MA6003D

DEPARTMENT OF MATHEMATICS, NIT CALICUT

(Electrical Engineering M.Tech-PE (EE63) and IPA (EE64))
1st Semester Midterm Examination, Monsoon 2022

Mathematical Methods for power Engineering

October, 2022 | Maximum Duration: 90 Minutes | Maximum Marks: 30

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READ INSTRUCTION BEFORE ANSWERING

- Attempt all 5 questions in PART I and each question carries 2 marks.
- Attempt all 4 questions in PART II and each question carries 5 marks
- Do not write anything on question paper, except name and registration number

PART I $(5 \times 2 = 10)$

1. Check whether the following mapping is linear transformation or not?

(a) $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T(x, y, z) = (1, y, z)$.

(b) $T : \mathbb{R}^3 \to \mathbb{R}^2$ be defined by $T(x, y, z) = (x + yz, 3z)$.

2. Determine whether the following set of vectors form a basis for $\mathbb{R}^3$ or not?

$$S = \{(2, 2, 0), (3, 0, 2), (2, -2, 2)\}.$$

3. Let $V = M_{n\times n}(\mathbb{R})$ be the set of all $n \times n$ matrices whose entries from $\mathbb{R}$. Verify the following subset $W \subset V$ is subspace or not?

$$W = \{A \in M_{n\times n}(\mathbb{R}) : A = A^T\},$$

where A^T means transpose of matrix A.

4. The standard weight of a special purpose brick is 5 kg and it contains two basic ingredients B_1 and B_2. B_1 costs Rs 5 per kg and B_2 costs Rs 8 per kg. Strength considerations dictate that the brick contains not more than 4 kg of B_1 and a minimum of 2 kg of B_2. Since the demand for the product is likely to be related to the price of the brick, find out graphically the minimum cost of the brick satisfying the above conditions.

5. Fill in the blanks with the correct word.

(a) A BFS (Basic Feasible Solution) of a LPP is said to be $-----$ if at least one of the basic variable is zero.

(b) Let there exist a BFS (Basic Feasible Solution) to the given LPP. If for an entering variable; $z_j - c_j$ most negative and the corresponding scalars $\alpha_{ij} \leq 0$ in the entire column (consider the case of maximization) then it has $-----$ solution.

PART II $(4 \times 5 = 20)$

1. Let linear mapping $T : \mathbb{R}^3 \to \mathbb{R}^4$ be defined by

$$T(x, y, z) = (x + 2y - z, y + z, x + y - 2z, -x + 6z).$$

Find the basis and dimension of Range of T and Kernel of T?

2. Solve the following:

$(3 + 2 = 5)$

(a) Let the linear mapping $T : \mathbb{R}^2 \to \mathbb{R}^3$ be defined by $T(x, y) = (2x + y, y - x, 3x + 4y)$. Find a matrix for T with respect to the ordered bases given by

$$B_1 = \{(1, -1), (0, 1)\} \text{ and } B_2 = \{(1, 1, 0), (0, 1, 1), (1, 0, 1)\}.$$

(b) Let $A = \begin{bmatrix} 1 & 2 \\ -1 & -5 \end{bmatrix}$. Find the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ corresponding to A by using the standard basis for $\mathbb{R}^2$.

3. Find the eigenvalues and the corresponding eigenvectors of the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$.

4. Use simplex method to solve the following linear programming problem. Is the solution unique? Why or why not? If not, give two different basic optimal solutions.

Maximize $Z = 4x_1 + 10x_2$

subjected to constraints

$$2x_1 + x_2 \leq 50; \quad 2x_1 + 5x_2 \leq 100; \quad x_1 + 3x_2 \leq 90; \quad x_1, x_2 \geq 0.$$