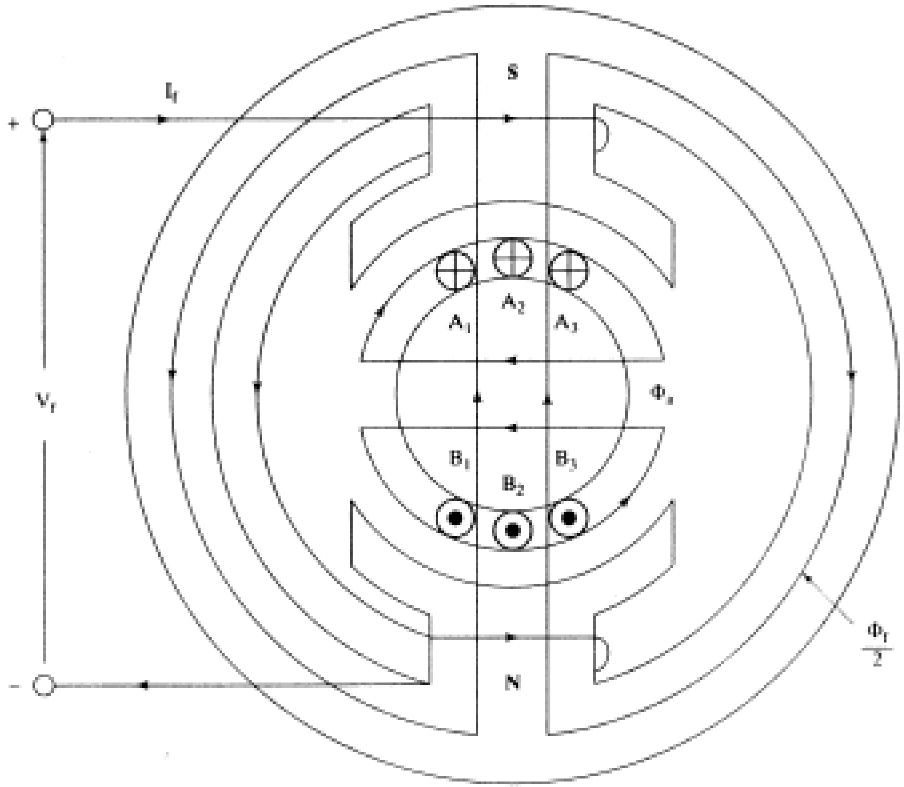
Electrical Drives

Lecture 9 (29-01-2024)

Modeling of DC Machines



Modeling of DC Machines

Induced EMF:

For a DC motor with P poles, Z armature conductors in a field with ϕ_f flux per pole and rotating at N rpm, induced emf is given by Faraday's Law (Neglecting sign)

$$e = Z\frac{\phi_f}{dt} = Z\frac{\phi_f}{t} \quad \cdots\cdots\cdots\cdots(1)$$

Where t is the time taken by the conductors to cut ϕ_f flux lines.

$$\therefore t = \frac{1}{2 * \text{frequency}} = \frac{1}{2\left(\frac{P}{2}\right)\left(\frac{N}{60}\right)} \quad \cdots\cdots\cdots\cdots(2)$$

flux change occurs for each pole pair.

Substituting eqn. (2) in eqn. (1), we get,

$$\therefore e == \frac{Z\phi_f PN}{60} \quad \cdots\cdots\cdots\cdots(3)$$

Modeling of DC Machines

If the armature conductors are divided into 'a' parallel paths

$$\therefore e == \frac{Z\phi_f PN}{60\,a} \quad \cdots\cdots\cdots\cdots(3)$$

There are two possible ways in which armature conductors can be arranged in slots: wave and lap. The values of 'a' for these two types of windings are

$$a = \begin{cases} P & \text{for lap} \\ 2 & \text{for wave} \end{cases}$$

Eqn. (3) can be written in a more compact form as:

$$\therefore e = K\phi_f\,\omega_m \quad \cdots\cdots\cdots\cdots(4)$$

$$\text{Where} \quad \omega_m = \frac{2\pi N}{60} \quad \text{and} \quad K = \left(\frac{P}{a}\right)Z\left(\frac{1}{2\pi}\right)$$

Modeling of DC Machines

If the field flux is constant, then the induced emf is proportional to the rotational speed and the constant of proportionality is known as induced emf or back emf constant. Then the induced emf is represented as

$$e = K_b \omega_m \quad \cdots\cdots\cdots\cdots (5)$$

Where K_b is the back emf constant given by:

$$K_b = K\phi_f \quad \text{volt} \Big/ \left(\frac{\text{rad.}}{\text{sec.}}\right)$$

$$\phi_f = \frac{N_f i_f}{\Re_m} \quad \cdots\cdots (6) \qquad K_b = \frac{K N_f i_f}{\Re_m} = M i_f \quad \cdots\cdots (7)$$

Where M is fictitious mutual inductance between field and armature windings.

Modeling of DC Machines

$$M = \frac{KN_f}{\Re_m} = \frac{P}{\pi}\frac{Z}{2a}\frac{N_f}{\Re_m} \cdots\cdots (8)$$

Now the emf equation can be re-written as:

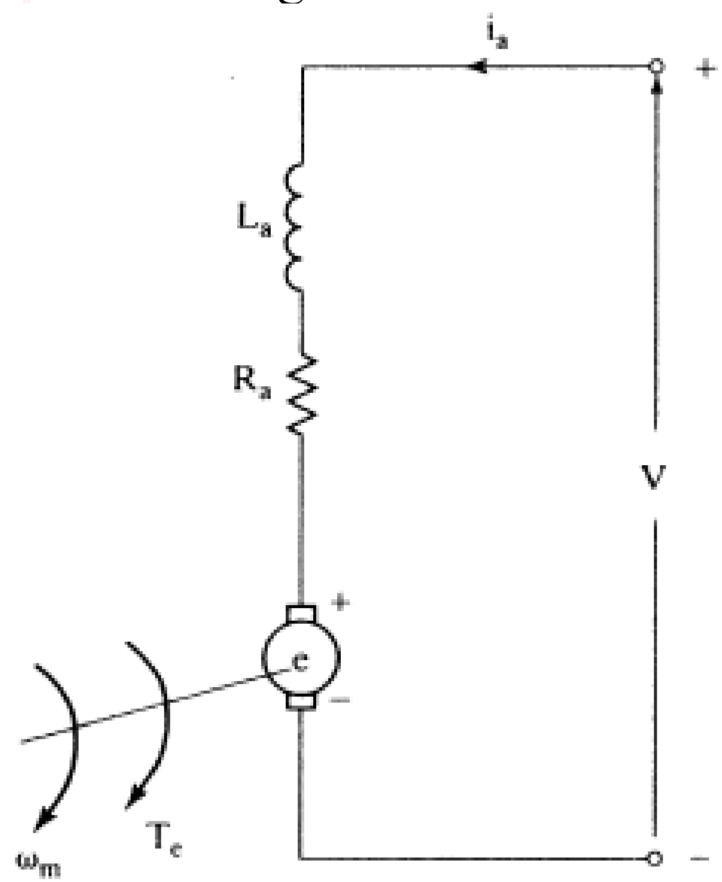
$$e = Mi_f\,\omega_m \quad \cdots\cdots\cdots\cdots (9)$$

The mutual inductance is a function of the field current and must be taken note to account for saturation of the magnetic material (stator laminations).

For operation within linear range, mutual inductance is assumed to be a constant in the machine.

Equivalent Circuit and Electromagnetic Torque

The equivalent circuit of a DC motor armature is based on the fact that the armature winding has a resistance R_a, self-inductance L_a and an induced emf e. This is shown in Fig. below:



Equivalent Circuit and Electromagnetic Torque

In the case of a motor, the input is electrical energy and the output is the mechanical energy, with an air gap torque of T_e at a rotational speed of ω_m.

The terminal relationship is written as:

$$v = e + R_a i_a + L_a \frac{di_a}{dt} \quad \cdots\cdots(10)$$

In steady state, armature current is constant and hence the rate of change of the armature current is zero.

Hence the armature voltage equation reduces to:

$$v = e + R_a i_a$$

The power balance is obtained by multiplying above equation by i_a.

$$vi_a = ei_a + R_a i_a^2$$

Equivalent Circuit and Electromagnetic Torque

The term $R_a i_a^2$ denotes the armature copper loss and vi_a is the total input power. Hence ei_a denotes the effective power that has been transferred from electrical to mechanical form, hereafter called air gap power, P_a.

The air gap power is expressed in terms of the electromagnetic torque and speed as:

$$P_a = \omega_m T_e = ei_a$$

Hence, the electromagnetic torque or air gap torque is represented as:

$$T_e = \frac{P_a}{\omega_m} = \frac{ei_a}{\omega_m}$$

By substituting for induced emf from eqn. (5), we get:

$$T_e = K_b i_a$$

The torque constant is equal to the back emf constant if it is expressed in volt-sec/rad. for a constant flux machine.

Electromechanical Modeling of Load

The DC motor drives a load, the modeling of load is to be carried out to complete the analysis for the motor-load system.

For simplicity, the load is modeled as a moment of inertia, J, kg-m^2, with a viscous friction, B_l, in N.m/(rad/sec).

Then the acceleration torque, T_a, in N-m drives the load and is given by:

$$T_a = T_e - T_l = J\frac{d\omega_m}{dt} + B_l\,\omega_m \quad \cdots\cdots(11)$$

Where T_l is the load torque.

Equations (10) and (11) constitute the dynamic model of the Dc motor with load.

State-Space Modeling

The Dynamic equations are cast in state-space form and are given by:

$$\begin{bmatrix} \dfrac{di_a}{dt} \\ \dfrac{d\omega_m}{dt} \end{bmatrix} = \begin{bmatrix} -\dfrac{R_a}{L_a} & -\dfrac{K_b}{L_a} \\ +\dfrac{K_b}{J} & -\dfrac{B_l}{J} \end{bmatrix} \begin{bmatrix} i_a \\ \omega_m \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L_a} & 0 \\ 0 & -\dfrac{1}{J} \end{bmatrix} \begin{bmatrix} v \\ T_l \end{bmatrix}$$

This can be represented compactly in the form:

$$\dot{X} = AX + BU$$

Where $X=[i_a\ \omega_m]^t$ and $U= =[v\ T_l]^t$. X is the state variable vector and U is the input vector. Even though load torque is a disturbance, for sake of compact representation it is included in the input vector.

State-Space Modeling

$$A = \begin{bmatrix} -\dfrac{R_a}{L_a} & -\dfrac{K_b}{L_a} \\ +\dfrac{K_b}{J} & -\dfrac{B_l}{J} \end{bmatrix} \qquad B = \begin{bmatrix} \dfrac{1}{L_a} & 0 \\ 0 & -\dfrac{1}{J} \end{bmatrix}$$

The roots of the system are evaluated from the A matrix; they are obtained by solving the system characteristic equation given by:

$$\lambda I - A = 0$$

The roots are:

$$\lambda_{1,2} = \frac{-\left(\dfrac{R_a}{L_a} + \dfrac{B_l}{J}\right) \pm \sqrt{\left(\dfrac{R_a}{L_a} + \dfrac{B_l}{J}\right)^2 - 4\left(\dfrac{R_a B_l}{JL_a} + \dfrac{K_b^2}{JL_a}\right)}}{2}$$

It is interesting to note that eigen values will always have a negative real part, indicating that the motor is stable on open-loop operation.

Block Diagrams and Transfer Functions

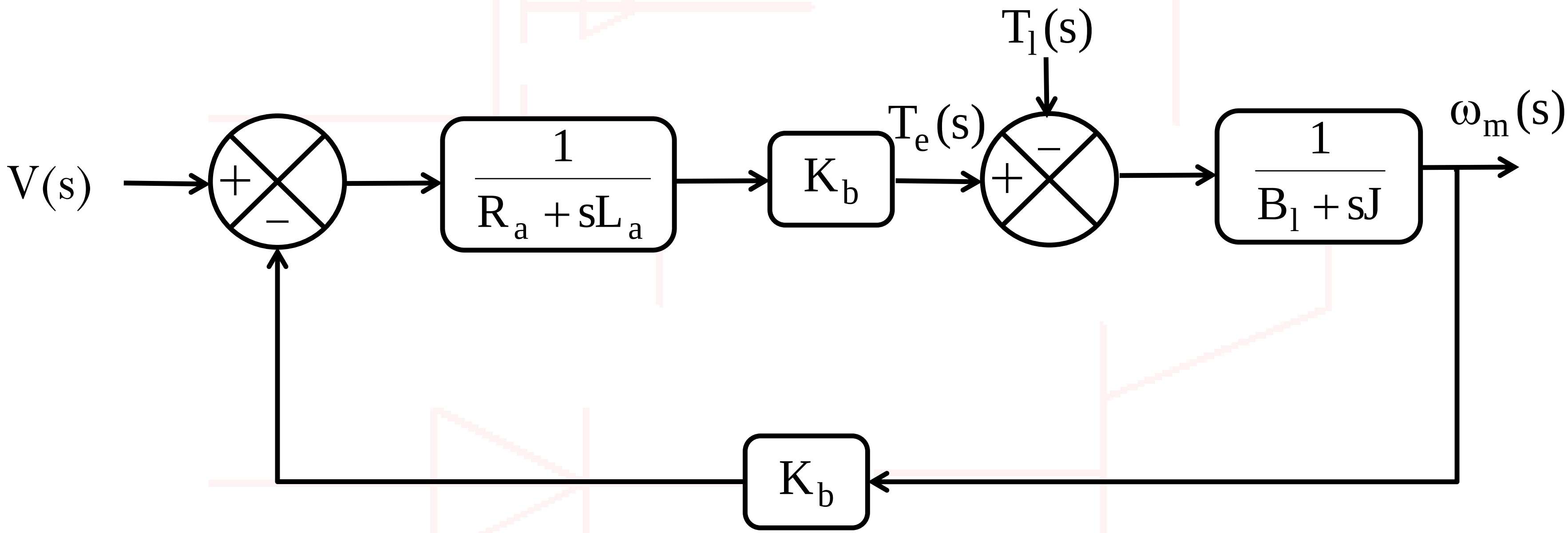
Taking Laplace Transforms of equations (10) , (11) and neglecting initial conditions, we get:

$$I_a(s) = \frac{V(s) - K_b \omega_m(s)}{R_a + sL_a}$$

$$\omega_m(s) = \frac{K_b I_a(s) - T_l(s)}{B_l + sJ}$$

Block Diagrams and Transfer Functions

In block diagram form, the relationships are represented as shown in figure below:



Block Diagram of DC motor with load

Block Diagrams and Transfer Functions

From block diagram, we can derive two transfer functions, namely:

$$\frac{\omega_m(s)}{V(s)} \quad \text{and} \quad \frac{\omega_m(s)}{T_l(s)}$$

$$G_{\omega V}(s) = \frac{\omega_m(s)}{V(s)} = \frac{K_b}{s^2(JL_a) + s(B_l L_a + JR_a) + (B_l L_a + K_b^2)}$$

$$G_{\omega l}(s) = \frac{\omega_m(s)}{T_l(s)} = \frac{-(R_a + sL_a)}{s^2(JL_a) + s(B_l L_a + JR_a) + (B_l L_a + K_b^2)}$$

Block Diagrams and Transfer Functions

Separately excited DC motor is a linear system, and hence the speed response due to the simultaneous voltage input and load torque disturbance can be written as a sum of their individual responses:

$$\omega_m(s) = G_{\omega V}(s) * V(s) + G_{\omega l}(s) * T_l(s)$$

Laplace inverse of $\omega_m(s)$ in above equation gives the time response of the speed for a simultaneous change in the input voltage and load torque.

The treatment so far based on a DC motor obtaining its excitation separately.

There are various forms of field excitation, that we have already discussed.

Problems

1. A DC motor is started directly from a 220V dc supply with no-load. Find its starting speed response and the time taken to reach 100rad/sec. DC Motor parameters are: R_a=0.5Ω, L_a=3mH, J=0.0167 kgm^2 and K_b=0.8V/rad/sec.

Problems

2. A separately-excited DC motor with the following parameters: R_a=0.5Ω, L_a=3mH and K_b=0.8V/rad/sec. is driving a load of J=0.0167kg-m^2, B=0.01N-m/rad/sec. with a load torque of 100N-m. Its armature is connected to a DC supply voltage of 220V and is given the rated field current. Find the speed of the motor.