Lecture 22

Dynamic Model of Induction Machine

WHY NEED DYNAMIC MODEL?

- In an electric drive system, the machine is part of the control system elements

- To be able to control the dynamics of the drive system, dynamic behavior of the machine need to be considered

- Dynamic behavior of IM can be described using dynamic model of IM

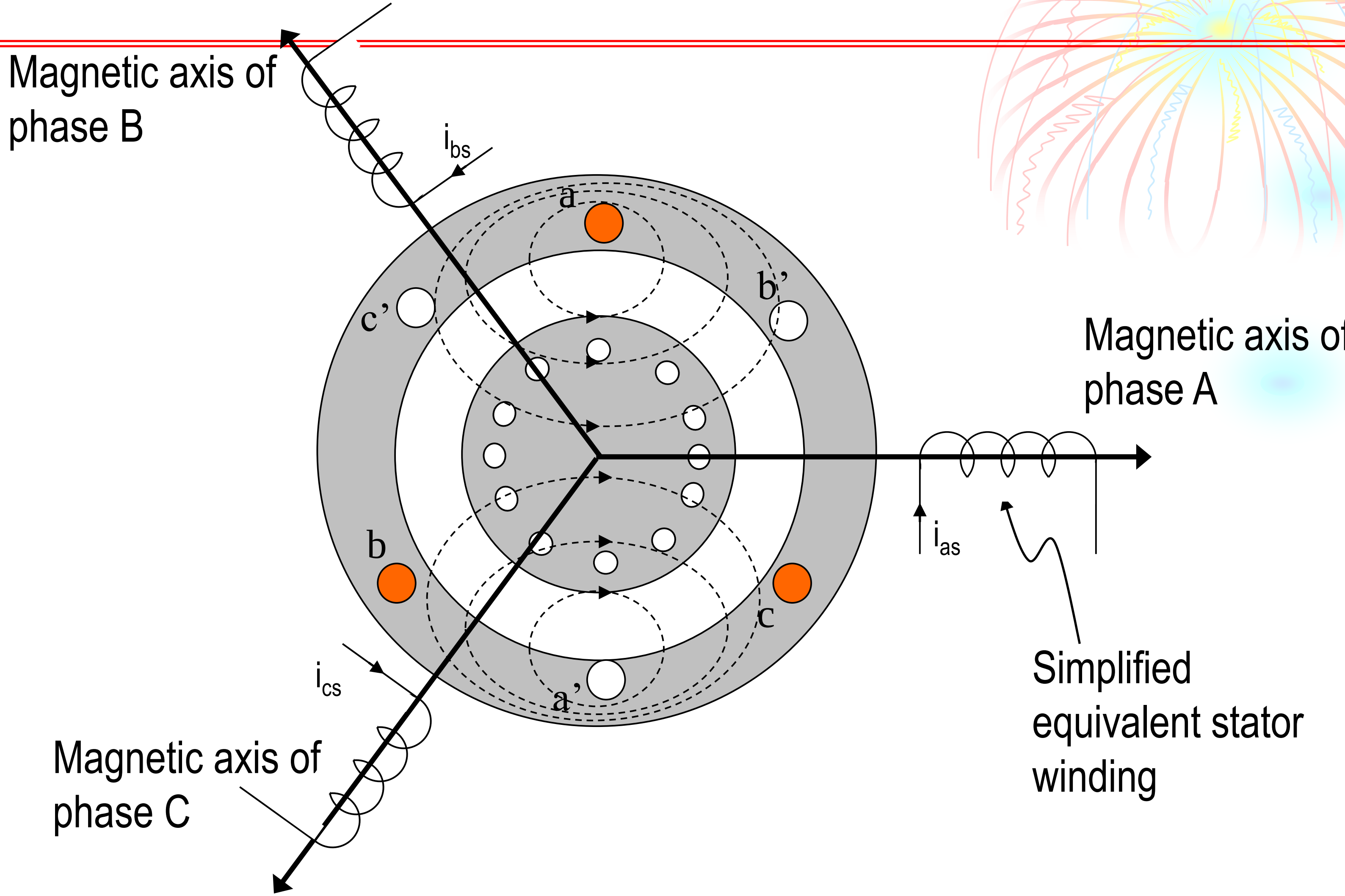
WHY NEED DYNAMIC MODEL?

- Dynamic model – complex due to magnetic coupling between stator phases and rotor phases

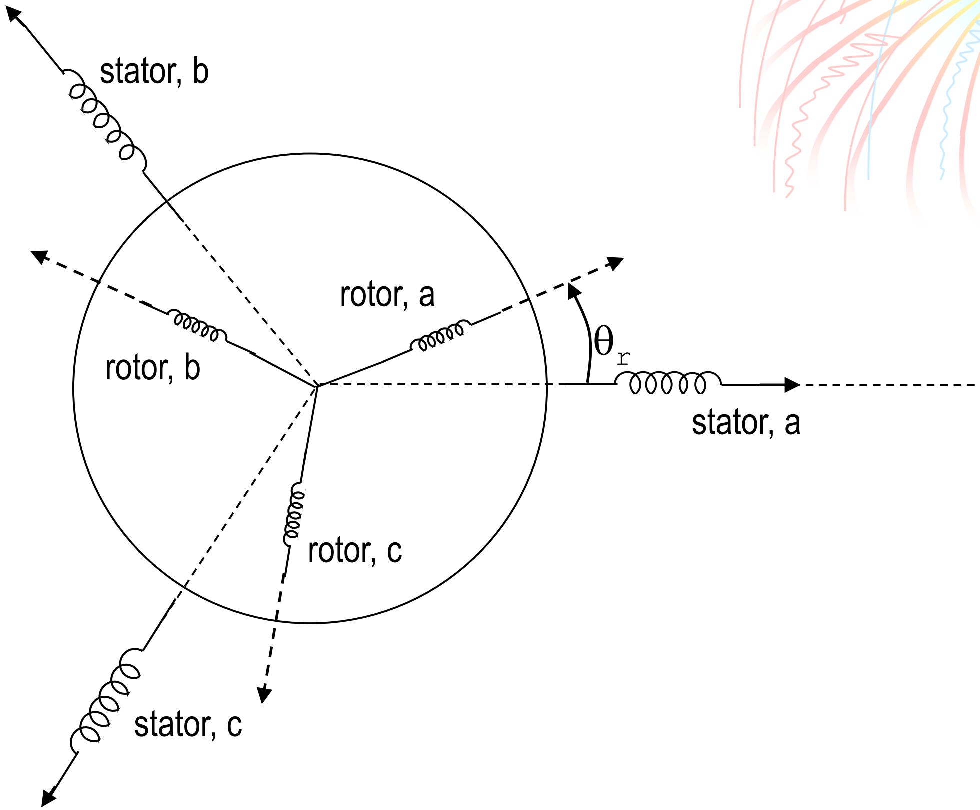
- Coupling coefficients vary with rotor position – rotor position vary with time

- Dynamic behavior of IM can be described by differential equations with time varying coefficients

DYNAMIC MODEL, 3-PHASE MODEL

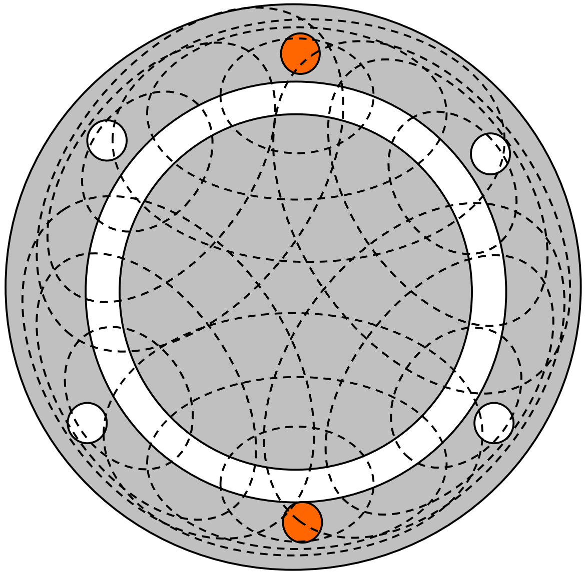


DYNAMIC MODEL – 3-phase model



DYNAMIC MODEL – 3-phase model

Let's look at phase *a*



Flux that links phase *a* is caused by:

- Flux produced by winding *a*
- Flux produced by winding *b*
- Flux produced by winding *c*

DYNAMIC MODEL – 3-phase model

Let's look at phase *a*

The relation between the currents in other phases and the flux produced by these currents that linked phase a are related by mutual inductances

- Flux produced by winding *b*
- Flux produced by winding *c*

DYNAMIC MODEL – 3-phase model

Let’s look at phase *a*

$$\Psi_{as} = \underline{\Psi_{as,s}} + \underline{\Psi_{as,r}}$$

$$L_{as} i_{as} + L_{abs} i_{bs} + L_{acs} i_{cs} \qquad L_{as,ar} i_{ar} + L_{as,br} i_{br} + L_{as,cr} i_{cr}$$

Mutual inductance between phase a and phase b of stator

Mutual inductance between phase a and phase c of stator

Mutual inductance between phase a of stator and phase a of rotor

Mutual inductance between phase a of stator and phase b of rotor

Mutual inductance between phase a of stator and phase c of rotor

DYNAMIC MODEL – 3-phase model

$$\mathbf{v_{abcs}} = R_s\mathbf{i_{abcs}} + d(\psi_{\mathbf{abcs}})/dt$$ - stator voltage equation

$$\mathbf{v_{abcr}} = R_{rr}\mathbf{i_{abcr}} + d(\psi_{\mathbf{abcr}})/dt$$ - rotor voltage equation

$$v_{abcs} = \begin{bmatrix} v_{as} \\ v_{bs} \\ v_{cs} \end{bmatrix} \qquad i_{abcs} = \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} \qquad \Psi_{abcs} = \begin{bmatrix} \Psi_{as} \\ \Psi_{bs} \\ \Psi_{cs} \end{bmatrix}$$

$$v_{abcr} = \begin{bmatrix} v_{ar} \\ v_{br} \\ v_{cr} \end{bmatrix} \qquad i_{abcr} = \begin{bmatrix} i_{ar} \\ i_{br} \\ i_{cr} \end{bmatrix} \qquad \Psi_{abcr} = \begin{bmatrix} \Psi_{ar} \\ \Psi_{br} \\ \Psi_{cr} \end{bmatrix}$$

- ψ_{abcs} flux (caused by stator and rotor currents) that links **stator windings**
- Ψ_{abcr} flux (caused by stator and rotor currents) that links **rotor windings**

DYNAMIC MODEL – 3-phase model

$$\Psi_{abcs} = \Psi_{abcs,s} + \Psi_{abcs,r}$$

$$\Psi_{abcr} = \Psi_{abcr,r} + \Psi_{abcr,s}$$

Flux linking stator winding due to stator current

$$\Psi_{abcs,s} = \begin{bmatrix} L_{as} & L_{abs} & L_{acs} \\ L_{abs} & L_{bs} & L_{bcs} \\ L_{acs} & L_{bcs} & L_{cs} \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix}$$

Flux linking stator winding due to rotor current

$$\Psi_{abcs,r} = \begin{bmatrix} L_{as,ar} & L_{as,br} & L_{as,cr} \\ L_{bs,ar} & L_{bs,br} & L_{bs,cr} \\ L_{cs,ar} & L_{cs,br} & L_{cs,cr} \end{bmatrix} \begin{bmatrix} i_{ar} \\ i_{br} \\ i_{cr} \end{bmatrix}$$

DYNAMIC MODEL – 3-phase model

Similarly we can write flux linking rotor windings caused by rotor and stator currents:

Flux linking rotor winding due to rotor current

$$\Psi_{abcr,r} = \begin{bmatrix} L_{ar} & L_{abr} & L_{acr} \\ L_{abr} & L_{br} & L_{bcr} \\ L_{acr} & L_{bcr} & L_{cr} \end{bmatrix} \begin{bmatrix} i_{ar} \\ i_{br} \\ i_{cr} \end{bmatrix}$$

Flux linking rotor winding due to stator current

$$\Psi_{abcr,s} = \begin{bmatrix} L_{ar,as} & L_{ar,bs} & L_{ar,cs} \\ L_{br,as} & L_{br,bs} & L_{br,cs} \\ L_{cr,as} & L_{cr,bs} & L_{cr,cs} \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix}$$

DYNAMIC MODEL – 3-phase model

- The self inductances consist of magnetising and leakage inductances

  $L_{as} = L_{ms} + L_{ls}$ $L_{bs} = L_{ms} + L_{ls}$ $L_{cs} = L_{ms} + L_{ls}$

The magnetizing inductance L_{ms}, accounts for the flux produce by the respective phases, crosses the airgap and links other windings

The leakage inductance L_{ls}, accounts for the flux produce by the respective phases, but does not cross the airgap and links only itself

DYNAMIC MODEL – 3-phase model

- It can be shown that the magnetizing inductance is given by

$$L_{ms} = \mu_o N_s^2 \left(\frac{rl}{g}\right)\left(\frac{\pi}{4}\right)$$

- It can be shown that the mutual inductance between stator phases is given by:

$$L_{abs} = L_{bcs} = L_{acs} = \mu_o N_s^2 \left(\frac{rl}{g}\right)\left(\frac{\pi}{4}\right)\cos 120^o$$

$$L_{abs} = L_{bcs} = L_{acs} = -\mu_o N_s^2 \left(\frac{rl}{g}\right)\left(\frac{\pi}{8}\right) = -\frac{L_{ms}}{2}$$

DYNAMIC MODEL – 3-phase model

- The mutual inductances between stator phases (and rotor phases) can be written in terms of magnetising inductances

$$\Psi_{abcs,s} = \begin{bmatrix} L_{ms} + L_{ls} & -\frac{L_{ms}}{2} & -\frac{L_{ms}}{2} \\ -\frac{L_{ms}}{2} & L_{ms} + L_{ls} & -\frac{L_{ms}}{2} \\ -\frac{L_{ms}}{2} & -\frac{L_{ms}}{2} & L_{ms} + L_{ls} \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix}$$

DYNAMIC MODEL – 3-phase model

The mutual inductances between the stator and rotor windings depends on **rotor position**

$$\Psi_{abcs,r} = \frac{N_r}{N_s} L_{ms} \begin{bmatrix} \cos\theta_r & \cos\left(\theta_r + {}^{2\pi}\!/_3\right) & \cos\left(\theta_r - {}^{2\pi}\!/_3\right) \\ \cos\left(\theta_r - {}^{2\pi}\!/_3\right) & \cos\theta_r & \cos\left(\theta_r + {}^{2\pi}\!/_3\right) \\ \cos\left(\theta_r + {}^{2\pi}\!/_3\right) & \cos\left(\theta_r - {}^{2\pi}\!/_3\right) & \cos\theta_r \end{bmatrix} \begin{bmatrix} i_{ar} \\ i_{br} \\ i_{cr} \end{bmatrix}$$

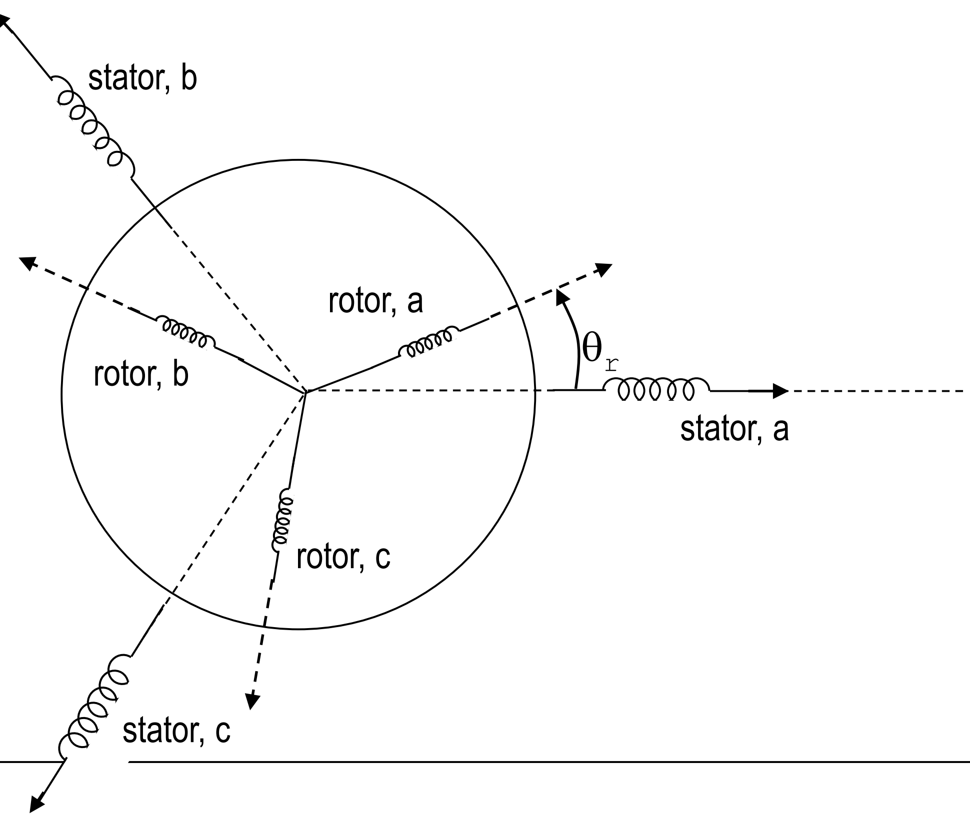
$$\Psi_{abcr,s} = \frac{N_r}{N_s} L_{ms} \begin{bmatrix} \cos\theta_r & \cos\left(\theta_r - {}^{2\pi}\!/_3\right) & \cos\left(\theta_r + {}^{2\pi}\!/_3\right) \\ \cos\left(\theta_r + {}^{2\pi}\!/_3\right) & \cos\theta_r & \cos\left(\theta_r - {}^{2\pi}\!/_3\right) \\ \cos\left(\theta_r - {}^{2\pi}\!/_3\right) & \cos\left(\theta_r + {}^{2\pi}\!/_3\right) & \cos\theta_r \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix}$$

DYNAMIC MODEL – 3-phase model

$$\Psi_{abcs,r} = \frac{N_r}{N_s} L_{ms} \begin{bmatrix} \cos\theta_r & \cos\left(\theta_r + \frac{2\pi}{3}\right) & \cos\left(\theta_r - \frac{2\pi}{3}\right) \\ \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos\theta_r & \cos\left(\theta_r + \frac{2\pi}{3}\right) \\ \cos\left(\theta_r + \frac{2\pi}{3}\right) & \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos\theta_r \end{bmatrix} \begin{bmatrix} i_{ar} \\ i_{br} \\ i_{cr} \end{bmatrix}$$

DYNAMIC MODEL – 3-phase model

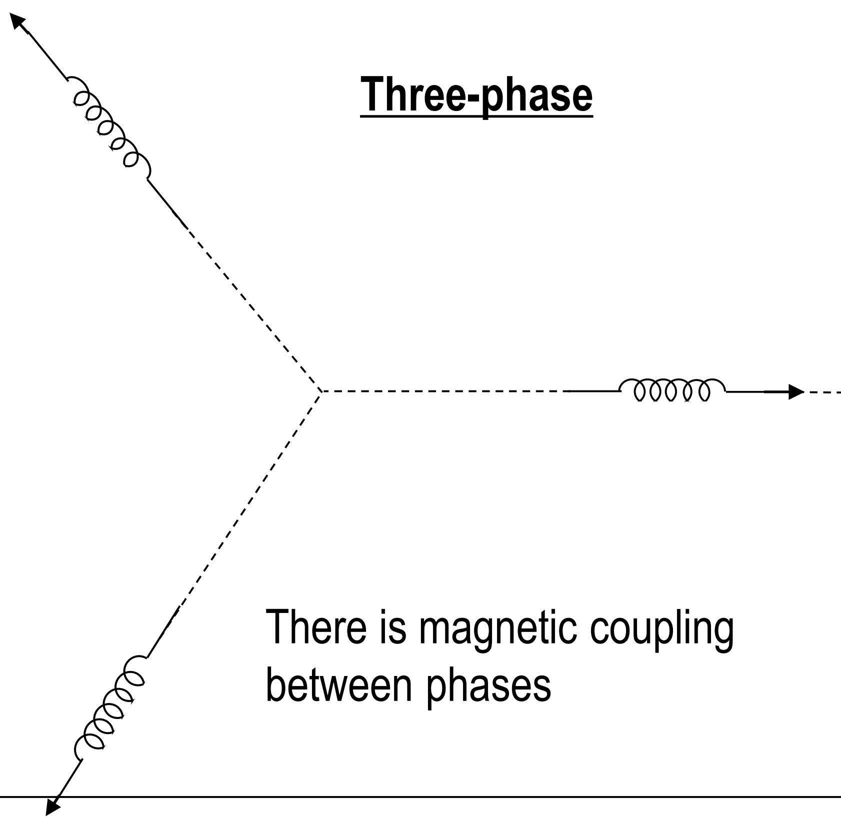
$$\Psi_{abcs,r} = \frac{N_r}{N_s} L_{ms} \begin{bmatrix} \cos\theta_r & \cos\left(\theta_r + 2\pi/3\right) & \cos\left(\theta_r - 2\pi/3\right) \\ \cos\left(\theta_r - 2\pi/3\right) & \cos\theta_r & \cos\left(\theta_r + 2\pi/3\right) \\ \cos\left(\theta_r + 2\pi/3\right) & \cos\left(\theta_r - 2\pi/3\right) & \cos\theta_r \end{bmatrix} \begin{bmatrix} i_{ar} \\ i_{br} \\ i_{cr} \end{bmatrix}$$



DYNAMIC MODEL, 2-PHASE MODEL

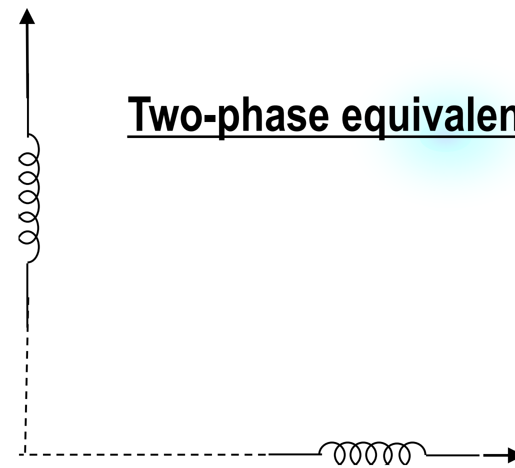
- It is **easier** to look on dynamic of IM using **two-phase** model. This can be constructed from the 3-phase model using Park's transformation

<u>Three-phase</u>



There is magnetic coupling between phases

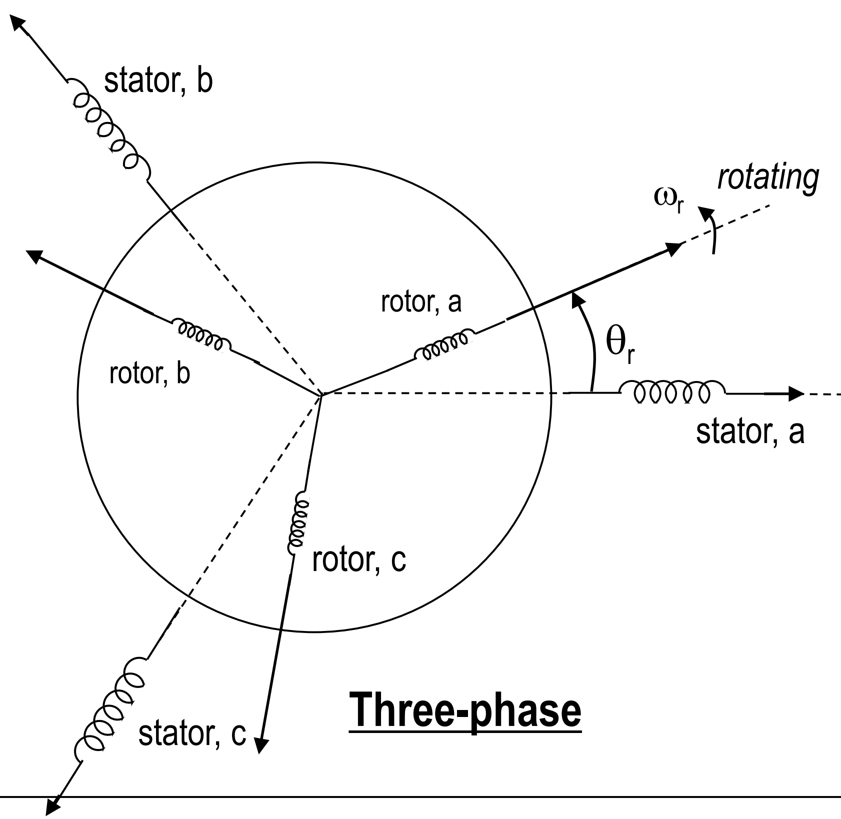
<u>Two-phase equivalent</u>



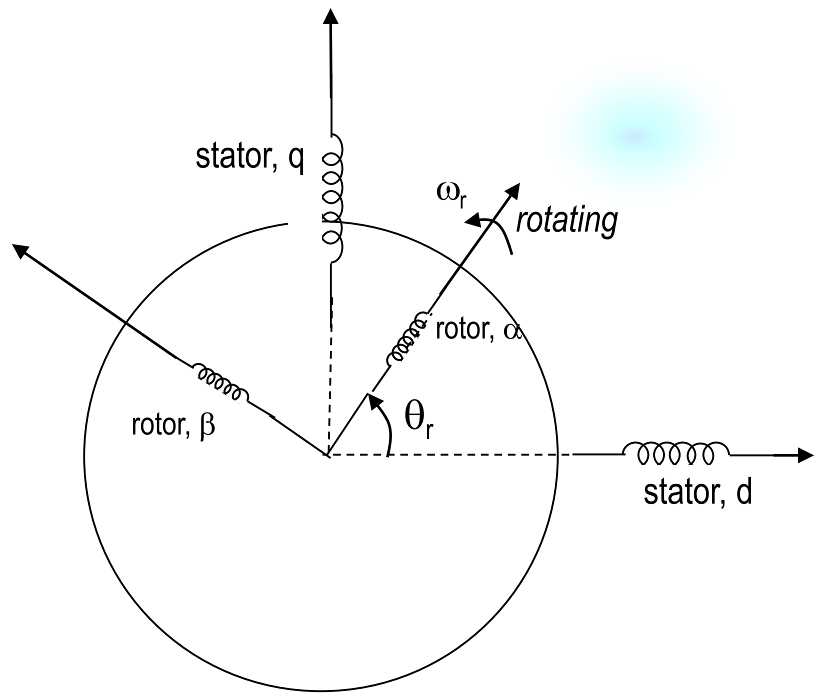
There is NO magnetic coupling between phases

DYNAMIC MODEL – 2-phase model

- It is **easier** to look on dynamic of IM using **two-phase** model. This can be constructed from the 3-phase model using Parks transformation



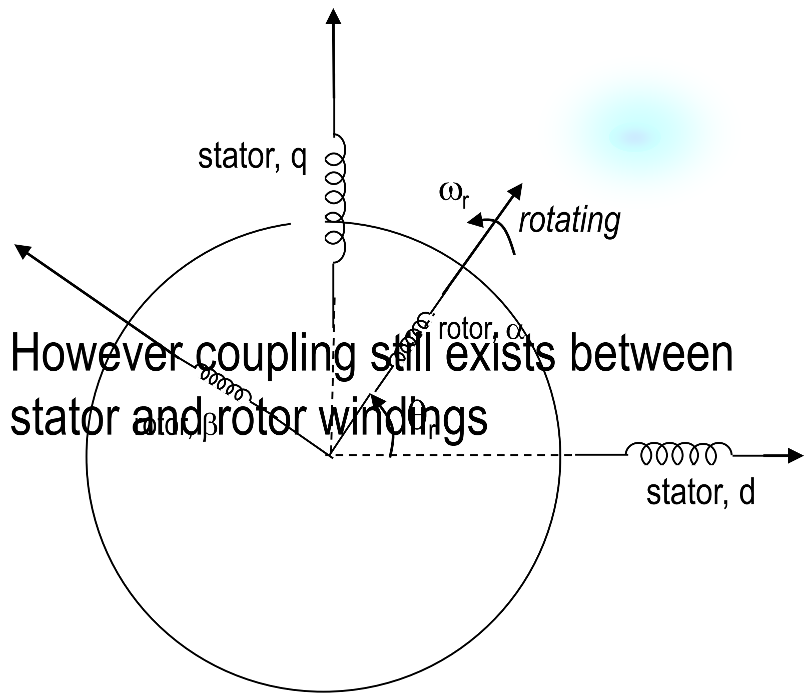
Three-phase



Two-phase equivalent

DYNAMIC MODEL – 2-phase model

- It is **easier** to look on dynamic of IM using **two-phase** model. This can be constructed from the 3-phase model using Parks transformation



However coupling still exists between stator and rotor windings

<u>Two-phase equivalent</u>

DYNAMIC MODEL – 2-phase model

- All the 3-phase quantities have to be transformed to 2-phase quantities

- In general if x_a, x_b, and x_c are the three phase quantities, the space phasor of the 3 phase systems is defined as:

$$\bar{x} = \frac{2}{3}\left(x_a + a x_b + a^2 x_c\right) \quad \text{, where} \quad a = e^{j2\pi/3}$$

$$\bar{x} = x_d + j x_q$$

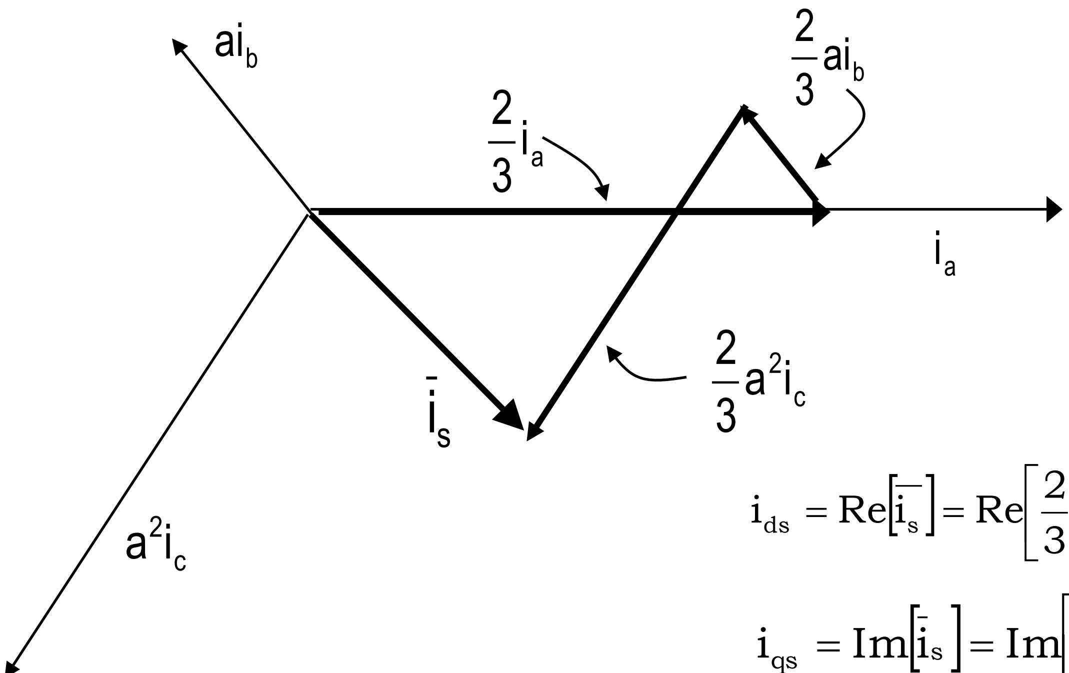
$$x_d = \mathrm{Re}[\bar{x}] = \mathrm{Re}\left[\frac{2}{3}\left(x_a + a x_b + a^2 x_c\right)\right] = \frac{2}{3}\left(x_a - \frac{1}{2}x_b - \frac{1}{2}x_c\right)$$

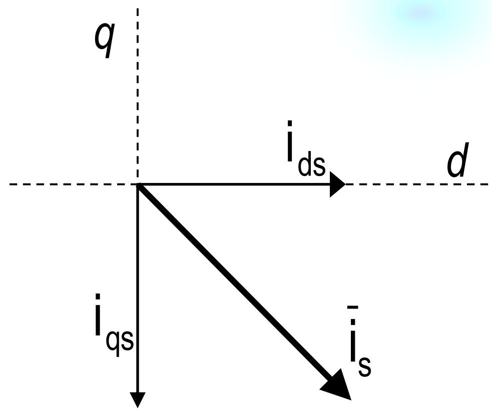
$$x_q = \mathrm{Im}[\bar{x}] = \mathrm{Im}\left[\frac{2}{3}\left(x_a + a x_b + a^2 x_c\right)\right] = \frac{1}{\sqrt{3}}\left(x_b - x_c\right)$$

DYNAMIC MODEL – 2-phase model

- All the 3-phase quantities have to be transformed into 2-phase quantities

$$\bar{i}_s = \frac{2}{3}\left(i_a + a i_b + a^2 i_c\right) \qquad \bar{i}_s = i_{ds} + j i_{qs}$$





$$i_{ds} = \mathrm{Re}\left[\bar{i}_s\right] = \mathrm{Re}\left[\frac{2}{3}\left(i_a + a i_b + a^2 i_c\right)\right] = \frac{2}{3}\left(i_a - \frac{1}{2}i_b - \frac{1}{2}i_c\right)$$

$$i_{qs} = \mathrm{Im}\left[\bar{i}_s\right] = \mathrm{Im}\left[\frac{2}{3}\left(i_a + a i_b + a^2 i_c\right)\right] = \frac{1}{\sqrt{3}}\left(i_b - i_c\right)$$

DYNAMIC MODEL – 2-phase model

- The transformation is given by:

$$\begin{bmatrix} i_d \\ i_q \\ i_o \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

For isolated neutral,

$i_a + i_b + i_c = 0$,

i.e. $i_o = 0$

$$\mathbf{i}_{dqo} = \mathbf{T}_{abc}\,\mathbf{i}_{abc}$$

The inverse transform is given by:

$$\mathbf{i}_{abc} = \mathbf{T}_{abc}^{-1}\,\mathbf{i}_{dqo}$$

DYNAMIC MODEL – 2-phase model

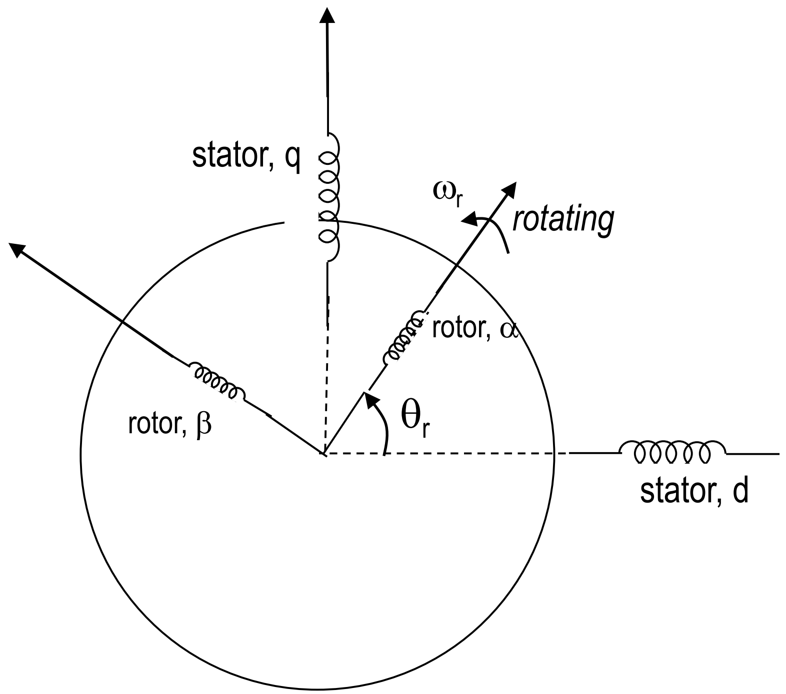
3-phase **IM equations :** *2-phase*

$$\mathbf{v}_{\mathbf{abcs}} = R_s \mathbf{i}_{\mathbf{abcs}} + d(\psi_{\mathbf{abcs}})/dt$$

$$\mathbf{v}_{\mathbf{abcr}} = R_{rr} \mathbf{i}_{\mathbf{abcr}} + d(\psi_{\mathbf{abr}})/dt$$

$$\mathbf{v}_{\mathbf{dq}} = R_s \mathbf{i}_{\mathbf{dq}} + d(\psi_{\mathbf{dq}})/dt$$

$$\mathbf{v}_{\alpha\beta} = R_{rr} \mathbf{i}_{\alpha\beta} + d(\psi_{\alpha\beta})/dt$$



DYNAMIC MODEL – 2-phase model

$$\mathbf{v_{dq}} = R_s\mathbf{i_{dq}} + d(\psi_{\mathbf{dq}})/dt$$ ⇐ Express in stationary frame

$$\mathbf{v_{\alpha\beta}} = R_r\mathbf{i}_{\alpha\beta} + d(\psi_{\alpha\beta})/dt$$ ⇐ Express in rotating frame

where,

$$\psi_{\mathbf{dq}} = \Psi_{dqs,s} + \Psi_{dqs,r}$$

$$\psi_{\alpha\beta} = \Psi_{\alpha\beta r,r} + \Psi_{\alpha\beta r,s}$$

$$\Psi_{dqs,s} = \begin{bmatrix} L_{dd} & L_{dq} \\ L_{qd} & L_{qq} \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix} \qquad \Psi_{dqs,r} = \begin{bmatrix} L_{d\alpha} & L_{d\beta} \\ L_{q\alpha} & L_{q\beta} \end{bmatrix} \begin{bmatrix} i_{\alpha r} \\ i_{\beta r} \end{bmatrix}$$

$$\Psi_{\alpha\beta r,r} = \begin{bmatrix} L_{\alpha\alpha} & L_{\alpha\beta} \\ L_{\beta\alpha} & L_{\beta\beta} \end{bmatrix} \begin{bmatrix} i_{\alpha r} \\ i_{\beta r} \end{bmatrix} \qquad \Psi_{\alpha\beta r,s} = \begin{bmatrix} L_{\alpha d} & L_{\alpha q} \\ L_{\beta d} & L_{\beta q} \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix}$$

Note that:

$$L_{dq} = L_{qd} = 0 \qquad L_{\beta\alpha} = L_{\alpha\beta} = 0$$

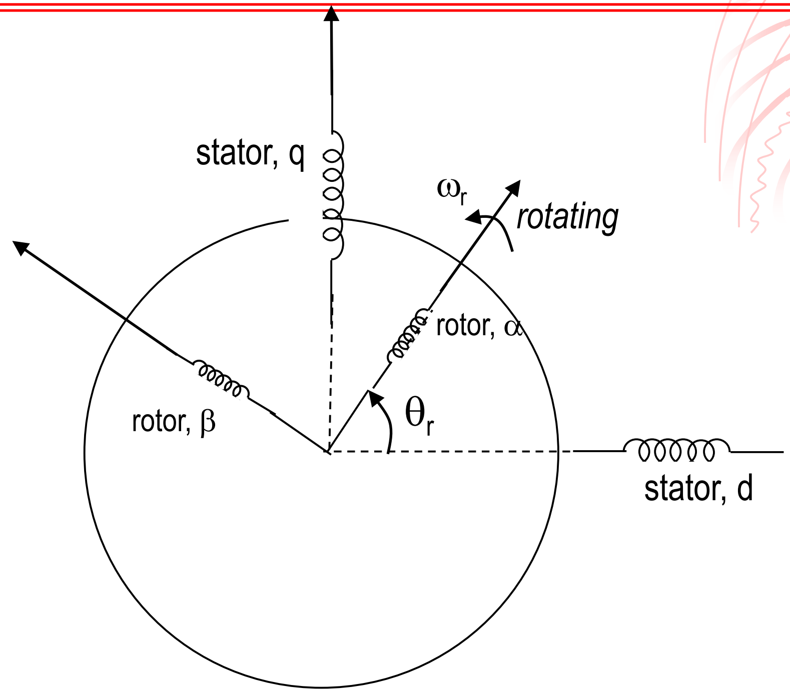
$$L_{dd} = L_{qq} \qquad L_{\beta\beta} = L_{\alpha\alpha}$$

The mutual inductance between stator and rotor depends on rotor position:

$$L_{d\alpha} = L_{\alpha d} = L_{sr} \cos \theta_r \qquad L_{q\beta} = L_{\beta q} = L_{sr} \cos \theta_r$$

$$L_{d\beta} = L_{\beta d} = - L_{sr} \sin \theta_r \qquad L_{q\alpha} = L_{\alpha q} = L_{sr} \sin \theta_r$$

DYNAMIC MODEL – 2-phase model



$$L_{d\alpha} = L_{\alpha d} = L_{sr} \cos \theta_r \qquad L_{q\beta} = L_{\beta q} = L_{sr} \cos \theta_r$$

$$L_{d\beta} = L_{\beta d} = - L_{sr} \sin \theta_r \qquad L_{q\alpha} = L_{\alpha q} = L_{sr} \sin \theta_r$$

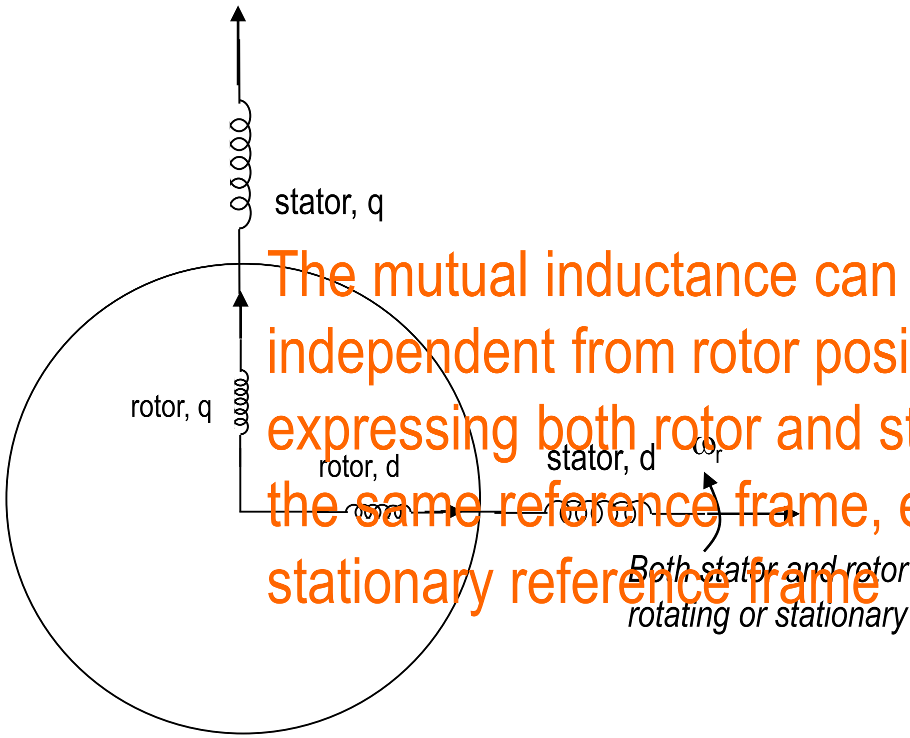
DYNAMIC MODEL – 2-phase model

In matrix form this an be written as:

$$\begin{bmatrix} v_d \\ v_q \\ v_\alpha \\ v_\beta \end{bmatrix} = \begin{bmatrix} R_s + sL_{dd} & 0 & sL_{sr}\cos\theta_r & -sL_{sr}\sin\theta_r \\ 0 & R_s + sL_{dd} & sL_{sr}\sin\theta_r & sL_{sr}\cos\theta_r \\ sL_{sr}\cos\theta_r & sL_{sr}\sin\theta_r & R_r + sL_{\alpha\alpha} & 0 \\ -L_{sr}\sin\theta_r & sL_{sr}\cos\theta_r & 0 & R_r + sL_{\beta\beta} \end{bmatrix} \cdot \begin{bmatrix} i_{sd} \\ i_{sq} \\ i_{r\alpha} \\ i_{r\beta} \end{bmatrix}$$

- The mutual inductance between rotor and stator depends on rotor position

DYNAMIC MODEL – 2-phase model



The mutual inductance can be made independent from rotor position by expressing both rotor and stator in the same reference frame, e.g. in the stationary reference frame

Magnetic path from stator linking the rotor winding independent of rotor position $\therefore$ mutual inductance independent of rotor position

DYNAMIC MODEL – 2-phase model

If the rotor quantities are referred to stator, the following can be written:

$$\begin{bmatrix} v_{sd} \\ v_{sq} \\ v_{rd} \\ v_{rq} \end{bmatrix} = \begin{bmatrix} R_s + sL_s & 0 & sL_m & 0 \\ 0 & R_s + sL_s & 0 & sL_m \\ sL_m & \omega_r L_m & R_r' + sL_r & \omega_r L_r \\ -\omega_r L_m & sL_m & -\omega_r L_r & R_r' + sL_r \end{bmatrix} . \begin{bmatrix} i_{sd} \\ i_{sq} \\ i_{rd} \\ i_{rq} \end{bmatrix}$$

L_m, L_r are the mutual and rotor self inductances referred to stator, and R_r' is the rotor resistance referred to stator

$L_s = L_{dd}$ is the stator self inductance

V_{rd}, v_{rq}, i_{rd}, i_{rq} are the rotor voltage and current referred to stator

DYNAMIC MODEL – 2-phase model

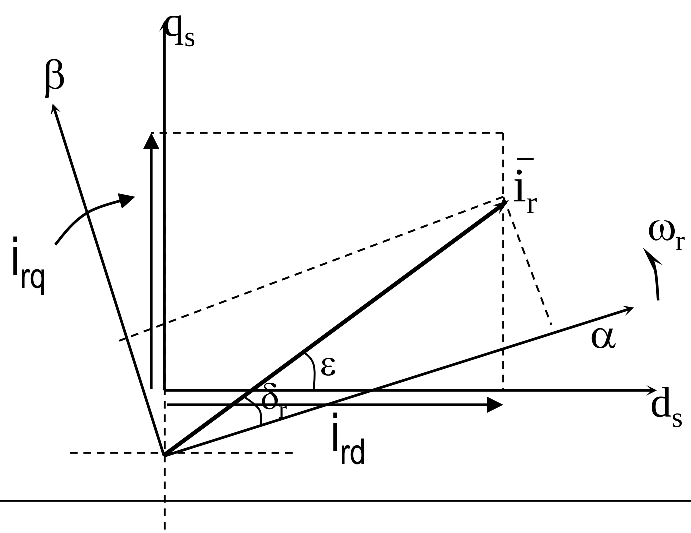
How do we express rotor current in stator (stationary) frame?

$$\bar{i}_r = \frac{2}{3}\left(i_{ra} + a i_{rb} + a^2 i_{rc}\right)$$

$\bar{i}_r$ is known as the **space vector** of the rotor current

In rotating frame this can be written as: $\bar{i}_r = i_r e^{j\delta_r}$

In stationary frame it be written as:



$$\bar{i}_r^{\,s} = i_r e^{j(\delta_r + \varepsilon)}$$

$$= \bar{i}_r e^{j\varepsilon}$$

$$= i_{rd} + j i_{rq}$$

DYNAMIC MODEL – 2-phase model

It can be shown that in a reference frame rotating at ω_g, the equation can be written as:

$$\begin{bmatrix} v_{sd} \\ v_{sq} \\ v_{rd} \\ v_{rq} \end{bmatrix} = \begin{bmatrix} R_s + sL_s & -\omega_g L_s & sL_m & -\omega_g L_m \\ \omega_g L_s & R_s + sL_s & \omega_g L_m & sL_m \\ sL_m & -(\omega_g - \omega_r)L_m & R_r' + sL_r & -(\omega_g - \omega_r)L_r \\ (\omega_g - \omega_r)L_m & sL_m & (\omega_g - \omega_r)L_r & R_r' + sL_r \end{bmatrix} \cdot \begin{bmatrix} i_{sd} \\ i_{sq} \\ i_{rd} \\ i_{rq} \end{bmatrix}$$

DYNAMIC MODEL – Space vectors

IM can be compactly written using space vectors:

$$\bar{v}_s^g = R_s \bar{i}_s^g + \frac{d\bar{\psi}_s^g}{dt} + j\omega_g \bar{\psi}_s^g$$

$$0 = R_r \bar{i}_r^g + \frac{d\bar{\psi}_r^g}{dt} + j(\omega_g - \omega_r)\bar{\psi}_r^g$$

$$\bar{\psi}_s^g = L_s \bar{i}_s^g + L_m \bar{i}_r^g$$

$$\bar{\psi}_r^g = L_r \bar{i}_r^g + L_m \bar{i}_s^g$$

All quantities are written in general reference frame

DYNAMIC MODEL – Torque equation

Product of voltage and current conjugate space vectors:

$$\bar{v}_s \bar{i}_s^* = \frac{2}{3}\left(v_{as} + a v_{bs} + a^2 v_{cs}\right)\frac{2}{3}\left(i_{as} + a^2 i_{bs} + a i_{cs}\right)$$

It can be shown that for $i_{as} + i_{bs} + i_{cs} = 0$,

$$\mathrm{Re}\left[\bar{v}_s \bar{i}_s^*\right] = \frac{2}{3}\boxed{\left(v_{as} i_{as} + v_{bs} i_{bs} + v_{cs} i_{cs}\right)}$$

$$P_{in} = \left(v_{as} i_{as} + v_{bs} i_{bs} + v_{cs} i_{cs}\right) = \frac{3}{2}\mathrm{Re}\left[\bar{v}_s \bar{i}_s^*\right]$$

DYNAMIC MODEL – Torque equation

$$P_{in} = \frac{3}{2}\mathrm{Re}\left[\bar{v}_s \bar{i}_s^*\right] = \frac{3}{2}\mathrm{Re}\left[(v_d + jv_q)(i_d - ji_q)\right] = \frac{3}{2}\left[v_d i_d + v_q i_q\right]$$

If $v = \begin{bmatrix} v_d \\ v_q \end{bmatrix}$ and $i = \begin{bmatrix} i_d \\ i_q \end{bmatrix}$

$$P_{in} = \frac{3}{2} i^t v$$

$$P_{in} = \left(v_{as} i_{as} + v_{bs} i_{bs} + v_{cs} i_{cs}\right) = \frac{3}{2}\mathrm{Re}\left[\bar{v}_s \bar{i}_s^*\right]$$

DYNAMIC MODEL – Torque equation

The IM equation can be written as:

$$\mathbf{v} = [R]\mathbf{i} + [L]s\mathbf{i} + [G]\omega_r\mathbf{i} + [F]\omega_g\mathbf{i}$$

The input power is given by:

$$p_i = \frac{3}{2}i^t V = \frac{3}{2}\left[i^t[R]i + i^t[L]si + i^t[G]\omega_r i + i^t[F]\omega_g i\right]$$

Power Losses in winding resistance

Rate of change of stored magnetic energy

Mech power

Power associated with ω_g – upon expansion gives zero

DYNAMIC MODEL – Torque equation

$$p_{mech} = \omega_m T_e = \frac{3}{2} i^t [G] \omega_r i$$

$$\begin{bmatrix} v_{sd} \\ v_{sq} \\ v_{rd} \\ v_{rq} \end{bmatrix} = \begin{bmatrix} R_s + sL_s & -\omega_g L_s & sL_m & -\omega_g L_m \\ \omega_g L_s & R_s + sL_s & \omega_g L_m & sL_m \\ sL_m & \boxed{-(\omega_g - \omega_r)L_m} & R_r' + sL_r & \boxed{-(\omega_g - \omega_r)L_r} \\ \boxed{(\omega_g - \omega_r)L_m} & sL_m & \boxed{(\omega_g - \omega_r)L_r} & R_r' + sL_r \end{bmatrix} \cdot \begin{bmatrix} i_{sd} \\ i_{sq} \\ i_{rd} \\ i_{rq} \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \omega_r L_m & 0 & \omega_r L_r \\ -\omega_r L_m & 0 & -\omega_r L_r & 0 \end{bmatrix} \cdot \begin{bmatrix} i_{sd} \\ i_{sq} \\ i_{rd} \\ i_{rq} \end{bmatrix}$$

DYNAMIC MODEL – Torque equation

$$p_{mech} = \omega_m T_e = \frac{3}{2} i^t [G] \omega_r i$$

$$\omega_m T_e = \frac{3}{2} \begin{bmatrix} i_{sd} \\ i_{sq} \\ i_{rd} \\ i_{rq} \end{bmatrix}^t \cdot \begin{bmatrix} 0 \\ 0 \\ L_m i_{sq} + L_r i_{rq} \\ -L_m i_{sd} - L_r i_{rd} \end{bmatrix} \omega_r$$

We know that $\omega_m = \omega_r / (p/2)$,

$$T_e = \frac{3}{2} \frac{p}{2} L_m \left(i_{sq} i_{rd} - i_{sd} i_{rq} \right)$$

DYNAMIC MODEL – Torque equation

$$T_e = \frac{3}{2}\frac{p}{2} L_m \,\mathrm{Im}\left(\bar{i}_s \bar{i}_r^*\right)$$

but $\quad \bar{\psi}_s = L_s \bar{i}_s + L_m \bar{i}_r \quad \Rightarrow L_m \bar{i}_r = \bar{\psi}_s - L_s \bar{i}_s$

$$T_e = \frac{3}{2}\frac{p}{2} \mathrm{Im}\left(\bar{i}_s \left(\psi_s^* - L_s i_s^*\right)\right)$$

$$\therefore \quad T_e = \frac{3}{2}\frac{p}{2} \mathrm{Im}\left(\bar{i}_s \psi_s^*\right)$$

$$T_e = \frac{3}{2}\frac{p}{2} L_m \left(i_{sq} i_{rd} - i_{sd} i_{rq}\right)$$

$$T_e = \frac{3}{2}\frac{p}{2} \bar{\psi}_s \times \bar{i}_s$$

DYNAMIC MODEL

Simulation

Re-arranging with stator and rotor currents as state space variables:

$$\begin{bmatrix} \dot{i}_{sd} \\ \dot{i}_{sq} \\ \dot{i}_{rd} \\ \dot{i}_{rq} \end{bmatrix} = \frac{1}{L_m^2 - L_r L_s} \begin{bmatrix} R_s L_r & -\omega_r L_m^2 i_{sq} & -R_r L_m & -\omega_r L_m L_r \\ \omega_r L_m^2 & R_s L_r & \omega_r L_m L_r & -R_r L_m \\ -R_s L_m & \omega_r L_m L_s & R_r L_s & \omega_r L_r L_s \\ -\omega_r L_m L_s & -R_s L_m & -\omega_r L_r L_s & R_r L_s \end{bmatrix} \cdot \begin{bmatrix} i_{sd} \\ i_{sq} \\ i_{rd} \\ i_{rq} \end{bmatrix} + \frac{1}{L_m^2 - L_r L_s} \begin{bmatrix} -L_r & 0 \\ 0 & -L_r \\ L_m & 0 \\ 0 & L_m \end{bmatrix} \cdot \begin{bmatrix} v_{sd} \\ v_{sq} \end{bmatrix}$$

The torque can be expressed in terms of stator and rotor currents:

$$T_e = \frac{3}{2}\frac{p}{2} L_m \left[i_{sq} i_{rd} - i_{sd} i_{rq} \right] \qquad T_e - T_L = J \frac{d\omega_m}{dt}$$