# Electrical Drives

## Lecture 11 (05-02-2024)

## CONVERTERS - Module 2

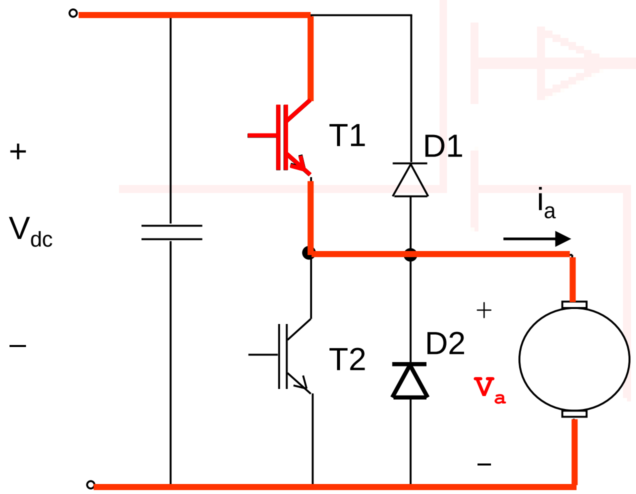
**AC-DC controlled rectifier**
- approximate model
- SIMULINK examples
  - open-loop
  - closed-loop

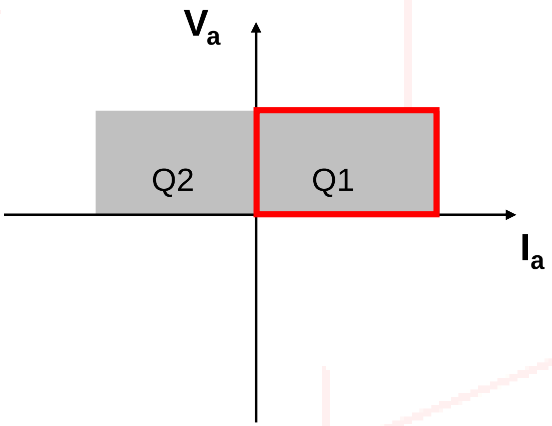
**Switch Mode DC-DC converter**
- 2-Q and 4-Q converters
  - Small signal modeling
    - unipolar
    - bipolar
- SIMULINK example

**Current-controlled for SM converters**
- Bridge converter
  - hysteresis
  - fixed frequency
- 3-phase VSI
  - hysteresis
  - fixed frequency
  - SVM-based

# Switch mode DC-DC converter

Two-quadrant converter

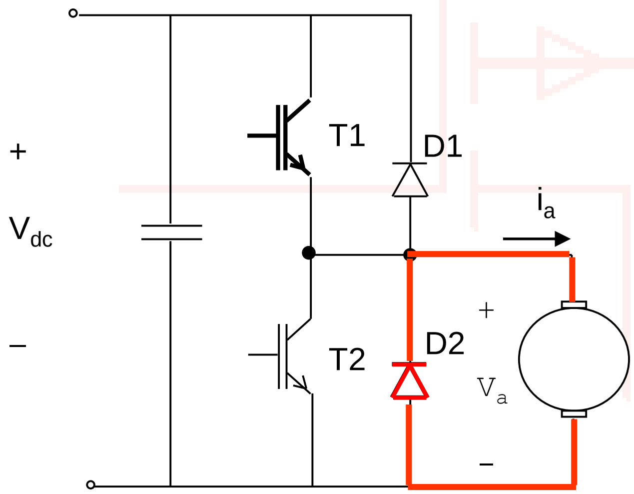


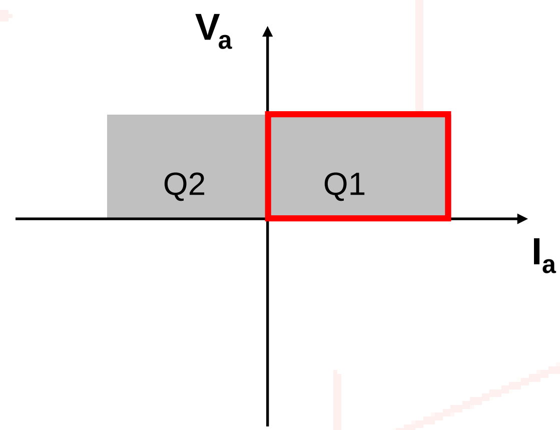




T1 conducts $\rightarrow$ $\mathbf{v_a = V_{dc}}$

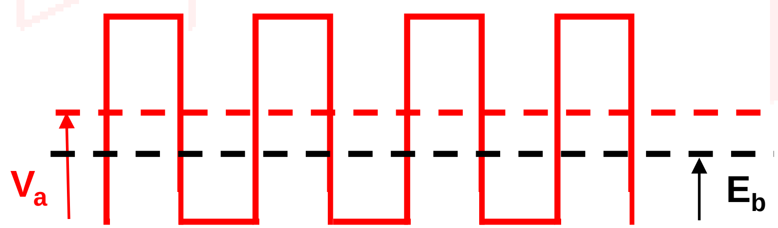
# Switch mode DC-DC converter

## Two-quadrant converter



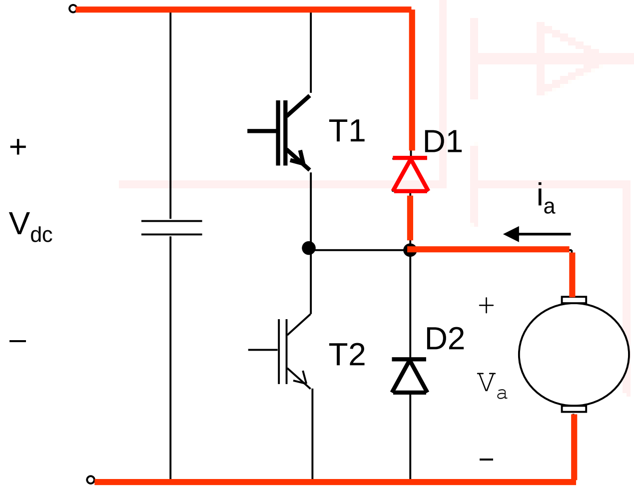

D2 conducts $\rightarrow$ $\mathbf{v_a = 0}$ T1 conducts $\rightarrow$ $\mathbf{v_a = V_{dc}}$

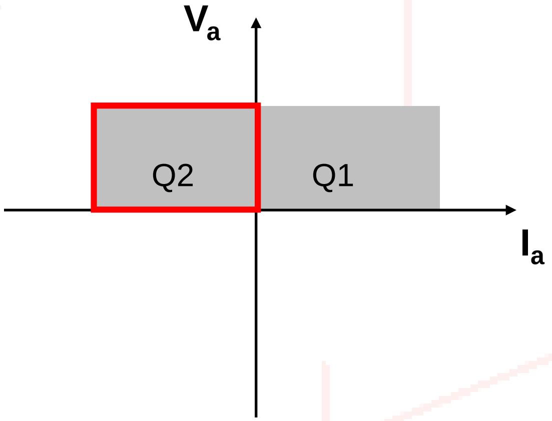



**Quadrant 1** The average voltage is made larger than the back emf

# Switch mode DC-DC converter

Two-quadrant converter

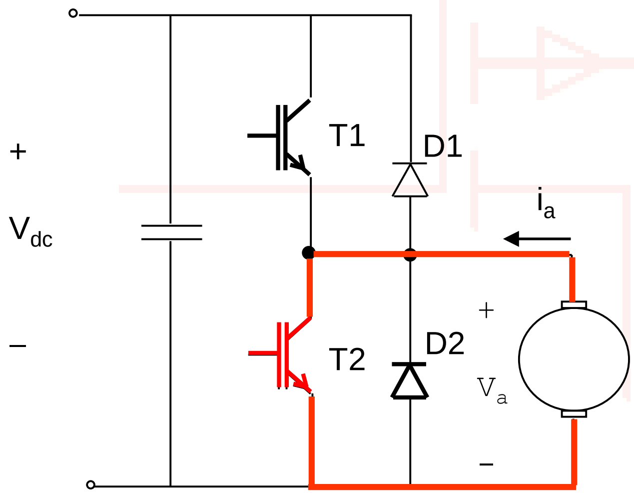


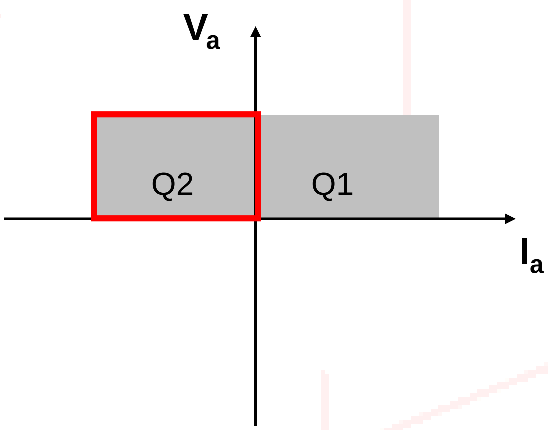




D1 conducts $\rightarrow$ $\mathbf{v_a = V_{dc}}$

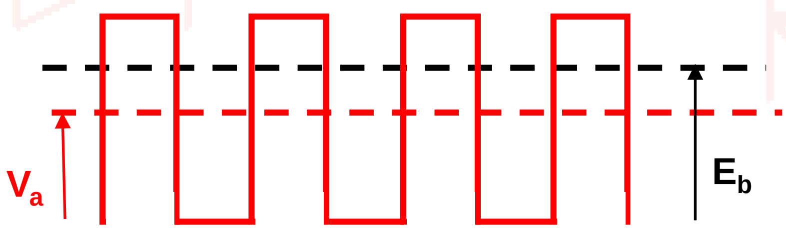
# Switch mode DC-DC converter

## Two-quadrant converter



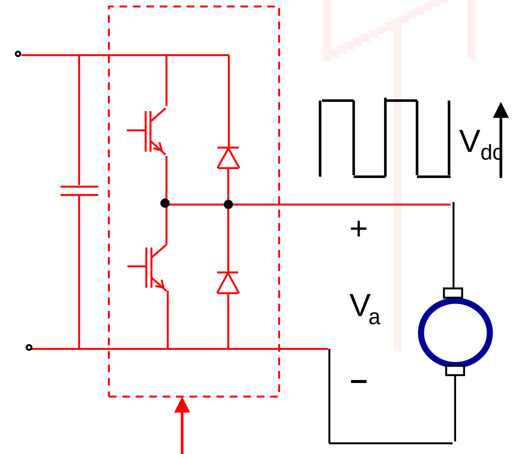

T2 conducts $\rightarrow$ $\mathbf{v_a = 0}$   D1 conducts $\rightarrow$ $\mathbf{v_a = V_{dc}}$



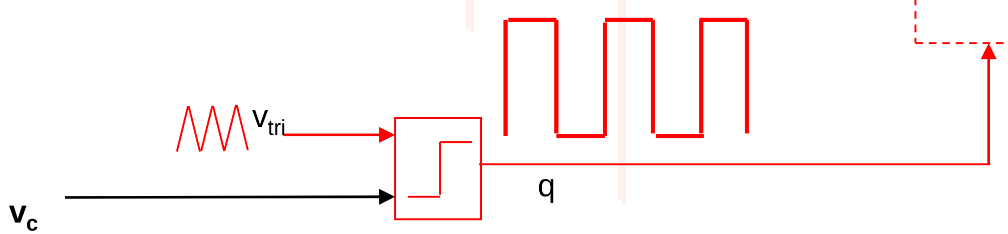

**Quadrant 2** The average voltage is made smaller than the back emf, thus forcing the current to flow in the reverse direction

# Switch mode DC-DC converter

## Two-quadrant converter

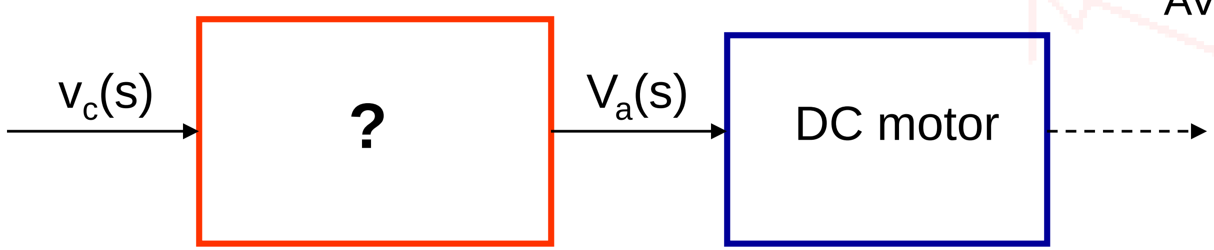



Switching signals obtained by comparing control signal with triangular wave



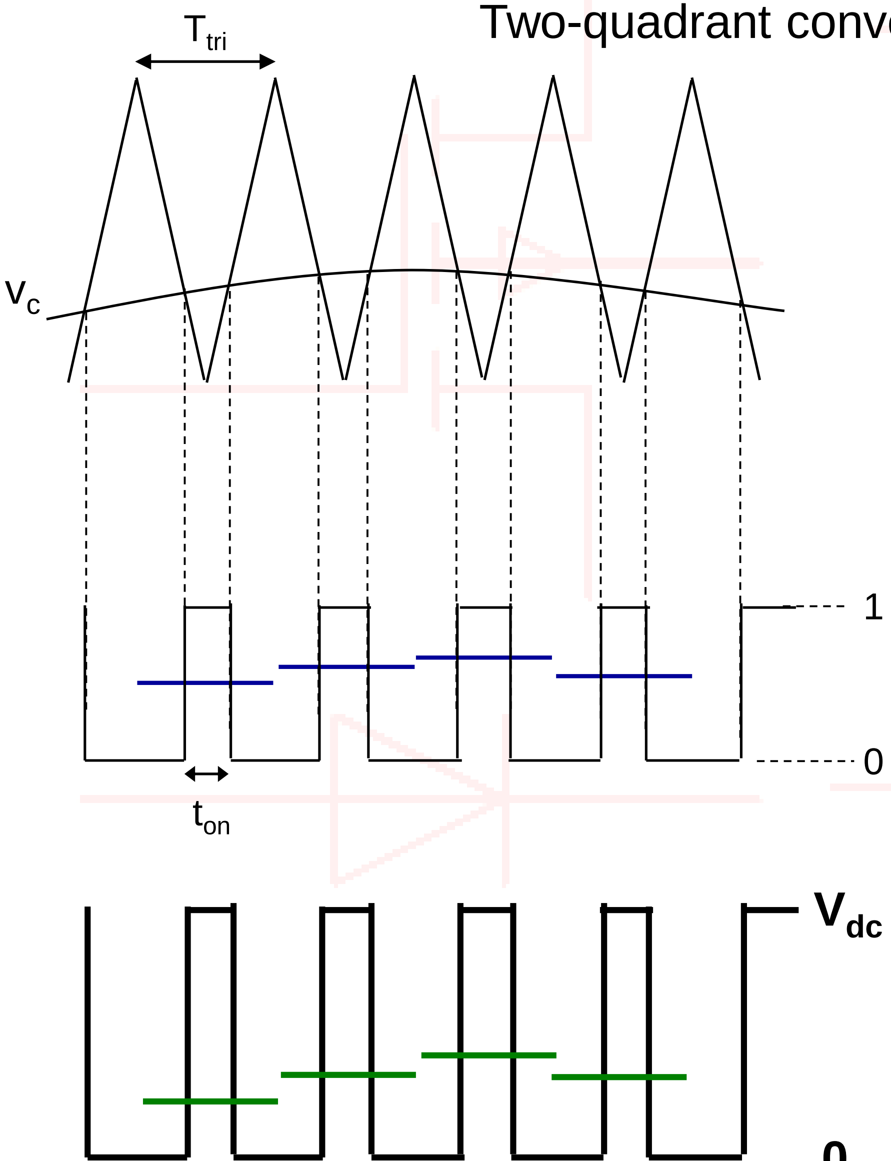

We want to establish a relation between $\mathbf{v_c}$ and $\underline{\mathbf{V_a}}$

AVERAGE voltage



# Switch mode DC-DC converter

## Two-quadrant converter




$$q = \begin{cases} 1 & V_c > V_{tri} \\ 0 & V_c < V_{tri} \end{cases}$$

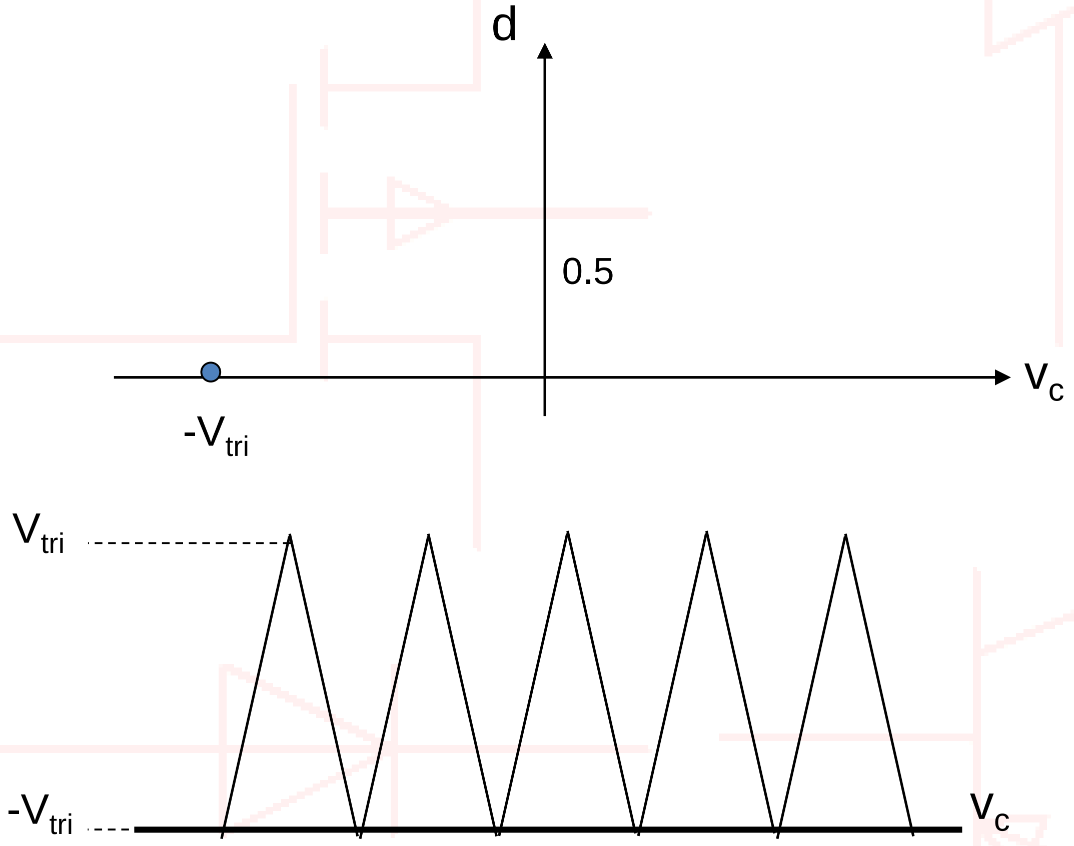
$$d = \frac{1}{T_{tri}} \int_t^{t+T_{tri}} q\,dt$$

$$= \frac{t_{on}}{T_{tri}}$$

$$V_a = \frac{1}{T_{tri}} \int_0^{dT_{tri}} V_{dc}\,dt = dV_{dc}$$

# Switch mode DC-DC converter

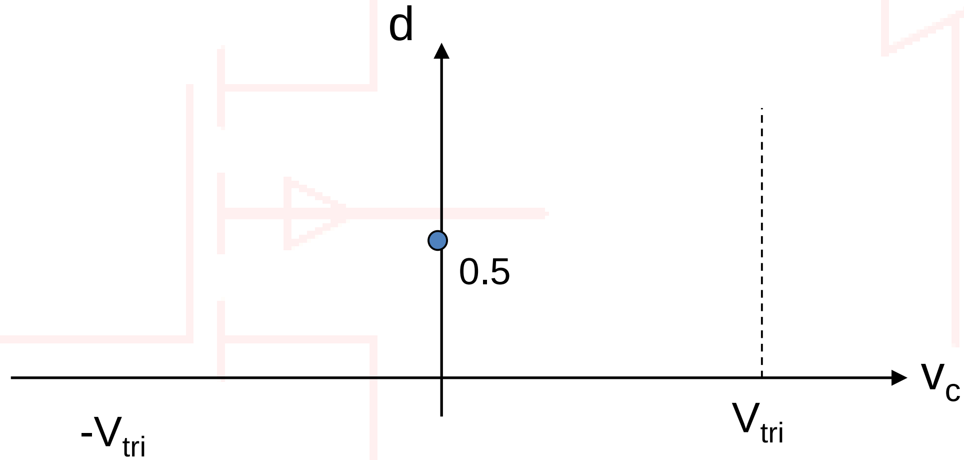
Two-quadrant converter

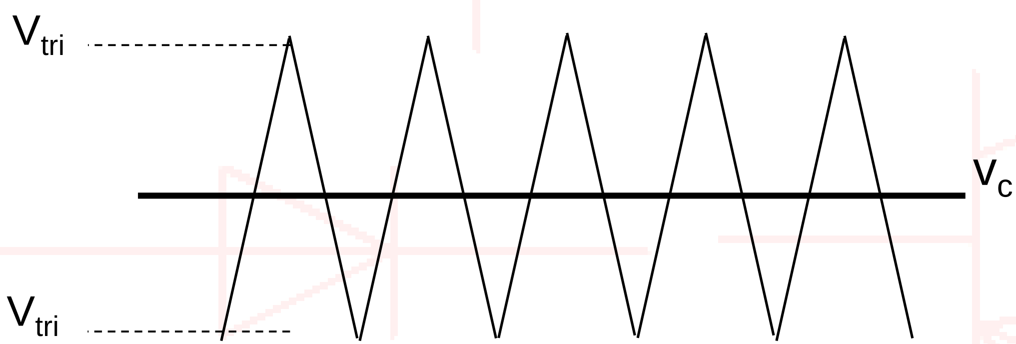



For $v_c = -V_{tri} \rightarrow d = 0$

## Switch mode DC-DC converter

### Two-quadrant converter



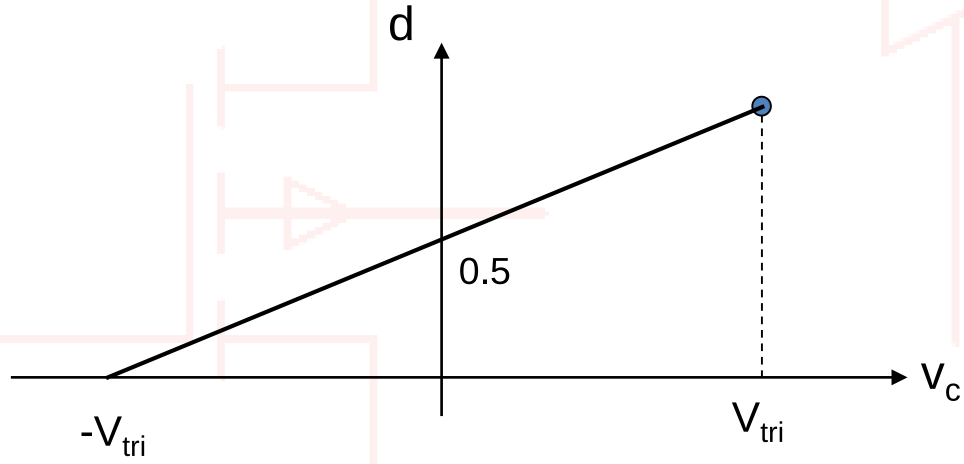

For $v_c = -V_{tri} \rightarrow d = 0$

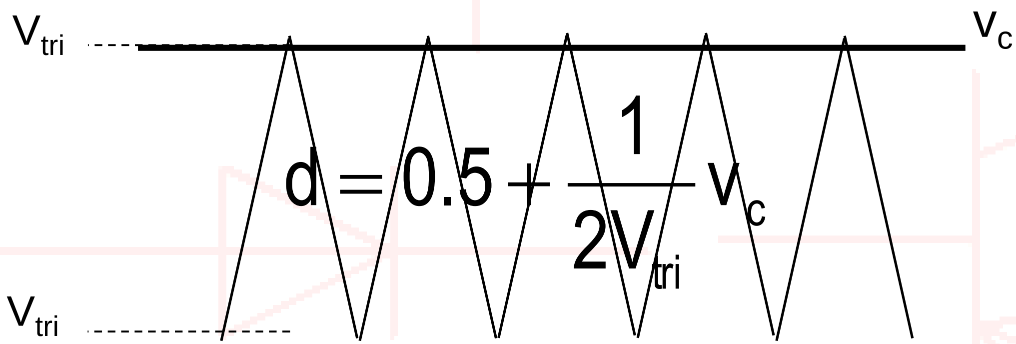
For $v_c = 0 \rightarrow d = 0.5$

For $v_c = V_{tri} \rightarrow d = 1$

## Switch mode DC-DC converter

Two-quadrant converter




$$d = 0.5 + \frac{1}{2V_{tri}} v_c$$

For $v_c = -V_{tri} \rightarrow \quad d = 0$

For $v_c = 0 \rightarrow \quad d = 0.5$

For $v_c = V_{tri} \rightarrow \quad d = 1$

## Switch mode DC-DC converter

Two-quadrant converter

Thus relation between $v_c$ and $V_a$ is obtained as:

$$V_a = 0.5V_{dc} + \frac{V_{dc}}{2V_{tri}} v_c$$

Introducing perturbation in $v_c$ and $V_a$ and separating DC and AC components:

**DC:**

$$V_a = 0.5V_{dc} + \frac{V_{dc}}{2V_{tri}} v_c$$

**AC:**

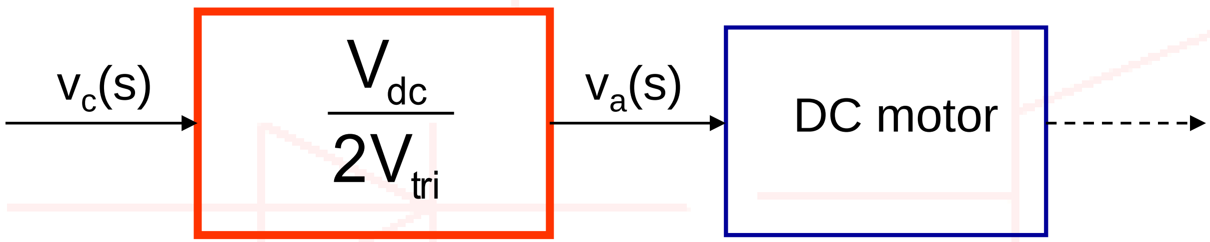
$$\tilde{v}_a = \frac{V_{dc}}{2V_{tri}} \tilde{v}_c$$

## Switch mode DC-DC converter

Two-quadrant converter

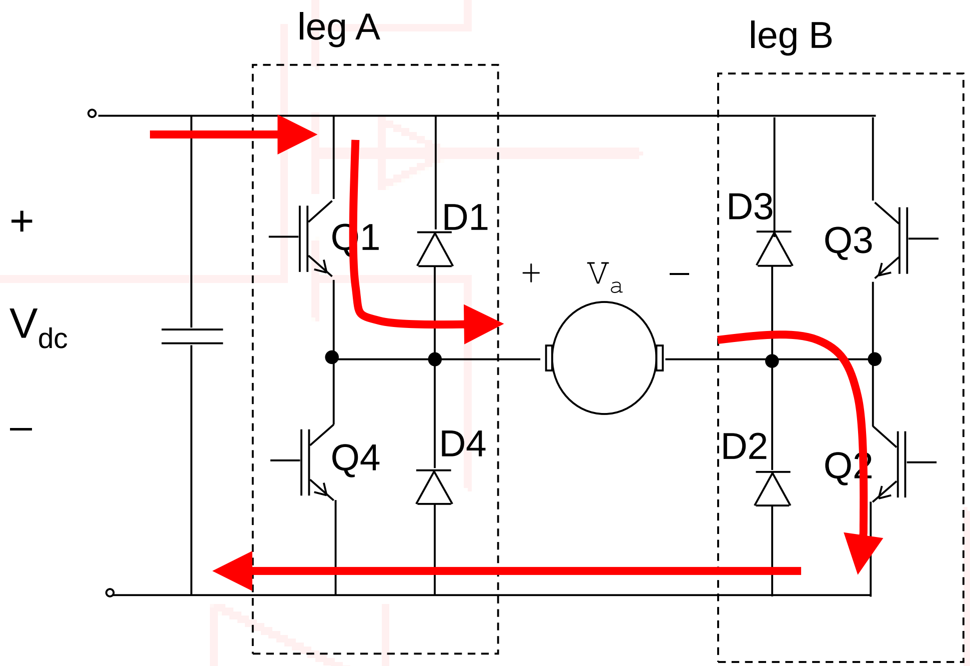
Taking Laplace Transform on the AC, the transfer function is obtained as:

$$\frac{v_a(s)}{v_c(s)} = \frac{V_{dc}}{2V_{tri}}$$



## Switch mode DC-DC converter

Four-quadrant converter

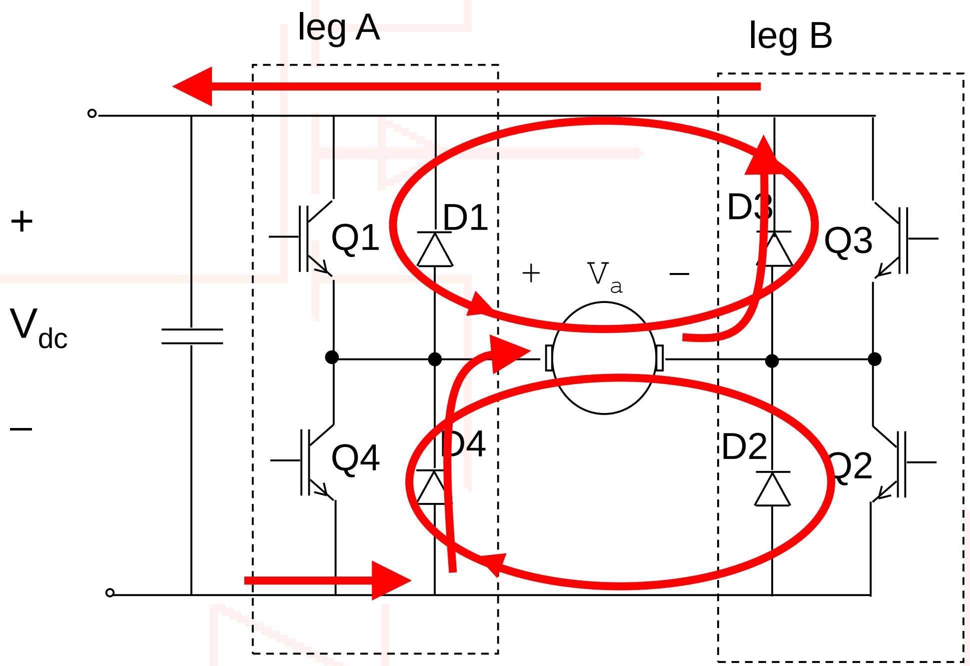



**Positive current**

**$v_a = V_{dc}$ when Q1 and Q2 are ON**

## Switch mode DC-DC converter

Four-quadrant converter




**Positive current**

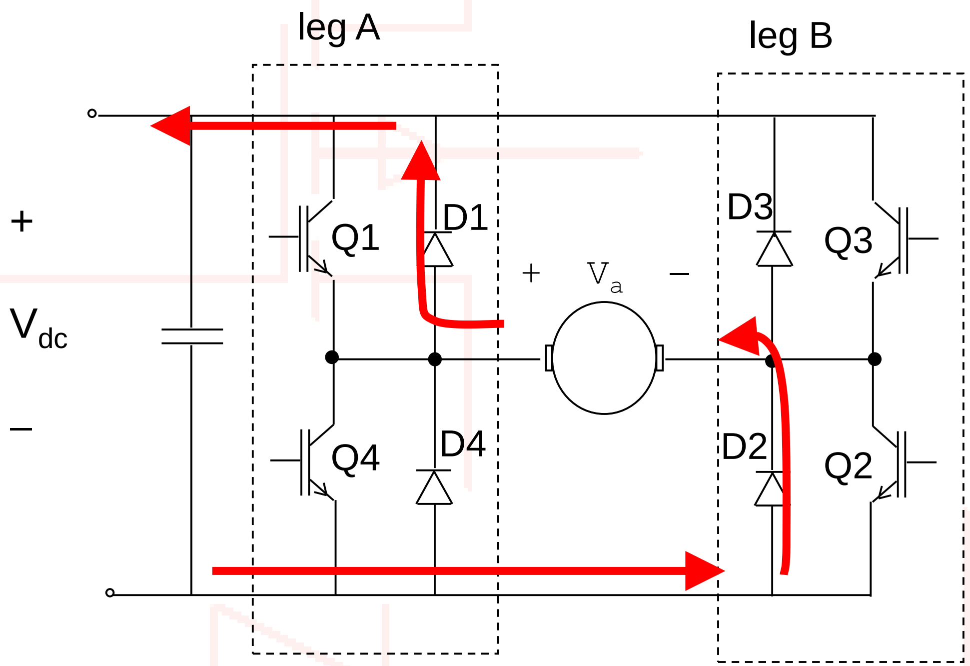
$v_a = V_{dc}$ when Q1 and Q2 are ON

$v_a = -V_{dc}$ when D3 and D4 are ON

$v_a = 0$ when current freewheels through Q and D

# Switch mode DC-DC converter

## Four-quadrant converter




**Positive current**

$v_a = V_{dc}$ when Q1 and Q2 are ON

$v_a = -V_{dc}$ when D3 and D4 are ON

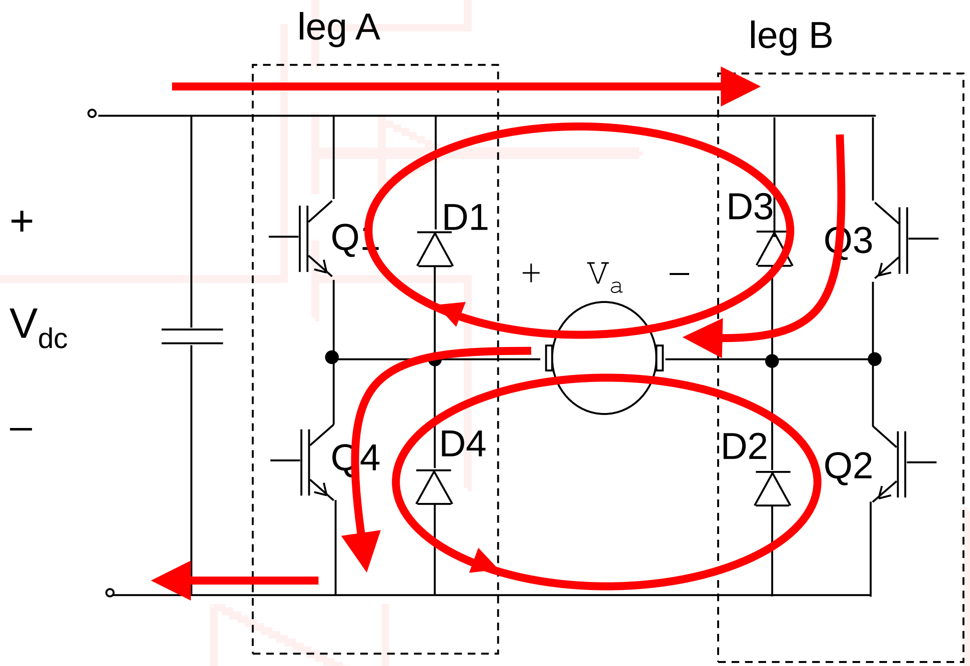
$v_a = 0$ when current freewheels through Q and D

**Negative current**

$v_a = V_{dc}$ when D1 and D2 are ON

# Switch mode DC-DC converter

## Four-quadrant converter




**Positive current**

$v_a = V_{dc}$ when Q1 and Q2 are ON

$v_a = -V_{dc}$ when D3 and D4 are ON

$v_a = 0$ when current freewheels through Q and D

**Negative current**

$v_a = V_{dc}$ when D1 and D2 are ON

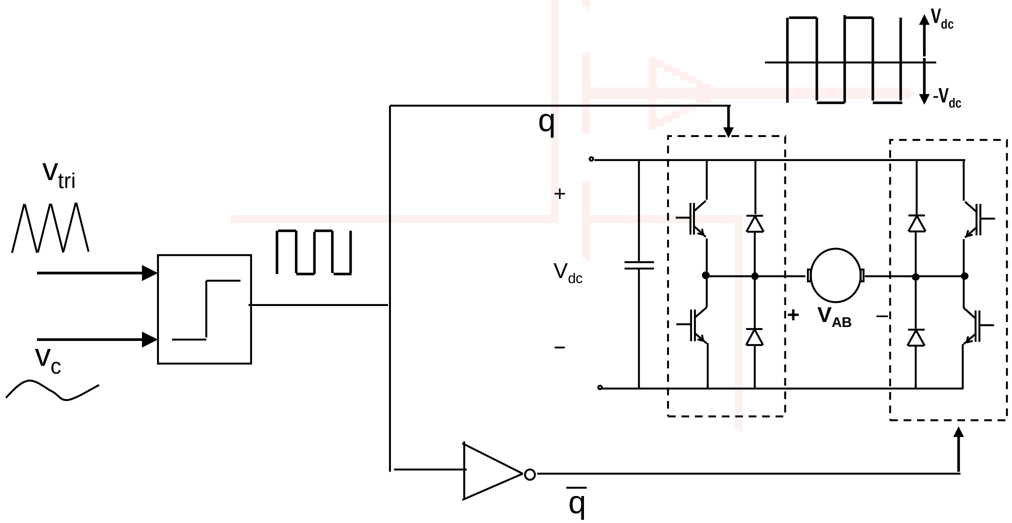
$v_a = -V_{dc}$ when Q3 and Q4 are ON

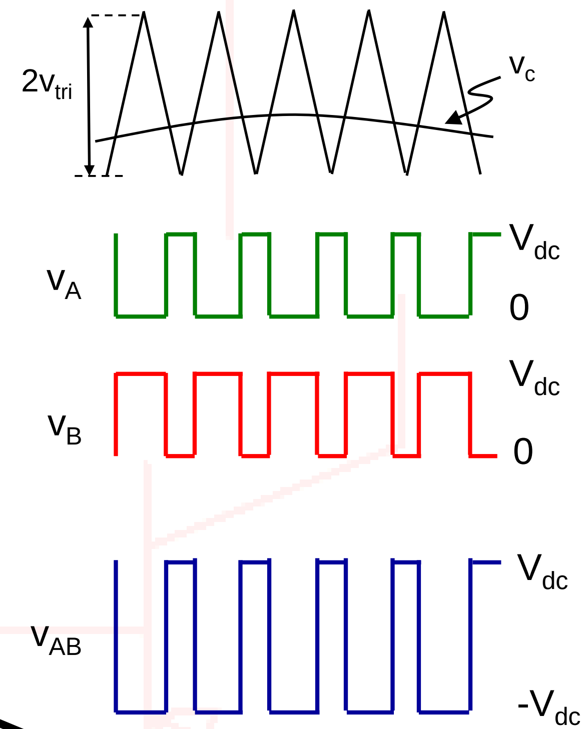
$v_a = 0$ when current freewheels through Q and D

# Switch mode DC-DC converter

## Four-quadrant converter

### Bipolar switching scheme




$$d_A = 0.5 + \frac{v_c}{2V_{tri}} \qquad d_B = 1 - d_A = 0.5 - \frac{v_c}{2V_{tri}}$$

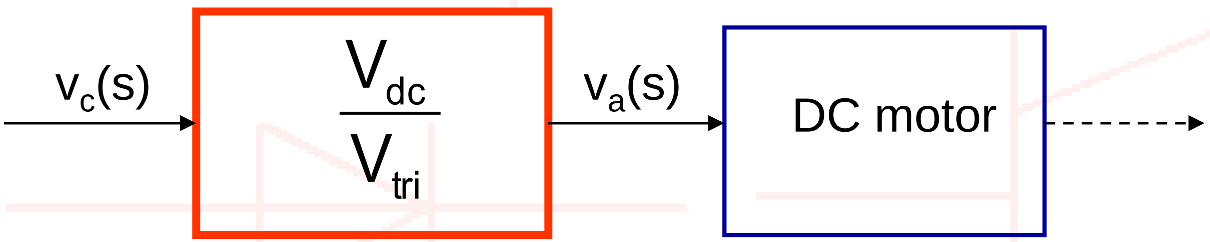
$$V_A = 0.5V_{dc} + \frac{V_{dc}}{2V_{tri}} v_c \qquad V_B = 0.5V_{dc} - \frac{V_{dc}}{2V_{tri}} v_c \qquad V_A - V_B = V_{AB} = \frac{V_{dc}}{V_{tri}} v_c$$

## Switch mode DC-DC converter

Four-quadrant converter

**Bipolar switching scheme**

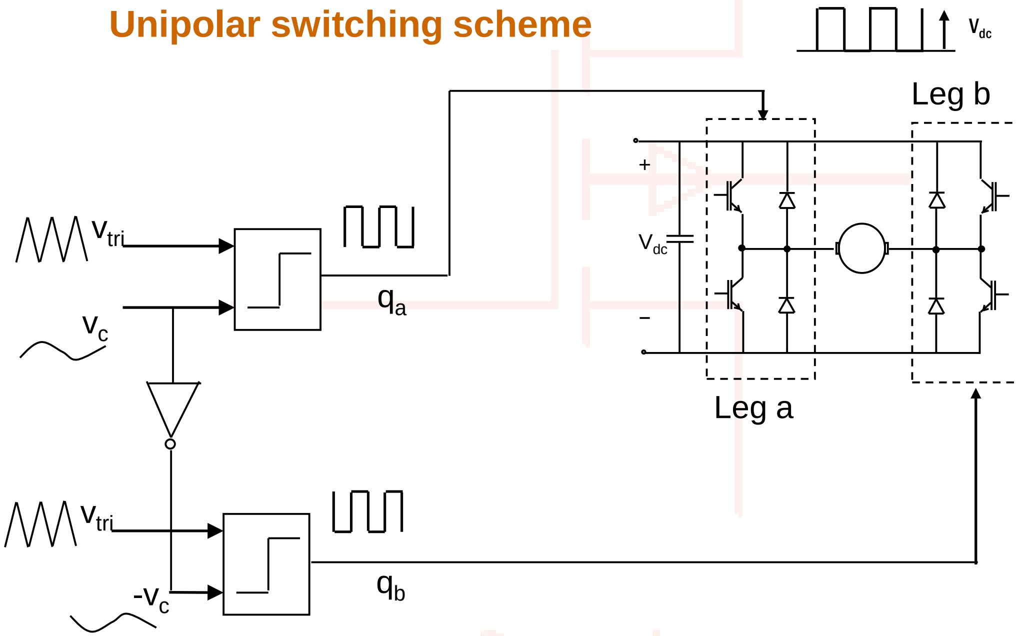
$$\frac{v_a(s)}{v_c(s)} = \frac{V_{dc}}{V_{tri}}$$

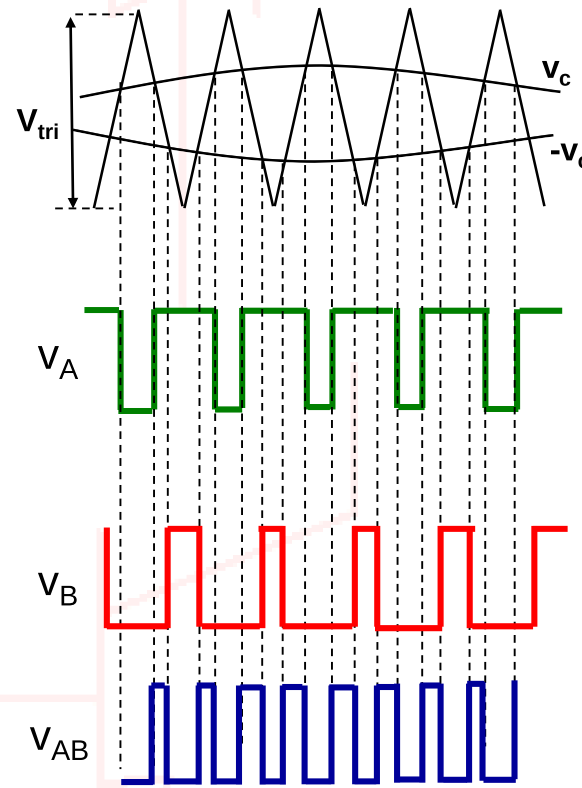


# Switch mode DC-DC converter

## Four-quadrant converter

### Unipolar switching scheme




$$d_A = 0.5 + \frac{v_c}{2V_{tri}} \qquad d_B = 0.5 + \frac{-v_c}{2V_{tri}}$$

$$V_A = 0.5V_{dc} + \frac{V_{dc}}{2V_{tri}} v_c \qquad V_B = 0.5V_{dc} - \frac{V_{dc}}{2V_{tri}} v_c \qquad V_A - V_B = V_{AB} = \frac{V_{dc}}{V_{tri}} v_c$$

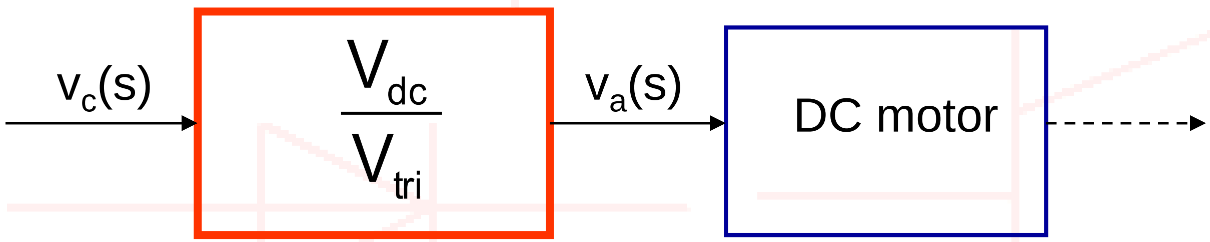
**The same average value we've seen for bipolar !**

# Switch mode DC-DC converter

## Four-quadrant converter

### Unipolar switching scheme

$$\frac{v_a(s)}{v_c(s)} = \frac{V_{dc}}{V_{tri}}$$

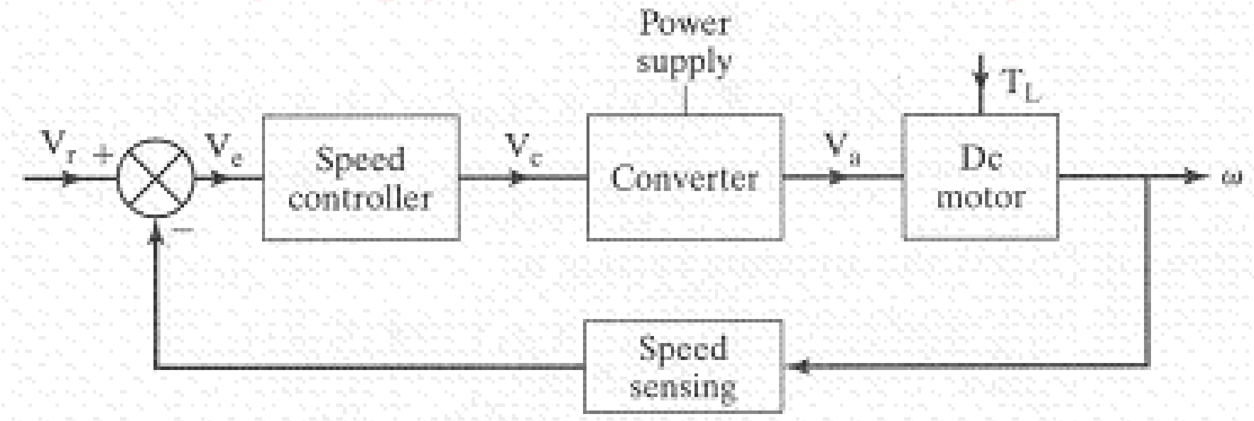


# Closed-Loop Control of DC Drives

- The speed of dc motors changes with the load torque.
- To maintain a constant speed, the armature (and or field) voltage should be varied continuously by varying the delay angle of ac-dc converters or duty cycle of dc-dc converters.
- In practical drive systems it is required to operate the drive at a constant torque or constant power; in addition, controlled acceleration and deceleration are required.
- Most industrial drives operate as closed-loop feedback systems.
- A closed-loop control system has the advantages of improved accuracy, fast dynamic response, and reduced effects of load disturbances and system nonlinearities.

# Closed-Loop Control of DC Drives

- The block diagram of a closed-loop converter-fed separately excited dc drive is shown in Figure .
- If the speed of the motor decreases due to the application of additional load torque, the speed error $V_e$ increases.
- The speed controller responses with an increased control signal $V_c$, change the delay angle or duty cycle of the converter, and increase the armature voltage of the motor.
- An increased armature voltage develops more torque to restore the motor speed to the original value.
- The drive normally passes through a transient period until the developed torque is equal to the load torque.



# Modelling of Converters

A control technique to overcome this nonlinear characteristic and the accompanying undesirable dynamic behavior is given in the following. The control input to determine the delay angle is modified to be

$$\alpha = \cos^{-1}\left(\frac{v_c}{V_{cm}}\right) = \cos^{-1}(v_{cn})$$

where $v_c$ is the control input and $V_{cm}$ is the maximum of the absolute value of the control voltage.

$$V_{dc} = \frac{3}{\pi}V_m \cos\alpha = \frac{3}{\pi}V_m \cos\left(\cos^{-1} v_{cn}\right)$$

$$= \left[\frac{3}{\pi}V_m\right]v_{cn} = \left[\frac{3}{\pi}\frac{V_m}{V_{cm}}\right]v_c = K_r v_c$$

where $v_{cn}$ is the normalized control voltage and $K_r$ is the gain of the converter.

# Modelling of Converters

$$K_r = \frac{3}{\pi}\frac{V_m}{V_{cm}} = \frac{3\sqrt{2}V}{\pi V_{cm}} = 1.35\frac{V}{V_{cm}}$$

where V is the rms line-to-line voltage.

Then the modified transfer characteristic is linear with a slope of $K_r$. The control voltage is normalized to keep its magnitude less than or equal to 1, to be able to obtain the inverse cosine of it.

# Transfer function model of 3ϕ Converter

The converter can be considered as a black box with a certain gain and phase delay for modeling and use in control studies. The gain of the linearized controller-based converter for a maximum control voltage $V_{cm}$ is

$$K_r = \frac{1.35V}{V_{cm}} .V/V$$

The converter is a sampled-data system. The sampling interval gives an indication of its time delay. Once a thyristor is switched on, its triggering angle cannot be changed. The new triggering delay can be implemented with the succeeding thyristor gating. In the meanwhile, the delay angle can be corrected and will be ready for implementation within 60°, i.e., the angle between two thryistors' gating. Statistically, the delay may be treated as one half of this interval; in time, it is equal to

$$T_r = \frac{60/2}{360} \times (\text{time period of one cycle}) = \frac{1}{12} \times \frac{1}{f_s}, \text{s}$$

# Transfer function model of 3ϕ Converter

For a 60-Hz supply-voltage source, note that the time delay is equal to 1.388 ms. The converter is then modeled with its gain and time delay as

$$G_r(s) = K_r e^{-T_r s}$$

and equation can also be approximated as a first-order time lag and given as

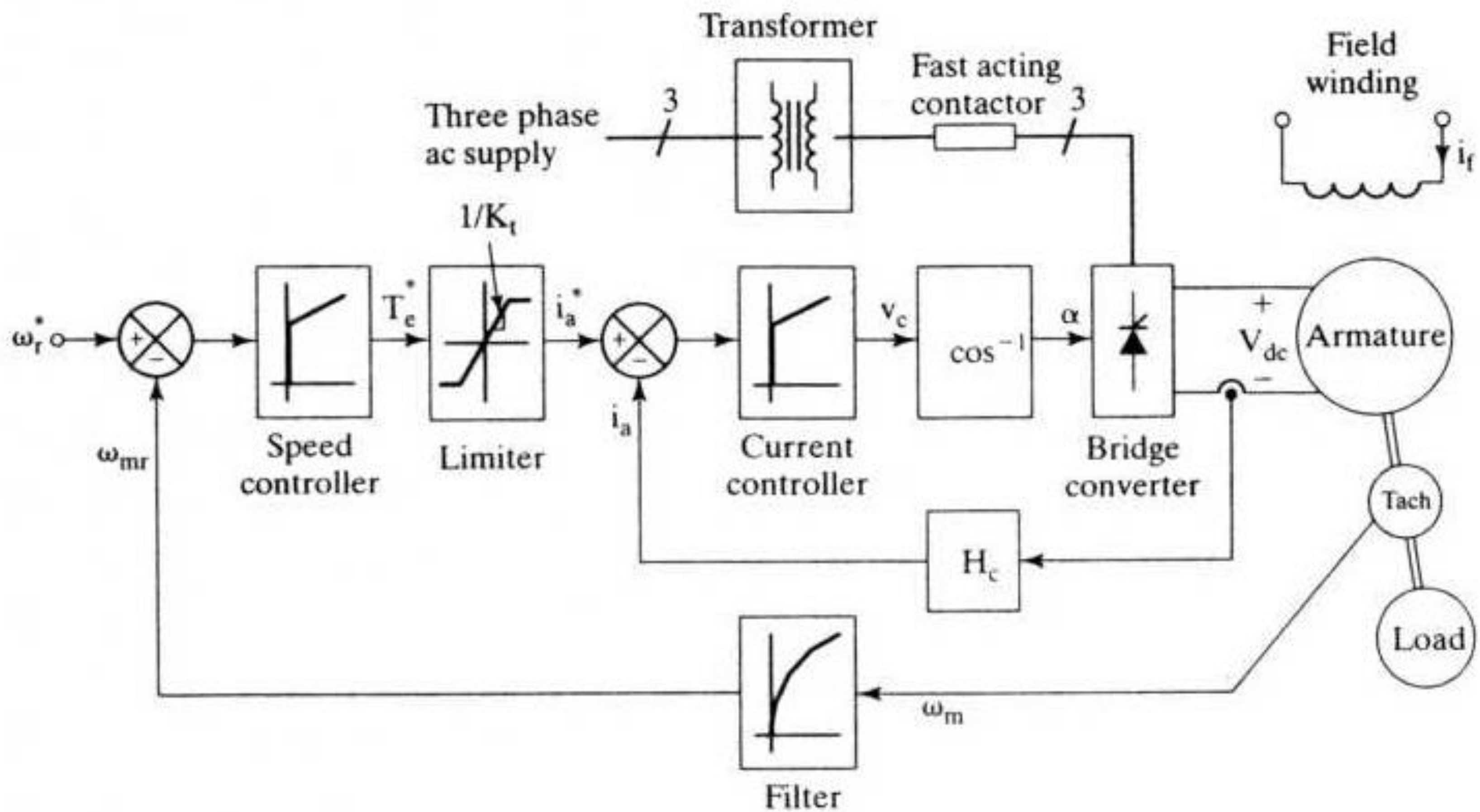
$$G_r(s) = \frac{K_r}{(1 + sT_r)}$$

Many low-performance systems have a simple controller with no linearization of its transfer characteristic. The transfer characteristic in such a case is nonlinear. Then, the gain of the converter is obtained as a small-signal gain given by

$$K_r = \frac{\delta V_{dc}}{\delta\alpha} = \frac{\delta}{\delta\alpha}\{1.35\ V \cos\alpha\} = -1.35\ V \sin\alpha$$

# 3ϕ Converter fed DC Motor Drive

The control schematic of a two-quadrant converter-controlled separately-excited dc motor drive is shown in Figure



# Transfer functions of the sub systems

## DC Motor and load

The dc machine contains an inner loop due to the induced emf. It is not physically seen; it is magnetically coupled. The inner current loop will cross this back-emf loop, creating a complexity in the development of the model. It is shown in Figure

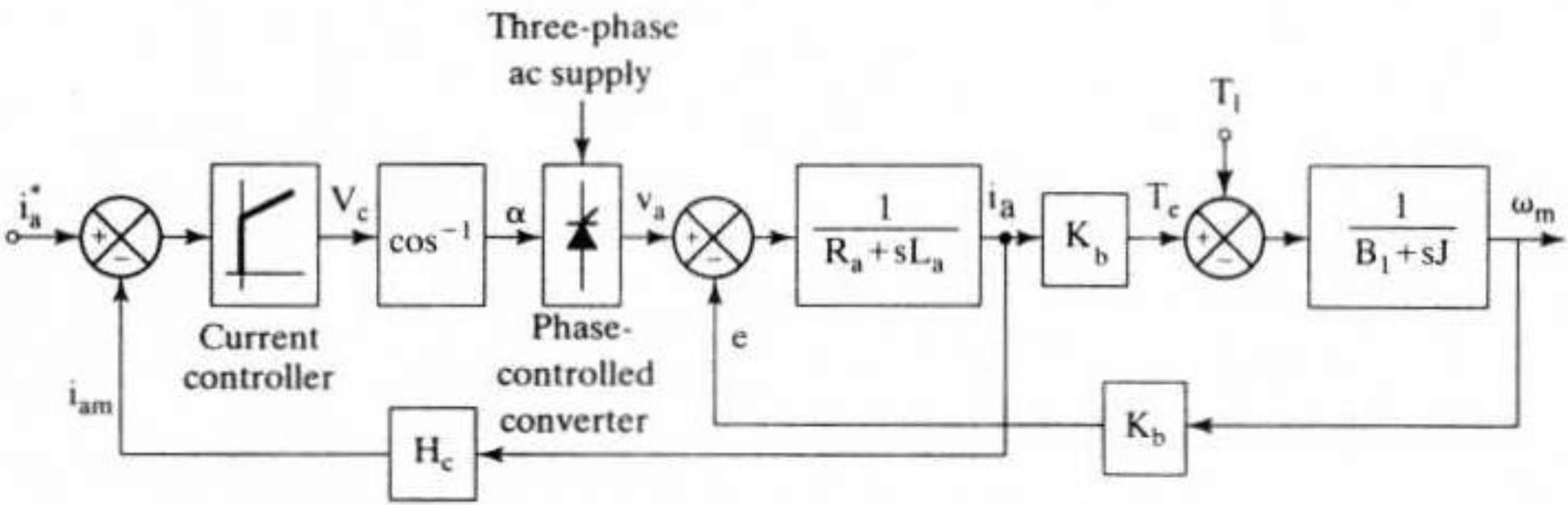



**Figure 3.27** DC motor and current-control loop

# DC Motor and load

The interactions of these loops can be decoupled by suitably redrawing the block diagram. The development of such a block diagram for the dc machine is shown in Figure 3.28, step by step. The load is assumed to be proportional to speed and is given as

$$T_l = B_l \omega_m$$

To decouple the inner current loop from the machine-inherent induced-emf loop, it is necessary to split the transfer function between the speed and voltage into two cascade transfer functions, first between speed and armature current and then between armature current and input voltage, represented as

$$\frac{\omega_m(s)}{V_a(s)} = \frac{\omega_m(s)}{I_a(s)} \cdot \frac{I_a(s)}{V_a(s)}$$

$$T_m = \frac{J}{B_t}$$

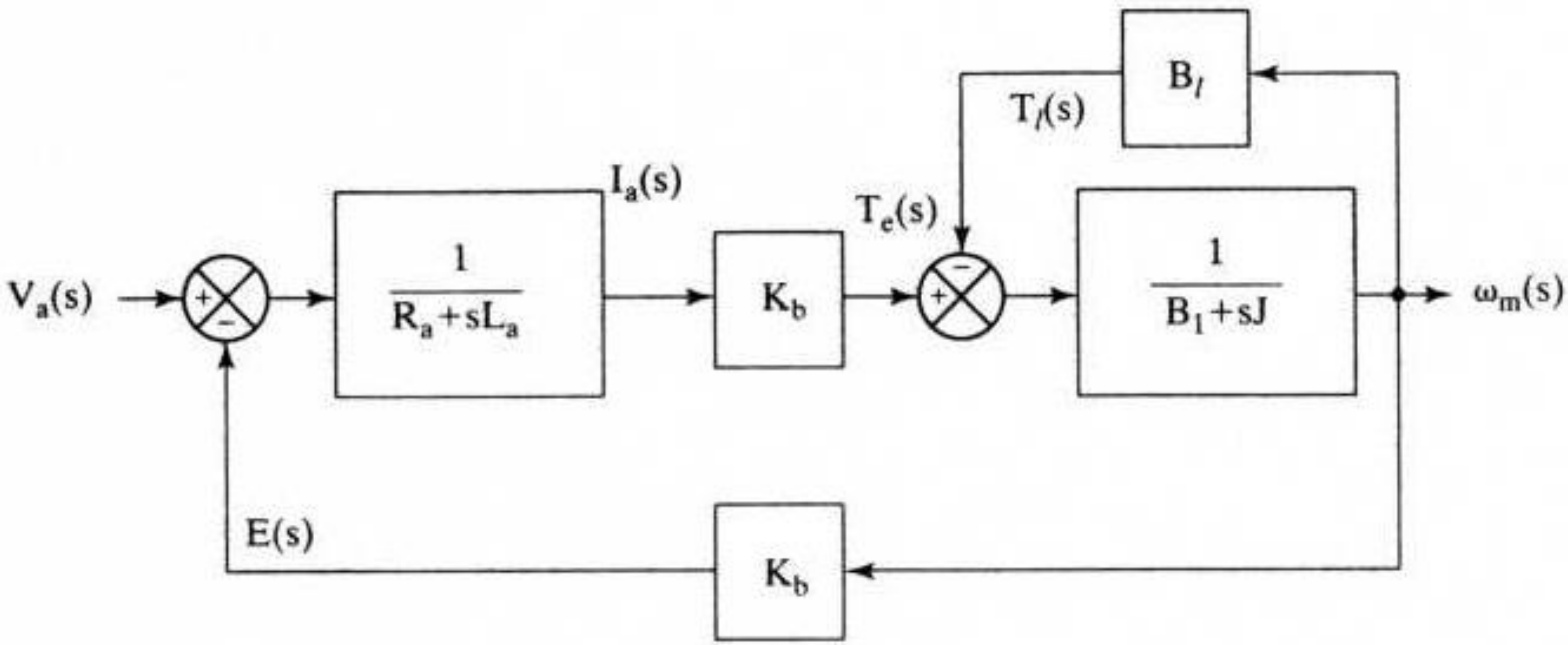
where

$$\frac{\omega_m(s)}{I_a(s)} = \frac{K_b}{B_t(1 + sT_m)}$$

$$B_t = B_1 + B_l$$

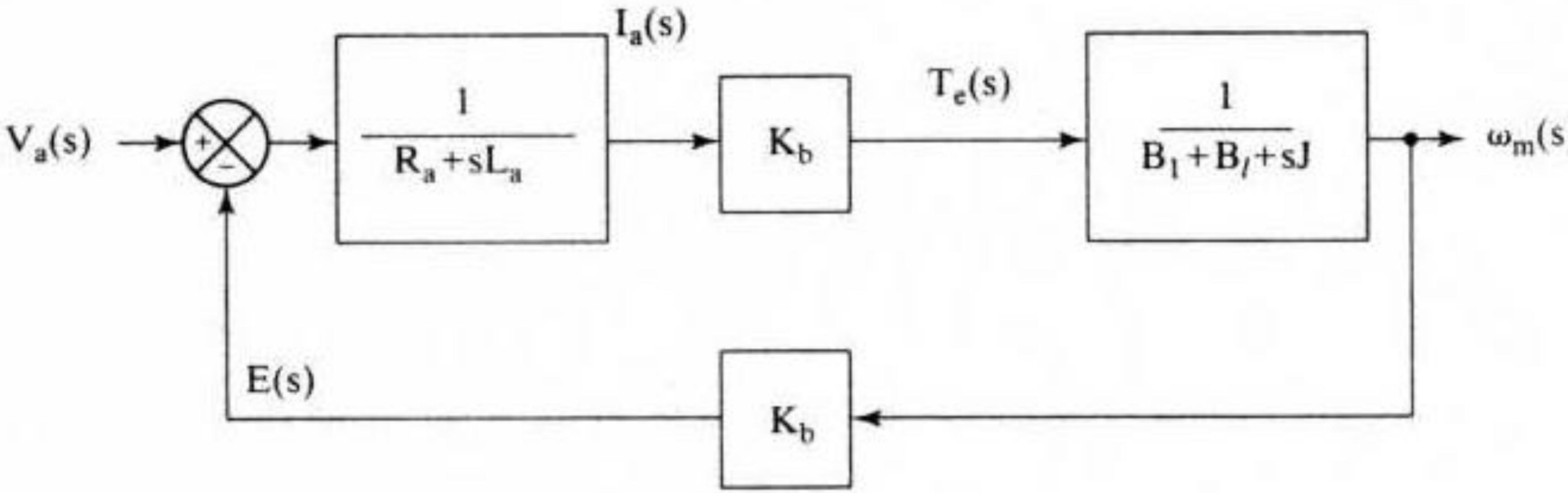
$$\frac{I_a(s)}{V_a(s)} = K_1 \frac{(1 + sT_m)}{(1 + sT_1)(1 + sT_2)}$$

# DC Motor and load



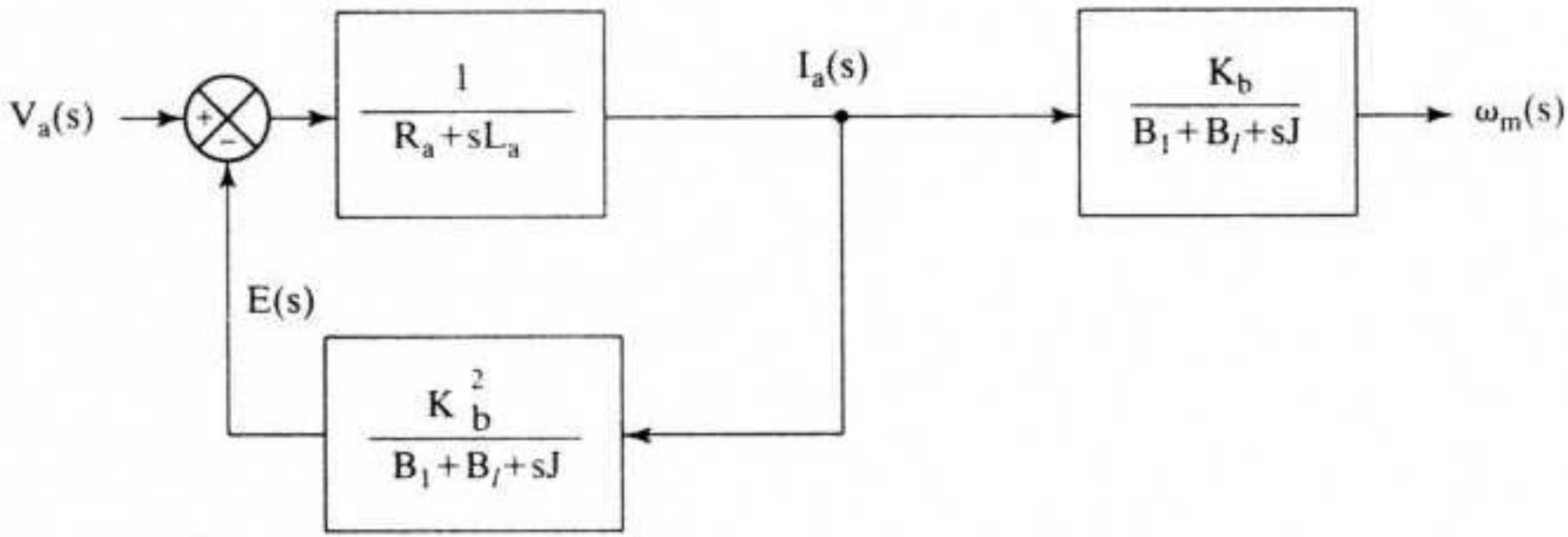

Step 1

# DC Motor and load

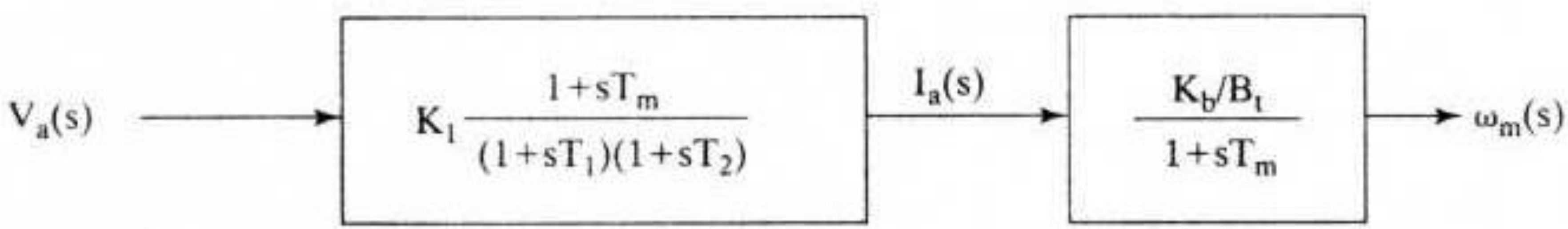



Step 2

# DC Motor and load




Step 3




Step 4

# DC Motor and load

$$-\frac{1}{T_1}, -\frac{1}{T_2} = -\frac{1}{2}\left[\frac{B_t}{J} + \frac{R_a}{L_a}\right] \pm \sqrt{\frac{1}{4}\left(\frac{B_t}{J} + \frac{R_a}{L_a}\right)^2 - \left(\frac{K_b^2 + R_a B_t}{J L_a}\right)}$$

$$K_1 = \frac{B_t}{K_b^2 + R_a B_t}$$

# Converter

The converter after linearization is represented as

$$G_r(s) = \frac{V_a(s)}{v_c(s)} = \frac{K_r}{1 + sT_r}$$

The delay time $T_r$ and gain are evaluated

# Current and Speed controllers

The current and speed controllers are of proportional-integral type. They are represented as

$$G_c(s) = \frac{K_c(1 + sT_c)}{sT_c}$$

$$G_s(s) = \frac{K_s(1 + sT_s)}{sT_s}$$

where the subscripts *c* and *s* correspond to the current and speed controllers, respectively. The K and T correspond to the gain and time constants of the controllers.

# Current feedback

The gain of the current feedback is $H_c$. No filtering is required in most cases. In the case of a filtering requirement, a low-pass filter can be included in the analysis. Even then, the time constant of the filter might not be greater than a millisecond.

# Speed feedback

Most high performance systems use a dc tachogenerator, and the filter required is low-pass, with a time constant under 10 ms. The transfer function of the speed feed-back filter is

$$G_\omega(s) = \frac{K_\omega}{1 + sT_\omega}$$

where $K_\omega$ is the gain and $T_\omega$ is the time constant.

The motor parameters and ratings are as follows:

220 V, 8.3 A, 1470 rpm, $R_a = 4\ \Omega$, $J = 0.0607\ kg - m_2$, $L_a = 0.072$ H, $B_t = 0.0869$ N·m / rad/sec, $K_b = 1.26$ V/rad/sec.

Obtain the motor transfer function: