Rectifiers

Classification of Rectifiers

1. **Uncontrolled rectifiers**

- **line frequency ac is converted into fixed voltage dc.**
  (uses uncontrollable devices like diodes)

- **2. Fully controlled rectifiers**
  line frequency ac is converted into variable voltage dc
  (uses controllable devices like SCRs, IGBT)

- **3. Half controlled converters**
  line frequency ac is converted into variable voltage dc
  (uses both uncontrollable and controllable devices)

Uncontrolled and Halfcontrolled Converters

- average output voltage is always positive.
- Power flow is from ac source to dc load
- Unidirectional converters
- operating points lie in the first quadrant of V_d - I_d Plane
- Single quadrant converters

Uncontrolled Rectifiers

APPLICATIONS

Switching power supplies

ac motor drives

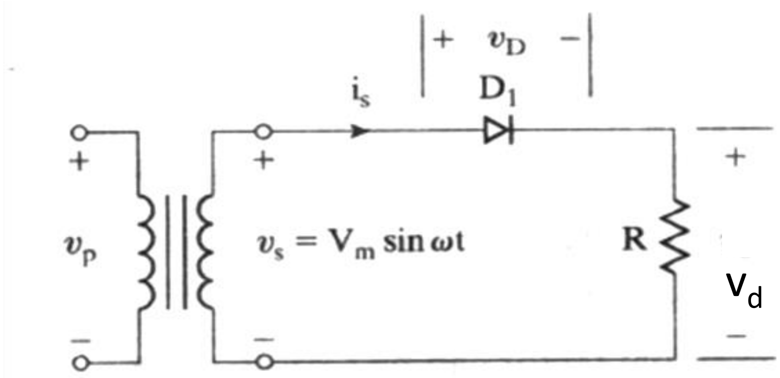
dc motor drives

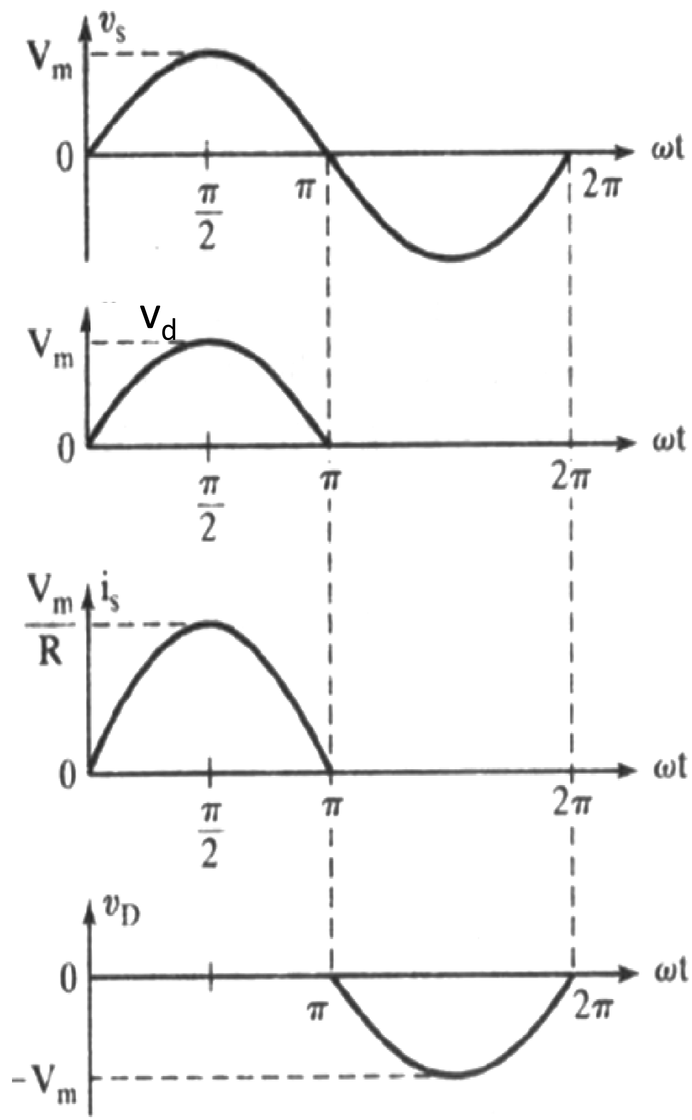
battery chargers

electrochemical processes

HVDC Transmission

Single phase Half wave rectifier - R load





$V_s = V_m \sin\omega t = \sqrt{2} V_s \sin\omega t$

V_s – RMS value of source voltage

0 - π D$_1$ conducts.

$v_d(\omega t) = v_s(\omega t)$

$i_d(\omega t) = v_d(\omega t)/R = i_s(\omega t)$

Diode voltage $v_{D1}(\omega t) = 0$

π -2π D$_1$ is off

$i_d(\omega t) = 0,$

$V_d(\omega t) = 0$

$i_s(\omega t) = 0,$

$v_{D1}(\omega t) = v_s(\omega t)$

Performance parameters

- **Average value of output voltage V_{davg}**

V_{davg} =

$$= \frac{1}{2\pi}\int_0^{2\pi} v_d d\omega t =$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} v_s d\omega t + \int_{\pi}^{2\pi} 0 d\omega t\right]$$

$$= \frac{1}{2\pi}\int_0^{\pi} \sqrt{2}\mathbf{v}_s \sin\omega t \; d\omega t = \frac{\sqrt{2}V_s}{\pi} = \frac{V_m}{\pi}$$

- **Average load current I_{davg}** $= \dfrac{\sqrt{2}V_s}{\pi R}$ = **Average current rating of diode**

- **Peak load current** $= \dfrac{\sqrt{2}V_s}{R}$

- Peak current rating of diode = $\dfrac{\sqrt{2}V_s}{R}$

RMS load voltage V_{dRMS}

$$V_{dRMS} = \sqrt{\frac{1}{2\pi}\int_0^{\pi}(v_d)^2} = \frac{V_s}{\sqrt{2}}$$

RMS load current $I_{dRMS} = \frac{V_s}{\sqrt{2}R}$

Load current Form factor $= \frac{I_{dRMS}}{I_{davg}} = 1.57$

RMS source current $I_{sRMS} = \frac{V_s}{\sqrt{2}R}$

- DC power delivered to the load $P_{dc} = V_{davg} \times I_{davg}$

$$= \frac{\sqrt{2}V_s}{\pi} \times \frac{\sqrt{2}V_s}{\pi R}$$

Transformer Utility factor TUF

DC power delivered /Transformer power rating

If transformer RMS current rating is same as I_{drms}, then

TUF =

$= P_{dc}/(V_s \times I_{dRMS}) =$ 29%

Power rating of transformer must be greater by 1/.29 (=3.5) times the dc load power rating

Rectifier efficiency η

= dc side load power/(ac load power + rectifier loss)

$$= \frac{dc\ side\ \text{load power}}{ac\ \text{side power} + rectifier\ \text{loss}} =$$

$$= \frac{{I_{davg}}^2 R}{{I_{dRMS}}^2 (R + R_f)} = \qquad = \frac{40.6}{1 + \frac{R_f}{R}} =$$

R_f ----forward resistance of diode

- **Ripple factor γ**

= RMS value of ripple content in load voltage / V_{davg}

$$\gamma = \frac{\sqrt{V_{dRMS}^{2} - V_{davg}^{2}}}{V_{davg}} = 1.21$$

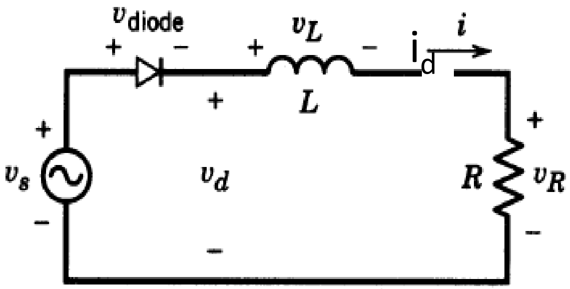
- **Input source pf** = $\dfrac{\text{source side actual power}}{\text{source side apparent power}}$

$$= \frac{P_{dc}}{\text{source side RMS voltage} \times \text{RMS current}}$$

$$= \frac{2V_s^{2}}{\pi^{2} R_L} \Big/ V_s \frac{V_s}{\sqrt{2} R_L} = 0.286$$

- **Peak Inverse voltage PIV**
- **Maximum instantaneous voltage that appears across the diode during the blocking state**
- = $\sqrt{2}V_s$

Single Phase Half wave uncontrolled Rectifier with R-L Load



During positive half cycle diode is on

$$v_d = v_s = \sqrt{2}V_s \sin \omega t = Ri_d(t) + L\frac{di_d(t)}{dt}$$

0 , **(di_d/dt) /** $_{\mathbf{t=0}}$ = 0

$$i_d(t) = \frac{V_m}{Z} sin\emptyset e^{-Rt/L} - \frac{V_m}{Z} sin(\omega t - \emptyset)$$

$$Z = \sqrt{R^2 + (\omega L)^2}$$

$$\emptyset = tan^{-1}(\omega L/R)$$

$$V_R(t) = i_d(t) \times R$$

$V_L(t)$ =L **(di_d/dt) =** $V_d(t)$ - $V_R(t)$

- During positive half cycle diode is on

$$V_d = V_s = Ri_d(t) + L\frac{d}{dt}(i_d(t))$$

$$V_m = \sqrt{2}V_s$$

$$\frac{V_m sin\omega}{s^2 + \omega^2} = RI_d(s) + L[sI_d(s)]$$

$$I_d(s) = \frac{V_m\omega}{(s^2 + \omega)^2(R + Ls)}$$

$$I_d(s) = \frac{k1}{(R + Ls)} + \frac{k2}{(s + j\omega)} + \frac{k3}{(s - j\omega)}$$

$$i_d(t) = \frac{V_m}{Z} sin\emptyset e^{-Rt/L} - \frac{V_m}{Z} sin(\omega t - \emptyset)$$

$$Z = \sqrt{R^2 + \omega^2} \qquad \emptyset = tan^{-1}(\omega L/R)$$

- 0-t₃

$$L\frac{d}{dt}\left(i_d(t)\right) = V_L$$

$$\int_0^{t_3} di_d(t) = \int_0^{t_3} \frac{V_L}{L}\,dt$$

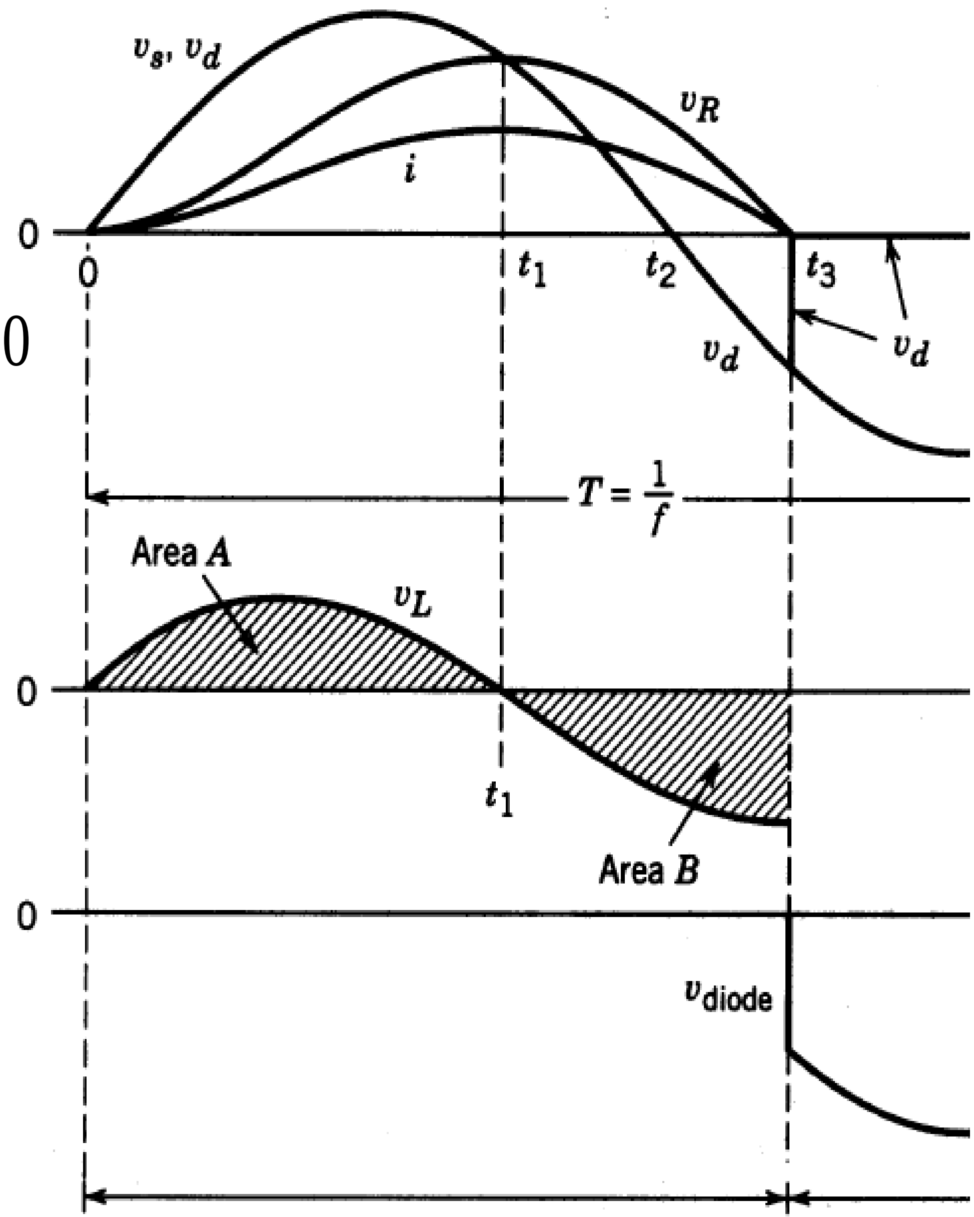
$$i_d /_{t=0} = 0, \qquad i_d /_{t=t_3} = 0$$

$$\int_0^{t_3} \frac{V_L}{L}\,dt = 0$$

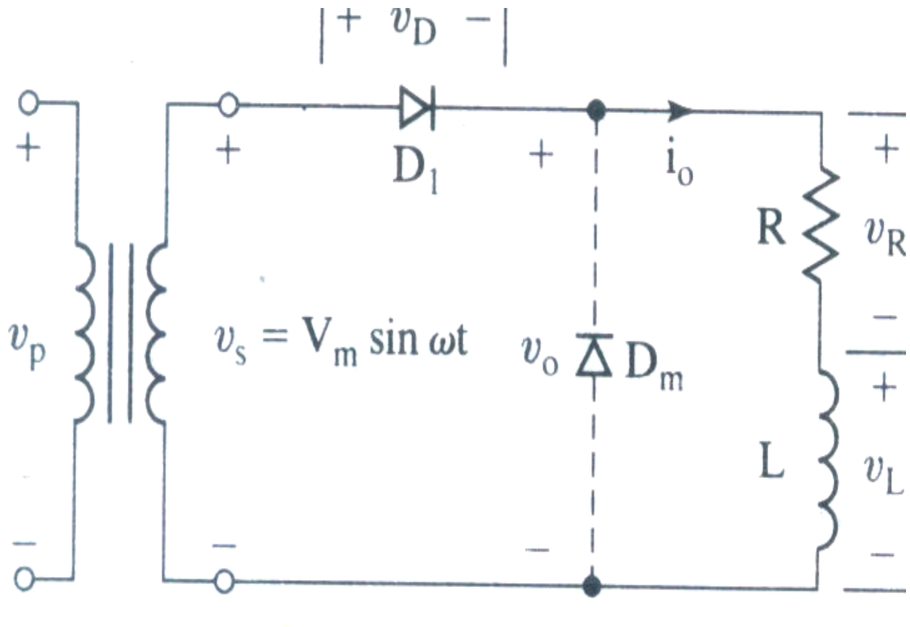
$$\int_0^{t_1} \frac{V_L}{L}\,dt + \int_{t_1}^{t_3} \frac{V_L}{L}\,dt = 0$$

$$\int_0^{t_1} \frac{V_L}{L}\,dt = -\int_{t_1}^{t_3} \frac{V_L}{L}\,dt$$

- **Area A = Area B**



R-L load with free wheeling diode



V_s , i_0, v_R, v_L, v_{D1}, v_{dm}

Average voltage (and current) can be increased by adding a freewheeling diode D_m .

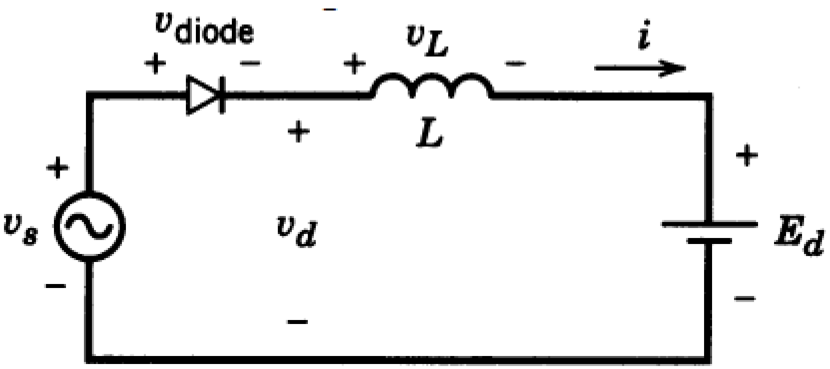
The effect of this diode is to prevent a negative voltage appearing across the load.

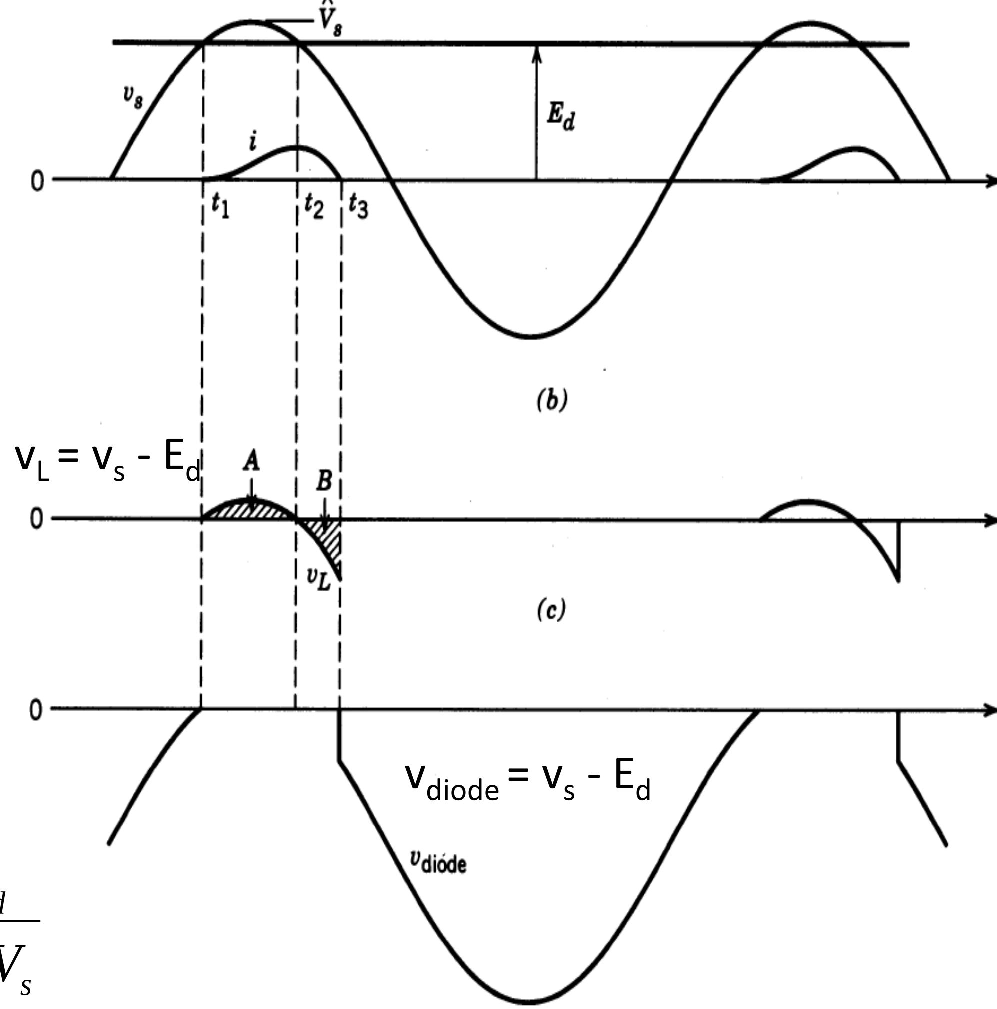
At $t_1 = \Pi/\omega$ the current from D_1 is transferred to D_m and this process is called commutation of diodes.

Continuity of the load current depends on its time constant $\tau = \omega L/R$.

L-E load

(dc motor with back emf)





- Diode is forward biased when $V_s \geq E_d$

At $\omega t = \omega t_1 = \alpha$; $v_s = E_d$

$\sqrt{2}V_s \sin \alpha = E_d$ $\qquad \alpha = \sin^{-1} \dfrac{E_d}{\sqrt{2}V_s}$

For $\omega t \geq \omega t_1$, D_1 is on, $V_{diode} = 0$

$\sqrt{2}V_s \sin \omega t = L di/dt + E_d$

i(t) =

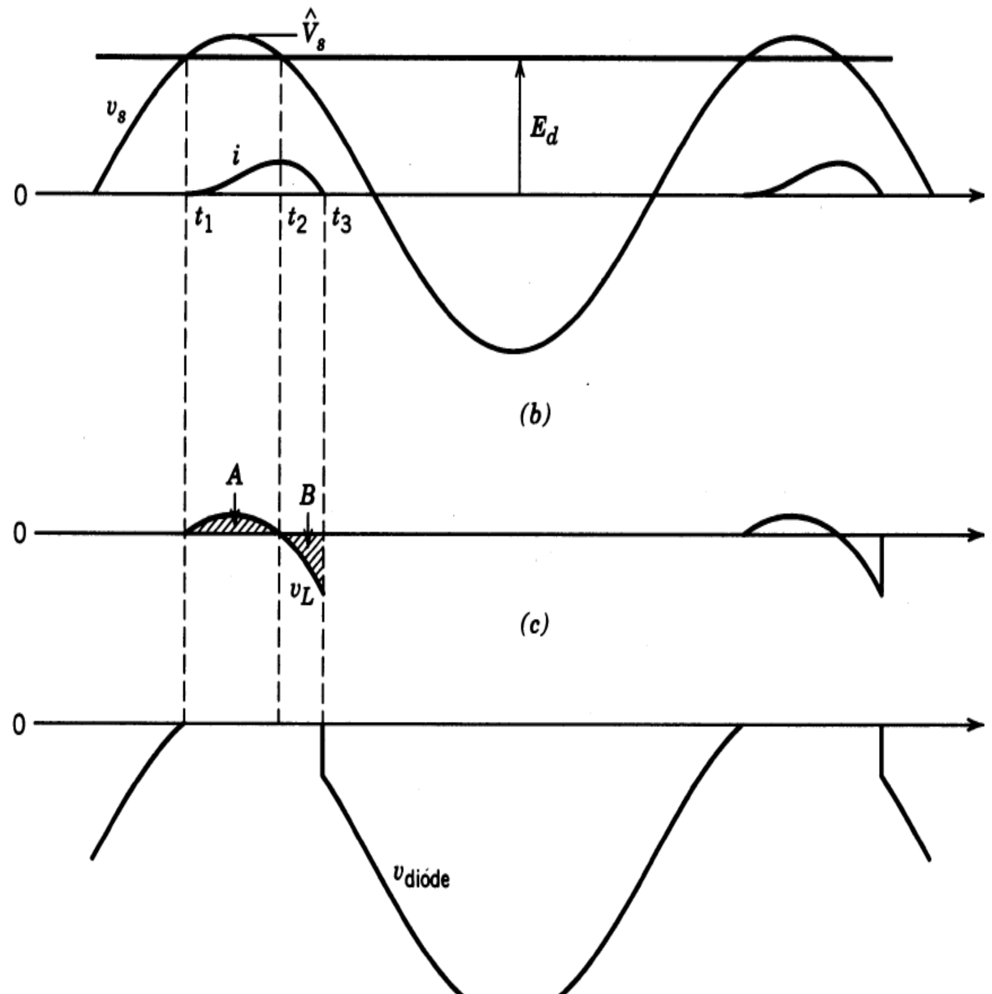
At $\omega t = \alpha$ $\qquad i = 0$

$\sqrt{2}V_s \sin \alpha = L di/dt + E_d$

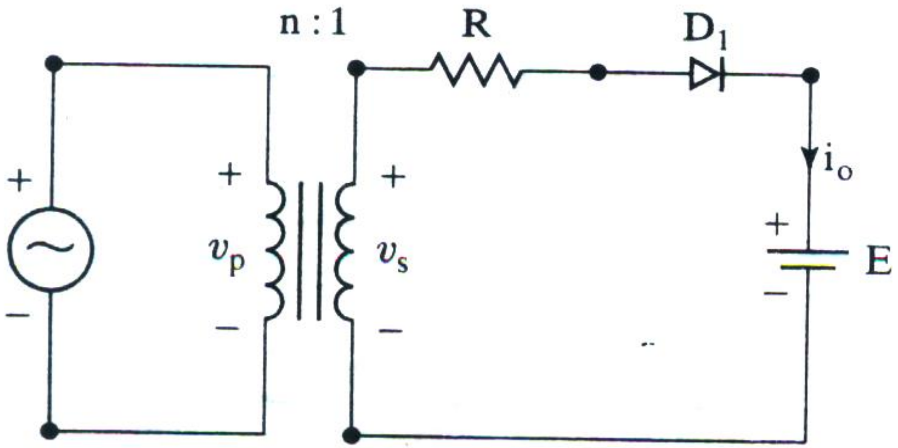
$di/dt \; /_{\omega t = \alpha} = (\sqrt{2}V_s \sin \alpha - E_d)/L = 0$

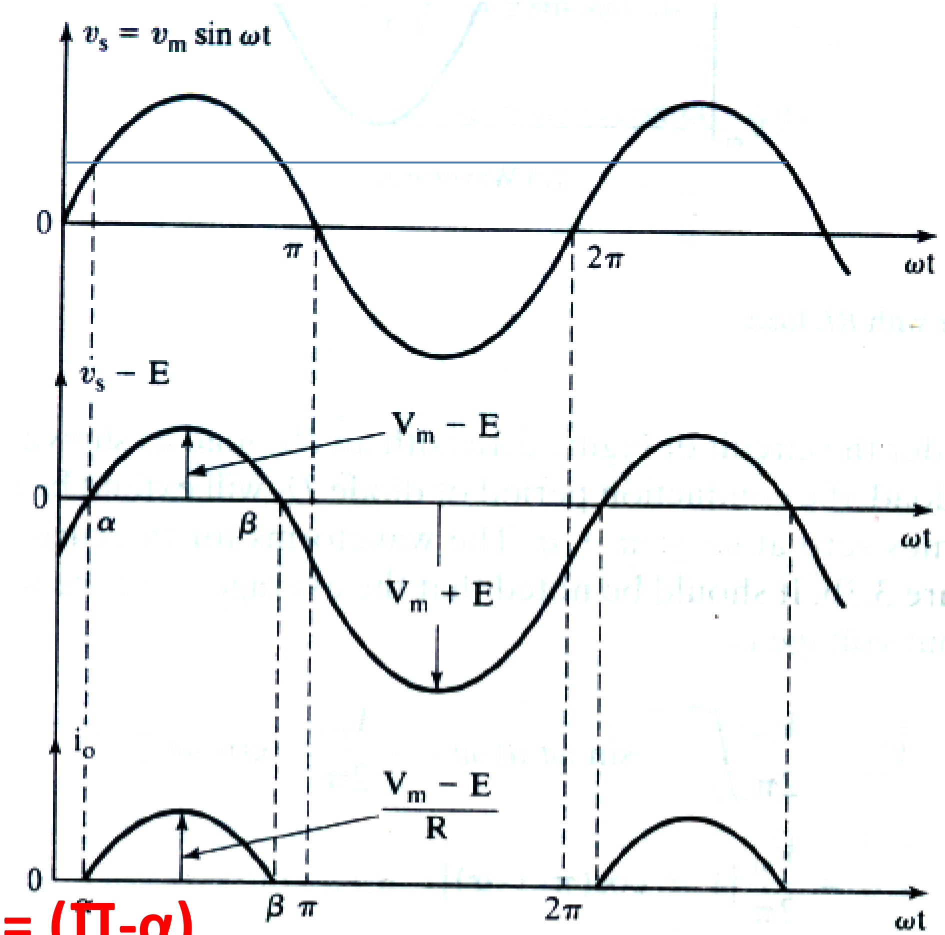
- **Current continues to flows for a while even after the input voltage has gone below the dc back-emf**

- $\underline{t_3 \rightarrow 2\pi/\omega}$
- $i = 0$
- $di/dt = 0, \quad v_L = 0$
- $V_{diode} = v_s - E_d$



Battery charging circuit





For Vs > E, diode D1 conducts.

$$\alpha = \sin^{-1} \frac{E}{V_m}$$

The charging current $i_0(t)$

D_1 is turned off when $i_o(t)=0$ at $\beta = (\pi-\alpha)$

$$i_0(t) = \frac{v_s - E}{R} = \frac{\sqrt{2}V_s \sin \omega t - E}{R} \quad \text{for } \alpha \le \omega t \le \beta$$

Performance parameters

- **Average charging current I_{0avg}**

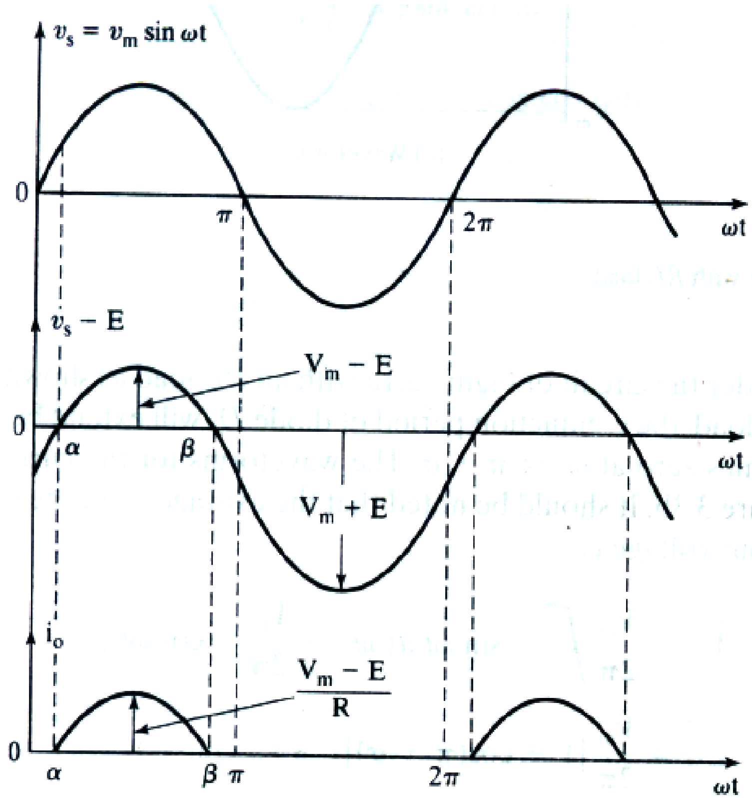
$$I_{0avg} = \frac{1}{2\pi}\int_{\alpha}^{\pi-\alpha} \frac{\sqrt{2}V_s \sin\omega t - E}{R}\, d\omega t$$

$$= \frac{1}{2\pi R}\left(2V_m \cos\alpha + 2E\alpha - \pi E\right)$$

- **Series connected resistance R =**

$$R = \frac{1}{2\pi I_{0avg}}\left(2V_m \cos\alpha + 2E\alpha - \pi E\right)$$

RMS value of Charging current $I_{0RMS}2$



$$I_{0RMS}^{\ 2} = \frac{1}{2\pi}\int_{\alpha}^{\pi-\alpha} \left(\frac{\sqrt{2}V_s \sin\omega t - E}{R}\right)^2 d\omega t$$

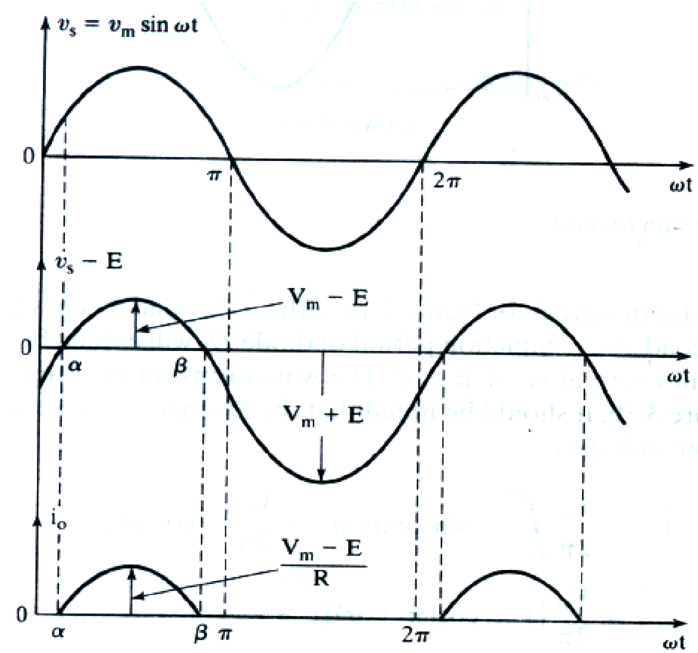
$$= \frac{1}{2\pi R^2}\left[\left(\frac{V_m^{\ 2}}{2} + E^2\right)(\pi - 2\alpha) + \frac{V_m^{\ 2}}{2}\sin 2\alpha - 4V_m E\cos\alpha\right]$$

- **Power delivered to the battery**

$$P_{dc} = E \times I_{0avg}$$

- **Power rating of resistance R,**

$$P_R = I_{0RMS}{}^2 \times R$$



- **Rectifier efficiency =**

Power delivered to battery / total input power

$$\eta = \frac{P_{dc}}{P_{dc} + P_R}$$

- **Peak inverse voltage of diode=** $PIV = V_m + E$

- A battery has a voltage of $E=12V$ and capacity 100Wh. The average charging current should be $I_{dc}=5A$. The primary input voltage is $V_p=120V$, 50Hz and the transformer has a turns ratio of 2:1.

Calculate

(a) conduction angle δ of the diode.

(b) current limiting resistor R

(c) power rating P_R of R (d) charging time

(e) the rectifier η and (f) PIV of diode

- Secondary voltage =
- α =
- Conduction angle δ=
- Average charging current = I_{dc} =

  R =

Power rating of resistor =

$I_{oRMS}^2 \; R$ =

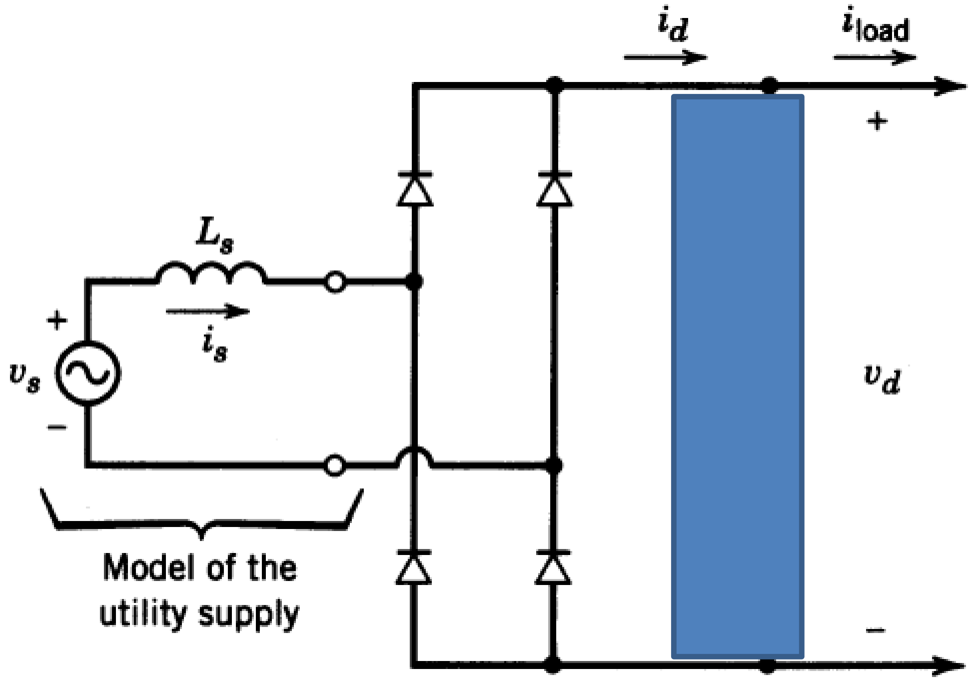
Power delivered to the battery =

$E \, I_{dc}$ =

η =

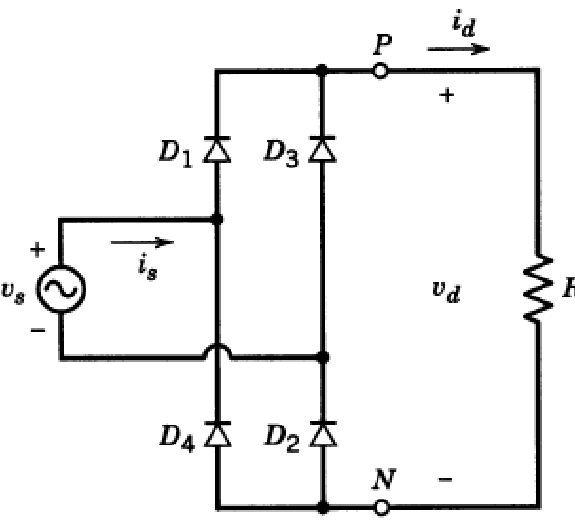
PIV =

Single phase Fullwave Bridge rectifiers



$$v_s = \sqrt{2}V_s \sin \omega t$$

Source inductance $L_s = 0$, Load is pure resistance



0 - Π

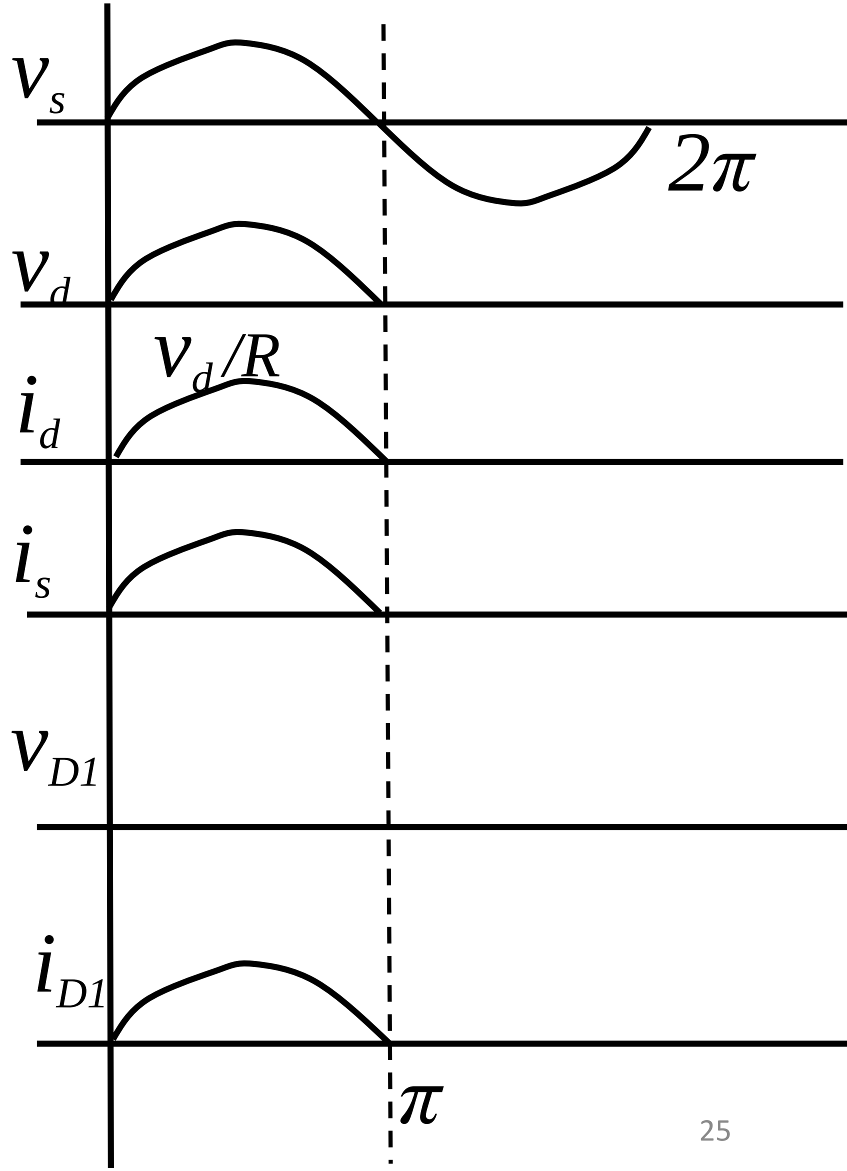
v_s is positive

D_1,D_2 forward biased
Current path is
V_s, D_1, load, D_2

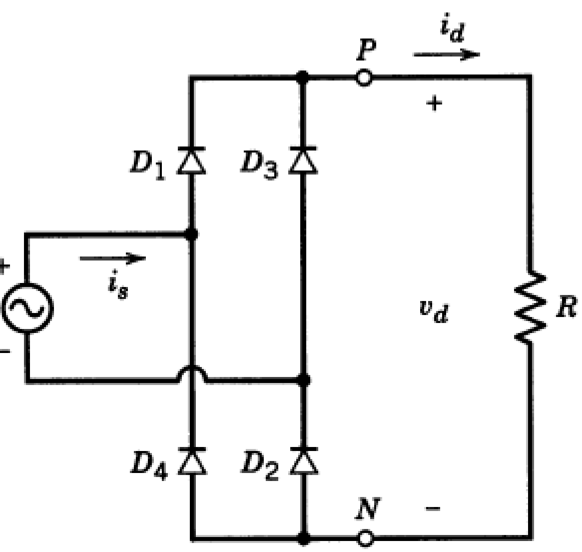
Load voltage $v_d = v_s$

Load current $i_d = v_d/R$

Source current $i_s = i_d$



Source inductance L_s = 0, Load is pure resistance



Π - 2 Π

V_s is negative

D_3,D_4 forward biased

Current path is

v_s, D_3, load, D_4

v_d =

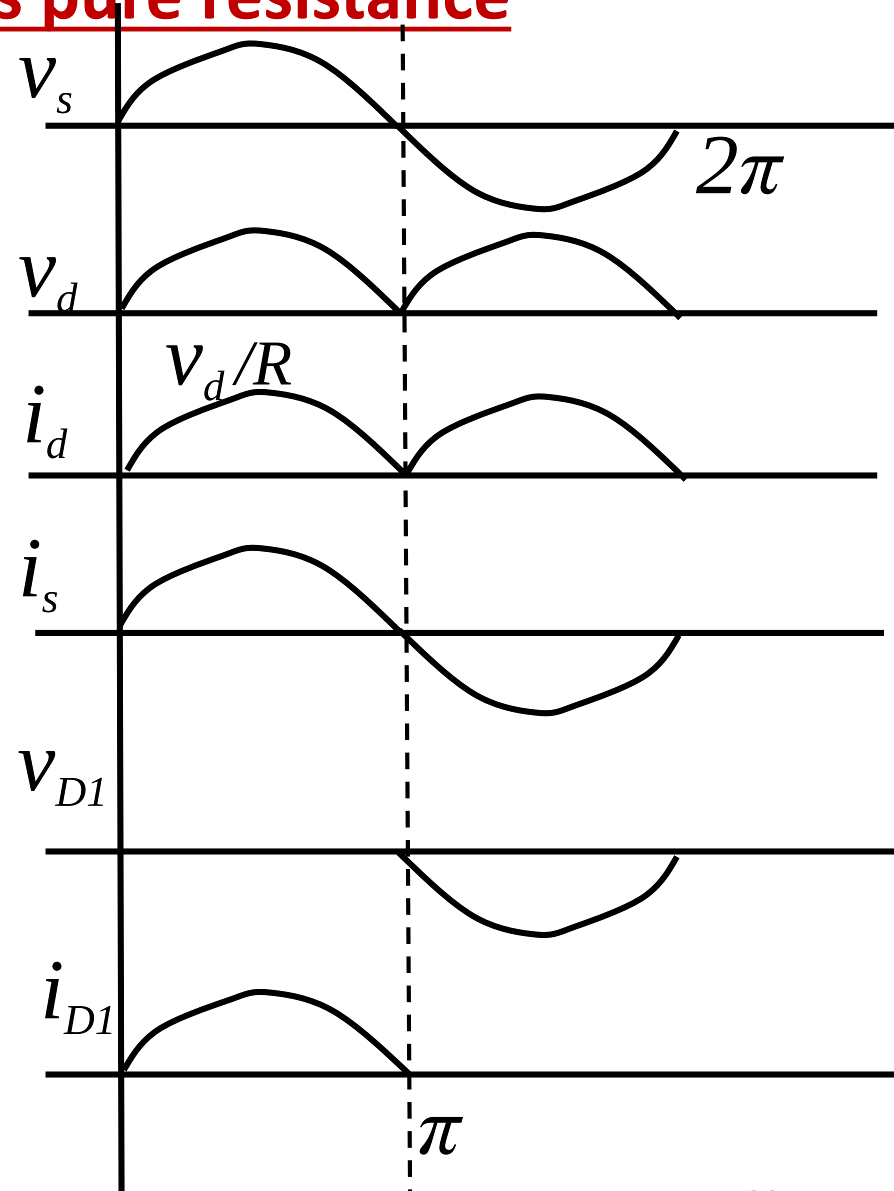
$-v_s$

i_d =

v_d/R

i_s =

$-i_d$



Performance parameters

- **Average load voltage V_{davg}**

$$V_{davg} = \frac{1}{\pi}\int_0^{\pi}(\sqrt{2}v_s \sin\omega t)\,d\omega t \qquad = \frac{2\sqrt{2}V_s}{\pi}$$

- **Average load current I_{davg}**

$$I_{davg} = \frac{2\sqrt{2}V_s}{\pi R}$$

- **RMS load voltage V_{dRMS}**

$$V_{dRMS} = \left[\frac{1}{\pi}\int_0^{\pi}(\sqrt{2}v_s \sin\omega t)^2\,d\omega t\right]^{1/2} = V_s$$

- **RMS load current $I_{dRMS} = V_s/R$**

- **Peak load current** $= \dfrac{\sqrt{2}V_s}{R}$

- **DC power developed in load**

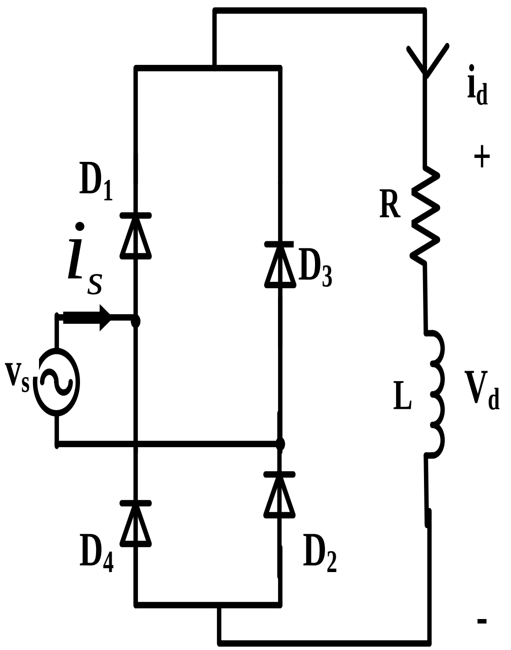
$$P_{dc} = V_{davg} \times I_{davg} = \frac{2\sqrt{2}V_s}{\pi} \times \frac{2\sqrt{2}V_s}{\pi R} = \frac{0.8113V_s^{\,2}}{R}$$

- **Transformer utility factor** $\dfrac{P_{dc}}{V_{dRMS} \times I_{dRMS}} = 81.13\%$

- **PIV** $= \sqrt{2}V_s$

- Ripple factor = $\sqrt{\dfrac{V_{dRMS}^{\,2} - V_{davg}^{\,2}}{V_{davg}^{\,2}}} = 0.482$

Source inductance $L_s = 0$, Load is R-L



- **0 - π**
- v_s is positive,
- D_1, D_2 forward biased
- $v_d = v_s$,

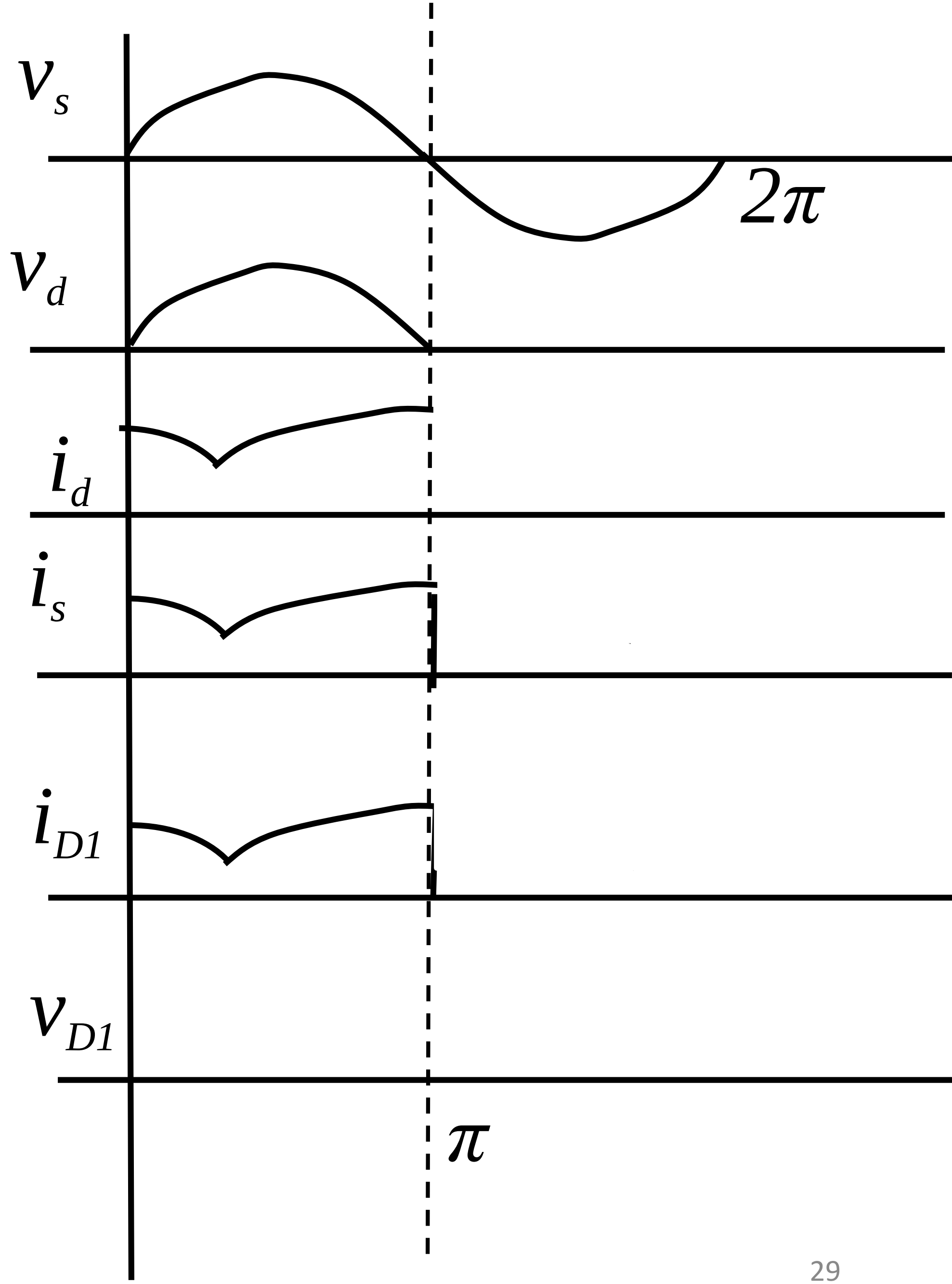
$$R i_d(\omega t) + L\frac{d i_d(\omega t)}{d\omega t} = \sqrt{2}V_s \sin \omega t$$

$$i_d(\omega t)\,/_{t=0} = +\,I_0$$

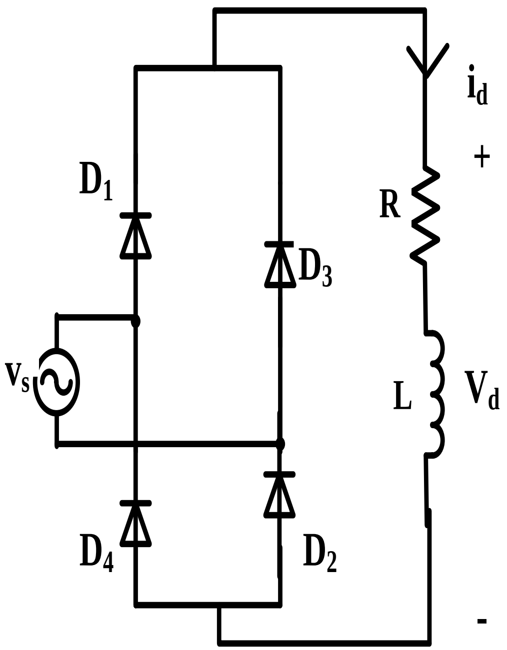
$$\frac{d i_d(\omega t)}{dt}\,/_{\omega t=0} \;\; is \;\; \frac{RI_0}{L}$$

$i_d(\omega\ t) =$

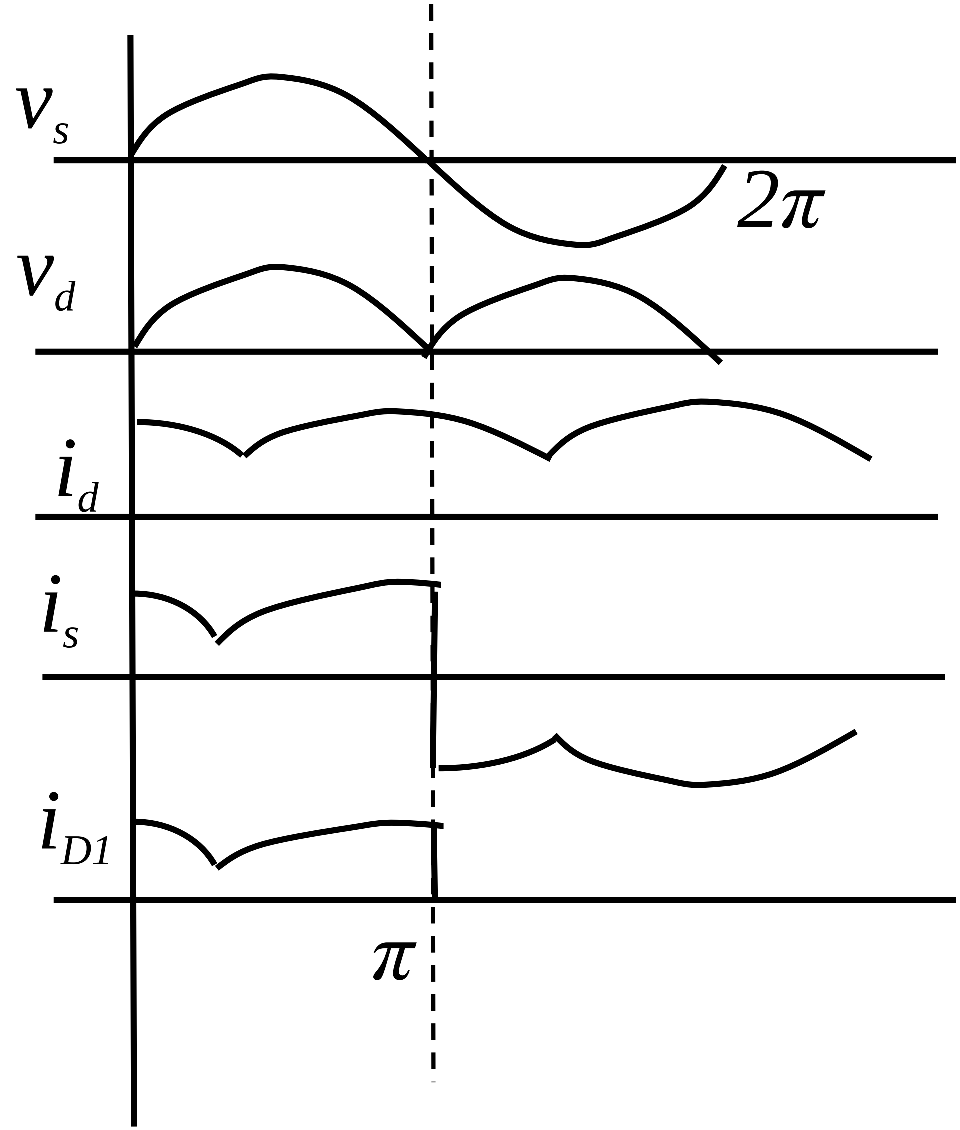
$i_s(\omega\ t) = i_d(\omega\ t)$



Source inductance $L_s = 0$, Load is R-L



- **π - 2 π**
- v_s is negative,
- D_3,D_4 forward biased
- $v_d = - v_s$,



$$Ri_d(\omega t) + L\frac{di_d(\omega t)}{d\omega t} = \sqrt{2}V_s \sin \omega t$$

$$i_d(\omega t) /_{\omega t = \pi} = + I_0$$

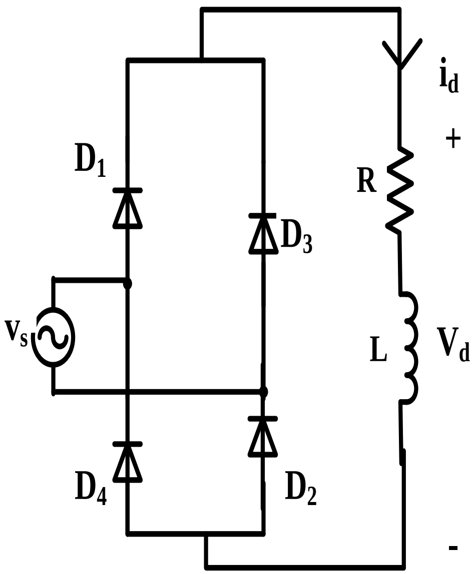
$$\frac{di_d(\omega t)}{dt}{}_{/\omega t=\pi} \quad is \quad \frac{-RI_0}{L}$$

$i_d(\omega\ t) =$

$i_s(\omega\ t) = -i_d(\omega\ t)$

$L_s = 0$, Load is highly inductive so that i_d is constant at I_d

$\omega L \gg R$



0 - Π

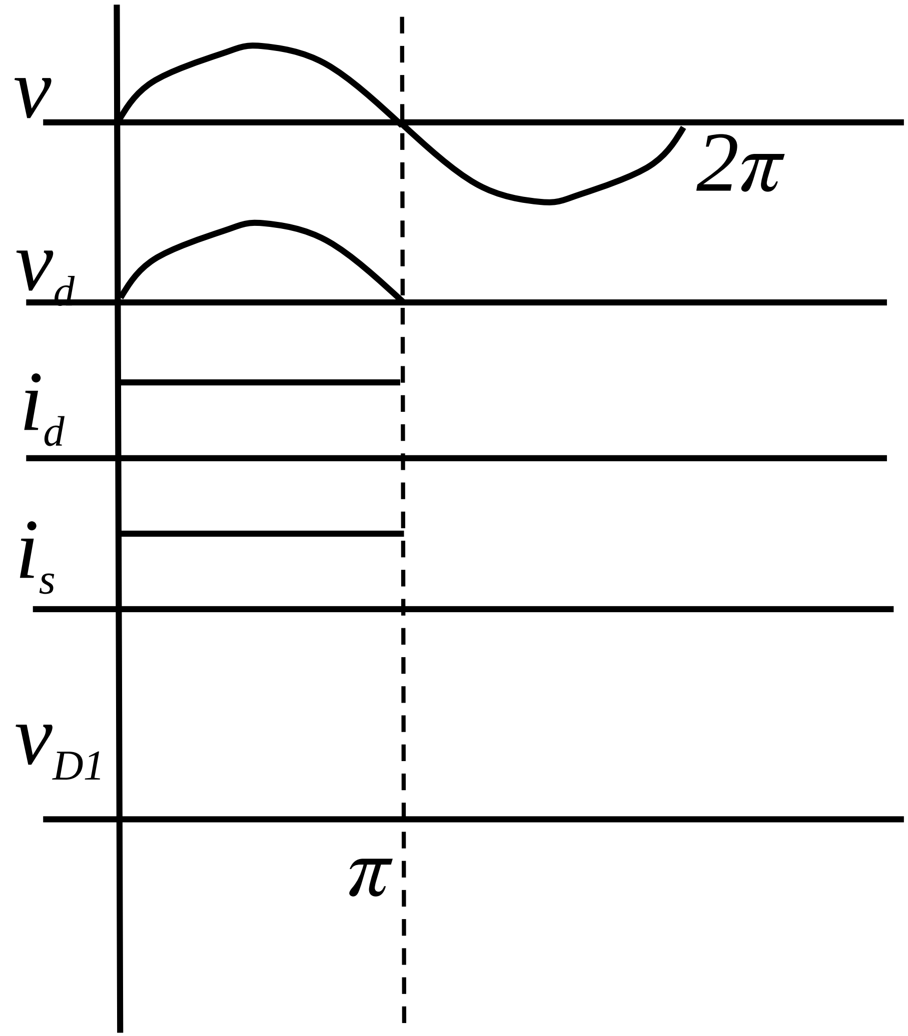
v_s is positive

D_1, D_2
forward
biased

$v_d = v_s$

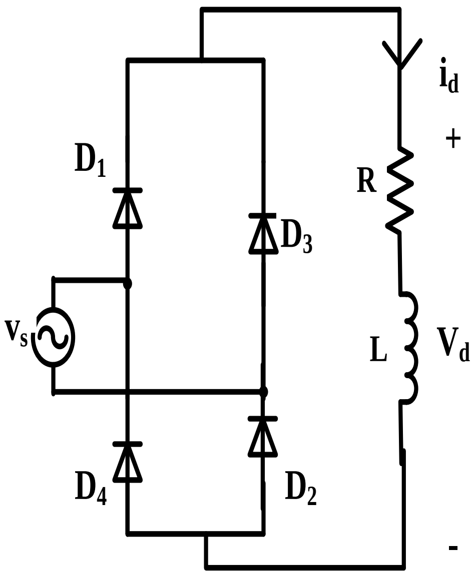
$i_d = I_d$

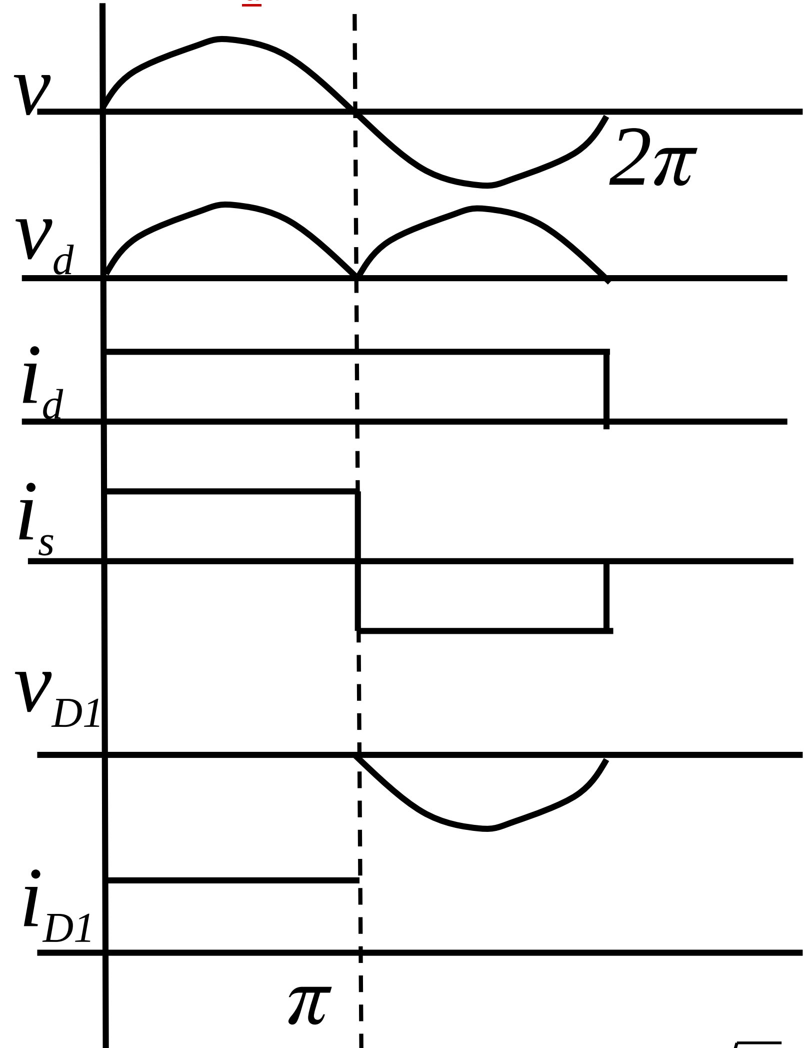
$i_s = +I_d$



$L_s = 0$, Load is highly inductive so that i_d is constant at I_d

- $\omega L \gg R$





$v_d = -v_s$

$I_d = I_d$

$i_s = -I_d$

π -2 π

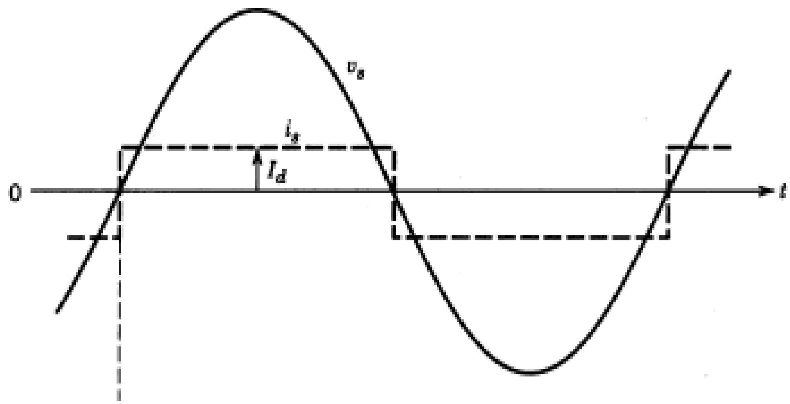
v_s is negative
D_3,D_4 forward
biased

Average output voltage

$$V_{davg} = \frac{1}{\pi}\int_0^{\pi}(\sqrt{2}v_s \sin \omega t)\,d\omega t = \frac{2\sqrt{2}V_s}{\pi}$$

$=0.9V_s$

Source inductance is zero. Hence transition from positive to negative value of source current is instantaneous.



$$i_s(t) = I_{dc} + \sum_{n=1}^{\infty} a_n \cos n\omega t + b_n \sin n\omega t$$

$$= I_{dc} + C_n \sin(n\omega t - \phi_n),$$

$$C_n = \sqrt{a_n^{\,2} + b_n^{\,2}}$$

$$\phi_n = \tan^{-1} a_n / b_n$$

$$I_{dc} = \frac{1}{2\pi}\int_0^{2\pi} i_s(t)\,d\omega t = \mathbf{0}$$

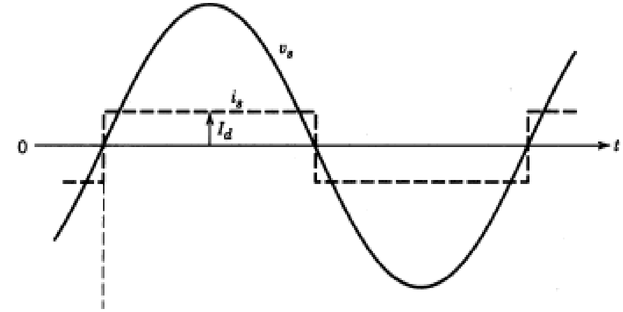
$$a_n = \frac{1}{\pi}\int_0^{2\pi} i_s(t)\cos(n\omega t)\,d\omega t = \mathbf{0}$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} i_s(t)\sin(n\omega t)\,d\omega t = = \frac{2I_d}{n\pi}[1-\cos n\pi] = \frac{4I_d}{n\pi} \; for\ n = 1, 3, 5, 7..$$

$$= 0 \; for\ n = 2, 4, 6, 8..$$

$$\phi_n = \tan^{-1} a_n / b_n = 0$$

$$i_s(t) = \sum_{n=1}^{\infty} \frac{4I_d}{n\pi} \sin n\omega t \qquad n = 1,3,5,....$$



$$i_s(t) = \frac{4I_d}{\pi}\left(\frac{\sin\omega t}{1} + \frac{\sin 3\omega t}{3} + \frac{\sin 5\omega t}{5} + ...\right)$$

$$i_{s1}(t) = \frac{4I_d}{\pi}\sin\omega t$$

$$i_{s5}(t) = \frac{4I_d}{5\pi}\sin 5\omega t$$

$$i_{s3}(t) = \frac{4I_d}{3\pi}\sin 3\omega t$$

RMS value of fundamental component of input current I_{s1}

$$I_{s1} = \frac{4I_d}{\pi\sqrt{2}} = 0.9 I_d$$

- **RMS value of nth harmonic of source current i_{sn}**

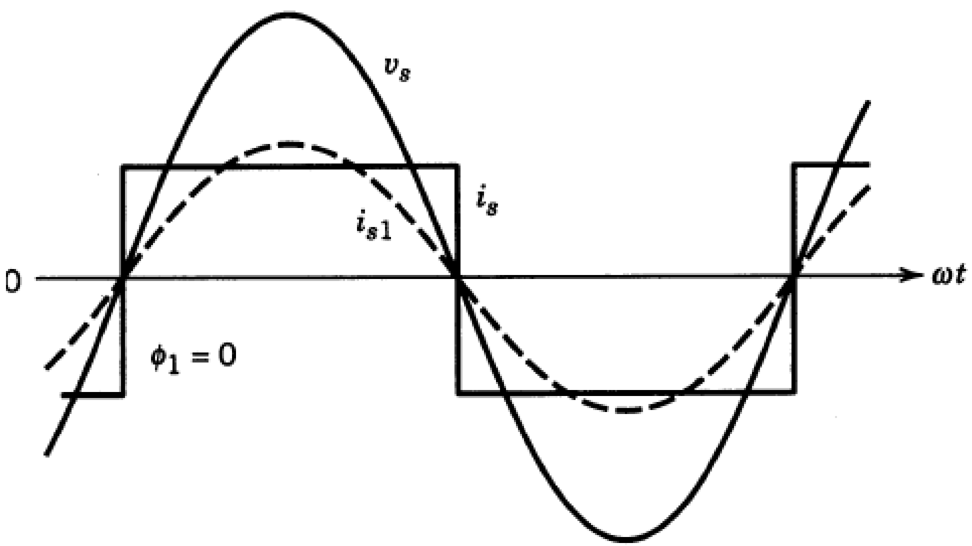
$$I_{sn} = \begin{cases} 0 & \text{for even values of n} \\ \dfrac{I_{s1}}{n} & \text{for odd values of n} \end{cases}$$

RMS value of total source current $I_s = \mathbf{I_d}$

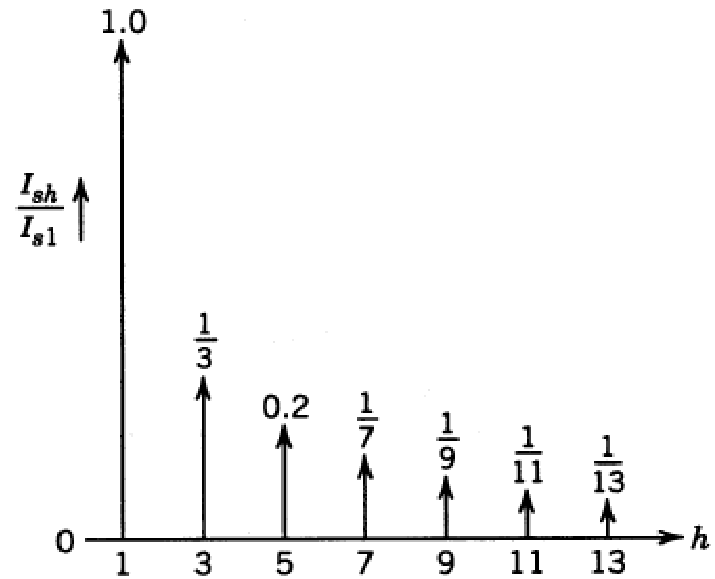
THD of source current $= \sqrt{\dfrac{\left(I_d{}^2 - I_{s1}{}^2\right)}{I_{s1}{}^2}}$ **= 48.4%**

- Displacement factor of source current DF = **$\cos\phi_1$= 1**

Harmonic components of Source Current



Source current and fundamental component



Harmonic components of I_s

AC source side Power factor = Actual power / Apparent power

Actual power $= \frac{1}{2\pi}\int_0^{2\pi} v_s i_s = \frac{1}{2\pi}\int_0^{2\pi} V_m \sin\omega_1 t\left[i_{s1} + i_{s3} + i_{s5} + ...\right]$

$$= \frac{1}{2\pi}\int_0^{2\pi} V_m \sin(\omega_1 t) I_{s1peak} \sin(\omega_1 t - \phi_1)$$

$$+ \frac{1}{2\pi}\int_0^{2\pi} V_m \sin(\omega_1 t) I_{s3peak} \sin(3\omega_1 t - \phi_3)$$

$$+ \frac{1}{2\pi}\int_0^{2\pi} V_m \sin(\omega_1 t) I_{s5peak} \sin(5\omega_1 t - \phi_5)$$

$$= \frac{1}{2\pi}\int_0^{2\pi} V_m \sin\omega_1 t \begin{bmatrix} I_{s1peak}\sin(\omega_1 t - \phi_1) + \\ I_{s3peak}\sin(3\omega_1 t - \phi_3) \\ + I_{s5peak}\sin(5\omega_1 t - \phi_5) + ... \end{bmatrix}$$

$$= \frac{V_m}{\sqrt{2}}\frac{I_{m1}}{\sqrt{2}}\cos\phi_1 = V_s I_{s1}\cos\phi_1$$

Apparent power $= V_s I_s$

Power Factor =

$$\frac{V_s I_{s1} \cos\phi_1}{V_s I_s}$$

$$= I_{s1} / I_s = \frac{2\sqrt{2} I_d}{\pi I_d}$$

Peak current rating of diode =

$$= I_d$$

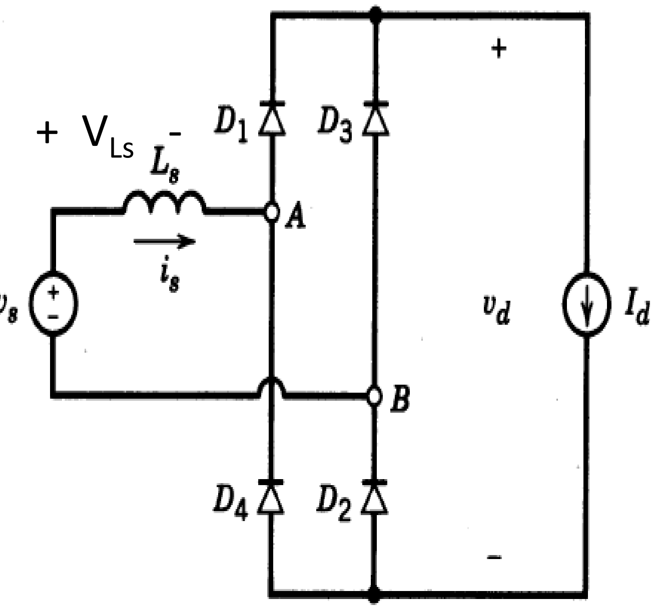
RMS current rating of diode =

$$= I_d / \sqrt{2}$$

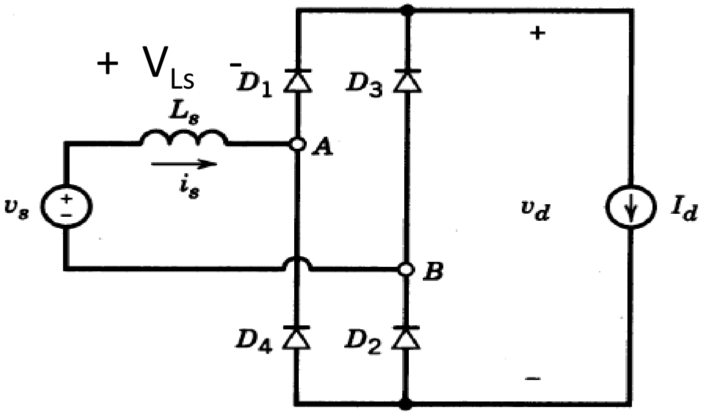
PIV of diode =

$$V_m$$

Effect of source inductance L_s



- **Due to source inductance the transition of ac side current from $+I_d$ to $-I_d$ is not instantaneous**
- **A finite time interval is required for the transition of current from outgoing diodes to incoming diodes.**
  - **This time interval is called current commutation period μ.**



- During the commutation
- interval μ, all four diodes conduct.

  $V_d = 0$

  Drop in the inductance v_{Ls}

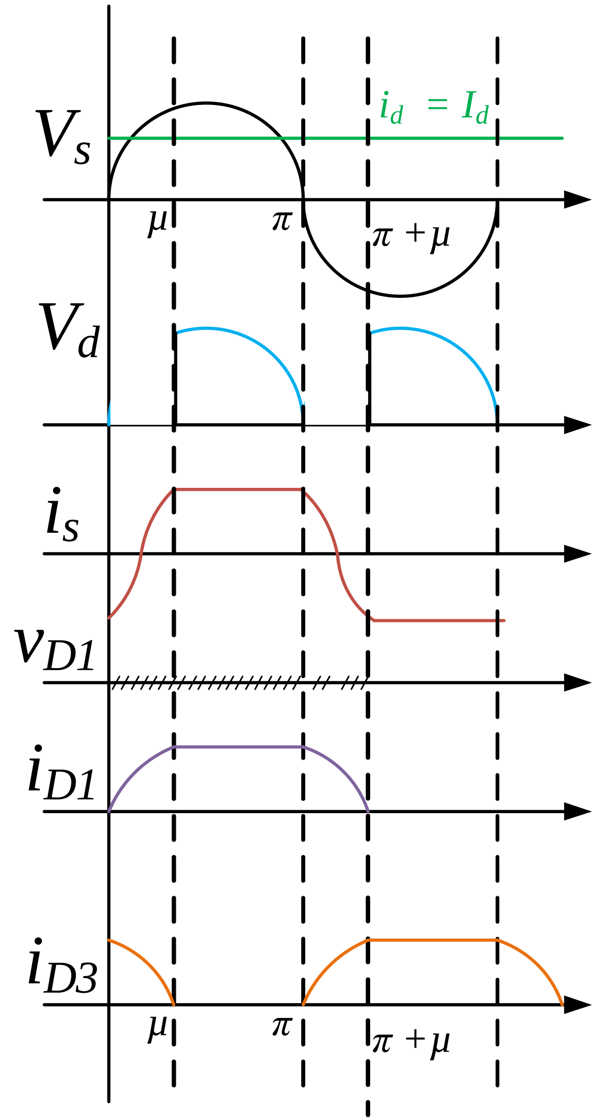
$$v_{LS} = \quad + v_s$$

$$L_s \frac{di_s(t)}{dt} = \sqrt{2} V_S \sin \omega t$$

$$\omega L_s \frac{di_s(t)}{d\omega t} = \sqrt{2} V_S \sin \omega t$$

$di_s =$

$$\frac{\sqrt{2} V_s}{\omega L_s} \sin \omega t \quad d(\omega t) = di_s$$



$$\frac{\sqrt{2}V_s}{\omega L_s}\sin \omega t \quad d(\omega t) = di_s$$

During *0* to *μ* i_s varies

from $-I_d$ to $+I_d$

$$\int_0^{\mu} \frac{\sqrt{2}V_s}{\omega L_s}\sin \omega t \quad d\omega t = \int_{-I_d}^{+I_d} di_s$$

$$\left| i_s \right|_{-I_d}^{+I_d} = \frac{\sqrt{2}V_s}{\omega L_s}\left[1 - \cos(\omega t)\right]$$

$$2I_d = \frac{\sqrt{2}V_s}{\omega L_s}(1 - \cos(\mu))$$

$$\cos\mu = 1 - \frac{\sqrt{2}\omega L_s I_d}{V_s}$$

commutation interval μ =

- **Average voltage V_{davg} =**

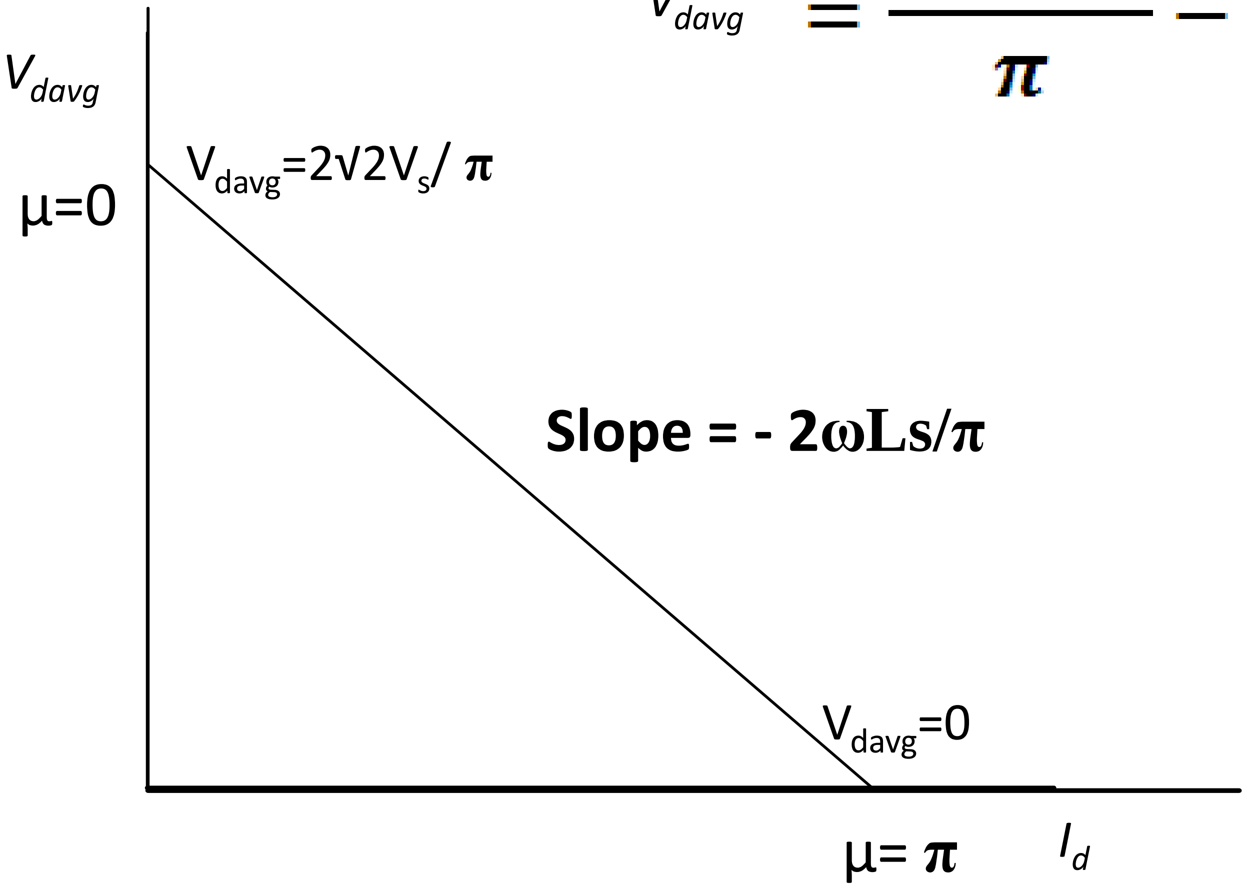
$$V_{davg} = \frac{1}{\pi}\int_{\mu}^{\pi}\sqrt{2}V_s \sin \omega t d(\omega t)$$

$$= \frac{\sqrt{2}\,V_s}{\pi}(1 + \cos\mu)$$

$$= \frac{2\sqrt{2}\,V_s}{\pi} - \left\{\frac{2\omega L_s I_d}{\pi}\right\}$$

Average output voltage is reduced by $\frac{2\omega L_s I_d}{\pi}$

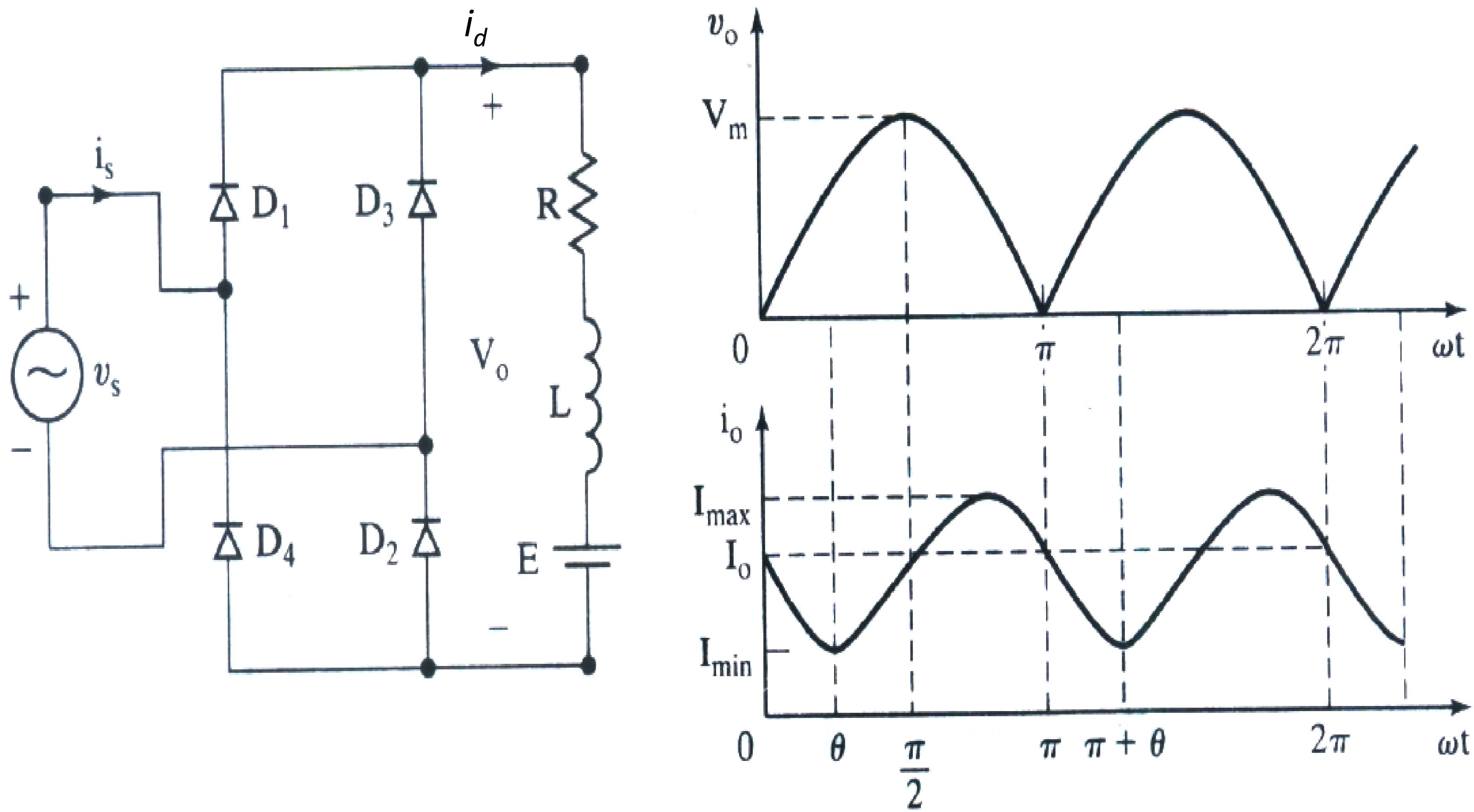
- $$V_{davg} = \frac{2\sqrt{2}\,V_s}{\pi} - \left\{\frac{2\omega L_s I_d}{\pi}\right\}$$



- **Large values of current or/and source Inductance result in larger value of μ and make average voltage zero.**

Bridge Rectifier with R-L-E load

- **Continuous current mode**



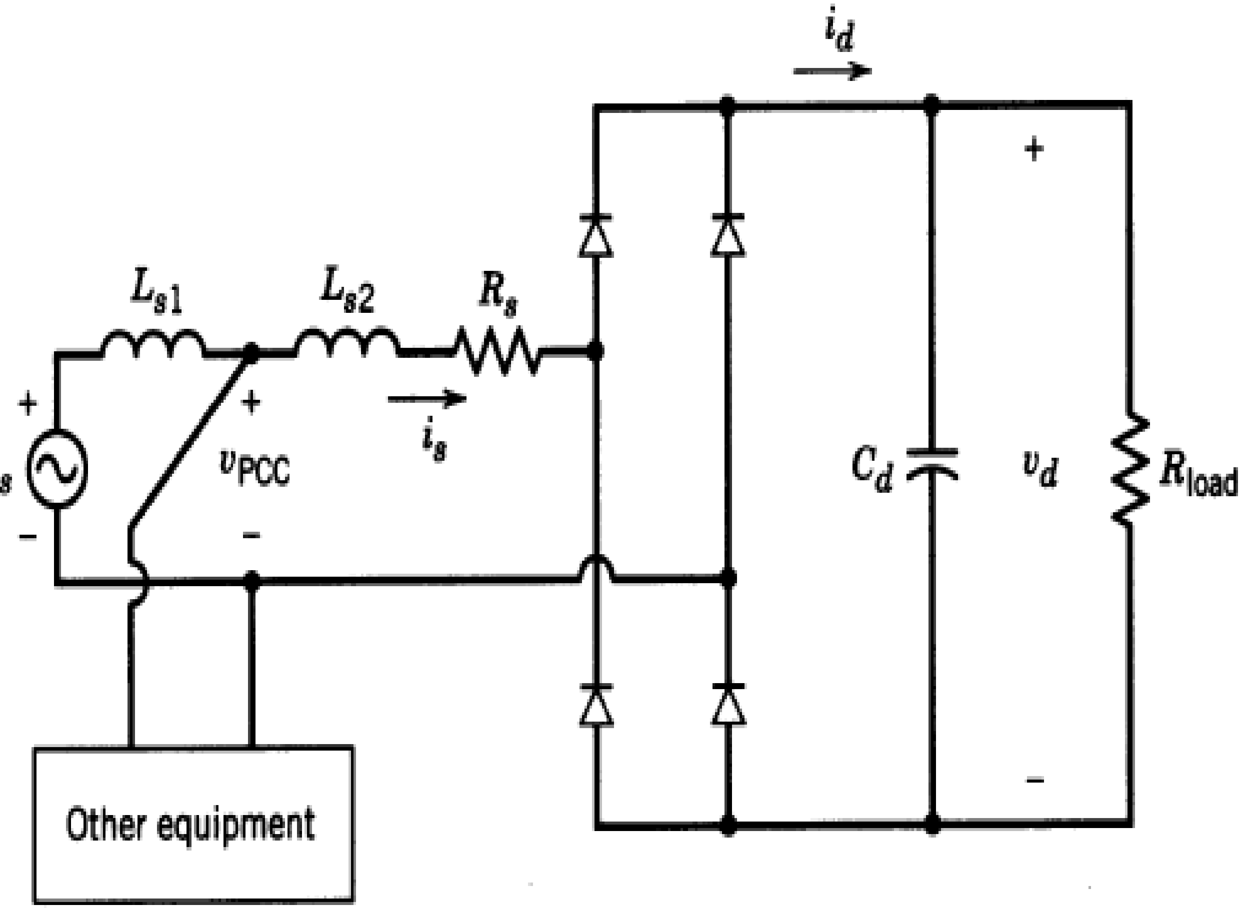
Effect of diode rectifier on utility voltage & Current

- **Line current distortion**

  Source current deviates significantly from the sinusoidal waveform.

- **Line voltage distortion**

  Distortion in the line current results in the distortion of line voltage waveform.



L_{s1} – Inductance of the utility side

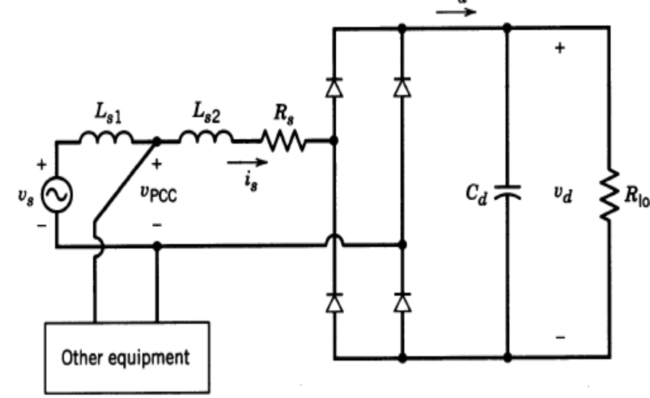
L_{s2}- Inductance due to the power electronic equipment

R_s-represents the diode resistance

PCC- point of common coupling where other loads and power electronic load are connected

V_{pcc} Voltage across the PE equipment and other loads at the point of common coupling

$$V_{pcc} = V_s - L_{s1}\frac{di_s}{dt}$$



- **Source current i_s**

$$i_s = i_{s1} + \sum_{h=3,5,....}^{\infty} i_{sh}$$

$$V_{pcc} = V_s - L_{s1}\frac{di_{s1}}{dt} - L_{s1}\sum_{h=1,3,5,...}^{\infty}\frac{di_{sh}}{dt}$$

$$= V_{pcc1} - L_{s1}\sum_{h=1,3,5,...}^{\infty}\frac{di_{sh}}{dt}$$

$$V_{pcc1} = V_s - L_{s1}\frac{di_{s1}}{dt}$$

- **Distortion in voltage at PCC due to harmonics in line current**

$$V_{pccdis} = L_{s1}\sum_{h=3,5,...}^{\infty}\frac{di_{sh}}{dt}$$

- **Voltage available at the point of common coupling is highly deviated from the ideal sine waveform**

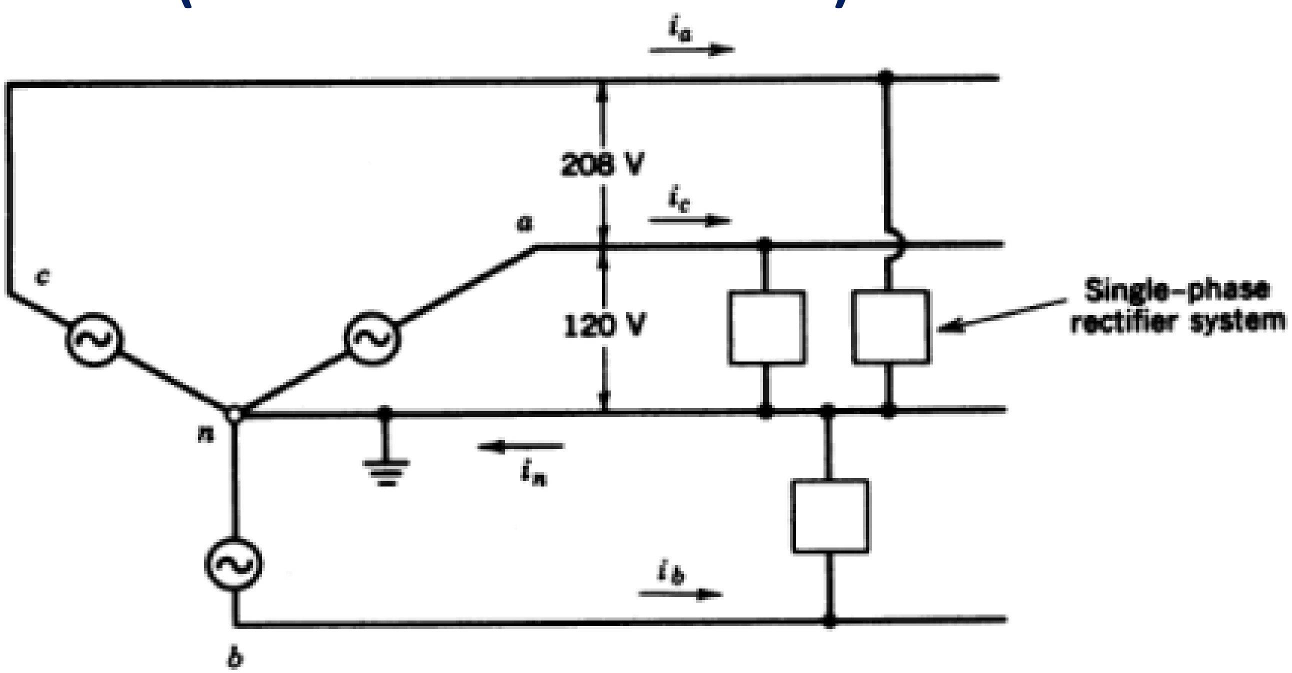
- **This is usually referred to voltage pollution in power system**

- **This is one of the reasons of power quality problems**

Effect of Single phase rectifiers on the neutral current in 3Φ 4 wire systems

- **When all the 3 phases are loaded equally or balanced, neutral current $i_n = 0$.**
- **When single phase rectifiers are connected, $i_n \neq 0$**

- **Consider identical rectifiers connected between each phase and neutral (balanced load condition)**



- **Phase a current**

$$i_a = i_{a1} + \sum_{n=2k+1}^{\infty} i_{an} \qquad \text{k=1,2,3,....}$$

- $i_a = \sqrt{2}I_{s1}sin(\omega t - \varphi_1) + \sum_{n=2k+1}^{\infty}\left(\sqrt{2}I_{sn} \quad \sin(n\omega t - \varphi_n) \quad \right)$

$$i_b = \sqrt{2}I_{s1}\sin(\omega t - \phi_1 - 120) + \sum_{n=2k+1}^{\infty}\sqrt{2}I_{sn}\sin(n\omega t - \phi_n - 120n)$$

$$i_c = \sqrt{2}I_{s1}\sin(\omega t - \phi_1 - 240) + \sum_{n=2k+1}^{\infty}\sqrt{2}I_{sn}\sin(n\omega t - \phi_n - 240n)$$

- **Neutral current** $i_N = i_a + i_b + i_c$

$$= i_{a1} + \sum_{n=2k+1}^{\infty} i_{an} + i_{b1} + \sum_{n=2k+1}^{\infty} i_{bn} + i_{c1} + \sum_{n=2k+1}^{\infty} i_{cn}$$

Sum of the three phase fundamental components and non-triplen harmonics is zero.

- $i_{a1} + i_{b1} + i_{c1} = 0$

$$i_N = \sum_{n=3(2k-1)}^{\infty} i_{an} + i_{bn} + i_{cn}$$

Neglecting all higher order harmonics,

$i_N = i_{a3} + i_{b3} + i_{c3} = 3\, i_{a3} \approx i_{a1}$

- **Thus, neutral wire carries 3 times 3rd harmonic current flowing through a phase**

- **neutral current is almost equal to or greater than the fundamental line current.**

Neutral current