materials → dia
→ para
→ fero, feri

$\phi = NiP$ (Wb).

$P = \mu A / l$

$\psi = N\phi = N^2 i P$

P = permeance
↓
(opposite of Reluctance)

Faraday - Lenz law

$$e = -\frac{d\psi}{dt} = -\frac{d}{dt}(N^2 P i)$$

if the no. of turns and the magnetic permeance do not chang with time then -

$$e = -N^2 P \frac{di}{dt} = -L\frac{di}{dt}.$$

$$e = -\frac{d(Li)}{dt}$$

$$= -L\frac{di}{dt} - i\frac{dL}{dt}$$

$$= -L\frac{di}{dt} - i\frac{\partial L}{\partial x}\cdot\frac{\partial x}{\partial t}$$ (If L changes with $x$)

↳ in any system when two different forces are acting then it exhibits oscillation. (when time const. mismatched.)
Here,

Active damping
Passive damping.

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# Electrodynamics


as plunger is moving there is a variable reluctance.
so; called dynamic eq$^n$.

$$V = Ri + \frac{d}{dt}(Li)$$

(comes due to movement of plunger.)

- if internal energy becomes fixed (const.) (or) Reached it's nominal value. ⇒ Steady State.
- If system is starts from 0 (or) some finite energy and reaching towards it's nominal value ⇒ Transient state.

$$V = Ri + L\frac{di}{dt} + i\frac{\partial L}{\partial x}\cdot\frac{dx}{dt}$$

$$Vi = Ri^2 + iL\frac{di}{dt} + i^2\frac{\partial L}{\partial x}\frac{dx}{dt}$$

$$Vi = Ri^2 + \frac{d}{dt}\left(\frac{1}{2}Li^2\right) + \frac{1}{2}i^2\frac{\partial L}{\partial x}\cdot\frac{dx}{dt}$$

$$\Rightarrow Vi = \underbrace{Ri^2}_{\text{power loss due to Resistance of the coil}} + \underbrace{\frac{d}{dt}\left(\frac{1}{2}Li^2\right)}_{\text{Rate of Change of Stored mag. energy.}} + \underbrace{\frac{1}{2}i^2\frac{\partial L}{\partial x}\cdot\frac{dx}{dt}}_{\text{Power necessary to acc.}}$$

$P_e = \frac{1}{2}i^2\frac{\partial L}{}$

electromechanical force $f_e = \frac{1}{2}i^2\frac{\partial L}{\partial x}$ K is the stiffness of the spring.

involves the square of the current, this implies that irrespective of the direction of the current through the coil the force on the plunger acts only in one dir$^n$.

$$m\frac{d^2x}{dt^2} + Kx - \frac{1}{2}i^2\frac{\partial L}{\partial x} = 0 .$$ (electrodynamic eq$^n$ of the plunger.)

Sol$^n$ at $t = 0.05s$.

Power loss. $i^2R = (5.8)^2 \times 1.6$

Rate of change of stored mag. energy $\Rightarrow \frac{d}{dt}\left(\frac{1}{2}Li^2\right)$

$$= Li\frac{di}{dt} + \frac{1}{2}i^2\frac{dL}{dt}.$$

$L = \frac{0.1276}{1-32.33x}$

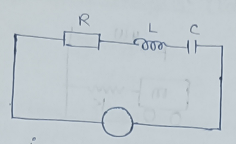
$$\frac{dL}{dt} = \frac{0.1276\,(32.33)\frac{dx}{dt}}{(1-32.33x)^2}$$

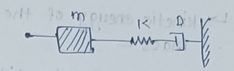
$$L = \frac{0.01276}{1-32.33x}$$

$$L = \frac{0.01276}{1-32.33(0.9)} = \frac{0.01276}{-28.097}$$

$$\Rightarrow \left[\frac{0.01276}{1-(32.33)(0.9)}\right](5.8)\cdot\underbrace{\left[\frac{6.2-5.8}{0.055-0.047}\right]}_{\left(\frac{di}{dt}\right)} + \frac{1}{2}\cdot(5.8)^2$$

* Electrodynamics




$E = N^2 P \int_0^i i \, di = \frac{1}{2} L i^2$ (J)

$E_c = \frac{1}{2} C V^2 = \frac{1}{2} \frac{q^2}{C}$

Kinetic energy $= \frac{1}{2} m v^2$

$= \frac{1}{2} J \omega^2$

$\int_0^x F dx = \int_0^x Kx \, dx = \frac{1}{2} K x^2$

$L\ddot{q} + R\dot{q} + \frac{q}{C} = V$ → electrical Network

$m\ddot{x} + D\dot{x} + Kx = F$ → mechanical system

↳ The analogy in general form: Lagrange's equation.

* Lagrangian Analysis (lossless system) :-

The Lagrangian

$\mathcal{L} = \underbrace{T}_{\text{Total K.E}} - V \rightarrow \text{P.E}$

Lagrangian eqn in general form for a system with 'n' degree of freedom.

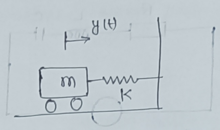
$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}_n}\right) - \frac{\partial \mathcal{L}}{\partial x_n} = 0$$

norm & diff type of norm

eigen vector → axis of rotation

in 3-D what are the different way of calculating distance?

• **Lagrangian Analysis** (Lossless system.)




↳ kinetic energy of the mass –

$$T = \frac{1}{2} m v^2 = \frac{1}{2} m (\dot{x})^2$$

↳ potential energy stored in the spring.

$$V = \int_0^x F\,dx = \int_0^x Kx\,dx = \frac{Kx^2}{2}$$

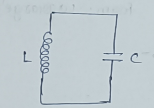
$$\frac{\partial T}{\partial \dot{x}} = \frac{\partial}{\partial \dot{x}}\left[\frac{1}{2} m (\dot{x})^2\right] = m\dot{x} = mv$$

$$\frac{\partial V}{\partial x} = \frac{\partial}{\partial x}\left(\frac{1}{2}Kx^2\right) = Kx = F$$

$$\mathcal{L} = T - V$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}_n}\right) - \frac{\partial \mathcal{L}}{\partial x_n} = 0$$

•




Newton's second law of motion –

$$\frac{d}{dt}(mv) = -F$$

Negative sign appears in this case because force is a restoring force.

$$\frac{\partial T}{\partial \dot{x}} = \frac{\partial}{\partial i}\left(\frac{1}{2}Li^2\right) = Li$$

$$\frac{\partial V}{\partial q} = \frac{\partial}{\partial q}\left(\frac{1}{2}\frac{q^2}{C}\right) = \frac{q}{C}$$

Substituting these values in Lagrange's equation

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}_n}\right) - \frac{\partial \mathcal{L}}{\partial x_n} = 0$$

$$\Rightarrow \frac{d}{dt}(Li) + \frac{q}{C} = 0$$

$$\Rightarrow L\frac{di}{dt} + \frac{q}{C} = 0$$

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## Lagrangian analysis (Lossy system)

The Lagrangian

$$\mathcal{L} = T - V$$

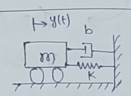
in order to include the effects of loss, a velocity-dependent (or current-dependent) function, the Rayleigh dissipation function is adopted.

$\boxed{\frac{\partial f}{\partial \dot{x}_n} = R_n \dot{x}_n}$ represents force (or voltage)

$$f = \sum_{n=1}^{K} \frac{1}{2} R_n (\dot{x}_n)^2$$

Lagrangian eqn

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}_n}\right) - \frac{\partial \mathcal{L}}{\partial x_n} + \frac{\partial f}{\partial \dot{x}_n} = V_n$$

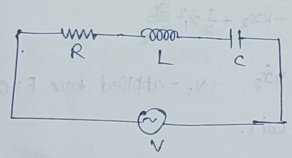



$$\mathcal{L} = T - V$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}_n}\right) - \frac{\partial \mathcal{L}}{\partial x_n} + \frac{\partial f}{\partial \dot{x}_n} = V_n$$

Substituting these values into Lagrange's equation—

$$\boxed{m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + Kx = F.}$$




$$\mathcal{L} = T - V$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}_n}\right) - \frac{\partial \mathcal{L}}{\partial x_n} + \frac{\partial f}{\partial \dot{x}_n} = V_n$$

$$\frac{\partial T}{\partial \dot{x}} = \frac{\partial}{\partial i}\left(\frac{1}{2} L i^2\right) = Li$$

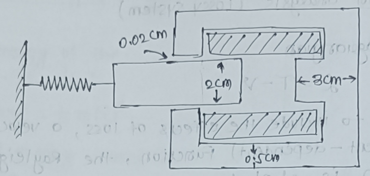
$$\frac{\partial V}{\partial q} = \frac{\partial}{\partial q}\left(\frac{1}{2}\frac{q^2}{c}\right) = \frac{q}{c}$$

$$\frac{\partial f}{\partial \dot{x}_n} = R_n \dot{x}_n$$

Substituting these values into Lagrange's eqn —

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{c} = V$$

*




Total kinetic energy

$T = \frac{1}{2}Li^2 + \frac{1}{2}mv^2$

elec. mech.

Total potential energy

$V = \frac{1}{2}Kx^2$

mech.

Dissipation function

$F = \frac{1}{2}Ri^2 + \frac{1}{2}Dv^2$

elec. mech.

$\mathcal{L} = T - V = \frac{1}{2}L\dot{x}_1^2 + \frac{1}{2}m\dot{x}_2^2 - \frac{1}{2}Kx_2^2$ D → viscous coeff.

$F = \frac{1}{2}R\dot{x}_1^2 + \frac{1}{2}D\dot{x}_2^2$

$\frac{\partial \mathcal{L}}{\partial \dot{x}_1} = L\dot{x}_1$

$\frac{\partial \mathcal{L}}{\partial x_1} = 0$

$\frac{\partial F}{\partial \dot{x}_1} = R_1\dot{x}_1$

$\frac{\partial \mathcal{L}}{\partial \dot{x}_2} = m\dot{x}_2$

$\frac{\partial \mathcal{L}}{\partial x_2} = -Kx_2 + \frac{1}{2}\dot{x}_1^2\frac{\partial L}{\partial x_2}$

$\frac{\partial F}{\partial \dot{x}_2} = D\dot{x}_2$

$V_2$ = applied force F = 0.

$V_n = V_1$ = voltage applied to the coil.

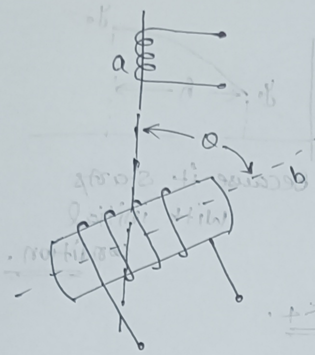
$L\frac{di}{dt} + Ri + i\frac{dL}{dt} = V_1$

(voltage eq$^n$ of the coil)

$m\ddot{x}_2 + D\dot{x}_2 + Kx_2 - \frac{1}{2}i^2\frac{\partial L}{\partial x_2}$

(Dynamic eq$^n$ of the plunger.)

## Doubly Excited coil:- (Lossy system)




- Total kinetic energy.

$$T = T_e + T_m$$

$$= \frac{1}{2}L_{11}i_1^2 + \frac{1}{2}L_{22}i_2^2 + i_1 i_2 M_{12} + \frac{1}{2}J\omega^2$$

$$T = \frac{1}{2}L_{11}\dot{x}_1^2 + \frac{1}{2}L_{22}\dot{x}_2^2 + \dot{x}_1\dot{x}_2 M_{12} + \frac{1}{2}J\dot{x}_3^2$$

- Total ~~kinet~~ potential energy

$$V = 0$$

- Dissipation function

$$f = \frac{1}{2}R_1 i_1^2 + \frac{1}{2}R_2 i_2^2 + \frac{1}{2}R_F\omega^2$$

$$= \frac{1}{2}R_1\dot{x}_1^2 + \frac{1}{2}R_2\dot{x}_2^2 + \frac{1}{2}R_F\dot{x}_3^2$$

- $x_1$ represent the charge $q_1$ through coil 1.
- $x_2$ represent the charge $q_2$ through coil 2.
- $x_3$ represent the displacement $\theta$ of the rotor.

$$\boxed{\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}_n}\right) - \frac{\partial \mathcal{L}}{\partial x_n} + \frac{\partial f}{\partial \dot{x}_n} = V_n}$$

$$\frac{\partial \mathcal{L}}{\partial \dot{x}_1} = \frac{\partial \mathcal{L}}{\partial i_1} = \frac{\partial T}{\partial i_1} = L_{11}i_1 + M_{12}i_2 \quad (\because V = 0)$$

$$\frac{\partial \mathcal{L}}{\partial \dot{x}_2} = \frac{\partial \mathcal{L}}{\partial i_2} = \frac{\partial T}{\partial i_2} = L_{22}i_2 + M_{12}i_1 \quad (\because V = 0)$$

$$\frac{\partial \mathcal{L}}{\partial \dot{x}_3} = \frac{\partial \mathcal{L}}{\partial \omega} = \frac{\partial T}{\partial \omega} = J\omega$$

$$\frac{\partial \mathcal{L}}{\partial x_3} = \frac{\partial \mathcal{L}}{\partial \theta} = \frac{1}{2}i_1^2\frac{\partial L_{11}}{\partial \theta} + \frac{1}{2}i_2^2\frac{\partial L_{22}}{\partial \theta} + i_1 i_2\frac{\partial M_{12}}{\partial \theta}$$

$$\frac{\partial f}{\partial \dot{x}_1} = \frac{\partial f}{\partial i_1} = R_1 i_1$$

$$\frac{\partial f}{\partial \dot{x}_2} = \frac{\partial f}{\partial i_2} = R_2 i_2$$

$$\frac{\partial f}{\partial \dot{x}_3} = \frac{\partial f}{\partial \omega} = R_F\omega$$

→ Collecting the eq$^n$ corresponding to n=1,2, and 3 successively, we have–

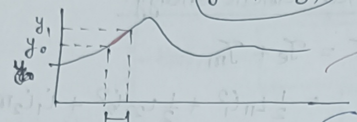
$$\begin{cases} V_1 = R_1 i_1 + P[L_{11}i_1 + M_{12}i_2] \\ V_2 = R_2 i_2 + P[L_{22}i_2 + M_{12}i_1] \\ 0 = R_F\omega + JP\omega - \left[\frac{1}{2}i_1^2\frac{\partial L_{11}}{\partial \theta} + \frac{1}{2}i_2^2\frac{\partial L_{22}}{\partial \theta} + i_1 i_2\frac{\partial M_{12}}{\partial \theta}\right] \end{cases}$$

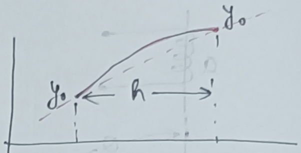
∵ in the third eq$^n$, $V_n = V_3 = 0$ if no torque is applied to the rotor.

→ These eq$^n$s give the complete electrodynamic description of a machine with two coil energised.

## * Runge kotta method (RK4)

$y' = f(x,y)$




Explicit method. → Because it starts with initial condition.

$$y_1 = y_0 + \frac{h}{6}\underbrace{(K_{10} + 2K_{20} + 3K_{30} + K_{40})}_{\text{Classical RK4.}}$$

$K_{ij}$

ith slope (ith iteration) — jth probe

**Ques** $y' + y = x^2$

$y(0) = 1$ → 0th sec.

$x(0) = 0$

$y' = x^2 - y$

slope $f(x,y) = x^2 - y$

$y(0.1) = ?$ → 0.1th sec.

$y(0.1) = y_1$

$= y_0 + \frac{h}{6}(K_{10} + 2K_{20} + 3K_{30} + K_{40})$

$h = 0.1$

$K_{10} = f(x_0, y_0)$

$K_{20} = f\left(x_0 + \frac{h}{2}, y_0 + \frac{hK_{10}}{2}\right)$

$K_{30} = f\left(x_0 + \frac{h}{2}, y_0 + \frac{hK_{20}}{2}\right)$

$K_{40} = f\left(x_0 + \frac{h}{2}, y_0 + hK_{30}\right)$

$K_0 = 0 - 1 = -1$

$K_{20} = \left((0.05)^2 + 1^2 - 0.05 \times (-1)\right)$

$= (-0.94)$

$K_{30} = -0.9505$

$K_{40} = -0.8949$

0.05 → Shift

$y(0.1) = y_1$

$= 1 + \frac{0.1}{6}\left(-1 + (2 \times -0.94) + (3 \times -0.9505) + (-0.8949)\right)$

$= 0.88931$

---

**Ques** $y' + y = x^2$ ; $y(0) = 1$ , $x(0) = 0$ . Find $y(0.1) = ?$

$\Rightarrow y' = x^2 - y$

$\Rightarrow f(x, y) = x^2 - y$

$y(0.1) = y_1 = y_0 + \frac{h}{6}(K_{10} + 2K_{20} + 3K_{30} + K_{40})$

$K_{10} = f(x_0, y_0)$

$K_{20} = f\left(x_0 + \frac{h}{2}, y_0 + \frac{hK_{10}}{2}\right)$

$K_{30} = f\left(x_0 + \frac{h}{2}, y_0 + \frac{hK_{20}}{2}\right)$

$K_{40} = f(x_0 + h, y_0 + hK_{30})$

$h = 0.1$ ; $f(x, y) = x^2 - y$

$K_{10} = f(0, 1) = 0 - 1 = -1$

$K_{20} = f(0.05, 0.95) = (0.05)^2 - 0.95$

$= -0.9475$

$K_{30} = f(0.05, 0.952625) = -0.950125$

$K_{40} = f(0.1, 0.9049875) = -0.8949875$

$\therefore y(0.1) = y_1 = y_0 + \frac{h}{6}(K_{10} + 2K_{20} + 3K_{30} + K_{40})$

$= 1 + \frac{0.1}{6}\left[-1 + 2(-0.9475) + 3(-0.950125) + (-0.8949875)\right]$

$= 0.889327$

* RK4 for 2nd order function :-

$$y'' = ay' + by + c \quad : \quad a, b \rightarrow f(x)$$

$$\boxed{y_{k+1} = y_k + \frac{h}{6}\left(k_{10} + 2k_{20} + 3k_{30} + k_{40}\right)}$$

Step-1

$$\frac{dy}{dx} = z = y' = f(x, y, z)$$

$$\frac{d^2y}{dx^2} = \frac{dz}{dx} = y'' = g(x, y, z)$$

Step-2

$$k_{10} = f(x_0, y_0\ z_0)$$

$$l_{10} = g(x_0 \cdot y_0, z_0)$$

$$k_{20} = f\left(x_0 + \frac{h}{2},\ y_0 + \frac{h}{2}k_{10},\ z_0 + \frac{h}{2}l_{10}\right)$$

$$l_{20} = g\left(x_0 + \frac{h}{2},\ y_0 + \frac{h}{2}k_{10},\ z_0 + \frac{h}{2}l_{10}\right)$$

$$k_{30} = f\left(x_0 + \frac{h}{2},\ y_0 + \frac{h}{2}k_{20},\ z_0 + \frac{h}{2}l_{20}\right)$$

$$l_{30} = g\left(x_0 + \frac{h}{2},\ y_0 + \frac{h}{2}k_{20},\ z_0 + \frac{h}{2}l_{20}\right)$$

$$k_{40} = f\left(x_0 + \frac{h}{2},\ y_0 + hk_{30},\ z_0 + h_8 l_{30}\right)$$

Step-3

$$y_{k+1} = \_\_\_\_$$

eg: $y'' = 1 + 2xy - x^2 y'$

$x_0 = 0$, $y_0 = 1$

$z_0 = 0$, $h = 0.1$

$z = y' = f(x, y, z)$

$\frac{dz}{dx} = y'' = g(x, y, z)$

$g(x, y, z) = 1 + 2xy - x^2 z$

$K_{10} = f(x_0, y_0, z_0)$

$K_{10} = z_0 = 0$

$l_{10} = g(x_0, y_0, z_0) = 1 + 0 - 0$

$l_{10} = 1$

$K_{20} = f\left(x_0 + \frac{h}{2}, y_0 + \frac{h}{2} K_{10}, z_0 + \frac{h}{2} l_{10}\right)$

$= z_0 + \frac{h}{2} l_{10}$

$= 0 + 0.05$

$= 0.05$

$l_{20} = g(0.05, 1, 0.05)$

$= 1 + 2 * (0.05) * 0 - (0.05)^2 * 0.05 + 2 * 1 * 0.05$

$= \cancel{0.999875}\ 1.09987$

$K_{30} = f(0.05, 1.0025, 0.054993)$

$= z$

$= 0.054993$

$l_{30} = f(0.05, 1.0025, 0.054993)$

$= 1 + 2 * 0.05 * 1.0025 - (0.05)^2 * 0.054993$

$= 1.1001$

$K_{40} = f(0.05, 1.005499, 0.11001)$

$= 0.11001$

So; $y_{k+1} = y_k + \frac{h}{6}(K_{10} + 2K_{20} + 3K_{30} + K_{40})$

$y_1 = y_0 + \frac{h}{6}(0 + 0.1 + 0.164979 + 0.11001)$

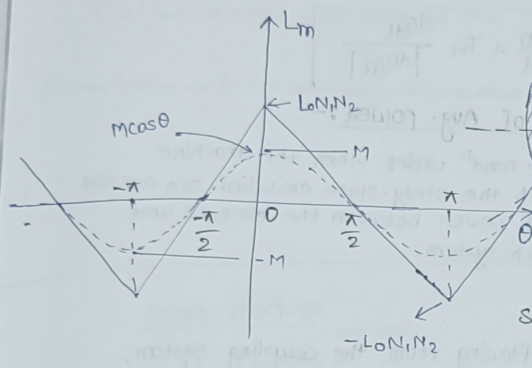
$= 1 + \frac{0.1}{6}(0.374989)$

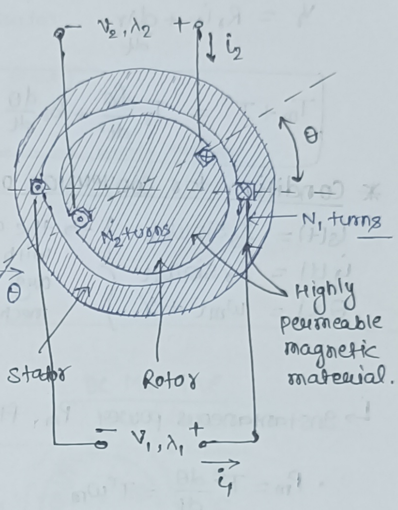
$= 1.00625$

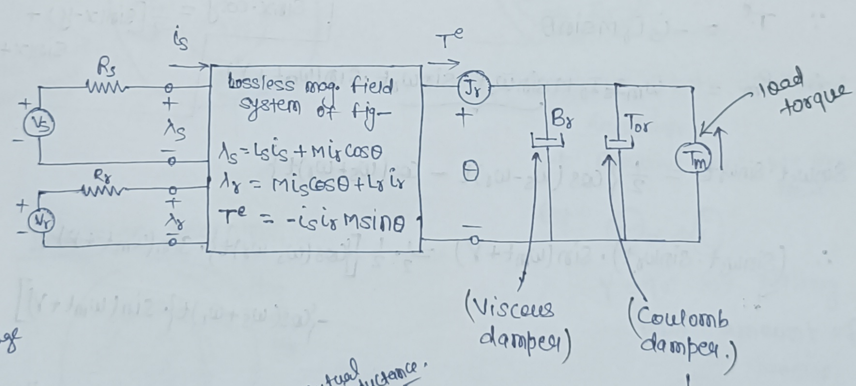
* Euler's Method :-

$$\boxed{y_{k+1} = y_k + h_0 y'}$$

## * Generalized machines/(smooth Airgap machines)




Stator flux linkage

$$\lambda_s = L_s i_s + L_{sr}(\theta) i_r,$$

Rotor flux linkage

$$\lambda_r = L_{sr}(\theta) i_s + L_r i_r,$$

$$T^e = i_s i_r \frac{dL_{sr}(\theta)}{d\theta}$$

Mutual inductance

$$L_{sr}(\theta) = M\cos\theta$$

so, $\lambda_s = L_s i_s + M i_r \cos\theta$

$$\lambda_r = M i_s \cos\theta + L_r i_r$$

$$T^e = -i_s i_r (\sin\theta)\cdot M$$

$$v_s = R_s i_s + \frac{d\lambda_s}{dt}$$

$$v_r = R_r i_r + \frac{d\lambda_r}{dt}$$

$$\boxed{T_m + T^e = J_r \frac{d^2\theta}{dt^2} + B_r \frac{d\theta}{dt} + T_{0r} \frac{d\theta/dt}{|d\theta/dt|}}$$

* **Condition for conversion of Avg. power:—**

$i_s(t) = I_s \sin\omega_s t$
$i_r(t) = I_r \sin\omega_r t$
$\theta(t) = \omega_m t + \gamma$

→ the condⁿ under which the machine with the stedy-state excitation can convert avg. power between the electrical and mech. system.

↳ Instantaneous power $P_m$, flowing from the coupling system,

$$\cdot\ P_m = T^e \frac{d\theta}{dt} = T^e \omega_m$$

$\sin x \cdot \sin y = \frac{1}{2}[\cos(x-y) - \cos(x+y)]$

$\sin x \cdot \cos y = \frac{1}{2}[\sin(x-y) + \sin(x+y)]$

$$\because\ T^e = -i_s i_r M \sin\theta$$

$$\therefore\ P_m = -\omega_m I_s I_r M \underbrace{\sin\omega_s t \cdot \sin\omega_r t} \cdot \sin(\omega_m t + \gamma)$$

$$\sin\omega_s t \cdot \sin\omega_r t = \frac{1}{2}\{\cos(\omega_s - \omega_r)t - \cos(\omega_s + \omega_r)t\}$$

$$\therefore\ (\sin\omega_s t \cdot \sin\omega_r t) \cdot \sin(\omega_m t + \gamma) = \frac{1}{2} \cdot \frac{1}{2}\left[\{\cos(\omega_s - \omega_r)t\} \cdot \sin(\omega_m t + \gamma) - \{\cos(\omega_s + \omega_r)t\} \sin(\omega_m t + \gamma)\right]$$

$$= \frac{1}{4}\left\{\sin[(\omega_m + \omega_r - \omega_s)t + \gamma] + \sin[(\omega_m + \omega_s - \omega_r)t + \gamma] - \sin[(\omega_m - \omega_s - \omega_r)t + \gamma] - \sin[(\omega_m + \omega_s + \omega_r)t + \gamma]\right\}$$

$$P_m = \frac{-\omega_m I_s I_r M}{4}\left[\sin\{(\omega_m + \omega_s - \omega_r)t + \gamma\} + \sin\{(\omega_m + \omega_r - \omega_s)t + \gamma\} - \sin\{(\omega_m + \omega_s + \omega_r)t + \gamma\} - \sin\{(\omega_m - \omega_s - \omega_r)t + \gamma\}\right].$$

$\omega_m \rightarrow$ mech. speed (·

$\omega_r \rightarrow$ freq$^n$ of supply generated or provided in Rotor

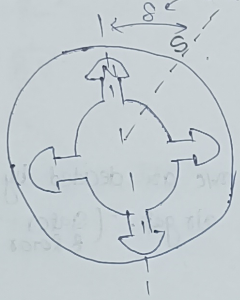
$\omega_s \rightarrow$ freq$^n$ of " given to Stator.

$T^e \rightarrow$ (due to force of attr. & repulsion b/w the field)

· Coulomb friction → friction on a moving body which is there in the moving medium.

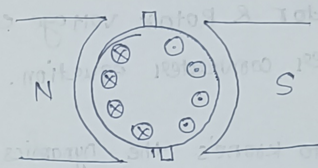
---

Synch. machine




Rotate b/c of magnetic attr$^n$.

DC Machine



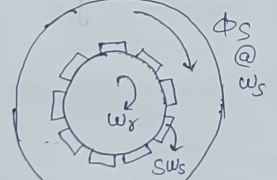

· Rotate b/c of both attr$^n$ and Repulsion

But Repulsive force we require more.

for $T \propto \phi i$

$\gamma = 90°$ for getting max$^m$ amount of torque.

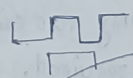
Induction machine




$$\omega_r + s\omega_s = \omega_s$$

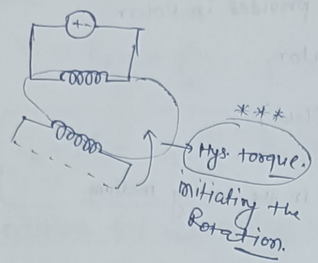
$\gamma$
- rotor pf
- Skewing.

Cogging.



Pole Slipping

Hysterisis torque




Polarization

we never use Cu bar for damper wdg. (Al bar is used why?)

electrical stress

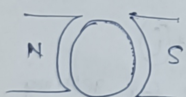
* primitive machines

Steps to designing a primitive machines models

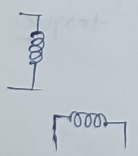
*① Develop the equivalent Krone's electric. model.
② Flux linkage equation
③ Stator & Rotor voltage equation
④ power converter equation.

. according to Krone's the Dynamics of elec. m/c are decided by the dynamics of Magnetic field (Flux) present in the air gap. (Stator & Rotor flux.)

For DC

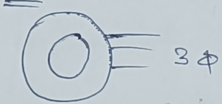



Both are Stationary fields.

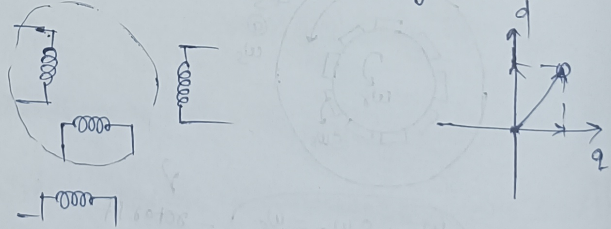


for dc to draw eq. model only two coils are required.

AC




Both fields are rotating




Here four coils are required for Ac. M/c's.

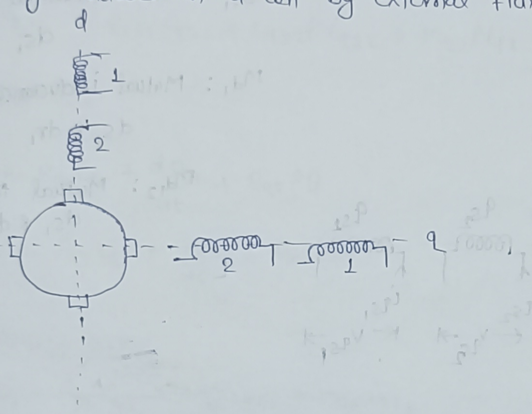
BK Bose
Sen gupta
~~Bimal K-bose~~
R krishnan

$d^s \quad q^s \rightarrow$ Stationary Ref. frame
$d^e \quad q^e \rightarrow$ Synchronous " "
$d^r \quad q^r \rightarrow$ Rotor " "
$d^c \quad q^c \rightarrow$ Arbitrary " "

## * primitive machine eqn. along stationary Axes :-

Fundamentals

- voltage induced in a coil by its own flux.
- Voltage induced in a coil by external flux.




- Voltage induced in a Stator coil

$$V = Ri + \frac{d\varphi}{dt} \quad \text{(transformer voltage)}.$$

- eqn for the applied voltage in a rotor coil will contain the terms

$$V = Ri + \frac{d\varphi}{dt} + BP\theta$$

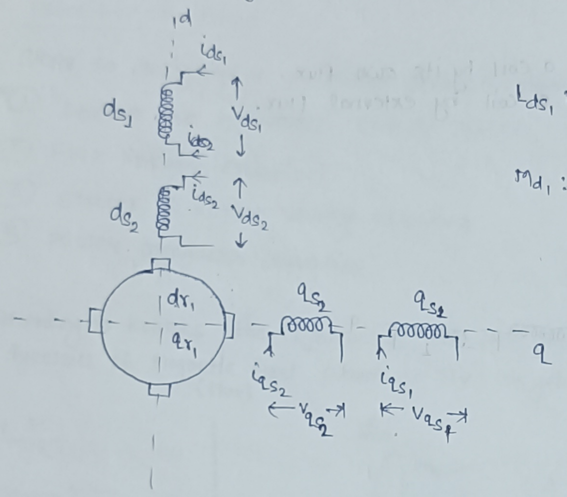
Res. drop + transformer voltage + generated voltage

$$e = \frac{d\varphi}{dt}$$
$$= \frac{d}{dt}(N\phi)$$
$$= N\frac{d}{dt}(\phi_m \sin\omega t)$$
$$= N\phi_m \omega \cos\omega t$$

$\hookrightarrow B\omega$

## Flux linkage

- the letter d and q stand for direct & quadrature axes
- s for stator coils, r for rotor coils.
- $M_{d1}$ is the mutual inductance b/w coil 1 on the stator and the rotor coil along the direct axis.
- $M_{d12}$ is the mutual inductance between coil 1 and 2 in the stator along the direct axis.




$L_{ds_1}$ : self inductance of $ds_1$

$M_{d_1}$ : Mutual inductance b/w $ds_1$ & $dr_1$

$M_{d_{12}}$ : Mutual inductance $ds_1$ & $ds_2$.

$$\psi_{dr} = L_{dr} i_{dr} + M_{d_1} \cdot i_{ds_1} + M_{d_2} i_{ds_2}$$

$$\psi_{qr} = L_{qr} i_{qr} + M_{q_1} i_{qs_1} + M_{q_2} i_{qs_2}$$

$$\psi_{ds_1} = L_{ds_1} i_{ds_1} + M_{d_1} i_{dr} + M_{d_{12}} i_{ds_2}$$

$$\psi_{qs_1} = L_{qs_1} i_{qs_1} + M_{q_1} i_{qr} + M_{q_{12}} i_{qs_2}$$

$$\psi_{ds_2} = L_{ds_2} i_{ds_2} + M_{d_2} i_{dr} + M_{d_{12}} i_{ds_1}$$

$$\psi_{qs_2} = L_{qs_2} i_{qs_2} + M_{q_2} i_{qr} + M_{q_{12}} i_{qs_1}$$

$L_{dr} \neq L_{qr}$ for salient pole machines.

↳ TWO HVDC lines in our country.

(DFIG)

Doubly Fed Ind. genr.

Kraemer drive

Scherbius "

$$\frac{d\psi_{dr}}{dt} = \frac{d}{dt}\left[L_{dr} i_{dr} + M_{d_1} \cdot i_{ds_1} + M_{d_2} \cdot i_{ds_2}\right]$$

$$= L_{dr} \cdot \frac{d}{dt}(i_{dr}) + M_{d_1} \frac{d}{dt}(i_{ds_1}) + M_{d_2} \cdot \frac{d}{dt}(i_{ds_2}).$$

$$\frac{d\psi_{qr}}{dt} = \frac{d}{dt}\left[L_{qr} \cdot i_{qr} + M_{q_1} i_{qs_1} + M_{q_2} i_{qs_2}\right]$$

$$= L_{qr} \cdot \frac{d}{dt}(i_{qr}) + M_{q_1} \frac{d}{dt}(i_{qs_1}) + M_{q_2} \frac{d}{dt}(i_{qs_2}).$$

$$\Rightarrow V_{dr} = R_{dr} \cdot i_{dr} + L_{dr} \frac{d}{dt}(i_{dr}) + M_{dr} \frac{d}{dt}(i_{ds_1}) + M_{d_2} \frac{d}{dt}(i_{ds_2})$$

$$+ \left(L'_{qr} i_{qr} + M'_{q_1} i_{qs_1} + M'_{q_2} i_{qs_2}\right) \frac{d}{dt}\theta$$

$$V_{dr} = R_{dr} i_{dr} + L_{dr} P i_{dr} + M_{dr} P i_{ds_1} + M_{d_2} P i_{ds_2} +$$

$$\left(L'_{qr} i_{qr} + M'_{q_1} i_{qs_1} + M'_{q_2} i_{qs_2}\right) P\theta.$$

$$V_{qr} = R_{qr} i_{qr} + L_{qr} P i_{qr} + M_{q_1} P i_{qs_1} + M_{q_2} P i_{qs_2}$$

$$- (L'_{dr} i_{dr}) P\theta - M'_{d_1} i_{ds_1} P\theta - M'_{d_2} i_{ds_2} P\theta.$$

$$V_{ds_1} = R_{ds_1} i_{ds_1} + L_{ds_1} P i_{ds_1} + M_{d_1} P i_{dr} + M_{d12} P i_{ds_2}$$

$$V_{ds_2} = R_{ds_2} i_{ds_2} + L_{ds_2} P i_{ds_2} + M_{d_2} P i_{dr} + M_{d12} P i_{ds_1}$$

$$V_{qs_1} = R_{qs_1} i_{qs_1} + L_{qs_1} \cdot P i_{qs_1} + M_{q_1} P i_{qr} + M_{q12} P i_{ds_2}$$

$$V_{qs_2} = R_{qs_2} i_{qs_2} + L_{qs_2} P i_{qs_2} + M_{q_2} P i_{qr} + M_{q12} P i_{qs_1}$$

$$\begin{bmatrix} V_{ds_1} \\ V_{ds_2} \\ V_{dr} \\ V_{qr} \\ V_{qs_1} \\ V_{qs_2} \end{bmatrix} = \begin{bmatrix} (R_{ds_1} + L_{ds_1}P) & (M_{d12}P) & (M_{d1}P) & 0 & 0 & 0 \\ M_{d12}P & (R_{ds_2} + L_{ds_2}P) & (M_{d2}P) & 0 & 0 & 0 \\ M_{d1}P & M_{d2}P & (R_{dr} + L_{dr}P) & -L'_{qr}P\theta & M'_{q1}P\theta & M'_{q2}P\theta \\ -M'_{d1}P\theta & -M'_{d2}P\theta & -L'_{dr}P\theta & (R_{qr} + L_{qr}P) & M_{q1}P & M_{q2}P \\ 0 & 0 & 0 & M_{q1}P & (R_{qs_1} + L_{qs_1}P) & M_{q12}P \\ 0 & 0 & 0 & M_{q2}P & M_{q12}P & (R_{qs_2} + L_{qs_2}P) \end{bmatrix} \begin{bmatrix} i_{ds_1} \\ i_{ds_2} \\ i_{dr} \\ i_{qr} \\ i_{qs_1} \\ i_{qs_2} \end{bmatrix}$$

<u>Primitive machine eqn</u>

$$[V] = [Z][i]$$

$$[V] = [R][i] + [L]P[i] + [G][i]P\theta$$

$$V = Ri + LPi + GiP\theta .$$

$$\cancel{\begin{bmatrix} V_{ds_1} \\ V_{ds_2} \\ V_{dr} \\ V_{qr} \\ V_{qs_1} \\ V_{qs_2} \end{bmatrix} = \overset{R}{\begin{bmatrix} R_{ds_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & R_{ds_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & R_{dr} & 0 & 0 & 0 \\ 0 & 0 & 0 & R_{qr} & 0 & 0 \\ 0 & 0 & 0 & 0 & R_{qs_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & R_{qs_2} \end{bmatrix}} \begin{bmatrix} i_{ds_1} \\ i_{ds_2} \\ i_{dr} \\ i_{qr} \\ i_{qs_1} \\ i_{qs_2} \end{bmatrix} + \begin{bmatrix} L_{ds_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & L_{ds_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{dr} & 0 & 0 & 0 \\ 0 & 0 & -L'_{dr} & L_{qr} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{qs_1} & 0 \\ 0 & 0 & & & & \end{bmatrix}}$$

$[V] = [R]\,[i] + [L]\,p\,[i] + [G]\,p\theta\,[i]$

$$\begin{bmatrix} V_{ds_1} \\ V_{ds_2} \\ V_{dr} \\ V_{qr} \\ V_{qs_1} \\ V_{qs_2} \end{bmatrix} = \begin{bmatrix} R_{ds_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & R_{ds_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & R_{dr} & 0 & 0 & 0 \\ 0 & 0 & 0 & R_{qr} & 0 & 0 \\ 0 & 0 & 0 & 0 & R_{qs_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & R_{qs_2} \end{bmatrix} \begin{bmatrix} i_{ds_1} \\ i_{ds_2} \\ i_{dr} \\ i_{qr} \\ i_{qs_1} \\ i_{qs_2} \end{bmatrix} + \begin{bmatrix} L_{ds_1} & M_{d12} & M_{d1} & 0 & 0 & 0 \\ M_{d12} & L_{ds_2} & M_{d2} & 0 & 0 & 0 \\ M_{d1} & M_{d2} & L_{dr} & 0 & 0 & 0 \\ 0 & 0 & 0 & L_{qr} & M_{q1} & M_{q2} \\ 0 & 0 & 0 & M_{q1} & L_{qs_1} & M_{q12} \\ 0 & 0 & 0 & M_{q2} & M_{q12} & L_{qs2} \end{bmatrix} p \begin{bmatrix} i_{ds_1} \\ i_{ds_2} \\ i_{dr} \\ i_{qr} \\ i_{qs_1} \\ i_{qs2} \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & L'_{qr} & M'_{q1} & M'_{q2} \\ -M'_{d1} & -M'_{d2} & -L'_{dr} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} p\theta \begin{bmatrix} i_{ds_1} \\ i_{ds2} \\ i_{dr} \\ i_{qr} \\ i_{qs_1} \\ i_{qs2} \end{bmatrix}$$

→ How it represents as a loss

Rate of change of stored magnetic energy. $\frac{d}{dt}\left(\frac{1}{2}Li^2\right) = i_L p i$

$$\underbrace{i_t^* V}_{\text{Power input to the motor}} = \underbrace{i_t^* R i}_{\text{dissipated as copper loss}} + i_t^* L p i + \underbrace{i_t^* G i p\theta}_{\text{Mechanical power at the output shaft.}}$$

$P_m = \omega T = T p\theta$

$T_0 = i_t^* G i = i_t^* B$ ∵ G is the torque matrix
B represents flux density in the air gap.

$[\, i_{ds_1}^* \quad i_{ds_2}^* \quad i_{dr}^* \quad i_{qr}^*$

$$i_t^* R i = i_{ds_1}^* R_{ds_1} i_{ds_1} + i_{ds_2}^* R_{ds_2} i_{ds_2} + i_{dr}^* R_{dr} i_{dr} + i_{qr}^* R_{qr} i_{qr} + i_{qs_1}^* R_{qs_1} i_{qs_1} + i_{qs_2}^* R_{qs_2} i_{qs_2}. \quad — (1)$$

$$\begin{aligned} i_t^* L P i = {} & i_{ds_1}^* L_{ds_1} P i_{ds_1} + i_{ds_2}^* M_{d12} P i_{ds_1} + i_{dr}^* M_{d1} P i_{ds_1} + \\ & i_{ds_1}^* M_{d12} P i_{ds_2} + i_{ds_2}^* L_{ds_2} P i_{ds_2} + i_{dr}^* M_{d2} P i_{ds_2} + \\ & i_{ds_1}^* M_{d1} P i_{dr} + i_{ds_2}^* M_{d2} P i_{dr} + i_{dr}^* L_{dr} P i_{dr} + \\ & i_{qr}^* L_{qr} P i_{qr} + i_{qs_1}^* M_{q1} P i_{qr} + i_{qs_2}^* M_{q2} P i_{qr} + \\ & i_{qr}^* M_{q1} P i_{qs_1} + i_{qs_1}^* L_{qs_1} P i_{qs_1} + i_{qs_2}^* M_{q12} P i_{qs_1} + \\ & i_{qr}^* M_{q2} P i_{qs_2} + i_{qs_1}^* M_{q12} P i_{qs_2} + i_{qs_2}^* L_{qs_2} P i_{qs_2} \quad — (2) \end{aligned}$$

<u>where</u>

$$i_t^* = [\, i_{ds_1}^* \;\; i_{ds_2}^* \;\; i_{dr}^* \;\; i_{qr}^* \;\; i_{qs_1}^* \;\; i_{qs_2}^* \,] \quad \text{and} \quad i = \begin{bmatrix} i_{ds_1} \\ i_{ds_2} \\ i_{dr} \\ i_{qr} \\ i_{ds_1} \\ i_{ds_2} \end{bmatrix}$$

<u>Now;</u>

$$\begin{aligned} i_t^* G i = {} & - i_{qr}^* M'_{d_1} i_{ds_1} \overset{\omega}{P\theta} - i_{qr}^* M'_{d_2} i_{ds_2} \overset{\omega}{P\theta} - i_{qr}^* L'_{dr} i_{dr} \overset{\omega}{P\theta} \\ & + \underbrace{i_{dr}^* L'_{qr} i_{qr} P\theta}_{G\omega} + \underbrace{i_{dr}^* M'_{q_1} i_{qs_1} P\theta}_{G\omega} + \underbrace{i_{dr}^* M'_{q_2} i_{qs_2} P\theta}_{G\omega}. \end{aligned}$$

$$\boxed{T_i = J P^2\theta + R_F P\theta - i_t^* G i}$$

$T_i$ represents mech. i/p torque

$R_F \Rightarrow$ mechanical coefficient representing dissipation due to friction and windage.

$J \Rightarrow$ Moment of inertia of the rotating mass ($kg\,m^2$).

$$\begin{bmatrix} V \\ T_i \end{bmatrix} = \begin{bmatrix} R + Lp & G i \\ -i_t^* G & Jp + R_F \end{bmatrix} \cdot \begin{bmatrix} i \\ \omega \end{bmatrix}$$

$$\begin{bmatrix} V_{ds_1} \\ V_{ds_2} \\ V_{dr} \\ V_{qr} \\ V_{qs_1} \\ V_{qs_2} \\ T_i \end{bmatrix} = \begin{bmatrix} (R_{ds_1} + L_{ds_1}P) & M_{d12}P & M_{d1}P & — & — & — & — \\ M_{d12}P & (R_{ds_2} + L_{ds_2}P) & M_{d2}P & — & — & — & — \\ M_{d1}P & M_{d2}P & (R_{dr} + L_{dr}P) & — & — & — & (L'_{qr} i'_{qr} + M'_{q1} i_{qs_1} + M'_{q2} i_{qs_2}) \\ — & — & — & (R_{qr} + L_{qr}P) & M_{q1}P & M_{q2}P & (-M'_{d1} i_{ds_1} - M'_{d2} i_{ds_2} - L_{dr} i_{dr}) \\ — & — & — & M_{q1}P & (R_{qs_1} + L_{qs_1}P) & M_{q12}P & — \\ — & — & — & M_{q2}P & M_{q12}P & (R_{qs_2} + L_{qs_2}P) & — \\ i_{qr}^* M'_{d1} & i_{qr}^* M'_{d2} & i_{qr}^* L'_{dr} & -i_{dr}^* L'_{qr} & -i_{dr}^* M'_{d1} & -i_{dr}^* M'_{q2} & (Jp + R_F) \end{bmatrix} \cdot \begin{bmatrix} i_{ds_1} \\ i_{ds_2} \\ i_{dr} \\ i_{qr} \\ i_{qs_1} \\ i_{qs_2} \\ P\theta \end{bmatrix}$$

approximate for 4

$$\begin{bmatrix} V_{ds_1} \\ V_{dr} \\ V_{qr} \\ V_{qs_1} \\ T_i \end{bmatrix} = \begin{bmatrix} (R_{ds_1} + L_{ds_1}P) & M_{d1}P & — & — & — \\ M_{d1}P & (R_{dr} + L_{dr}P) & — & — & (L'_{qr} i'_{qr} + M'_{q1} i_{qs_1} + \cancel{M'_{q2} i_{q2}}) \\ — & — & (R_{qr} + L_{qr}P) & M_{q1}P & (-M'_{d1} i_{ds_1} - L'_{dr} i_{dr}) \\ — & — & M_{q1}P & (R_{qs_1} + L_{qs_1}P) & — \\ i_{qr}^* M'_{d1} & + i'_{qr} L'_{dr} & -i_{dr}^* L'_{qr} & -i_{dr}^* M'_{q2} & (Jp + R_F) \end{bmatrix} \begin{bmatrix} i'_{ds_1} \\ i'_{dr} \\ i'_{qr} \\ i'_{qs_1} \\ P\theta \end{bmatrix}$$

* Compute the electromechanical torque developed

$$\boxed{W = 0.85 \text{ P.U}}$$

$$T_e = -i'^*_{qr} M'_{d1} i'_{ds1} P\theta - i'^*_{qr} L'_{dr} i'_{dr} P\theta + i'^*_{dr} L'_{qr} i'_{qr} P\theta + i'^*_{dr} M'_{q1} i'_{qs1} P\theta$$

$$i'^*_{qr} = i'_{qr}$$

$$T_e = -(0.345)(0.499)(0.846)\cdot(0.85) - (0.345)(0.592)(0.773)(0.85) + (0.773)(0.397)(0.345)(0.85) + (0.773)(0.374)(0.512)(0.85)$$

$$= \cancel{-0.12379 - 0.134195 + 0.0899928 + 0.1167341}$$

$$T_e = -0.04218 \text{ PU}$$

(we are getting small value because the current value is very small.

$$T_m - T_e = J \cdot \frac{d^2\theta}{dt^2} + R_f \cdot \frac{d\theta}{dt}$$

$\Delta T$

$$\Delta T(s) = s^2 J\,\theta(s) + s R_f\,\theta(s)$$

$$\frac{\theta(s)}{\Delta T(s)} = \frac{1}{Js^2 + R_f s}$$

$$\frac{\theta(s)}{\Delta T(s)} = \frac{1}{s(Js + R_f)}$$