2 = R_2 i_2 + p[L_{22} i_2 + M_{12} i_1]$$

$$0 = R_F \omega + Jp\omega - \left[\underbrace{\tfrac{1}{2} i_1^2 \frac{\partial L_{11}}{\partial \theta} + \tfrac{1}{2} i_2^2 \frac{\partial L_{22}}{\partial \theta} + i_1 i_2 \frac{\partial M_{12}}{\partial \theta}}_{\text{Mechanical Torque Produced}} \right]$$

Solution!



$$L\frac{di}{dt} + Ri + i\frac{dL}{dt} = V_1$$

$$m\ddot{x}_2 + D\dot{x}_2 + Kx_2 - \tfrac{1}{2}i^2\frac{\partial L}{\partial x_2} = 0$$

How to know nature of plunger movement or supply current?

The product of the dependent variables, current and velocity in the voltage equations and the square of the current in the dynamical equation, make it difficult to derive a closed form solution.

By considering small perturbations about the operating point these equations can be linearised and a solution obtained. This is standard practice in considering small oscillations in electrical machines. Linearisation of the equations where fairly large displacements are involved leads to considerable errors and in such cases the standard practice is to obtain numerical solutions.

Euler's Method

Runge-Kutta Method

Euler’s Method

$$y' = f(x, y)$$

‘h’ is a small interval defined

$$x_{n+1} - x_n$$

$$y_{n+1} = y_n + h y'_n$$

One of the major disadvantages of Euler's method is that the interval or the step length h must be very small in order to provide correct results. Selecting large step sizes often leads to divergence or numerical instability.

Runge-Kutta Method

Order 2

$$y_{n+1} = y_n + \tfrac{1}{2}(k_1 + k_2)$$

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf(x_n + h \quad , y_n + k_1)$$

Order 4

$$y_{n+1} = y_n + \tfrac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf(x_n + h/2, y_n + k_1)$$

$$k_3 = hf(x_n + h/2, y_n + k_2)$$

$$k_4 = hf(x_n + h, y_n + k_3)$$

$$pi = -\frac{R}{L}i - \frac{1}{L}i\frac{\partial L}{\partial x}v + \frac{V}{L}$$

$$pv = -\frac{D}{m}v - \frac{Kx}{m} + \frac{1}{2m}i^2\frac{\partial L}{\partial x}$$

$$px = v$$

Euler's Method

$$y' = f(x, y)$$

$$y_{n+1} = y_n + h y'_n$$

'h' is a small interval defined

$$x_{n+1} - x_n$$

Runge-Kutta Method

$$y_{n+1} = y_n + \tfrac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

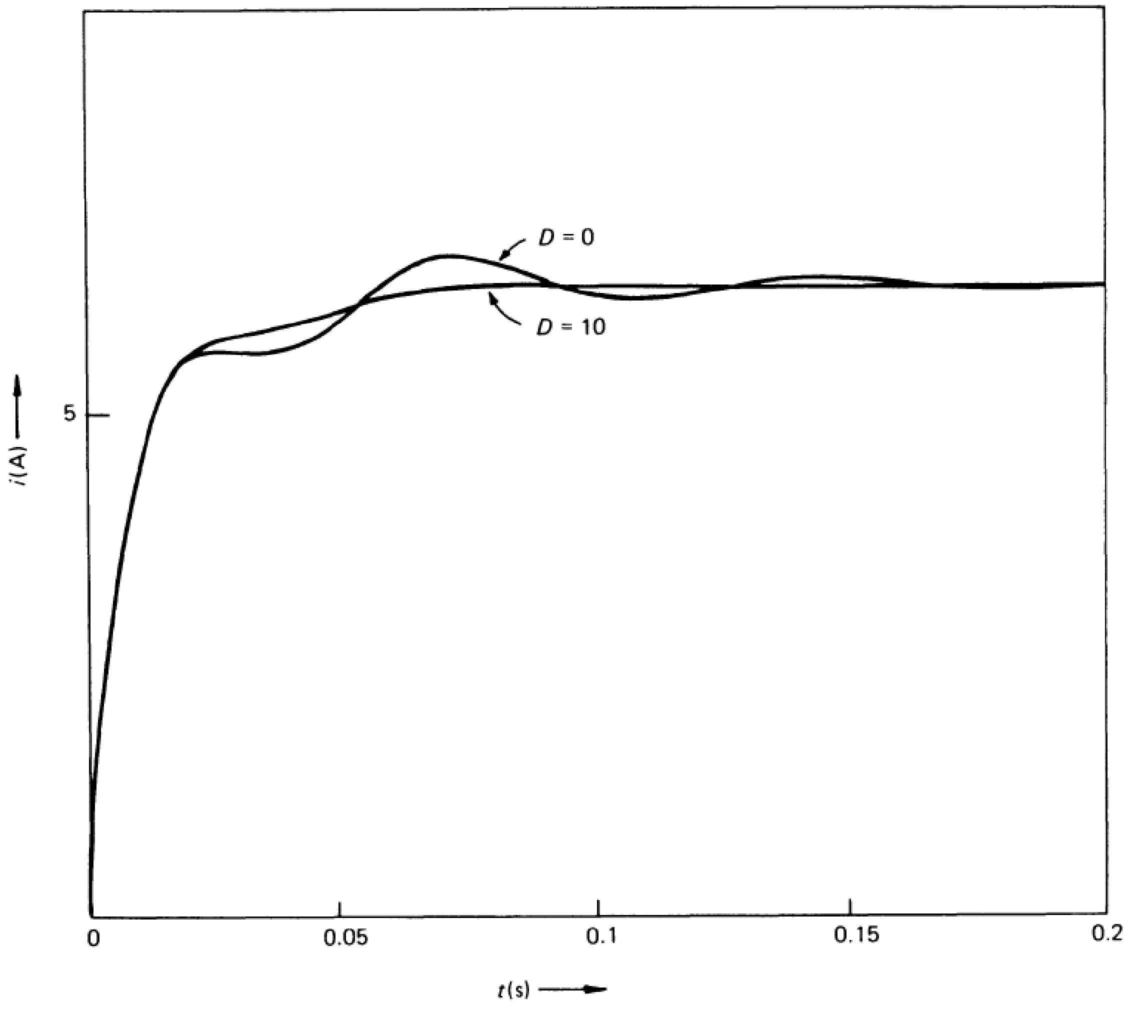
$$k_1 = hf(x_n, y_n)$$

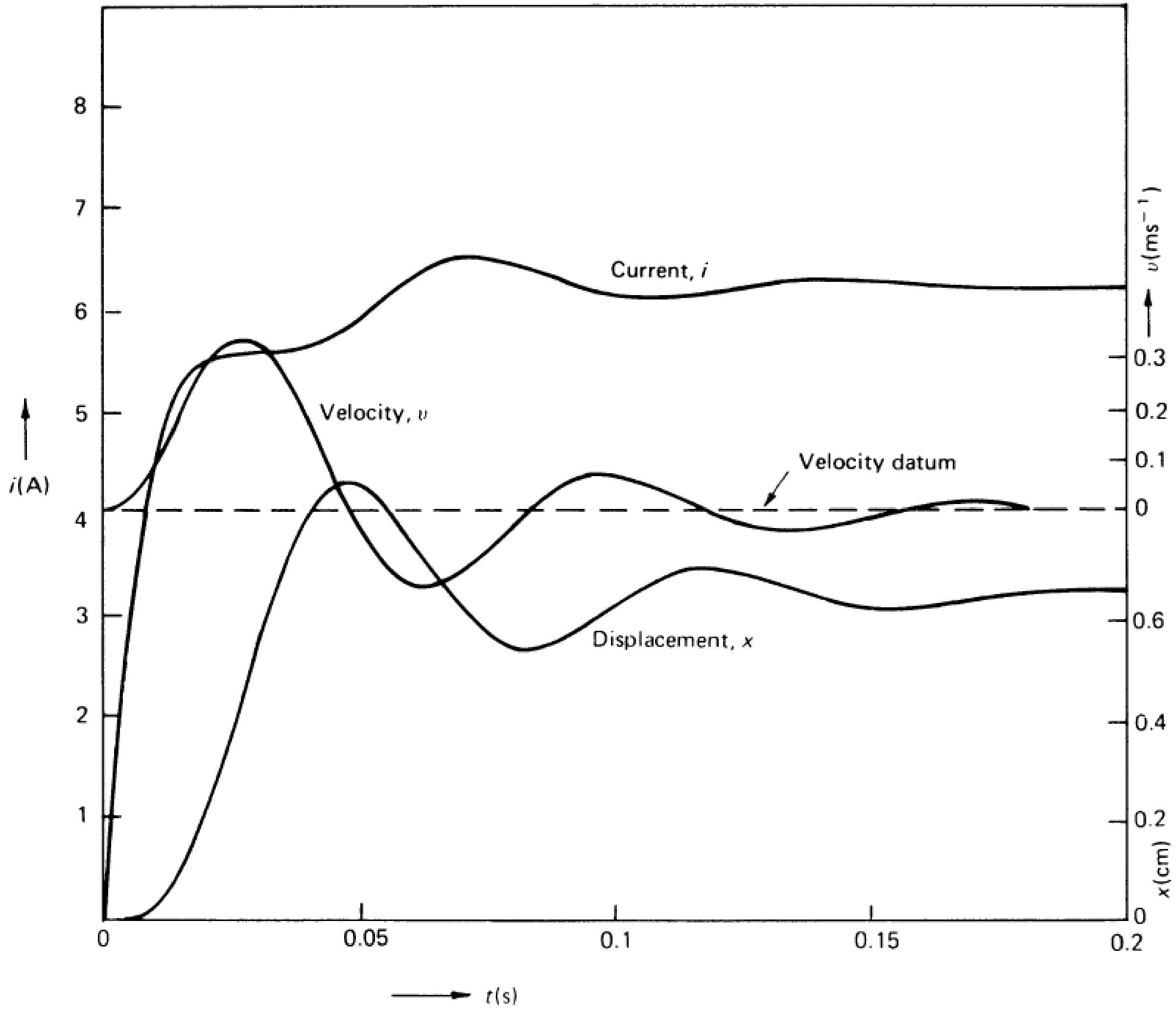
$$k_2 = hf(x_n + h/2,\ y_n + k_1)$$

$$k_3 = hf(x_n + h/2,\ y_n + k_2)$$

$$k_4 = hf(x_n + h,\ y_n + k_3)$$

Solution using RK4





Tutorial Problems #1

Use the Runge-Kutta method with h=0.1 to find approximate values for the solution of the problem at x=0.1, 0.2.

$$y' + 2y = x^3 e^{-2x}, \quad y(0) = 1$$

$$f(x,y) = -2y + x^3 e^{-2x},\ x_0 = 0,\ \text{and } y_0 = 1.$$

x=0.1

$$\begin{aligned}
k_{10} &= f(x_0, y_0) = f(0,1) = -2, \\
k_{20} &= f(x_0 + h/2, y_0 + hk_{10}/2) = f(.05, 1 + (.05)(-2)) \\
&= f(.05, .9) = -2(.9) + (.05)^3 e^{-.1} = -1.799886895, \\
k_{30} &= f(x_0 + h/2, y_0 + hk_{20}/2) = f(.05, 1 + (.05)(-1.799886895)) \\
&= f(.05, .910005655) = -2(.910005655) + (.05)^3 e^{-.1} = -1.819898206, \\
k_{40} &= f(x_0 + h, y_0 + hk_{30}) = f(.1, 1 + (.1)(-1.819898206)) \\
&= f(.1, .818010179) = -2(.818010179) + (.1)^3 e^{-.2} = -1.635201628, \\
y_1 &= y_0 + \frac{h}{6}(k_{10} + 2k_{20} + 2k_{30} + k_{40}), \\
&= 1 + \frac{.1}{6}(-2 + 2(-1.799886895) + 2(-1.819898206) - 1.635201628) = .818753803,
\end{aligned}$$

Cont...

x=0.2

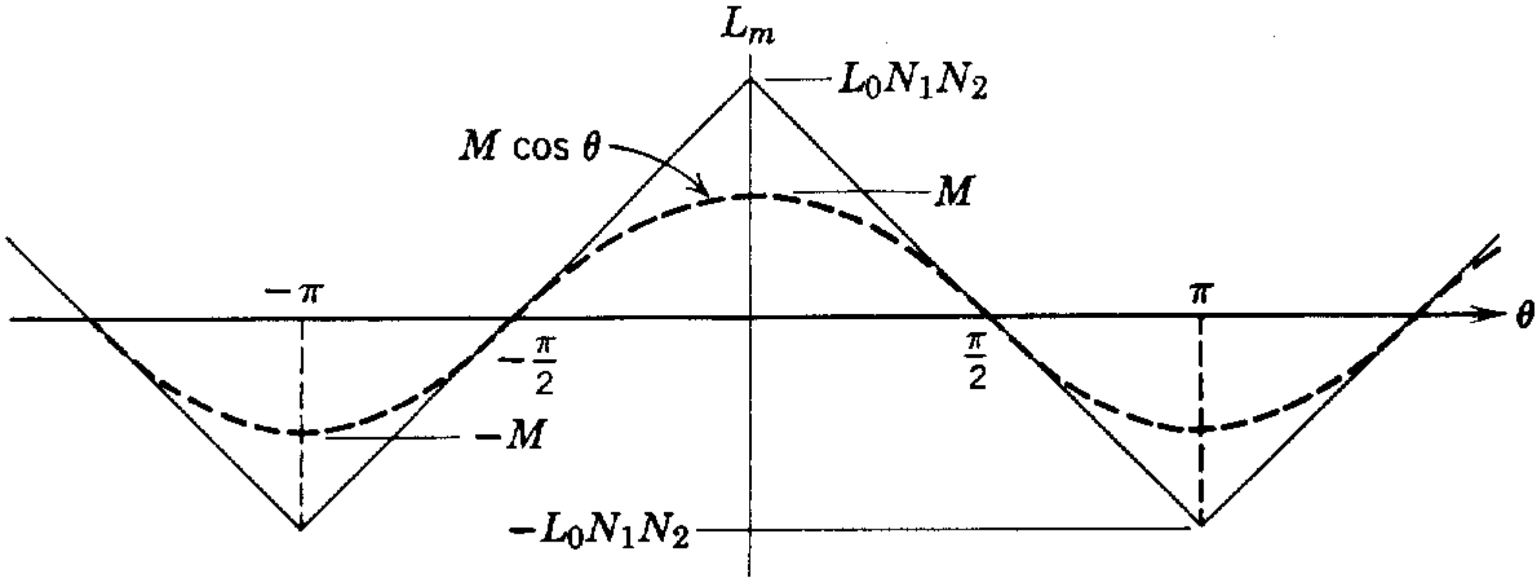
$$
\begin{aligned}
k_{11} &= f(x_1, y_1) = f(.1, .818753803) = -2(.818753803)) + (.1)^3 e^{-.2} = -1.636688875,\\
k_{21} &= f(x_1 + h/2, y_1 + hk_{11}/2) = f(.15, .818753803 + (.05)(-1.636688875))\\
&= f(.15, .736919359) = -2(.736919359) + (.15)^3 e^{-.3} = -1.471338457,\\
k_{31} &= f(x_1 + h/2, y_1 + hk_{21}/2) = f(.15, .818753803 + (.05)(-1.471338457))\\
&= f(.15, .745186880) = -2(.745186880) + (.15)^3 e^{-.3} = -1.487873498,\\
k_{41} &= f(x_1 + h, y_1 + hk_{31}) = f(.2, .818753803 + (.1)(-1.487873498))\\
&= f(.2, .669966453) = -2(.669966453) + (.2)^3 e^{-.4} = -1.334570346,\\
y_2 &= y_1 + \frac{h}{6}(k_{11} + 2k_{21} + 2k_{31} + k_{41}),\\
&= .818753803 + \frac{.1}{6}(-1.636688875 + 2(-1.471338457) + 2(-1.487873498) - 1.334570346)\\
&= .670592417.
\end{aligned}
$$

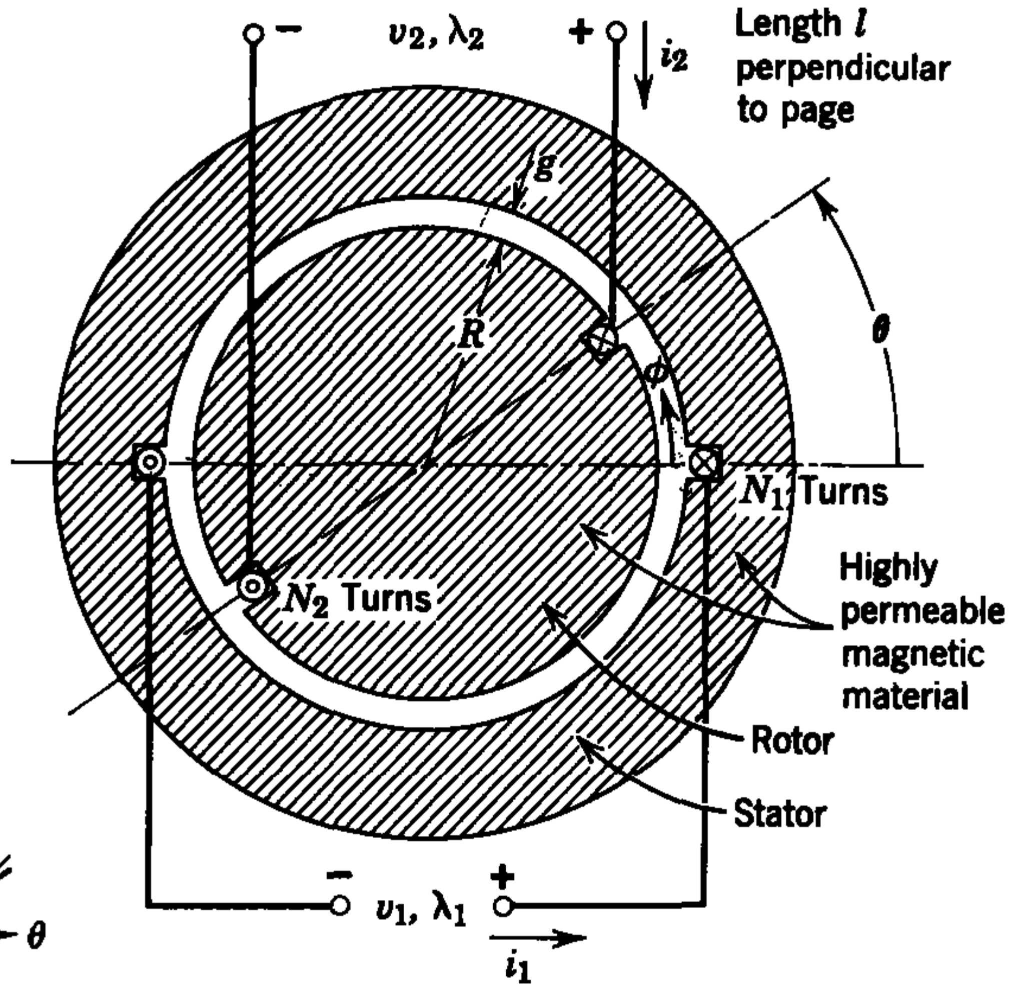
Generalized Machines

- The most numerous and the most widely used electromechanical device in existence is the magnetic field type rotating machine.
- Rotating machines occur in many different types, depending on the nature of the electrical and mechanical systems to be coupled and on the coupling characteristics desired.
- The primary purpose of most rotating machines is to convert energy between electrical and mechanical systems, either for electric power generation or for the production of mechanical power to do useful tasks.

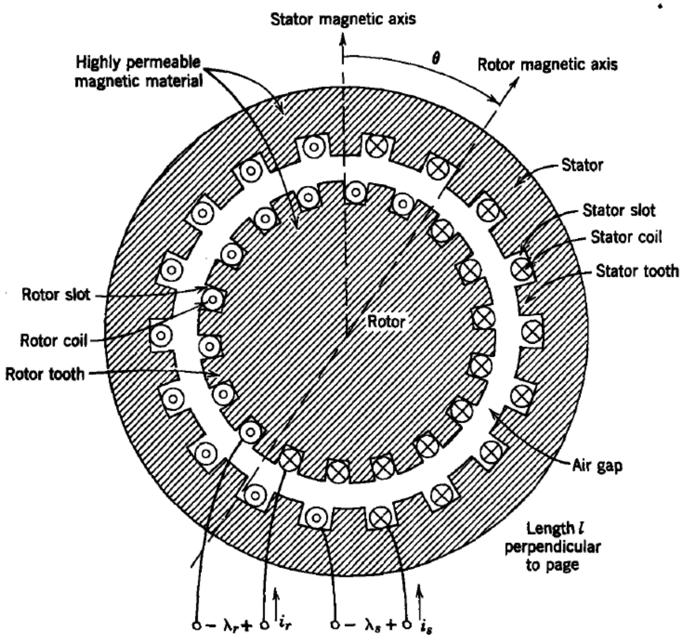
Air gap field, and hence the inductances associated with, play a vital role in the analysis of the electromechanical coupling systems in rotating machines

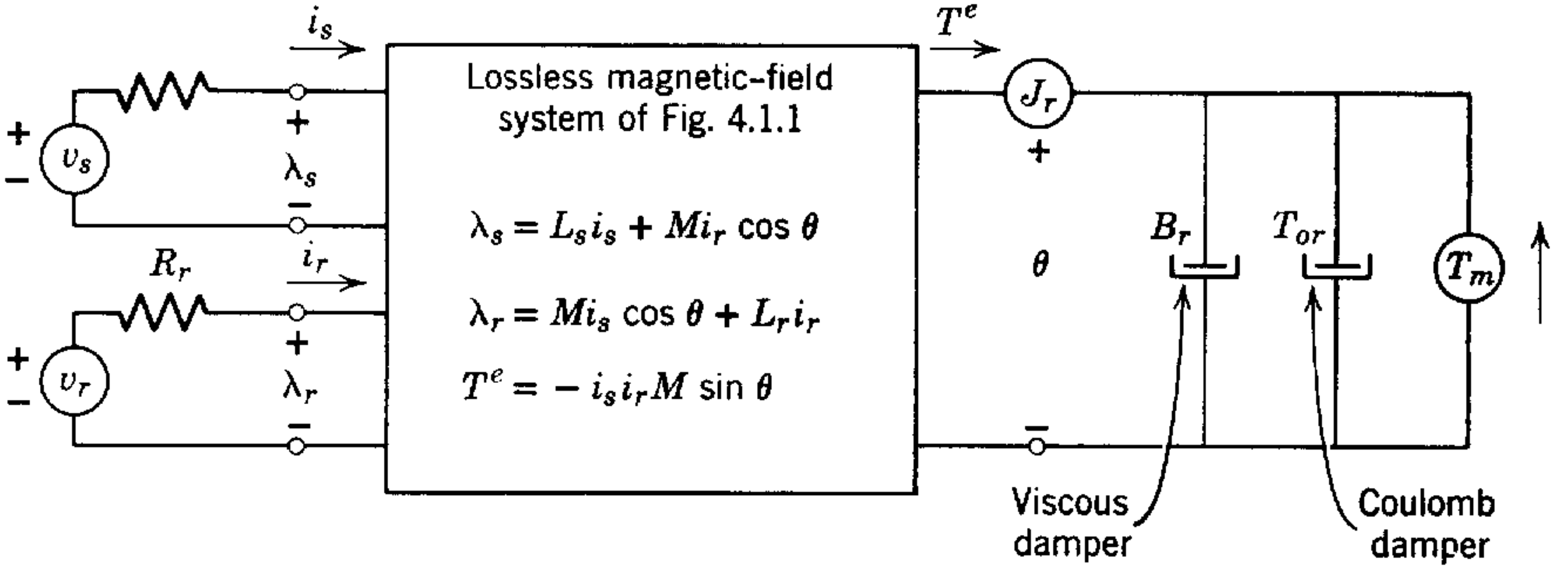
Generalized Machines/ Smooth Airgap Machines





Generalized Machines





$$\lambda_s = L_s i_s + L_{sr}(\theta) i_r,$$
$$\lambda_r = L_{sr}(\theta) i_s + L_r i_r,$$
$$T^e = i_s i_r \frac{dL_{sr}(\theta)}{d\theta},$$

$$L_{sr}(\theta) = M\cos\theta.$$

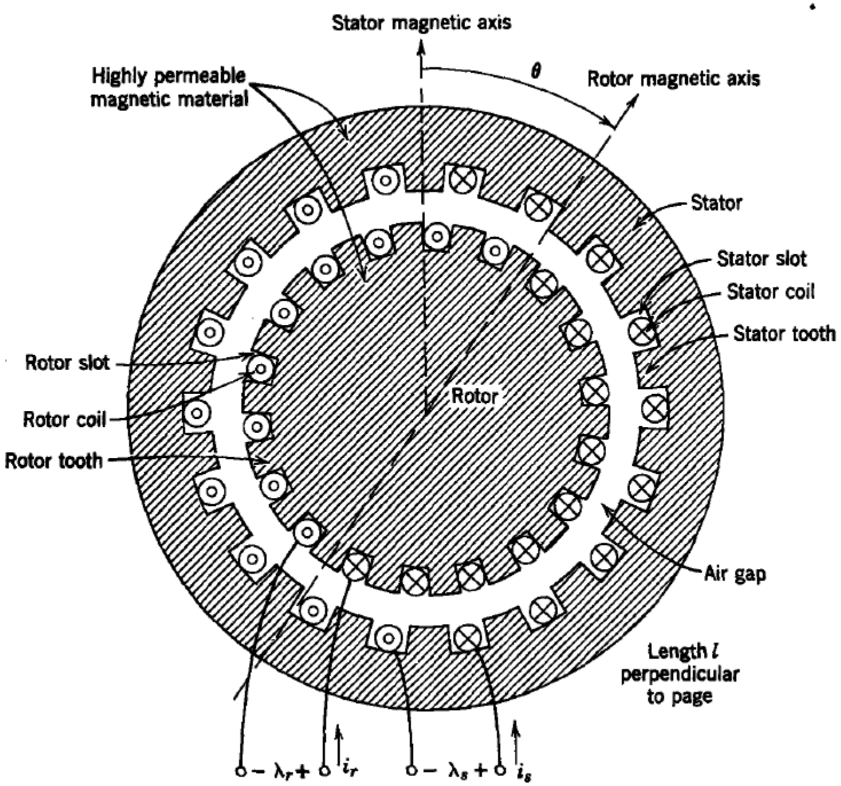
$$\lambda_s = L_s i_s + M i_r \cos\theta,$$
$$\lambda_r = M i_s \cos\theta + L_r i_r,$$
$$T^e = -i_s i_r M \sin\theta.$$

$$v_s = R_s i_s + \frac{d\lambda_s}{dt}$$
$$v_r = R_r i_r + \frac{d\lambda_r}{dt}$$
$$T_m + T^e = J_r \frac{d^2\theta}{dt^2} + B_r \frac{d\theta}{dt} + T_{or} \frac{d\theta/dt}{|d\theta/dt|}.$$

Generalized Machines/ Conditions for Conversion of Average Power

$$i_s(t) = I_s \sin \omega_s t,$$
$$i_r(t) = I_r \sin \omega_r t,$$
$$\theta(t) = \omega_m t + \gamma,$$

We need to find the conditions under which the machine with the steady-state excitations like this can convert average power between the electrical and mechanical systems



instantaneous power p_m, flowing from the coupling system,

$$p_m = T^e \frac{d\theta}{dt} = T^e \omega_m.$$

$$T^e = -i_s i_r M \sin \theta.$$

$$p_m = -\omega_m I_s I_r M \sin \omega_s t \sin \omega_r t \sin (\omega_m t + \gamma).$$