* Char. of ideal switching device:—

1.) ON state

↳ it must ~~of~~ have ability to carry high forward Current.

$$I_f \to \infty$$

↳ $V_f \to 0$

↳ $R_f \to 0$

↳ $P_{ON} \to 0$

2.) Off state

- $V_{BR} \to \infty$ (it must withstand high forward and Reverse voltages.)

$I_{off} \to 0$

$R_{off} \to \infty$

$P_{off} \to 0$

3.) $t_{on} \to 0$

$t_{off} \to 0$

turn on and turn off process must occur instantaneously so that it can be used for high freqⁿ applications.

4.) for turn and turn off process,

$P_G \to 0$ (low gate drive power)

$V_G \to 0$ (low gate drive voltage)

$I_G \to 0$ (low gate drive current.)

5.) Both turn on and turn off must be controllable.

it must turn on with a +ve gate signal

must be turned off with -ve (or) zero gate signal.

6.) For turning on and off, it should Required a pulse signal only.

A small

7.)

8.) Must have high $\frac{dv}{dt} \to \infty$

9.) Must have high $\frac{di}{dt} \to \infty$
it must be capable of having

10.) it must have the ability to sustain any fault current for a long time period
$\hookrightarrow i^2 t \to 0$

11.) Low price.

Classification of power electronics converter.
On the Basis of i/p & o/p side power condition ____

1.) ac-dc converters (Rectifier)

a.) Uncontrolled
fixed voltage ac $\longrightarrow$ fixed voltage dc

b.) Controlled

2.) ac — ac converter (ac choppers

3.) dc - dc converters (Choppers)

Fixed dc voltage to controlled dc voltage

4.) dc - ac converters (inverter)

uncontrolled dc voltage to controlled ac voltage & frequency.

5.) Static switches (contactors.)

Classification (on the basis of switching of devices)

1.) Line commutated (naturally commutated) converters

2.) switching (forced commutated) converters

3.) resonant converters (zero voltage / zero current switching)

* outline of the course

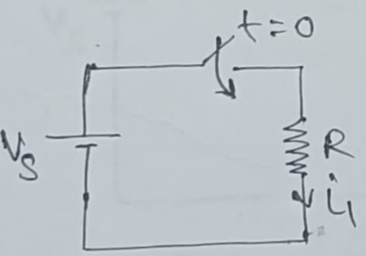
1.) converter circuits

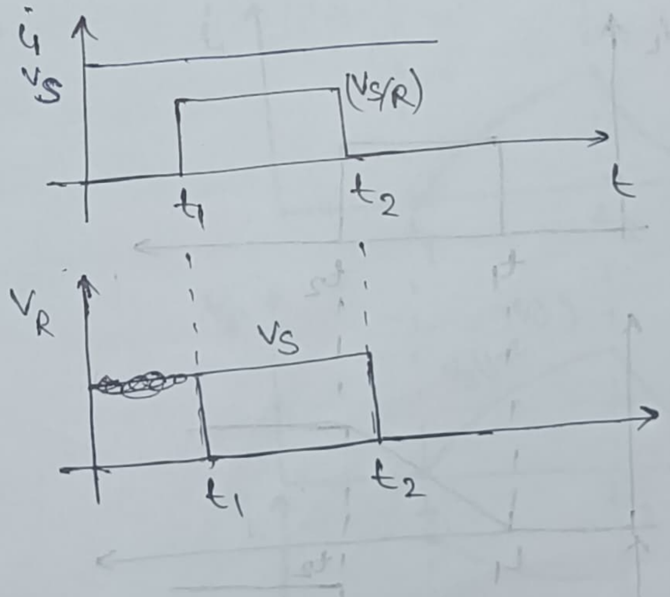
ac - dc converters

dc - ac converters.

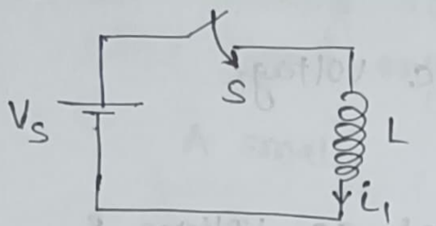
* R - ckt.

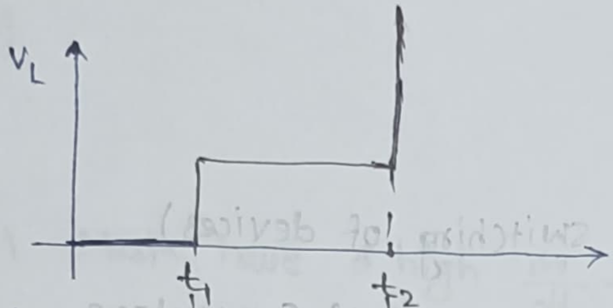
switch is closed at $t = t_1$ and opened at $t = t_2$

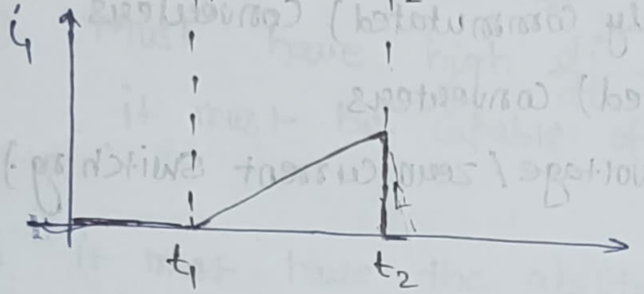


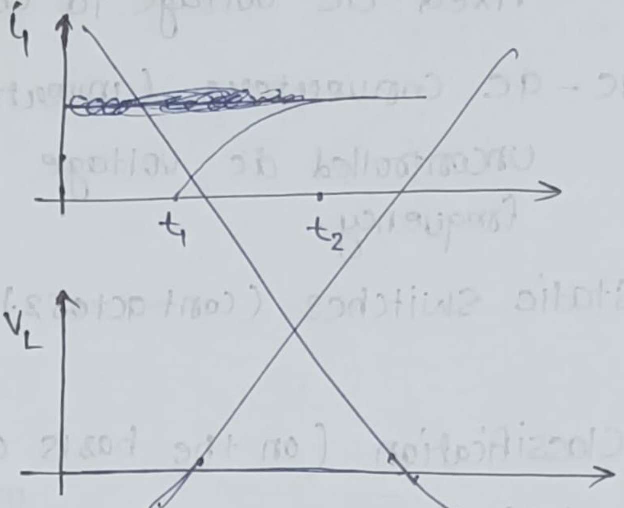


L-ckt.









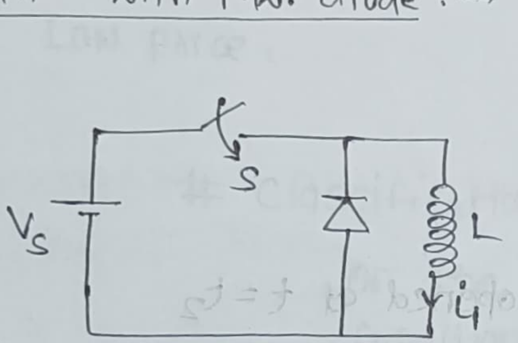
* at $t = t_2$ current becomes instantly zero.

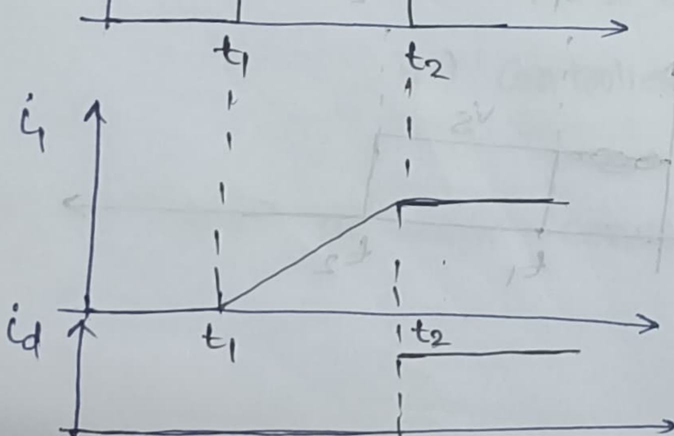
So, $\left(\frac{di}{dt}\right) \rightarrow$ $-ve$ and very large.

$V_L \rightarrow \infty$

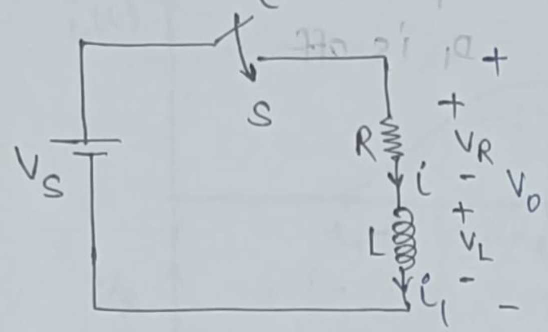
$\frac{di}{dt} = \frac{I}{\sim 0}$

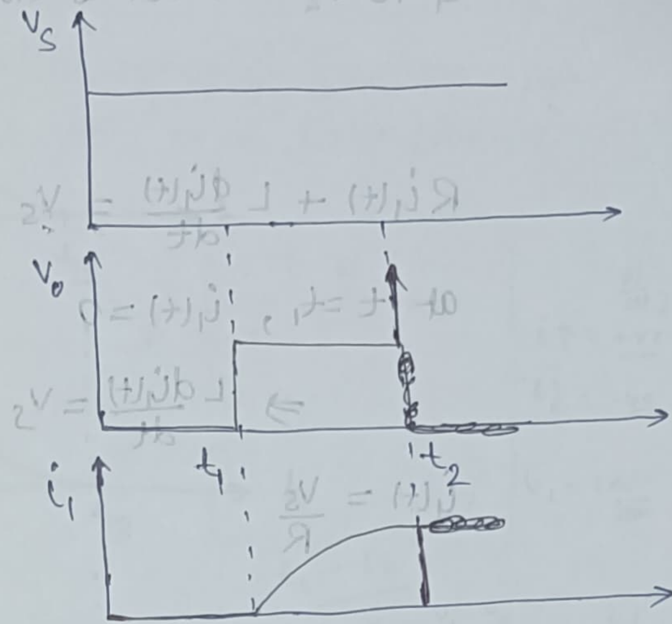
L-Kt. With F.W. diode :-





R-L ckt.





$V_0 = V_R + V_L$

at $t = t_1$

$V_S = V_R + V_L = V_0$

$V_0 = V_S = R\,i(t) + L\dfrac{di(t)}{dt}$

$\Rightarrow V_S = R\,I(S) + L(S\,I(S) - I(0))$

$I(S)[R + LS] = V_S$

$I(S) = \dfrac{V_S}{S + (R/L)}$

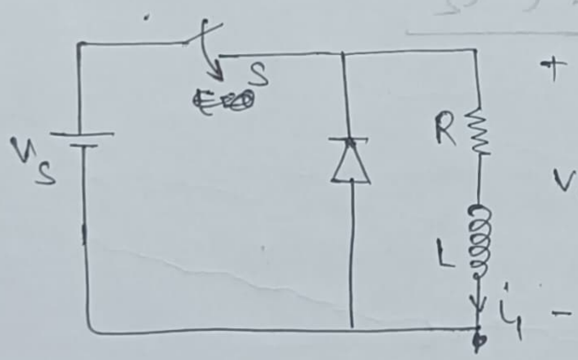
$i(0^-) = i(0^+) = \left(\dfrac{V_S}{R}\right)$

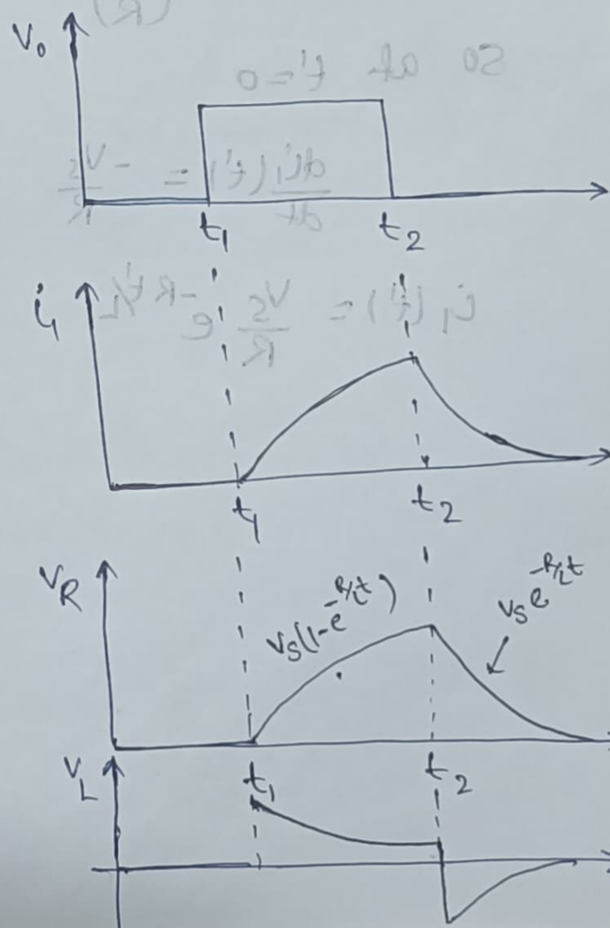
$i(t) = i(\infty) + [i(0^+) - i(\infty)]\,e^{-t/\tau}$

$\cancel{V_S = R\,I(S) + L\,S\,I(S)}$

$\cancel{I_S\,(R + LS)}\ S$

$\Rightarrow \quad i(t) = \dfrac{V_S}{R}\left(1 - e^{-R/L\,t}\right)$





- for $t > t_2$

  Diode is conducting —

  $V_0 = 0$

  $R\,i'(t) + L\dfrac{di'(t)}{dt} = 0$

  $\Rightarrow \quad i'(t) = \dfrac{V_S}{R}\,e^{-R/L\,t}$

t_1 to t_2 switch is closed $\quad V_{D_1} = -V_S$

D_1 is off

$$R i_1(t) + L \frac{di_1(t)}{dt} = V_S$$

at $t = t_1$, $i_1(t) = 0$

$$\Rightarrow L \frac{di_1(t)}{dt} = V_S$$

$$i_1(t) = \frac{V_S}{R}$$

For $t > t_2$ switch is open

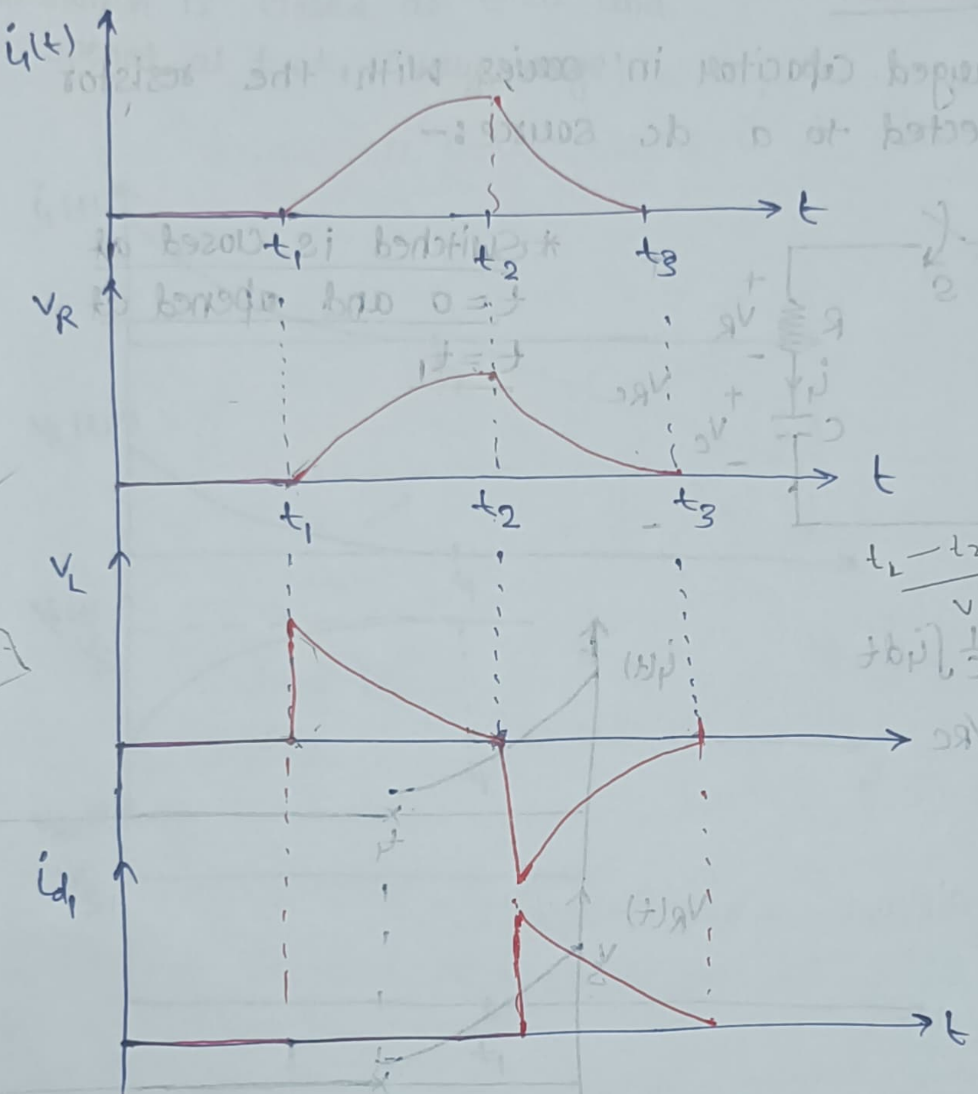
$$R i_1(t') + L \frac{di_1(t')}{dt} = 0$$

$$i_1(t'=0) = \left(\frac{V_S}{R}\right)$$

so at $t'=0$ $\qquad t' = t - t_2$

$$\frac{di_1(t')}{dt} = -\frac{V_S}{R}$$

$$i_1(t') = \frac{V_S}{R} e^{-Rt'/L}$$



$\frac{di}{dt}$ 2

$i\uparrow = \underline{+ve}$

$i\downarrow = \underline{-ve}$

$V_L = L\frac{di}{dt}$

$\underline{t_1 - t_2}$

$V_L = V_s - V_R$

$i\downarrow$

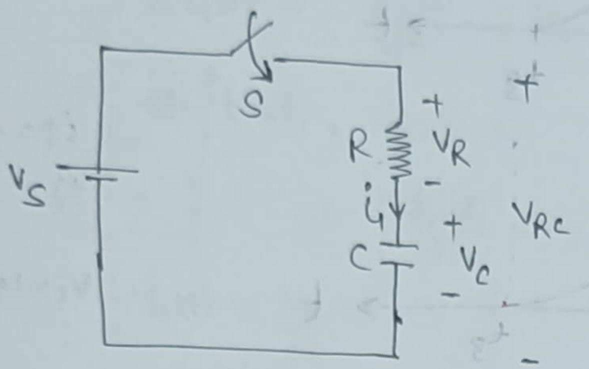
$L\frac{di}{dt} = -ve$

$\frac{di}{dt}$

$V_L = -V_R$

* R-C circuit :-

• An uncharged capacitor in series with the resistor is connected to a dc source:-



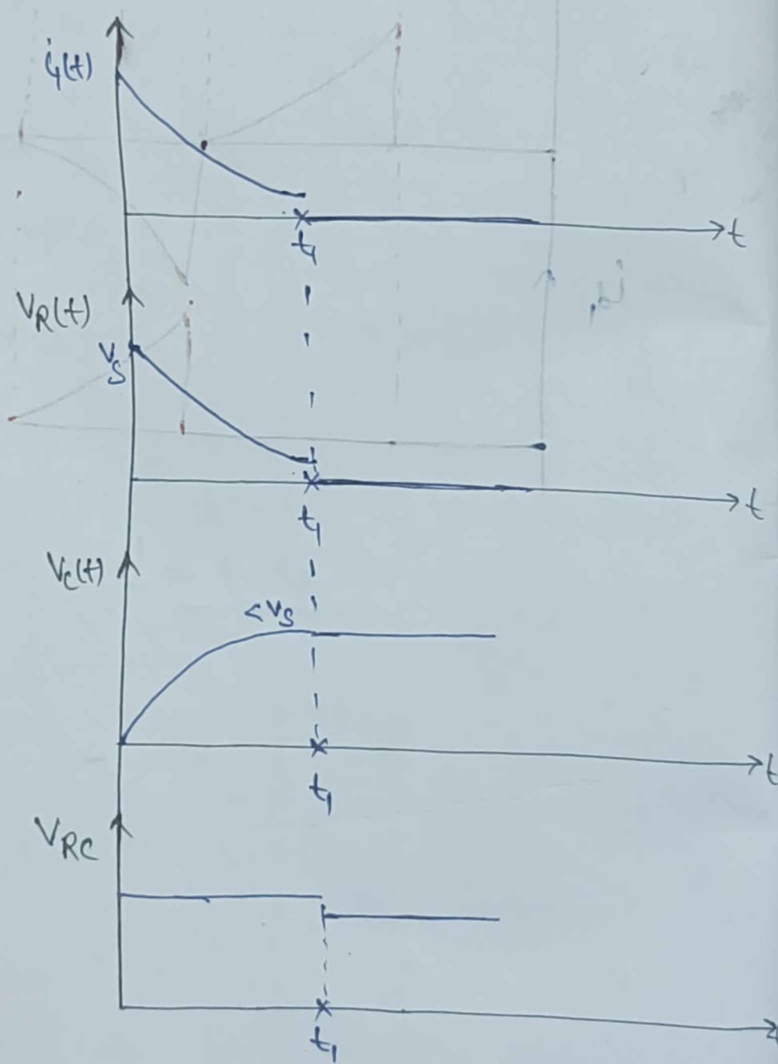
* Switched is closed at $t=0$ and opened at $t=t_1$

$$V_s = Ri_1 + \frac{1}{C}\int i_1 dt$$

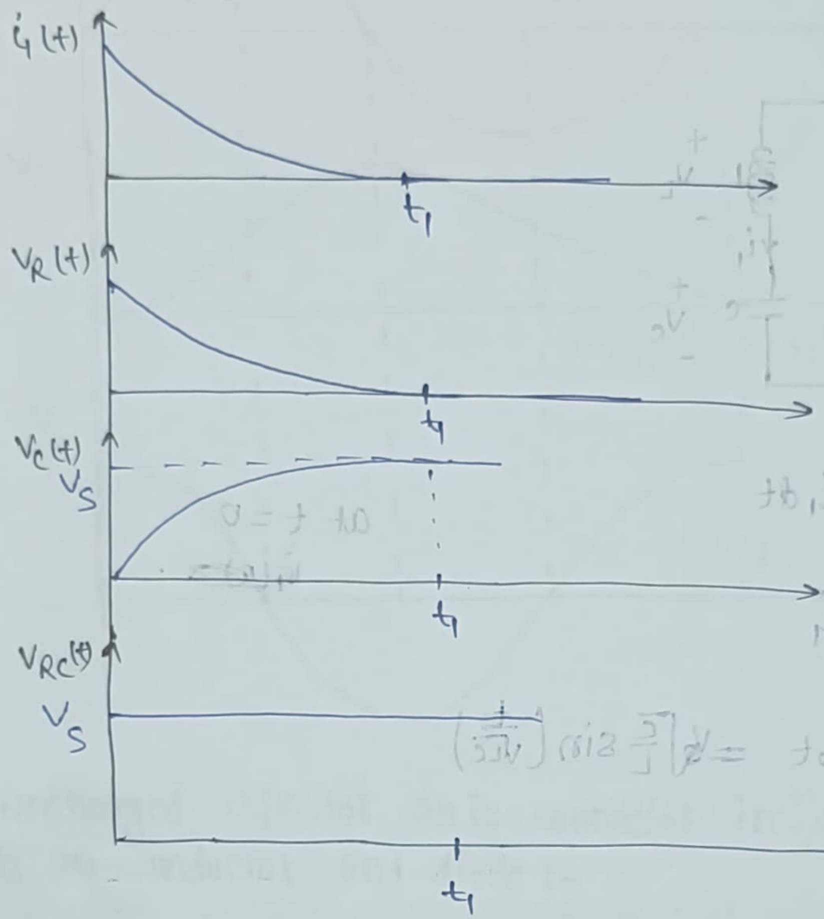
$$i_1 = \frac{V_s}{R} e^{-t/RC}$$

$$V_c = V_s - V_R$$

$$V_R = Ri_1$$

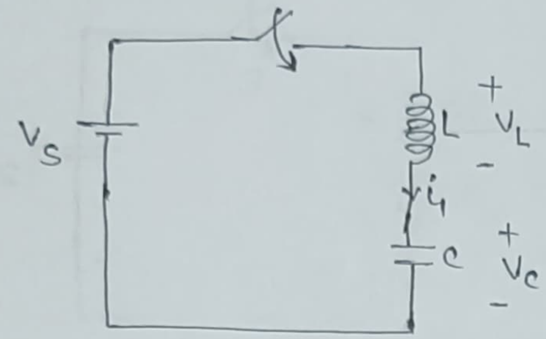


* switch is closed at $t=0$ and opened at $t=t_1$ (very large)



* L-C circuit :-

An uncharged capacitor in series with an inductor is connected to a dc source.



$$V_S = L\frac{di_1}{dt} + \frac{1}{C}\int i_1\, dt$$

at $t = 0$

$$0 = \frac{d^2 i_1}{dt^2} + \frac{1}{LC} i_1$$

$$i_1(t) = V_S\sqrt{\frac{C}{L}}\sin\omega_0 t = V_S\sqrt{\frac{C}{L}}\sin\left(\frac{t}{\sqrt{LC}}\right)$$

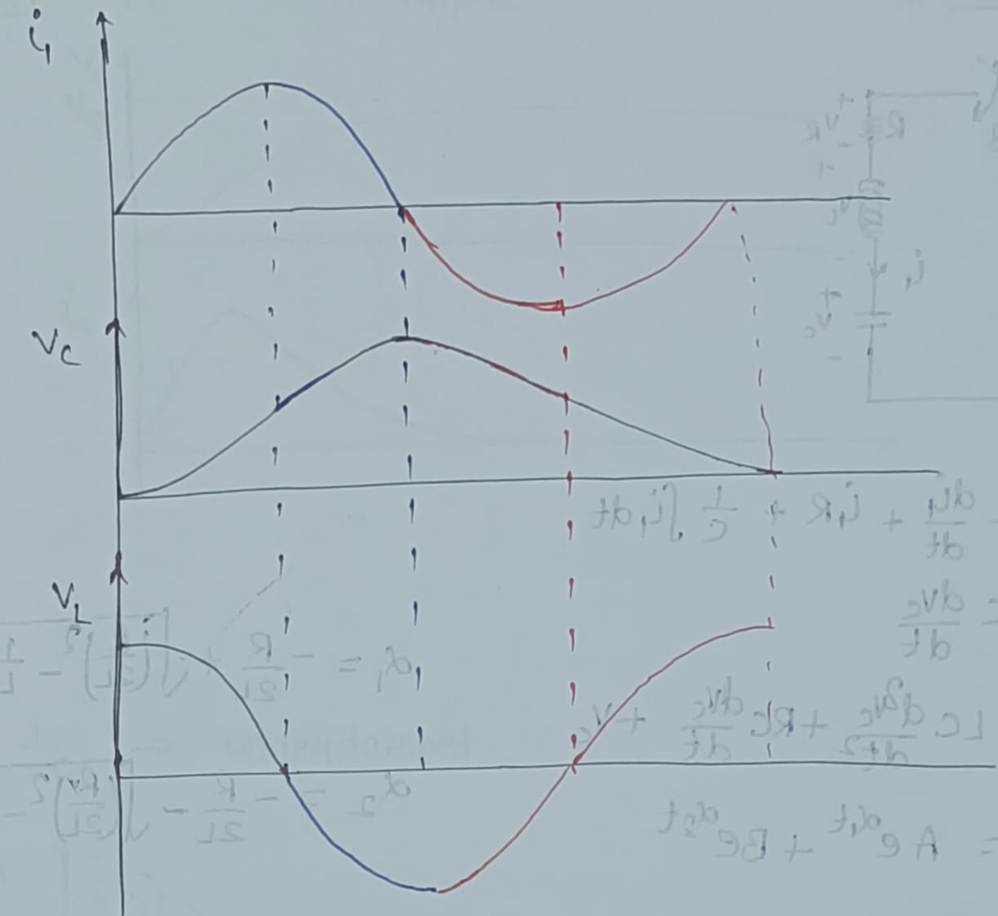
$$V_L(t) = L\frac{di_1(t)}{dt}$$

$$= \cancel{L}\cdot\left\{V_S\cdot\sqrt{\frac{C}{L}}\cdot\frac{1}{\sqrt{LC}}\cos\left(\frac{t}{\sqrt{LC}}\right)\right\}$$

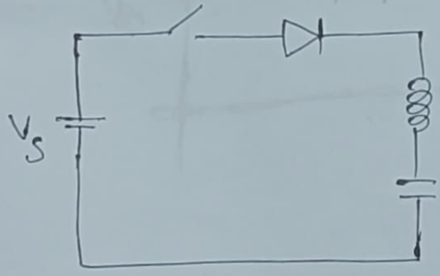
$$= V_S\cos\omega_0 t = V_S\cos\left(\frac{1}{\sqrt{LC}}\right)t$$

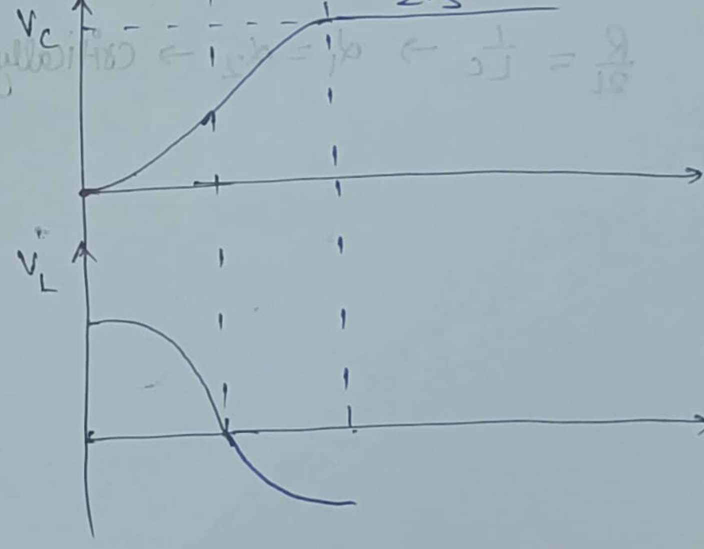
$$V_C(t) = V_S - V_L$$

$$= V_S(1 - \cos\omega_0 t) = V_S\left(1 - \cos\frac{1}{\sqrt{LC}}t\right)$$

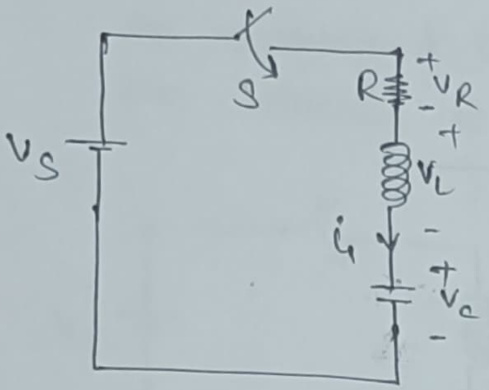


* An uncharged capacitor ~~an~~ is connected in series with an inductor and diode :-





* R-L-C ckt :-



$$V_S = L\frac{di_1}{dt} + i_1 R + \frac{1}{c}\int i_1 dt$$

$$i_1 = c\frac{dV_c}{dt}$$

$$V_S = LC\frac{d^2V_c}{dt^2} + RC\frac{dV_c}{dt} + V_c$$

$$V_C = Ae^{\alpha_1 t} + Be^{\alpha_2 t}$$

$$\alpha_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$\alpha_2 = -\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

there are three conditions

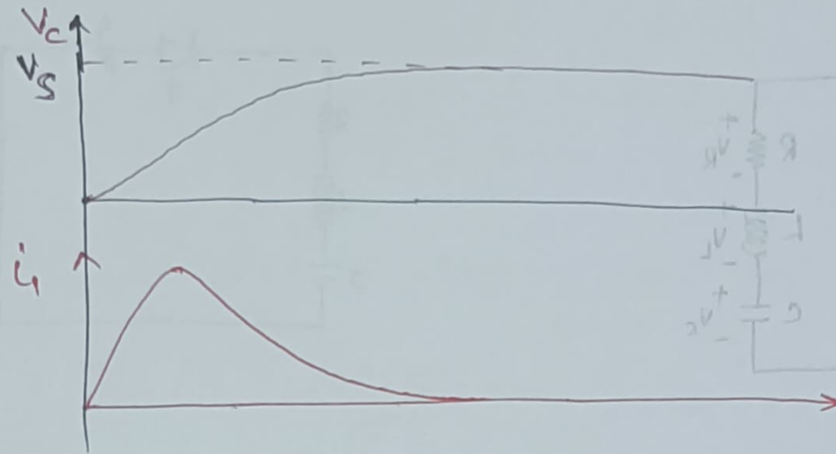
(1.) $\frac{R}{2L} = \frac{1}{LC} \rightarrow \alpha_1 = \alpha_2 \rightarrow$ critically damped

(2) $\frac{R}{2L} > \frac{1}{LC} \longrightarrow$ overdamped.

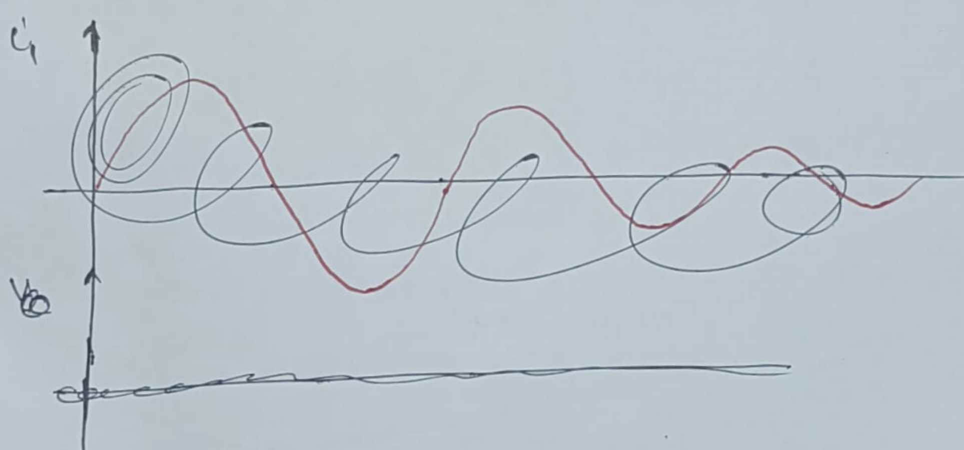
(3) $\frac{R}{2L} < \frac{1}{LC} \longrightarrow$ underdamped.

$\frac{R}{2L} = \frac{1}{LC} \rightarrow \alpha_1 = \alpha_2 \rightarrow$ critically damped.

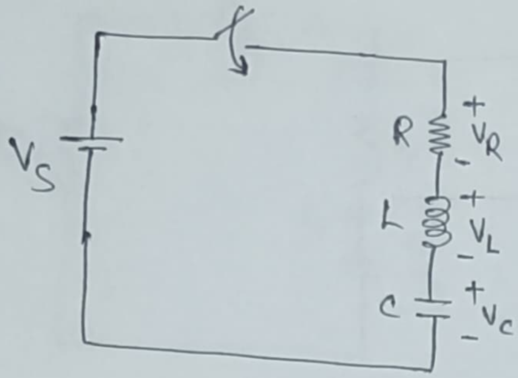
$\frac{R}{2L} > \frac{1}{LC} \rightarrow$ overdamped.



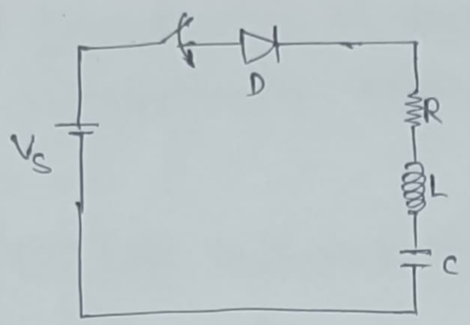
$\frac{R}{2L} < \frac{1}{LC} \rightarrow$ underdamped



* R-L-C circuit



* R-L-C circuit with diode :-



04/09/23

Rectifiers :-

* Classification of Rectifiers

① Uncontrolled Rectifiers :- Line freq^n AC is converted into fixed voltage dc. (uses dide)

② Fully controlled Rectifiers :- Line freq^n AC is converted into variable voltage dc. (used devices are SCRs, IGBT etc)

③ Half controlled Converters :-

Line freq^n AC $\longrightarrow$ variable voltage dc. (used device diode, SCRs etc)

* Uncontrolled and halfcontrolled converters :-

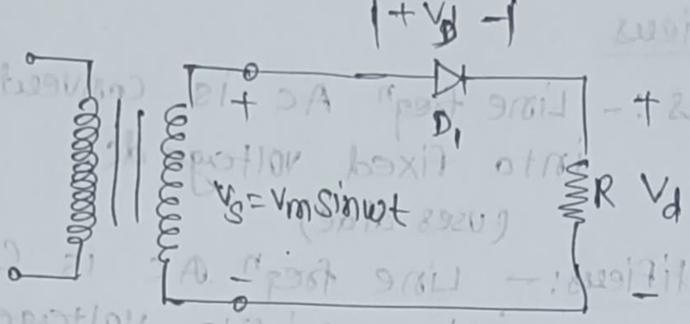
↳ avg. o/p voltage always +ve.
↳ power flow is from ~~dc~~ ac source to dc load.
↳ unidirectional converters.
↳ operating points lies in the 1st quad. of avg. voltage V_d → avg. current I_d plane.
↳ single quadrant converters.

• Uncontrolled Rectifier :-

Appl^n :-

↳ switching power supplies
↳ ac motor drives
↳ dc motor drives
↳ Battery chargers
↳ electrochemical process.
↳ HVDC transmission.

* 1-Φ HWR - R load



$V_s = V_m \sin\omega t = \sqrt{2} V_s \sin\omega t$

$V_s \rightarrow$ RMS value of source voltage

0 — π ; D_1 conducts.

$V_d(\omega t) = V_s(\omega t)$

$i_d(\omega t) = \frac{V_d(\omega t)}{R} = i_s(\omega t)$

Diode voltage

$V_{D_1}(\omega t) = 0$

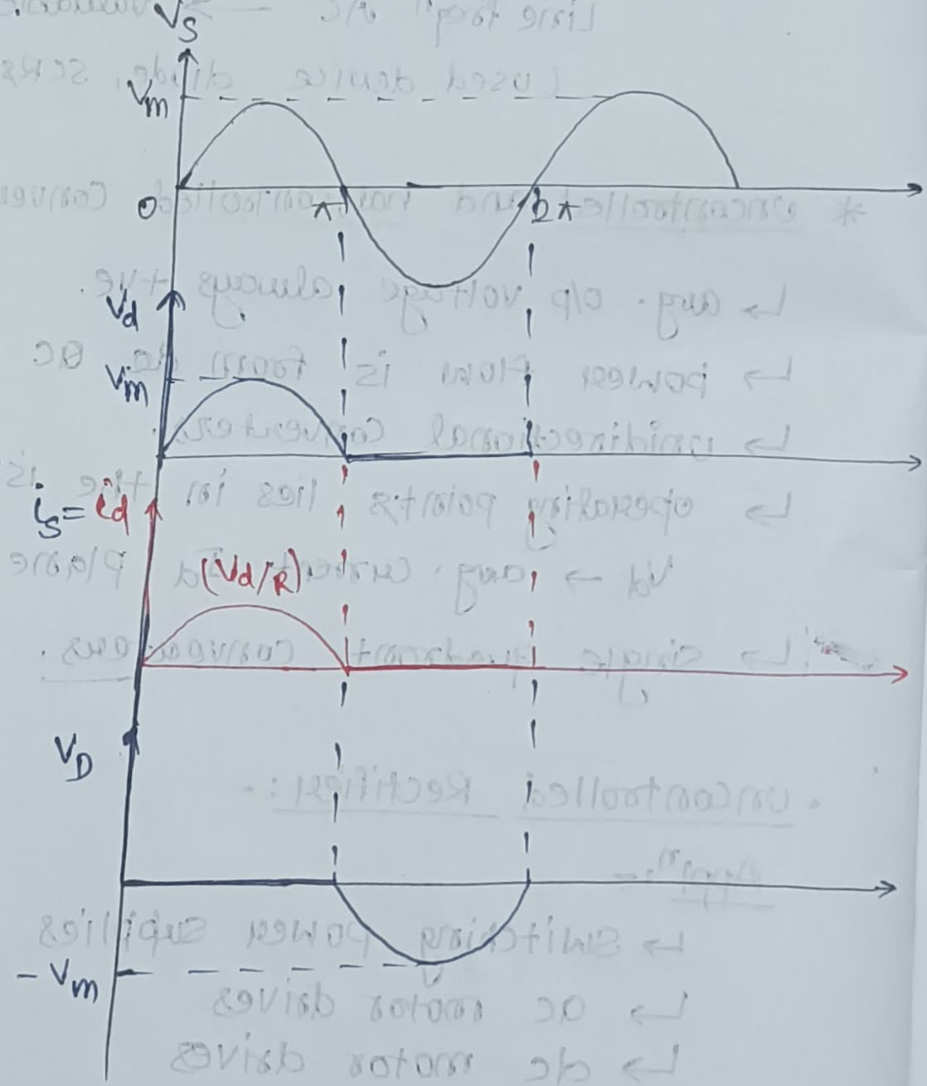
π - 2π D_1 is off

$i_d(\omega t) = 0$

$V_d(\omega t) = 0$

$i_s(\omega t) = 0$

$V_{D_1}(\omega t) = V_s(\omega t)$



Performance parameters :-

- Avg. value of output voltage $(V_d)_{avg}$.

$$(V_d)_{avg} = \frac{1}{2\pi}\int_0^{2\pi} V_d \, d\omega t$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} V_s \, d\omega t + \int_{\pi}^{2\pi} 0 \cdot d\omega t\right]$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} V_m \sin\omega t \, d\omega t\right]$$

$$= \frac{1}{2\pi}\left[V_m \left[-\cos\omega t\right]_0^{\pi}\right]$$

$$= \frac{1}{2\pi}\left[V_m - (-1-1)\right]$$

$$\boxed{(V_d)_{avg} = \left(\frac{V_m}{\pi}\right)}$$

- Avg. Load current $(I_d)_{avg}$.

$$(I_d)_{avg} = \frac{\sqrt{2}\, V_s}{\pi R} = \text{Avg. current Rating of diode} = \left(\frac{V_m}{\pi R}\right)$$

- peak load current $= \frac{\sqrt{2}\, V_s}{R} = \frac{V_m}{R}$

↳ peak current Rating of diode $= \frac{\sqrt{2} V_s}{R}$

* RMS load voltage $(V_d)_{rms}$

$$(V_d)_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} (V_d)^2 d\omega t} = \left(\frac{V_s}{\sqrt{2}}\right) = \frac{V_m}{2}$$

* Rms load current

$$I_{dRms} = \left(\frac{V_s}{\sqrt{2} R}\right) = \frac{V_m}{2R}$$

- **load current form factor**

$$\boxed{\text{load F.F} \quad \frac{(I_d)_{rms}}{(I_d)_{avg}} = 1.54}$$

$$\frac{\frac{V_m}{2R}}{\frac{V_m}{\pi R}} = \left(\frac{\pi}{2}\right) = 1.57$$

- **RMS source current**

$$(I_s)_{rms} = \left(\frac{V_s}{\sqrt{2}R}\right)$$

- **DC power delivered to the load**

$$P_{dc} = (V_d)_{avg} * (I_d)_{avg} = \frac{\sqrt{2}V_s}{\pi} * \frac{\sqrt{2}V_s}{\pi R}$$

$$= \frac{V_m}{\pi} * \frac{V_m}{\pi R}$$

- **Transformer utility factor 'TUF'**

DC power delivered / Transformer power Rating.

↳ If transformer RMS current Rating is same as $(I_d)_{rms}$, then —

$$TUF = \frac{(P_{dc})}{V_s * (I_d)_{rms}}$$

$$= \frac{\frac{\sqrt{2}V_s}{\pi} \cdot \frac{\sqrt{2}V_s}{\pi R}}{V_s * \frac{V_s}{\sqrt{2}R}}$$

$$= 29\%$$

↳ power Rating of transformer must be greater by $1/0.29 = (3.5)$ times the dc load power Rating.

- Rectifier efficiency

$$= \frac{\text{dc side load power}}{(\text{ac load power} + \text{rectifier loss})} = \frac{P_{dc}}{(P_{ac}) + P_{rec}}$$

↳ diode loss

$$= \frac{\text{dc side load power}}{(\text{ac side power} + \text{rectifier loss})}$$

$$= \frac{(I_d)^2_{avg} * R}{(I_d)^2_{RMS} (R+R_f)}$$

$$= \frac{\left(\frac{\sqrt{2} V_s}{\pi R}\right)^2 * R}{\left(\frac{V_s}{\sqrt{2} R}\right)^2 (R+R_f)}$$

$$= \frac{\left(\frac{4}{\pi^2}\right)}{\left(1+\frac{R_f}{R}\right)}$$

$$= \frac{40.6}{\left(1+\frac{R_f}{R}\right)} \%$$

For $R_f = 0$ Rectification $\eta = 40.6\%$.

- Ripple Factor γ:

$$\gamma = \frac{\text{RMS value of ripple content in load voltage}}{(V_d)_{avg}}$$

$$= \frac{\sqrt{V_{dRMS}^2 - V_{davg}^2}}{(V_d)_{avg}}$$

$$RF = \frac{V_{ac}}{V_{dc}} = \frac{\sqrt{V_{rms}^2 - V_{avg}^2}}{V_{avg}}$$

$$= \sqrt{(FF)^2 - 1}$$

$$FF = \frac{rms}{avg}$$

$$\gamma = 1.21$$

- <u>Input source PF</u> $=$ $\dfrac{\text{Source side actual power}}{\text{Source side apparent power}}$

TUF

$$= \frac{P_{dc}}{\text{source side RMS voltage} \times \text{RMS current}}$$

$$PF = \frac{P_{act}}{P_{app}} = \frac{V_{rms} I_{rms}}{V_S I_S}$$

$$= \frac{\dfrac{2V_s^2}{\pi^2 R}}{\cancel{\sqrt{2}}V_s \times \cancel{\dfrac{\sqrt{2}V_s}{R}}\ \dfrac{V_s}{\sqrt{2}R}}$$

$$\frac{\left(\frac{V_m}{\pi}\right)^2 \frac{1}{R}}{\frac{V_m}{\sqrt{2}} \times \left(\frac{V_m}{2R}\right)} = \left(\frac{2\sqrt{2}}{\pi^2}\right) = 0.286$$

$$= \frac{2\sqrt{2}}{\pi^2}$$

$$= \underline{0.286}$$

- <u>Peak inverse voltage PIV:</u>
  - Maximum instantaneous voltage that appears accross the diode during the blocking time

$$= \sqrt{2}V_s = \underline{V_m}$$

- Input source PF = $\frac{\text{Source side actual power}}{\text{Source side apparent power}}$

TUF

$= \frac{P_{dc}}{\text{Source side RMS voltage} \times \text{RMS current}}$

$PF = \frac{P_{ac}}{P_{app}} = \frac{V_{rms} I_{rms}}{V_S I_S}$

$= \frac{\frac{2V_S^2}{\pi^2 R}}{\cancel{\sqrt{2}}V_S * \frac{V_S}{\sqrt{2}R}}$

$\frac{\left(\frac{V_m}{\pi}\right)^2 \times \left(\frac{1}{R}\right)}{\frac{V_m}{\sqrt{2}} * \left(\frac{V_m}{2R}\right)}$

$= \frac{2\sqrt{2}}{\pi^2}$

$= \left(\frac{2\sqrt{2}}{\pi^2}\right)$

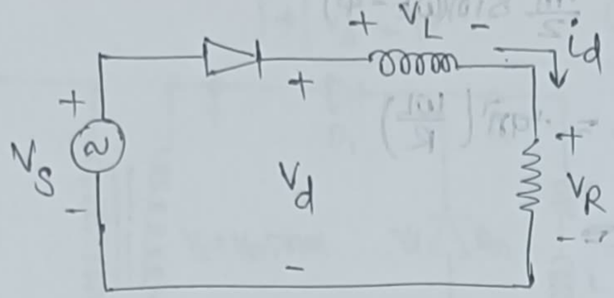
$= 0.286$

$= 0.286$

- Peak inverse voltage PIV:
  - Maximum instantaneous voltage that appears accross the diode during the blocking time

    $= \sqrt{2}V_S = V_m$

* 1-Φ HW Uncontrolled Rectifier with R-L load :-



during positive half cycle

diode → on

$V_d = V_s = \sqrt{2} V_s \sin\omega t$

$= R i_d(t) + L \frac{d i_d(t)}{dt}$

$i_d(0) = 0$

& $L \cdot \frac{d i_d(0)}{dt} = 0$

$\Rightarrow V_d = V_s = R i_d(t) + L \frac{d i_d(t)}{dt}$

$$\boxed{\begin{aligned} V_s(S) &= R I_d(S) + LS I_d(S) \\ V_s(S) &= I_d(S)[R + LS] \\ I_d(S) &= \frac{V_s(S)}{R+LS} \\ &= \frac{V_s(S)/L}{(S+\frac{R}{L})} \\ &= V_s(1 - e^{-R/L\, t}) \end{aligned}}$$

In case of RL load,

$i(t) = C e^{-t/\tau} + \frac{V_m}{|Z|}\sin(\omega t - \phi)$

by using initial condⁿ $i(0) = 0$

$0 = C + \frac{V_m}{|Z|}\sin(-\phi)$

$\Rightarrow C = \frac{V_m}{|Z|}\sin\phi$

$\therefore i(t) = \frac{V_m}{|Z|}\sin\phi\, e^{-t/\tau} + \frac{V_m}{|Z|}\sin(\omega t - \phi)$

$\left.\begin{aligned} |Z| &= \sqrt{R^2 + (\omega L)^2} \\ \phi &= \tan^{-1}\left(\frac{\omega L}{R}\right) \end{aligned}\right\}$

$\frac{V_m \omega}{S^2 + \omega^2} = R I_d(S) + LS I_d(S)$

$I_d(S) = \frac{V_m \omega}{(S^2 + \omega^2)(R + LS)}$

$I_d(S) = \frac{K_1}{R+LS} + \frac{K_2}{(S+j\omega)} + \frac{K_3}{(S-j\omega)}$

$\Rightarrow i_d(t) = \frac{V_m}{Z}\sin\phi \cdot e^{-R/L\, t} - \frac{V_m}{Z}\sin(\omega t - \phi)$

$Z = \sqrt{R^2 + (\omega L)^2}$

$$\Rightarrow \boxed{i_d(t) = \frac{V_m}{Z}\sin\phi\, e^{-\frac{R}{L}t} - \frac{V_m}{Z}\sin(\omega t-\phi)}$$

$$Z = \sqrt{R^2+(\omega L)^2} \quad ; \quad \phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$V_R(t) = i_d(t) * R$$

$$V_L(t) = L\,\frac{di_d(t)}{dt} = V_d(t) - V_R(t)$$

0 to t_3

$$L\frac{di_d(t)}{dt} = V_L$$

$$\int_0^{t_3} di_d(t) = \int_0^{t_3}\frac{V_L}{L}dt$$

$$i_d\big|_{t=0} = 0$$

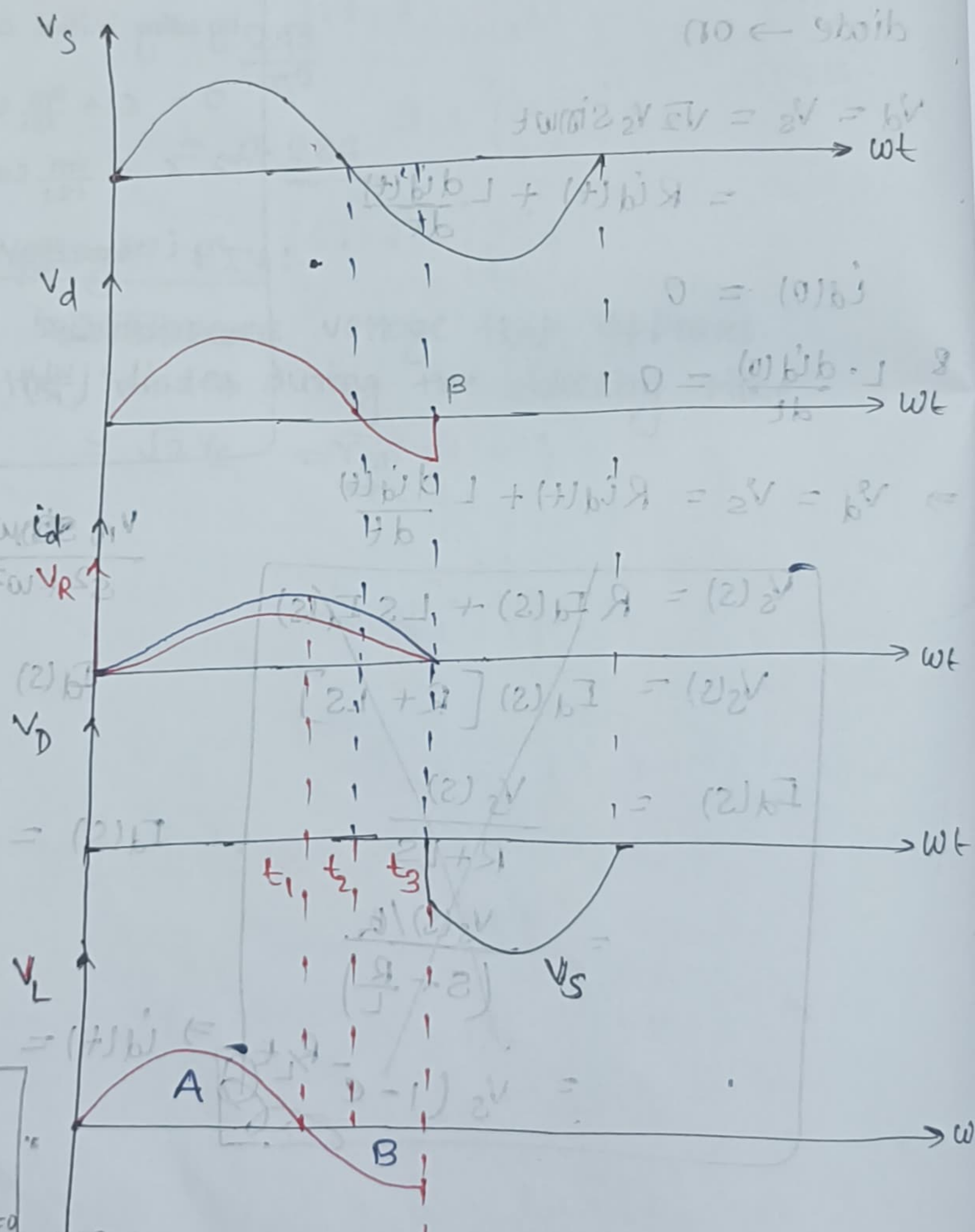
$$i_d\big|_{t=t_3} = 0$$

$$\Rightarrow \int_0^{t_3}\frac{V_L}{L}dt = 0$$

$$\Rightarrow \int_0^{t_1}\frac{V_L}{L}dt + \int_{t_1}^{t_3}\frac{V_L}{L}dt = 0$$

means area(A) = area(B).

$$\int_0^{t_1}\frac{V_L}{L}dt = -\int_{t_1}^{t_3}\frac{V_L}{L}dt$$



at $\omega t = \beta$, $i = 0$

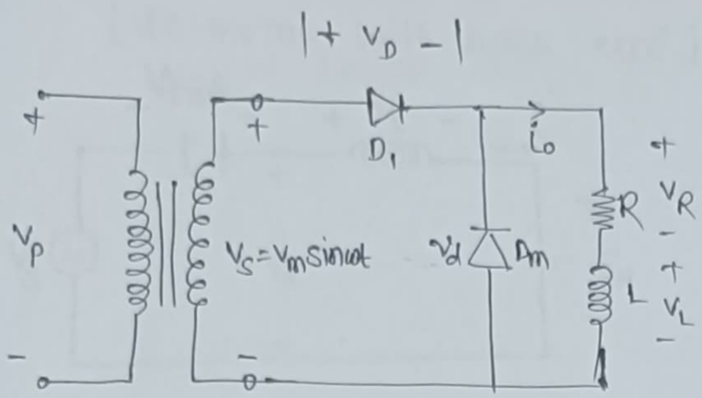
$\Rightarrow t = \beta/\omega$

$$\Rightarrow i\left(\frac{\beta}{\omega}\right) = \frac{V_m}{|Z|}\sin\phi\, e^{-R/\omega L} + \frac{V_m}{|Z|}\sin(\beta-\phi) = 0$$

(3) 0 = ______

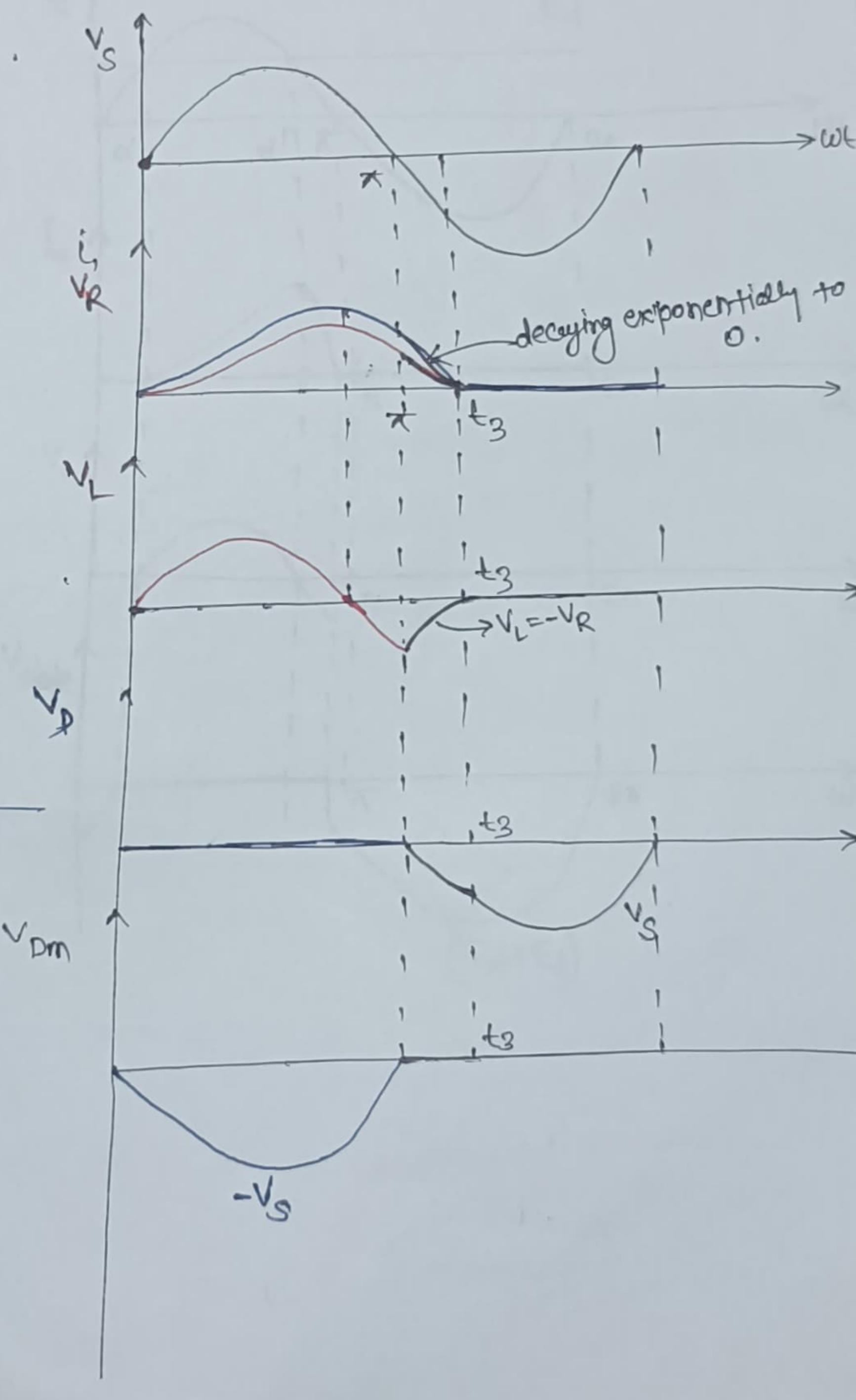
Solve to find β.

* R-L Load With free wheeling diode :-



$V_s, i_o, V_R, V_L, V_{D_1}, V_{dm}$

during 0 to π : D_1 is in Forward biased.



for $t > t_3$

both D_1 and D_m are in off condn.

$i = 0$

$V_R = 0$

$V_L = 0$ $\left(\because i = 0 \;\therefore \frac{di}{dt} = 0\right)$

$V_D = V_s$ ✓

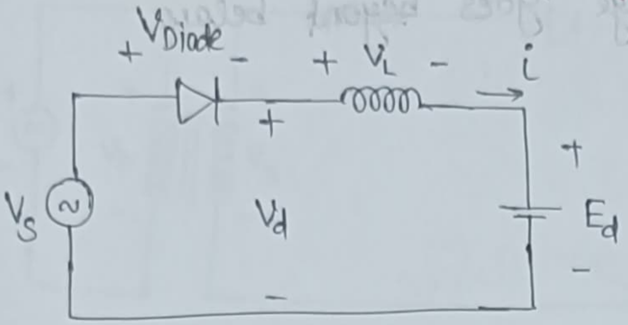
$V_{Dm} = 0$

↳ The effect of this f.w diode is to prevent a negative voltage appears accross the load.

↳ at

* L-E load

(dc motor with back emf.)



diode is F.B. when

$V_s \geq E_d$

at $\omega = \omega t_1 = \alpha$

$V_s = E_d$

$\sqrt{2} V_s \sin\alpha = E_d$

$\alpha = \sin^{-1}\left(\frac{E_d}{\sqrt{2} V_s}\right)$

for $\omega t \geq \omega t_1$

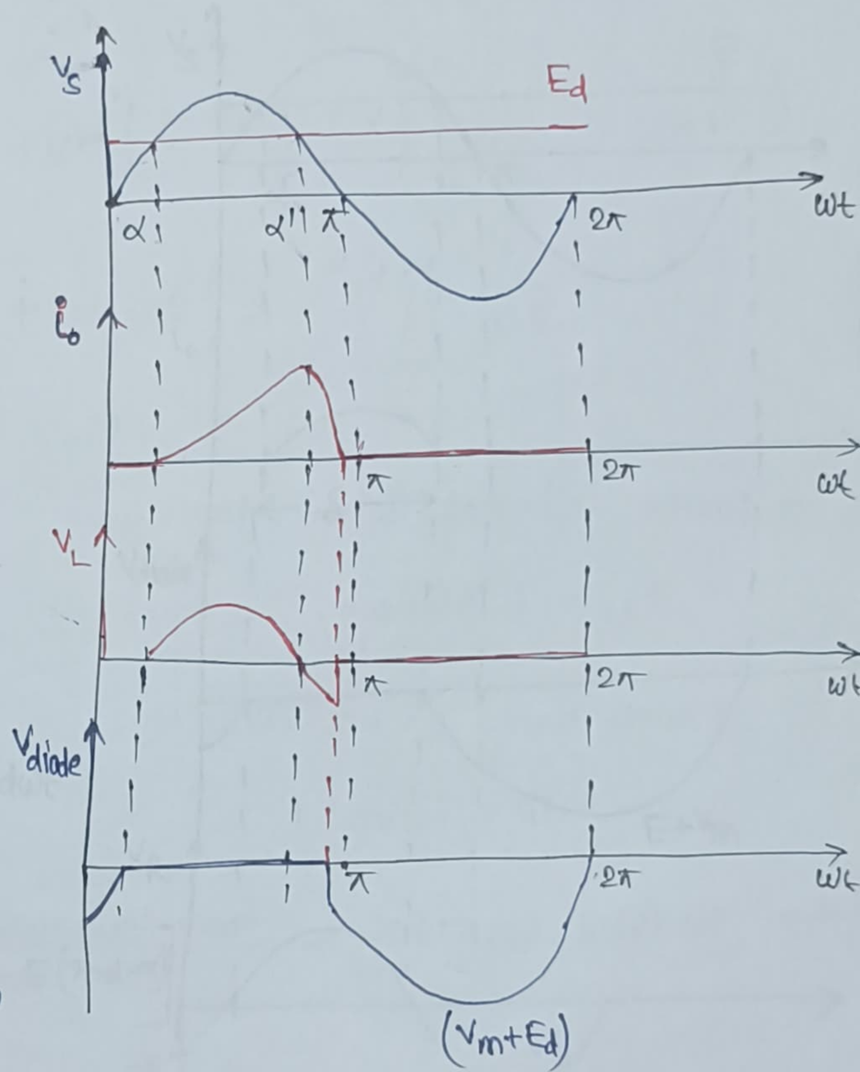
D_1 is on, $V_{diode} = 0$

$\sqrt{2} V_s \sin\omega t = L\frac{di}{dt} + E_d$

at $\omega t = \alpha$, $i = 0$

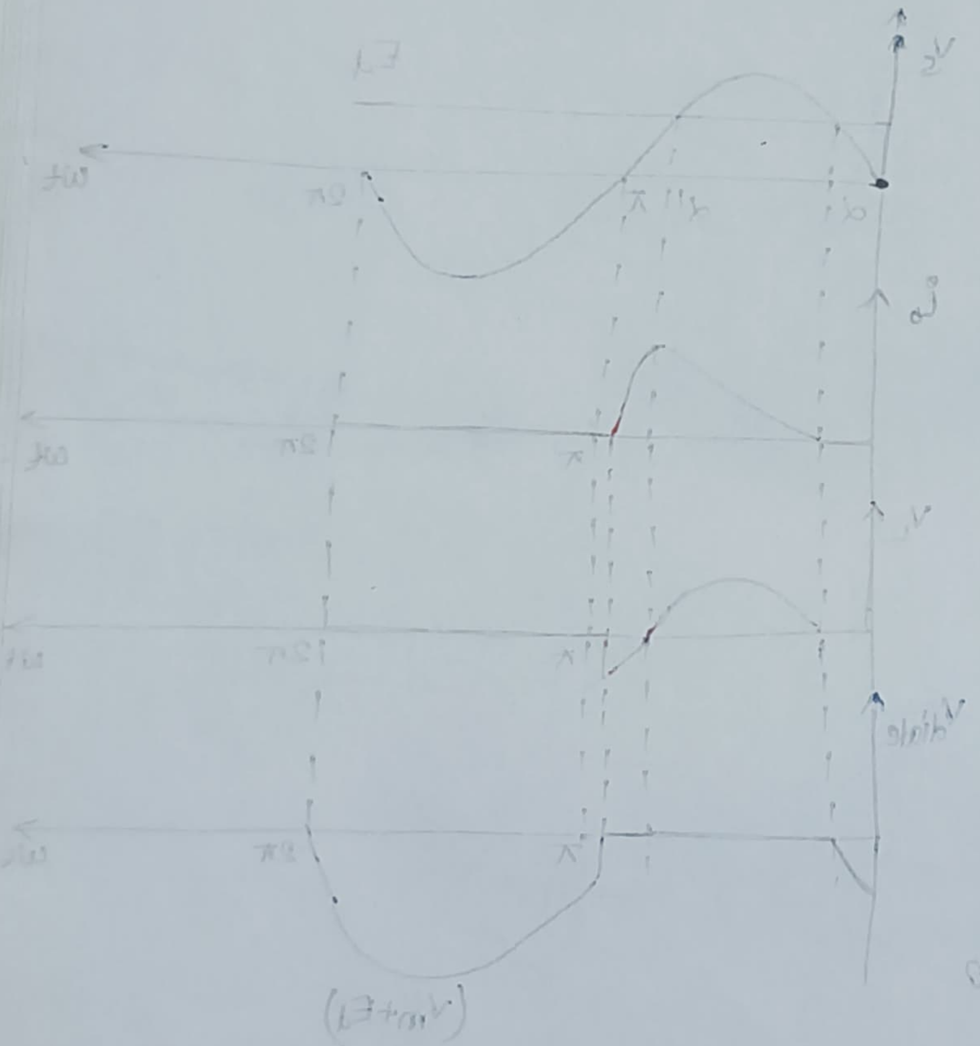
$\Rightarrow \sqrt{2} V_s \sin\alpha = L\frac{di}{dt} + E_d$

$\left.\frac{di}{dt}\right|_{\omega t = \alpha} = \frac{\sqrt{2} V_s \sin\alpha - E_d}{L} = 0$

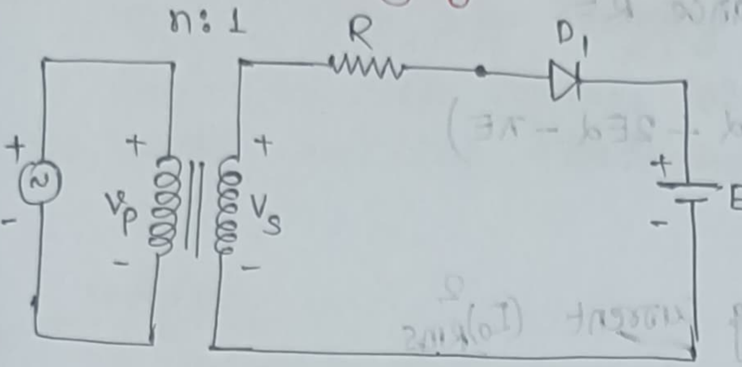


↳ Current continues to flow for a while even after the i/p voltage goes ~~beyond~~ below the dc back-emf.

- $t_3 \rightarrow 2\pi/\omega$
- $i = 0$
- $\frac{di}{dt} = 0,\ V_L = 0$



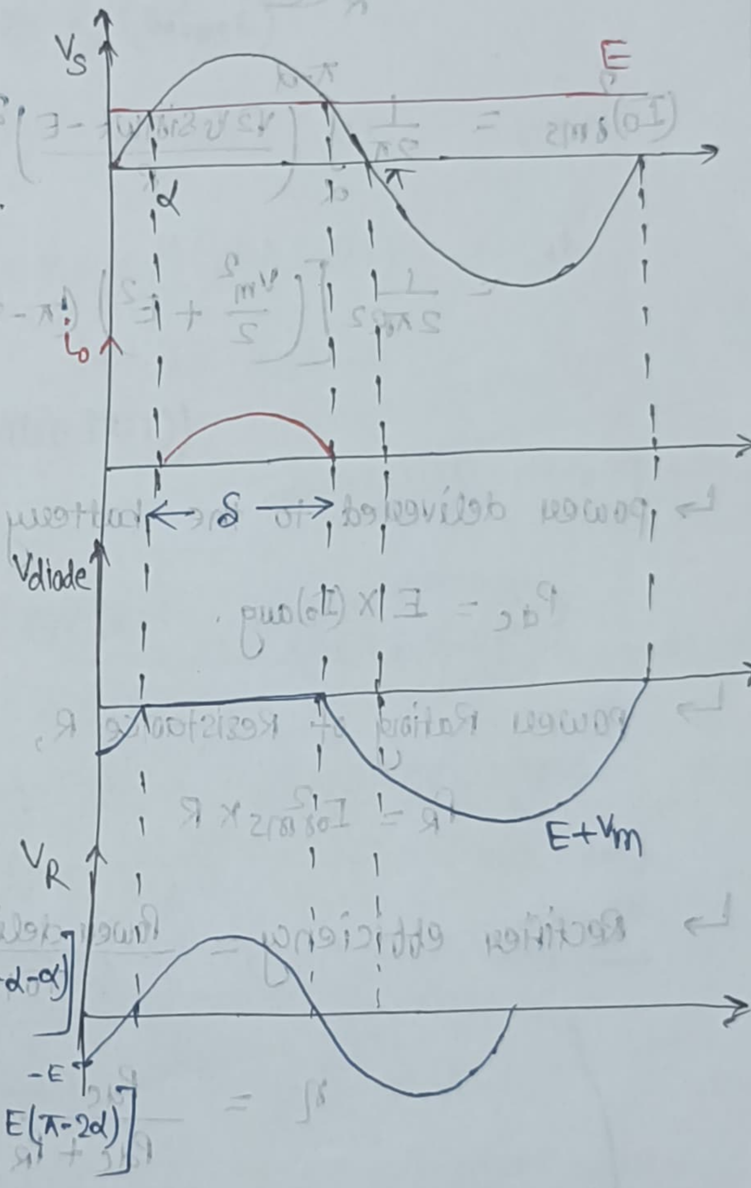
* Battery charging circuit :-



For $V_s > E$,
diode D_1 conducts.

$$i_o = \frac{V_R}{R} = \left(\frac{V_s - E}{R}\right)$$

$$\alpha = \sin^{-1}(E/V_m)$$



• Avg. charging current

$(i_o)_{avg} = ?$

$$(I_o)_{avg} = \frac{1}{2\pi}\int_{\alpha}^{\pi-\alpha}\left(\frac{\sqrt{2}V_s \sin\omega t - E}{R}\right) d\omega t$$

$$= \frac{1}{2\pi R}\left[\int_{\alpha}^{\pi-\alpha}\sqrt{2}V_s \sin\omega t \, d\omega t - E(\pi-\alpha-\alpha)\right]$$

$$= \frac{1}{2\pi R}\left[\sqrt{2}V_s\cdot\left(\cos(\alpha) - \cos(\pi-\alpha)\right) - E(\pi - 2\alpha)\right]$$

$$= \frac{1}{2\pi R}\left(2V_m\cos\alpha + 2E\alpha - \pi E\right)$$

- Series connected Resistance R =

$$R = \frac{1}{2\pi (I_o)_{avg}} (2V_m \cos\alpha + 2E\alpha - \pi E)$$

- RMS value of charging current $(I_o)^2_{RMS}$

$$\cancel{(I_o)_{rms}} = \frac{\cancel{(V_o)^2_{rms} - E}}{R}$$

$$(I_o)^2_{rms} = \frac{1}{2\pi} \int_{\alpha}^{\pi-\alpha} \left(\frac{\sqrt{2} V_s \sin\omega t - E}{R} \right)^2 d\omega t$$

$$= \frac{1}{2\pi R^2} \left[\left(\frac{V_m^2}{2} + E^2 \right) (\pi - 2\alpha) + \frac{V_m^2}{2} \sin 2\alpha - 4 V_m E \cos\alpha \right]$$

↳ power delivered to the battery

$$P_{dc} = E \times (I_o)_{avg}.$$

↳ power Rating of Resistance R,

$$P_R = I^2_{orms} \times R$$

↳ Rectifier efficiency = $\frac{\text{Power delivered to battery}}{\text{Total input power}}$

$$\eta = \frac{P_{dc}}{P_{dc} + P_R}$$

↳ PIV of diode = $\underline{V_m + E}$

Ques) A battery has a voltage of $E = 12V$ and capacity 100 wh. The avg. charging current should be $I_{dc} = 5A$. The primary input voltage is $V_p = 120V$, 50 Hz. and the transformer has turns ratio of 2:1.

Calculate

(a) conduction angle δ of the diode $(180° - 2\alpha)$
(b) current limiting Resistor R.
(c) Power Rating P_R of R $(I_{rms}^2 R)$
(d) Charging time
(e) the rectifier η
(f) PIV of diode.

$V_p = 120V$

so; $V_s = 60V$ (∵ turns ratio (2:1))

so, $\alpha = \sin^{-1}\left(\frac{E}{V_m}\right)$

$\alpha = \cancel{\sin^{-1}\left(\frac{12}{60}\right)}$ $\sin^{-1}\left(\frac{12}{\sqrt{2}*60}\right)$ $\alpha \rightarrow 8.13°$

$\boxed{\alpha = 11.53° \simeq 12°}$ $= 8.13°$

(a) conduction angle $\delta = (180° - 2\alpha)$
$= (180° - 2 \times 12)$
$= 156°$

Conduction angle
$\delta = (180° - 2\alpha)$
$= 163.739°$

(b) $R = \frac{1}{2\pi (I_0)_{avg}} (2V_m \cos\alpha + 2E\alpha - \pi E)$

$= \frac{1}{2\pi * 5} (2*60*\cos(12°) + 2*12*(0.2094) - \pi*12)$

$= \frac{1}{10\pi} [117.377 + 5.0256 - 37.6991]$

$R = 2.696\,\Omega$ (4.26)

$$(I_o)^2_{rms} = \frac{1}{2\pi(2.696)^2}\left[\left(\frac{(60)^2}{2} + (12)^2\right)(2.7227) + \frac{(60)^2}{2}\sin(24°) - 4*60*12*\cos(12°)\right]$$

$$= \frac{1}{\quad}\left[5292.92880 + 732.12595 - 2817.065\right]$$

$$= \frac{3207.9897}{45.6688}$$

$$(I_o)^2_{rms} = 70.2446 \ \cancel{W}.$$

(286.4 W)

$$P_R = (70.2446) * 2.696$$

$$= 189.3796 \ W.$$

- power delivered

(b)

$$(I_o)_{avg} = \frac{1}{2\pi R}\left[2V_m\cos\alpha + 2E\alpha - \pi E\right] =$$

$$\Rightarrow R = \frac{1}{2\pi (I_o)_{avg}}\left[2V_m\cos\alpha + 2E\alpha - \pi E\right] = \frac{1}{2\pi * 5}\left[2*60*0.9899 + 2*12*0.1418 - \pi*12\right]$$

$$= \left(\frac{133.78}{10\pi}\right)$$

$$\simeq 4.2585\ \Omega$$

(c) Power Rating of Resistor: $P_R = (I_o)^2_{rms} * R$

$$(I_o)^2_{rms} = \frac{1}{2\pi}\int_{\alpha}^{\pi-\alpha}\left(\frac{\sqrt{2}V_m\sin\omega t - E}{R}\right)^2 d\omega t$$

$$= \frac{1}{2\pi R^2}\left[\left(\frac{V_m^2}{2} + E^2\right)(\pi - 2\alpha) + \frac{V_m^2}{2}\sin 2\alpha - 4V_m E\cos\alpha\right]$$

$$= \frac{1}{2\pi(4.2585)^2}\left[\left\{\left(\frac{\sqrt{2}*60}{2}\right)^2 + (12)^2\right\}\left(\pi - \frac{0.28379}{0.1487}\right) + \left(\frac{\sqrt{2}*60}{2}\right)^2 \sin(8.527) - 4*\sqrt{2}*60*12\cos(4.26)\right]$$

$=$

(d) charging time $\Rightarrow$ $\delta = (\pi - 2\alpha)$

$$\omega t = \delta = 2.8577$$

$$t = \frac{2.8577}{2\pi*50} = \underline{9.09\ m\,sec.}$$

(e) $$\eta_{rec.} = \left(\frac{P_{dc}}{P_{dc} + P_R}\right) = \frac{E*(I_0)_{avg}}{E*(I_0)_{avg} + (286.4)}$$

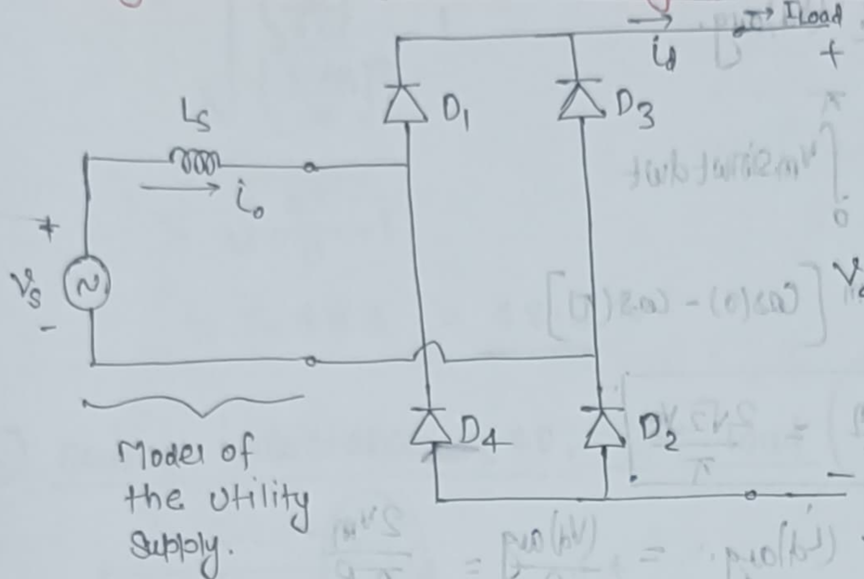
$$= \frac{12*5}{12*5 + 286.4} = \underline{17\%}$$

(f) $PIV = V_m + E$

$$= \sqrt{2}*60 + 12$$

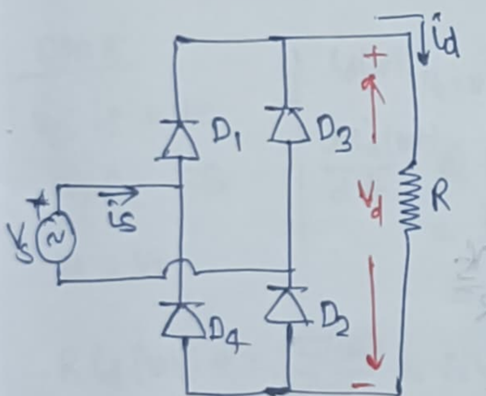
$$= \underline{96.8528V}$$

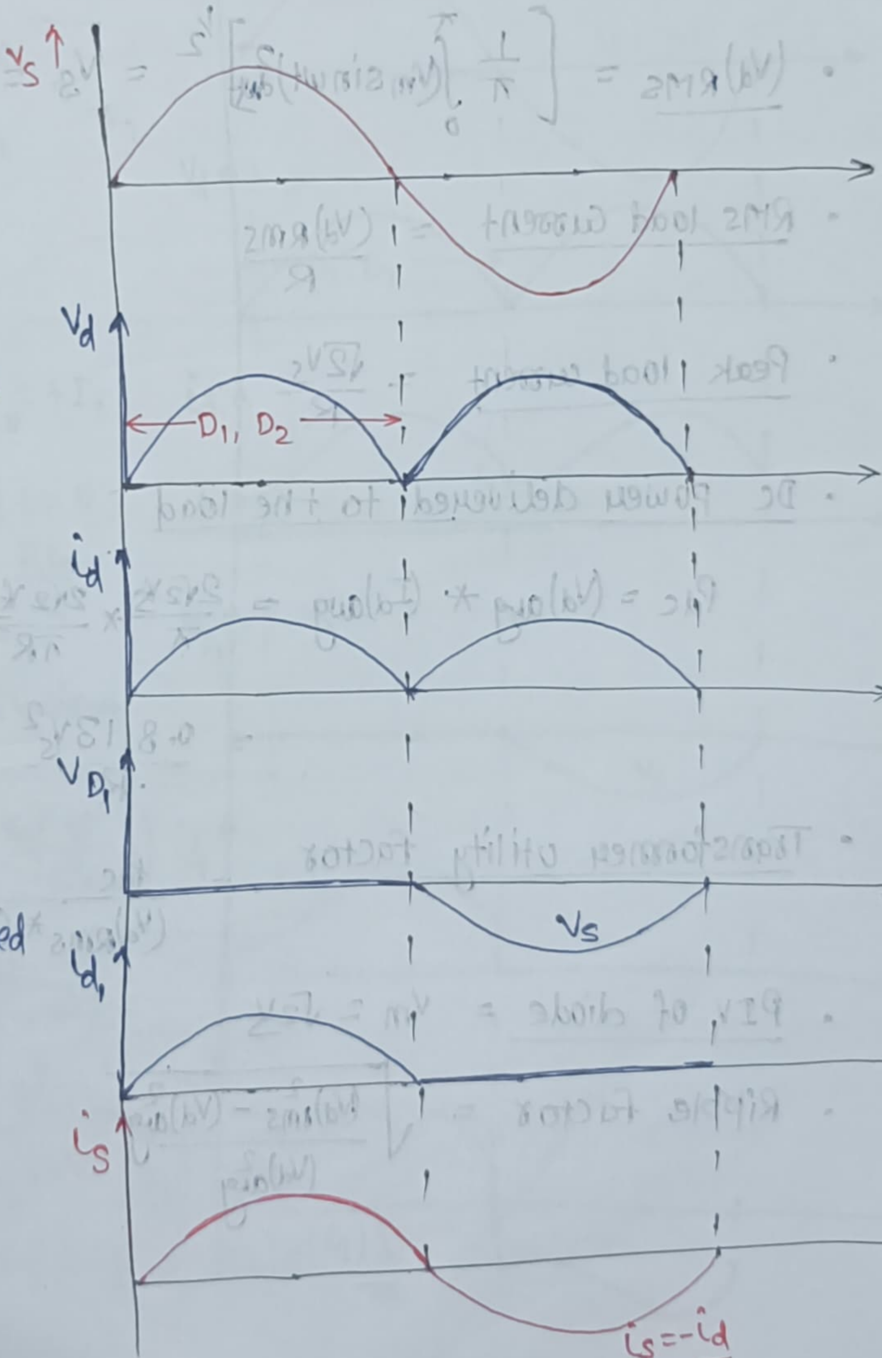
* Single-phase Fullwave Bridge Rectifiers :-



$v_s = \sqrt{2} V_s \sin\omega t$.

① source inductance $L_s = 0$, Load is pure resistance.





0 to π.

v_s is +ve.

D_1, D_2 is forward biased

Current path is

v_s, D_1, Load, D_2

Load voltage v_d = ___

- Avg. load voltage $(V_d)_{avg}$.

$$(V_d)_{avg} = \frac{1}{\pi}\int_0^{\pi} V_m \sin\omega t \, d\omega t$$

$$= \frac{1}{\pi} V_m \left[\cos(0) - \cos(\pi)\right]$$

$$\boxed{(V_d)_{avg} = \frac{2V_m}{\pi} = \frac{2\sqrt{2}V_s}{\pi}}$$

- Avg. load current $(i_d)_{avg} = \frac{(V_d)_{avg}}{R} = \frac{2V_m}{\pi R}$

- $(V_d)_{RMS} = \left[\frac{1}{\pi}\int_0^{\pi}(V_m \sin\omega t)^2 d\omega t\right]^{1/2} = V_s = \left(\frac{V_m}{\sqrt{2}}\right)$

- RMS load current $= \frac{(V_d)_{RMS}}{R}$

- Peak load current $= \frac{\sqrt{2}V_s}{R}$

- DC power delivered to the load

$$P_{dc} = (V_d)_{avg} * (I_d)_{avg} = \frac{2\sqrt{2}V_s}{\pi} * \frac{2\sqrt{2}V_s}{\pi R}$$

$$= \frac{0.8113 V_s^2}{R}$$

- Transformer utility factor $= \frac{P_{dc}}{(V_d)_{Rms} * (I_d)_{Rms}} = 81.13\%$

- PIV of diode $= V_m = \sqrt{2}V_s$

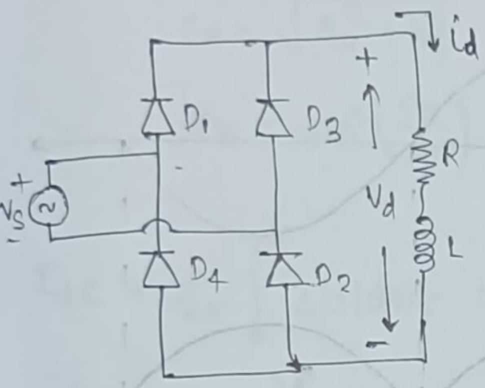
- Ripple factor $= \sqrt{\frac{(V_d)_{rms}^2 - (V_d)_{avg}^2}{(V_d)_{avg}^2}}$

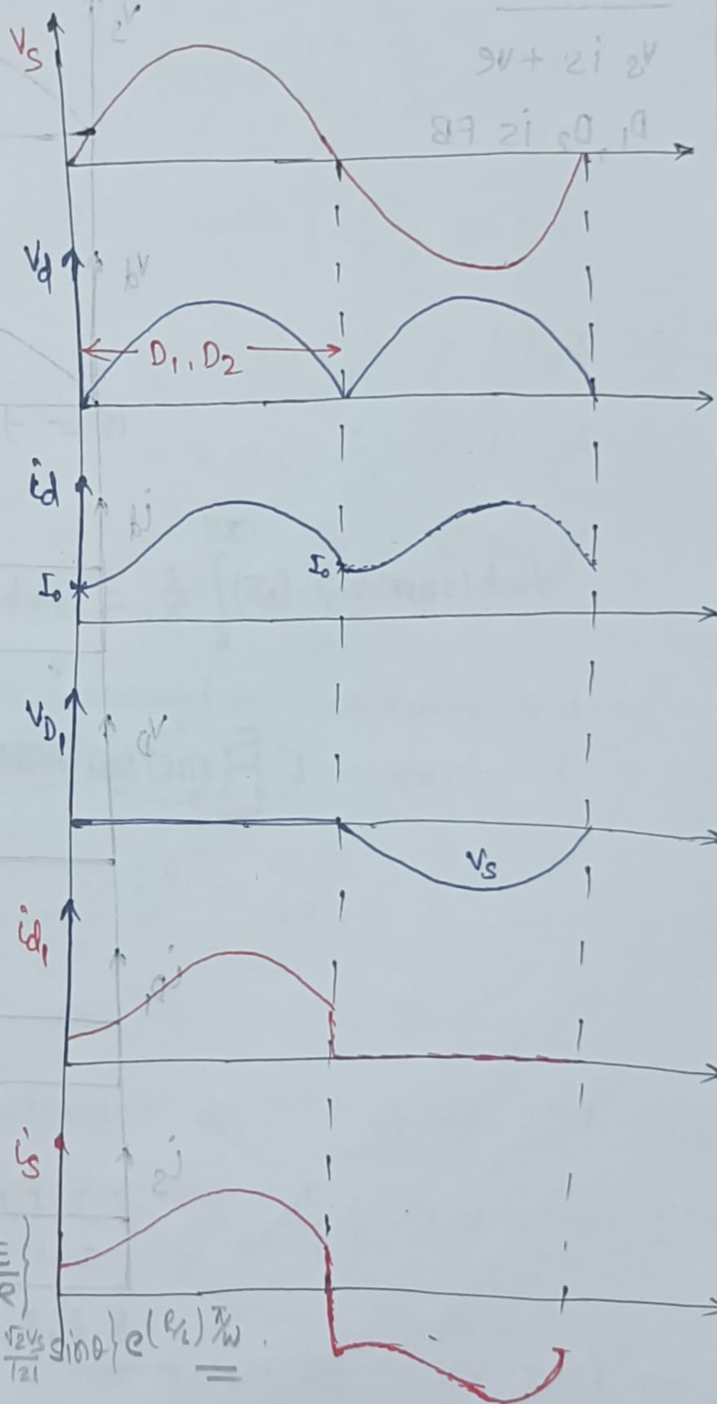
$$= \sqrt{\frac{\left(\frac{V_m}{\sqrt{2}}\right)^2}{\left(\frac{2V_m}{\pi}\right)^2} - 1}$$

$$= \sqrt{\frac{\pi^2}{8} - 1}$$

$$= 0.483 = 48.3\,\%$$

(2) Source inductance $L_s = 0$, R-L load





0 to π

v_s is +ve

D_1, D_2 FB

$v_d = v_s$

$i_d(\omega t)\big|_{t=0} = +I_0$

$\frac{d\, i_d(\omega t)}{dt}$ @ $t = 0$ $= \frac{RI_0}{L}$

$$R\, i_d(\omega t) + L\frac{d\, i_d(\omega t)}{d\omega t} = \sqrt{2}V_s \sin\omega t$$

$$R\, I_D(s) + Ls\, I_D(s) = \sqrt{2}V_s\left(\frac{\omega}{s^2+\omega^2}\right)$$

$$I_D(s)\left[R + Ls\right] = \sqrt{2}V_s\left(\frac{\omega}{s^2+\omega^2}\right)$$

$$I_D(s) = \sqrt{2}V_s\left(\frac{\omega}{s^2+\omega^2}\right)\left(\frac{1}{R+Ls}\right)$$

$$I_D(t) = \left\{\frac{\sqrt{2}V_s}{|Z|}\sin(\omega t - \theta) + A_1 e^{-(R/L)t} - \frac{E}{R}\right\}$$

@ $t = 0$ $\;I_D(0) = I_0 \rightarrow A = \left\{I_0 + \frac{E}{R} - \frac{\sqrt{2}V_s}{|Z|}\sin\theta\right\} e^{(R/L)\pi/\omega}$

⇒

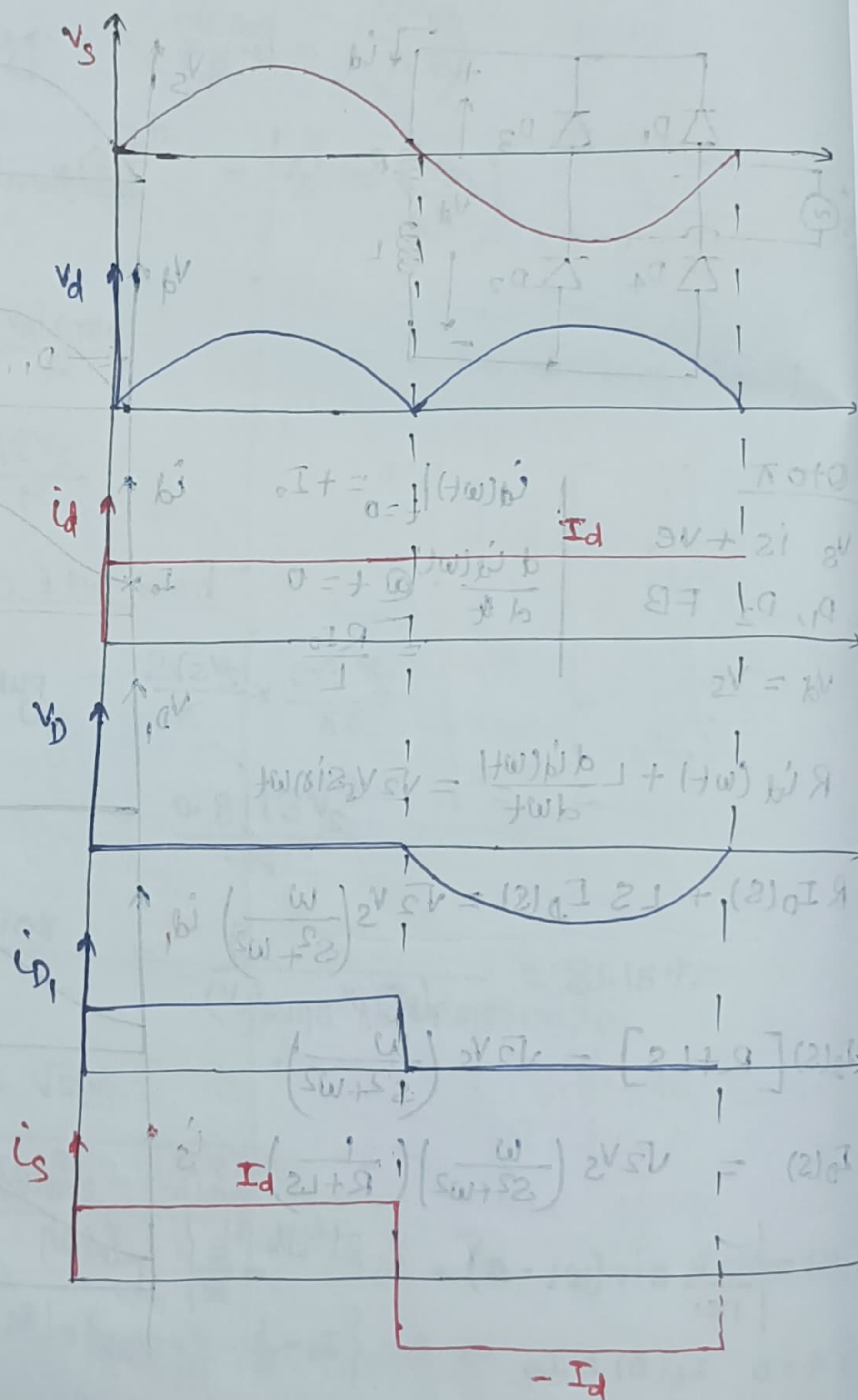
- π to 2π
- V_s is $-ve$
- D_3, D_4 is forward biased
- $V_d = -V_s$

* $L_s = 0$ and Load is highly inductive so that i_d is constant at I_d $(\omega L = R)$

0 to π

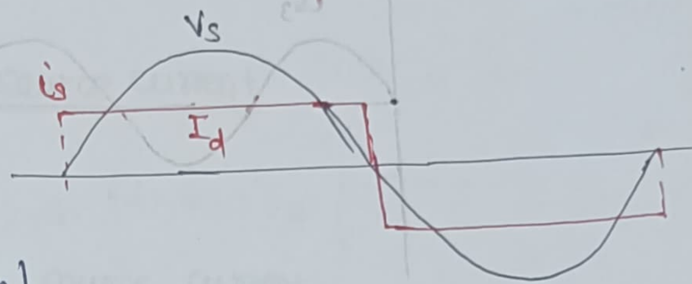
V_s is $+ve$

D_1, D_2 is FB



↳ source inductance is zero, Hence transition from +ve to -ve value of source current is instantaneous.

$$i_s(t) = I_{dc} + \sum_{n=1}^{\infty} a_n \cos n\omega t + b_n \sin n\omega t$$



$$= I_{dc} + C_n \sin(n\omega t - \phi_n)$$

$$C_n = \sqrt{a_n^2 + b_n^2}$$

$$\phi_n = \tan^{-1}\left(\frac{a_n}{b_n}\right)$$

$$I_{dc} = \frac{1}{2\pi}\int_0^{2\pi} i_s(t)\, d\omega t = 0$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} i_s(t) \cos(n\omega t)\, d\omega t = 0$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} i_s(t) \sin(n\omega t)\, d\omega t = \frac{1}{\pi}\int_0^{2\pi} (I_d) \cdot \sin(n\omega t)\, d\omega t$$

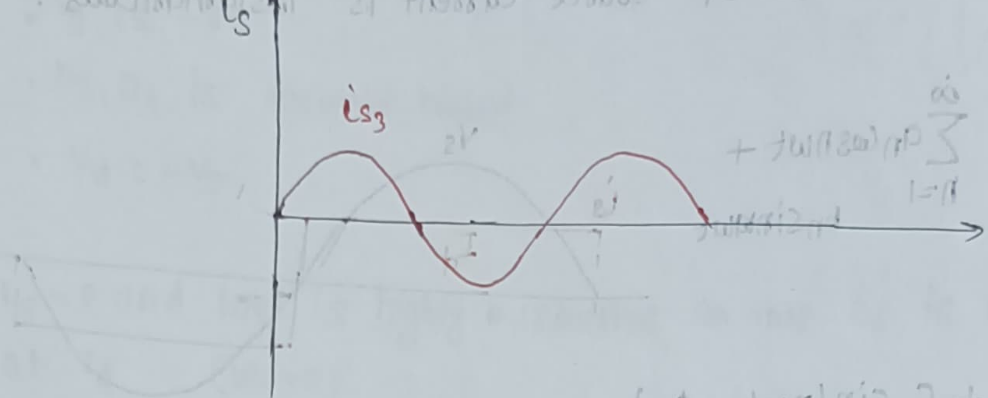
$$= \frac{1}{\pi}\frac{i_s(t)}{n}\left[\cos(0) - \cancel{\cos(2\pi)}\cos(2n\pi)\right]$$

$$= \frac{i_s(t)}{n\pi}\left[\right.$$

$$b_n = \frac{2 I_d}{n\pi}\left[1 - \cos n\pi\right]$$

$$b_n = \frac{4 I_d}{n\pi} \text{ for } n = 1, 3, 5, 7.$$

$$= 0 \text{ for } n = 2, 4, 6, 8.$$



$$i_s(t) = \sum_{n=1}^{\infty} \frac{4I_d}{n\pi} \sin n\omega t \quad : n = 1, 3, 5 \ldots$$

$$i_s(t) = \frac{4I_d}{\pi}\left[\sin\omega t + \frac{\sin 3\omega t}{3} + \frac{\sin 5\omega t}{5} + \ldots\right]$$

$$\left.\begin{aligned} I_{s_1}(t) &= \frac{4I_d}{\pi}\sin\omega t \\ I_{s_3}(t) &= \frac{4I_d}{3\pi}\sin 3\omega t \\ I_{s_5}(t) &= \frac{4I_d}{5\pi}\sin 5\omega t. \end{aligned}\right\}$$

↳ RMS value of fundamental component of i/p current I_{s_1}

$$I_{s_1} = \frac{4I_d}{\pi\sqrt{2}} = 0.9\,I_d \qquad I_{s_3} = \frac{4I_d}{3\pi\sqrt{2}} = 0.3\,I_d$$

$$I_{s5} = \frac{4I_d}{5\pi\sqrt{2}} = 0.18\,I_d.$$

↳ RMS values of nth harmonic of source current i_{sn}

$$i_{sn} = \begin{cases} 0 & \text{for even values of } n \\ \frac{I_{s1}}{n} & \text{for odd value of } n \end{cases}$$

↳ RMS value of total source current $I_s = I_d$

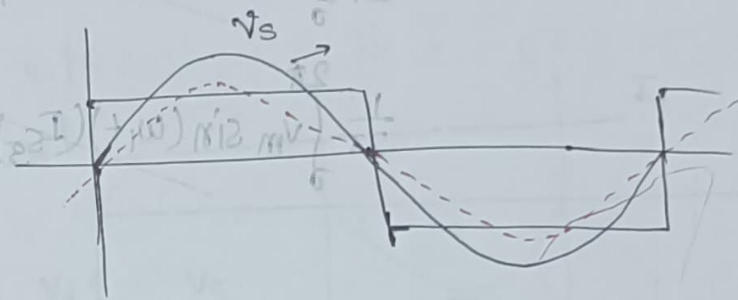
* Total harmonic distortion (THD)

$$THD = \sqrt{\frac{(I_d^2 - I_{s_1}^2)}{I_{s_1}^2}} = 48.4\%$$

* <u>Displacement factor of source current</u>

$$DF = \cos\phi_1 = 1$$

* <u>Harmonic components of source current</u>



* <u>AC source side power factor</u>

$$= \frac{\text{Actual power}}{\text{Apparent power}}$$

$$= \frac{V_{s_1} I_{s_1} \cos\phi_1}{V_s I_s}$$

$$\text{actual power} = \frac{1}{2\pi}\int_0^{2\pi} v_s I_s = \frac{1}{2\pi}\left[\int_0^{2\pi} V_m \sin\omega t \left[I_{s_1} + I_{s_3} + \cdots\right]\right]$$

$$= \frac{1}{2\pi}\int_0^{2\pi} V_m \sin\omega_1 t \left[(I_{s_1})_{Peak} \sin(\omega_1 t - \phi_1) + (I_{s_3})_{Peak} \sin(\omega_3 t - \phi_3) + (I_{s_5})_{Peak} \sin(\omega_5 t - \phi_5) + \dots \right.$$

$$= \frac{1}{2\pi}\int_0^{2\pi} V_m \sin(\omega_1 t)(I_{s_1})_{Peak} \sin(\omega_1 t - \phi_1) +$$

$$\cancel{\frac{1}{2\pi}\int_0^{2\pi} V_m \sin(\omega_1 t)(I_{s_3})_{Peak} \sin(3\omega_1 t - \phi_3)} \rightarrow 0 \; +$$

$$\cancel{\frac{1}{2\pi}\int_0^{2\pi} V_m \sin(\omega_1 t)(I_{s_5})_{Peak} \sin(5\omega_1 t - \phi_5)} \rightarrow 0 \; + \dots$$

$$= \frac{V_m}{\sqrt{2}} \cdot \frac{I_{s_1}}{\sqrt{2}} \cos\phi_1$$

Actual power $= V_s \cdot I_{s_1} \cdot \cos\phi_1$

apparent power $= V_s I_s$

$\therefore$ power factor $= \dfrac{\cancel{V_s} I_{s_1} \cos\phi_1}{\cancel{V_s} I_s}$

power factor $= \dfrac{I_{s_1}}{I_s} = \dfrac{2\sqrt{2} I_d}{\pi I_d} = \underline{\underline{0.9}}$

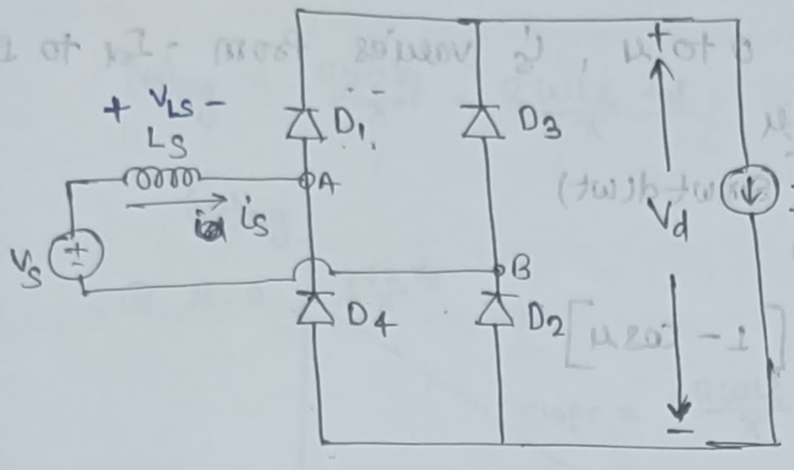
* peak current rating of diode $= I_d$

* Rms current Rating of diode $= \left(\dfrac{I_d}{\sqrt{2}}\right)$

* PIV of diode $= V_m$.

* Effect of source inductance :-

11/09/23



↳ due to source inductance the transition of ac side current from $+I_d$ to $-I_d$ is not instantaneous.

↳ A finite interval is required for the transition of current from outgoing diodes to incoming diodes.

↳ During commutation interval u, all four diodes conducts

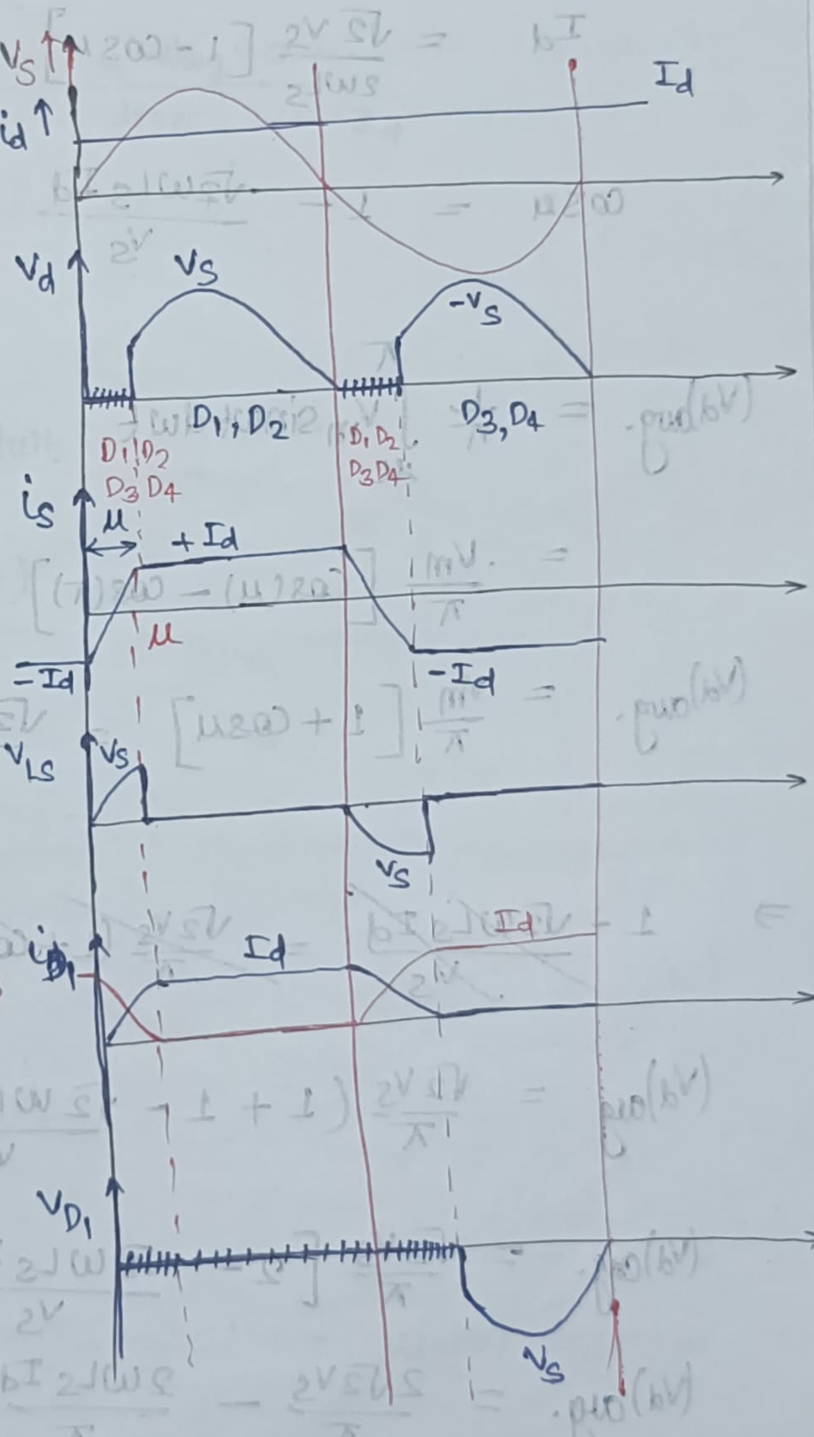
$V_d = 0$

- Drop in the inductance

$V_{LS} = +V_s$

$$L_S \cdot \frac{di_s(t)}{dt} = \sqrt{2} V_s \sin\omega t$$

$$\omega L_S \frac{di_s(t)}{d\omega t} = \sqrt{2} V_s \sin\omega t$$



$$di_s(t) = \frac{\sqrt{2}V_s}{\omega L_s} \sin\omega t \, d\omega t$$

during the interval 0 to μ, i_s varies from $-I_d$ to I_d

$$\int_{-I_d}^{I_d} di_s(t) = \frac{\sqrt{2}V_s}{\omega L_s} \int_0^{\mu} \sin\omega t \, d(\omega t)$$

$$2I_d = \frac{\sqrt{2}V_s}{\omega L_s}\left[1-\cos\mu\right]$$

$$I_d = \frac{\sqrt{2}V_s}{2\omega L_s}\left[1-\cos\mu\right]$$

$$\cos\mu = 1 - \frac{\sqrt{2}\omega L_s I_d}{V_s}$$

$$(V_d)_{avg.} = \frac{1}{\pi}\int_{\mu}^{\pi} V_m \sin\omega t \, d\omega t$$

$$= \frac{V_m}{\pi}\left[\cos(\mu) - \cos(\pi)\right]$$

$$(V_d)_{avg.} = \frac{V_m}{\pi}\left[1+\cos\mu\right] \Rightarrow \frac{\sqrt{2}V_s}{\pi}(1+\cos\mu)$$

$$\Rightarrow \cancel{1 - \frac{\sqrt{2}\omega L_s I_d}{V_s} = \frac{\sqrt{2}V_s}{\pi}(1+\cos\mu)}$$

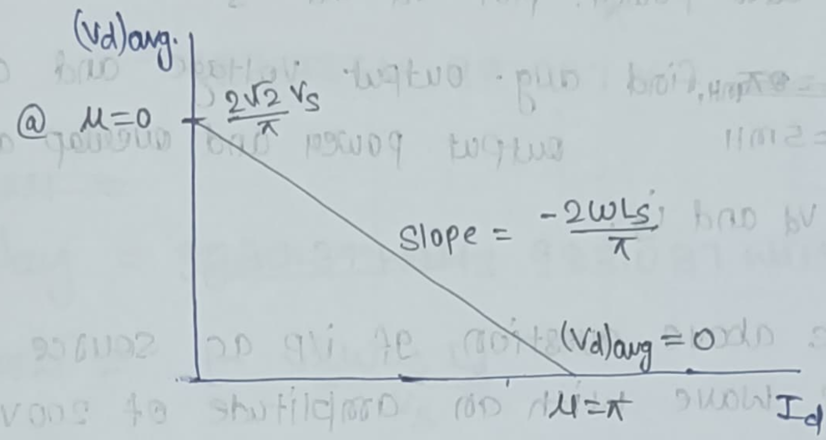
$$(V_d)_{avg} = \frac{\sqrt{2}V_s}{\pi}\left(1 + 1 - \frac{\sqrt{2}\omega L_s I_d}{V_s}\right)$$

$$(V_d)_{avg.} = \frac{\sqrt{2}V_s}{\pi}\left[2 - \frac{\sqrt{2}\omega L_s I_d}{V_s}\right]$$

$$(V_d)_{avg.} = \frac{2\sqrt{2}V_s}{\pi} - \frac{2\omega L_s I_d}{\pi}$$

↳ so avg. o/p voltage is reduced by $\frac{2\omega L_s I_d}{\pi}$

$$(V_d)_{avg} = \frac{2\sqrt{2}V_s}{\pi} - \frac{2\omega L_s I_d}{\pi}$$



during 'μ'

$$V_{Ls} = L_s \frac{di_s}{dt} = \sqrt{2}V_s \sin\omega t$$

$$di_s = \frac{\sqrt{2}}{\omega L_s} V_s \sin\omega t \, d\omega t$$

$$i_s = \frac{\sqrt{2}V_s}{\omega L_s}(-\cos\omega t)$$

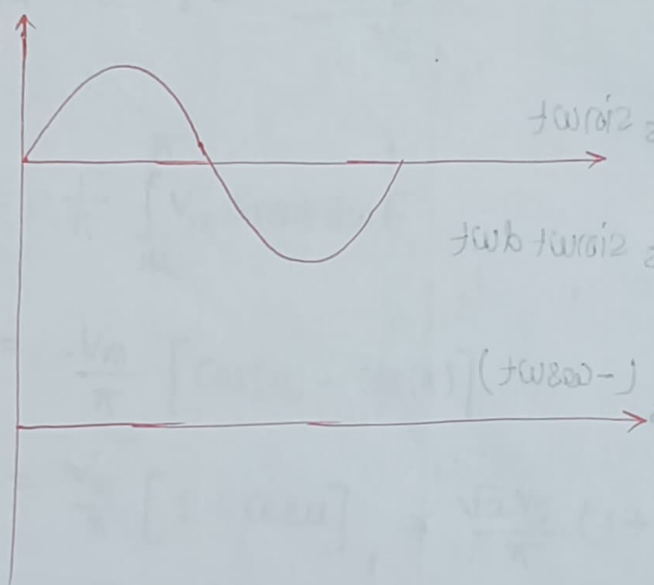
Ques ① A single phase uncontrolled bridge rectifier connected to a 120V ac source applies a constant load current of 10A.

(a) If $L_s = 0$, find avg. output voltage and avg. output ~~curr~~ power. plot V_d and i_s.

(b) If ~~$L_s = 0.7mH$~~ $L_s = 5mH$, find avg. output voltage and avg. output power and overlap angle. Plot V_d and i_s.

② Repeat the above question if i/p ac source is of square wave with an amplitude of 200V.

Soln:-



$$(V_d)_{avg.} = \frac{2\sqrt{2}V_s}{\pi}$$

$$= \frac{2\sqrt{2} * 120}{\pi} = 108.037 \text{ V.}$$

$(I_d) = 10$ A

avg. O/P Power = 1080.37 Watts.

(b) For $L_s = 5mH$

$$(V_d)_{avg} = \frac{2\sqrt{2}V_s}{\pi} - \frac{2\pi f L_s I_d}{\pi}$$

$$= 108.037 - \frac{\cancel{2}\times 2\cancel{\pi}\times \cancel{50}\times 5\times 10^{-3}\times \cancel{10}}{\cancel{\pi}}$$

$$= \cancel{103.037\,V}\ 98.037\,V$$

~~Cosμ =~~

$$(P_0)_{avg} = \cancel{9830.37\text{ Watts}}\ 983.037\text{ Watts}$$

$$\cos\mu = 1 - \frac{\sqrt{2}\,\omega L_s I_d}{V_s}$$

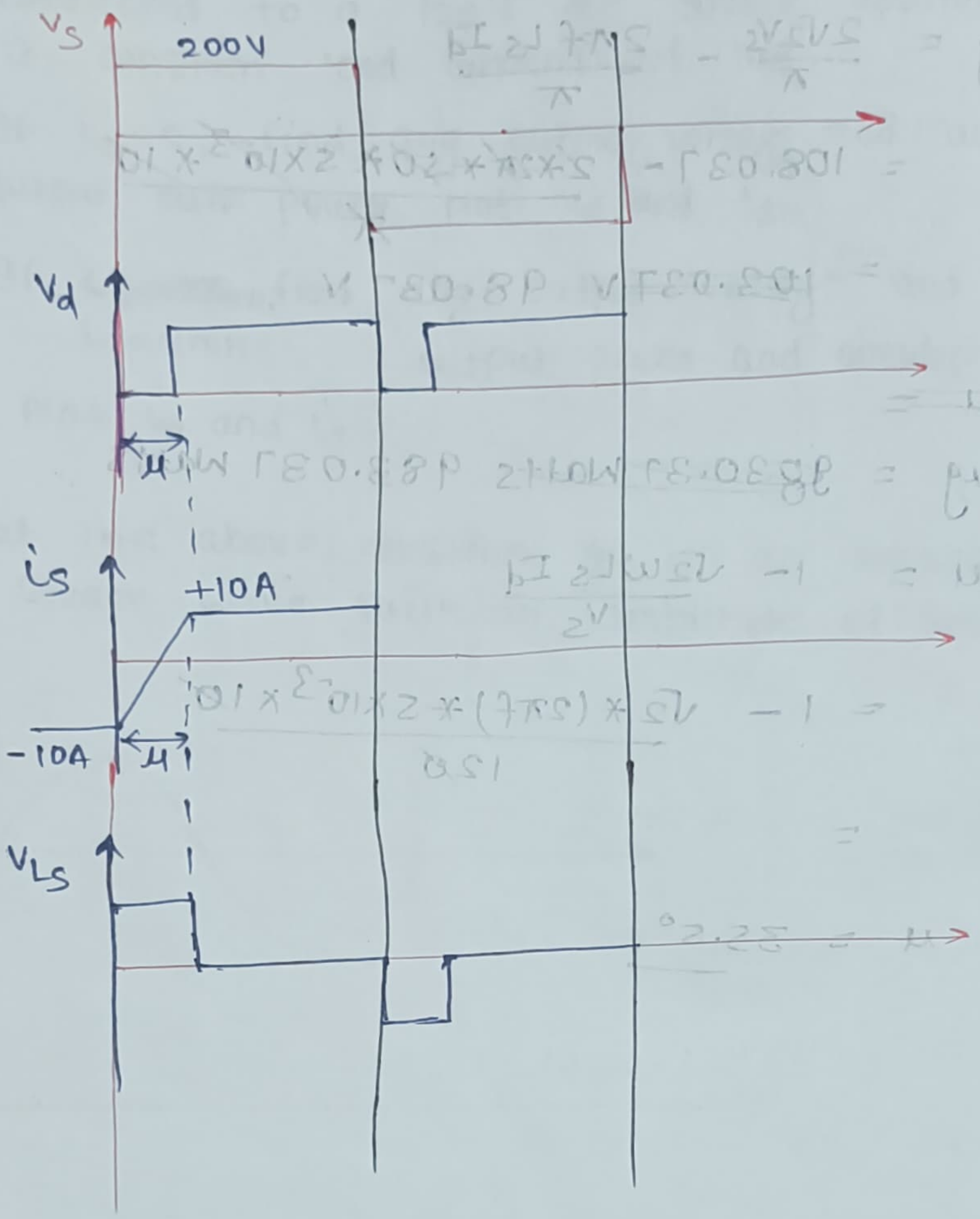
$$= 1 - \frac{\sqrt{2}\times(2\pi f)\times 5\times 10^{-3}\times 1\cancel{0}}{12\cancel{0}}$$

$$=$$

$$\mu = 35.5^\circ$$

② ~~Solⁿ~~

(2) Sol^n:-



a) Without source inductance.

$$(V_d)_{avg.} = \underline{200\,V}$$

$$(P_d)_{avg.} = 2000 \text{ watts.}$$

b) With source inductance

$$\cos\mu = 1 - \frac{\sqrt{2}\,\omega L_s I_d}{V_s}$$

$$= 1 - \frac{\sqrt{2} * (2\pi f) * 10 \times 10^{-3} \times 10}{200}$$

$\mu = 27.26°$

$$\cancel{(V_d)_{avg.} = \left(\frac{180° - 27.26°}{180°}\right) * \frac{2\pi}{\pi} * 200}$$

$$\cancel{= 533.1631} \quad \cancel{169.711°}$$

$$L\frac{di_s}{dt} = 200$$

$$di_s = \frac{200}{\omega L}\, d\omega t$$

$$\int_{-10}^{10} di_s = \frac{200}{\omega L}\left[t\right]_0^{\mu}$$

$$\cancel{20} = \frac{200\,\cancel{10}}{2\pi f \times 5\times10^{-3}} * \mu$$

$$\mu = \pi\times 50\times 10^{-3} = 0.157 \text{ rad} \cong 9°$$

$$(V_d)_{avg.} = \frac{1}{\pi}\int_{9°}^{180°} 200V$$

$$= \frac{200}{\pi}\left[180° - 9°\right]$$

$$= \frac{200}{\pi}(171°)$$

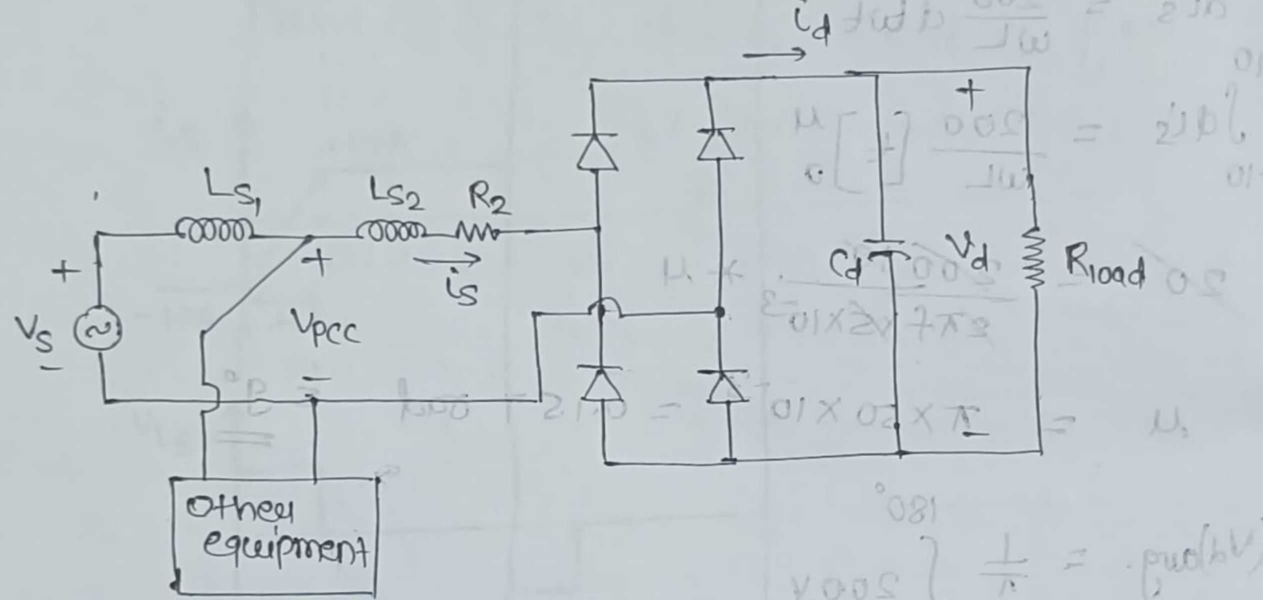
$$= \frac{200}{\pi} * \frac{171}{180} \times \pi = 190 \text{ Volt.}$$

$$(P_d)_{avg.} = 1900 \text{ watts.}$$

3-φ diode rectifier circuits :-

* Effect of diode - Rectifier on Utility voltage and current :-

- **Line current distortion :** source current deviates significantly from the sinusoidal waveform.



$L_{s_1} \rightarrow$ inductance of the utility side.
$L_{s_2} \rightarrow$ " due to the power elec. equipments.
$R_s \rightarrow$ diode Resistance.

PCC $\rightarrow$ point of common coupling where other load & PE loads are connected

$V_{pcc} \rightarrow$ Voltage accross the

$$V_{pcc} = V_s - L_{s_1} \cdot \frac{di_s}{dt}$$

- Source current i_s

$$i_s = i_{s_1} + \sum_{h=3,5\ldots}^{\infty} i_{sh}$$

$$V_{pcc} = \underbrace{V_s - L_{s_1} \cdot \frac{di_{s_1}}{dt}} - L_{s_1} \sum_{h=3,5,\ldots}^{\infty} \frac{di_{sh}}{dt}$$

$$V_{pcc} = V_{pcc1} - L_{s_1} \sum_{h=3,5,\ldots}^{\infty} \frac{di_{sh}}{dt} \qquad V_{pcc1} = V_s - L_{s_1} \cdot \frac{di_{s_1}}{dt}$$

↳ Distortion in voltage at PCC due to harmonics in line current –

$$V_{pccdis} = L_{s_1} \sum_{h=3,5,\ldots}^{\infty} \frac{di_{sh}}{dt}$$

↳ Voltage available at the point of common coupling is heighly deviated from the ideal sine waveform.

↳ This is reffered to voltage pollution in power system.

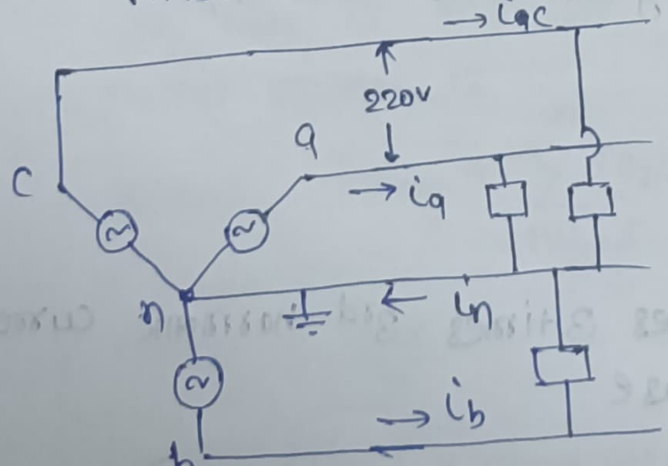
↳ This is one of the reasons of power quality problems.

* Effect of single phase Rectifiers on the neutral current in 3-ϕ – 4 wire systems :-

↳ When all the 3-phases are loaded equally (or) balanced neutral current $i_n = 0$

↳ When 1-ϕ rectifier are connected ~~to~~ as load to 3ϕ, $i_n \neq 0$

↳ Consider identical rectifiers connected between each phase and neutral (balanced load condⁿ.)



• Phase a current

$$i_a = i_{a_1} + \sum_{n=2k+1}^{\infty} i_{an} \qquad k = 1, 2, 3 \ldots$$

$$i_a = \sqrt{2} I_{s_1} \sin(\omega t - \phi_1) + \sum_{n=2k+1}^{\infty} \sqrt{2} I_{sn} \sin(n\omega t - \phi_n)$$

Sly

$$i_b = \sqrt{2} I_{s_1} \sin(\omega t - \phi_1 - 120) + \sum_{n=2k+1}^{\infty} \sqrt{2} I_{sn} \sin(n\omega t - \phi_n - 120n)$$

$$i_c = \sqrt{2} I_{s_1} \sin(\omega t - \phi_1 - 240) + \sum_{n=2k+1}^{\infty} \sqrt{2} I_{sn} \sin(n\omega t - \phi_n - 240n)$$

• Neutral current

$$i_N = i_a + i_b + i_c$$

$$= i_{a_1} + \sum_{n=2k+1}^{\infty} i_{an} + i_{b_1} + \sum_{n=2k+1}^{\infty} i_{bn} + i_{c_1} + \sum_{n=2k+1}^{\infty} i_{cn}$$

∵ sum of the 3-φ fundamental components and non-triplen harmonics is zero.

• $i_{a_1} + i_{b_1} + i_{c_1} = 0$

$$\Rightarrow i_N = i_{a_3} + i_{a_9} + i_{a_{15}} + \ldots + i_{b_3} + i_{b_9} + \ldots + i_{c_3} + i_{c_9} + \ldots$$

Neglecting all the heigher order harmonics

$$\boxed{i_N = i_{a_3} + i_{b_3} + i_{c_3}}$$

$$i_N = 3 i_{a_3} \simeq i_{a_1}$$

↳ Thus neutral wire carries 3 times 3rd harmonic current flowing through a phase

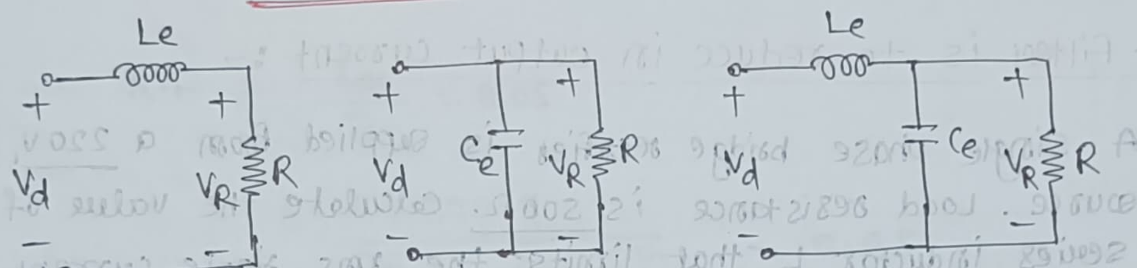
↳ Neutral current is almost equal to or greater than the fundament line current. ~~RMS~~ V.

Rectifier circuit design :-

↳ Determination of the rating of semiconductor devices
• avg current, rms current, Peak current and PIV.

↳ Design of filters at the output (dc side) to remove voltage harmonics. These are called dc Filters.
They are – L, C. and LC type.

DC - Filters



↳ Design of filters at the input (ac side) to remove current harmonics.
These filters are called as ac filters and are usually of LC type.

DC Filters :-

↳ To design a dc filter circuit, knowledge of the magnitude and frequency of harmonics at the dc side is required.

⇒ output voltage is

$$V_d(\omega t) = (V_d)_{avg} + \sum_{n=1,3,5..}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$

$$\therefore (V_d)_{avg} = \frac{2V_m}{\pi}$$

$$a_n = \frac{1}{(\pi/2)} \int_0^{\pi} V_m \sin\omega t \cos n\omega t \, d\omega t = \frac{4V_m}{\pi} \sum_{n=2,4,6..}^{\infty} \frac{-1}{(n-1)(n+1)}$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} V_m \sin\omega t \cdot \sin n\omega t \, d\omega t = 0$$

$$\cancel{V_d(t) = \frac{2V_m}{\pi} - \frac{4V_m}{\pi}\sum \frac{1}{n}}$$

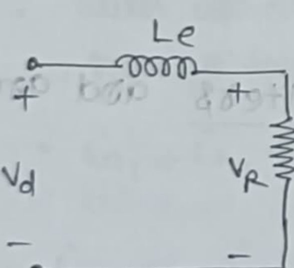
$$V_d(t) = \frac{2V_m}{\pi} - \frac{4V_m}{3\pi}\cos 2\omega t - \frac{4V_m}{15\pi}\cos 4\omega t - \frac{4V_m}{35\pi}\cos 6\omega t$$

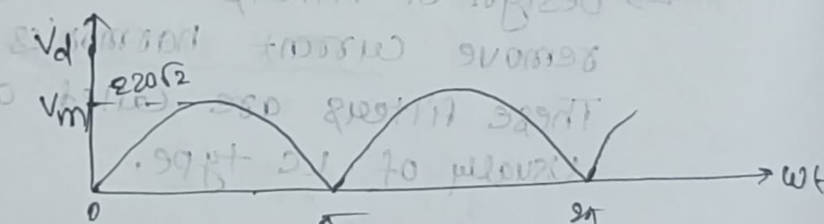
↳ output voltage of the full wave rectifier contains only even harmonics.

↳ second harmonics at 100 Hz is most dominant.

* L- Filter is to reduce in output current :-

↳ A single phase bridge rectifier is supplied from a 220 V, 50 Hz source. Load resistance is 500 Ω. Calculate the value of series inductor L that limits the rms ripple current to be less than 5% of I_{DC}.





$$V_d(t) = \frac{2V_m}{\pi} - \frac{4V_m}{3\pi}\cos 2\omega t - \frac{4V_m}{15\pi}\cos 4\omega t - \frac{4V_m}{35\pi}\cos 6\omega t - \dots$$

$$i_d(t) = \frac{2V_m}{\pi R} - \frac{\frac{4V_m}{3\pi}\cos 2\omega t - \frac{4V_m}{15\pi}\cos 4\omega t - \frac{4V_m}{35\pi}\cos 6\omega t}{\sqrt{R^2 + (n\omega L)^2}\ \angle \tan^{-1}\left(\frac{n\omega L}{R}\right)}$$

$$= \frac{2V_m}{\pi R} - \frac{4V_m}{\pi\sqrt{R^2 + (n\omega L)^2}}\left[\frac{1}{3}\cos(2\omega t - \theta_2) + \frac{1}{15}\cos(4\omega t - \theta_4) + \dots\right]$$

L 0.396

$$\theta_n = \tan^{-1}\left(\frac{n\omega L}{R}\right)$$

- DC value of output current

$$I_{dc} = \frac{2V_m}{\pi R}$$

- Rms value of Ripple current

$$I_{ac}^2 = \frac{(4V_m)^2}{2\pi^2 \sqrt{(R^2 + (2\omega L)^2}} \left(\frac{1}{3}\right)^2 + \frac{(4V_m)^2}{2\pi^2 (R)^2 + (4\omega L)^2} \left(\frac{1}{15}\right)^2 + \text{---}$$

Neglecting all harmonics other than predominant second harmonic

$$I_{ac}^2 = \frac{(4V_m)^2}{2\pi^2 (R^2 + (2\omega L)^2)} \left(\frac{1}{3}\right)^2$$

$$R.F = \frac{I_{ac}}{I_{dc}} = 0.05$$

$$= \frac{\dfrac{93.37089}{\sqrt{(500)^2 + (200\pi L)^2}}}{0.3961} = 0.05$$

$$\Rightarrow \frac{93.37089}{\sqrt{(500)^2 + (200\pi L)^2}} = 0.019805$$

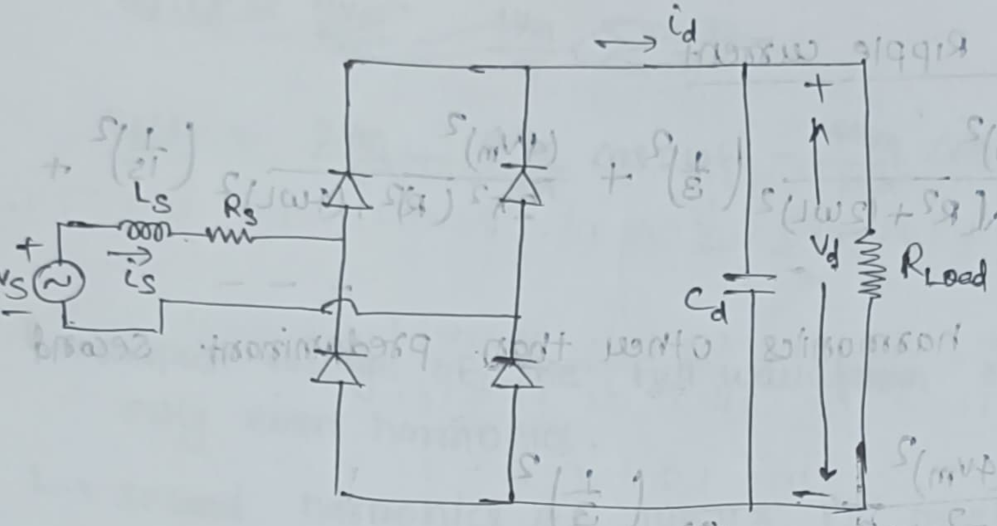
$$(500)^2 + (200\pi L)^2 = (4714.51)^2$$

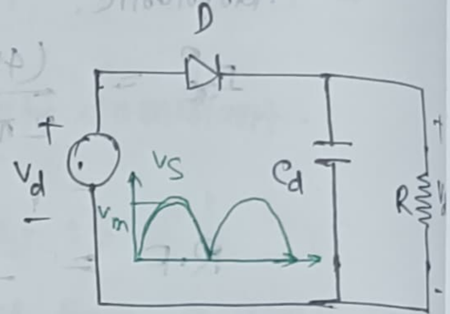
$$200\pi L = 4687.92$$

$$L = 7.4629\text{ H}$$

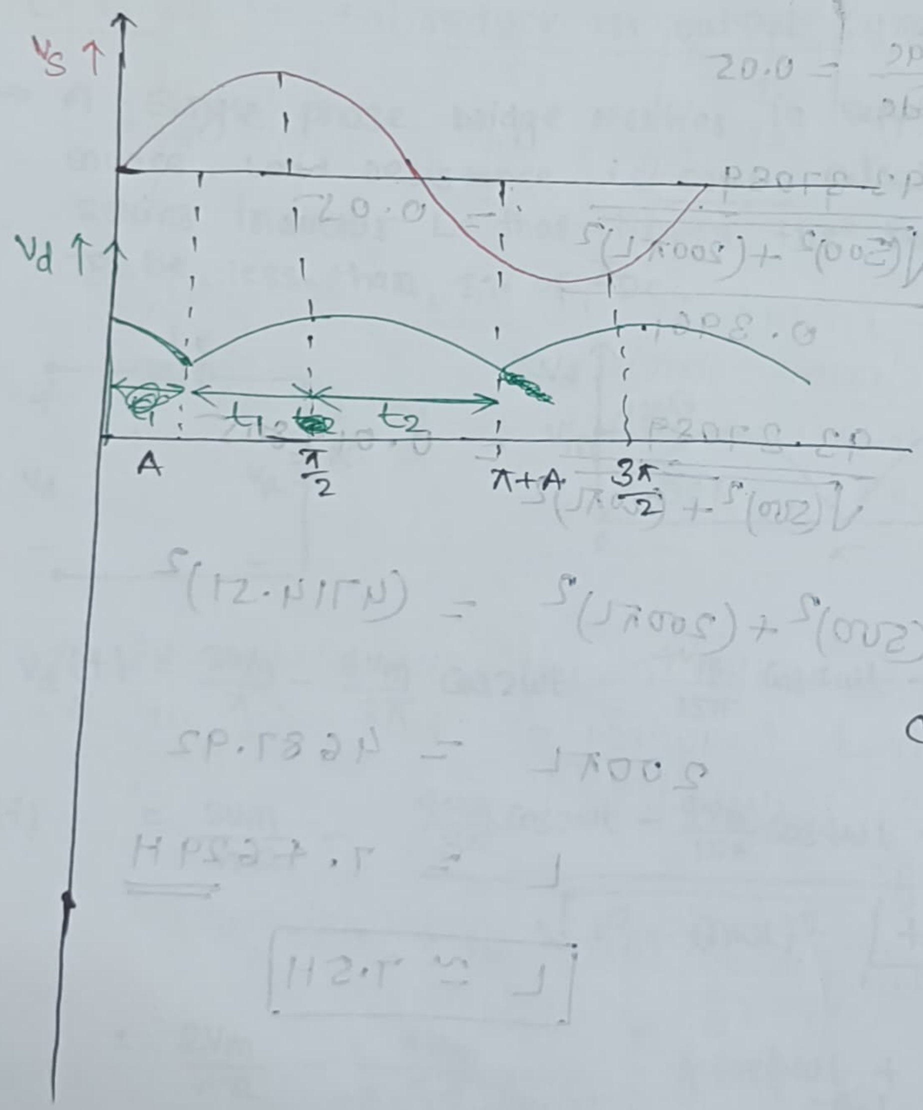
$$\boxed{L \simeq 7.5\text{H}}$$

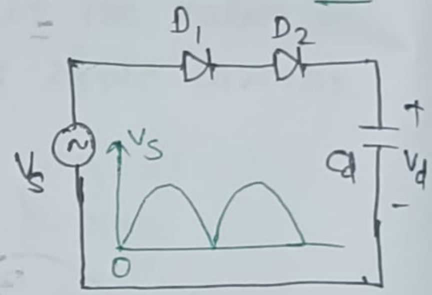
* Diode bridge rectifier with a capacitor filter to limit the amount of output voltage ripple :-



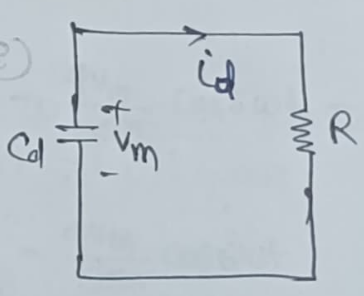


(a) ckt. model.

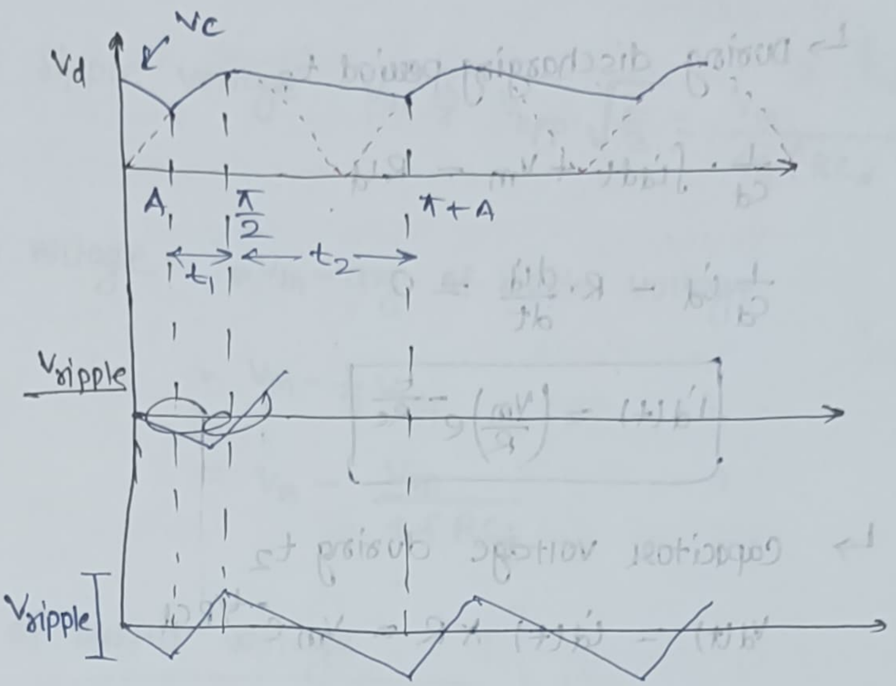




charging



discharging



waveform for full wave rectifier.

- $0 \rightarrow A$

  $V_s < V_d$ all diodes are off

  capacitor discharges to load R.

  load current $i_o = \frac{V_c}{R}$; $V_d = V_c$

- $A \rightarrow \frac{\pi}{2}$, $V_s > V_c$

  diodes D_1 and D_2 conduct (during t_1)

  $V_d = V_c = V_s$

  Capacitor gets charged to V_m.

- $\frac{\pi}{2} \rightarrow \pi + A$ $V_s < V_c$

  all diodes are off (during t_2)

  capacitor discharges to load R.

  $V_d = V_c$, $i_d = \frac{V_c}{R}$

$\pi + A$ to $\frac{3\pi}{2}$. $-V_s < V_c$: capacitor gets charge to V_m.

$V_d = V_c$, $i_d = \frac{V_c}{R}$

↳ During discharging period t_2

$$\frac{1}{C_d}\int i_d\,dt + V_m = R i_d$$

$$\frac{1}{C_d} i_d - R\cdot\frac{di_d}{dt} = 0$$

$$\boxed{i_d(t) = \left(\frac{V_m}{R}\right)e^{-\frac{t}{RC}}}$$

↳ Capacitor voltage during t_2

$$V_d(t) = i_d(t)\times R = V_m e^{-t/RC_d}$$

max^m capacitor/load voltage to V_m.

minimum capacitor/load voltage $(V_d)_{min} = V_m e^{-t_2/RC_d}$

↳ Peak to peak ripple voltage

$$V_{rpp} = V_m - V_{dmin} = V_m - V_m e^{-t_2/RC_d}$$

$$= V_m\left(1 - e^{-t_2/RC_d}\right)$$

$$\cong V_m\left[1 - \left(1 - \frac{t_2}{RC_d}\right)\right]$$ when $RC_d >> t_2$

If C_d is selected as very large then t_1 is very small as compare to t_2.

So $t_2 \simeq \left(\frac{T}{2}\right)$

So; $$V_{pp} = \frac{V_m T}{2(RC_d)} = \left(\frac{V_m}{2fRC_d}\right)$$

• Rms value of ripple voltage $V_{ac} \Rightarrow V_{rpp} \cdot \sqrt{\frac{2}{3}} = \frac{V_m}{\sqrt{6} f R C_d}$

avg. load voltage = V_m − avg. of ripple voltage

$$= V_m - \frac{1}{2} V_{rpp}$$

$$= V_m - \frac{V_m}{4 f R C_d}$$

14/09/23

↳ Ripple factor of output voltage :

$$\boxed{RF = \frac{\text{Rms of Ripple voltage}}{\text{Avg. output voltage}}}$$

$$= \frac{\cancel{V_m} \left(\frac{1}{\sqrt{6} f R C_d} \right)}{\cancel{V_m} \left(1 - \frac{1}{4 f R C_d} \right)}$$

$$= \frac{1}{\sqrt{6} \cancel{f R C_d}} \times \frac{4 \cancel{f R C_d}}{(4 f R C_d - 1)}$$

$$= \frac{4}{\sqrt{6}} \left(\frac{1}{4 f R C_d - 1} \right)$$

•
$$\boxed{RF = \frac{2\sqrt{\frac{2}{3}}}{(4 f R C_d - 1)}}$$

• C_e is selected such that RF is within the permissible limit.

$$\boxed{C_d = \frac{1}{4 f R} \left[1 + \frac{2\sqrt{\frac{2}{3}}}{R.F.} \right]}$$

Ques) A 1-ϕ bridge rectifier supplied from a 220V, 50Hz source. The load Resistance is $R = 500\Omega$. Design a C-filter so that Ripple factor of the o/p voltage is less than 5%.

Solⁿ

$V_m = 220\sqrt{2}$

$f = 50\,Hz$

$R = 500\,\Omega$

$R.F = 0.05$

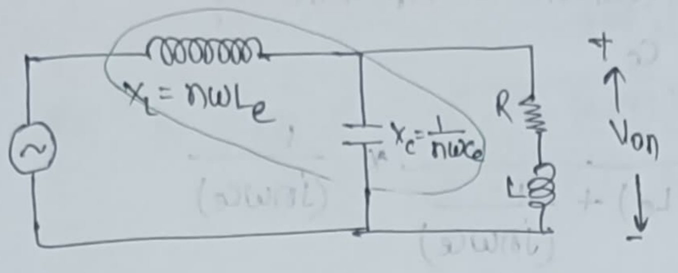
$$C_d = \frac{1}{4fR}\left[1 + \frac{2\sqrt{\frac{2}{3}}}{R.F}\right]$$

$$= \frac{1}{4 \times 50 \times 500}\left[1 + \frac{2\sqrt{\frac{2}{3}}}{0.05}\right]$$

$$= 0.0003365\ F$$

$$= 336\ \mu F.$$

* LC Filter to limit the output ripple :-



Rectifier output

$$V_d(\omega t) = \frac{2V_m}{\pi} - \frac{4V_m}{3\pi}\cos 2\omega t - \frac{4V_m}{15\pi}\cos 4\omega t - \frac{4V_m}{35\pi}\cos 6\omega t - \cdots$$

↳ • For nth harmonic to pass through the capacitor,

• the capacitance impedance must be much smaller than that of the load.

$$\boxed{\sqrt{R^2+(n\omega L)^2} >> \frac{1}{n\omega C_e}}$$

this condition is satisfied

When the most dominant second harmonic,

$$\sqrt{R^2+(2\omega L)^2} = \frac{10}{2\omega C_e}$$

• C_e is selected such that this condition is satisfied.

$$C_e = \frac{5}{\omega\sqrt{R^2+(2\omega L)^2}}$$

$$C_e = \frac{10}{4\pi f\sqrt{R^2+(4f\pi L)^2}}$$

↳ load impedance must be very large compared to Ce.

For nth harmonic current, load behaves as open circuit.

↳ ~~The~~ RI

The Rms value of harmonic component of voltage accross the Ce:

$$V_{on} = (V_d)_{in} * \frac{1}{(jwnL_e) + \frac{1}{(jnwC_e)}} * \frac{1}{(jnwC_e)}$$

$$= (V_d)_{in} * \frac{1}{-(nw)^2 L_e C_e + 1}$$

$$V_{on} = (V_d)_{in} * \frac{1}{1 - n^2 w^2 L_e C_e}$$

Total harmonic Rms value of ripple voltage due to all harmonics —

$$(V_o)_{ripple} = \left[\sum_{n=2,4,6}^{\infty} (V_{on})^2 \right]^{1/2}$$

considering only the second harmonic —

$$(V_o)_{ripple} = V_{o2} = \frac{-1}{(2w)^2 L_e C_e - 1} (V_d)_2$$

$$\boxed{V_{d_2} = \frac{4V_m}{3\pi\sqrt{2}} \qquad V_{dc} = \frac{2V_m}{\pi}}$$

$$R.f. = \left(\frac{V_{o2}}{V_{dc}} \right)$$

$$= \frac{\frac{2\cancel{4}V_m}{3\cancel{\pi}\sqrt{2}} \left(\frac{1}{1-(2w)^2 L_e C_e} \right)}{\frac{\cancel{2V_m}}{\cancel{\pi}}}$$

$$\boxed{R.f. = \frac{\sqrt{2}}{3} \left(\frac{1}{1-(2w)^2 L_e C_e} \right)}$$

$$R.F. = \frac{\sqrt{2}}{3}\left(\frac{1}{1-(2\omega)^2\,\frac{10}{4\pi f\sqrt{R^2+(4\pi f L)^2}}\times Le}\right)$$

$$R.F = \frac{\sqrt{2}}{3}\left[\frac{\sqrt{R^2+(4\pi f L)^2}}{\sqrt{R^2+(4\pi f L)^2}-10Le}\right]$$

$$\frac{3\,R.F}{\sqrt{2}} = \frac{\sqrt{R^2+(4\pi f L)^2}}{\sqrt{R^2+(4\pi f L)^2}-10Le}$$

$$\frac{3}{\sqrt{2}}(R.f.) = \frac{1}{1-\frac{10Le}{\sqrt{R^2+(4\pi f L)^2}}}$$

$$1-\frac{10Le}{\sqrt{R^2+(4\pi f L)^2}} = \frac{\sqrt{2}}{3R.F}$$

$$1-\frac{\sqrt{2}}{3R.F} = \frac{10Le}{\sqrt{R^2+(4\pi f L)^2}}$$

$$\boxed{Le = \frac{\sqrt{R^2+(4\pi f L)^2}}{10}\left[1-\frac{\sqrt{2}}{3RF}\right]}$$

Ques An LC filter is used to reduce the amount of ripple content at the output of a 1-ϕ full wave rectifier, the load Resistance $R = 40\Omega$ & load inductance $L = 10mH$ & source is 220V, 50Hz. Design an LC filter so that R.F of the o/p is 10%.

Soln

$R = 40\Omega$

$L = 10mH$

$V_m = 220\sqrt{2}$

$f = 50Hz$

$$C_e = \frac{10}{4\pi f\sqrt{(40)^2 + (4\pi * 50 * 10 \times 10^{-3})^2}}$$

$$= \frac{10}{4 * \pi * 50 \cdot \sqrt{}}$$

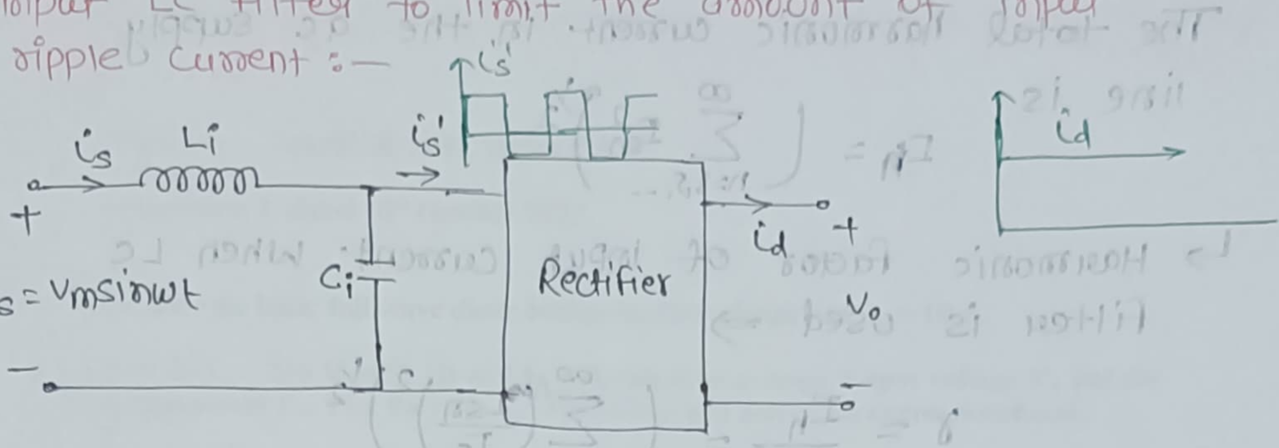
$$C_e = 393\,\mu F$$

$$L_e = \frac{\sqrt{R^2 + (4\pi f L)^2}}{10}\left[1 - \frac{\sqrt{2}}{3 * 0.1}\right]$$

$$= \frac{\sqrt{(40)^2 + (200\pi * 10^{-2})^2}}{10}\left[1 - \frac{\sqrt{2}}{0.3}\right]$$

$$=$$

* input LC filter to limit the amount of input ripple current :—



↳ i_{sn} is the nth harmonic current in the source.

↳ i_{sn}' is the nth harmonic current at the input of rectifier

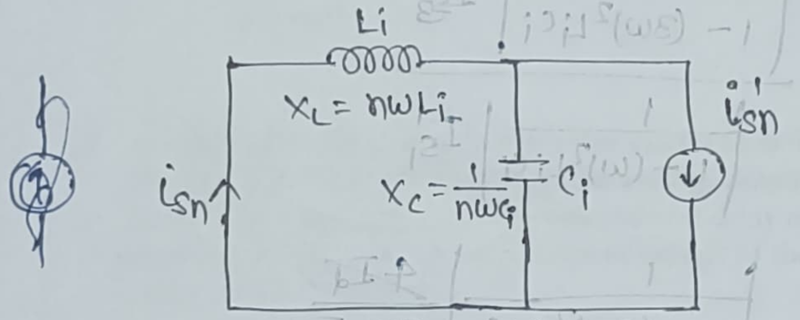


fig. — eq. ckt. for nth harmonic current.

by current divider rule

$$i_{sn} = \frac{i_{sn}' * \frac{1}{jn\omega C_i}}{(jn\omega L_i) + \frac{1}{(jn\omega C_i)}}$$

$$= i_{sn}' * \frac{1}{1 - (n\omega)^2 L_i C_i}$$

$$= \left| \frac{1}{1 - (n\omega)^2 L_i C_i} \right| i_{sn}'$$

The total harmonic current in the ac supply line is

$$I_h = \left(\sum_{n=3,5,\ldots}^{\infty} I_{sn}^2 \right)^{1/2}$$

↳ Harmonic factor of input current when LC filter is used ⇒

$$\gamma = \frac{I_h}{I_{s1}} = \left(\sum_{n=3,5}^{\infty} \left(\frac{I_{sn}}{I_{s1}} \right)^2 \right)^{1/2}$$

By considering only the third harmonic —

$$\gamma = \frac{I_3}{I_{s1}} = \left(\frac{I_{s3}}{I_{s1}} \right)$$

$$= \frac{\left| \frac{1}{1-(3\omega)^2 L_i C_i} \right| I'_{s3}}{\left| \frac{1}{1-(\omega)^2 L_i C_i} \right| I'_{s1}}$$

$$= \frac{\left| \frac{1}{1-(3\omega)^2 L_i C_i} \right| \frac{4 I_d}{3\pi \sqrt{2}}}{\left| \frac{1}{1-\omega^2 L_i C_i} \right| \cdot \frac{4 I_d}{\pi \sqrt{2}}}$$

$$=$$

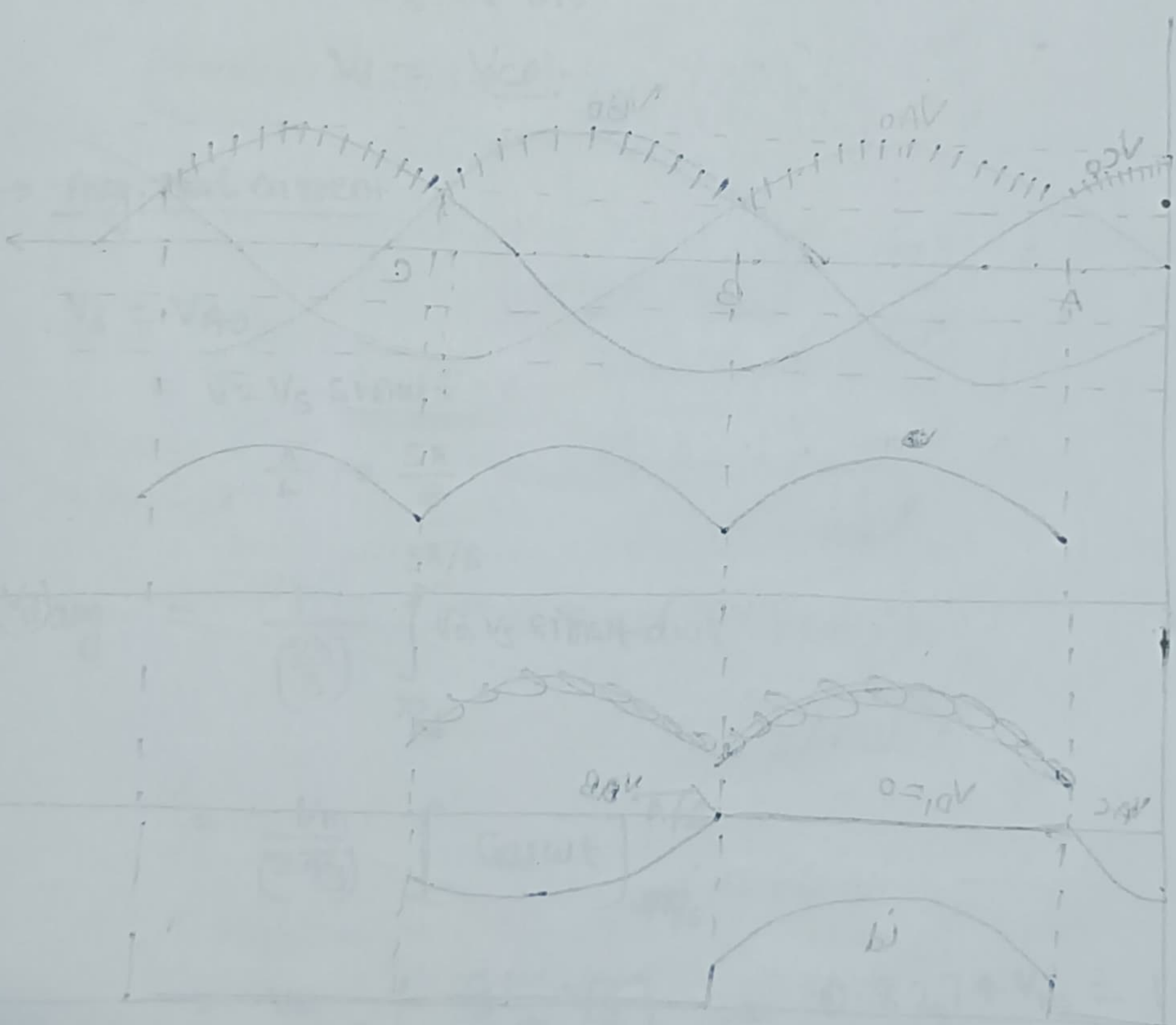
For specified value of Harmonic factor of i/p current γ, L_i, C_i can be calculated.

Choose a convenient value of C_i and then obtain the value of L_i.

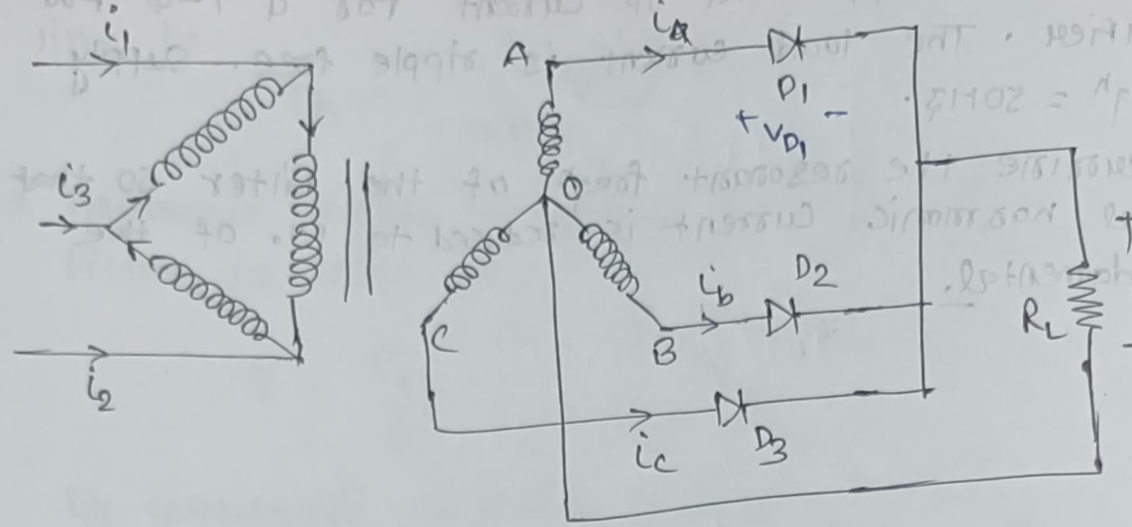
$$* \boxed{\text{Filter Resonant freq}^n = \frac{1}{\sqrt{L_i C_i}}}$$

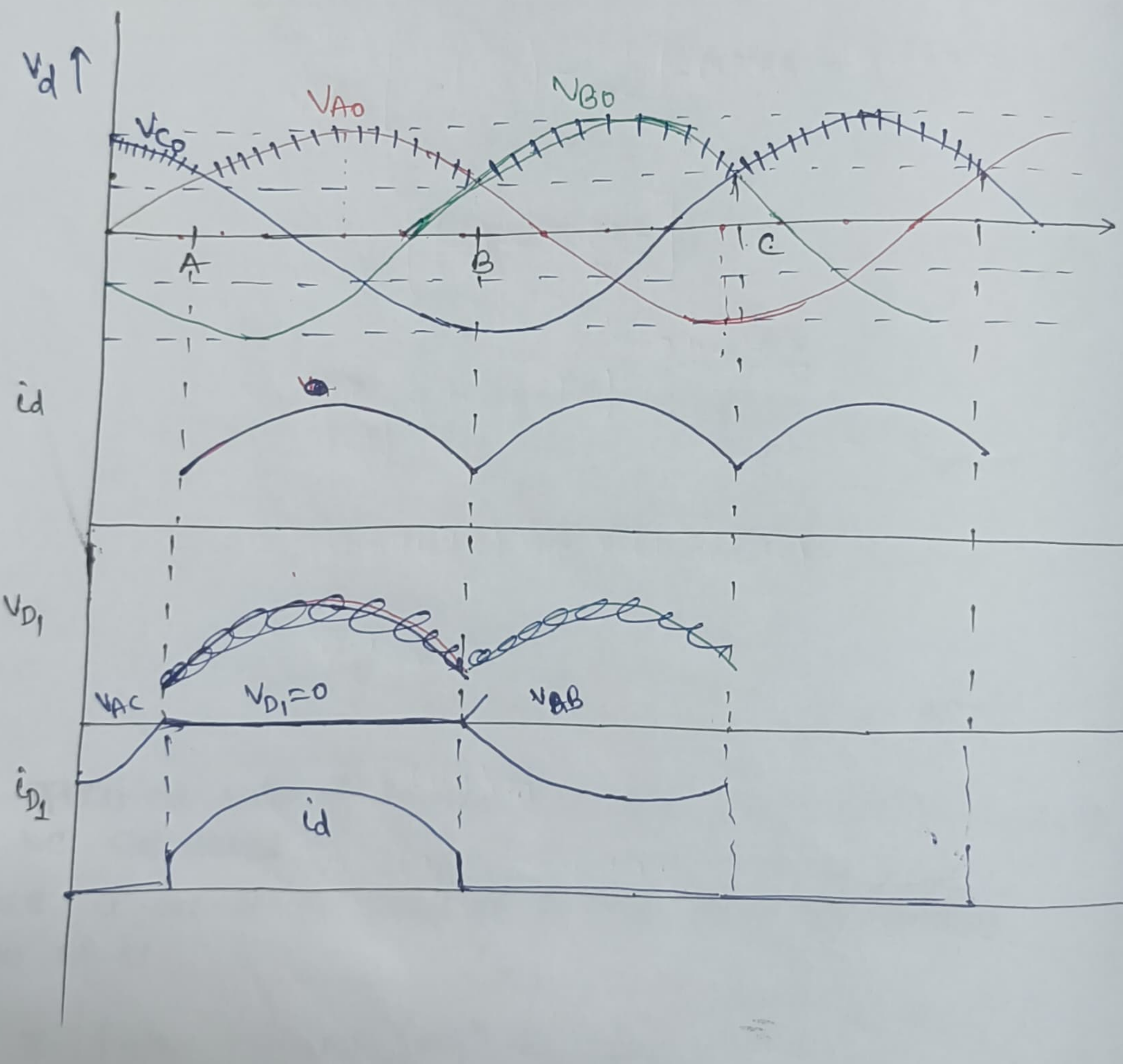
Ques An LC filter is to be designed to limit the amount of ripple in i/p current for a 1-Φ FW Rectifier. The load current is ripple free. supply freqⁿ = 50Hz.

determine the resonant freqⁿ of the filter so that total harmonic current is reduced to 1% of the fundamental.



* 3-Φ Half wave uncontrolled Rectifier Circuit :-





- $A \to B$: V_{AO} is most positive
  D_1 is ON
  $V_d = V_{AO}$

For time greater than B
$V_{BO} > V_{AO}$
$\therefore$ D_2 becomes F.B $\to$ ON.

- $B \to C$: $D_2 \to$ on
  $V_d = V_{BO}$

- $C \to A$: V_{CO} is most positive
  D_3 is ON
  $V_d = V_{CO}$

↳ Avg. load current

$V_d = V_{AO}$
$= \sqrt{2} V_s \sin\omega t$
$\frac{\pi}{6} \to \frac{5\pi}{6}$

$$(V_d)_{avg} = \frac{1}{\left(\frac{2\pi}{3}\right)} \int_{\pi/6}^{5\pi/6} \sqrt{2} V_s \sin\omega t \, d\omega t$$

$$= \frac{V_m}{(2\pi/3)} \left[\cos\omega t \right]_{5\pi/6}^{\pi/6}$$

$$= \frac{V_m}{\left(\frac{2\pi}{3}\right)} \cdot \left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \right) = 0.8274 V_m = \underline{\underline{1.17 V_s}}$$

$$= \frac{V_m}{\left(\frac{2\pi}{3}\right)} (\sqrt{3}) = \left(\frac{3\sqrt{3}}{2\pi}\right) V_m = \frac{3\sqrt{3}}{2\pi} \sqrt{2} V_s = \left(3 \sqrt{\frac{3}{2}} \frac{V_s}{\pi}\right)$$

↳ RMS value of load voltage

$$(V_d)_{rms} = \left[\frac{1}{\frac{2\pi}{3}} \int_{\pi/6}^{5\pi/6} V_m^2 \sin^2\omega t \, d\omega t \right]^{1/2}$$

$$= \frac{V_m^2}{\left(\frac{2\pi}{3}\right)} \int_{\pi/6}^{5\pi/6} \left(\frac{1 - \cos 2\omega t}{2} \right) d\omega t$$

$$= \left[\frac{V_m^2}{(2\pi/3)} \cdot \frac{1}{2} \left[\left(\frac{5\pi}{6} - \frac{\pi}{6} \right) - \frac{1}{2} \left(\sin \frac{5\pi}{3} - \sin \frac{\pi}{3} \right) \right] \right]^{1/2}$$

$$= \frac{V_m}{\sqrt{\frac{2\pi}{3}}} \left[\frac{1}{\cancel{2}} \left\{ \left(\frac{\cancel{2}\pi}{3} \right) - \frac{1}{2} \left(-0.866 - 0.866 \right) \right\} \right]^{1/2}$$

$$= \frac{V_m}{\sqrt{\left(\frac{2\pi}{3}\right)}} \left[\underbrace{\left(\frac{\pi}{3} \right) + \frac{0.866}{2}}_{1.2166} \right]$$

$$= \frac{V_m}{\sqrt{\left(\frac{2\pi}{3}\right)}} \cdot (1.38318) = 0.84065 V_m$$

$$= (0.69098) V_m * (1.38318)$$

$$= \cancel{0.955}$$

$$= 0.8406 V_m$$

$$= 1.188 V_s \simeq 1.19 V_s.$$

↳ load voltage f.f. $= \dfrac{1.19 V_s}{1.177 V_s} \approx 1.01$

↳ Ripple factor $= \gamma = \dfrac{\sqrt{V_{d\,rms}^2 - V_{d\,avg}^2}}{(V_d)_{avg}} \approx \underline{0.185}$

↳ Ripple freqⁿ $= 3 *$ (supply freqⁿ)

↳ PIV diode $= \sqrt{3}\, V_s \sqrt{2} = \sqrt{6}\, V_s$

$= \underline{(\sqrt{3}\, V_m)}$

↳ as load is Resistive,

Avg. load current $= (I_d)_{avg} = \dfrac{(V_d)_{avg}}{R_L}$

$(i_{d1})_{avg} \Rightarrow$

↳ Avg. diode current Rating $= \dfrac{1}{3}\left\{\dfrac{(V_d)_{avg}}{R_L}\right\}.$

↳ Rms load current $= (I_d)_{rms} = (1.01)(I_d)_{avg}.$

↳ Total load power $= P = (I_d)_{rms}^2 R_L.$

Let I_D is the Rms value of current through a diode.

$\boxed{3 I_D^2 R_L = (I_d)_{rms}^2 R_L} \leftarrow$ total power flow through transformer.

~~$I_D = 0.58 (I_d)_{avg}$~~. $\boxed{I_D = 0.58\,(I_d)_{rms}}$

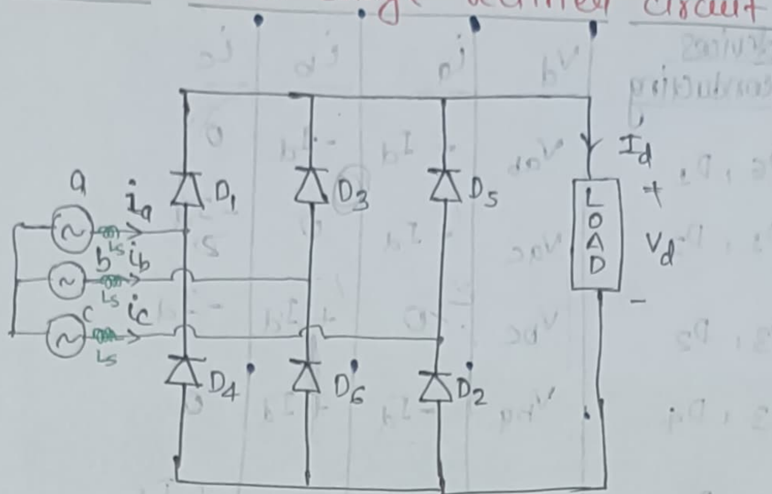
↳ Each diode must carry an Rms value of current equal to 58% of avg. load current.

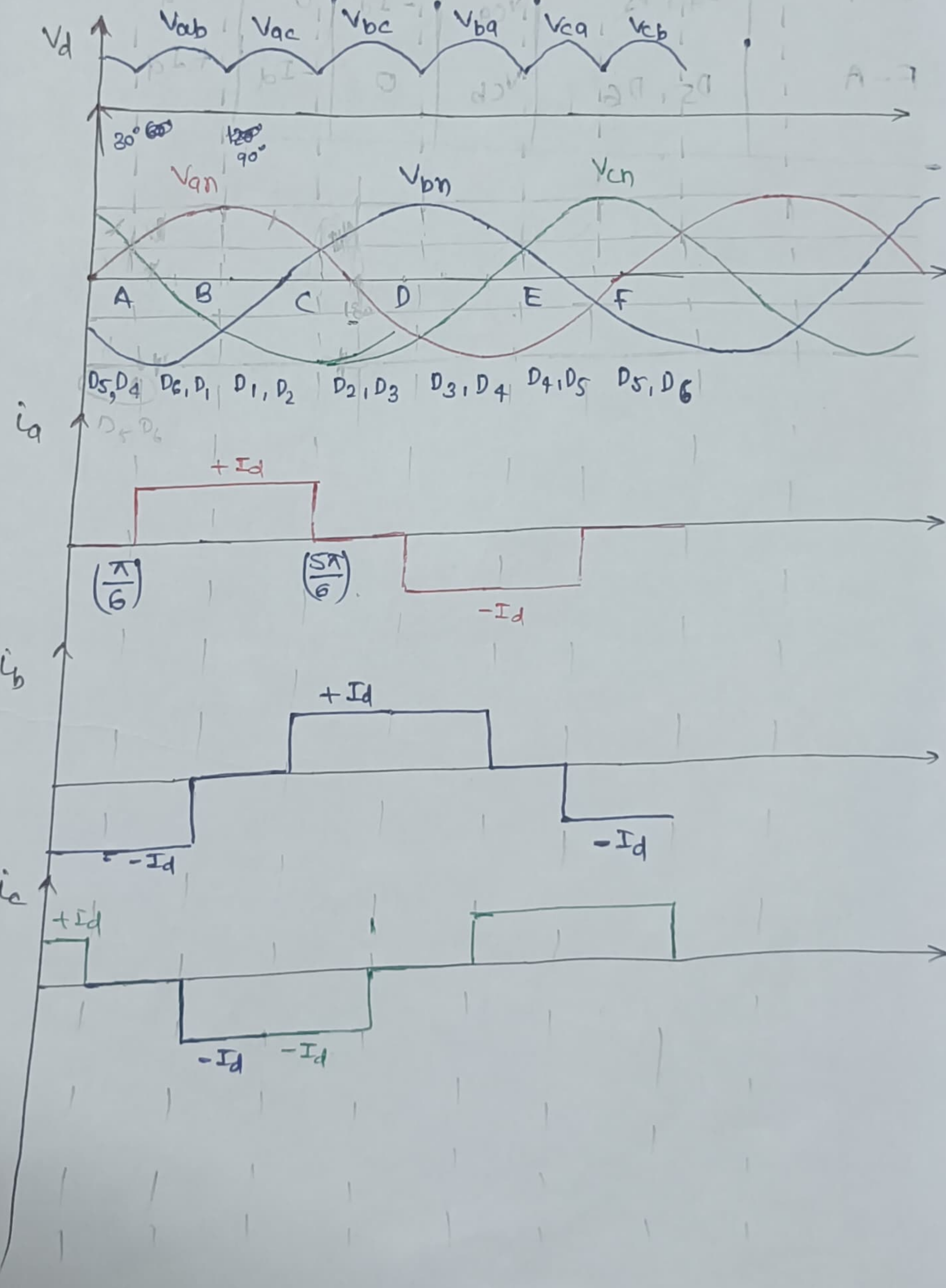
- If the rated transformer secondary RMS current is choosen same as that of diode RMS current.
- Transformer secondary voltampere $= 3V_S I_D$

$$= 3\,\frac{(V_d)_{avg}}{1.17} \times 0.58\,(I_d)_{avg}$$

$$= 1.5\,(V_d)_{avg} \cdot (I_d)_{avg}$$

* 3-Φ Fullwave Diode bridge rectifier circuit.





interval	devices conducting	V_d	i_a	i_b	i_c
A - B	D_6, D_1	V_{ab}	$+I_d$	$-I_d$	0
B - C	D_1, D_2	V_{ac}	$+I_d$	0	$-I_d$
C - D	D_3, D_2	V_{bc}	0	$+I_d$	$-I_d$
D - E	D_3, D_4	V_{ba}	$-I_d$	$+I_d$	0
E - F	D_4, D_5	V_{ca}	$-I_d$	0	$+I_d$
F - A	D_5, D_6	V_{cb}	0	$-I_d$	$+I_d$

* Avg. load voltage

$$V_d = \frac{1}{(2\pi/6)} \int_{\pi/6}^{\pi/2} (V_{ab})\, d\omega t$$

$$= \frac{1}{\left(\frac{2\pi}{6}\right)} \int_{\pi/6}^{\pi/2} [V_m \sin\omega t - V_m \sin(\omega t - 120^\circ)]\, d\omega t$$

$$= \frac{V_m}{\left(\frac{2\pi}{6}\right)} \left[(\cos\omega t)_{\pi/2}^{\pi/6} - \{\cos(\omega t - 120^\circ)\}_{\pi/2}^{\pi/6} \right]$$

$$\left(\frac{\sqrt{3}}{2} - (0 - 0.866)\right) \quad \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = \sqrt{3}$$

$$\frac{\sqrt{3}}{(\pi/3)} V_m = \frac{\sqrt{3}}{(\pi/3)} \sqrt{2} V_s$$

$$= \left(\frac{3}{\pi}\right) \sqrt{2}\, V_{LL}$$

$$= 1.35\, V_{LL}$$ (V_{LL} is the Rms value of line to line voltage)

* Rms value of current :-

$$(I_d)_{rms} = \left(\sqrt{\frac{2}{3}}\right) I_d$$

- instantaneous value of fundamental component of i_a :-

$$i_{a_1} = a_1 \cos\omega_1 t + b_1 \sin\omega_1 t$$

$$a_1 = \frac{1}{\pi/2} \int_{\pi/6}^{5\pi/6} I_d \cos\omega t\, d\omega t = 0$$

$$b_1 = \frac{1}{\pi/2} \int_{\pi/6}^{5\pi/6} I_d \sin\omega t\, d\omega t$$

$$= \frac{2}{\pi} \left[I_d (\cos\omega t)_{5\pi/6}^{\pi/6} \right]$$

$$= \frac{2 I_d}{\pi} \left[\frac{\sqrt{3}}{2} - \left(-\frac{\sqrt{3}}{2}\right) \right]$$

$$= \frac{2 I_d}{\pi} (\sqrt{3})$$

$$b_1 = \frac{2\sqrt{3}}{\pi} I_d$$

$$\phi_1 = \tan^{-1}\left(\frac{a_1}{b_1}\right) = 0$$

$$i_{a_1} = b_1 \sin\omega t$$

$$i_{a_1} = \frac{2\sqrt{3}}{\pi} I_d \sin\omega t$$

↳ Rms value of fundamental component of source line current

$$= \frac{\left(\frac{2\sqrt{3}}{\pi}\right)}{\sqrt{2}} I_d$$

$$= \underline{\underline{\frac{\sqrt{6}}{\pi} I_d}}$$

↳ Rms value of h^{th} harmonic components of line current i_{ah} :-

$$i_{ah} = \left(\frac{i_{a_1}}{h}\right) \qquad h = 5, 7, 11, 13, \text{ etc.}$$

- even and triplen harmonics are absent.

↳ Source side apparent power $= 3 V_s I_a$

$$= 3 V_s \sqrt{\frac{2}{3}} I_d$$

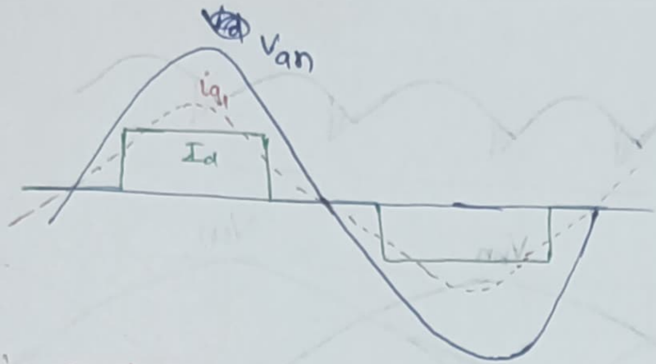
↳ source side active power $= 3 V_s I_{a_1} \cos\phi_1$

$$= 3 V_s \frac{\sqrt{6}}{\pi} I_d.$$

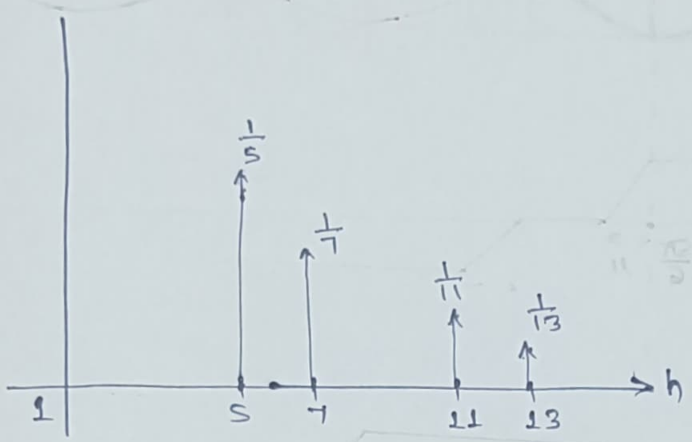
↳ source side power factor $= \dfrac{\cancel{3} \cancel{V_s} \frac{\sqrt{6}}{\pi} \cancel{I_d}}{\cancel{3} \cancel{V_s} \sqrt{\frac{2}{3}} \cancel{I_d}}$

$$= \frac{\sqrt{6}}{\pi} * \frac{\sqrt{3}}{\sqrt{2}} = \left(\frac{3}{\pi}\right)$$

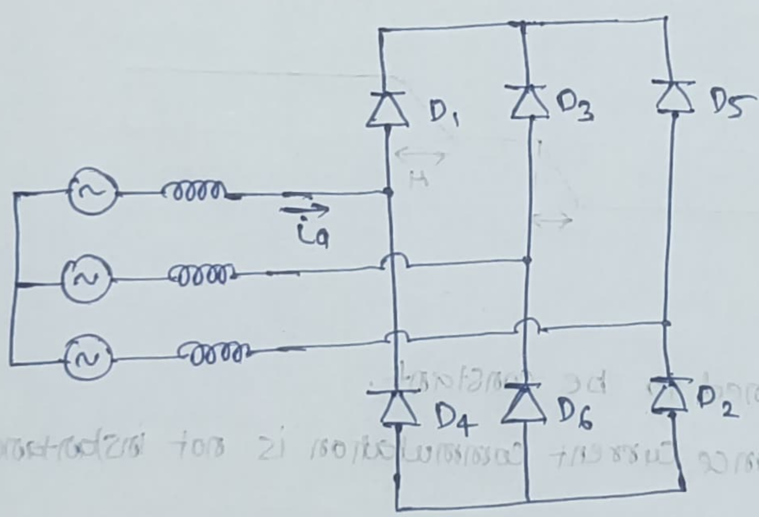
$$= \underline{0.955}$$

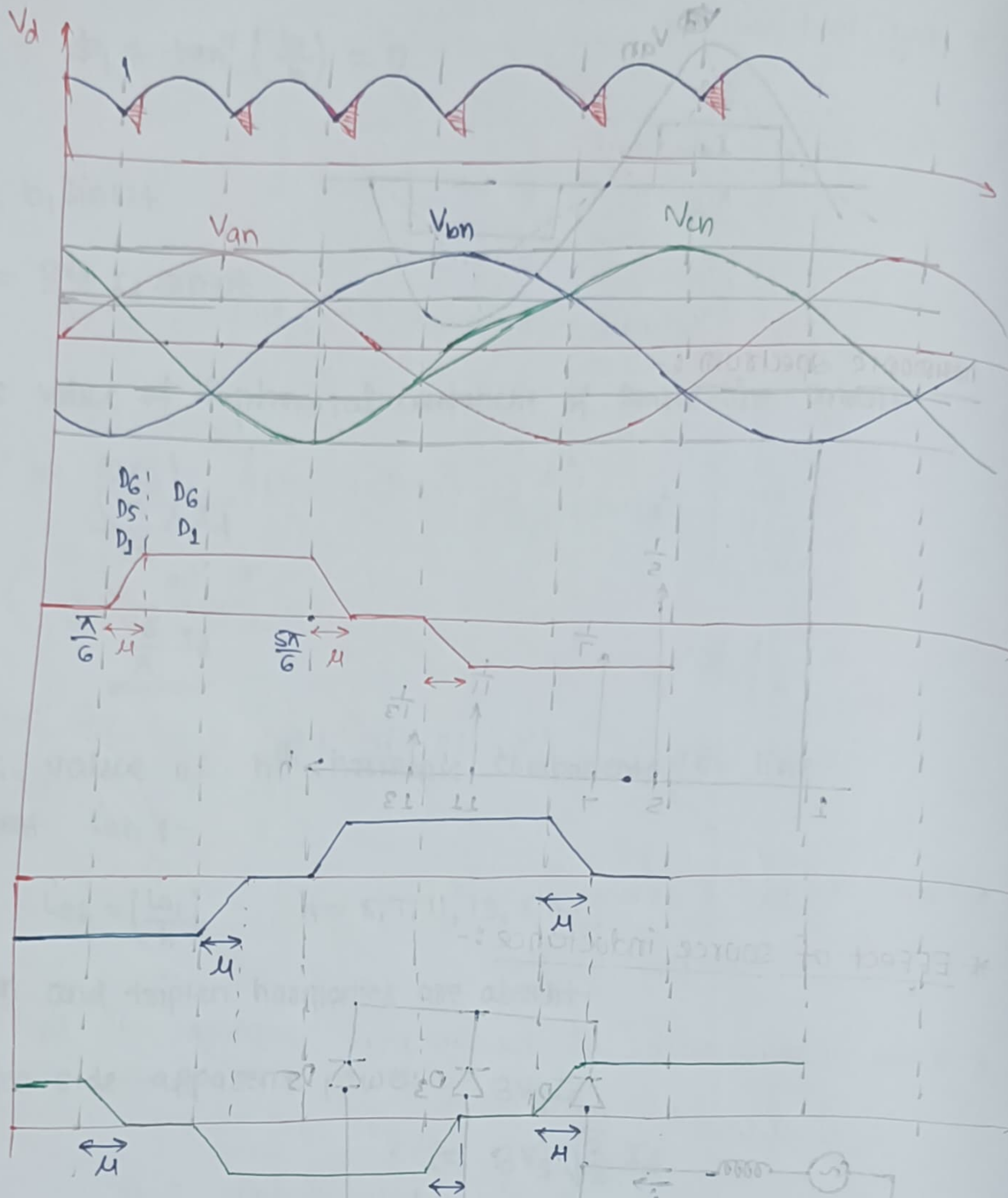


harmonic spectrum :-



* EFFect of source inductance :-



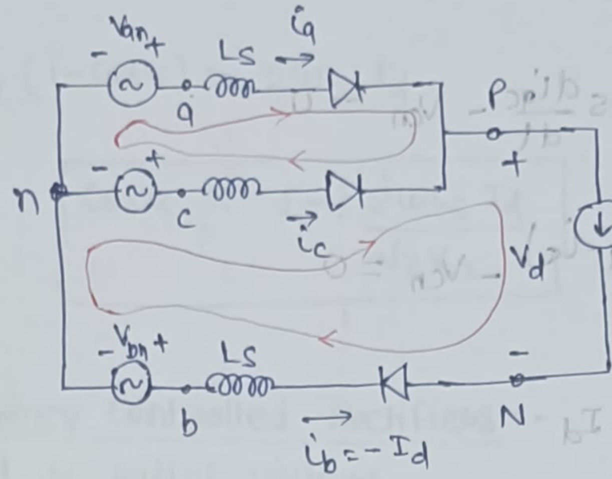


↳ output current is assumed to be constant.

↳ Due to source inductance, current commutation is not instantaneous.

- For $\omega t < \pi/6$, D_5 and D_6 were conducting.
- For $\omega t > \pi/6$, V_{an} is greater than V_{cn}.
- D_1 starts conducting. Due to L_s, i_a rises from zero to I_d through D_1 gradually in a period μ i.e. during $\left(\frac{\pi}{6}\right)$ to $\left(\frac{\pi}{6}+\mu\right)$
- During same interval, i_c decreases from I_d to zero through D_5.
- During $\left(\frac{\pi}{6}\right)$ to $\left(\frac{\pi}{6}+\mu\right)$, D_5, D_6 and D_1 conduct.

During $\left(\frac{\pi}{6}\right)$ to $\left(\frac{\pi}{6}+\mu\right)$ eq. ckt :-



eq. ckt. diagram during μ.

$$-V_{an} + L_s\frac{di_a}{dt} + V_d + L_s\frac{di_b}{dt} + V_{bn} = 0$$

$\Rightarrow$ ~~Van = d~~ $\quad V_d = V_{an} - V_{bn} - L_s\frac{di_a}{dt}$ (during commutation period.)

• avg. output voltage $= V_d = \frac{1}{\left(\frac{2\pi}{6}\right)}\left[\int_{\pi/6}^{\pi/6+\mu}\left(V_{an} - V_{bn} - L_s\frac{di_a}{dt}\right)d\omega t + \int_{\pi/6+\mu}^{\pi/2}(V_{an} - V_{bn})\,d\omega t\right]$

$$= \frac{1}{(2\pi/6)}\left[\int_{\pi/6}^{\pi/2}(V_{an} - V_{bn})\,d\omega t + \int_{\pi/6}^{\pi/6+\mu}\left(-\omega L_s\frac{di_a}{d\omega t}\right)d\omega t\right]$$

$$= (1.35\,V_{LL}) + \frac{1}{\left(\frac{2\pi}{6}\right)}\cdot\left\{-\omega L_s i_a\right.$$

$$= 1.35\,V_{LL} - \cancel{\frac{\omega L_s i_a \mu}{\left(\frac{2\pi}{6}\right)}}\left(\frac{3\omega L_s}{\pi}\right) I_d$$

* overlap angle μ

$$V_{an} - L_s \frac{di_a}{dt} + L_s \frac{di_c}{dt} - V_{cn} = 0$$

$$V_{an} - L_s \frac{d(i_a - i_c)}{dt} - V_{cn} = 0$$

$$i_c + i_a = I_d$$

$$V_{an} - V_{cn} = L_s \frac{d}{dt}(i_a - i_c)$$

$$= L_s \frac{d}{dt}(i_a - I_d + i_a)$$

$$= L_s \frac{d}{dt}(2i_a - I_d)$$

$$= L_s \frac{d}{dt}(2i_a) - 0 \qquad \left(\because \frac{dI_d}{dt} = 0\right)$$

$$V_{an} - V_{cn} = 2L_s \frac{di_a}{dt}$$

$$(V_{an} - V_{cn})\, d\omega t = \omega 2L_s\, di_a$$

$$\int_{\pi/6}^{\pi/6+\mu} (V_{an} - V_{cn})\, d\omega t = \int_0^{I_d} 2\omega L_s\, di_a$$

$$\int_{(\pi/6)}^{(\pi/6+\mu)} (V_m \sin\omega t - V_m \sin(\omega t - \cancel{240^\circ}\ {+120^\circ}))\, d\omega t = 2\omega L_s I_d$$

$$V_m\left[\cos\frac{\pi}{6} - \cos\left(\frac{\pi}{6}+\mu\right) - \cos\left(\frac{5\pi}{6}\right) + \cos\left(\frac{5\pi}{6}+\mu\right)\right] = 2\omega L_s I_d$$

$$V_m\left(2\cos\frac{\pi}{6} - 2\cos\left(\frac{\pi}{6}+\mu\right)\right) = 2\omega L_s I_d$$

$$2V_m\left[\cos\frac{\pi}{6} - \left\{\cos\frac{\pi}{6}\cdot\cos\mu - \sin\frac{\pi}{6}\cdot\sin\mu\right\}\right] = 2\omega L_s I_d$$

$$2V_m \frac{\sqrt{3}}{2}(1-\cos\mu) = 2\omega L_s I_d$$

$$\sqrt{3}V_m(1-\cos\mu) = 2\omega L_s I_d$$

$$\boxed{\cos\mu = 1 - \frac{2\omega L_s I_d}{\sqrt{2}V_{LL}}}$$

6/09/23

* Linear frequency Controlled Rectifiers :-

- Controlled dc output voltage
  - ↳ varies from +ve max^m to -ve max^m.
- controllable devices like thyristors, IGBT BJT are used.
- Commutation of the devices depends on the ac side line freq^n.
- Hence the name line freq^n rectifiers.
- DC side current is always +ve.
- DC side voltage is +ve (or) -ve.
- +ve voltage and +ve current at dc side indicates rectification mode. power flows from ac side to dc side.

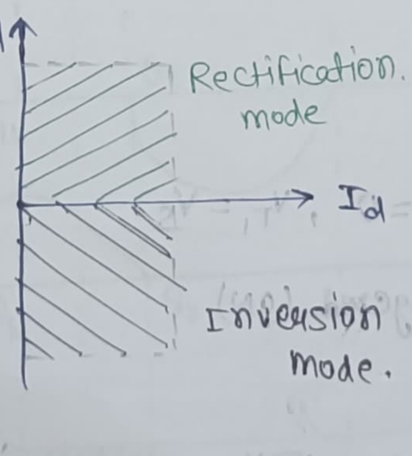
⇒ operating point lie in the first quadrant of $V_d - I_d$ plane

⇒ V_d (avg. voltage of dc side)

⇒ I_d (avg. current at DC side)

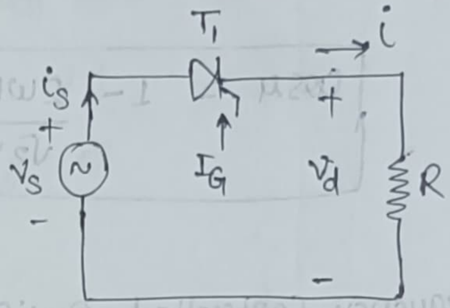
- +ve current and -ve voltage at dc side indicates inversion mode. power flow from dc side to ac side

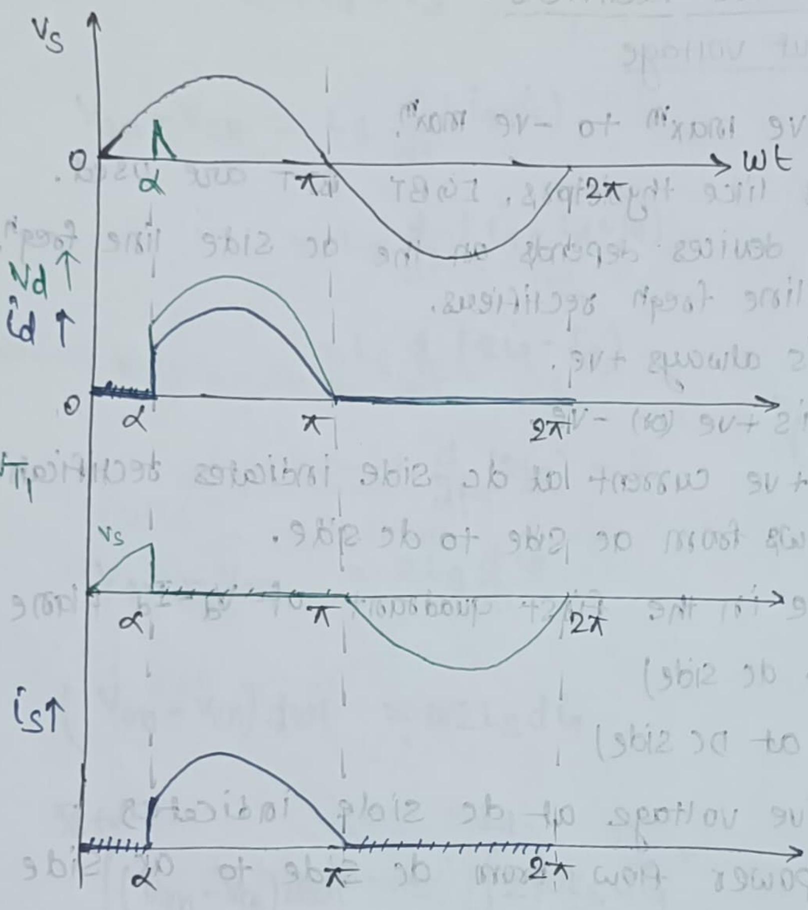
⇒ operating points lie in the fourth quadrant of $V_d - I_d$ plane.



* Single phase Half wave controlled rectifier :-

* R-load :-





From $0 \to \alpha$

T_1 is off

$i = 0$, $V_d = 0$, $i_s = 0$; $V_{T_1} = V_s$

At $\omega t = \alpha$, T_1 is triggered (ON)

↳ $\alpha \to \pi$

T_1 is ON

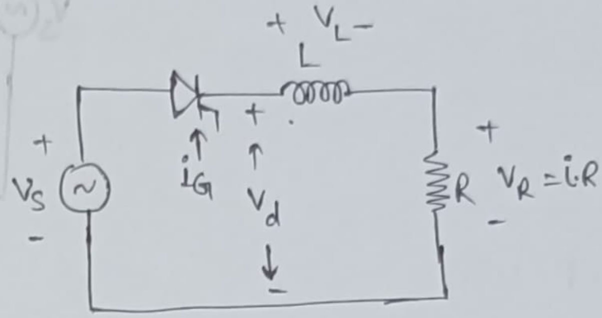
- $V_d = V_s$, $i = \frac{V_d}{R} = \frac{V_s}{R} = \frac{\sqrt{2} V_s}{R} \sin \omega t$, $i_s = i$, $V_T = 0$

• at $\omega t = \pi$, $i = 0$. T_1 becomes off

↳ $\pi - 2\pi$

T_1 is off

$i = 0$, $V_d = 0$, $V_{T_1} = V_s$; $i_s = 0$



* R-L load :-

$0 \to \alpha$ (device is off)

$i = 0$, $V_d = 0$, $V_L = 0$, $V_R = 0$

$V_T = V_s$.

$\alpha \to \theta_2$ (Thyristor is on)

$$V_d = Ri(t) + L\frac{di(t)}{dt} = V_s = \sqrt{2}V_s \sin\omega t$$

$i =$

$V_R = Ri(t)$

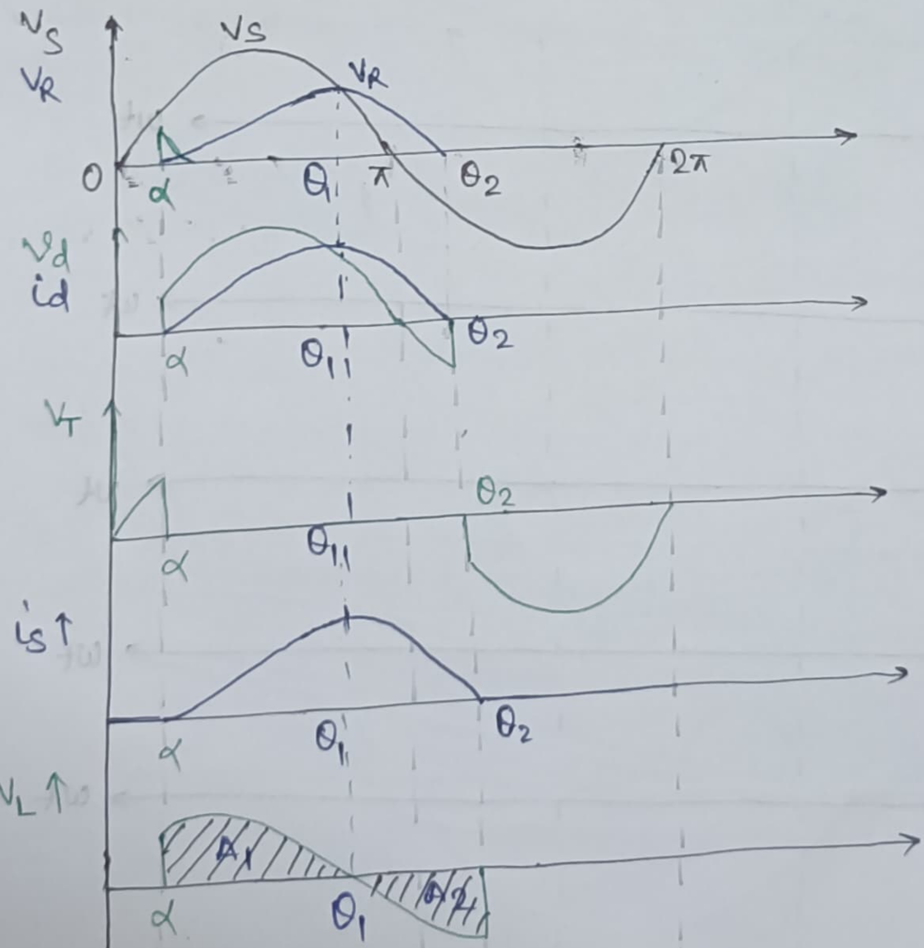
$V_L(t) = V_s - V_R(t)$

at $\omega t = \theta_1$

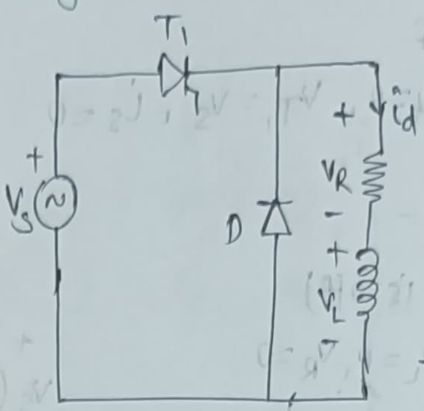
$\frac{di}{dt} = 0$, $V_L = 0$, $V_R = V_s$

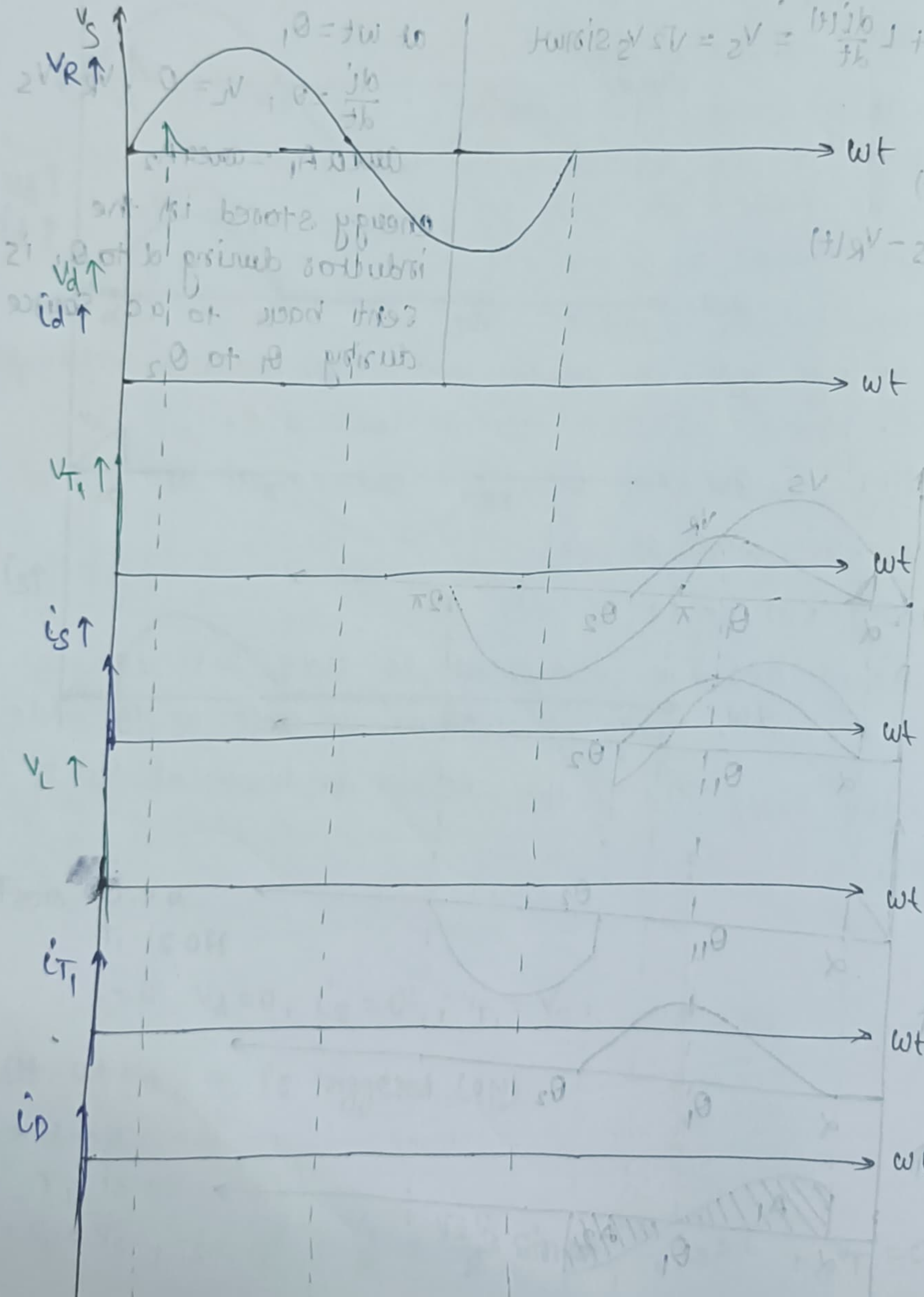
area A_1 = area A_2

energy stored in the inductor during α to θ_1 is sent back to ac source during θ_1 to θ_2.

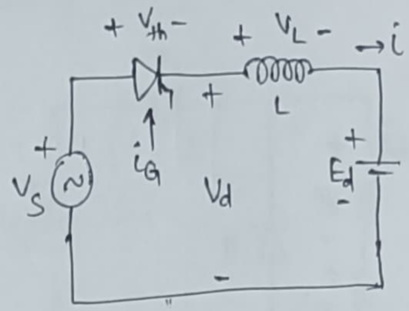


* R-L load with a freewheeling diode:-

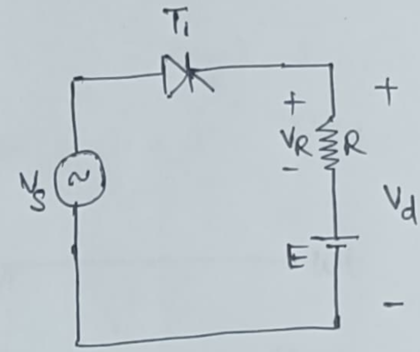


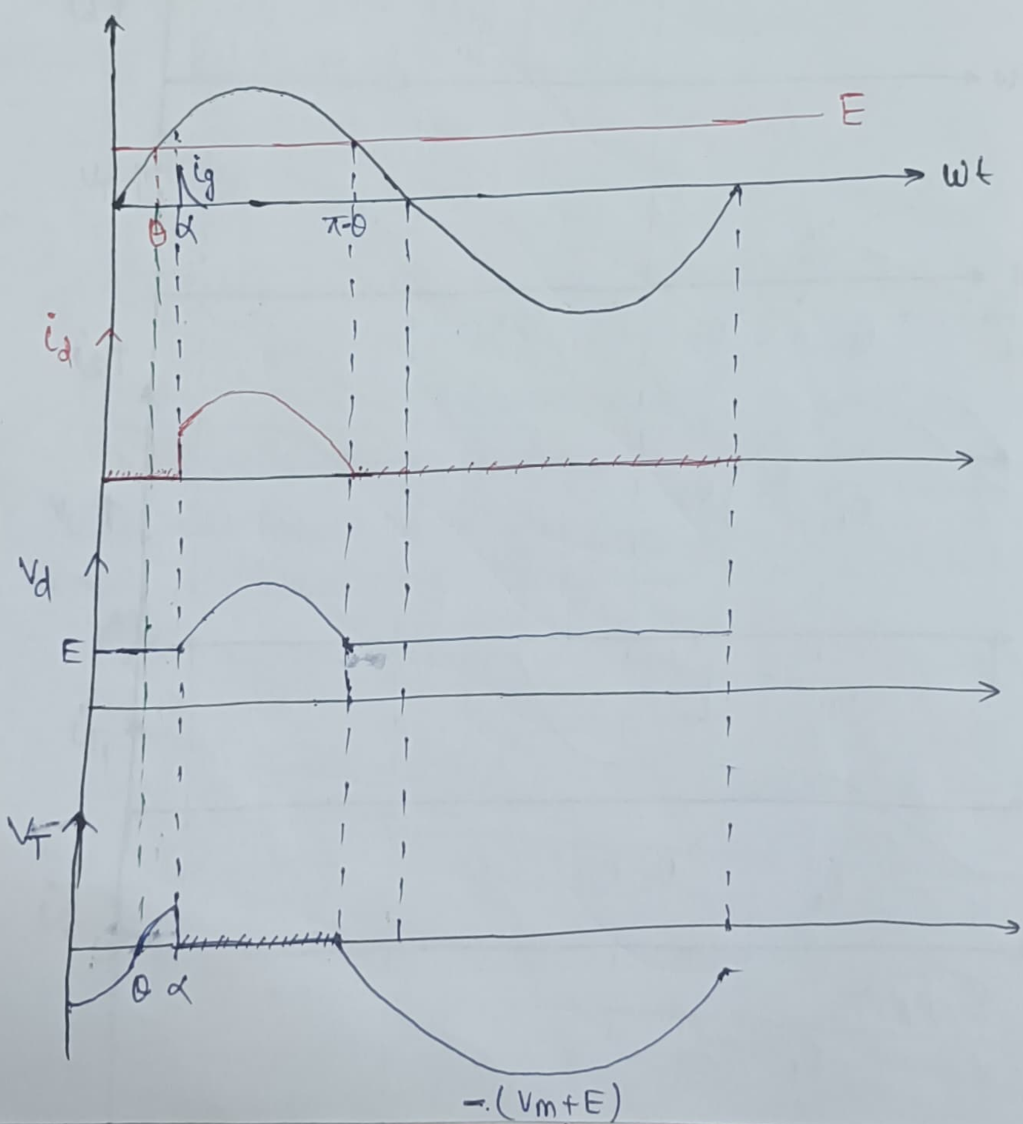


* L-E load :-



* R-E Load :-

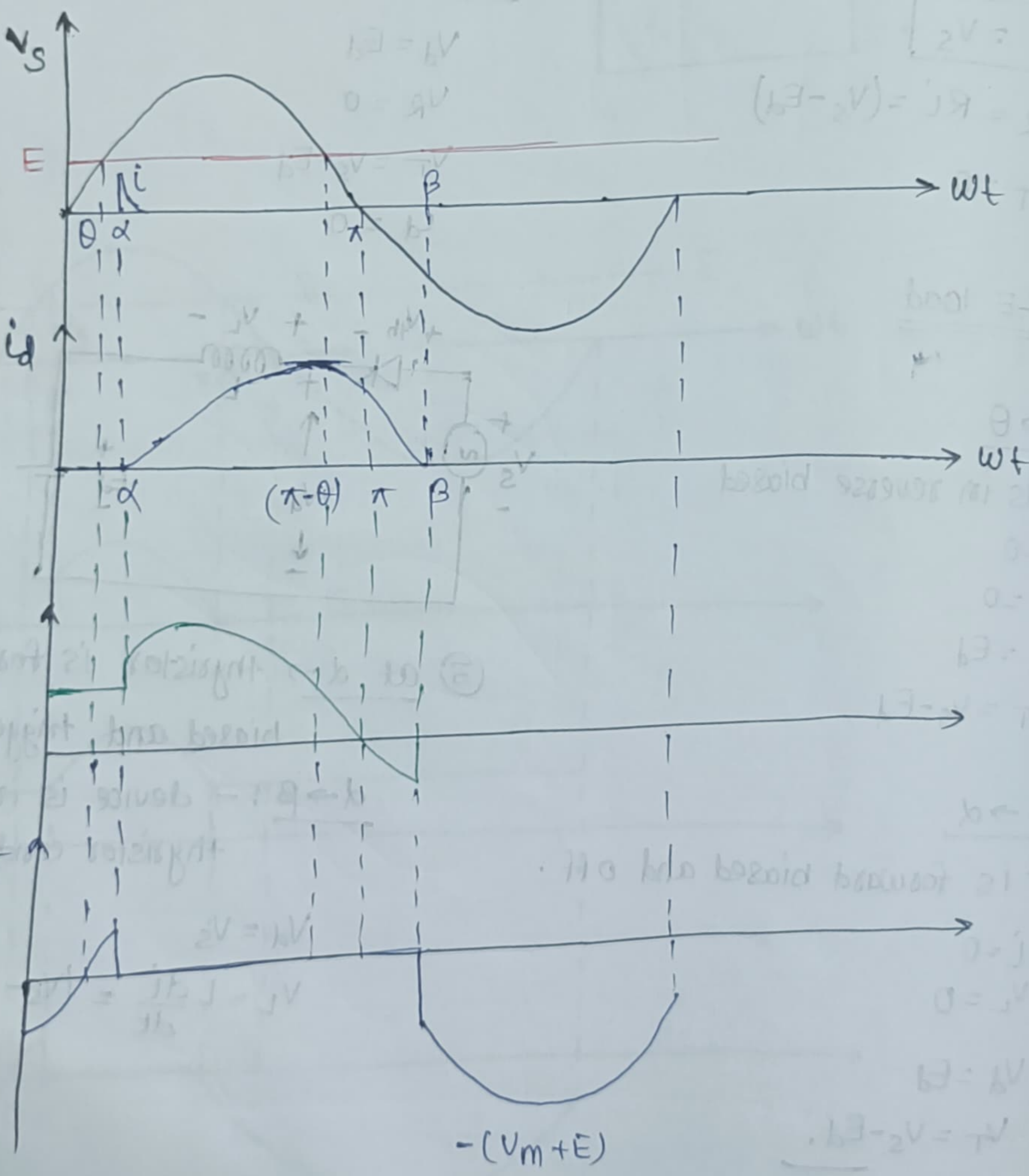




$$i_d = \frac{1}{\omega L} \cdot \int_{\alpha}^{\beta} (V_s - E_d)\, d\omega t$$

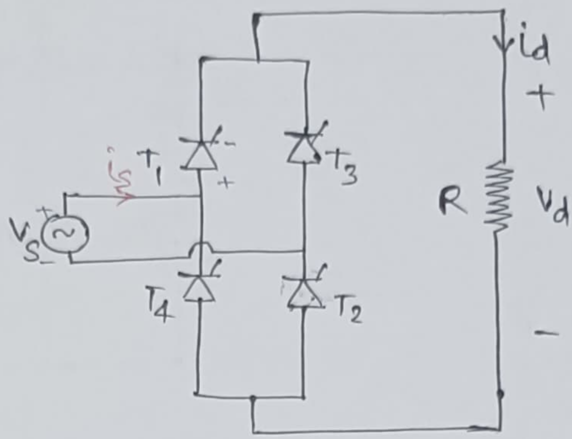
$$i_d = \frac{1}{\omega L} \left[\int_{\alpha}^{\beta} (V_m \sin\omega t\, d\omega t - E_d \int_{\alpha}^{\beta} d\omega t \right]$$

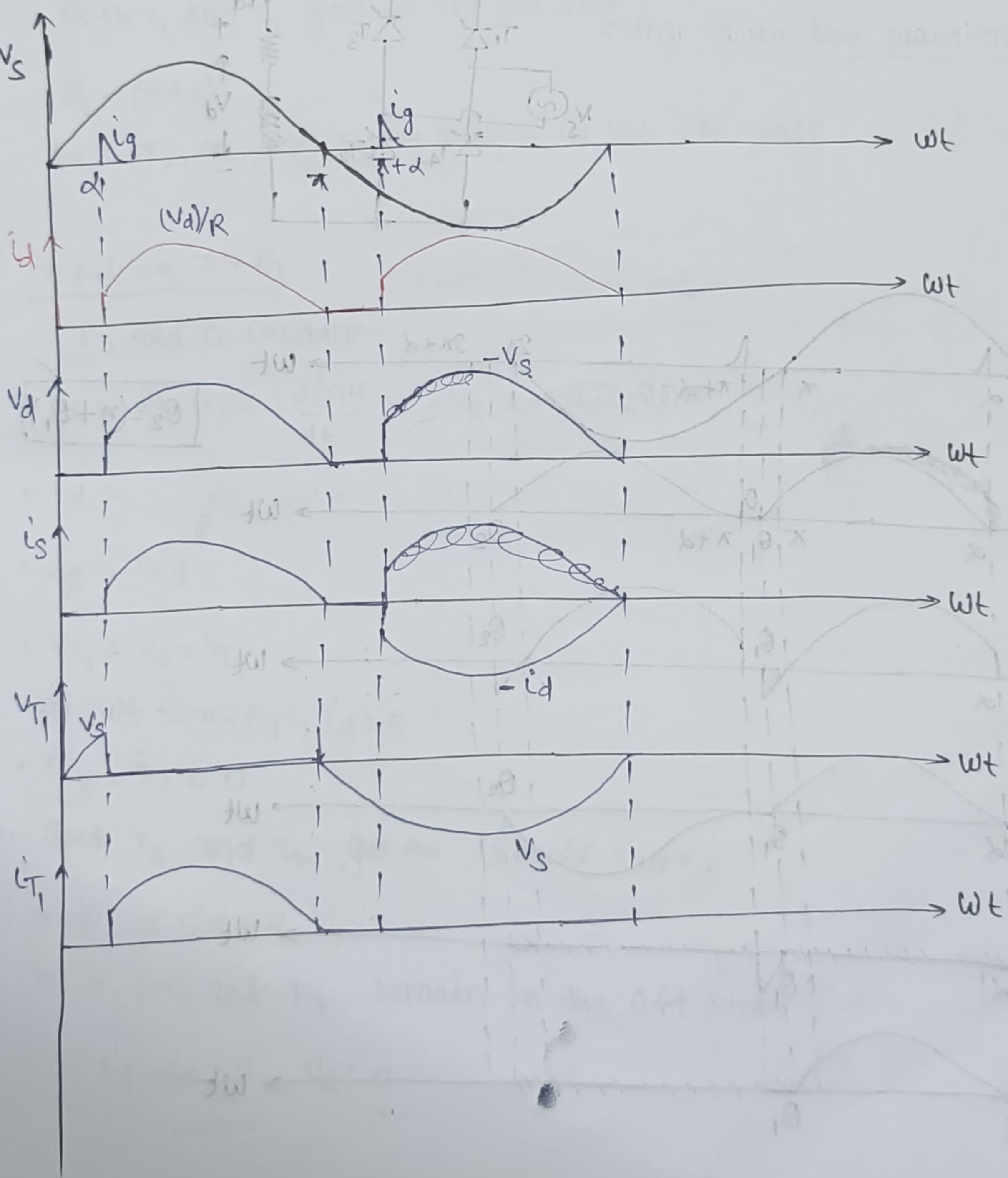
$$V_T = 0$$



* Single-phase fully controlled bridge Rectifier :-

* R-load





$0 \to \alpha$

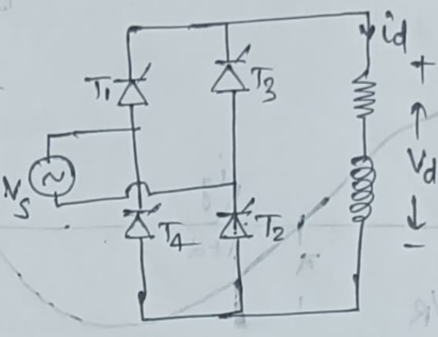
↳ all devices are in off state.

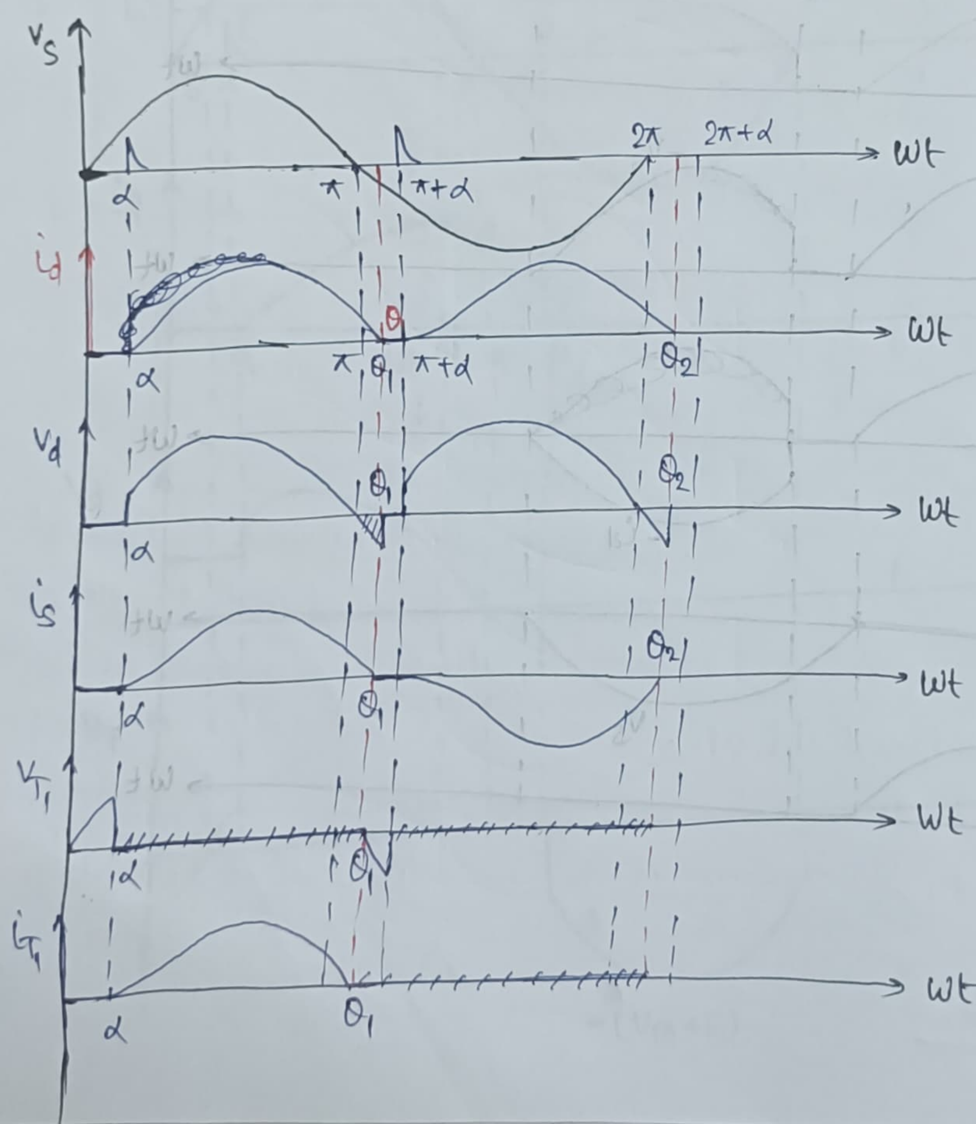
$\alpha \to \pi$

T_1 and T_2 gets turned on.

* R-L Load :-

• Discontinuous load current mode:-





$\boxed{\theta_2 = (\pi + \theta_1)}$

① $\alpha \to \theta_1 \to T_1$ and T_2 conducts

$V_d = V_s = R i_d(t) + L\frac{di_d(t)}{dt} = \sqrt{2}V_s \sin\omega t$

$i_d =$

$i_s = i_d$

$V_{T_1} = 0,\ V_{T_2} = 0,\ V_{T_3} = -V_s$

② at $\omega t = \theta_1,\ i_d = 0$

$i_{T_1} = i_{T_2} = 0$

Both T_1 and T_2 goes to the off state.

$\theta_1 \to (\pi + \alpha)$

$\hookrightarrow T_1, T_2, T_3$, and T_4 → remain in the off state.

③ $\pi + \alpha \to \pi + \theta_1$

T_3 and T_4 conduct

$V_d = R i_d(t) + L\frac{di_d(t)}{dt} = -V_s = -\sqrt{2}V_s \sin\omega t$

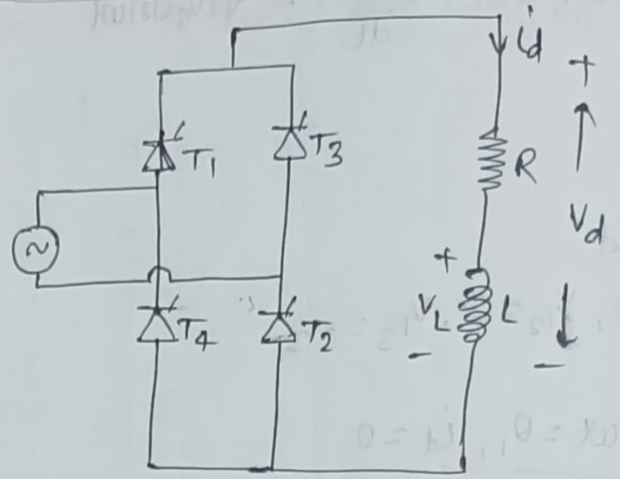
- $i_d =$
- $i_s = -i_d$
- $V_{T_1} = V_s = V_{T_2}$
- at $\omega t = \pi + \theta_1,\ i_d = 0$
- $i_{T_3} = i_{T_4} = 0$
- Both T_3 and T_4 go to the off state.

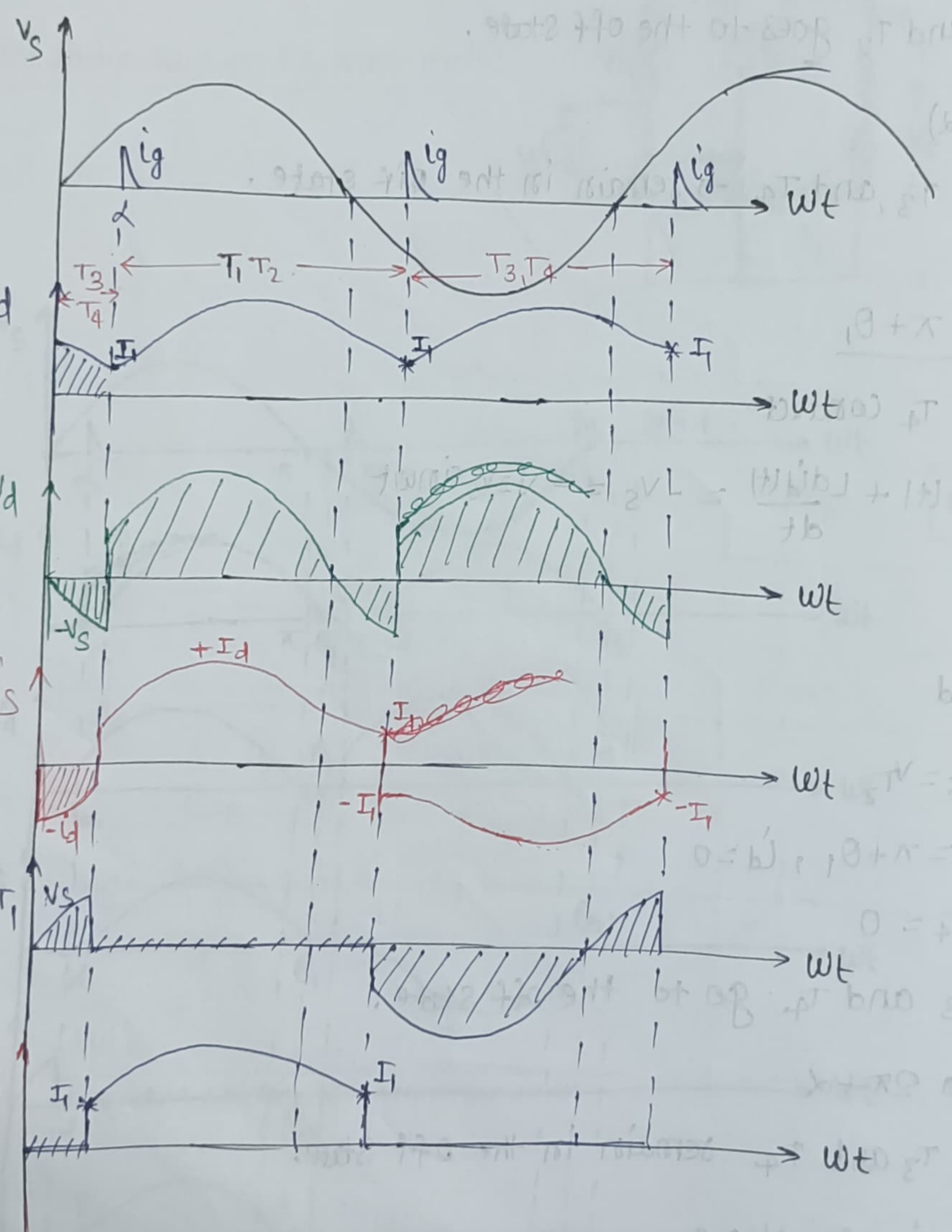
④ $\pi + \theta_1 \to 2\pi + \alpha$

T_1, T_2, T_3 and T_4 remain in the off state.

$i_s = i_d = 0,\ V_d = 0.$

* R-L load continuous current mode:-





↳ For $\omega t < \alpha$, T_3 and T_4 were conducting

at $\omega t = \alpha$, $i_d = i_1$ $(\neq 0)$

T_1 and T_2 are turned on.

T_3 and T_4 are line commutated

↳ $\alpha \rightarrow (\pi + \alpha)$

T_1 and T_2 conduct

T_3 and T_4 $\rightarrow$ turned off.

- $V_d = V_s = \sqrt{2} V_s \sin\omega t = R i_d + L \frac{di_d}{dt}$

@ $\omega t = \alpha$, $i_d = I_1$

$$\Rightarrow R I_1 + L \frac{dI_d}{dt}\Big|_{\omega t = \alpha} = \sqrt{2} V_s \sin\alpha .$$

H.W

$i_d =$ ____

$i_s = i_d$

$V_{T_3} = -V_s = V_{T_4}$.

↳ at $\pi + \alpha$, T_3 and T_4 are turned on

- T_1 and T_2 are line commutated.

↳ $(\pi + \alpha) \rightarrow (2\pi + \alpha)$

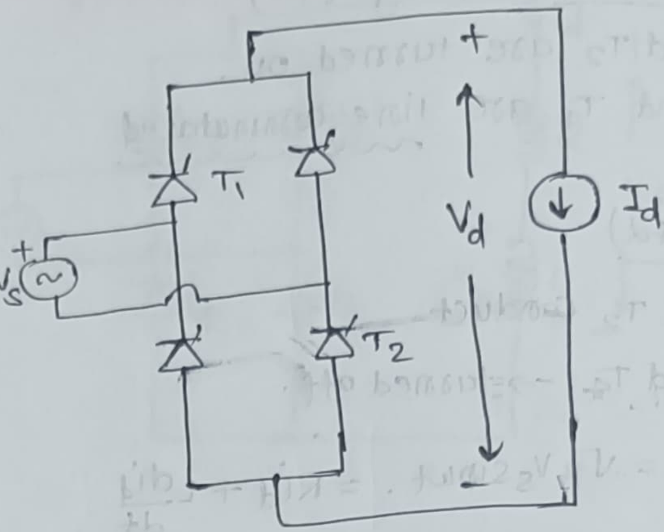
T_3 and T_4 conduct

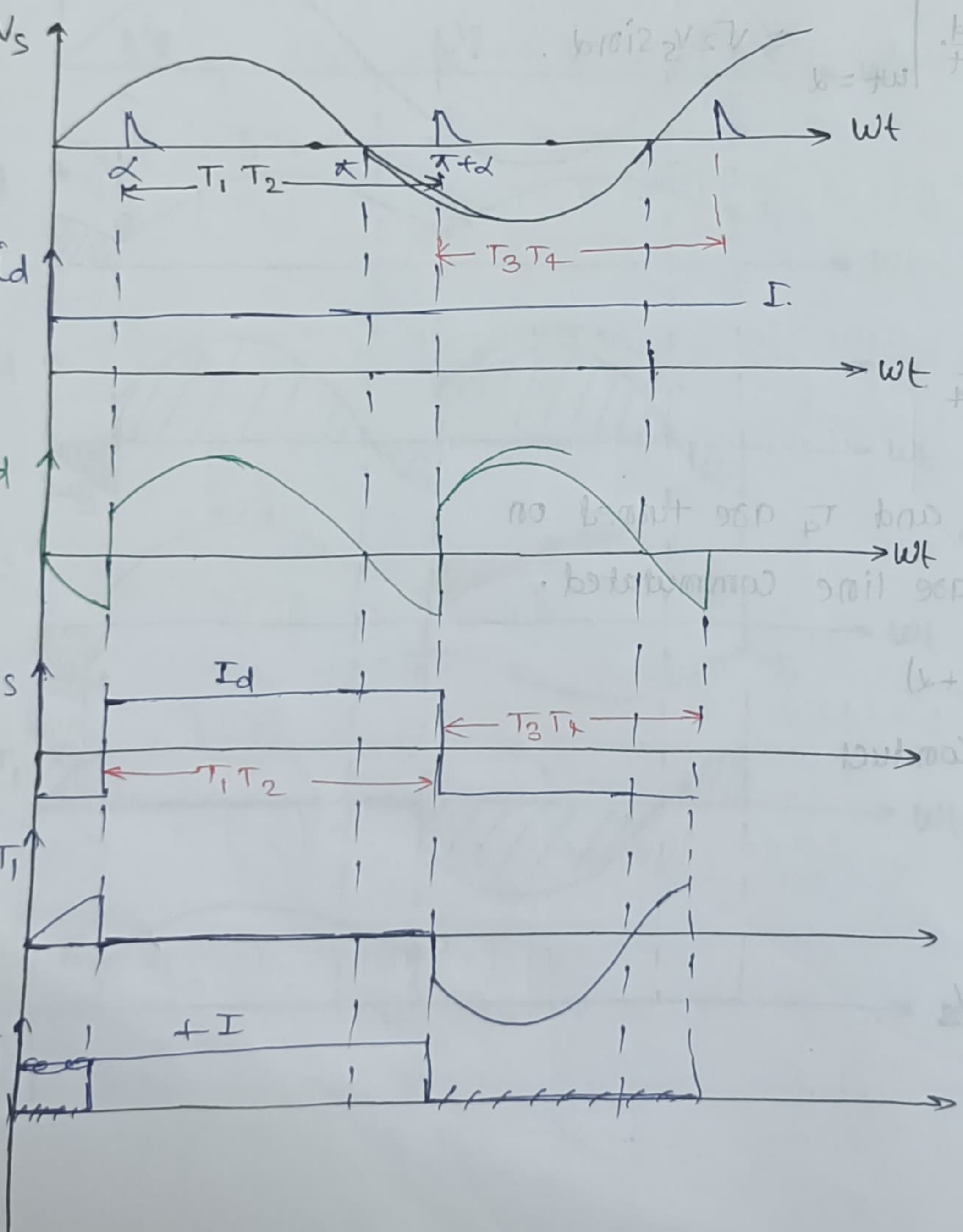
$V_d = -V_s$

$i_s = -i_d$

$V_{T_1} = V_{T_2} = V_s$

* R-L Load, constant load current I_d :-





- $\alpha \to \pi+\alpha$

T_1 and T_2 conduct.
T_3 and T_4 are line commutated.

$V_d = V_s$
$i_s = i_d$
$V_{T3} = -V_s = V_{T4}$

- $\pi+\alpha$ to $2\pi+\alpha$
- T_3 and T_4 conduct
- T_1 and T_2 are line commutated
- $V_d = -V_s$
- $i_s = -I_d$
- $V_{T_1} = V_s = V_{T_2}$

* performance parameters:-

* DC side

- $$(V_d)_{avg.} = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha}(V_m \sin\omega t)\, d\omega t$$

$$= \frac{2V_m}{\pi}\cos\alpha = \frac{2\sqrt{2}V_s\cos\alpha}{\pi} = 0.9\, V_s \cos\alpha .$$

- $$(V_d)_{rms} = \left[\frac{1}{\pi}\int_{\alpha}^{\pi+\alpha}(V_m^2 \sin^2\omega t)\, d\omega t\right]^{1/2} = V_s$$

$$= \left[\frac{1}{\pi}V_m^2 \int\left(\frac{1-\cos 2\omega t}{2}\right) d\omega t\right]^{1/2}$$

- Avg. power through converter

$$(P_d)_{avg} = (V_d)_{avg} I_d = 0.9\, V_s I_d \cos\alpha .$$

- Ripple freq^n = 2 * (line freq^n)
- Voltage Ripple factor $K_V = \sqrt{\frac{(V_d)^2_{rms} - (V_d)^2_{avg}}{(V_d)^2_{avg}}}$

$$= \sqrt{\frac{(V_d)^2_{rms}}{(V_d)^2_{avg}} - 1} = \sqrt{}$$

- $i_s(t) = a_0 + \sum_{1,2,3\ldots}^{\infty} a_n \cos n\omega t + b_n \sin n\omega t \qquad (a_0 = 0)$

$$i_s(t) = \sum_{1}^{\infty} C_n \sin(n\omega t + \phi_n) \quad ; \quad C_n = \sqrt{a_n^2 + b_n^2}$$

$$\phi_n = \tan^{-1}\left(\frac{a_n}{b_n}\right)$$

$$i_{s_1}(t) = C_1 \sin(\omega t + \phi_1) \quad ; \quad C_1 = \sqrt{a_1^2 + a_2^2}$$

$$a_1 = \frac{1}{\pi}\left[\int_{\alpha}^{\pi+\alpha} I_d \cos\omega t \, d\omega t + \int_{\pi+\alpha}^{2\pi+\alpha} -I_d \cos\omega t \, d\omega t\right]$$

$$= \cancel{\frac{1}{\pi}\left[I_d(\sin\alpha - \sin(\pi+\alpha)) + I_d(\sin(2\pi+\alpha) - \sin(\pi+\alpha))\right]}$$

$$= \cancel{\frac{4 I_d}{\pi}}$$

$$= \frac{1}{\pi}\left[I_d(\sin(\pi+\alpha) - \sin\alpha) - I_d(\sin(2\pi+\alpha) - \sin(\pi+\alpha))\right]$$

$$= -\frac{4 I_d}{\pi}\sin\alpha .$$

$$b_1 = \frac{1}{\pi}\left[\int_{\alpha}^{\pi+\alpha} I_d \sin\omega t \, d\omega t + \int_{\pi+\alpha}^{2\pi+\alpha} -I_d \sin\omega t \, d\omega t\right] = \frac{4 I_d}{\pi}\cos\alpha .$$

$$\therefore C_1 = \sqrt{\left(\frac{4 I_d}{\pi}\right)^2 (\sin^2\alpha + \cos^2\alpha)}$$

$$C_1 = \frac{4 I_d}{\pi}$$

$$\phi_1 = \tan^{-1}\left(\frac{a_1}{b_1}\right) = -\alpha$$

$$\therefore \boxed{i_{s_1} = \frac{4 I_d}{\pi} \cdot \sin(\omega t - \alpha)}$$

- Rms value of $i_{s_1} \Rightarrow \left(\frac{2\sqrt{2} I_d}{\pi}\right)$.

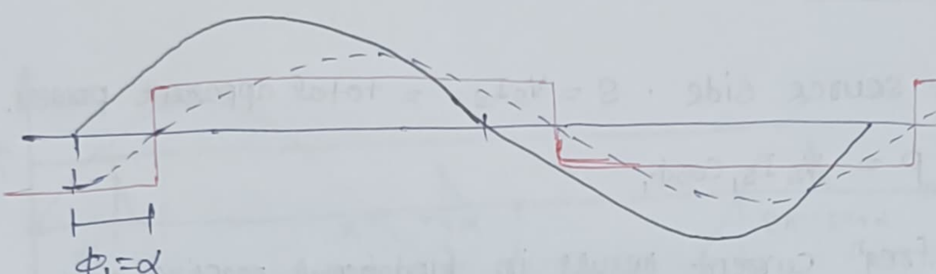
• RMS value of n^{th} harmonics $\Rightarrow$ $I_{sn} = \left(\frac{I_{s1}}{n}\right)$

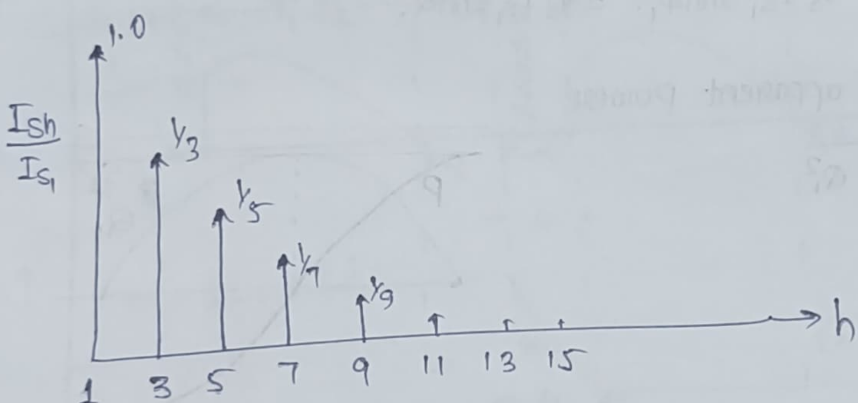
$n = 3, 5, 7 \ldots$

• RMS value of total source current i_s.

$I_s = I_d$

$\therefore \quad I_{sh} = \begin{cases} 0 & \text{for even value of } h \\ \frac{I_{s1}}{h} & \text{for odd value of } h \end{cases}$





Harmonic spectrum.

Total harmonic distortion THD. $= \sqrt{\frac{I_s^2 - I_{s1}^2}{I_{s1}^2}} = \sqrt{\frac{I_d^2}{\frac{8 I_d^2}{\pi^2}} - 1}$

$= \sqrt{\left(\frac{\pi^2}{8} - 1\right)}$

$= 0.4843$

$= 48.43\%$

• Displacement factor $DF = \cos(-\alpha) = \cos\alpha$.

• Ac side power factor $= \dfrac{\text{Actual power}}{\text{Apparent power}} = \dfrac{\cancel{V_S} I_{S_1} \cos\phi_1}{\cancel{V_S} I_S}$

$$= \frac{\left(\frac{2\sqrt{2}\,\cancel{I_d}}{\pi}\right)\cos\alpha}{\cancel{I_d}}$$

$$= \frac{2\sqrt{2}}{\pi}\cos\alpha$$

$$= 0.9\cos\alpha$$

• power at the source side. $S = V_S I_S$ = total apparent power.

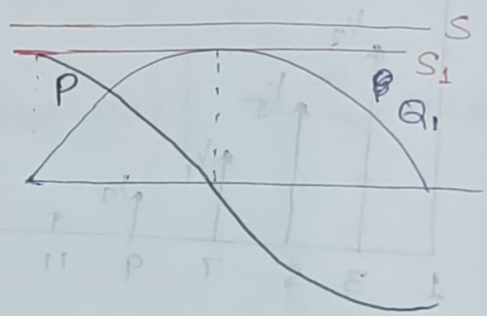
• Active power $P = V_S I_{S_1} \cos\phi_1$

↳ Fundamental freqⁿ current result in fundamental reactive volt amperes —

$$Q_1 = V_S I_{S_1} \sin\phi_1 = V_S I_{S_1} \sin\alpha.$$

↳ Fundamental freqⁿ apparent power

• $S_1 = V_S I_{S_1} = \sqrt{P^2 + Q_1^2}$



• $(V_d)_{avg\,\alpha}$ = Avg. o/p voltage for a triggering angle α.

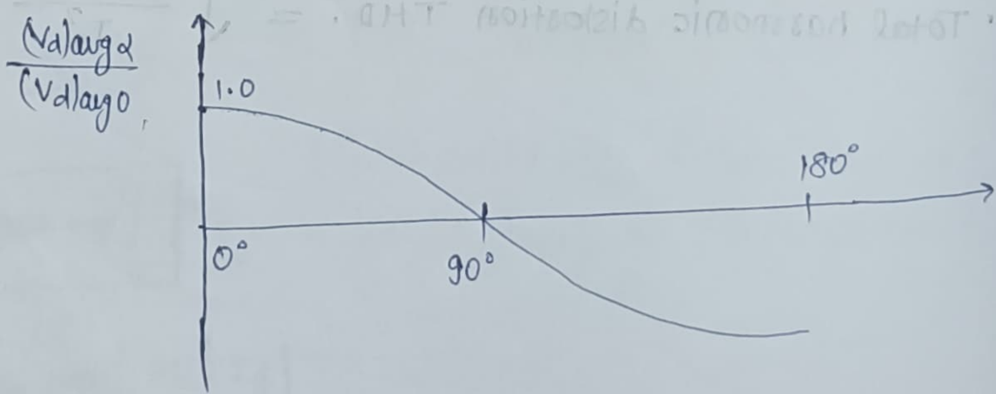


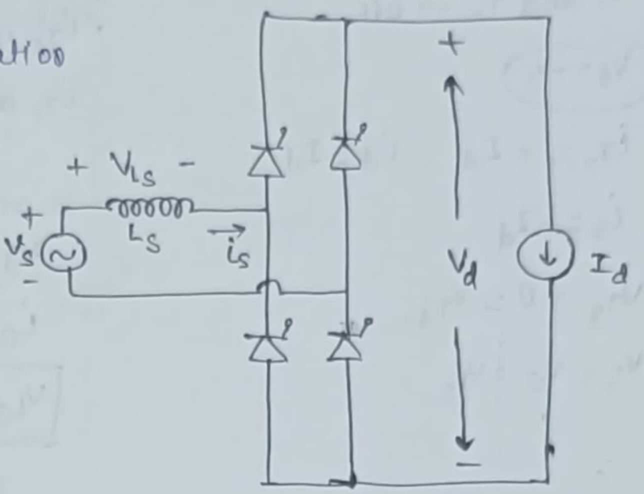
fig. - Normalized Vd as a function of α.

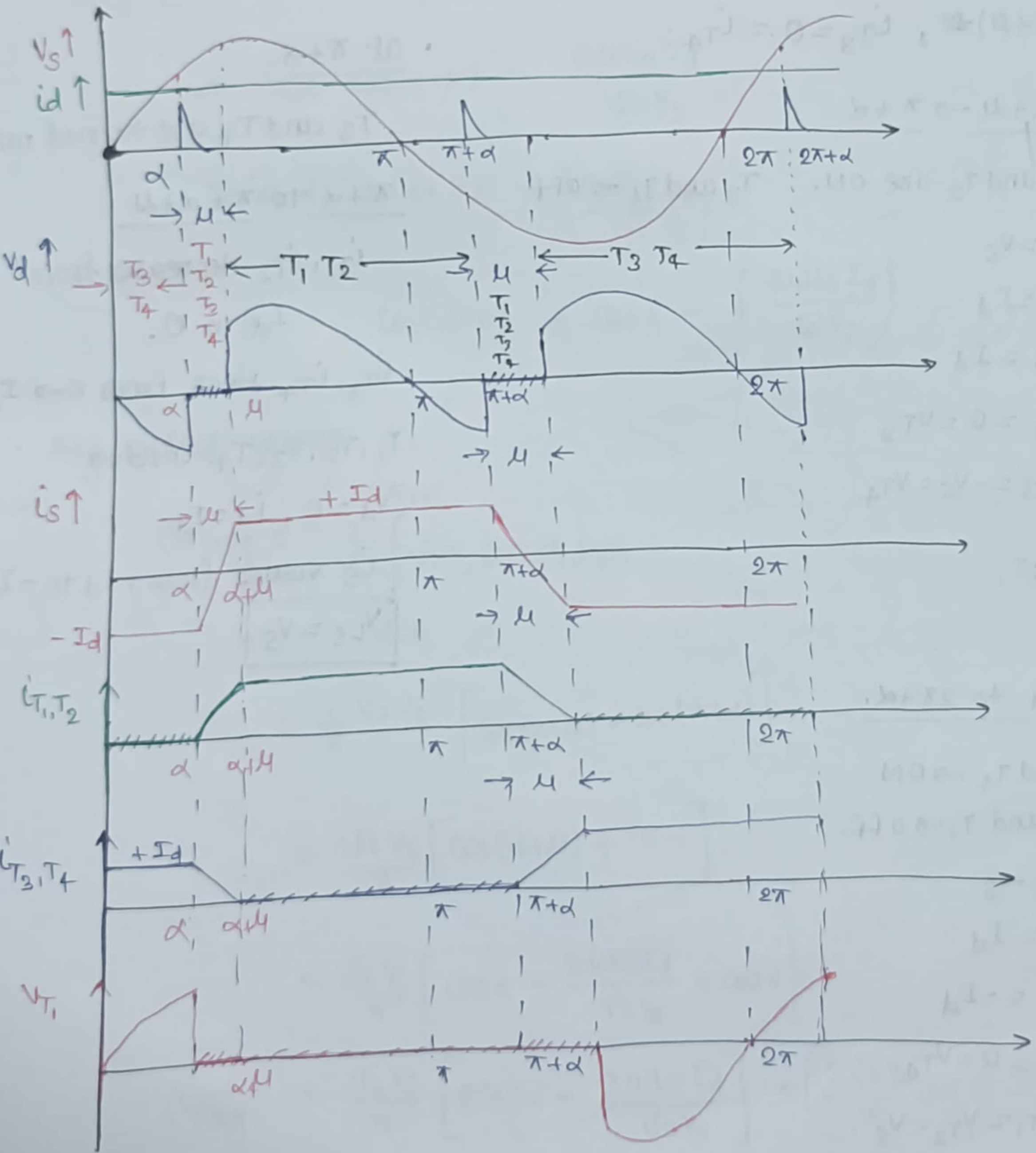
06/10/23

* Effect of source inductance :-

• Constant load current I_d.

↳ The current commutation takes a finite time called μ (or) commutation interval.





during 0 to α

T_3 and $T_4 \rightarrow$ ON

T_1 and $T_2 \rightarrow$ off.

$V_d = -V_s$

$i_{T_3, T_4} = I_d$, $i_d = I_d$

$i_s = -I_d$

$V_{T_3} = 0 = V_{T_4}$

$V_{T_1} = V_S = V_{T_2}$

at α, T_1 and T_2 are turned ON

$\alpha \rightarrow \alpha + \mu$

- i_{T_3} and i_{T_4} decreases from I_d to 0
- i_{T_1} and i_{T_2} rise from 0 to I_d

T_1, T_2, T_3, T_4 are on.

$V_d = 0$, $i_d = I_d$

i_s varies from $-I_d$ to I_d.

$\boxed{V_{LS} = V_S}$

- At $(\alpha + \mu)$, $i_{T_3} = 0 = i_{T_4}$.

from $\alpha + \mu \rightarrow \pi + \alpha$

T_1 and T_2 are ON. T_3 and $T_4 \rightarrow$ off

$V_d = V_S$

$i_d = I_d$

$i_s = I_d$

$V_{T_1} = 0 = V_{T_2}$

$V_{T_3} = -V_S = V_{T_4}$

V_{LS} -

- at $\pi + \alpha$

T_3 and T_4 are turned ON.

$\pi + \alpha$ to $\pi + \alpha + \mu$

i_{T_1}, i_{T_2} decreases from I_d to 0.

i_{T_3}, i_{T_4} rises, from $0 \rightarrow I_d$

T_1, T_2, T_3, T_4 conduct

$V_d = 0$, $i_d = I_d$

i_s varies from $+I_d$ to $-I_d$

$\boxed{V_{LS} = V_S}$

- $\pi + \alpha + \mu$ to $2\pi + \alpha$.

- T_3 and $T_4 \rightarrow$ ON
- T_2 and $T_1 \rightarrow$ off.
- $V_d = -V_S$
- $i_d = I_d$
- $i_s = -I_d$
- $V_{T_3} = 0 = V_{T_4}$
- $V_{T_1} = V_{T_2} = V_S$.
- $V_{LS} = 0$

during the commutation period μ, T_1, T_2, T_3 and T_4 are ON.

$$V_{LS} = V_S$$

$$\sqrt{2} V_S \sin\omega t = L_S \frac{d i_s(t)}{dt} = \omega L_S \frac{d i_s(t)}{d\omega t}$$

$$\frac{\sqrt{2} V_S}{\omega L_S} \sin\omega t \, d\omega t = d i_s(t)$$

integrate both sides from α to $\alpha + \mu$

$$\int_{\alpha}^{\alpha+\mu} \frac{\sqrt{2} V_S}{\omega L_S} \sin\omega t \, d\omega t = \int_{-I_d}^{I_d} d i_s(t)$$

$$\frac{\sqrt{2} V_S}{\omega L_S} \left[\cos(\alpha) - \cos(\alpha+\mu)\right] = 2 I_d$$

$$\Rightarrow \cos\alpha - \cos(\alpha+\mu) = \frac{2\omega L_S I_d}{\sqrt{2} V_S}$$

$$\Rightarrow \cos\alpha - \cos(\alpha+\mu) = \left(\frac{2\omega L_S I_d}{\sqrt{2} V_S}\right)$$

$$\Rightarrow \cos(\alpha+\mu) = \cos\alpha - \left(\frac{2\omega L_S I_d}{\sqrt{2} V_S}\right)$$

- Avg. output voltage

$$(V_d)_{avg.} = \frac{1}{\pi} \int_{\alpha+\mu}^{\pi+\alpha} \sqrt{2} V_S \sin\omega t \, d\omega t$$

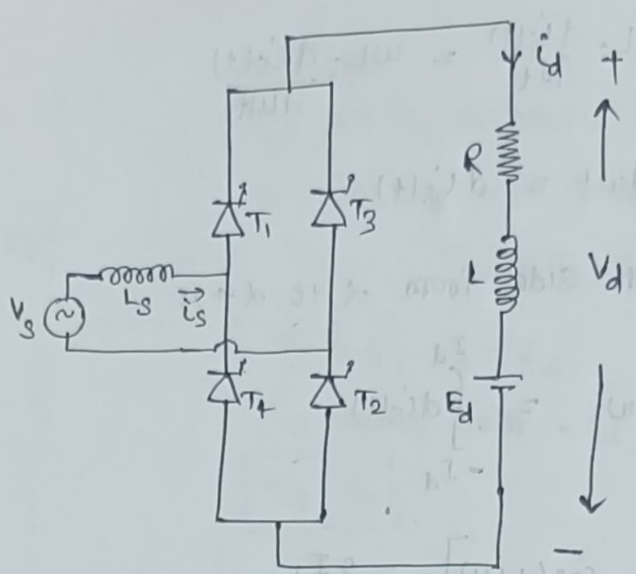
$$= \frac{1}{\pi} \sqrt{2} V_S \left[\cos(\alpha+\mu) - \cos(\pi+\alpha)\right]$$

$$= \frac{\sqrt{2} V_S}{\pi} \left[\cos(\alpha+\mu) + \cos\alpha\right]$$

$$= \frac{\sqrt{2} V_S}{\pi} \left[\cos\alpha - \frac{2\omega L_S I_d}{\sqrt{2} V_S} + \cos\alpha\right]$$

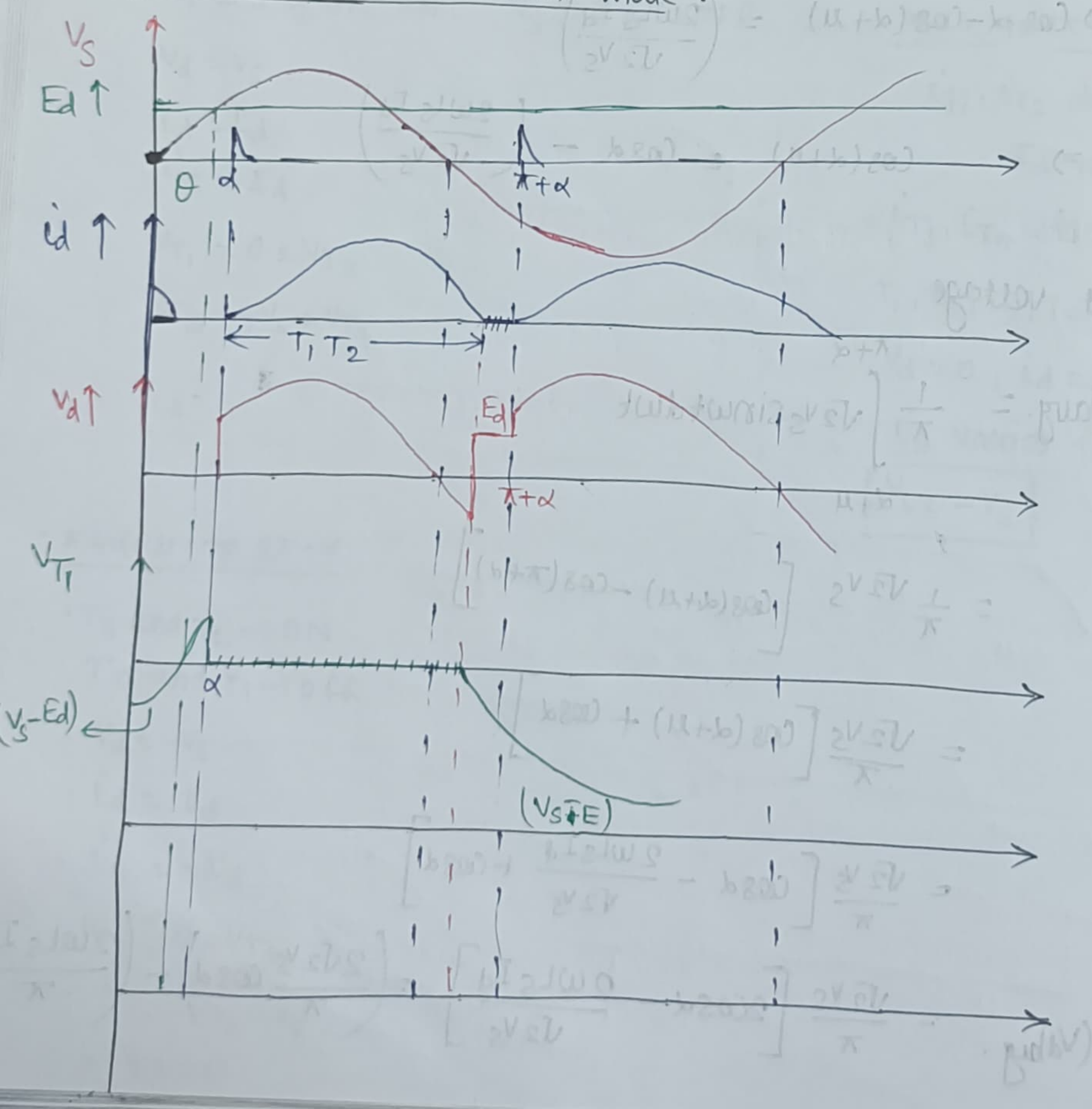
$$(V_d)_{avg.} = \frac{\sqrt{2} V_S}{\pi} \left[2\cos\alpha - \frac{2\omega L_S I_d}{\sqrt{2} V_S}\right] = \left(\frac{2\sqrt{2} V_S}{\pi} \cos\alpha\right) - \left(\frac{2\omega L}{}\right.$$

* Practical thyristor converters :-



- Shape of the load current depends on the load parameters and firing angle.

① Discontinuous load current mode :-



When T_1 and T_2 are conducting,

$$i_d(\omega t) = \frac{\sqrt{2}V_s}{Z}\sin(\omega t - \phi) - \frac{E_d}{R} + Ke^{-\frac{R}{L}t}$$

at $\omega t = \alpha$, $i_d = 0$

$Z = \sqrt{R^2 + \omega^2 L^2}$

$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$

$$0 = \frac{\sqrt{2}V_s}{Z}\sin(\alpha - \phi) - \frac{E_d}{R} + Ke^{-\frac{R}{L}\frac{\alpha}{\omega}}$$

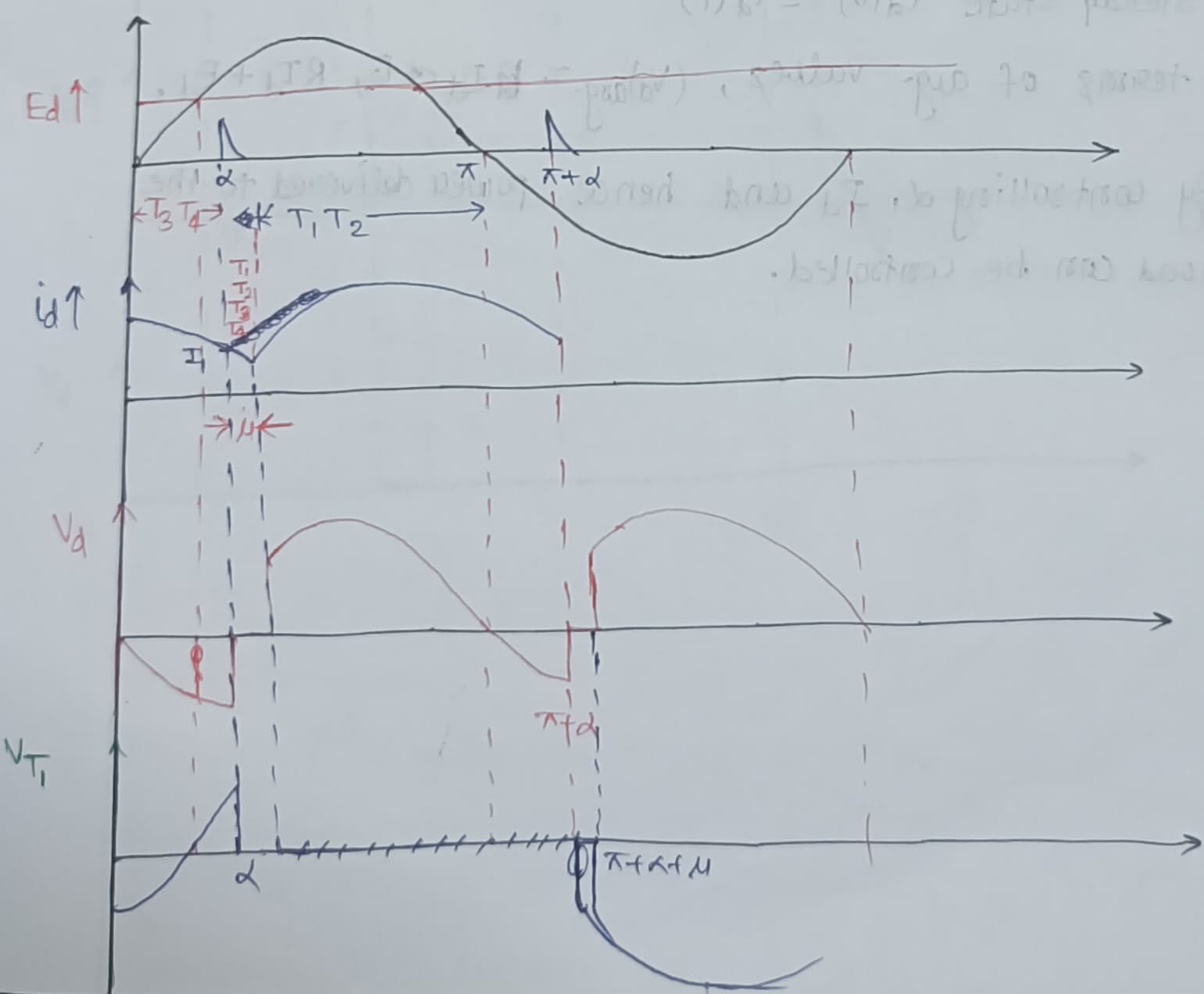
$$\therefore K = \left\{\frac{E_d}{R} - \frac{\sqrt{2}V_s}{Z}\sin(\alpha-\phi)\right\}\cdot e^{\frac{R\alpha}{L\omega}}$$

- $\boxed{m = \left(\frac{E_d}{\sqrt{2}V_s}\right)}$

$$i_d(\omega t) = \frac{\sqrt{2}V_s}{Z}\sin(\omega t - \phi) - \frac{E_d}{R} + \left\{\frac{E_d}{R} - \frac{\sqrt{2}V_s}{Z}\sin(\alpha-\phi)\right\}e^{\frac{R\alpha}{L\omega}}\cdot e^{-\frac{R}{L}t}$$

$$= \frac{\sqrt{2}V_s}{Z}\sin(\omega t - \phi) - \frac{E_d}{R} + \left\{\frac{E_d}{R} - \frac{\sqrt{2}V_s}{Z}\sin(\alpha-\phi)\right\}\cdot e^{\frac{R}{L}\left(\frac{\alpha}{\omega} - t\right)}$$

2.) Continuous current mode:-



- Avg. value of v_d is approximately $\longrightarrow$

$$(V_d)_{avg.} \simeq \frac{2\sqrt{2}V_s}{\pi}\cos\alpha - \frac{2\omega L_s (i_d)_{min.}}{\pi}$$

↳ $(I_d)_{min.}$ is min^m. value of i_d that occurs at $\omega t = \alpha$.

instantaneous value of load voltage —

$$(v_d) = R \cdot i_d + L\frac{di_d}{dt} + E_d$$

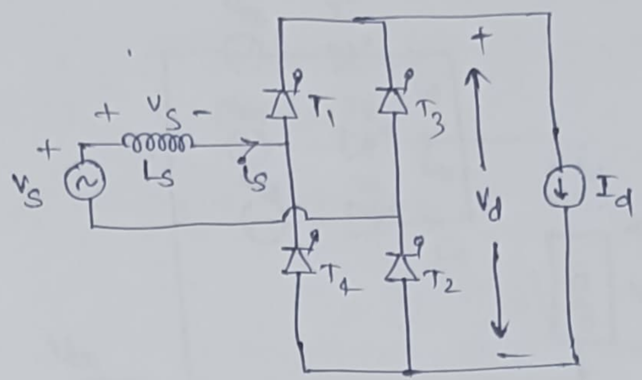
$$\frac{1}{T}\int_0^T v_d\,dt = \frac{R_d}{T}\int_0^T i_d\,dt + \frac{L}{T}\int_{I_d(0)}^{I_d(T)} di_d + E_d$$

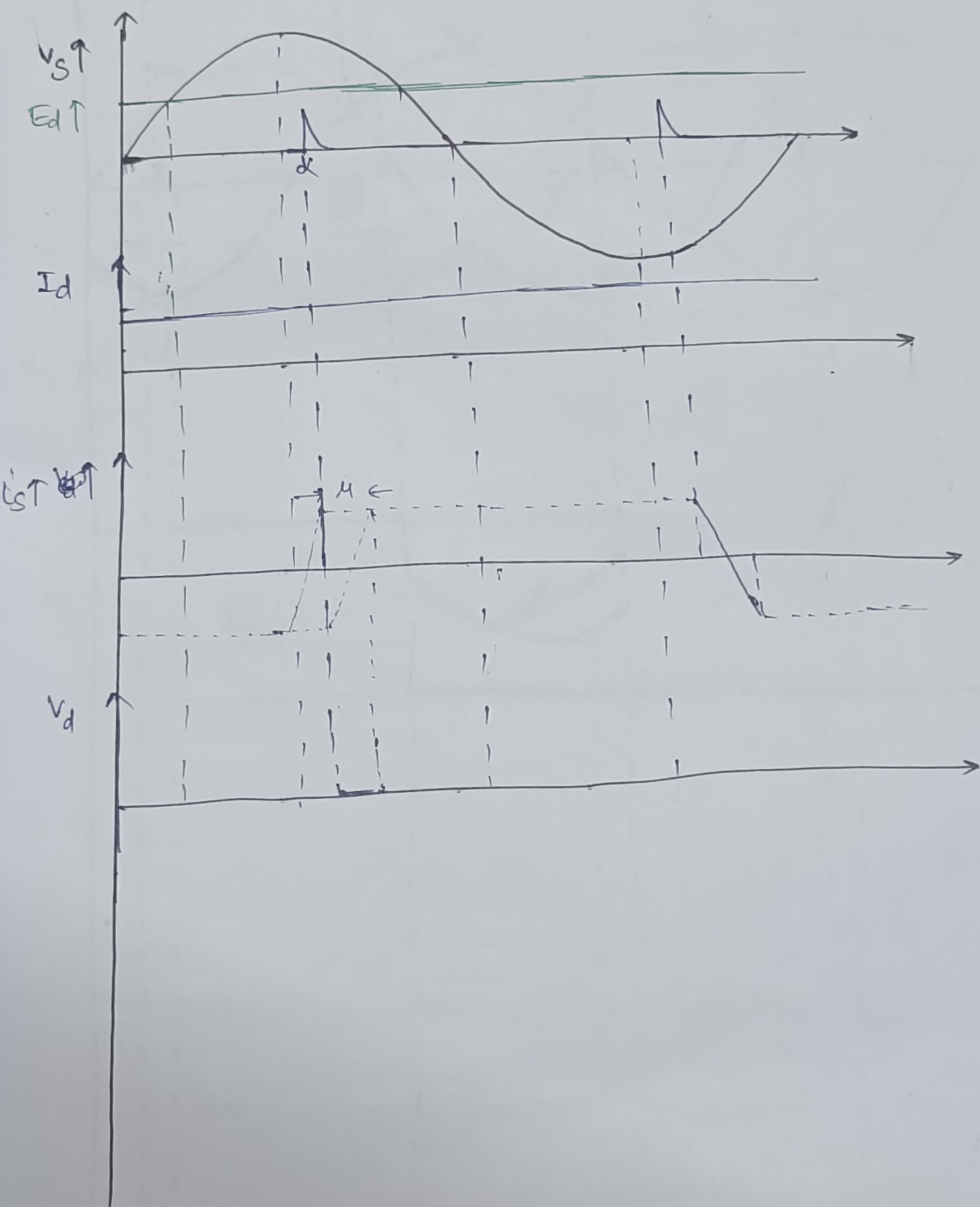
$$\frac{1}{T}\int_0^T v_d\,dt = \frac{R}{T}\int_0^T i_d\,dt + E_d$$

- in steady state $i_d(0) = i_d(T)$
- in terms of avg. values, $(V_d)_{avg.} = \cancel{R E_d + E_d}\ R I_d + E_d$.

↳ By controlling α, I_d and hence power delivered to the load can be controlled.

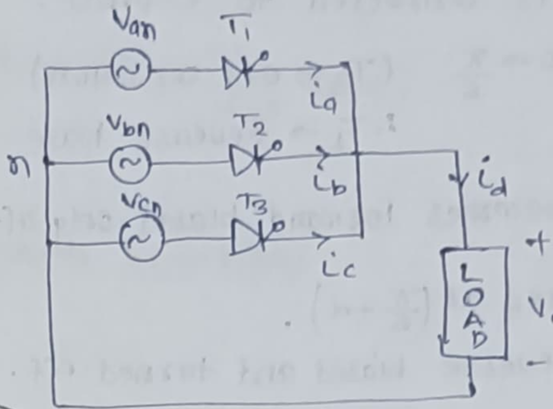
* Inverter mode of rectifier Bridge :-

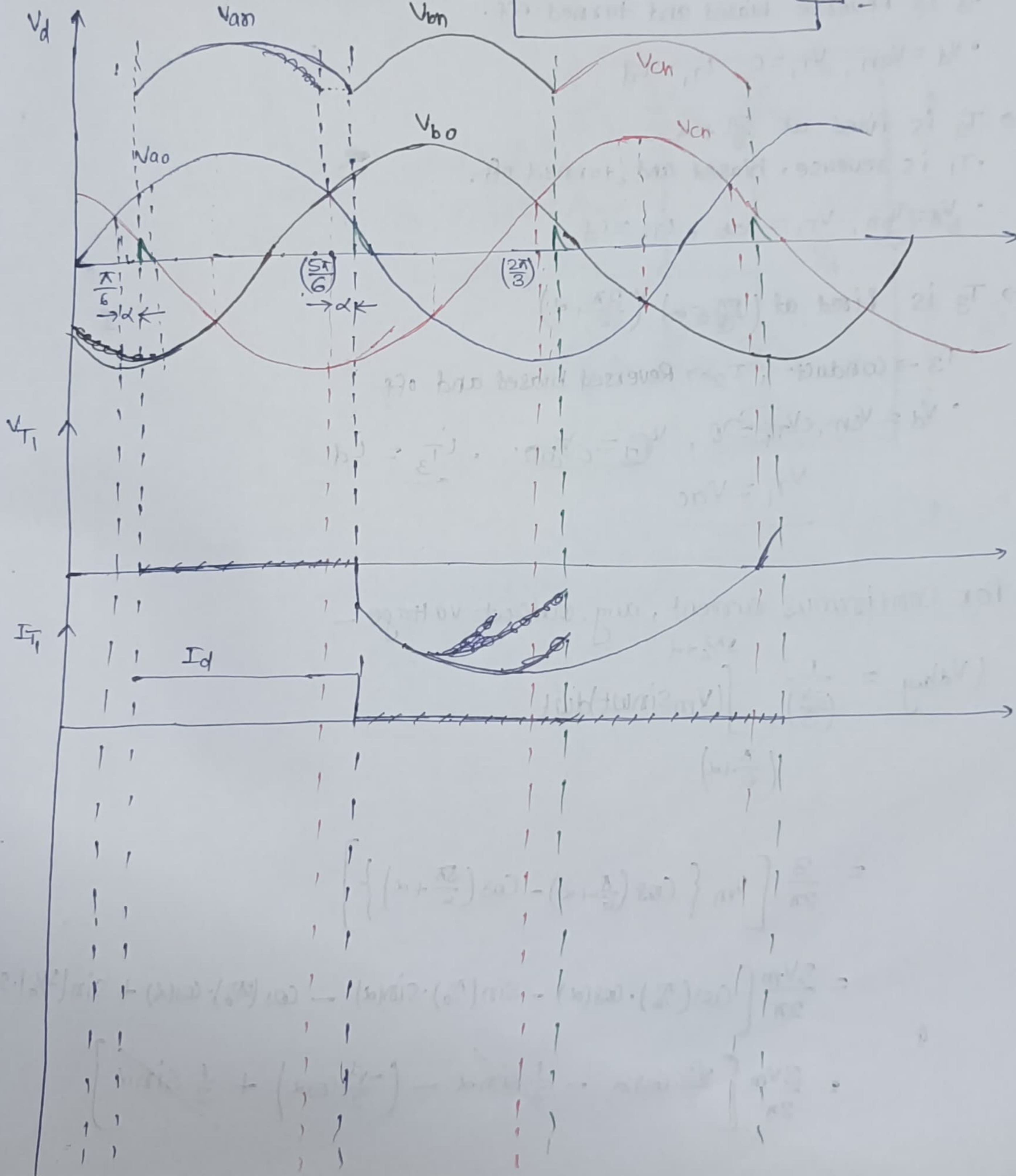




* Three-phase half wave controlled Rectifier :-

$\boxed{i_d \to \text{constant.}}$





↳ initially let T_3 alone is in on state
so load is connected to c-phase.

during $0 \to \frac{\pi}{6}$ ($T_3 \to$ ON condition)

$\therefore T_1 \to$ Reverse biase ($\because V_{cn} > V_{an}$).

so, T_1 becomes forward biased only after $\frac{\pi}{6}$. (so gate signal provided after $\frac{\pi}{6}$).

↳ T_1 is fired at $\left(\frac{\pi}{6}+\alpha\right)$.

T_3 is Reverse biased and turned off.

- $V_d = V_{an}$, $V_{T_1} = 0$, $i_{T_1} = i_d$

↳ T_2 is fired at $\frac{5\pi}{6}+\alpha$

- T_1 is reverse biased and turned off.
- $V_d = V_{bn}$, $V_{T_1} = V_{ab}$, $i_{T_2} = i_d$

↳ T_3 is fired at $\left(\frac{3\pi}{2}+\alpha\right)$

- $T_3 \to$ conduct, $T_2 \to$ Reversed biased and off
- $V_d = V_{cn}$, $i_{T_3} = i_d$

$V_{T_1} = V_{ac}$

- for continuous current, avg. output voltage:-

$$(V_d)_{avg.} = \frac{1}{\left(\frac{2\pi}{3}\right)} \int_{\left(\frac{\pi}{6}+\alpha\right)}^{5\pi/6+\alpha} (V_m \sin\omega t)\, d\omega t$$

$$= \frac{3}{2\pi}\left[V_m\left\{\cos\left(\frac{\pi}{6}+\alpha\right) - \cos\left(\frac{5\pi}{6}+\alpha\right)\right\}\right]$$

$$= \frac{3V_m}{2\pi}\left[\cos(\pi/6)\cdot\cos(\alpha) - \sin(\pi/6)\cdot\sin(\alpha) - \cos(5\pi/6)\cdot\cos(\alpha) + \sin(5\pi/6)\cdot\sin\alpha\right]$$

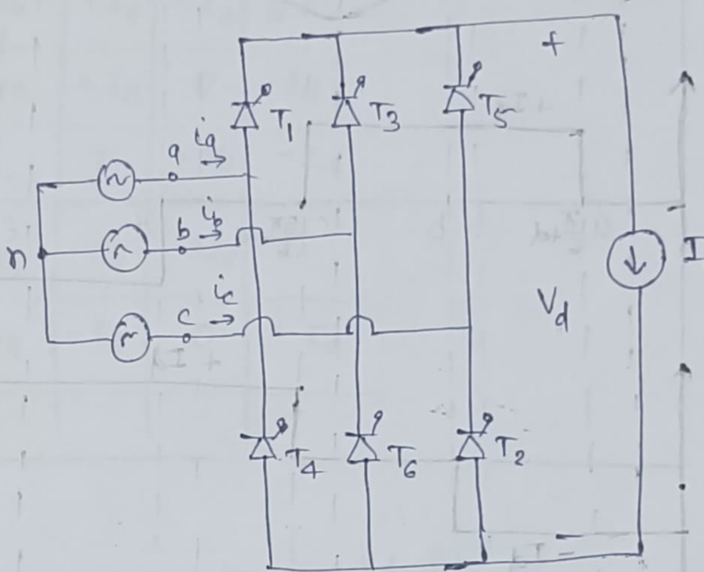
$$= \frac{3V_m}{2\pi}\left[\frac{\sqrt{3}}{2}\cos\alpha - \cancel{\frac{1}{2}\sin\alpha} - \left(-\frac{\sqrt{3}}{2}\cos\alpha\right) + \cancel{\frac{1}{2}\sin\alpha}\right]$$

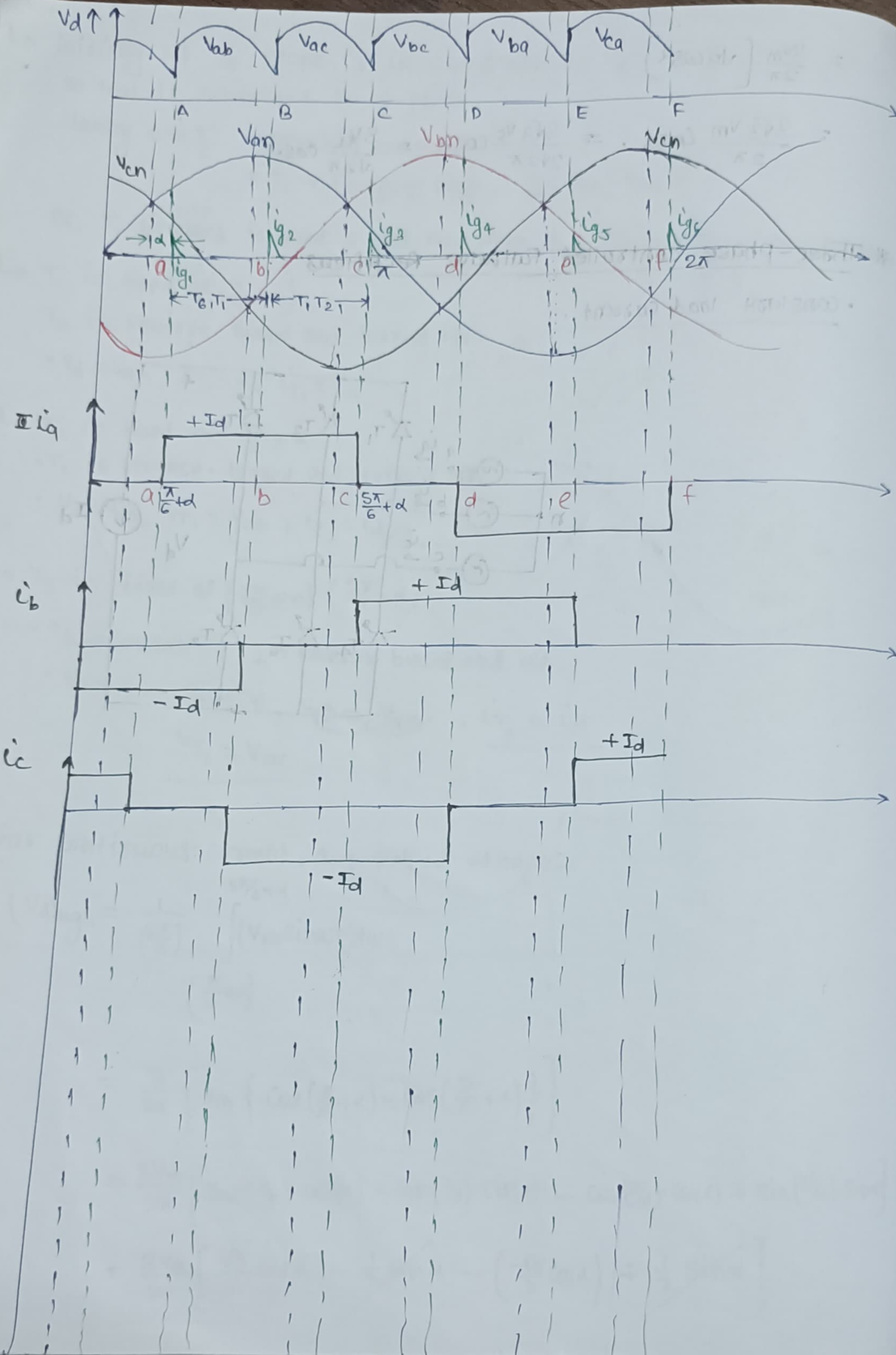
$$= \frac{3V_m}{2\pi}\left[\sqrt{3}\cos\alpha\right]$$

$$= \frac{3\sqrt{3}\,V_m}{2\pi}\cos\alpha = \frac{3\sqrt{3}\,V_s}{2\sqrt{2}\,\pi}\cos\alpha = \frac{3V_{LL}}{\sqrt{2}\,\pi}\cos\alpha.$$

Three-phase controlled Fullwave Rectifiers

- constant load current.





V_d
V_{ab}
V_{ac}
V_{bc}
V_{ba}
V_{ca}
A
B
C
D
E
F
V_{an}
V_{bn}
V_{cn}
V_{cn}
α
i_{g1}
i_{g2}
i_{g3}
i_{g4}
i_{g5}
i_{g6}
a
b
c
d
e
f
π
2π
T_6, T_1
$T_1 T_2$
i_a
$+I_d$
$\frac{\pi}{6}+\alpha$
$\frac{5\pi}{6}+\alpha$
i_b
$+I_d$
$-I_d$
i_c
$+I_d$
$-I_d$

- constant load current.
- earliest instant at which T_1 can be turned on is at $\pi/6$.
- Hence α is measured from $\pi/6$.

(at point b, c phase is most negative so we give i_{g2} to turn on the T_2.)
sly for every T.

Interval	Devices	V_d	i_a	i_b	i_c
$A \to B$	T_6, T_1	V_{ab}	$+I_d$	$-I_d$	0
$B \to C$	T_1, T_2	V_{ac}	$+I_d$	0	$-I_d$
$C \to D$	T_3, T_2	V_{bc}	0	$+I_d$	$-I_d$
$D \to E$	T_3, T_4	V_{ba}	$-I_d$	$+I_d$	0
$E - F$	T_5, T_4	V_{ca}	$-I_d$	0	$+I_d$

$$(V_0)_{avg} = \frac{1}{\left(\frac{\pi}{3}\right)} \int_{\left(\frac{\pi}{6}+\alpha\right)}^{\left(\frac{\pi}{2}+\alpha\right)} V_m \sin\omega t \, d\omega t - V_m \sin(\omega t - 120)\, d\omega t$$

$$= \frac{3V_m}{\pi}\left[-\cos\omega t\right]_{\pi/6+\alpha}^{\frac{\pi}{2}+\alpha}$$

$$= \frac{3V_m}{\pi}\left[\cos\left(\pi/6+\alpha\right) - \cos\left(\frac{\pi}{2}+\alpha\right)\right] = \frac{3V_m}{\pi}\left[\cos\frac{\pi}{6}\cdot\cos\alpha - \sin\frac{\pi}{6}\cdot\sin\alpha - \cos\frac{\pi}{2}\cdot\cos\alpha + \sin\frac{\pi}{2}\cdot\sin\alpha\right]$$

$$= \frac{3V_m}{\pi}\left[\right.$$

$$(V_0)_{avg} = \frac{3\sqrt{6}V_s}{\pi}\cos\alpha = 1.35\, V_{LL}\cos\alpha.$$

$$(V_0)_{avg.} = \frac{3\sqrt{3}V_m}{\pi}\cos\alpha = \frac{3V_{mL}}{\pi}\cos\alpha$$

• Avg. output power $= 1.35 V_{LL} I_d \cos\alpha$.

↳ <u>AC side current i_a:-</u>

$$= \sum_{n=1}^{\infty} a_n \cos n\omega t + b_n \sin n\omega t$$

$$i_{a_1} = a_1 \cos\omega t + b_1 \sin\omega t$$

$$a_1 = \frac{1}{\pi}\left[\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} I_d \cos\omega t \, d\omega t + \int_{\frac{7\pi}{6}+\alpha}^{\frac{11\pi}{6}+\alpha} -I_d \cos\omega t \, d\omega t\right]$$

$$= \frac{1}{\pi}\left[I_d\left\{\sin\left(\frac{5\pi}{6}+\alpha\right) - \sin\left(\frac{\pi}{6}+\alpha\right)\right\} - I_d\left\{\sin\left(\frac{11\pi}{6}+\alpha\right) - \sin\left(\frac{7\pi}{6}+\alpha\right)\right\}\right]$$

$$= \frac{1}{\pi}\left[I_d\left\{\sin\frac{5\pi}{6}\cdot\cos\alpha + \cos\frac{5\pi}{6}\cdot\sin\alpha - \sin\frac{\pi}{6}\cdot\cos\alpha - \cos\frac{\pi}{6}\cdot\sin\alpha\right\}\right.$$

$$\left. - I_d\left\{\sin\frac{11\pi}{6}\cdot\cos\alpha + \cos\frac{11\pi}{6}\cdot\sin\alpha - \sin\frac{7\pi}{6}\cdot\cos\alpha - \cos\frac{7\pi}{6}\cdot\sin\alpha\right\}\right]$$

$$= -\frac{2\sqrt{3}\, I_d}{\pi} \sin\alpha$$

$$b_1 = \frac{1}{\pi}\left[\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} I_d \sin\omega t \, d\omega t + \int_{\frac{7\pi}{6}+\alpha}^{\frac{11\pi}{6}+\alpha} -I_d \sin\omega t \, d\omega t\right]$$

$$= \frac{2\sqrt{3}\, I_d}{\pi} \cos\alpha.$$

$$\therefore C_1 = \sqrt{a_1^2 + b_1^2} = \frac{2\sqrt{3}\, I_d}{\pi} \; ; \; \phi_1 = -\alpha.$$

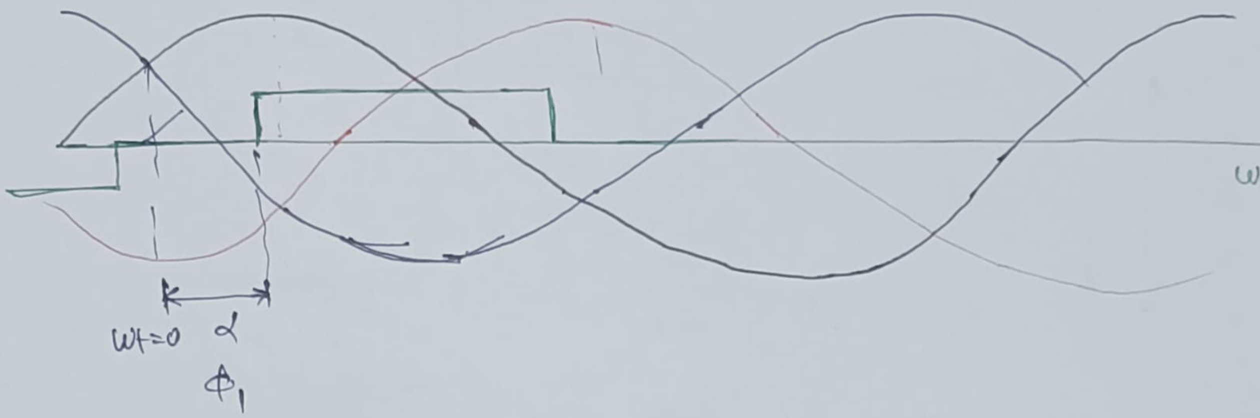
$$\therefore \boxed{i_{a_1} = \frac{2\sqrt{3}\, I_d}{\pi} \sin(\omega t - \alpha)}$$

↳ RMS value of fundamental component $I_{a1} = \frac{2\sqrt{3}}{\sqrt{2}\pi} I_d = \frac{\sqrt{6} I_d}{\pi}$

$= 0.78 I_d$

↳ RMS value of harmonic components

$$I_{ah} = \left(\frac{I_{a1}}{h}\right) \quad ; \quad h = 6n \pm 1 \; , \; n = 1, 2, 3 \ldots\ldots$$



↳ Total RMS value of phase current $I_a = \sqrt{\frac{2}{3}} I_d = 0.816 I_d$

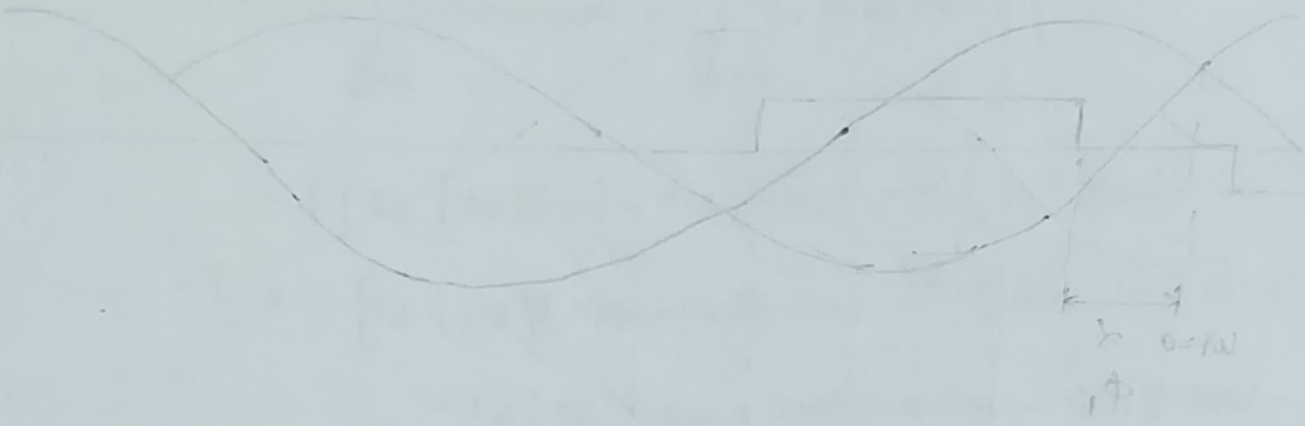
- THD $= \frac{\sqrt{I_a^2 - I_{a1}^2}}{I_{a1}} = 31.08\%$

- DPF $= \cos\phi_1 = \cos\alpha$

- power factor $= \frac{\text{actual power}}{\text{apparent power}} = \cancel{\frac{3\sqrt{6} V_s \cos\alpha \, I_d}{\pi \, V_s \sqrt{6} I_d}} \quad \frac{\sqrt{3} V_{LL} I_{a1} \cos\alpha}{\sqrt{3} V_{LL} I_a}$

$= \boxed{\frac{3}{\pi}\cos\alpha}$

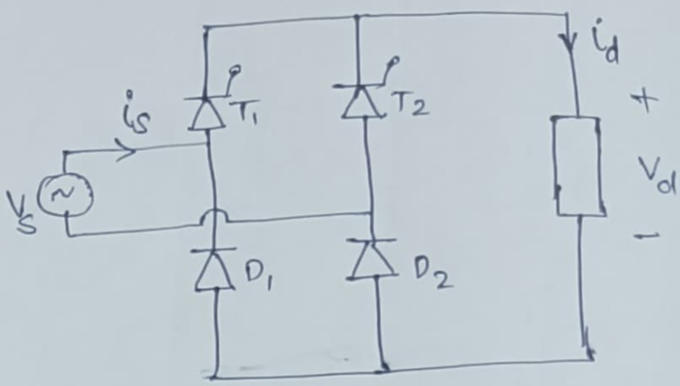
* Effect of source inductance:-

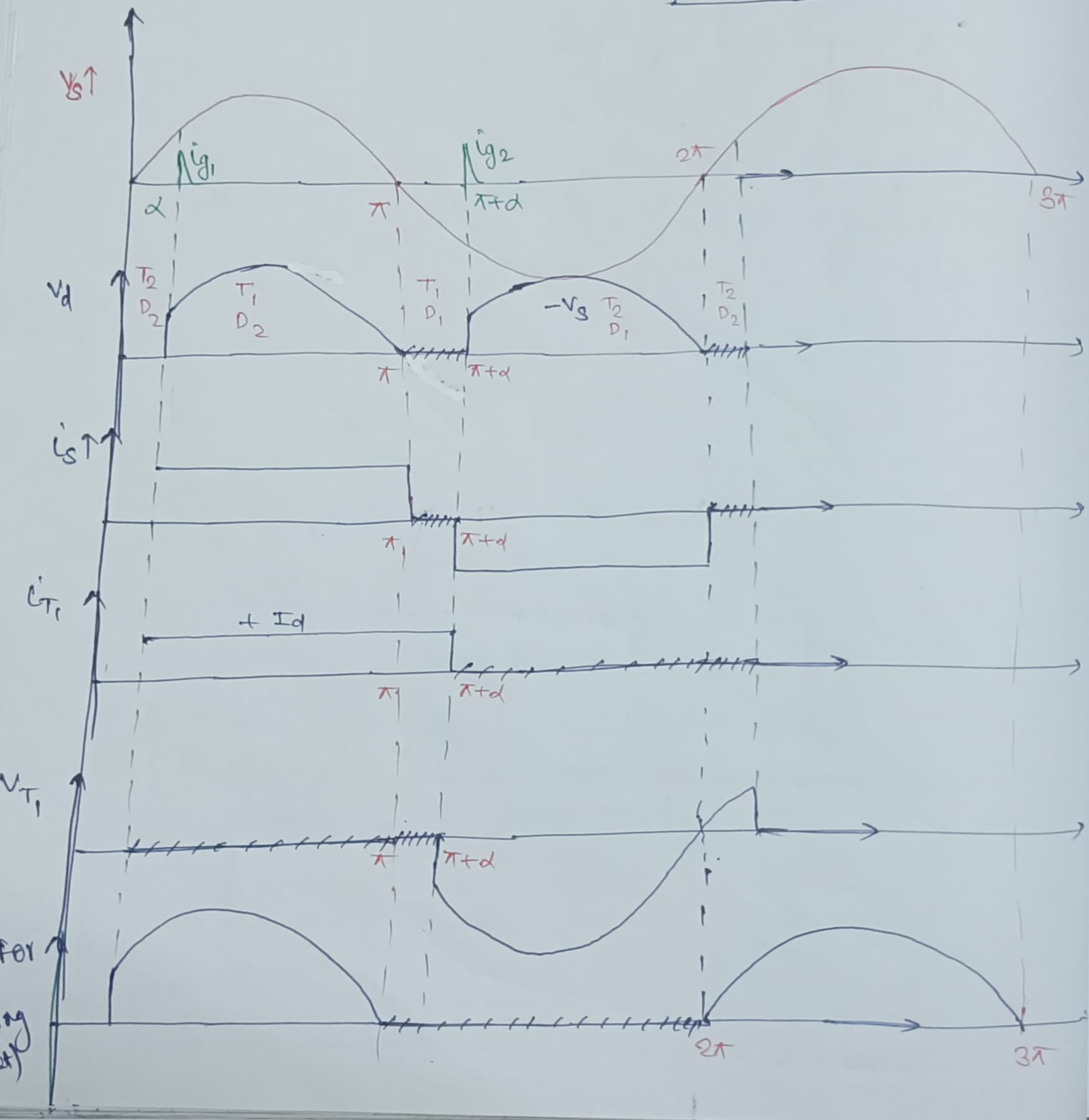


* Single Quadrant Converters – Half controlled Converters :-

Symmetrical configuration :-

load current i_d is constant at I_d.





at α: T_1 is turned on

$\alpha \to \pi$: T_1 and D_2 conduct.

$V_d = V_s$

$i_s = +I_d$

$i_{T_1} = I_d$ $\quad i_{T_2} = 0$, $i_{D_1} = 0$

$i_{D_2} = I_d$

$\pi \to \pi+\alpha$

T_1, D_1 - conduct

$V_d = 0$

$i_s = 0$

$i_{T_1} = I_d$

$i_{D_2} = 0$

$i_{D_1} = I_d$

$V_{T_1} = 0$

$\pi+\alpha \to 2\pi$

T_2, D_1 conduct.

$V_d = -V_s$

$i_s = -I_d$

$i_{T_1} = 0$, $i_{T_2} = I_d$

$i_{D_2} = 0$.

$2\pi \to 2\pi+\alpha$.

T_2, D_2 conduct

$V_d = 0$

$i_s = 0$

$i_{T_1} = 0$, $i_{T_2} = +I_D$

$i_{D_2} = I_D$.

$V_{T_1} = V_s$

Half waving effect :-

suppose i_{g_2} is missing at $(\pi+\alpha)$ then $T_1 D_1$ continue to conduct upto $wt = 2\pi$ giving a load voltage ~~upto~~ equal to zero. at $wt = 2\pi$, D_2 is forward biased and the conducting devices are T_1 and D_2 so the load voltage is entire positive voltage of source voltage.

under such cond^n the rectifier behaves similar to a uncontrolled half wave rectifier.

This phenomenon is called half waving effect at α: T_1 is turned on

$\alpha \to \pi$: T_1 and D_2 conduct.

$V_d = V_s$

$i_s = +I_d$

$i_{T_1} = I_d$ $\quad i_{T_2} = 0$, $i_{D_1} = 0$

$i_{D_2} = I_d$

$\pi \to \pi+\alpha$

T_1, D_1 - conduct

$V_d = 0$

$i_s = 0$

$i_{T_1} = I_d$

$i_{D_2} = 0$

$i_{D_1} = I_d$

$V_{T_1} = 0$

$\pi+\alpha \to 2\pi$

T_2, D_1 conduct.

$V_d = -V_s$

$i_s = -I_d$

$i_{T_1} = 0$, $i_{T_2} = I_d$

$i_{D_2} = 0$.

$2\pi \to 2\pi+\alpha$.

T_2, D_2 conduct

$V_d = 0$

$i_s = 0$

$i_{T_1} = 0$, $i_{T_2} = +I_D$

$i_{D_2} = I_D$.

$V_{T_1} = V_s$

* Half waving effect :-

suppose i_{g_2} is missing at $(\pi+\alpha)$ then $T_1 D_1$ continue to conduct upto $\omega t = 2\pi$ giving a load voltage ~~upto~~ equal to zero.

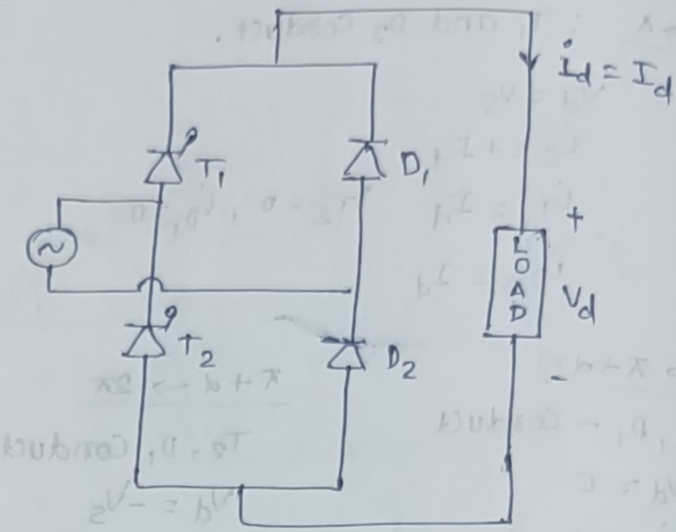
at $\omega t = 2\pi$, D_2 is forward biased and the conducting devices are T_1 and D_2 so the load voltage is entire positive voltage of source voltage.

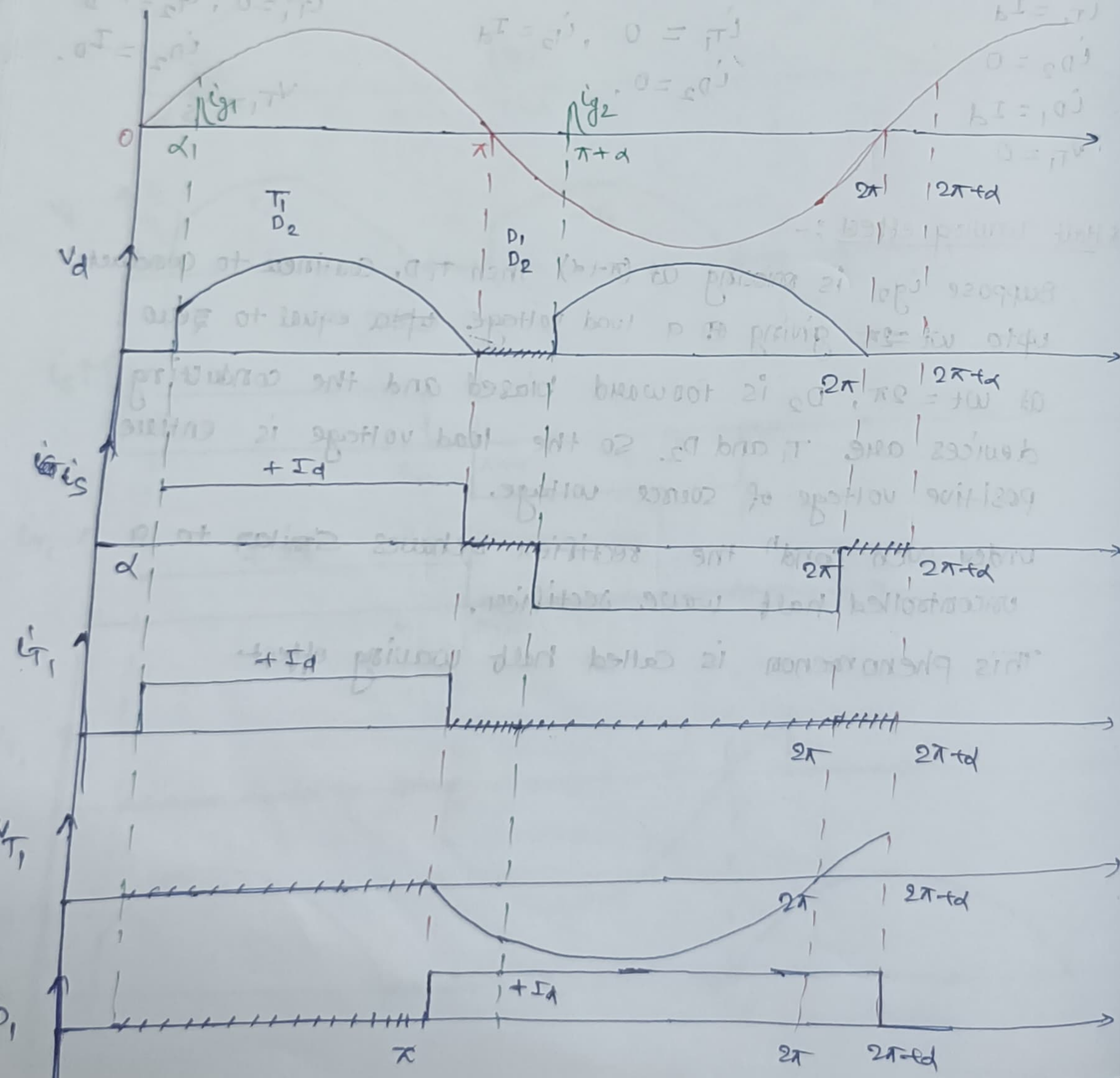
under such cond^n the rectifier behaves similar to a uncontrolled half wave rectifier.

This phenomenon is called half waving effect

Asymmeterical configration

Q→





$\alpha \to \pi$

T_1, D_2 conduct

$V_d = V_s$

$i_s = +I_d$ $\quad V_{T_1} = 0$

$i_{T_1} = I_d$

$i_{D_2} = I_d$

$\pi \to \pi+\alpha$

D_1, D_2 conduct

$V_d = 0$

$i_s = 0$ $\quad V_{T_1} = V_s$

$i_{T_1} = 0$

$i_{D_2} = i_{D_1} = I_d$

$\pi+\alpha \to 2\pi$

T_2, D_1 conduct

$V_d = -V_s$

$i_s = -I_d$

$i_{T_1} = 0$; $V_{T_1} = V_s$

$i_{D_1} = +I_d$

2π to $2\pi+\alpha$

$D_2 \to$ ON

$D_1 D_2 \to$ conducts

$V_d = 0$

$i_s = 0$

$i_{T_1} = 0$ $\quad V_{T_1} = V_s$

$i_{D_1} = i_{D_2} = +I_d$

↳ prior to $(\pi+\alpha)$ D_1 and D_2 are conducting if i_{g_2} is missing these devices continue to conduct until $(2\pi+\alpha)$. ~~So that~~ Then the output voltage is similar to that of Half wave controlled rectifier.

Therefore the possibility of half waving effect is avoided.

⇒ For asymmetric configuration one more gate pulse is required (i.e, two different gate pulse required.)

* performance parameters

① $(V_d)_{avg.} = \frac{1}{\pi}\int_{\alpha}^{\pi} V_m \sin\omega t \, d\omega t = \frac{V_m}{\pi}\left[\cos\alpha - (-1)\right] = \frac{V_m}{\pi}\left[1+\cos\alpha\right]$

② Avg. power through converter

$P = (V_d)_{avg.} I_d = \frac{V_m}{\pi}(1+\cos\alpha) I_d = \frac{V_m I_d}{\pi}\left[1+\cos\alpha\right]$

③ $(I_T)_{avg} = \frac{1}{2\pi}\left[(I_d)(\pi-\alpha)\right]$

$(I_D)_{avg} = \frac{1}{2\pi}\left[I_d(\pi+\alpha)\right]$

$(I_T)_{rms} =$

$(I_D)_{rms} =$

$\therefore PIV = V_m = \sqrt{2} V_s$.

$$i_s(t) = a_0 + \sum_{1,2,3}^{\infty} a_n \cos n\omega t + b_n \sin n\omega t.$$

$$a_0 = 0$$

$$a_1 = \frac{1}{\pi}\left[\int_{\alpha}^{\pi} I_d \cos\omega t \, d\omega t + \int_{\pi+\alpha}^{2\pi} -I_d \cos\omega t \, d\omega t\right]$$

$$= \frac{-2I_d}{\pi} \sin\alpha$$

$$b_1 = \frac{1}{\pi}\left[\int_{\alpha}^{\pi} I_d \sin\omega t \, d\omega t + \int_{\pi+\alpha}^{2\pi} -I_d \sin\omega t \, d\omega t\right]$$

$$= \frac{2I_d}{\pi}(1 + \cos\alpha)$$

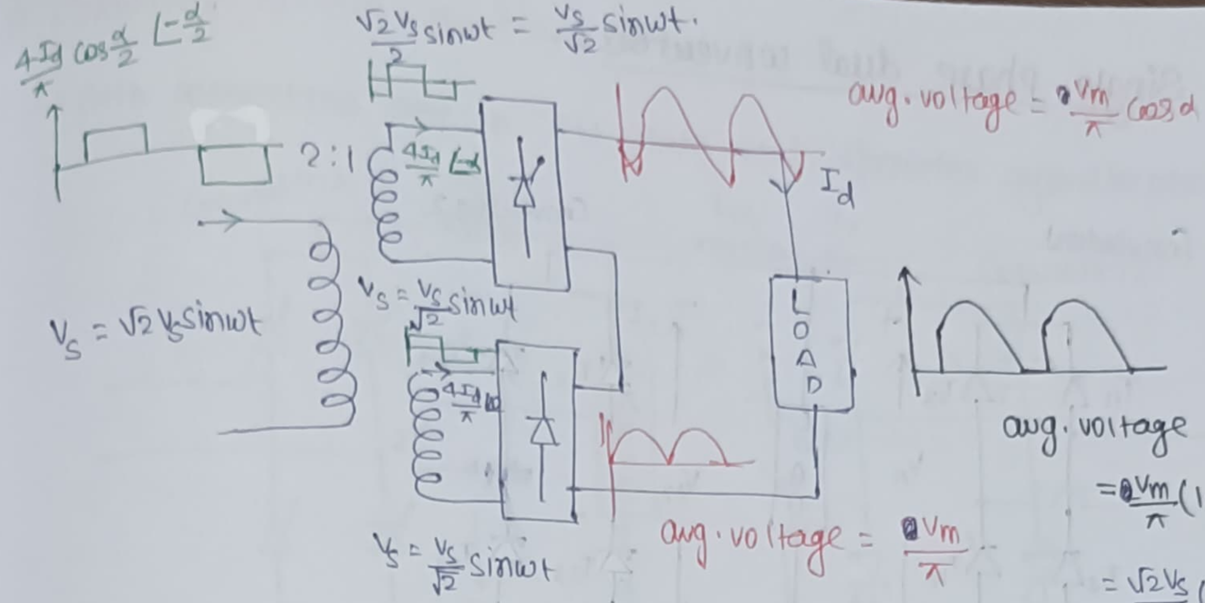
$$C_1 = \sqrt{a_1^2 + b_1^2}$$

$$= \sqrt{\frac{4I_d^2}{\pi^2}\sin^2\alpha + \frac{4I_d^2}{\pi}(1+\cos\alpha)^2}$$

$$= \frac{2I_d}{\pi}\left[\sqrt{2 + 2\cos\alpha}\right]$$

$$= \frac{2I_d}{\pi}\left[\sqrt{\cancel{2}(\cancel{1}+2\cos\frac{\alpha}{2})}\right]$$

$$= \frac{2I_d 2\cos\frac{\alpha}{2}}{\pi} = \frac{4I_d}{\pi}\cos\frac{\alpha}{2}$$



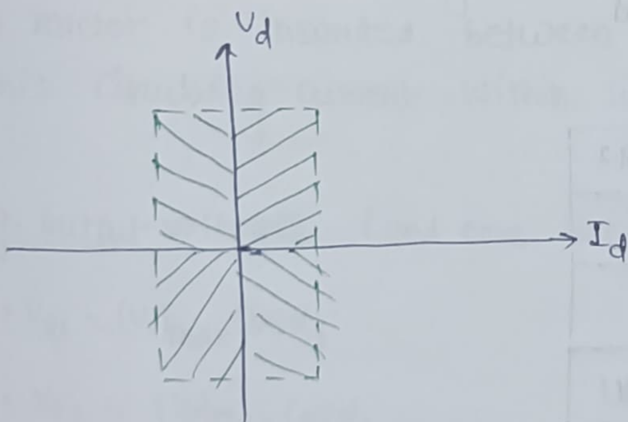
avg. voltage

$= \frac{2V_m}{\pi}(1+\cos\alpha)$

$= \frac{\sqrt{2}V_s}{\pi}(1+\cos\alpha)$

- uncontrolled Rectifier and Half controlled Rectifiers $\Rightarrow$ single quadrant (1^{st}) of $V_d - I_d$ plane.
- Controlled Rectifier $\Rightarrow$ double quadrant (1^{st} & 4^{th}) in $V_d - I_d$ plane.

Four quadrant converters :-

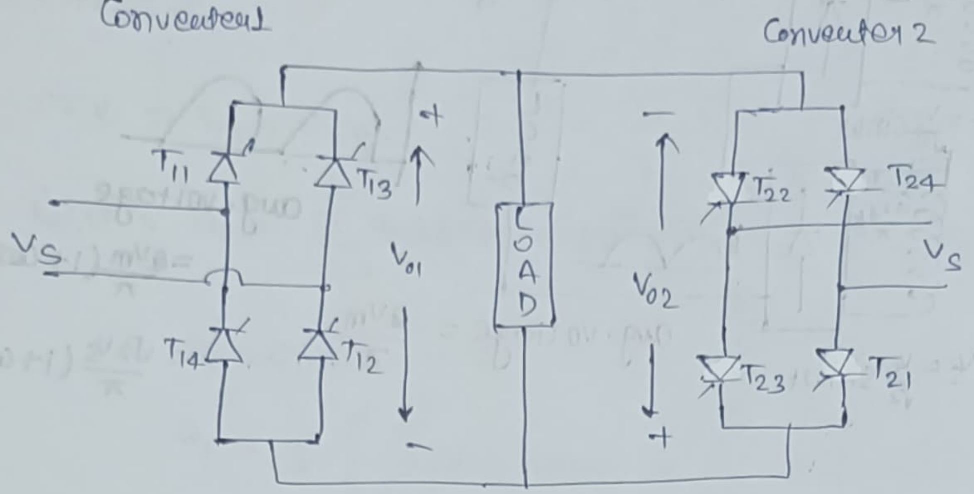


Two fully controlled converter are connected back to back

$\Downarrow$

Dual converter.

* Single-phase dual converters :-



↳ Each converter has the ability to conduct current in one direction only.

↳ Load current is bidirectional.

* Non-circulating current operation :-

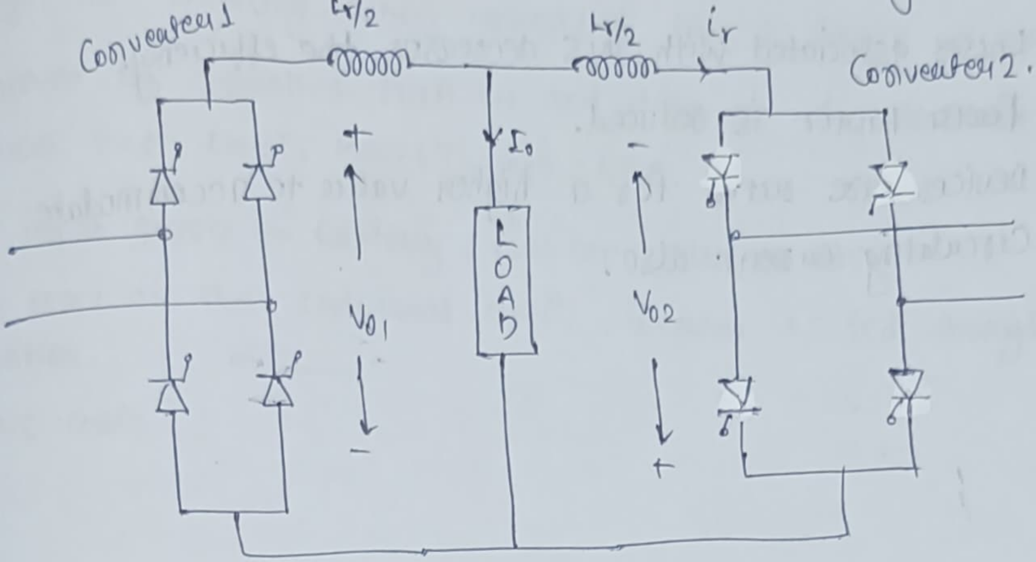
↳ only one converter operates at a time and carries the entire load current.

V_o	I_o	Converter 1	Converter 2
+ve	+ve	Rectifier	----
-ve	+ve	Inverter	---
-ve	-ve	----	Rectifier
+ve	-ve	----	Inverter

↳ No current limiting reactor is required.

* Circulating current operation :-

↳ Both converters are in ON state and operates simultaneously.



For both (+ve) load current and voltage the Inverter1 behaves as a rectifier and Inverter2 behaves as Inverter.

- Avg. voltage of both converters are equal.
- Instantaneous voltages are different.
- This leads to a circulating current between the converters
- A reactor is inserted between the converters to limit this circulating current within limits.

<u>Avg. output voltages</u> (and one behaves as inverter and other will act as a rectifier.)

- $V_{01} = (V_0)_{max} \cos\alpha_1$
- $V_{02} = (V_0)_{max} \cos\alpha_2$
- $V_{01} = -V_{02} = V_0$

$\cos\alpha_1 + \cos\alpha_2 = 0$

$\Rightarrow \underline{\alpha_1 + \alpha_2 = 180^\circ}$

↳ At any instant one converter operates as rectifier while the other one operates as inverter.

↳ To reverse the current, the role of the converters are interchanged

↳ Response is very fast

Drawbacks

- ↳ A reactor is needed.
- ↳ Losses associated with this decreases the efficiency. Power factor is reduced.
- ↳ Devices are rated for a higher value to accommodate circulating current also.

* Switch mode DC-ac Converter (Inverter)

↳ An inverter is a ckt. or system that delivers ac power at desired voltage and frequency when energized from a source of dc power.

↳ Achieved by controlled turn ON and turn off of controllable devices like IGBT, MOSFET, GTO, BJT.

↳ DC input source → Battery, fuel cell, solar cell etc.

↳ In most of the Industrial appl^n. inverter is fed through a rectifier.

↳ This conf

* Areas for applications

- variable speed ac motor drives.
- Industrial heating.
- Aircraft / spacecraft power supplies.
- uninterruptable power supplies.
- HVDC systems.
- Static VAR compensators.
- Active harmonic filters.
- Electric vehicles.

* Classifications of inverters

① voltage source inverters (VSI)

- Input dc source is a voltage source.

② current source inverters (CSI)

- input dc source is a current source.

— usually used in high power ac drives.

* Classification of VSI on the basis of output voltage control.

① Square wave inverters

- output voltege waveform is similar to a square wave.
- Input dc voltage is controlled to get controlled magnitude of ac output.
- the inverter has to control only the frequency.

② PWM inverters

- input dc voltage is constant in magnitude.
- the inverter has to control both the freqⁿ and magnitude.

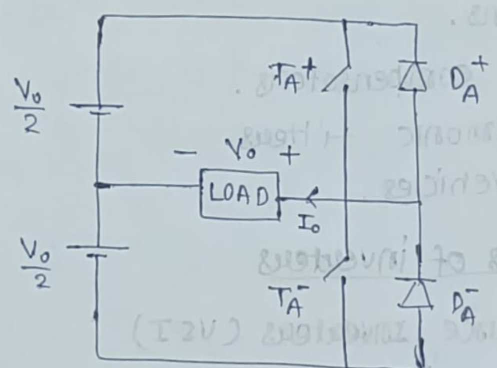
* Square wave inverters

① single phase bridge Inverter :

a) Half Bridge inverter circuit.

↳ period of inverter is selected as $T = 1/f$

f: required output frequency.



R-load

- For $t<0$

Let T_A^- was conducting

- load current path —

↳ lower source, load and T_A^-

at $t=0$, T_A^- is turned off forcibly and T_A^+ is turned ON

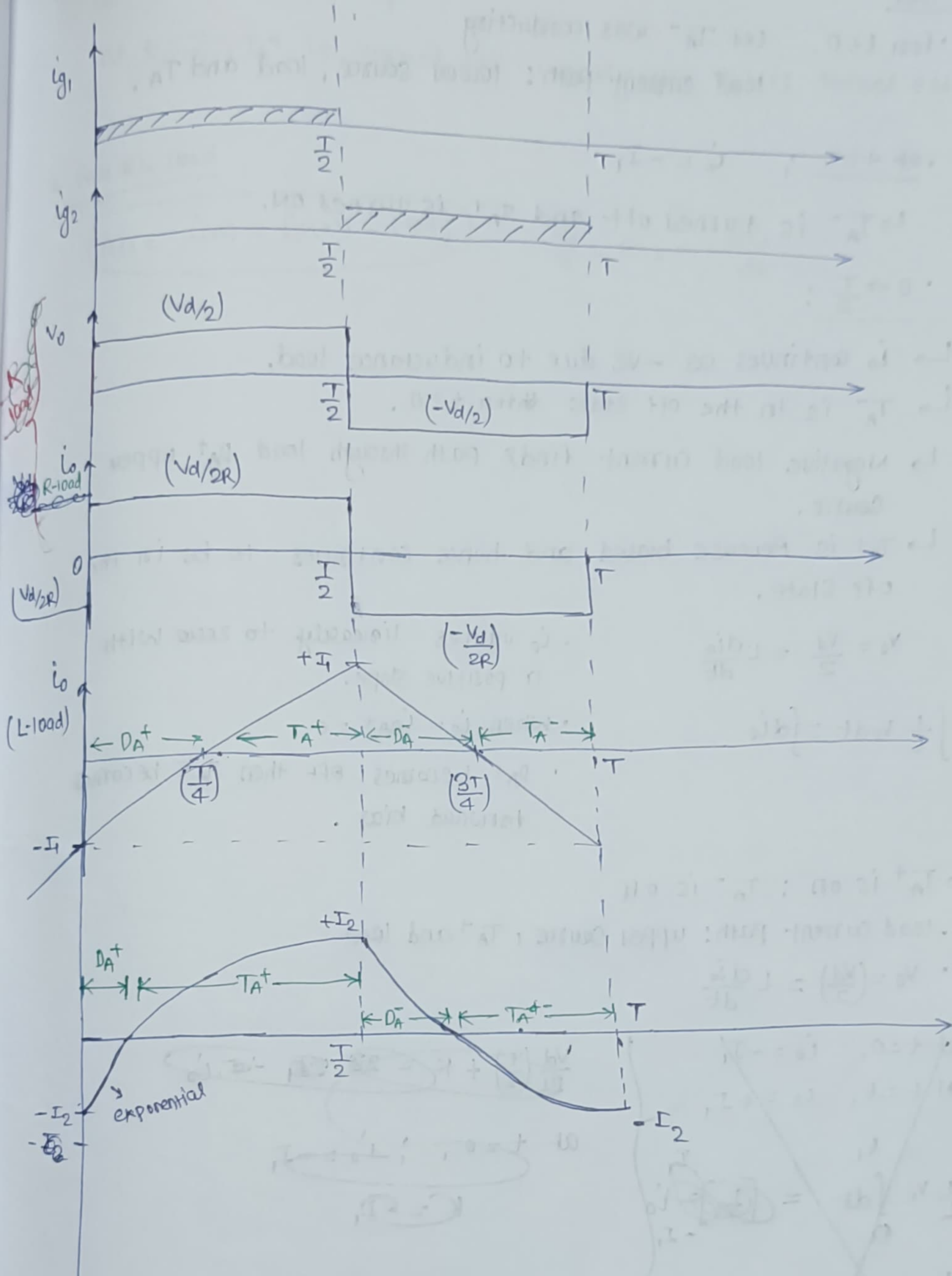
- $0 \to T/2$

$T_A^+ \to$ ON $T_A^- \to$ off

- load current path — upper source, T_A^+ and load.

- $T/2 \to T$: T_A^- ON, $T_A^+ \to$ off.

load current path →



i_{g_1}
T/2
T
i_{g_2}
T/2
T
V_0
R-load
(Vd/2)
T/2
(-Vd/2)
T
i_o
R-load
(Vd/2R)
0
(Vd/2R)
T/2
T
(-Vd/2R)
i_o
(L-load)
$+I_1$
D_A^+
T_A^+
D_A^-
T_A^-
(T/4)
(3T/4)
T
$-I_1$
$+I_2$
D_A^+
T_A^+
D_A^-
T_A^-
T
T/2
$-I_2$
exponential
$-I_2$

L-load

- for $t<0$, Let T_A^- was conducting
  load current path: lower source, load and T_A.

- at $t=0$; $i_o = -I_1$
  ↳ T_A^- is turned off and T_A^+ is turned ON.

- $0 \rightarrow \frac{T}{2}$:

↳ i_o continues as -ve due to inductance load.
↳ T_A^- is in the off state ~~th~~ for $t>0$.
↳ Negative load current finds path through load D_A^+ upper source.
↳ T_A^+ is Reverse biased and hence continues to be in the off state.

$$v_o = \frac{V_d}{2} = L\frac{di_o}{dt}$$

$$\int \frac{1}{L} v_o dt = \int di_o$$

- i_o varies linearly to zero with a positive slope.
- when $i_o = i_{DA^+} = 0$
- D_A^+ becomes off then T_A^+ becomes forward bias.

↳ T_A^+ is on ; T_A^- is off
- load current path: upper source, T_A^+ and load
- $V_o = \left(\frac{V_d}{2}\right) = L\frac{di_o}{dt}$

~~at $t=0$, $i_o = -I_1$~~
~~at $t=t_1$ $i_o = +I_1$~~

~~$\frac{1}{L} v_o \int_0^{t_1} dt = \int_{-I_1}^{I_1} di_o$~~

~~$\frac{V_d}{2L} t_1 + K = 2I_1$~~

~~$\frac{V_d}{2L}\left(\frac{T}{2}\right) = 2I_1$~~

$\frac{V_d}{2L}\left(\frac{T}{2}\right) + K = i_o$

at $t=0$; $i_o = -I_1$

$K = -I_1$

. i_0 is zero at $t = T/4$

. at $t = \frac{T}{2}$, T_A^+ is turned off forcibly at T_A^- is turned ON.

* For RL load

$$i(t) = i(\infty) + [i(0^+) - i(\infty)]$$

$$=$$

$$\frac{V_d}{2} = R i_0 + L \frac{di_0}{dt}$$