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BRANCH – POWER ELECTRONICS

SUBJECT – SMRC END SEMESTER EXAM

Ques 1. a) **Duty Ratio** – Duty ratio (d) is defined only over a full switching cycle. Hence when duty ratio varies with time what we get is a sequence of values – one value in T seconds. That is 'd' is a discrete sequence on 't'.

Duty Ratio function – To allow duty ratio to vary with time and get ~~reasons~~ retain continuous time nature of the model to define a continuous function of time is called duty ratio function d(t) by using local averaging.

Switching function –

$$q(t) = \begin{cases} 1 & \text{, when switch is ON} \\ 0 & \text{, when switch is off} \end{cases}$$

If duty ratio is kept constant, the cycle average of q(t) as well as the local average of q(t) are equal to d. If duty ratio varies with time the cyclic average of q(t), $q_{cav}(n) = d(n)$ where both are discrete sequences

$$\boxed{d(t) = \frac{1}{T} \int_{t-T}^{t} q(t)\, dt.}$$

b> State space Averaged Model

State space averaged value of state variables and output variables in a PE Converter will be nearly equal to the local average values of corresponding variables if,

all the time constant of all the linear Circuit structures involved are $\geq 10T$.

$$\dot{X} = A_1 X + B_1 U$$
$$\dot{Y} = C_1 X + D_1 U$$

Exact local Average Model

$$x_{ma}(t) = \frac{1}{T}\int_{t-T}^{t} x(t)\,dt$$

~~A~~ sliding
local Averaging is the method choosen for developing continuous time models for switched PE Converters.

Relation

$$\dot{\bar{X}} = [d(t)A_1 + (1-d(t))A_2]\bar{X} + [d(t)B_1 + (1-d(t))B_2]U$$
$$\dot{\bar{Y}} = [d(t)C_1 + (1-d(t))C_2]\bar{X} + [d(t)D_1 + (1-d(t))D_2]U$$

Conditions

i) If all the time constants of all the structures in a switched system are $>>> T$

ii) all the natural oscillation frequencies in all the structures in a switched system are $<< \frac{1}{T}$

iii) all inputs to the switched systemm remain virtually constant over T.

iv) all the variables in the switched system do not deviate significantly within a local averaging window from their local average values.

Then both the above equation is the state space averaged description of the switched system. State variables in the state space averaged model are nothing but local average values of original state variables in the switched system. Output variables in the state space averaged model is nothing but the local average value of output variables in the original system.

Switched Model

$$\dot{x} = [A_1 q(t) + (1-q(t)] A_2]x + [B_1 q(t) + (1-q(t))B_2]U$$

$$\dot{y} = [C_1 q(t) + (1-q(t)) C_2] x + [D_1 q(t) + (1-q(t)]D_2]U$$

local Average

$$\dot{\bar{x}} = [A_1 d(t) + (1-d(t)] A_2]\bar{x} + [B_1 d(t) + (1-d(t)] B_2]\bar{U}$$

$$\dot{\bar{y}} = [C_1 d(t) + (1-d(t)] C_2]\bar{x} + [D_1 d(t) + (1-d(t)] D_2]\bar{U}$$

$\bar{U} = U \rightarrow$ i.e, input to the converter is U.

$$\dot{x} = [A_1 d(t) + (1-d(t)) A_2] \bar{x} + [B_1 d(t) + (1-d(t)) B_2] u$$

$$\dot{y} = [C_1 d(t) + (1-d(t)) C_2] \bar{x} + [D_1 d(t) + (1-d(t)) D_2] u$$

~~[illegible]~~

$$\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dv_c}{dt} \end{bmatrix} = \begin{bmatrix} \frac{-[r_c + q(t) r_s + r_L + (1-q(t)) r_d]}{L} & -\frac{1}{L} \\ 1/C & 0 \end{bmatrix} \begin{bmatrix} i_L \\ v_c \end{bmatrix} + \begin{bmatrix} \frac{q(t)}{L} & \frac{1-q(t)}{L} & \frac{r_c}{L} \\ 0 & 0 & -1/C \end{bmatrix} \begin{bmatrix} V_{in} \\ V_r \\ i_o \end{bmatrix}$$

$$[v_o] = [r_c \quad 1] \begin{bmatrix} i_L \\ v_c \end{bmatrix} + [0 \quad 0 \quad -r_c] \begin{bmatrix} V_{in} \\ V_r \\ i_o \end{bmatrix}$$

Approximation

Product of local average = local avg of product

$$\begin{bmatrix} \frac{d\bar{i}_L}{dt} \\ \frac{d\bar{v}_c}{dt} \end{bmatrix} = \begin{bmatrix} \frac{-[r_d + r_d d(t) + (1-d(t)) r_d + r_L]}{L} & -1/L \\ 1/C & 0 \end{bmatrix} \begin{bmatrix} \bar{i}_L \\ \bar{v}_c \end{bmatrix} + \begin{bmatrix} \frac{d(t)}{L} & \frac{-[1-d(t)]}{L} & \frac{r_c}{L} \\ 0 & 0 & -1/C \end{bmatrix} \begin{bmatrix} \bar{V}_{in} \\ V_r \\ \bar{i}_o \end{bmatrix}$$

$$[\bar{v}_o] = [r_c \quad 1] \begin{bmatrix} \bar{i}_L \\ \bar{v}_c \end{bmatrix} + [0 \quad 0 \quad -r_c] \begin{bmatrix} \bar{V}_{in} \\ V_r \\ \bar{i}_o \end{bmatrix}$$

Ques 2.

a) All the three types of Compensators Commonly employed in voltage Mode Control of SMPS have a common factor $1/s$ in their transfer functions. This is because in all the three cases $1/s$ term Contributes a fixed $90°$ phase delay at all frequencies.

Before this

$$|K(j\omega_{co})| = \frac{1}{|G(j\omega_{co})|}$$

Phase Delay Angle of $K(j\omega_{co})$ = $180°$ − Phase Delay of $G(j\omega_{co})$ − PM

After $1/s$ term, we replace above as,

$$\boxed{\text{Phase delay of } K'(j\omega_{co}) = 90° - \text{Phase Delay of } G(j\omega_{co}) - PM}$$

Ques 3. a) Difference factors that lead to a choice of gain crossover frequency in design of Compensators –

i) Bandwidth

ii) Phase Margin

$$\frac{\Delta V_o(s)}{\Delta V_{ref}(s)} = \frac{G(s)\,K(s)}{1+G(s)\,K(s)}$$ → Close loop Transfer function.

For getting a close loop bandwith, we are using a Continuous time state space averaged model for designing the control system. The bandwidth of the control loop must be within the validity range of frequency for the modelling technique.

frequency range one which state space average modeling is accurate enough is upto 1/10 of switching frequency, $f_s/10$ is the band width we can hope for.

The phase delay contributed by modulator, additional harmonics injected into output, possible subharmonics switching pattern etc cannot be ignored. Hence close loop bandwidth possible is about $f_s/10$.

The loop gain crossover frequency is usually specified as $f_s/10$ due to the above reasons.

b. **Origin of right - half zero in the control to output transfer function**

In Boost Converter, the transfer function has a zero in the right half of the s - plane.

The zero is located at $s = \frac{(1-d_o)^2 V_o}{L I_o}$ and is very much operating point dependent through d_o and I_o. The numerator factor in the transfer function contributes lag angle to the bode plot of the transfer function. The zero location varies with input voltage and load current.

Due to right half zero

→ Contributes lag angle to the bode plot

→ Stability changes

→ transient response affected

→ damping of transients varies.

As $\omega_n = \frac{1-d_o}{LC}$

$$f = \frac{\frac{r_L}{1-d_o} + \frac{d_o r_s}{1-d_o} + r_d + r_c}{2\sqrt{L/C}}$$

Ques 4.

a) When current mode control is implemented in Buck / Buck derived converters there will be line rejection near to 100 %. but not 100%. So what happen due to slight changes in Iref there will not tracking of v_o.

This changes the average value of i_L.

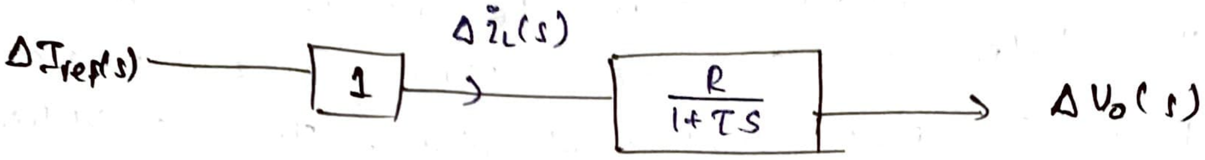
With properly choose compensator slope

$$m_c = \frac{m_2}{2} = \frac{V_o}{2L}$$

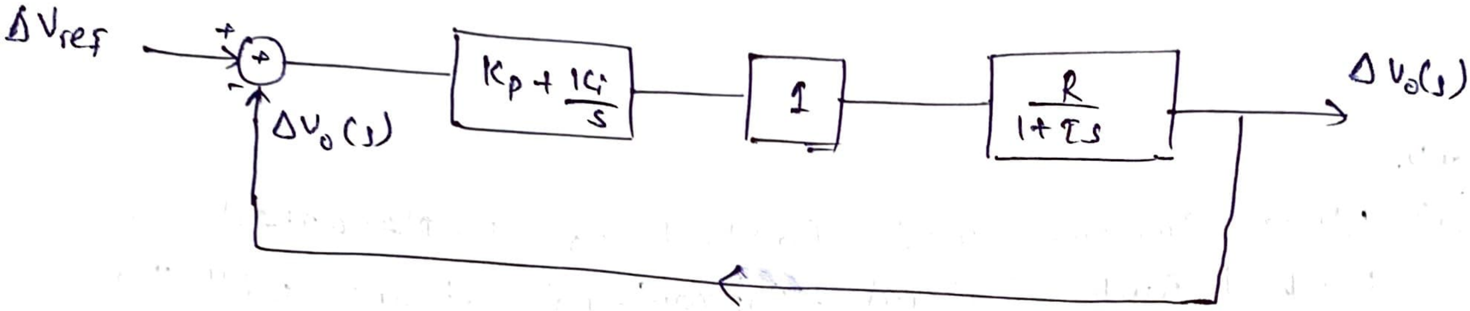
$$i_L = I_{ref} - \frac{m_2}{2} T_s$$

$$= I_{ref} - \frac{V_o}{2 f_s L}$$

So, Δi_L is simply ΔI_{ref}



$$\tau = RC$$

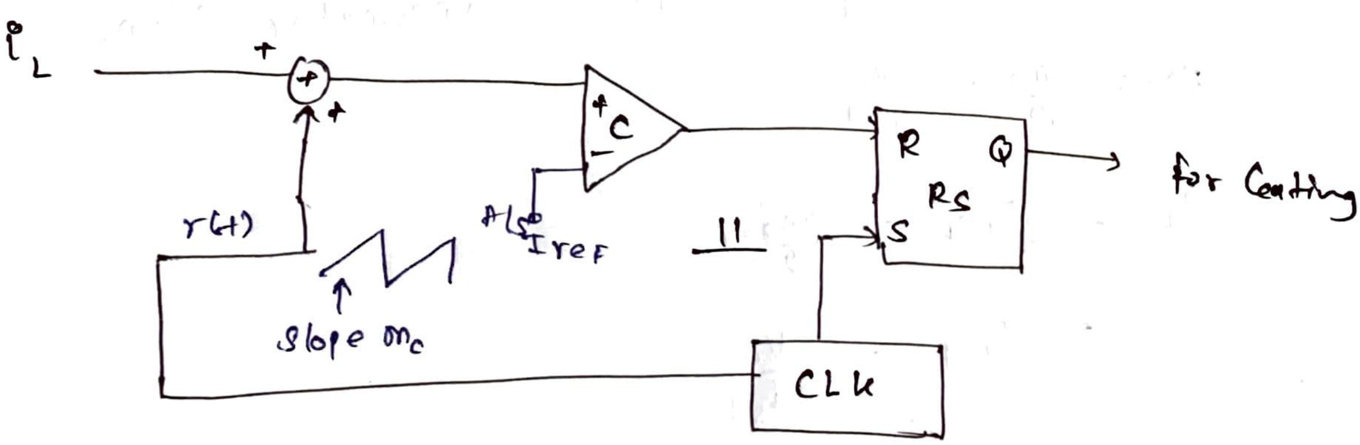


$$\frac{\Delta V_o(s)}{\Delta V_{o\,ref}(s)} = \frac{\frac{K_p R}{\tau}\left(s + \frac{K_i}{K_p}\right)}{s^2 + \left(\frac{1+K_p R}{\tau}\right)s + \frac{K_i R}{\tau}}$$

$$K_p = \frac{a\tau}{R} = aC$$

$$K_i = \frac{a}{R} = a/R$$

$$\frac{\Delta V_o(s)}{\Delta V_{ref}(s)} = \frac{a}{s+a}$$



$$\Delta I_n = (-1)^n \left(\frac{m_2 - m_c}{m_1 + m_c} \right)^n \Delta I_0$$

We want $\frac{m_2 - m_c}{m_1 + m_c} < 1$

$$m_2 - m_1 < 2m_c$$

m_c has to be $> \frac{m_2 - m_1}{2}$

$$\therefore \boxed{m_c \geq \frac{m_2}{2}\left(1 - \frac{m_1}{m_2}\right)}$$

$$\boxed{m_c \geq \frac{m_2}{2}\left(\frac{2d-1}{d}\right)}$$

$m_c \geq \frac{V_o}{2L}$ is required for Buck

$$\therefore y = x + m_1 T_{ON}$$

$$= x + \frac{m_1 (I_{ref} - x)}{m_1 + m_c}$$

$$\boxed{y = m_2 T_s - \frac{m_2}{m_1 + m_2}(I_{ref} - x) + x}$$

$$x = y - m_2 T_{off}$$

$$\boxed{x = y - m_2\left(T_s - \frac{(I_{ref} - x)}{m_1 + m_c}\right)}$$

b. **Subharmonic instability in CMC**

When some disturbance occured and clocking is done earlier that is before reaching I_{min}, SR flip flop is set and MOSFET is turned ON.

Reduction in ON time first cycle $= \frac{\Delta I_0}{m_1}$

Increase in off time in first cycle $= \frac{\Delta I_0}{m_1}$

$$\Delta I_L = \frac{\Delta I_1}{m_1} \cdot m_2$$

In general

$$\therefore I_n = (-1)^n \left(\frac{m_2}{m_1}\right)^n \Delta I_0$$

$$\boxed{S = (-1)^n \left(\frac{m_2}{m_1}\right)^n \Delta I_0}$$

Ques 5. a) functions of the under- voltage lockout, latch and flip flop.

We have a regulator in SG 3525. It will regulate the voltage inside the IC. Hence the under voltage lockout is straight away supplied from Vcc. This lockout circuit control the value of Vcc.

When value of Vcc is less than 7.5 then under voltage lockout circuit is high & Block voltage inside Ic.

When value of Vcc is more than 7.5 then under voltage lockout circuit is low.

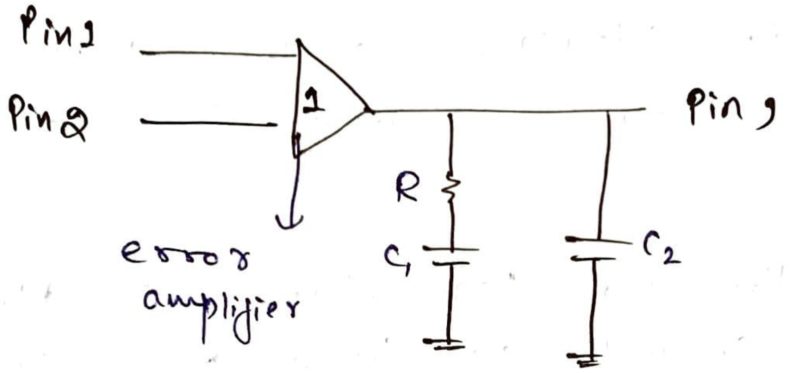
Latch.

Due to sudden disturbance ~~eg~~ or EMI there might be one extra duty cycle in switching cycle & to avoid this we make use of Latch.

flip flop

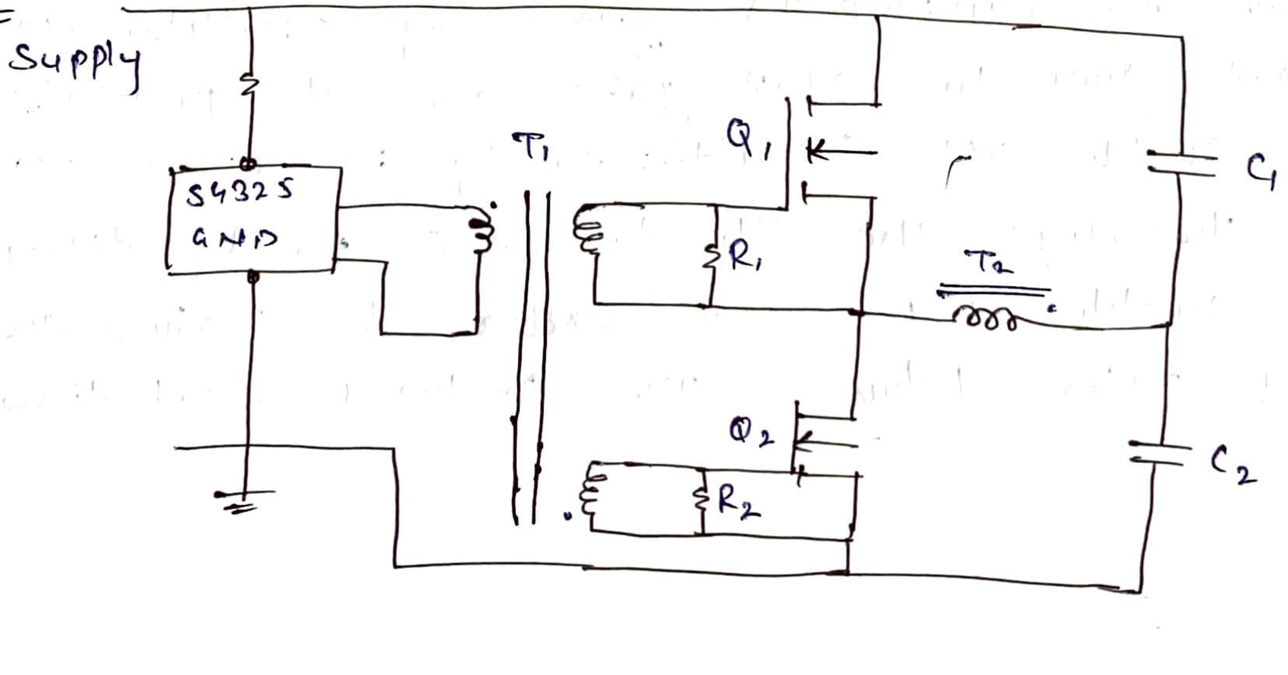
Input to flip flop is given by oscillator and output goes to NOR gate and it prevents the switch ON simultaneously in other words it channelize the o/p terminal ~~to~~ by doing one switch off. In order to reduce the duty ratio below 50%. Produces dead time.

b. Using error amplifier we can design a compensator. We can use compensation pins and find i/p terminal of error amplifier to design a type-II compensator.



c. Current mode control can be implemented with the help of shut down pin. When load current is above desired values, sensor senses the inductor current and compare it with the reference current. & the output of compensator is stored in flip flop and flip flop is ~~cause~~ connected to shut down pins which are used for controlling switching of MOSFET & for resting flip flop.

d)



low power transformer can be designed and driven with the help of SG 3525A. During dead time switching takes place automatically. This switching is generated by flip flop when both primary winding ends are grounded.

Ques 6. Buck Converter

Given

$V_{in} = 110V$

$V_{op} = 48V$

$I_{op} = 4.8A$

$f_s = 100 kHz$

$L = 250\mu H / 0.2\Omega$

$C = 47\mu F / 0.2\Omega$

$r_s = r_d = 0.2\Omega$

$V_r = 0.5V$

a) Control to Output Transfer function

$$\begin{bmatrix} \Delta\dot{\bar{i}}_L \\ \Delta\dot{\bar{v}}_c \end{bmatrix} = \begin{bmatrix} -\frac{[r_c + d_o r_s + (1-d_o)r_d + r_L]}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} \Delta\bar{i}_L \\ \Delta\bar{v}_c \end{bmatrix} + \begin{bmatrix} \frac{d_o}{L} & -\frac{[1-d_o]}{L} & \frac{r_c}{L} \\ 0 & 0 & -\frac{1}{C} \end{bmatrix} \begin{bmatrix} \Delta\bar{v}_{in} \\ 0 \\ \Delta\bar{i}_o \end{bmatrix}$$

$$+ \begin{bmatrix} \frac{(r_d - r_s)\bar{i}_L}{L} + \frac{\bar{V}_{in} + V_r}{L} \\ 0 \end{bmatrix} \Delta d(t)$$

$$[\Delta\bar{v}_o] = [r_c \quad 1] \begin{bmatrix} \Delta\bar{i}_L \\ \Delta\bar{v}_c \end{bmatrix} + [0 \quad 0 \quad -r_c] \begin{bmatrix} \Delta\bar{v}_{in} \\ 0 \\ \Delta\bar{i}_o \end{bmatrix}$$

The converter will reach a steady state which can be solved by substituting 0 value for state derivaties. The result will be,

$$V_o = d_o V_{in} + (1-d_o)V_r + [r_L + d_o r_s + r_d(1-d_o)]i_o$$

$$48 = d_o \times 110 + (1-d_o)0.5 + [0.2 + 0.2d_o + 0.2(1-d_o)]4.8$$

$$48 = 109.5(d_o + 0.0221)$$

$$\boxed{d_o = 0.4162}$$

Transfer function

$$\frac{\Delta \bar{V}_o(s)}{\Delta d(s)} = \frac{\left(\frac{V_{in}+V_r}{LC}\right)(1+r_c s C)}{s^2 + \frac{r_{eff}}{L}s + \frac{1}{LC}}$$

$$r_{eff} = r_c + r_L + d_o r_s + (1-d_o) r_d$$
$$= 0.2 + 0.2 + 0.4162 \times 0.2 + (1-0.4162)0.2$$
$$= 0.6\,\Omega$$

$$\frac{\Delta \bar{V}_o(s)}{\Delta d(s)} = \frac{\left(\frac{110+0.5}{250\mu \times 47\mu}\right)(1+0.2 \times s \times 47\mu)}{s^2 + \frac{0.6}{250\mu}s + \frac{1}{250\mu \times 47\mu}}$$

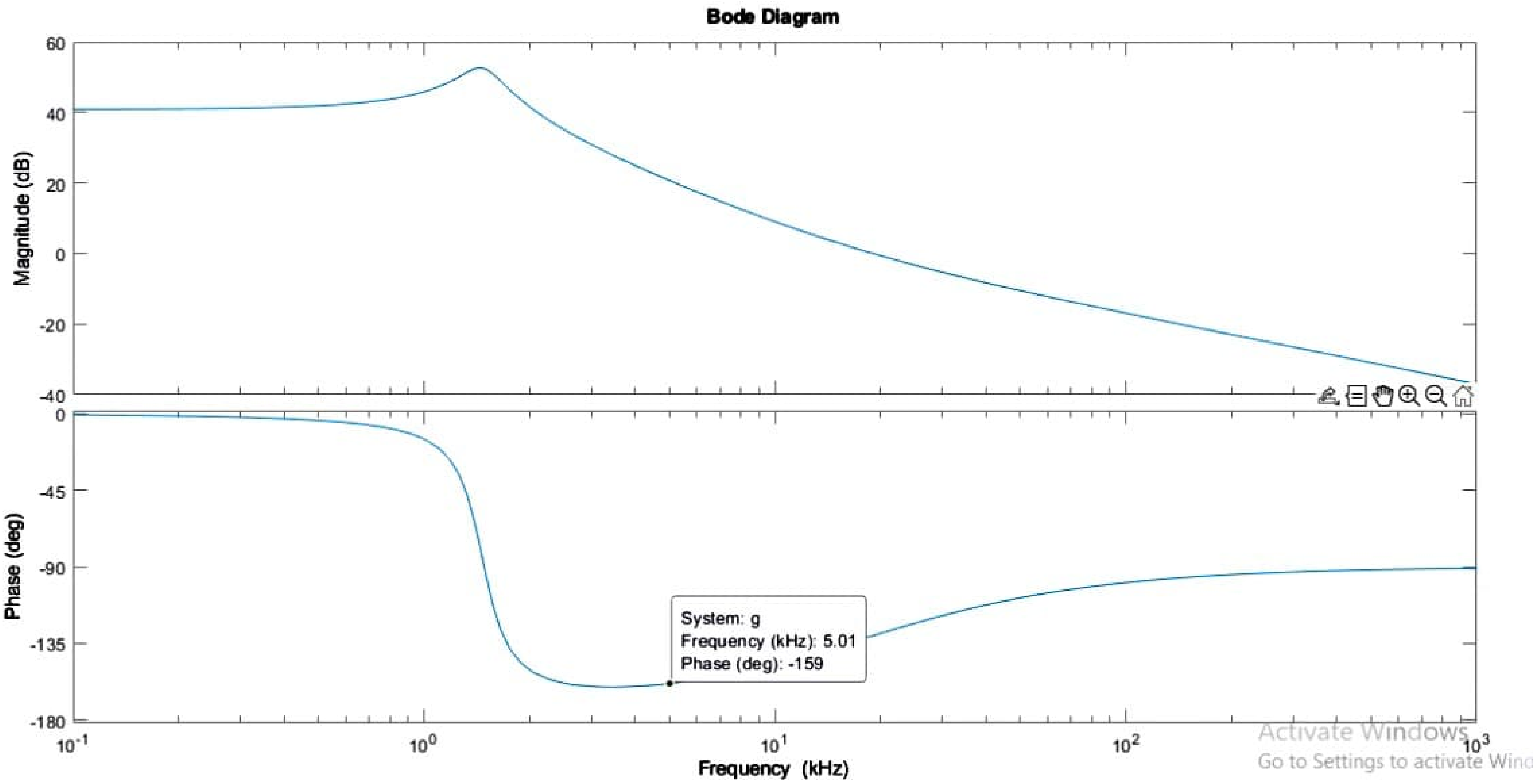
$$\boxed{\frac{\Delta V_o(s)}{\Delta d(s)} = \frac{9404.25 \times 10^6 (1 + 9.4 \times 10^{-6} s)}{s^2 + 2400s + 85.10 \times 10^6}}$$

b. $W_{gc} \rightarrow$
$f_{co} = 5\,KHz$
Phase Margin = ? for Type 1, Type 2, Type 3.

for Type I

When we plot this above transfer function,
ie, $\frac{\Delta V_o(s)}{\Delta d(s)}$ ~~then~~ in MATLAB then at $f_{co} = 5\,kHz$
We get phase margin as $-ve$.

so, It is not possible to design Type 1 Compensator at $f_{co} = 5\,kHz$.


Bode Diagram
Magnitude (dB)
60
40
20
0
-20
-40
Phase (deg)
0
-45
-90
-135
-180
System: g
Frequency (kHz): 5.01
Phase (deg): -159
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Frequency (kHz)

for Type II

Phase delay Angle of $K'(j\omega_{co})$ = 90° − Phase delay angle of $G(j\omega_{co})$ − PM

∴ Phase delay of $G(j\omega_{co})$ + P.M ≤ 170°

from the Bode plot we can see that phase delay of $G(j\omega_{co})$ at $f_{co} = 5\,kHz$ = +159°

∴ $PM = 170 - 159°$

$$\boxed{PM = 11°}$$

for type III

Phase delay of $G(j\omega_{co})$ + PM ≤ 250°

from the bode plot $\angle G(j\omega_{co}) = -159°$

Delay = 159°

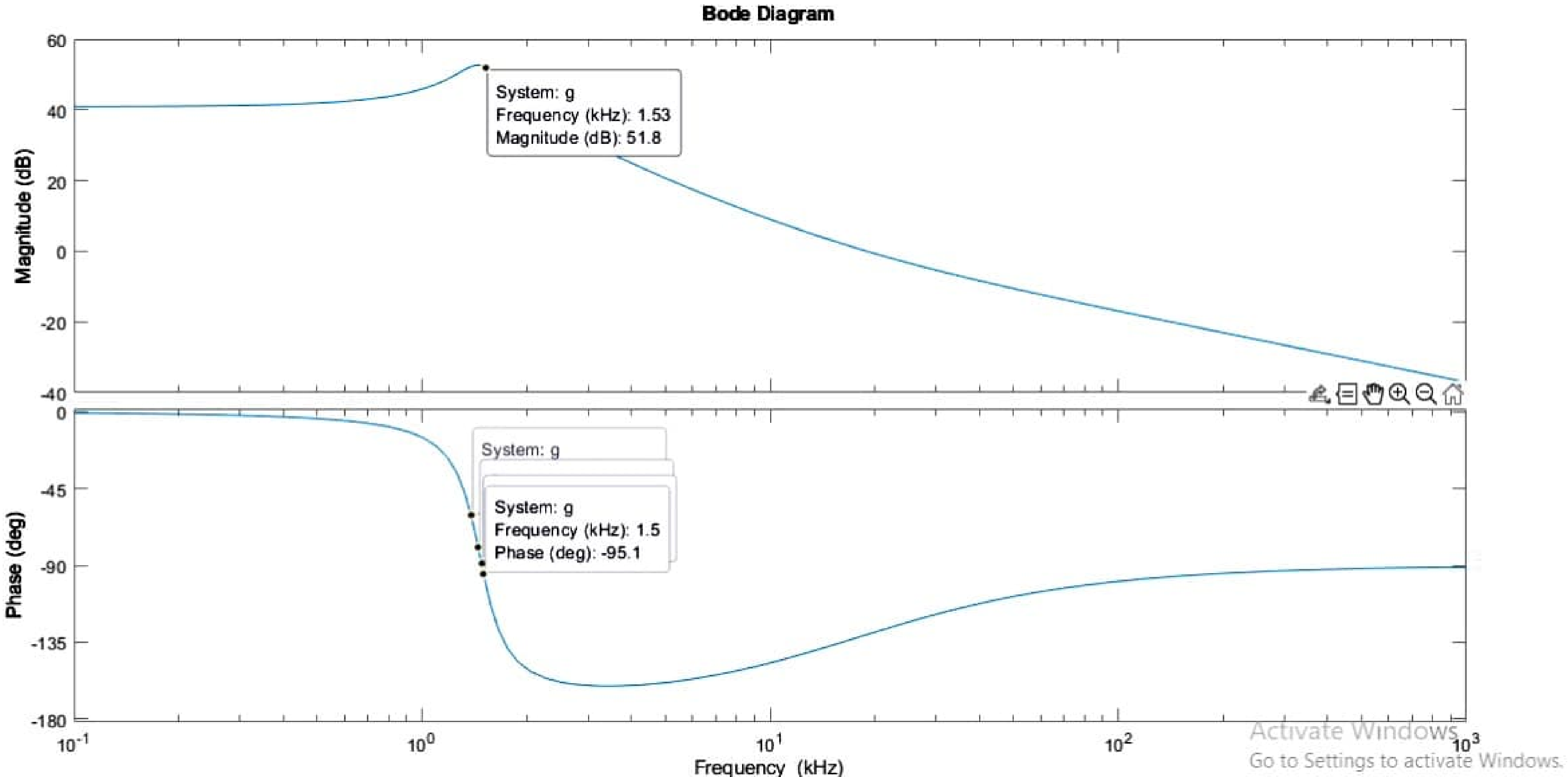
$PM = 250° - 159°$

$$\boxed{PM = 91°}$$

c) Design Type 2 Compensator for $f_{co} = 1.5\,kHz$ and $PM = 60°$.

$$K(s) = \frac{k}{s}\frac{(1+T_2 s)}{(1+T_p s)} = \frac{k}{s} \cdot K'(s)$$

$$\omega_2 = \frac{\omega_{co}}{k}, \quad \omega_p = k\,\omega_{co}$$


Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (kHz)
System: g
Frequency (kHz): 1.53
Magnitude (dB): 51.8
System: g
System: g
Frequency (kHz): 1.5
Phase (deg): -95.1
Activate Windows
Go to Settings to activate Windows.

$$k = \tan\left\{\frac{\text{Phase Delay angle of } G(j\omega_{co}) + PM}{2}\right\}$$

~~$k = \tan \{$~~

From the Bode plot at $f_{co} = 1.5\,kHz$ the phase delay of $G(j\omega_{co}) = 95^\circ$

$|G(j\omega_{co})| = 51.8\,dB$

$20\log|M| = 51.8$

$|M| = 389.04$

$$k = \tan\left\{\frac{95.1 + 60}{2}\right\}$$

$k = 4.51$

$|K'(j\omega_{co})| = k = 4.51$

$$K \times 4.51 = \frac{\omega_{co}}{|G(j\omega_{co})|} = \frac{2\pi \times 1.5 \times 10^3}{389.04}$$

$$\boxed{K = 5.368}$$

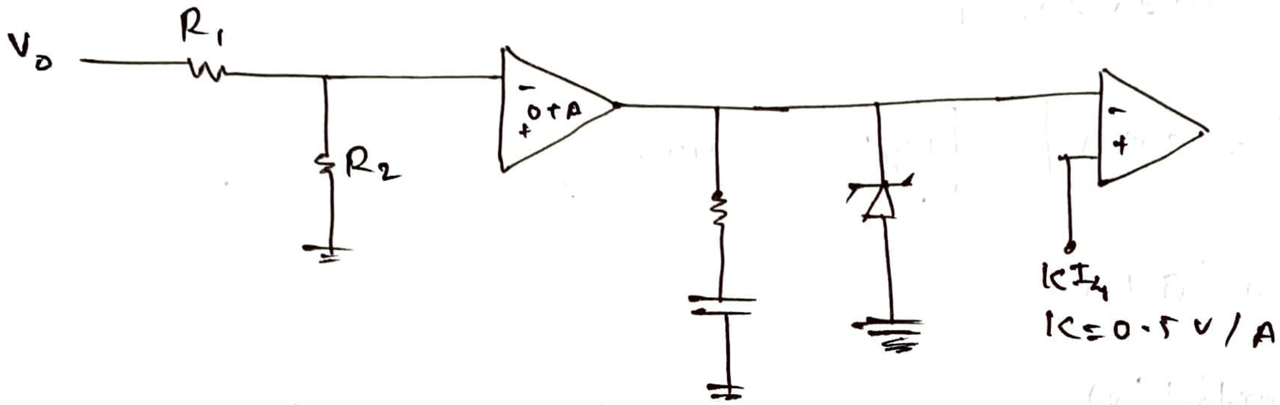
$$T_2 = \frac{k}{\omega_{co}} = \frac{4.51}{2\pi \times 1.5 \times 10^3} = 0.478 \times 10^{-3}\,\text{sec.}$$

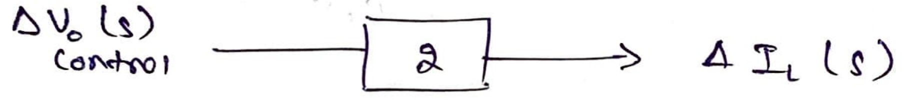
$$T_p = \frac{1}{k\,\omega_{co}} = \frac{1}{4.51 \times 2\pi \times 1.5 \times 10^3} = 23.5\,\mu\,\text{sec.}$$

$$\therefore \boxed{K(s) = \frac{5.368}{s}\,\frac{(1 + 0.478 \times 10^{-3} s)}{(1 + 23.5 \times 10^{-6} s)}}$$

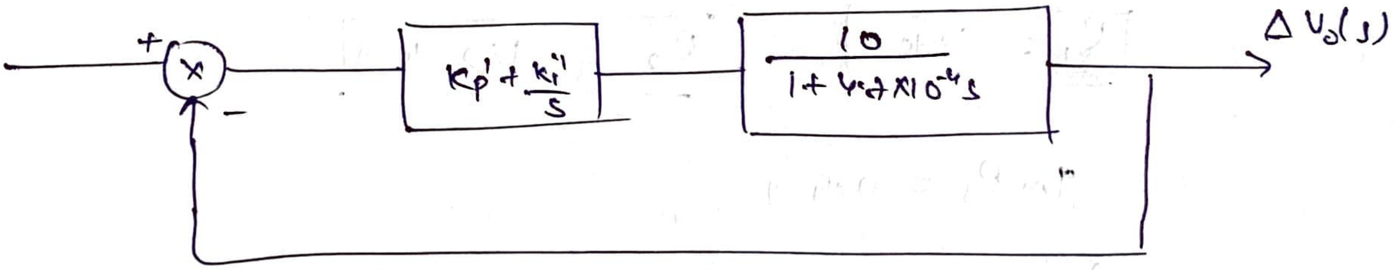
d. OTA with $g_m = 2\,ms$

$V_{oref} = 2.5\,V$ for $V_o = 48\,V$





Sensing gain = 0.5 V/A



$K_p' = 2K_p$ $\quad K_i' = 2K_i$.

$$\tau = \frac{1\,msec}{2.2} = 0.4545\,msec$$

$$a = \frac{1}{0.4545} = 2200$$

$$K_p' = a\tau$$

$$= 2200 \times 47\times10^{-6}$$

$$K_p' = 0.1034$$

$$K_i' = \frac{a}{R} = \frac{2200}{10} = 220$$

$K_p = 0.0517$ $\qquad$ $K_i = 110$

$48\alpha = 2.5$

$\boxed{\alpha = 0.05208}$

$\boxed{K_p'' = 0.99}$ $\qquad$ $\boxed{K_i'' = 2112.13}$

from OTA

$$\frac{\Delta V_o \text{control}(s)}{\Delta V_d(s)} = -\alpha\left(g_m R_s + \frac{g_m}{C_s S}\right)$$

$$\alpha = \frac{R_1 R_2}{R_1 + R_2} = \frac{2.5}{48} = \frac{10}{192}$$

$\boxed{R_1 = 10k\Omega}$, $\boxed{R_2 = 182\ k\Omega}$

$$g_m R_s = 0.99$$

$$R_s = \frac{0.99}{2\times10^{-3}}$$

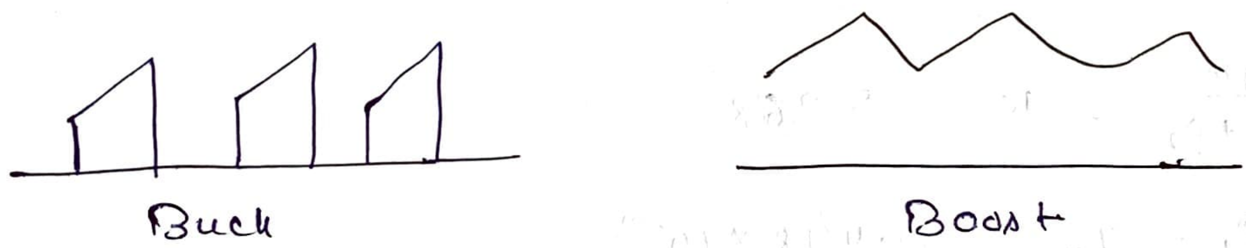
$\boxed{R_s = 495\ \Omega}$

$$\frac{g_m}{C_s} = 2112.13$$

$\boxed{C_s = 946\ pF}$

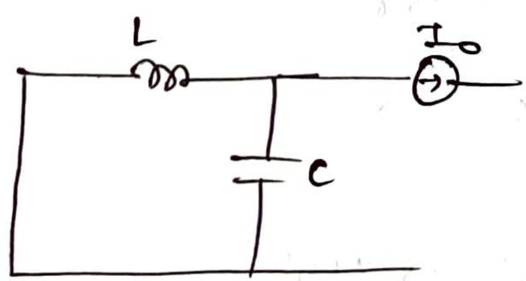
Ques 7. Input filter instability Problem

Input Current drawn in Buck & Boost Converter are as shown



This control have higher value of ripple in frequency of AC current.

→ Thus Iac provides instability in the system. So make use of LC filter in Converter.



In above figure most of I_o flows through C but some portion do flow through L which is reflected in input grid.

→ So if switching frequency is ~~higher~~ higher which is fed to AC grid which is not acceptable according to the standards set by government.

→ Hence we need to change value of L, to reduce AC component in current.

→ for Higher harmonics Capacitor Behaves as ESR rather than short Circuit.

→ So a AC Capacitor of higher quality need to be used in parallel with the electrolytic Capacitor to reduce these frequency in I_o.

→ In Battery power DC - DC Converter due to I_o battery life is degraded and that is what we don't want so input filtering is made to be employed.