Monsoon
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EE6303D Dynamics of Electrical Machines (DEM)

Module 4

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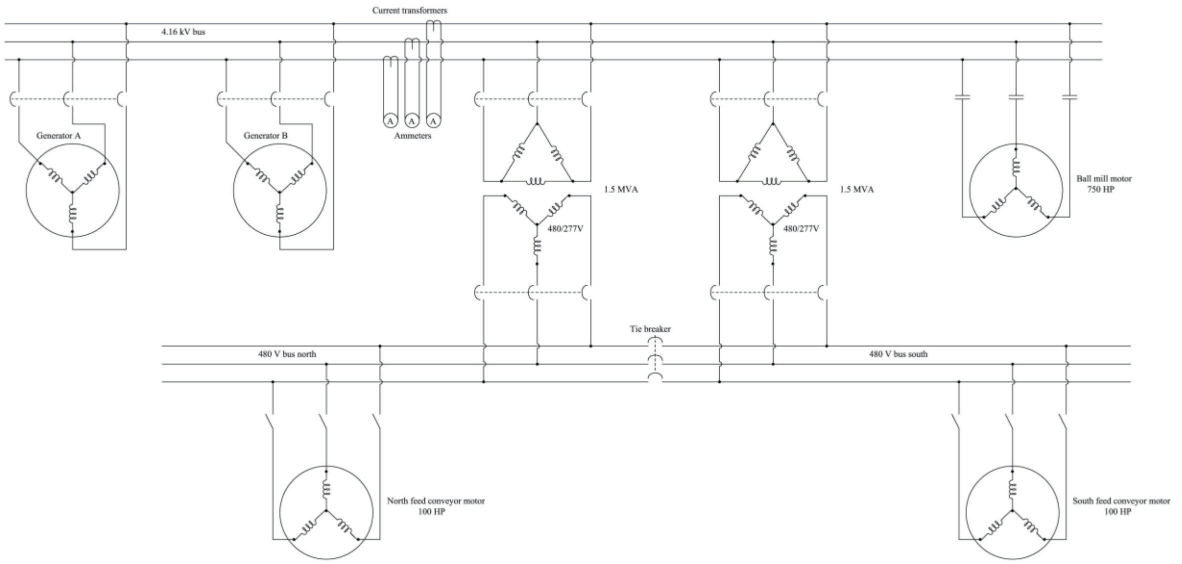
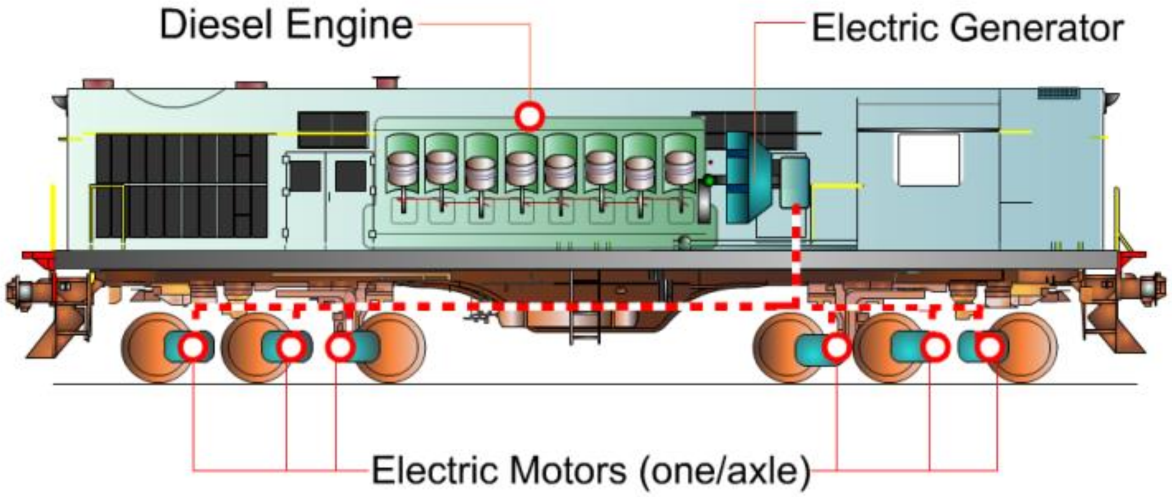
Interconnected Machines



In industrial processes electrical machines may be used singly, in groups of identical machines, or in groups of different machines interconnected to achieve some required overall operating characteristics. A simple example of this is the use of a pilot exciter and main exciter to provide controlled and stable field current for a large alternator in a power station, all machine rotors being on one shaft. In steel production, groups of machines may operate in parallel or in controlled sequence.



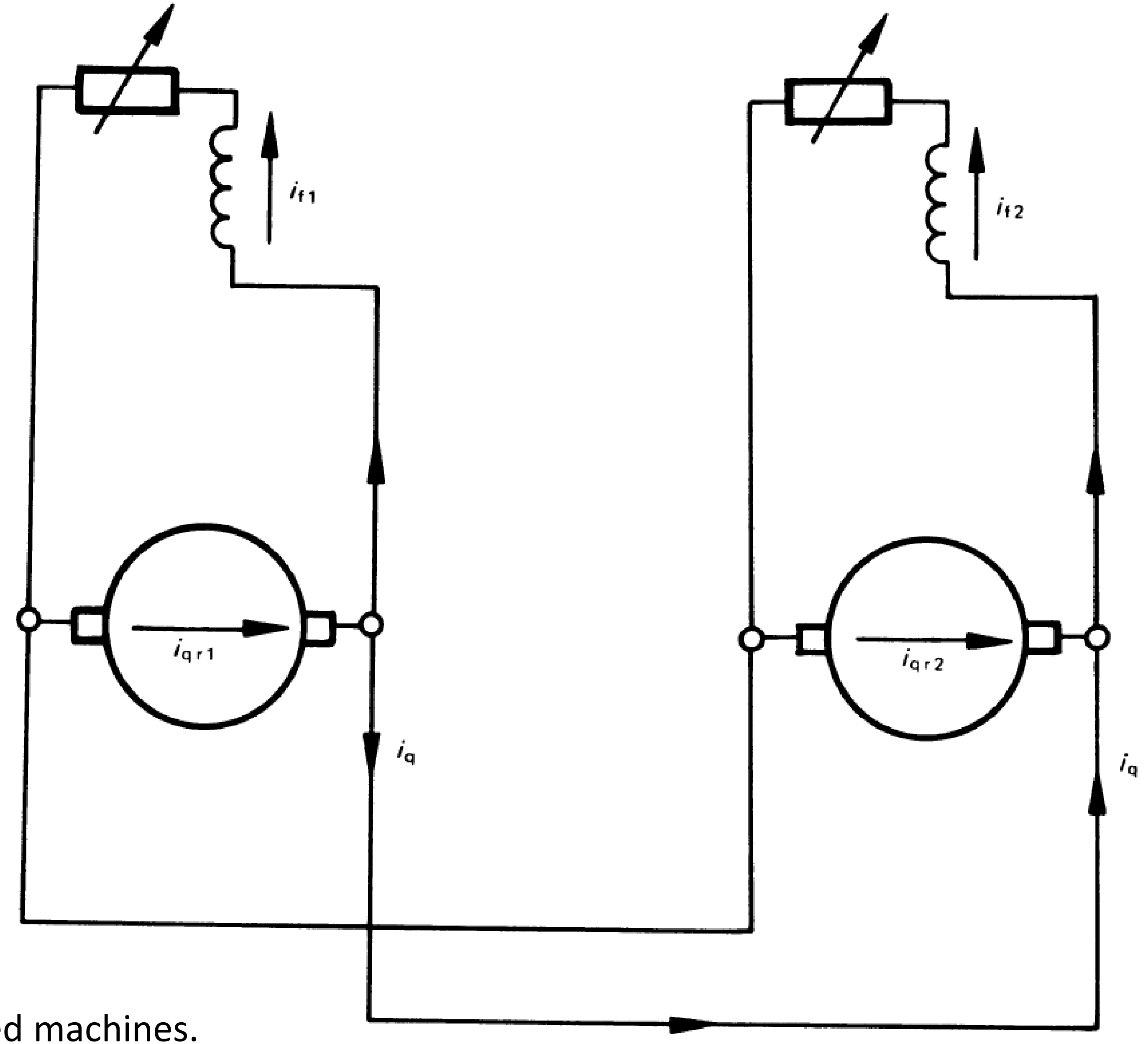
Interconnected Machines

In the machines which we have investigated in the foregoing analysis of torque equations, the choice of d and q axes was obvious. However, where machines of different types are interconnected, we must consider carefully the choice of reference axes. For example, the armature voltage and torque equations of synchronous machines are usually expressed along Park's axes, namely in a direct axis along the rotor field pole and a quadrature axis lying between the field poles. In the ideal synchronous machine in steady state operation, all values of voltage, current, and torque along these axes are constant with respect to time, and the operator $p(= d/dt)$ in the equations becomes zero. With the induction motor the usual reference frame is fixed to the stator with the direct axis along one phase and the quadrature axis orthogonal to this position. The rotor axes are also fixed in this position in space. Along these axes, in steady state operation, the p operator in the equations becomes $j\omega$, where $\omega = 2\pi f$. Obviously if these machines are operating together, it is essential that we select reference axes such that the operator p has the same steady-state and transient significance for all time dependent behaviour. A reference frame for the induction motor, which rotates uniformly with the synchronous flux wave gives equations of the same form as those of Park for the synchronous machine and a complex arrangement of such machines may then be treated mathematically as a single coupled dynamical system. In simple cases of a few coupled machines it may be possible in a computer programme to treat each machine separately with its own frame of reference, the computer making the adjustments in values when transformations between the reference axes become necessary as the machines interact.

For many purposes it is desirable to programme the overall dynamical behaviour of interconnected machines, using the **combined equations of performance**; for example when using the state A-matrices and in the study of stability conditions and eigenvalues.

Interconnected Machines

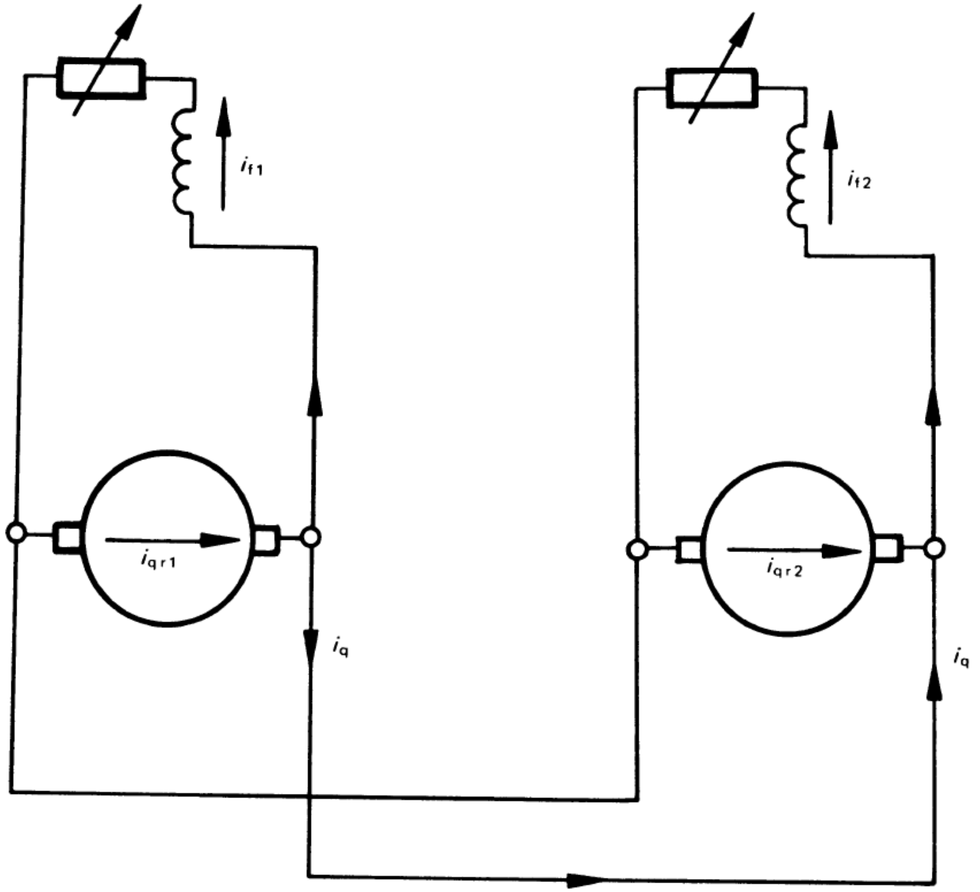
Case I: dc generator-motor system



assume that the prime mover driving the shunt generator is very large and runs at constant speed whatever the load on the generator

electrically coupled machines.

Interconnected Machines



Considering impressed voltages on each machine, in accordance with Kron's convention, the equations for each machine are identical in form

generator

$$\begin{bmatrix} V_{f1} \\ V_{a1} \end{bmatrix} = \begin{array}{c|cc} & ds1 & qr1 \\ \hline ds1 & R_{f1}+L_{f1}p & \\ qr1 & -M_{d1}p\theta & R_{a1}+L_{a1}p \end{array} \cdot \begin{bmatrix} i_{f1} \\ i_{a1} \end{bmatrix}$$

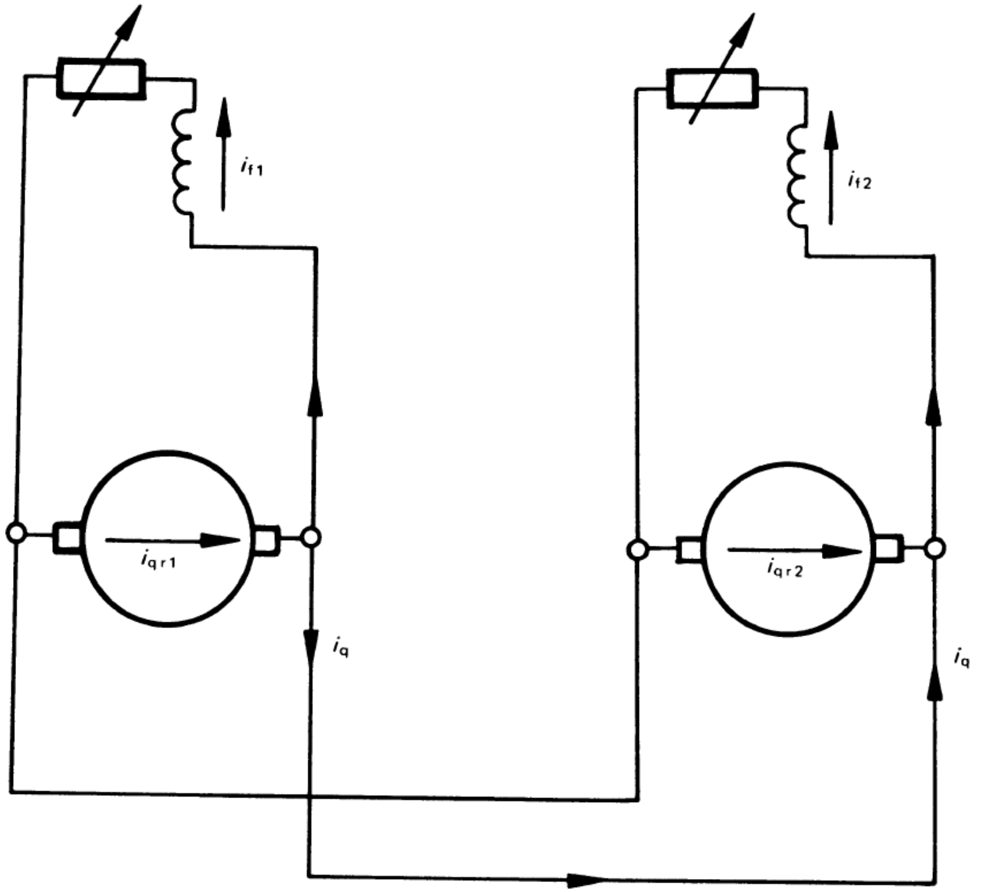
$$V_1 = Z_1 i_1$$

motor

$$\begin{bmatrix} V_{f2} \\ V_{a2} \end{bmatrix} = \begin{array}{c|cc} & ds2 & qr2 \\ \hline ds2 & R_{f2}+L_{f2}p & \\ qr2 & -M_{d2}p\theta & R_{a2}+L_{a2}p \end{array} \cdot \begin{bmatrix} i_{f2} \\ i_{a2} \end{bmatrix}$$

$$V_2 = Z_2 i_2$$

Interconnected Machines



$$i_{ds1} = i_{f1}$$
$$i_{qr1} = i_{f1} + i_q$$
$$i_{ds2} = i_{f2}$$
$$i_{qr2} = i_{f2} - i_q$$

$$\mathbf{C} = \begin{array}{c} \\ ds1 \\ qr1 \\ ds2 \\ qr2 \end{array} \begin{array}{c} \begin{array}{ccc} f1 & f2 & q \end{array} \\ \left[\begin{array}{ccc} 1 & & \\ 1 & & 1 \\ & 1 & \\ & +1 & -1 \end{array}\right] \end{array}$$

combined matrix is given by

$$\mathbf{Z}' = \mathbf{C}_t \mathbf{Z} \mathbf{C}$$

$$\mathbf{Z} = \begin{array}{c|cccc} & ds1 & qr1 & ds2 & qr2 \\ \hline ds1 & R_{f1} + L_{f1}p & & & \\ qr1 & -M_{d1}p\theta & R_{a1} + L_{a1}p & & \\ ds2 & & & R_{f2} + L_{f2}p & \\ qr2 & & & -M_{d2}p\theta & R_{a2} + L_{a2}p \end{array}$$

Interconnected Machines

$$
\mathbf{Z}' = \begin{array}{c|c|c|c|}
 & \text{f1} & \text{f2} & \text{q} \\
\hline
\text{f1} & R_{f1} + L_{f1}p - M_{d1}p\theta + R_{a1} + L_{a1}p & & R_{a1} + L_{a1}p \\
\hline
\text{f2} & & R_{f2} + L_{f2}p - M_{d2}p\theta + R_{a2} + L_{a2}p & -R_{a2} - L_{a2}p \\
\hline
\text{q} & -M_{d1}p\theta + R_{a1} + L_{a1}p & M_{d2}p\theta - R_{a2} - L_{a2}p & R_{a1} + L_{a1}p + R_{a2} + L_{a2}p \\
\hline
\end{array}
$$

In the steady state, terms containing the operator p become zero

$$
\mathbf{Z}'_{\text{s-state}} = \begin{array}{c|c|c|c|}
 & \text{f1} & \text{f2} & \text{q} \\
\hline
\text{f1} & R_{f1} + R_{a1} - M_{d1}p\theta & & R_{a1} \\
\hline
\text{f2} & & R_{f2} + R_{a2} - M_{d2}p\theta & -R_{a2} \\
\hline
\text{q} & -M_{d1}p\theta + R_{a1} & M_{d2}p\theta - R_{a2} & R_{a1} + R_{a2} \\
\hline
\end{array}
$$

Interconnected Machines

Torque generated in each machine is given by $i^*_t G i$ where, from matrix

$$
\begin{array}{cc}
 & \begin{array}{ccc} f1 & f2 & q \end{array} \\
\begin{array}{c} f1 \\ \mathbf{G} = f2 \\ q \end{array} & \left[\begin{array}{ccc} -M_{d1} & & \\ & -M_{d2} & \\ -M_{d1} & M_{d2} & \end{array}\right]
\end{array}
$$

generated torque

$$T = -i_{f1} M_{d1}(i_q + i_{f1}) + i_{f2} M_{d2}(i_q - i_{f2})$$

The first term is the torque generated by current in the generator, to oppose the torque impressed upon the shaft by the prime mover. The second term gives the torque generated by the motor to meet the load torque impressed upon the motor shaft.

Interconnected Machines

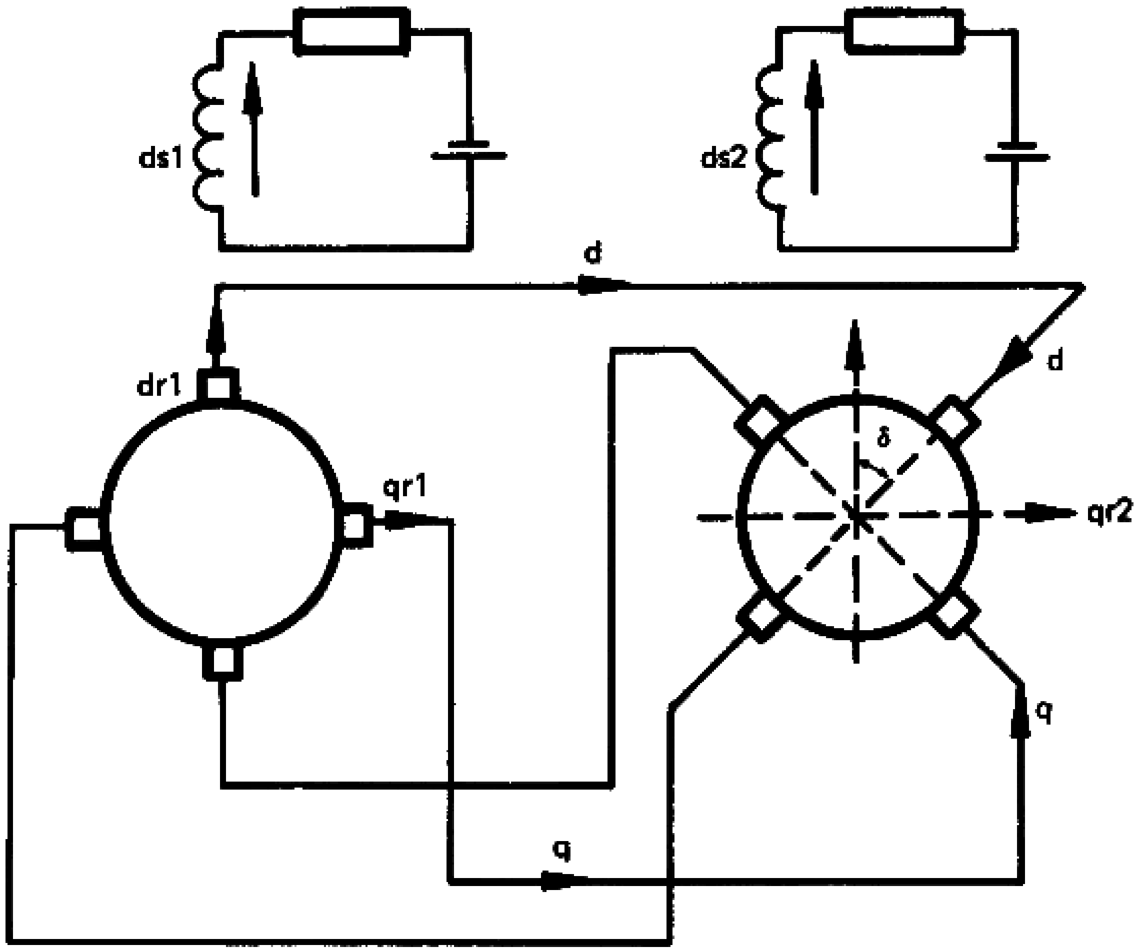
The new voltage vector is given by $\mathbf{V}' = \mathbf{C}_t \mathbf{V}$

$$\mathbf{V} = \begin{array}{c} \mathrm{ds1} \\ \mathrm{qr1} \\ \mathrm{ds2} \\ \mathrm{qr2} \end{array} \begin{bmatrix} 0 \\ V_{qr1} \\ 0 \\ V_{qr2} \end{bmatrix}$$

$$\mathbf{V}' = \begin{array}{c} \mathrm{f1} \\ \mathrm{f2} \\ \mathrm{q} \end{array} \begin{bmatrix} V_{qr1} \\ V_{qr2} \\ V_{qr1} - V_{qr2} \end{bmatrix} = \begin{array}{c} \mathrm{f1} \\ \mathrm{f2} \\ \mathrm{q} \end{array} \begin{bmatrix} V_{f1} \\ V_{f2} \\ V_q \end{bmatrix}$$

Interconnected Machines

Case II: alternator supplying a synchronous motor

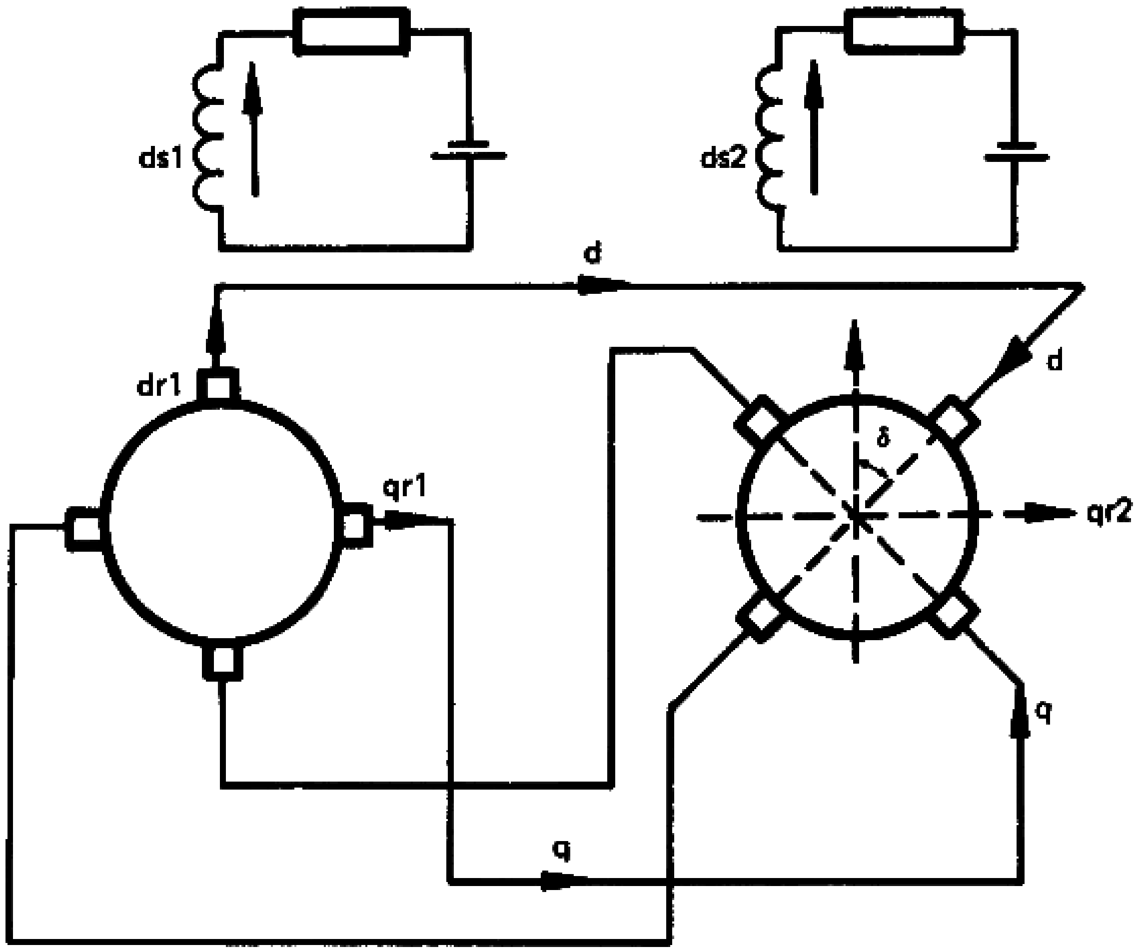


$$C = \begin{array}{c|cccc} & ds1 & d' & q' & ds2 \\ \hline ds1 & 1 & & & \\ dr1 & & 1 & & \\ qr1 & & & 1 & \\ ds2 & & & & 1 \\ dr2 & & -\cos\delta & \sin\delta & \\ qr2 & & -\sin\delta & -\cos\delta & \end{array}$$

Due to the shaft load on the motor, it operates with the armature direct and quadrature axes lagging those of the alternator by the load angle δ. Our assumption of synchronous operation implies that pδ is zero.

Interconnected Machines

Case II: alternator supplying a synchronous motor



$$\begin{array}{c} \\ \begin{bmatrix} V_{ds} \\ V_{dr} \\ V_{qr} \end{bmatrix} \end{array} \begin{array}{c} \\ \begin{matrix} ds \\ = dr \\ qr \end{matrix} \end{array} \begin{array}{c} \begin{matrix} \quad ds \quad & \quad dr \quad & \quad qr \quad \end{matrix} \\ \begin{bmatrix} R_{ds} + L_{ds}p & M_d p & \\ M_d p & R_{dr} + L_{dr}p & L_{qr}p\theta \\ -M_d p\theta & -L_{dr}p\theta & R_{qr} + L_{qr}p \end{bmatrix} \end{array} \cdot \begin{array}{c} \\ \begin{bmatrix} i_{ds} \\ i_{dr} \\ i_{qr} \end{bmatrix} \end{array}$$

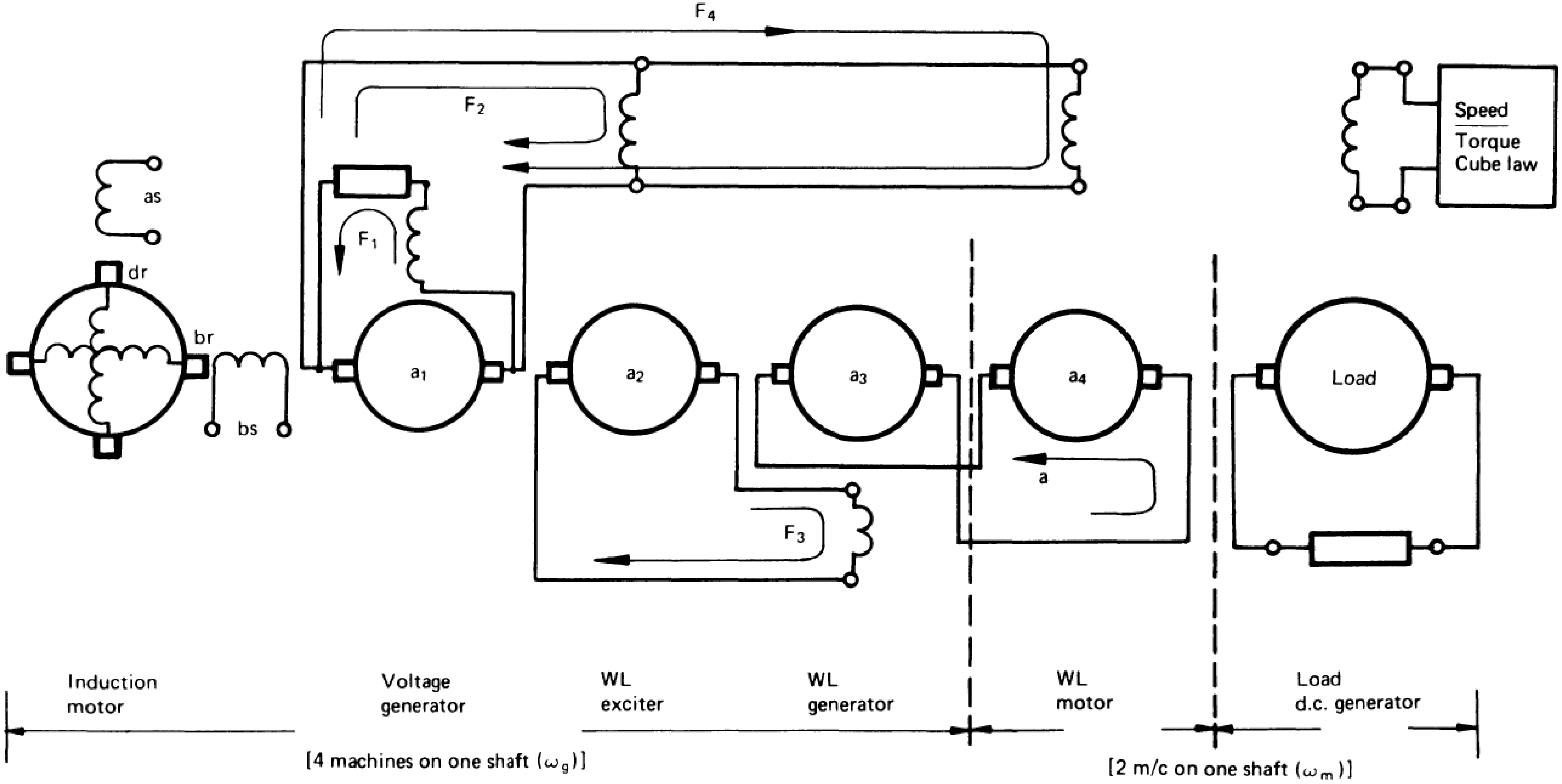
$$\mathbf{Z} = \begin{bmatrix} \mathbf{Z}_{alt} & \\ & \mathbf{Z}_{motor} \end{bmatrix} \equiv \begin{bmatrix} \mathbf{Z}_1 & \\ & \mathbf{Z}_2 \end{bmatrix}$$

$$\mathbf{Z}' = \mathbf{C}_t \mathbf{Z} \mathbf{C}$$

Estimate Z', Steady state impedance matrix, Torque equation

Interconnected Machines

Case III: Ward-Leonard five-machine system



Interconnected Machines

Case III: Ward-Leonard five-machine system

$$
\begin{array}{c|c|c|c|c|c|}
 & \text{ds} & \text{dr} & \text{qr} & \text{qs} & \text{s} \\
\hline
\text{ds} & R_s + L_s p & Mp & & & \\
\hline
\text{dr} & Mp & R_r + L_r p & L_r p\theta & Mp\theta & \\
\hline
\mathbf{Z} = \text{qr} & -Mp\theta & -L_r p\theta & R_r + L_r p & Mp & \\
\hline
\text{qs} & & & Mp & R_s + L_s p & \\
\hline
\text{s} & Mi_{qr} & L_r i_{qr} & -L_r i_{dr} & -Mi_{dr} & Jp + R_F \\
\hline
\end{array}
$$

induction motor

DC machine

$$
\begin{array}{c|c|c|c|}
 & \text{F} & \text{a} & \text{s} \\
\hline
\text{F} & R_f + L_f p & & \\
\hline
\mathbf{Z} = \text{a} & -Mp\theta & R_a + L_a p & \\
\hline
\text{s} & Mi_a & & Jp + R_F \\
\hline
\end{array}
$$

Interconnected Machines

Case III: Ward-Leonard five-machine system

$\mathbf{C} =$

		F_1	F_2	F_3	F_4	a	as	ar	br	bs	sA	sB
	F_1	1										
VG	a1	1	1		1							
	s1										−1	
	F_2		1									
WLE	a2			1								
	s2										−1	
	F_3			1								
WLG	a3					1						
	s3										−1	
	F_4				1							
WLM	a4					−1						
	s4											1
	as						1					
	ar							1				
IM	br								1			
	bs									1		
	s5										1	

Interconnected Machines

Hunting analysis of interconnected machines

Hunting is a sustained state of sinusoidal perturbation in speed, about the normal constant mean value. The hunting characteristics of machines or systems differ from those in the steady state or under transient conditions.

Z

The problem of small oscillations is encountered mainly with machines in which there is some form of feedback. This may consist of signals generated within the machine inherently, due to the configuration of windings. Synchronous or other types of motor with feedback are prone to small oscillations or hunting. These small oscillations may be (1) forced, or (2) self-excited, as in many physical systems. A diesel alternator may be subjected to forced oscillations due to cyclic variations in torque from the prime mover. It is essential therefore to determine the natural frequencies of the system and to see if any are close to the frequency of impressed cyclic variation. Self-excited oscillations are fairly common in power systems. Usually two kinds of self-excited oscillations are encountered. One is a high-frequency electrical oscillation, to which the machine rotors, because of their high inertia, cannot respond.

Interconnected Machines

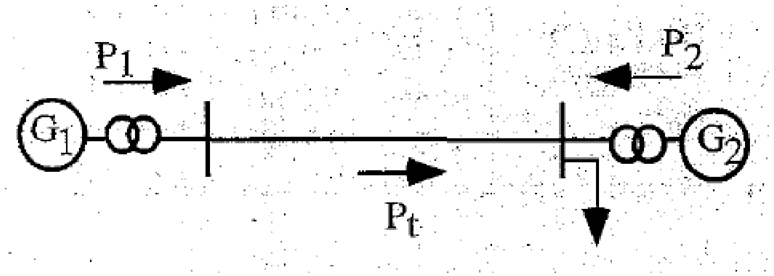
Hunting analysis of interconnected machines

C. Jing, J. D. McCalley and M. Kommareddy, "An energy approach to analysis of interarea oscillations in power systems," in IEEE Transactions on Power Systems, vol. 11, no. 2, pp. 734-740, May 1996

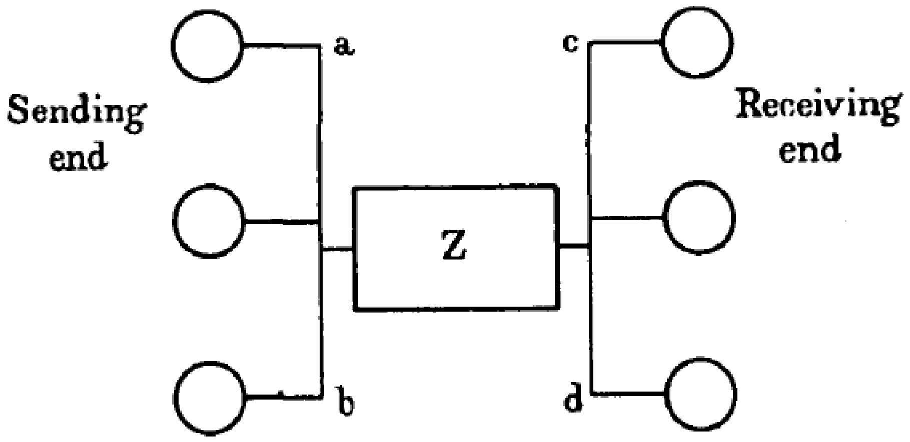
Y. Yu, S. Grijalva, J. J. Thomas, L. Xiong, P. Ju and Y. Min, "Oscillation Energy Analysis of Inter-Area Low-Frequency Oscillations in Power Systems," in IEEE Transactions on Power Systems, vol. 31, no. 2, pp. 1195-1203, March 2016

Interconnected Machines

Hunting analysis of interconnected machines

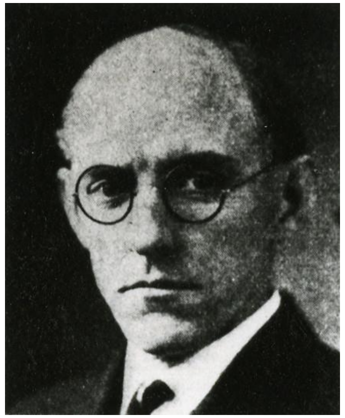


- Modes, Modal analysis
- Motion modes
- Energy modes
- Mode shapes
- Participation factor



There are many more "energy modes" than "motion modes"

Large Signal Transient Analysis



Robert E. Doherty

Theorem of Constant Flux Linkages or Doherty's Law

In any group of coupled electromagnetic circuits

$$e = ri + \frac{d\lambda}{dt}$$

$$\int_{t_1}^{t_2} \frac{d\lambda}{dt} dt = \lambda(t_2) - \lambda(t_1) = \int_{t_1}^{t_2} (e - ri) dt$$

If e, r, and i remain finite, it follows that the flux linkages can not change suddenly (i.e., have a discontinuity in the flux) but must change continuously since the flux is the result of an integral. That is, a discontinuity is possible only in the time derivative of the flux.

If the circuit is suddenly short-circuited so that e = 0 and r is very small, the integral on the right hand side will always be small. Indeed, if at t = t1 the voltage impression the circuit suddenly becomes zero and if r = 0, then for any t2 > t1,

$$\lambda(t_2) = \lambda(t_1)$$

Hence, the total flux linkages tend to remain constant if a circuit is short-circuited or if the voltage impressed on a closed circuit is zero both before and after a sudden voltage change in any other (coupled) circuit.

Large Signal Transient Analysis

Let λas(0), λbs(0), and λcs(0) be the flux linkages of the three stator phases as, bs and cs **at the moment the short-circuit** occurs. From Doherty's Law, if 'e' and 'r' are both zero, then for any time thereafter,

$$\lambda_{as}(t) = \lambda_{as}(0)$$

$$\lambda_{bs}(t) = \lambda_{bs}(0)$$

$$\lambda_{cs}(t) = \lambda_{cs}(0)$$

$$\theta_r = \omega_e t + \alpha$$

d–q frame is rotating at synchronous speed with an arbitrary initial angle

$$\lambda_{qs} = \frac{2}{3}\left[\lambda_{as}\cos(\omega_e t + \alpha) + \lambda_{bs}\cos\left(\omega_e t + \alpha - \frac{2\pi}{3}\right) + \lambda_{cs}\cos\left(\omega_e t + \alpha + \frac{2\pi}{3}\right)\right]$$

$$\lambda_{ds} = \frac{2}{3}\left[\lambda_{as}\sin(\omega_e t + \alpha) + \lambda_{bs}\sin\left(\omega_e t + \alpha - \frac{2\pi}{3}\right) + \lambda_{cs}\sin\left(\omega_e t + \alpha + \frac{2\pi}{3}\right)\right]$$

Large Signal Transient Analysis

$$\lambda_{qs} = \lambda_{qs}(0)\cos\omega_e t - \lambda_{ds}(0)\sin\omega_e t$$

$$\lambda_{ds} = \lambda_{qs}(0)\sin\omega_e t + \lambda_{ds}(0)\cos\omega_e t$$

$$\lambda_{fr} = \lambda_{fr}(0)$$

$$\lambda_{qs}(0) = \frac{2}{3}\left[\lambda_{as}(0) - \frac{1}{2}\lambda_{bs}(0) - \frac{1}{2}\lambda_{cs}(0)\right]\cos\alpha - \frac{1}{\sqrt{3}}[\lambda_{cs}(0) - \lambda_{bs}(0)]\sin\alpha$$

$$\lambda_{ds}(0) = \frac{2}{3}\left[\lambda_{as}(0) - \frac{1}{2}\lambda_{bs}(0) - \frac{1}{2}\lambda_{cs}(0)\right]\sin\alpha + \frac{1}{\sqrt{3}}[\lambda_{cs}(0) - \lambda_{bs}(0)]\cos\alpha$$

Large Signal Transient Analysis

$$\lambda_{qs} = L_{qs} i_{qs}$$

$$\lambda_{ds} = L_{ds} i_{ds} + L_{md} i_{fr}$$

$$\lambda_{fr} = L_{fr} i_{fr} + L_{md} i_{ds}$$

$$i_{ds} = \frac{-L_{md}\lambda_{fr}(0)}{L_{fr}\left[L_{ds} - \frac{L_{md}^2}{L_{fr}}\right]} + \frac{[\lambda_{ds}(0)\cos\omega_e t + \lambda_{qs}(0)\sin\omega_e t]}{\left[L_{ds} - \frac{L_{md}^2}{L_{fr}}\right]}$$

$$i_{qs} = \frac{\lambda_{qs}(0)\cos\omega_e t - \lambda_{ds}(0)\sin\omega_e t}{L_{qs}}$$

$$i_{fr} = \frac{L_{ds}\lambda_{fr}(0)}{L_{fr}\left[L_{ds} - \frac{L_{md}^2}{L_{fr}}\right]} - \frac{L_{md}[\lambda_{qs}(0)\sin\omega_e t + \lambda_{ds}(0)\cos\omega_e t]}{L_{fr}\left[L_{ds} - \frac{L_{md}^2}{L_{fr}}\right]}$$

Large Signal Transient Analysis

Transient Reactance

$$x_d' \stackrel{\Delta}{=} \omega_e\left[L_{ds} - \frac{L_{md}^2}{L_{fr}}\right]$$

voltage behind transient reactance

$$E_q' \stackrel{\Delta}{=} \frac{\omega_e L_{md}}{L_{fr}}\lambda_{fr}(0) = \frac{x_{md}}{x_{fr}}\psi_{fr}(0)$$

$$i_{ds} = \frac{-E_q'}{x_d'} + \frac{\psi_{ds}(0)\cos\omega_e t + \psi_{qs}(0)\sin\omega_e t}{x_d'}$$

$$i_{qs} = \frac{\psi_{qs}(0)\cos\omega_e t - \psi_{ds}(0)\sin\omega_e t}{x_{qs}}$$

$$i_{fr} = \frac{x_{ds}E_q'}{x_d' x_{md}} - \frac{(x_{ds} - x_d')}{x_d'}\frac{\psi_{ds}(0)\cos\omega_e t + \psi_{qs}(0)\sin\omega_e t}{x_{md}}$$

Ref Book:
'Analysis of Synchronous Machines' by T.A. Lipo