Electrical Drives

Lecture 13 (12-02-2024)

Design a speed-controlled dc motor drive maintaining the field flux constant. The motor parameters and ratings are as follows:

220 V, 8.3 A, 1470 rpm, $R_a = 4\ \Omega$, $J = 0.0607$ kg − m_2, $L_a = 0.072$ H, $B_t = 0.0869$ N·m / rad/sec, $K_b = 1.26$ V/rad/sec.

The converter is supplied from 230V, 3-phase ac at 60 Hz. The converter is linear, and its maximum control input voltage is ±10 V. The tachogenerator has the transfer function $G_\omega(s) = \dfrac{0.065}{(1 + 0.002s)}$. The speed reference voltage has a maximum of 10V. The maximum current permitted in the motor is 20 A.

Solution (i) Converter transfer function:

$$K_r = \frac{1.35\ V}{V_{cm}} = \frac{1.35 \times 230}{10} = 31.05\ V/V$$

$$V_{dc}(\max) = 310.5\ V$$

The rated dc voltage required is 220 V, which corresponds to a control voltage of 7.09 V. The transfer function of the converter is

$$G_r(s) = \frac{31.05}{(1 + 0.00138s)}\ V/V$$

(ii) Current transducer gain: The maximum safe control voltage is 7.09 V, and this has to correspond to the maximum current error:

$$i_{max} = 20 \text{ A}$$

$$H_c = \frac{7.09}{I_{max}} = \frac{7.09}{20} = 0.355 \text{ V/A}$$

(iii) Motor transfer function:

$$K_1 = \frac{B_t}{K_b^2 + R_a B_t} = \frac{0.0869}{1.26^2 + 4 \times 0.0869} = 0.0449$$

$$-\frac{1}{T_1}, -\frac{1}{T_2} = -\frac{1}{2}\left[\frac{B_t}{J} + \frac{R_a}{L_a}\right] \pm \sqrt{\frac{1}{4}\left(\frac{B_t}{J} + \frac{R_a}{L_a}\right)^2 - \left(\frac{K_b^2 + R_a B_t}{J L_a}\right)}$$

$$T_1 = 0.1077 \text{ sec} \qquad T_2 = 0.0208 \text{ sec}$$

$$T_m = \frac{J}{B_t} = 0.7 \text{ sec}$$

The subsystem transfer functions are

$$\frac{I_a(s)}{V_a(s)} = K_1\frac{(1 + sT_m)}{(1 + sT_1)(1 + sT_2)} = \frac{0.0449(1 + 0.7s)}{(1 + 0.0208s)(1 + 0.1077s)}$$

$$\frac{\omega_m(s)}{I_a(s)} = \frac{K_b/B_t}{(1 + sT_m)} = \frac{14.5}{(1 + 0.7s)}$$

(iv) Design of current controller:

$$T_c = T_2 = 0.0208 \text{ sec}$$

$$K = \frac{T_1}{2T_r} = \frac{0.1077}{2 \times 0.001388} = 38.8$$

$$K_c = \frac{KT_c}{K_1 H_c K_r T_m} = \frac{38.8 \times 0.0208}{0.0449 \times 0.355 \times 31.05 \times 0.7} = 2.33$$

(v) Current-loop approximation:

$$\frac{I_a(s)}{I_a^*(s)} = \frac{K_i}{(1 + sT_i)}$$

where

$$K_i = \frac{K_{fi}}{H_c} \cdot \frac{1}{(1 + K_{fi})}$$

$$K_{fi} = \frac{K_c K_r K_1 T_m H_c}{T_c} = 38.8$$

$$\therefore K_i = \frac{27.15}{28.09} \cdot \frac{1}{0.355} = 2.75$$

$$T_i = \frac{T_3}{1 + K_{fi}} = \frac{0.109}{1 + 38.8} = 0.0027 \text{ sec}$$

The validity of the approximations is evaluated by plotting the frequency response of the closed-loop current to its command, with and without the approximations. This is shown in Figure 3.34. From this figure, it is evident that the approximations are quite valid in the frequency range of interest.

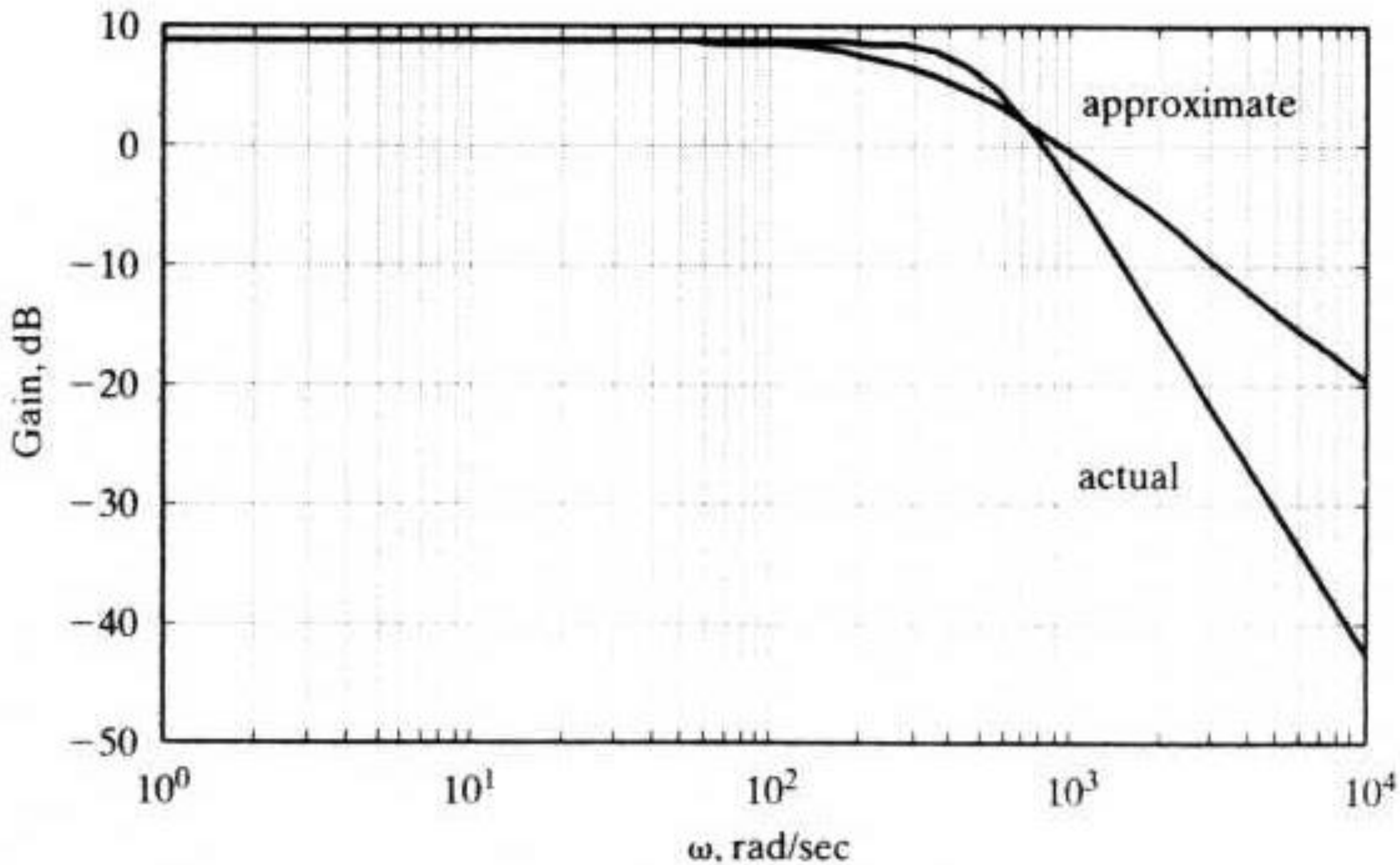

Figure 3.34 Frequency response of the current-transfer functions with and without approximation

(vi) Speed-controller design:

$$T_4 = T_i + T_\omega = 0.0027 + 0.002 = 0.0047 \text{ sec}$$

$$K_2 = \frac{K_i K_b H_\omega}{B_t T_m} = \frac{2.75 \times 1.26 \times 0.065}{0.0869 \times 0.7} = 3.70$$

$$K_s = \frac{1}{2K_2 T_4} = \frac{1}{2 \times 3.70 \times 0.0047} = 28.73$$

$$T_s = 4T_4 = 4 \times 0.0047 = 0.0188 = \text{sec}$$

The frequency responses of the speed to its command are shown in Figure 3.35 for cases with and without approximations. That the model reduction with the approximations has given a transfer function very close to the original is obvious from this figure. Further, the

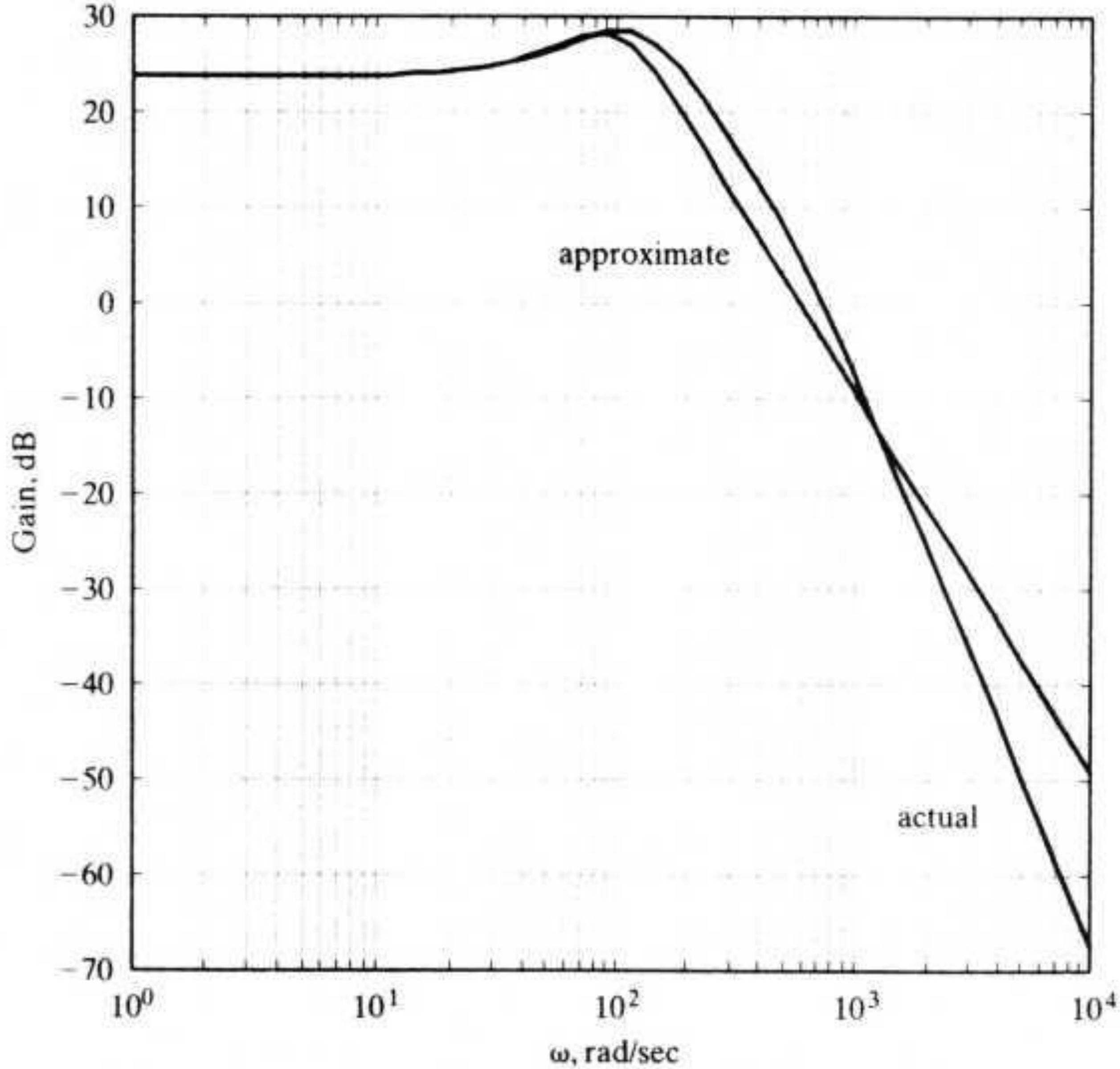

Figure 3.35 Frequency response of the speed-transfer functions with and without approximation

smoothing of the overshoot by the cancellation of the zero with a pole at $-1/4T_4$ is shown
in Figu ... order for
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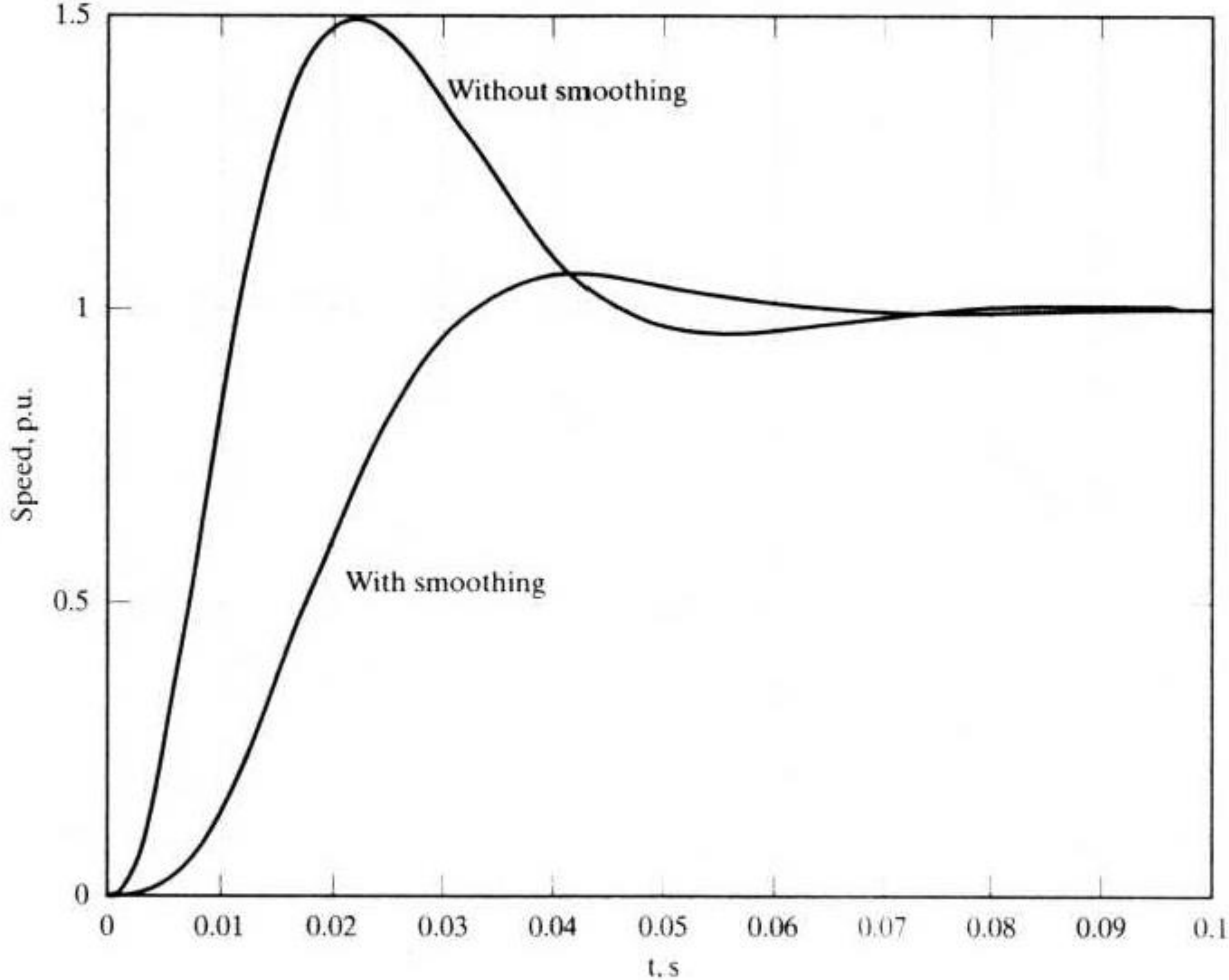

Figure 3.37 Time response of the speed controller

Assume motor poles are complex. Develop a design procedure for the current controller.

Solution If motor poles are complex, then the procedure outlined above is not applicable for the design of the current and speed controllers. One alternative is as follows: the current controller is designed by using the symmetric optimum criterion that was applied in the earlier speed controller design. The steps are given below.

Assuming $(1 + sT_m) \cong sT_m$ leads to the following current-loop transfer function:

$$\frac{i_a(s)}{i_a^*(s)} = \frac{K_2 \frac{K_c}{T_c}(1 + sT_c)}{b_0 + b_1 s + b_2 s^2 + b_3 s^3}$$

where

$$K_2 = K_1 K_r T_m$$

$$b_0 = 1 + K_2 \frac{K_c}{T_c} H_c$$

$$b_1 = T_1 + T_2 + T_r + K_2 K_c H_c$$

$$b_2 = (T_1 + T_2)T_r + T_1 T_2$$

$$b_3 = T_1 T_2 T_r$$

Applying symmetric-optimum conditions,

$$b_1^2 = 2b_0b_2$$

$$(T_1 + T_2 + T_r + K_2K_cH_c)^2 = 2\left(1 + K_2\frac{K_c}{T_c}H_c\right)((T_1 + T_2)T_r + T_1T_2)$$

$$b_2^2 = 2b_1b_3$$

$$((T_1 + T_2)T_r + T_1T_2)^2 = 2(T_1 + T_2 + T_r + K_2K_cH_c)(T_1T_2T_r)$$

but $T_r << T_1, T_2$,

$$\therefore (T_1 + T_2)T_r << T_1T_2$$

$$\frac{(T_1T_2)^2}{2(T_1T_2T_r)} = T_1 + T_2 + T_r + K_2K_cH_c$$

$$\frac{T_1T_2}{2T_r} \cong K_2K_cH_c\left[\because T_1 + T_2 + T_r << \frac{T_1T_2}{2T_r}\right]$$

$$\therefore K_c = \frac{T_1T_2}{2T_r}\frac{1}{K_2H_c}.$$

Also, $$\left(T_1 + T_2 + T_r + \frac{T_1T_2}{2T_r}\right)^2 = 2\left(1 + \frac{T_1T_2}{2T_rT_c}\right)((T_1 + T_2)T_r + T_1T_2)$$

$$\frac{(T_1T_2)^2}{4T_r^2} \cong 2T_1T_2\left(1 + \frac{T_1T_2}{2T_rT_c}\right)$$
$$\frac{1}{4T_r^2} \cong \frac{1}{T_rT_c}$$

$$T_c \cong \frac{4T_r^2}{T_r} \cong 4T_r.$$

The next step is to obtain the first-order approximation of the current-loop transfer function for the synthesis of the speed controller. Since the time constant T_c is known, the first-order approximation of the current loop is written as

$$\frac{i_a(s)}{i_a^*(s)} \cong \frac{K_i}{1 + sT_i}$$

where the steady-state gain is obtained from the exact transfer function, by setting $s = 0$, as

$$K_i = \frac{\dfrac{K_2K_c}{T_c}}{1 + \dfrac{K_2K_cH_c}{T_c}}$$

and $T_i = T_c$.

From this point, the speed-controller design follows the symmetric-optimum procedure outlined earlier for the case with the real motor poles.