राष्ट्रीय प्रौद्योगिकी संस्थान कालिकट

NATIONAL INSTITUTE OF TECHNOLOGY CALICUT

Monsoon 2023-2024

# EE6302E Dynamics of Electrical Machines (DEM)

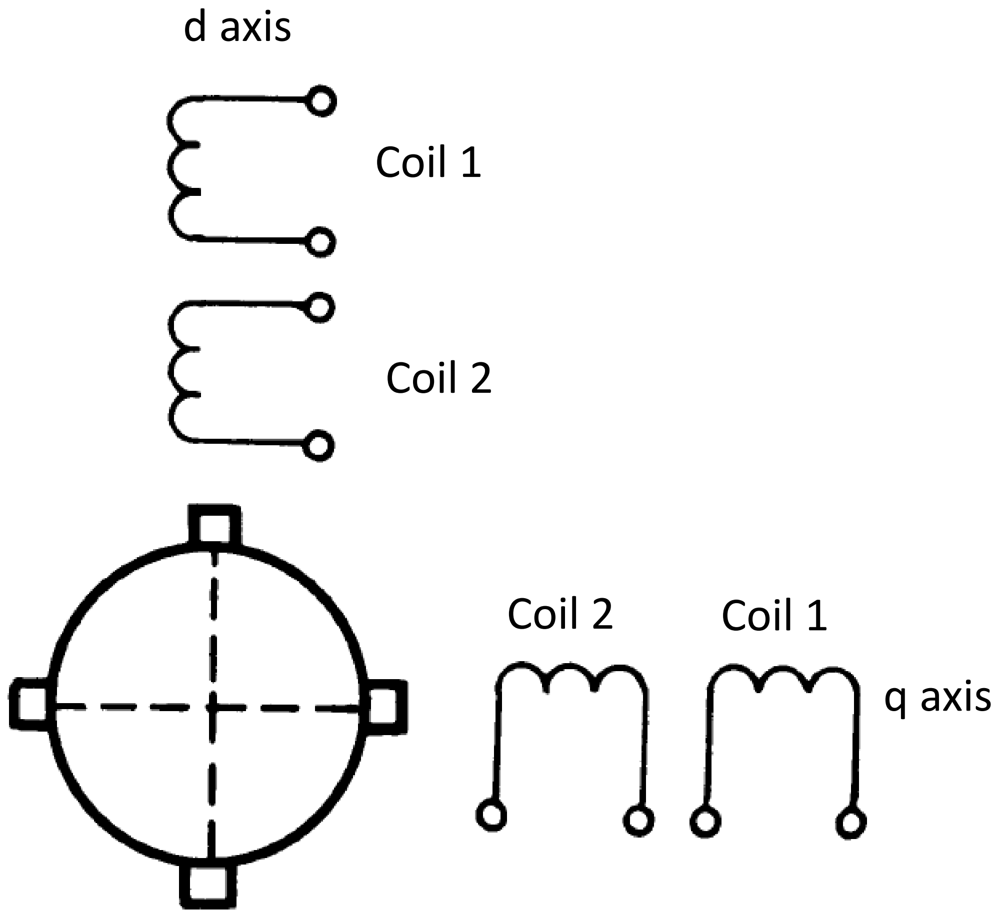
## Module 2


**References**

1. *D P Sengupta & J.B. Lynn, Electrical Machine Dynamics, The Macmillan Press Ltd., 1980.*
2. *R Krishnan, Electric Motor Drives, Modeling, Analysis and Control, Pearson Education, 2001.*
3. *P.C. Kraus, Analysis of Electrical Machines, McGraw Hill Book Company,1987*

# PRIMITIVE MACHINE EQUATIONS ALONG STATIONARY AXES

**Rotor Voltages**




$$V_{dr} = R_{dr} i_{dr} + \frac{d\psi_{dr}}{dt} + B_{qr} p\theta$$

$$V_{qr} = R_{qr} i_{qr} + \frac{d\psi_{qr}}{dt} - B_{dr} p\theta$$

<u>flux density terms</u>

$$B_{qr} = L'_{qr} i_{qr} + M'_{q1} i_{qs1} + M'_{q2} i_{qs2}$$

$$B_{dr} = L'_{dr} i_{dr} + M'_{d1} i_{ds1} + M'_{d2} i_{ds2}$$

$$V_{dr} = R_{dr} i_{dr} + L_{dr} p i_{dr} + M_{d1} p i_{ds1} + M_{d2} p i_{ds2} + L'_{qr} i_{qr} p\theta + M'_{q1} i_{qs1} p\theta + M'_{q2} i_{qs2} p\theta$$

$$V_{qr} = R_{qr} i_{qr} + L_{qr} p i_{q2} + M_{q1} p i_{qs1} + M_{q2} p i_{qs2} - L'_{dr} i_{dr} p\theta - M'_{d1} i_{ds1} p\theta - M'_{d2} i_{ds2} p\theta$$

# PRIMITIVE MACHINE EQUATIONS ALONG STATIONARY AXES

$$[\mathbf{V}] = [\mathbf{Z}][\mathbf{i}]$$

$$[\mathbf{V}] = [\mathbf{R}][\mathbf{i}] + [\mathbf{L}]p[\mathbf{i}] + [\mathbf{G}][\mathbf{i}]p\theta$$

$$\mathbf{V} = \mathbf{R}\mathbf{i} + \mathbf{L}p\mathbf{i} + \mathbf{G}\mathbf{i}p\theta$$

$$\mathbf{R} = \begin{array}{c|cccccc|}
 & \text{ds1} & \text{ds2} & \text{dr} & \text{qr} & \text{qs1} & \text{qs2} \\
\hline
\text{ds1} & R_{ds1} & & & & & \\
\text{ds2} & & R_{ds2} & & & & \\
\text{dr} & & & R_r & & & \\
\text{qr} & & & & R_r & & \\
\text{qs1} & & & & & R_{qs1} & \\
\text{qs2} & & & & & & R_{qs2} \\
\hline
\end{array}$$

$$\mathbf{L} = \begin{array}{c|cccccc|}
 & \text{ds1} & \text{ds2} & \text{dr} & \text{qr} & \text{qs1} & \text{qs2} \\
\hline
\text{ds1} & L_{ds1} & M_{d12} & M_{d1} & & & \\
\text{ds2} & M_{d12} & L_{ds2} & M_{d2} & & & \\
\text{dr} & M_{d1} & M_{d2} & L_{dr} & & & \\
\text{qr} & & & & L_{qr} & M_{q1} & M_{q2} \\
\text{qs1} & & & & M_{q1} & L_{qs1} & M_{q12} \\
\text{qs2} & & & & M_{q2} & M_{q12} & L_{qs2} \\
\hline
\end{array}$$

$$\mathbf{G} = \begin{array}{c|cccccc|}
 & \text{ds1} & \text{ds2} & \text{dr} & \text{qr} & \text{qs1} & \text{qs2} \\
\hline
\text{ds1} & & & & & & \\
\text{ds2} & & & & & & \\
\text{dr} & & & & L'_{qr} & M'_{q1} & M'_{q2} \\
\text{qr} & -M'_{d1} & -M'_{d2} & -L'_{dr} & & & \\
\text{qs1} & & & & & & \\
\text{qs2} & & & & & & \\
\hline
\end{array}$$

$$\begin{bmatrix} V_{ds1} \\ V_{ds2} \\ V_{dr} \\ V_{qr} \\ V_{qs1} \\ V_{qs2} \end{bmatrix} =
\begin{array}{c|cccccc|}
 & \text{ds1} & \text{ds2} & \text{dr} & \text{qr} & \text{qs1} & \text{qs2} \\
\hline
\text{ds1} & R_{ds1} + L_{ds1}p & M_{d12}p & M_{d1}p & & & \\
\text{ds2} & M_{d12}p & R_{ds2} + L_{ds2}p & M_{d2}p & & & \\
\text{dr} & M_{d1}p & M_{d2}p & R_r + L_{dr}p & L'_{qr}p\theta & M'_{q1}p\theta & M'_{q2}p\theta \\
\text{qr} & -M'_{d1}p\theta & -M'_{d2}p\theta & -L'_{dr}p\theta & R_r + L_{qr}p & M_{q1}p & M_{q2}p \\
\text{qs1} & & & & M_{q1}p & R_{qs} + L_{qs1}p & M_{q12}p \\
\text{qs2} & & & & M_{q2}p & M_{q12}p & R_{qs2} + L_{qs2}p \\
\hline
\end{array}
\cdot
\begin{bmatrix} i_{ds1} \\ i_{ds2} \\ i_{dr} \\ i_{qr} \\ i_{qs1} \\ i_{qs2} \end{bmatrix}$$

# SQUIRREL CAGE INDUCTION MOTOR MODEL FROM PRIMITIVE MACHINE EQUATIONS

**Step 1**

Remove coil 2 and corresponding rows and column

Replace the operator p by $j\omega_1$

$p\theta = \omega_r$

$$\omega_r = \omega_1 (1 - s)$$

$$\begin{bmatrix} V_{ds1} \\ V_{dr} \\ V_{qr} \\ V_{qs1} \end{bmatrix} = \begin{array}{c|c|c|c|c} & ds1 & dr & qr & qs1 \\ \hline ds1 & R_{ds1} + j\omega_1 L_{ds1} & j\omega_1 M_{d1} & & \\ dr & j\omega_1 M_{d1} & R_r + j\omega_1 L_{dr} & \omega_1 (1-s) M_{q1} & \omega_1 (1-s) M_{q1} \\ qr & -\omega_1 (1-s) M_{d1} & -\omega_1 (1-s) L_{dr} & R_r + j\omega_1 L_{qr} & j\omega_1 M_{q1} \\ qs1 & & & j\omega_1 M_{q1} & R_{qs1} + j\omega_1 L_{qs1} \end{array} \begin{bmatrix} i_{ds1} \\ i_{dr} \\ i_{qr} \\ i_{qs1} \end{bmatrix}$$

# SQUIRREL CAGE INDUCTION MOTOR MODEL FROM PRIMITIVE MACHINE EQUATIONS

**Step 2**

$$R_{ds1} = R_{qs1} = R_1, \qquad R_r = R_2$$

$$L_{ds1} = L_{qs1} = L_1, \qquad L_{dr} = L_{qr} = L_2$$

$$M_{d1} = M_{q1} = M$$

$$M'_{d1} = M_{d1} = M$$

$$M'_{q1} = M_{q1} = M$$

stator coils which are symmetrically distributed

air gap is uniform

$$\begin{bmatrix} V_{ds1} \\ V_{dr} \\ V_{qr} \\ V_{qs1} \end{bmatrix} = \begin{array}{c|cccc} & ds1 & dr & qr & qs1 \\ \hline ds1 & R_1 + jX_1 & jX_m & & \\ dr & jX_m & R_2 + jX_2 & (1-s)X_2 & (1-s)X_m \\ qr & -(1-s)X_m & -(1-s)X_2 & R_2 + jX_2 & jX_m \\ qs1 & & & jX_m & R_1 + jX_1 \end{array} \begin{bmatrix} i_{ds1} \\ i_{dr} \\ i_{qr} \\ i_{qs1} \end{bmatrix}$$

flux wave is sinusoidally distributed in space and hence the coefficients of mutual inductance for transformer and generated voltages are the same

# SQUIRREL CAGE INDUCTION MOTOR MODEL FROM PRIMITIVE MACHINE EQUATIONS

**Step 3**

$$
\begin{array}{c}
\begin{bmatrix} V_1 \\ 0 \\ 0 \\ -jV_1 \end{bmatrix}
\end{array}
=
\begin{array}{c|cccc}
 & ds1 & dr & qr & qs1 \\
\hline
ds1 & R_1 + jX_1 & jX_m & & \\
dr & jX_m & R_2 + jX_2 & (1-s)X_2 & (1-s)X_m \\
qr & -(1-s)X_m & -(1-s)X_2 & R_2 + jX_2 & jX_m \\
qs1 & & & jX_m & R_1 + jX_1
\end{array}
\cdot
\begin{bmatrix} I_1 \\ I_2 \\ -jI_2 \\ -jI_1 \end{bmatrix}
$$

net m.m.f. they produce rotates at synchronous speed

During balanced operation, net MMF

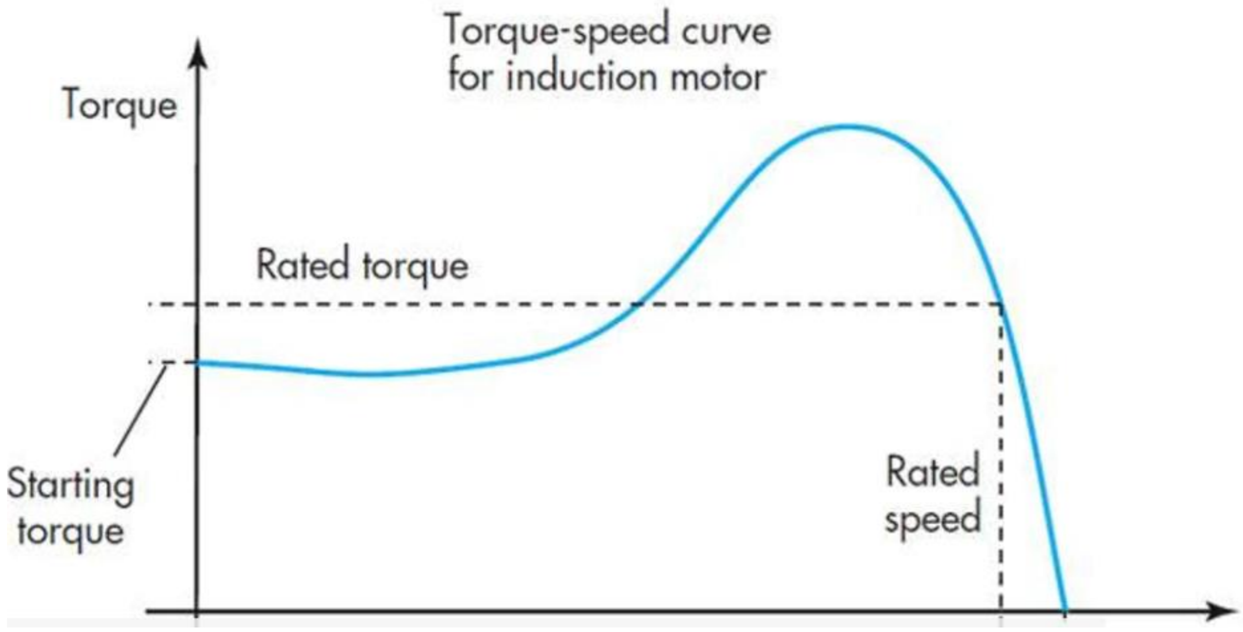
$$N_1(I_1^2 \sin^2 \omega_1 t + I_1^2 \cos \omega_1 t)^{\frac{1}{2}}$$

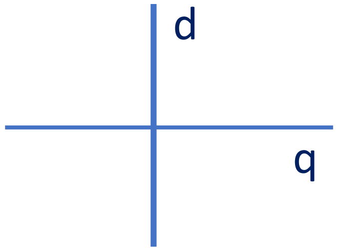
rotor voltages are zero since the rotor coils in an induction motor are short-circuited

# SQUIRREL CAGE INDUCTION MOTOR MODEL FROM PRIMITIVE MACHINE EQUATIONS

$$T = -\mathbf{i}^*\mathbf{G}\mathbf{i}$$

$$= -i_{qr}^* M_d i_{ds} + i_{dr}^* M_q i_{qs}$$



# Squirrel Cage Induction Motor: Stability Analysis

**Assignment Q1**
**Marks: 10**
**Last date: before 12.12.2023**

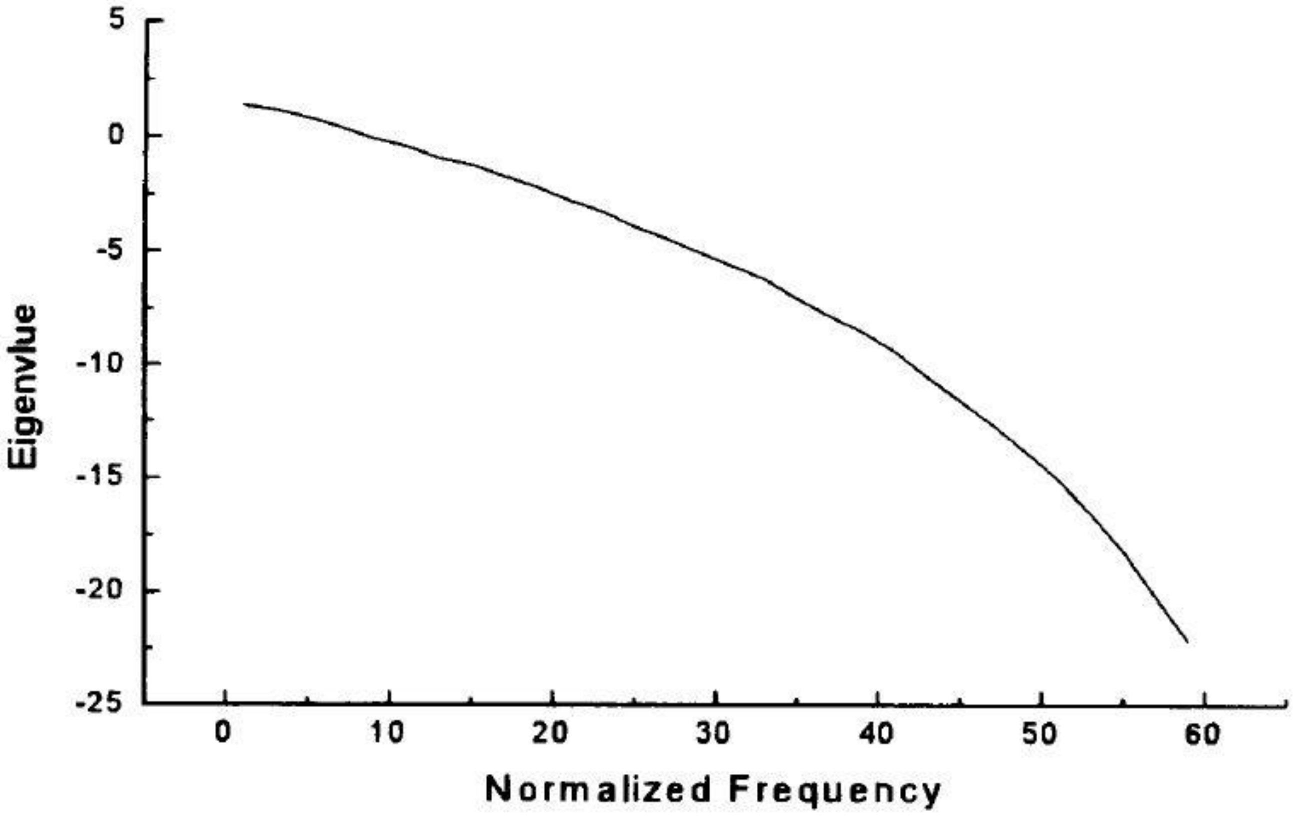
***Develop the state space model for the induction machine with following parameters. Estimate eigen values and eigen vector using any simulation platform. Infer on the stability***

| Machines Parameters | Value | Per Unit Value |
|---|---|---|
| Horse Power (Hp) | 50 hp | - |
| Voltage ($V_L$) | 460 V | - |
| Frequency (Hz) | 60 Hz | - |
| Stator Resistance ($r_s$) | 0.087 Ω | 0.015336 |
| Stator Reactance ($X_{ls}$) | 0.302 Ω | 0.053235 |
| Mutual Reactance ($X_M$) | 13.08 Ω | 2.30569 |
| Equivalent Rotor Resistance ($r'_r$) | 0.302 Ω | 0.040191 |
| Equivalent Rotor Reactance ($X'_{lr}$) | 0.228 Ω | 0.053235 |
| Moment of Inertia (J) | 1.662 Ω | - |

M. B. Uddin, M. N. Pramanik and S. A. Reza, "Low frequency stability study of a three-phase induction motor," *2007 7th International Conference on Power Electronics*, 2007, pp. 1115-1120

*Ref. Section 5.10 of "Electrical Machine Dynamics" by Sengupta for State space model example.*
*Various currents and rotor speed are to be taken as State variables.*

# Squirrel Cage Induction Motor: Stability Analysis

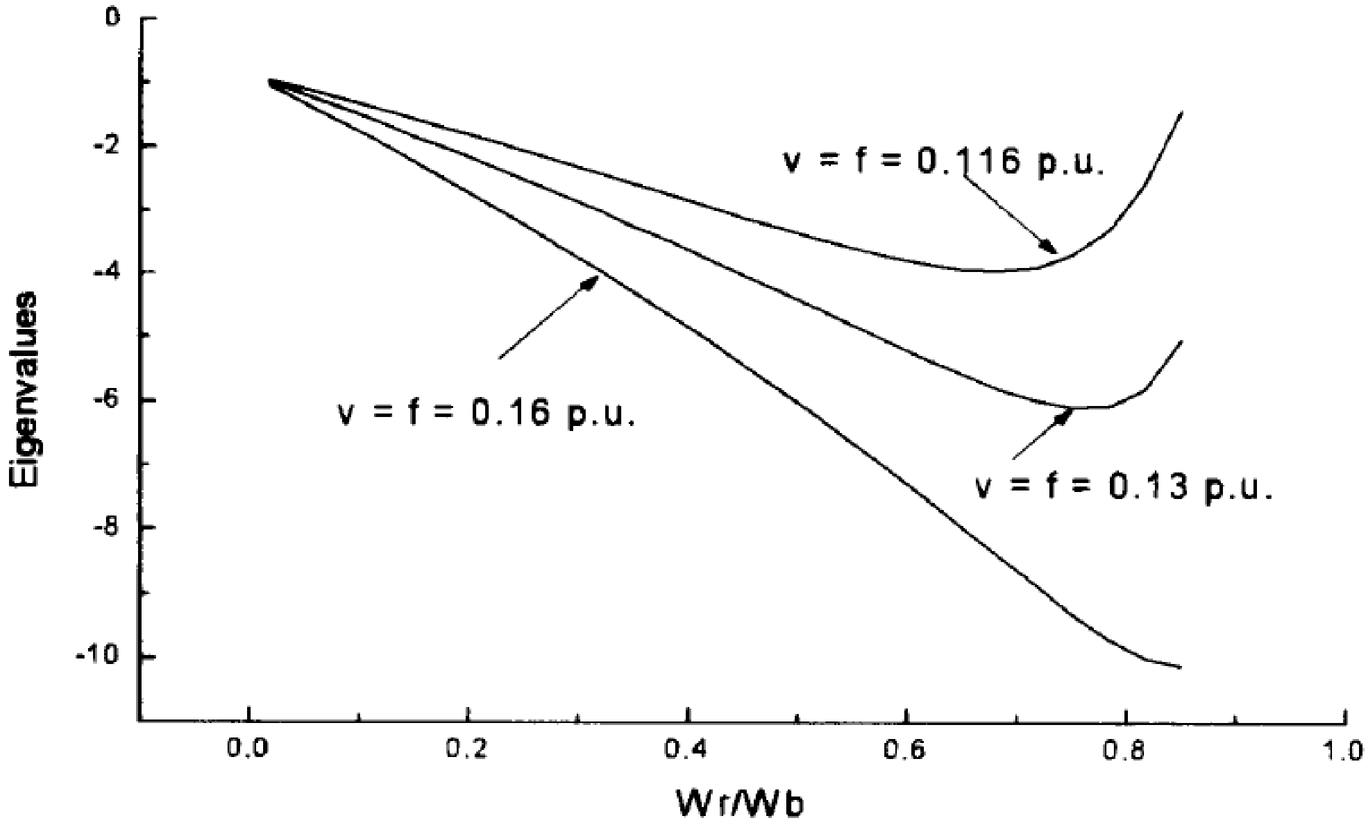



M. B. Uddin, M. N. Pramanik and S. A. Reza, "Low frequency stability study of a three-phase induction motor," *2007 7th International Conference on Power Electronics*, 2007, pp. 1115-1120

The excursion of eigen values for the rated operating condition of induction motor are depicted in Fig. Which indicate that operation becomes unstable at lower frequency. In each step of computation the applied voltage is decreased linearly with frequency. The eigen values are found to cross the boundary at low frequency of 0.116 p. u. (7 Hz) while the corresponding stator voltage is also 0.116 p. u. indicating unstable operation below this frequency.

The straight line parallel to the x-axis passing through zero value of the ordinate forms the boundary between the stable and unstable region of operation. The lower part of the boundary represents the stable region and the upper part signifies the unstable region of operation.

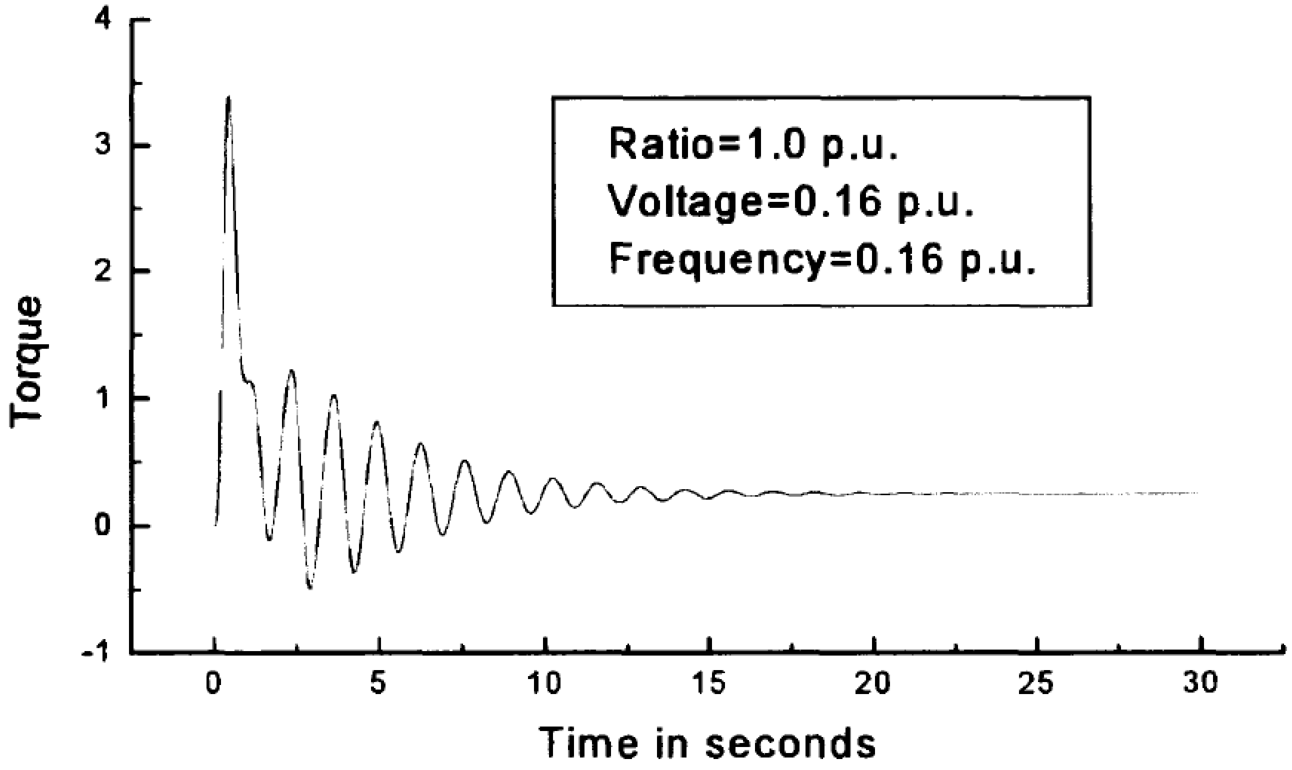
# Squirrel Cage Induction Motor: Stability Analysis

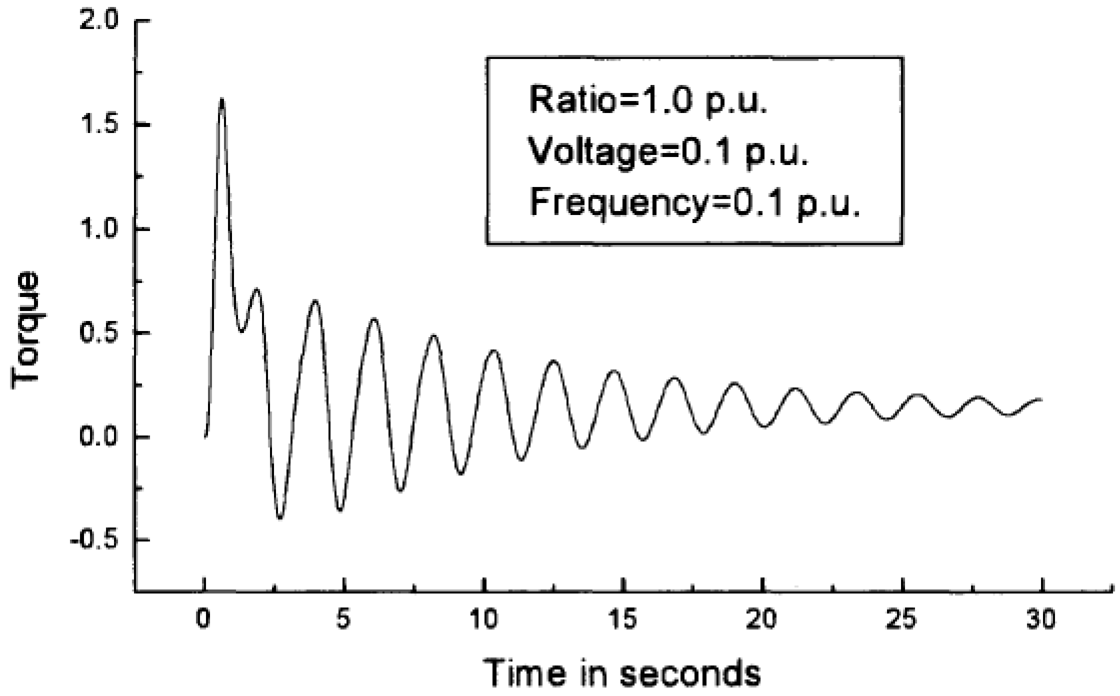



M. B. Uddin, M. N. Pramanik and S. A. Reza, "Low frequency stability study of a three-phase induction motor," *2007 7th International Conference on Power Electronics*, 2007, pp. 1115-1120

The effect of amplitude of the stator voltage on stability is illustrated in Fig. These curves are obtained by decreasing the stator voltage and frequency keeping volt/Hz ratio constant. The reduction of voltage and frequency simultaneously is indicating the induction motor trends to be unstable at lower frequency

# Squirrel Cage Induction Motor: Stability Analysis

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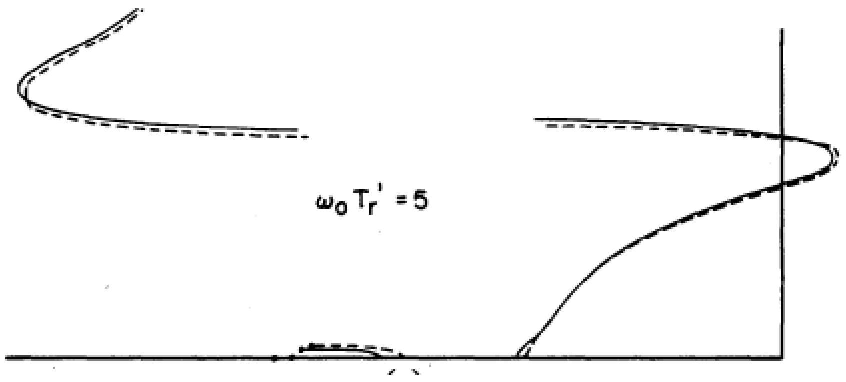
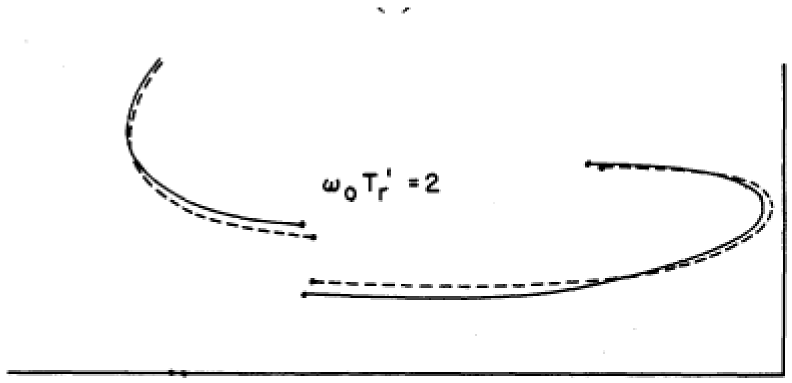
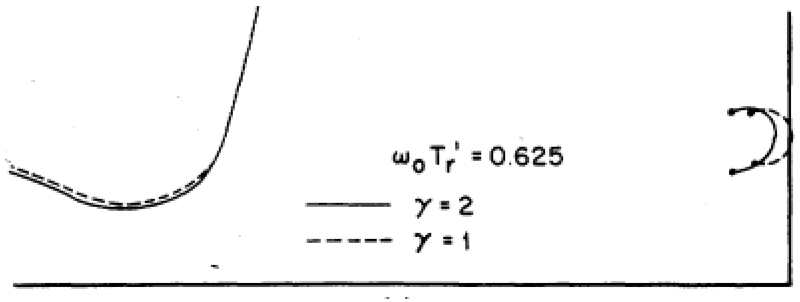
# Squirrel Cage Induction Motor: Stability Analysis



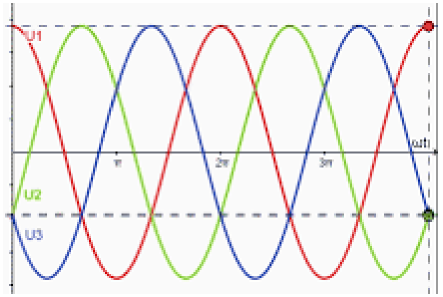



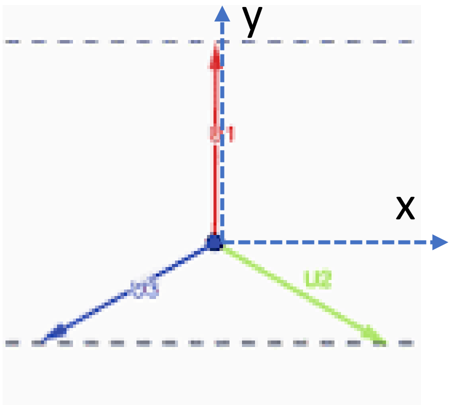

The leftward shift of the overall root locus in the region of low damping is perhaps the most significant effect of transient saturation.

Modern high efficiency machines which have increased pu magnetizing reactance and are designed to operate at reduced flux levels are likely to have larger and stronger regions of instability and to exhibit poorer damped transient response than conventional machines.

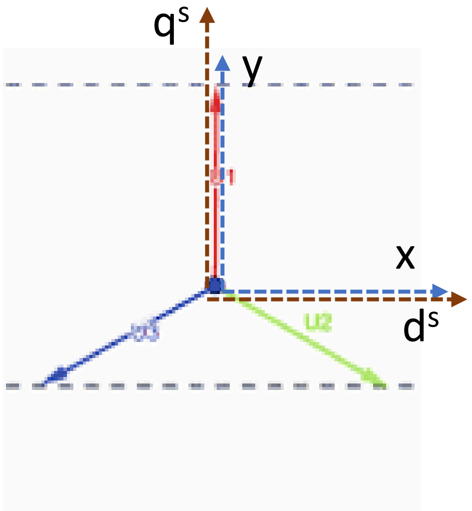
# Reference Frame

A three phase voltage (or current or any balanced sinusoidal oscillation) can be represented as rotating vectors in multiple ways (different perspectives)





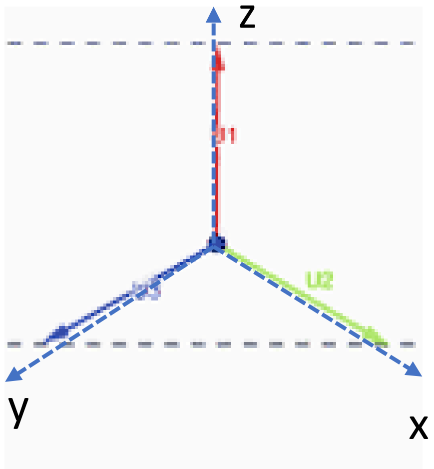

Two stationary axes (x and y) and three rotating vectors



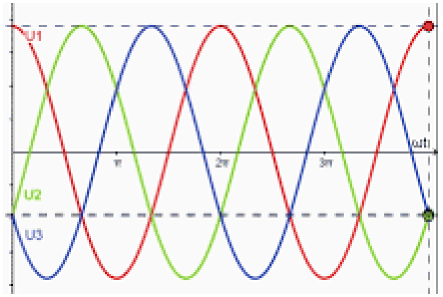

- The three rotating vectors can be mapped to two stationary axes ($d^s$ and $q^s$). Since axes are stationary, the mapped quantities will be sinusoidal.
- That is, three-phase voltage ($V_R$, $V_Y$, $V_B$) can be represented in their equivalent two phase components ($V_d^s$, $V_q^s$ phase shifted by 90 deg). This mapping is called Clarke's transformation.
- By changing the magnitudes of direct and quadrature axis components (the mapped components), we could realize three-phase components of any magnitude and phase angle at any particular instant (i.e. by increasing quadrature component magnitude alone in this case, we could generate phase lead to three phase set in comparison with original set. Also change in magnitude).

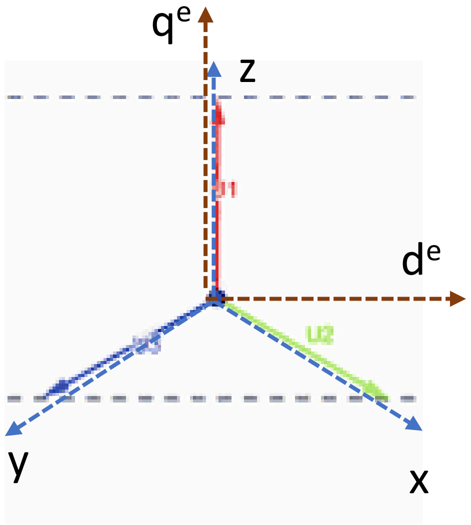
- In developing control for three phase system, this approach facilitates easier implementation than using three-phase quantities directly (since number of variables reduces)

# Reference Frame




Three rotating axes and three stationary vectors in those axes






- The **three stationary vectors (constant) in three rotating axes** can be mapped to **two rotating axes** ($d^e$ and $q^e$). Since axes are rotating, the **mapped quantities also will be constant (dc)** (assuming $d^e$-$q^e$ axes rotating in synchronism with that of three-phase).
- That is, three-phase voltage ($V_R$, $V_Y$, $V_B$) can be represented in their equivalent two phase components ($V_d^e$, $V_q^e$, phase shifted by 90 deg). This mapping is called Park's transformation.
- By changing the magnitudes of direct and quadrature axis components (the mapped components), we could realize three-phase components of any magnitude and phase angle at any particular instant (i.e. by increasing quadrature component magnitude alone in this case, we could generate phase lead to three phase set in comparison with original set. Also change in magnitude).

- In developing control for three phase system, this approach facilitates easier implementation than Clarke's transformation (since number of variables reduces and becomes DC (lower bandwidth requirement))

# Fortescue’s Transformation

- This transformation is known as the method of symmetrical components and developed by Fortescue.
- This transformation states that N unbalanced phasors can be represented by N systems of N balanced phasors.
- It uses a complex transformation to decouple the abc phase variables.
- The method of symmetrical components is used to simplify analysis of unbalanced three phase power systems under both normal and abnormal conditions.
- It is used to decouple an unbalanced three-phase network into three simpler sequence (zero, positive and negative) networks.

# Fortescue's Transformation

- The method of symmetrical components is expressed as follows

$$\boxed{[\mathbf{f}_{012}] = [\mathbf{T}_{012}][\mathbf{f}_{abc}]}$$

$$\boxed{[\mathbf{f}_{abc}] = [\mathbf{T}_{012}]^{-1}[\mathbf{f}_{012}]}$$

$$\boxed{[\mathbf{f}_{012}] = \begin{bmatrix} f_0 \\ f_1 \\ f_2 \end{bmatrix}}$$

$$\boxed{[\mathbf{f}_{abc}] = \begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix}}$$

- Variable *f* may be the currents, voltages or fluxes and the transformation and its inverse are given by

$$\boxed{[\mathbf{T}_{012}] = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}}$$

$$\boxed{[\mathbf{T}_{012}]^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}}$$

where

$$a = e^{j\frac{2\pi}{3}}$$

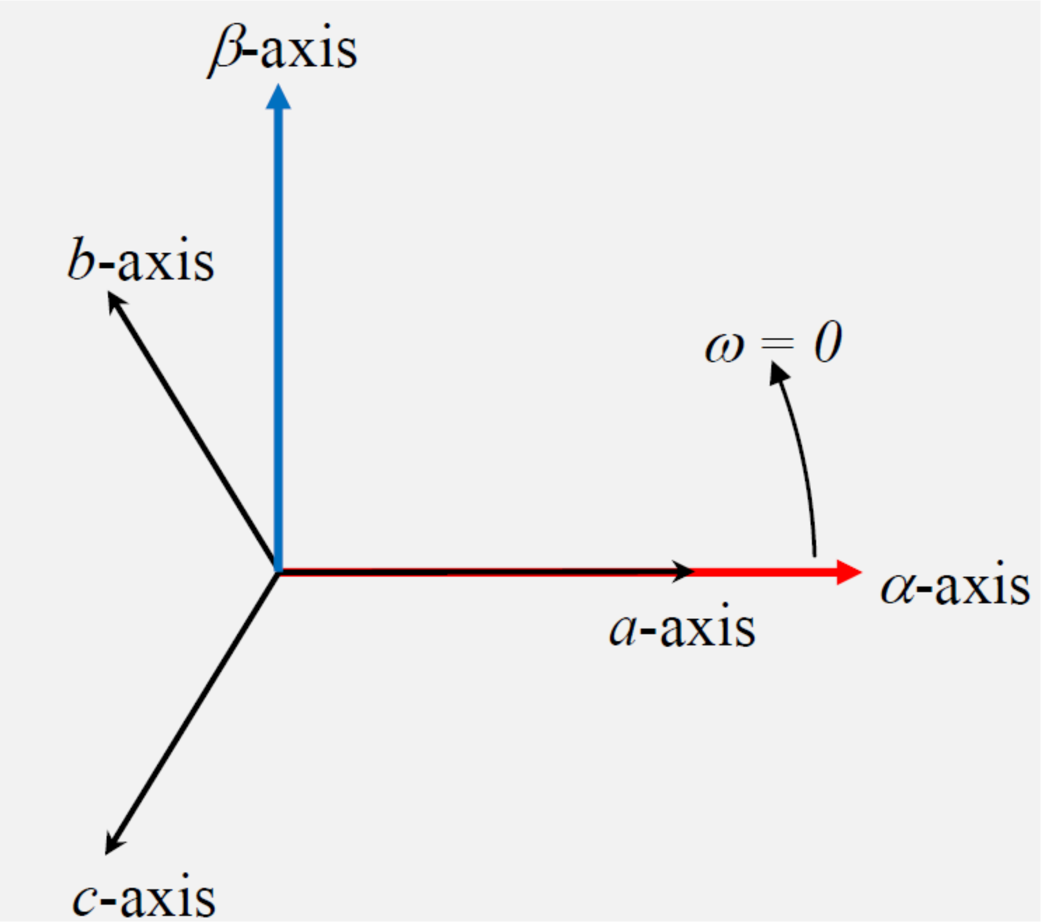
# Clarke's Transformation

- The stationary two-phase variables of Clarke's transformation are denoted as α and β.

- As shown below, the α -axis coincides with the phase a-axis and the α-axis leads the β -axis by π/2.

- A third variable known as the zero-sequence component is also included.

- Clarke's transformation is not power-invariant (i.e. the values of power before and after the transformation are not the same.)



# Clarke's Transformation

Clarke's transformation is expressed as follows

$$\boxed{\left[\mathbf{f}_{\alpha\beta 0}\right]=\left[\mathbf{T}_{\alpha\beta 0}\right]\left[\mathbf{f}_{abc}\right]}$$

$$\boxed{\left[\mathbf{f}_{abc}\right]=\left[\mathbf{T}_{\alpha\beta 0}\right]^{-1}\left[\mathbf{f}_{\alpha\beta 0}\right]}$$

$$\boxed{\left[\mathbf{f}_{\alpha\beta 0}\right]=\begin{bmatrix} f_\alpha \\ f_\beta \\ f_0 \end{bmatrix}}$$

$$\boxed{\left[\mathbf{f}_{abc}\right]=\begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix}}$$

Similarly variable *f* may be the currents, voltages or fluxes and the transformation and its inverse are given by

$$\boxed{\left[\mathbf{T}_{\alpha\beta 0}\right]=\frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}}$$

$$\boxed{\left[\mathbf{T}_{\alpha\beta 0}\right]^{-1}=\begin{bmatrix} 1 & 0 & 1 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 1 \end{bmatrix}}$$

# Clarke's Transformation

**Power-Invariant Property**

$$\begin{cases} v_a = V_m \cos(\omega t) \\ v_b = V_m \cos(\omega t - 2\pi/3) \\ v_c = V_m \cos(\omega t - 4\pi/3) \end{cases}$$

$$\begin{cases} i_a = I_m \cos(\omega t) \\ i_b = I_m \cos(\omega t - 2\pi/3) \\ i_c = I_m \cos(\omega t - 4\pi/3) \end{cases}$$

at $\omega t = 0$

$$\begin{cases} v_a = V_m \\ v_b = \frac{-1}{2} V_m \\ v_c = \frac{-1}{2} V_m \end{cases}$$

$$\begin{cases} i_a = I_m \\ i_b = \frac{-1}{2} I_m \\ i_c = \frac{-1}{2} I_m \end{cases}$$

Three Phase Power

$$P = v_a i_a + v_b i_b + v_c i_c = \frac{3}{2} V_m I_m$$

Clarke's Transformed quantities

$$\begin{cases} v_\alpha = V_m \\ v_\beta = 0 \\ v_0 = 0 \end{cases}$$

$$\begin{cases} i_\alpha = I_m \\ i_\beta = 0 \\ i_0 = 0 \end{cases}$$

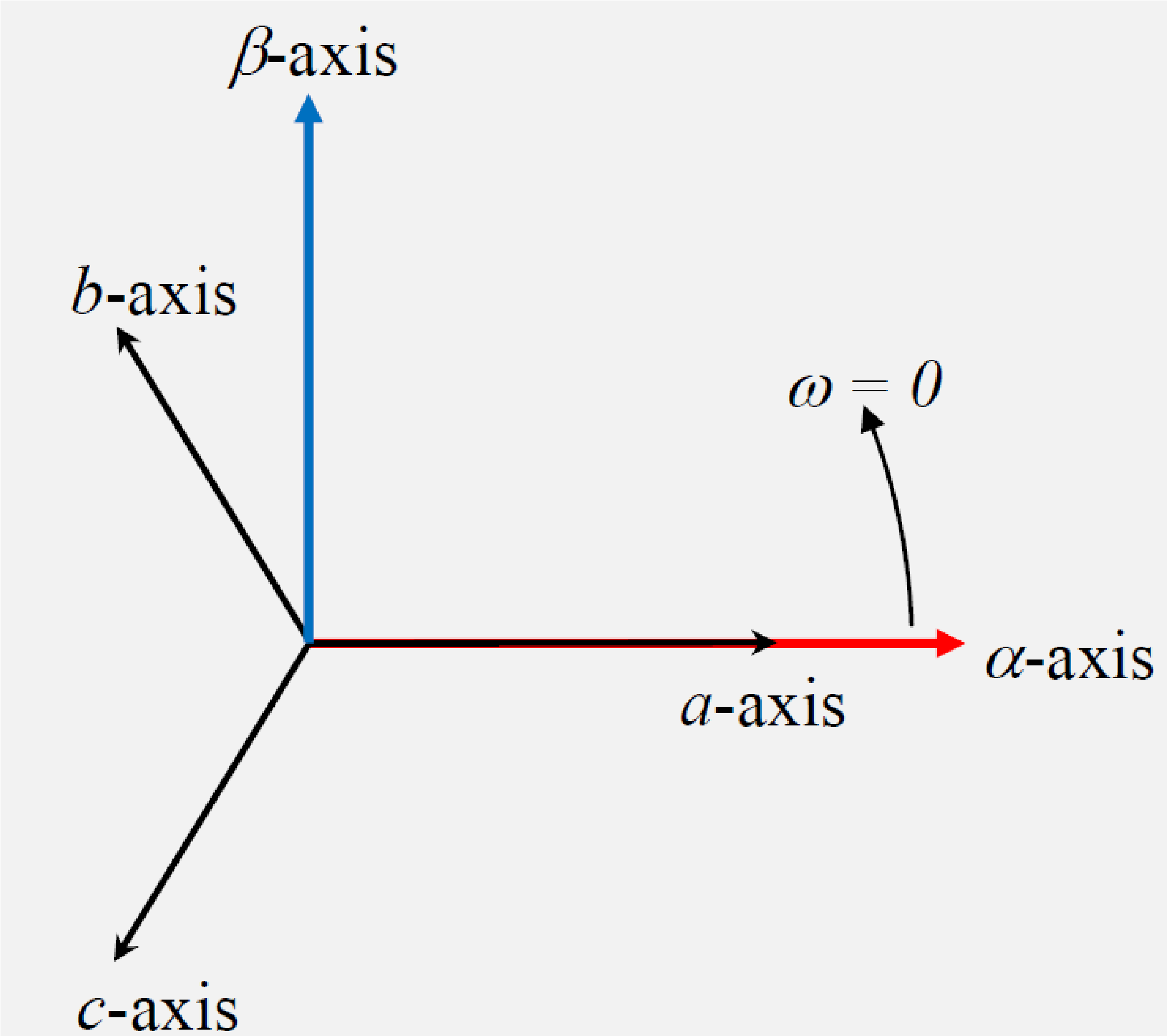
**Clarke's transformation is not power-invariant**

Power in $\alpha\beta$ frame

$$P = v_\alpha i_\alpha + v_\beta i_\beta + v_0 i_0 = V_m I_m$$

# Concordia’s Transformation

- Concordia’s transformation is similar to Clarke’s transformation.
- The only difference is that Concordia’s transformation is power-invariant (i.e. the values of power before and after the transformation are identical.
- To have the power-invariant property, the transformation matrix must be orthogonal.
- A matrix is orthogonal if its inverse and its transpose are the same



# Concordia's Transformation

$$\left[\mathbf{f}_{\alpha\beta 0}\right] = \left[\mathbf{T}_{\alpha\beta 0}\right]\left[\mathbf{f}_{abc}\right]$$

$$\left[\mathbf{f}_{abc}\right] = \left[\mathbf{T}_{\alpha\beta 0}\right]^{-1}\left[\mathbf{f}_{\alpha\beta 0}\right]$$

$$\left[\mathbf{f}_{\alpha\beta 0}\right] = \begin{bmatrix} f_\alpha \\ f_\beta \\ f_0 \end{bmatrix}$$

$$\left[\mathbf{f}_{abc}\right] = \begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix}$$

$$\left[\mathbf{T}_{\alpha\beta 0}\right] = \sqrt{\frac{2}{3}}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\left[\mathbf{T}_{\alpha\beta 0}\right]^{-1} = \sqrt{\frac{2}{3}}\begin{bmatrix} 1 & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

# Concordia's Transformation

**Power-Invariant Property**

$$\begin{cases} v_a = V_m \cos(\omega t) \\ v_b = V_m \cos(\omega t - 2\pi/3) \\ v_c = V_m \cos(\omega t - 4\pi/3) \end{cases}$$

$$\begin{cases} i_a = I_m \cos(\omega t) \\ i_b = I_m \cos(\omega t - 2\pi/3) \\ i_c = I_m \cos(\omega t - 4\pi/3) \end{cases}$$

at ωt = 0

$$\begin{cases} v_a = V_m \\ v_b = \frac{-1}{2} V_m \\ v_c = \frac{-1}{2} V_m \end{cases}$$

$$\begin{cases} i_a = I_m \\ i_b = \frac{-1}{2} I_m \\ i_c = \frac{-1}{2} I_m \end{cases}$$

Three Phase Power

$$P = v_a\, i_a + v_b\, i_b + v_c\, i_c = \tfrac{3}{2} V_m I_m$$

Concordia's Transformed quantities

$$\begin{cases} v_\alpha = \sqrt{\tfrac{3}{2}} V_m \\ v_\beta = 0 \\ v_0 = 0 \end{cases}$$

$$\begin{cases} i_\alpha = \sqrt{\tfrac{3}{2}} I_m \\ i_\beta = 0 \\ i_0 = 0 \end{cases}$$

**Concordia's transformation is power-invariant**

Power in αβ frame

$$P = v_\alpha\, i_\alpha + v_\beta\, i_\beta + v_0\, i_0 = \tfrac{3}{2} V_m I_m$$

# Induction Motor Model in α-β frame

**Voltage Equations in the stationary reference frame**

$$v_{s\alpha} = R_s i_{s\alpha} + s\varphi_{s\alpha}$$
$$v_{s\beta} = R_s i_{s\beta} + s\varphi_{s\beta}$$
$$v_{r\alpha} = 0 = R_r i_{r\alpha} + s\varphi_{r\alpha} + \omega_r \varphi_{r\beta}$$
$$v_{r\beta} = 0 = R_r i_{r\beta} + s\varphi_{r\beta} - \omega_r \varphi_{r\alpha}$$

where 's' indicates the differential operator (d/dt)
[**NOTE: its instead of usual *p* operator. not jω**].

**stator and rotor fluxes equations**

$$\varphi_{s\alpha} = L_s i_{s\alpha} + L_m i_{r\alpha}$$
$$\varphi_{s\beta} = L_s i_{s\beta} + L_m i_{r\beta}$$
$$\varphi_{r\alpha} = L_r i_{r\alpha} + L_m i_{s\alpha}$$
$$\varphi_{r\beta} = L_r i_{r\beta} + L_m i_{s\beta}$$

*In these equations, Rs, Rr, Ls, and Lr are, respectively, the resistors and the inductances of the stator windings and the rotor windings, Lm is the mutual inductance and ωr=p.Ωr is the rotor speed (with p is the pairs poles number). Additionally, ωs is the synchronous frequency.*

# Induction Motor Model in α-β frame

**Electromagnetic torque** $$\Gamma_{em} = \frac{3}{2} p \left( \varphi_{s\alpha} i_{s\beta} - \varphi_{s\beta} i_{s\alpha} \right)$$

For the complete model of the induction machine, the flux expressions are replaced in the voltage equations. We obtain a mechanical equation and four electrical equations in terms of the stator currents, rotor fluxes components, and the electric speed of induction machine as well

$$\frac{di_{s\alpha}}{dt} = -\frac{1}{\sigma L_s}\left(R_s + \frac{1}{T_r}\frac{L_m^2}{L_r}\right) i_{s\alpha} + \frac{1}{\sigma L_s}\left(\frac{L_m}{L_r}\frac{1}{T_r}\right)\varphi_{r\alpha} + \frac{1}{\sigma L_s}\left(\frac{L_m}{L_r}\right)\omega_r \varphi_{r\beta}$$

$$\frac{di_{s\beta}}{dt} = -\frac{1}{\sigma L_s}\left(R_s + \frac{1}{T_r}\frac{L_m^2}{L_r}\right) i_{s\beta} - \frac{1}{\sigma L_s}\left(\frac{L_m}{L_r}\right)\omega_r \varphi_{r\alpha} + \frac{1}{\sigma L_s}\left(\frac{L_m}{L_r}\frac{1}{T_r}\right)\varphi_{r\beta}$$

$$\frac{d\varphi_{r\alpha}}{dt} = \frac{L_m}{T_r} i_{s\alpha} - \frac{1}{T_r}\varphi_{r\alpha} - \omega_r \varphi_{r\beta}$$

$$\frac{d\varphi_{r\beta}}{dt} = \frac{L_m}{T_r} i_{s\alpha} + \omega_r \varphi_{r\alpha} - \frac{1}{T_r}\varphi_{r\beta}$$

$$\omega_m = p\,\Omega_m ; \omega_r = [\omega_s - \omega_m]; \sigma = 1 - \frac{L_m^2}{L_s L_r}; T_r = \frac{L_r}{R_r}; T_s = \frac{L_s}{R_s}$$

# Induction Motor Model in α-β frame

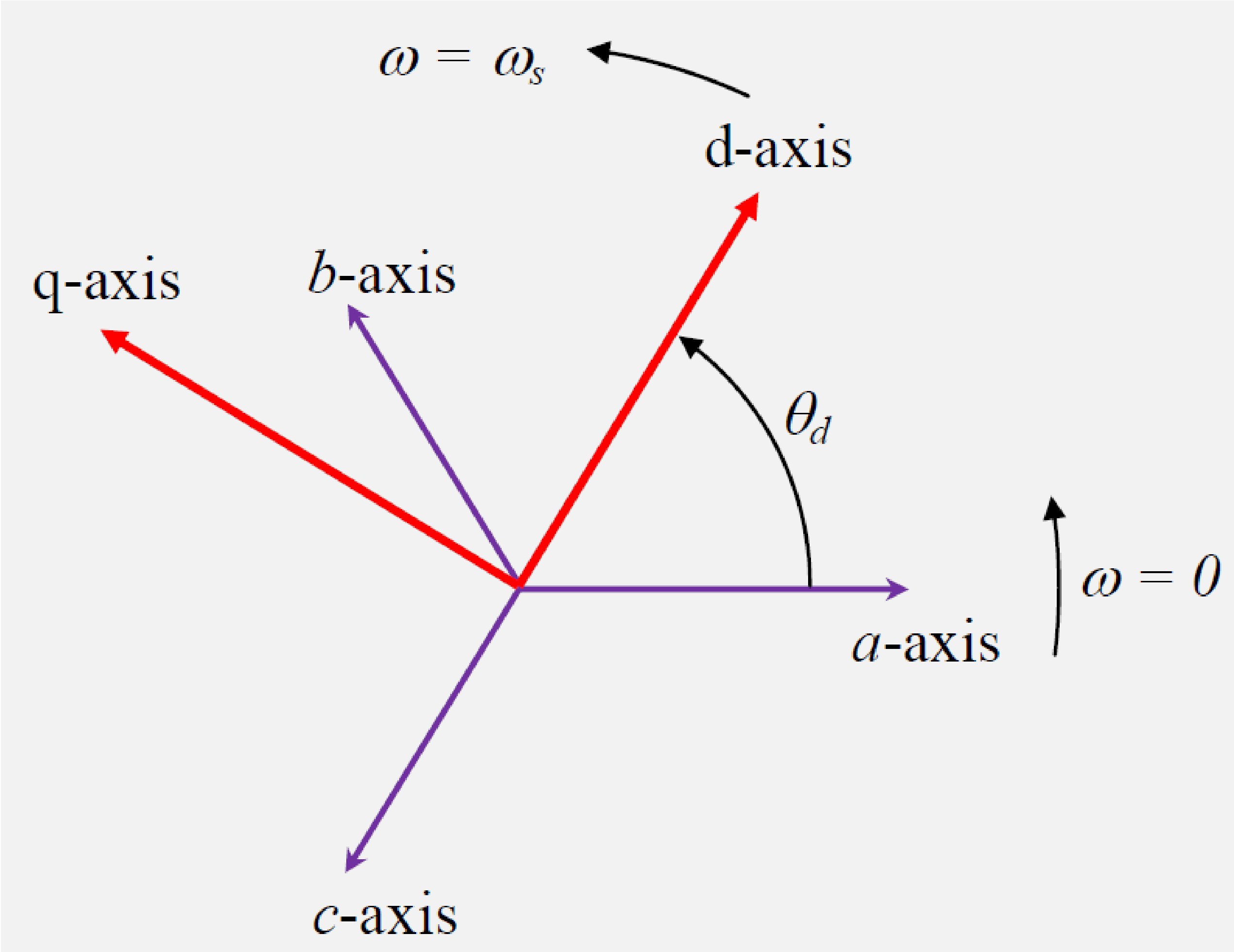
$$\Gamma_{em} - \Gamma_r = f\Omega_m + J\frac{d\Omega_m}{dt}$$

where Γem is the electromagnetic torque [N.m] and Γr is the resistive torque imposed by the machine shaft [N. m]. Ω: Mechanical speed, *f*: damping coefficient

**Inference:** **Modeling the machine in this way (α-β frame) that reduces the number of quantities that we need to know in order to simulate machine operation. In fact, only the instantaneous values of the stator voltages and the resistive torque must be determined in order to impose them on the machine. Therefore, we do not need to know the stator frequency value, or the slip as in the case of the model whose equations are written in the reference frame rotating in synchronism**

# Park's Transformation

The d-axis is leading the q-axis by 90 electrical degrees; and the angle between the d-axis w.r.t. the a-axis is used



# Park's Transformation

$$\boxed{\left[\mathbf{f}_{dq0}\right]=\left[\mathbf{T}_{dq0}(\theta_d)\right]\left[\mathbf{f}_{abc}\right]}$$

$$\boxed{\left[\mathbf{f}_{dq0}\right]=\begin{bmatrix} f_d \\ f_q \\ f_0 \end{bmatrix}} \qquad \boxed{\left[\mathbf{f}_{abc}\right]=\begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix}}$$

$$\boxed{\left[\mathbf{T}_{dq0}(\theta_d)\right]=\frac{2}{3}\begin{bmatrix} \cos\theta_d & \cos(\theta_d-2\pi/3) & \cos(\theta_d+2\pi/3) \\ -\sin\theta_d & -\sin(\theta_d-2\pi/3) & -\sin(\theta_d+2\pi/3) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}}$$

$$\boxed{\theta_d=\omega t+\theta_0}$$

**Park's transformation is not power-invariant. But it can be made power invariant by replacing 2/3 with √(2/3)**

# Induction Motor Model in d-q frame (Park's Frame) Synchronized to Rotating Field

**Voltage Equations**

$$v_{sd} = R_s i_{sd} + \frac{d\varphi_{sd}}{dt} - \omega_s \varphi_{sq}$$

$$v_{sq} = R_s i_{sq} + \frac{d\varphi_{sq}}{dt} + \omega_s \varphi_{sd}$$

$$v_{rd} = 0 = R_r i_{rd} + \frac{d\varphi_{rd}}{dt} - \omega_r \varphi_{rq}$$

$$v_{rq} = 0 = R_r i_{rq} + \frac{d\varphi_{rq}}{dt} + \omega_r \varphi_{rd}$$

$\omega_s$= Synchronous speed in electrical rad/sec

$\omega_r$ = slip speed in electrical rad/sec

# Induction Motor Model in d-q frame (Park's Frame) Synchronized to Machine Rotor

**Voltage Equations**

$$v_{sd} = R_s i_{sd} + \frac{d\varphi_{sd}}{dt} - \omega_s \varphi_{sq}$$

$$v_{sq} = R_s i_{sq} + \frac{d\varphi_{sq}}{dt} + \omega_s \varphi_{sd}$$

$$v_{rd} = 0 = R_r i_{rd} + \frac{d\varphi_{rd}}{dt}$$

$$v_{rq} = 0 = R_r i_{rq} + \frac{d\varphi_{rq}}{dt}$$

$\omega_s = \omega_m$ = rotor speed in electrical rad/sec

**Voltage Equations in the stationary reference frame**

$$v_{s\alpha} = R_s i_{s\alpha} + s\varphi_{s\alpha}$$
$$v_{s\beta} = R_s i_{s\beta} + s\varphi_{s\beta}$$
$$v_{r\alpha} = 0 = R_r i_{r\alpha} + s\varphi_{r\alpha} + \omega_r \varphi_{r\beta}$$
$$v_{r\beta} = 0 = R_r i_{r\beta} + s\varphi_{r\beta} - \omega_r \varphi_{r\alpha}$$

**Voltage Equations in the dq frame synchronized to mag field**

$$v_{sd} = R_s i_{sd} + \frac{d\varphi_{sd}}{dt} - \omega_s \varphi_{sq}$$
$$v_{sq} = R_s i_{sq} + \frac{d\varphi_{sq}}{dt} + \omega_s \varphi_{sd}$$
$$v_{rd} = 0 = R_r i_{rd} + \frac{d\varphi_{rd}}{dt} - \omega_r \varphi_{rq}$$
$$v_{rq} = 0 = R_r i_{rq} + \frac{d\varphi_{rq}}{dt} + \omega_r \varphi_{rd}$$

**Voltage Equations in the dq frame synchronized to rotor**

$$v_{sd} = R_s i_{sd} + \frac{d\varphi_{sd}}{dt} - \omega_s \varphi_{sq}$$
$$v_{sq} = R_s i_{sq} + \frac{d\varphi_{sq}}{dt} + \omega_s \varphi_{sd}$$
$$v_{rd} = 0 = R_r i_{rd} + \frac{d\varphi_{rd}}{dt}$$
$$v_{rq} = 0 = R_r i_{rq} + \frac{d\varphi_{rq}}{dt}$$

# Tutorial #1

**An induction motor specifications are as follows; 5 hp, 200V, 3-phase star connected 4 pole 60 Hz. Rs=0.277Ω, Rr=0.183 Ω, Lm=0.0538 H, Ls=0.0553 H, Lr=0.056 H. Calculate stator and rotor currents using stator-reference frame model. $\omega_r$=140 rad/sec**

$$\boxed{\left[\mathbf{T}_{\alpha\beta 0}\right] = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}}$$

$$\boxed{\left[\mathbf{T}_{\alpha\beta 0}\right]^{-1} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \end{bmatrix}}$$

$$\begin{aligned} v_{s\alpha} &= R_s i_{s\alpha} + s\varphi_{s\alpha} \\ v_{s\beta} &= R_s i_{s\beta} + s\varphi_{s\beta} \\ v_{r\alpha} &= 0 = R_r i_{r\alpha} + s\varphi_{r\alpha} + \omega_r \varphi_{r\beta} \\ v_{r\beta} &= 0 = R_r i_{r\beta} + s\varphi_{r\beta} - \omega_r \varphi_{r\alpha} \end{aligned}$$

# Tutorial #2

**Assignment Q2**
**Marks: 10**
**Last date: before 12.12.2023**

**Calculate electromagnetic torque generated in an induction motor with specifications as follows; 2000 hp, 1400 RPM 2300V, 3-phase star connected 4 pole 60 Hz, full load slip of 0.03746. Rs=0.02Ω, Rr=0.12 Ω, Xm=50 Ω, Xls=Xlr=0.32 Ω. Use synchronous reference frame**

$$\boxed{\left[\mathbf{T}_{dq0}(\theta_d)\right]=\frac{2}{3}\begin{bmatrix} \cos\theta_d & \cos\left(\theta_d-2\pi/3\right) & \cos\left(\theta_d+2\pi/3\right) \\ -\sin\theta_d & -\sin\left(\theta_d-2\pi/3\right) & -\sin\left(\theta_d+2\pi/3\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}}$$

$$\boxed{\theta_d=\omega t+\theta_0}$$

**Te= 3/2 p (λds iqs- λqs ids)**

**p=P/2=Pole pairs**

$$v_{sd}=R_s i_{sd}+\frac{d\varphi_{sd}}{dt}-\omega_s\varphi_{sq}$$

$$v_{sq}=R_s i_{sq}+\frac{d\varphi_{sq}}{dt}+\omega_s\varphi_{sd}$$

$$v_{rd}=0=R_r i_{rd}+\frac{d\varphi_{rd}}{dt}-\omega_r\varphi_{rq}$$

$$v_{rq}=0=R_r i_{rq}+\frac{d\varphi_{rq}}{dt}+\omega_r\varphi_{rd}$$

# Nonlinearities In Machine Equations

## Saturation

Saturation of machine causes lower inductance and hence lower flux linkage. It can also generate more higher order harmonics due to current waveform distortion. The inductance and torque matrices have been assumed to have constant coefficients in Park's axes, but they are no longer constant when we consider saturation, their values depending on various currents. During disturbances the currents may vary over wide ranges. In computation we have to start with a certain set of inductances and then alter their values in short time steps as the calculation proceeds, according to the saturation characteristic of the magnetic path.

## Space Harmonics

In most studies relating to synchronous machines, it is assumed that the flux wave is sinusoidally distributed in space. This assumption is reasonable for induction motors and cylindrical-rotor synchronous machines of good design. It is not always a valid assumption, however, and the effects of space harmonics, which produce voltage time-harmonics, electrical noise and parasitic torques, must in certain cases be taken into account.