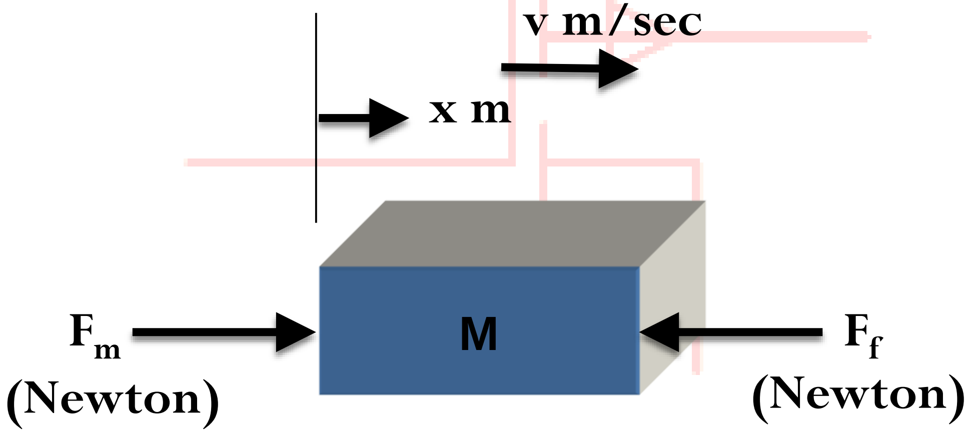
# Dynamics of Electrical Drives

Lecture 2 (05-01-2024)

# Fundamental Torque Equations

## Elementary principles of mechanics – Linear Motion




Newton's law

$$F_m - F_f = \frac{d(Mv)}{dt}$$

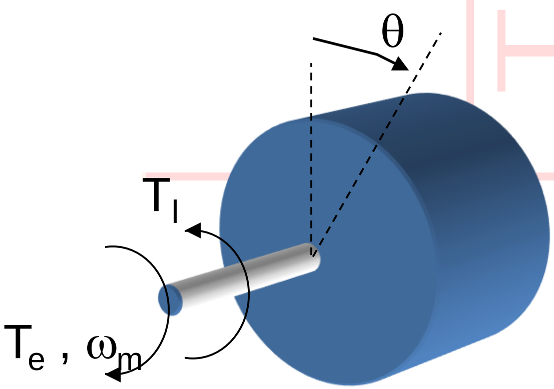
**Linear motion, constant *M***

$$F_m - F_f = M\left(\frac{dv}{dt}\right) = M\left(\frac{d^2x}{dt^2}\right) = Ma$$

- First order differential equation for speed
- Second order differential equation for displacement

# Fundamental Torque Equations

## Elementary principles of mechanics – Rotational Motion




- Normally is the case for electrical drives

$$\mathbf{T_e - T_l = \frac{d(J\omega_m)}{dt}}$$

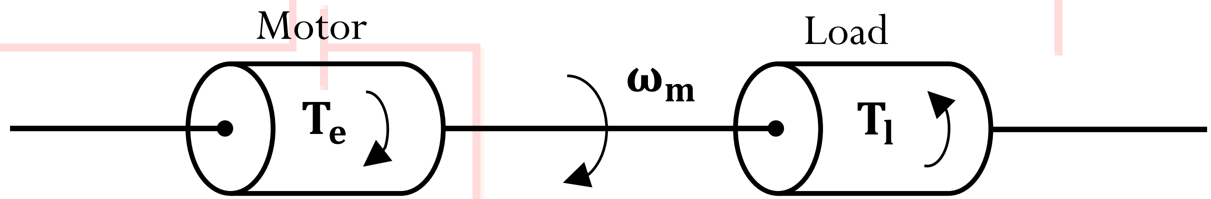
With constant J,

$$\mathbf{T_e - T_l = J\left(\frac{d\omega_m}{dt}\right) = J\left(\frac{d^2\theta}{dt^2}\right)}$$

- First order differential equation for angular frequency (or velocity)
- Second order differential equation for angle (or position)

# Fundamental Torque Equations

## Equivalent motor-load system




$$T_e - T_l = \frac{d(J\omega_m)}{dt}$$

$$T_e - T_l = J\frac{d\omega_m}{dt} + \omega_m\frac{dJ}{dt}$$

For drives with constant inertia, $\frac{dJ}{dt}=0$

$$T_e = T_l + J\frac{d\omega_m}{dt}$$

# Fundamental Torque Equations

## Equivalent motor-load system

$$T_e = T_l + J\frac{d\omega_m}{dt}$$

Electromagnetic Torque

Load Torque

Dynamic Torque

- Electromagnetic torque – developed torque by the motor
- Load Torque
- Dynamic Torque – Present during transient operations
    - Acceleration
    - Deceleration

# Fundamental Torque Equations

## Equivalent motor-load system

$$T_e = T_l + J\frac{d\omega_m}{dt}$$

$$\omega_m T_e = \omega_m T_l + \omega_m J\frac{d\omega_m}{dt} \Rightarrow \underbrace{p_D}_{\text{Driving Power}} = \underbrace{p_l}_{\text{Load Power}} + \underbrace{J\frac{d\omega_m}{dt}}_{\text{Change in KE}}$$

- A step change in speed requires an infinite driving power
- Therefore $\omega$ is a continuous variable

# Fundamental Torque Equations

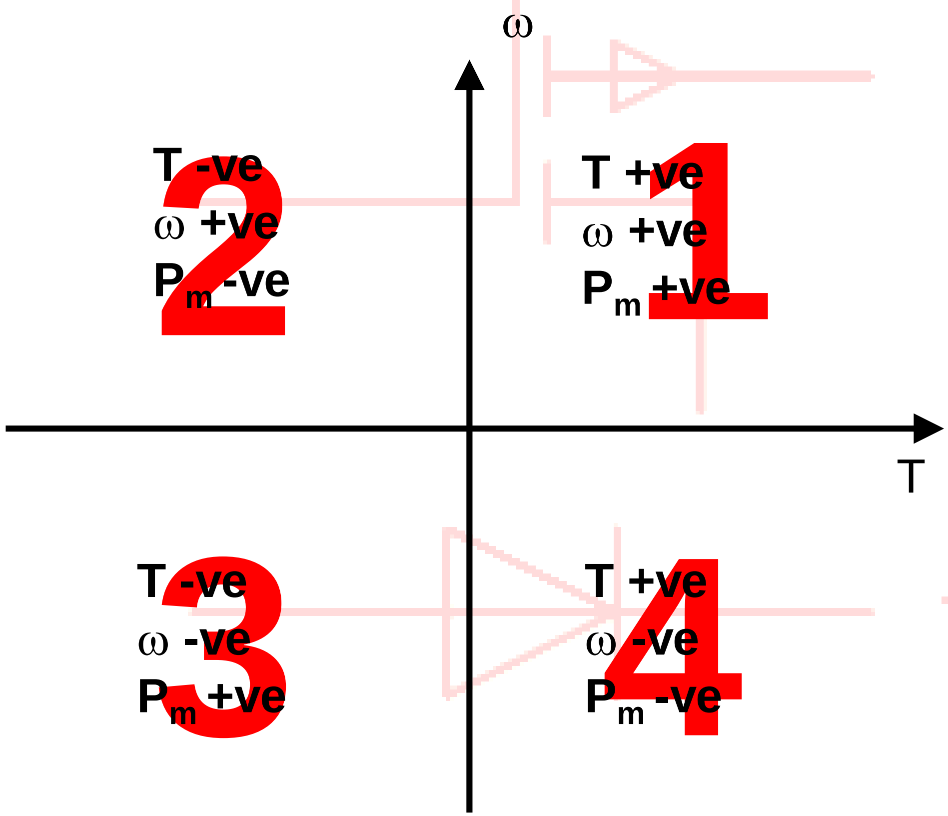
A drive system that require fast acceleration must have

- large motor torque capability
- small overall moment of inertia

As the motor speed increases, the kinetic energy also increases. During deceleration, the dynamic torque changes its sign and thus helps motor to maintain the speed. This energy is extracted from the stored kinetic energy:

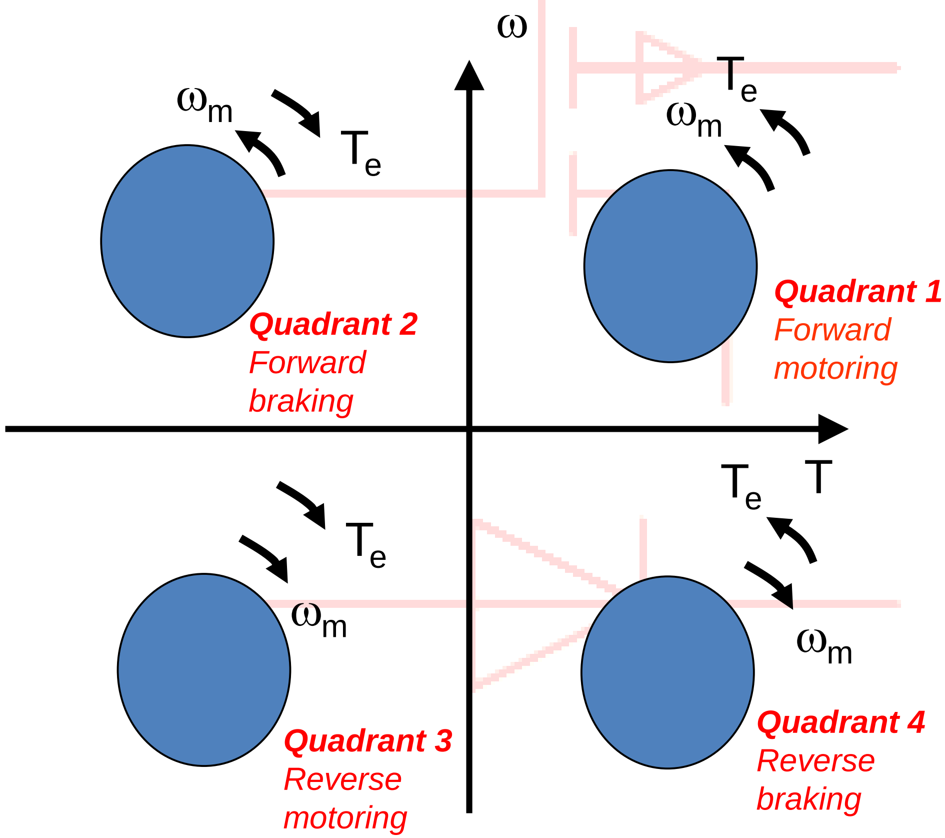
J is purposely increased to do this job !

# Torque-speed quadrant of operation




- Quadrant of operation is defined by the speed and torque of the motor
- Most rotating electrical machines can operate in 4 quadrants
- Not all converters can operate in 4 quadrants

# Torque-speed quadrant of operation

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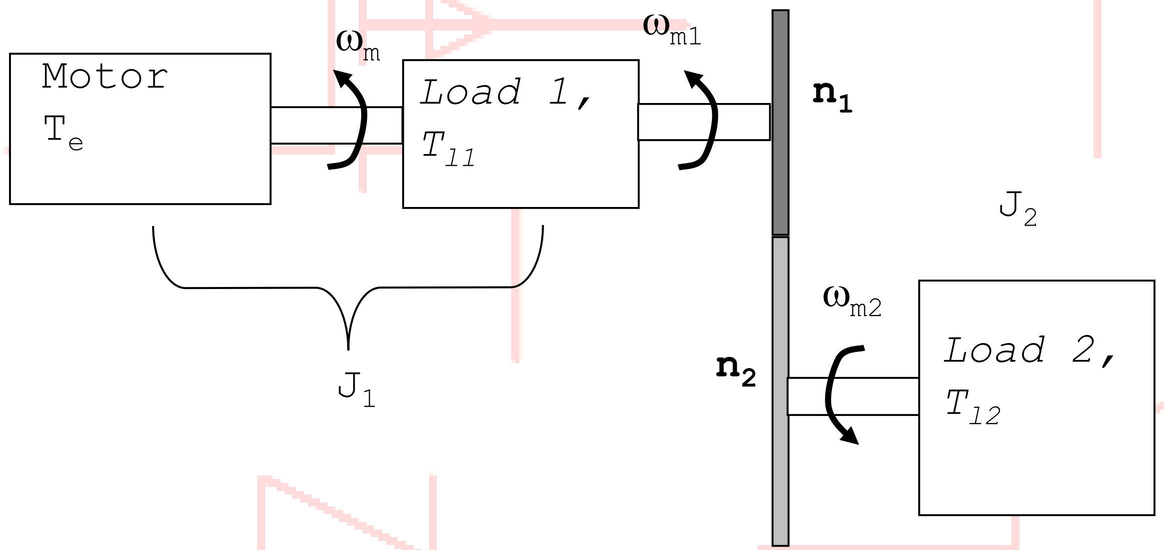
## Loads with rotational motion




$$\frac{\omega_{m2}}{\omega_{m1}} = \frac{n_1}{n_2} = a_2$$ $a_2$ is the gear tooth ratio

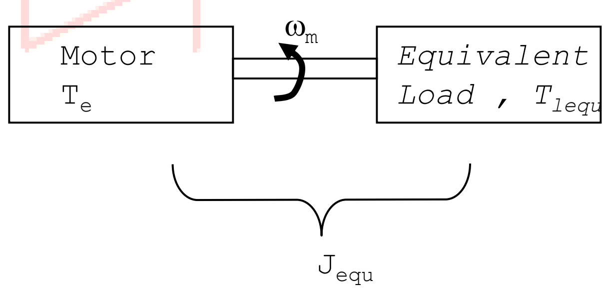
## Loads with rotational motion

If the losses in transmission are neglected, then the KE due to equivalent MOI must be the same as KE of various moving parts

$$\frac{1}{2}\mathbf{J_{equ}}\boldsymbol{\omega_m}^2 = \frac{1}{2}\mathbf{J_1}\boldsymbol{\omega_{m1}}^2 + \frac{1}{2}\mathbf{J_2}\boldsymbol{\omega_{m2}}^2$$

$$\mathbf{J_{equ}} = \mathbf{J_1} + \mathbf{a_2^2 J_2},$$

**as** $\frac{\boldsymbol{\omega_{m1}}}{\boldsymbol{\omega_m}} = \mathbf{1}$ **and** $\frac{\boldsymbol{\omega_{m2}}}{\boldsymbol{\omega_{m1}}} = \mathbf{a_2}$



# Loads with rotational motion

Power at the motor and loads must be the same. If transmission efficiency of the gears is $\eta_2$, then

$$T_{lequ}\omega_m = T_{l1}\omega_{m1} + \frac{T_{l2}\omega_{m2}}{\eta_2}$$

$$T_{lequ} = T_{l1} + \frac{T_{l2}a_2}{\eta_2}$$

# Loads with rotational motion

If in addition to load directly coupled to the motor with MOI $J_0$ there are m other loads of MOIs $J_1$ , $J_2$ , $J_3$ …… $J_m$ and gear teeth ratios $a_1$, $a_2$,…$a_m$, then the equivalent MOI $J_{equ}$ is given by

$$\mathbf{J_{equ} = J_0 + a_1^2 J_1 + a_2^2 J_2 + \ldots\ldots + a_m^2 J_m}$$

If in addition to load directly coupled to the motor with torque $T_{l0}$ there are m other loads with torques $T_{l1}$, $T_{l2}$, $T_{l3}$, …….. $T_{lm}$ coupled through gears with gear teeth ratios $a_1$, $a_2$,…$a_m$ and transmission efficiencies $\eta_1$, $\eta_2$,…$\eta_m$ then the equivalent torque $T_{lequ}$ is given by

$$\mathbf{T_{lequ} = T_{l0} + \frac{a_1 T_{l1}}{\eta_1} + \frac{a_2 T_{l2}}{\eta_2} + \ldots + \frac{a_m T_{lm}}{\eta_m}}$$

## Problem - 1

A motor drives two loads. One has rotational motion. It is coupled to the motor through a reduction gear with a = 0.1 and efficiency of 90%. The load has a MOI of 10 $kgm^2$ and a load torque of 10Nm. Other load has translational motion and consists of 1000kg weight to be lifted up at an uniform speed of 1.5m/s. Coupling between this load and the motor has an efficiency of 85%. Motor has a MOI of 0.2 $kgm^2$ and runs at a constant speed of 1420rpm. Determine the equivalent MOI referred to the motor shaft and power delivered by the motor.