DEPARTMENT OF MATHEMATICS, NIT CALICUT

MA6003D MATHEMATICAL METHODS FOR POWER ENGINEERING

Deadline: November 30, 2022 Assignment - 02 Maximum Marks: 05

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1. Find an optimal solution to the following linear program using (a) simplex algorithm (b) algebraic method.

$$\text{Min. } Z = x_1 + 2x_2 + 3x_3, \text{ s.t. } x_1 + 3x_2 + 6x_3 = 6, x_1, x_2, x_3 \geq 0.$$

2. Two consecutive simplex tableaus of a LPP are

B.V.	x_1	x_2	x_3	x_4	x_5	X_B
$Z-$ row	A	-1	3	0	0	
x_4	B	C	D	1	0	6
x_5	-1	2	E	0	1	1

B.V.	x_1	x_2	x_3	x_4	x_5	X_B
$Z-$ row	0	-4	J	K	0	
x_1	G	2/3	2/3	1/3	0	F
x_5	H	8/3	-1/3	1/3	1	3

Find the values of A to K?

3. Solve the following system of equations using both Big-M method and two phase method:

   (a) Min. $Z = 3x_1 + 5x_2$, s.t. $x_1 + 3x_2 \geq 3$; $x_1 + x_2 \geq 2 x_1, x_2 \geq 0$.

   (b) Max $Z = 4x_2 - 3x_1$, s.t. $x_1 - x_2 \geq 2; 2x_1 - x_2 \geq 2, x_1, x_2 \geq 0$.

4. Write the duals of the following problems:

   (a) Min.$Z = x_1 - 2x_2 + 4x_3 - 3x_4$, s.t. $x_1 + x_2 - 3x_3 + x_4 = 9$; $3x_1 + 5x_2 + 2x_3 - 7x_4 \leq 5$, $x_1 - 3x_2 + 5x_4 \geq 8, x_1, x_2, x_3, x_4 \geq 0$.

   (b) Min$Z = 2x_1 + x_2 + x_3$, s.t. $x_1 + x_2 - x_3 \geq 1; -2x_1 + x_3 \leq 0, x_1 - x_2 + x_3 = 2, x_1 \geq 0, x_2 \leq 0, x_3$ is unrestricted in sign.

   (c) Min$Z = 6x_1 + 3x_2$, s.t. $6x_1 - 3x_2 + x_3 \geq 2; 3x_1 + 4x_2 + x_3 \geq 5, x_1, x_2, x_3 \geq 0$.

   (d) Max$Z = x_1 + x_2$, s.t. $2x_1 + x_2 = 5; 3x_1 - x_2 = 6, x_1, x_2$ is unrestricted.

5. Describe the dual simplex method. Using it solve:

   (a) Min.$Z = 2x_1 + x_2$, s.t. $3x_1 + x_2 \geq 3$; $4x_1 + 3x_2 \geq 6$, $x_1 + 2x_2 \leq 3, x_1, x_2 \geq 0$.

(b) Min$Z = x_1 + 4x_2 + 3x_4$, s.t. $x_1 + 2x_2 - x_3 + x_4 \geq 3; -2x_1 + x_2 + 4x_3 + x_4 \geq 3, x_1, x_2, x_3, x_4 \geq 0$.

6. Find maximum and minimum of the following functions (if exist):

   (a) $f(x) = 2x^3 - 15x^2 + 36x + 1$.

   (b) $f(x, y) = \frac{1}{3}x^3 + y^2 + 2xy - 6x - 3y + 4$.

   (c) A square sheet of cardboard with each side 5 inches is to be used to make an open-top box by cutting a small square of cardboard from each of the corners and bending up the sides. What is the side length of the small squares if the box is to have as large a volume as possible?

7. Solve the following problems using Lagrange multiplier method:

   (a) Optimize $9-8x_1-6x_2-4x_3+2x_1^2+2x_2^2+x_3^2+2x_1x_2+2x_1x_3$ subject to $x_1+x_2+2x_3 = 3$.

   (b) Optimize $x_1 + x_2$ subject to $x_1^2 + x_2^2 = 2$.

   (c) Optimize $x_1^2 + x_2^2 + x_3^2$ subject to $3x + y + z = 5$ and $x + y + z = 1$.

8. Solve the following problems using Kuhn-Tucker conditions:

   (a) Minimize $(x_1 - 2)^2 + (x_2 - 1)^2$ subject to $x_1 + x_2 \leq 2$ and $x_1^2 \leq x_2$.

   (b) Maximize $-(x_1 - 5)^2 - (x_2 - 5)^2$ subject to $x_1^2 + x_2 \leq 9$ and $x_1, x_2 \geq 0$.

   (c) Minimize $(x_1 - 4)^2 + (x_2 - 4)^2$ subject to $2x_1 + 3x_2 \geq 6$, $12 - 3x_1 - 2x_2 \geq 0$, and $x_1, x_2 \geq 0$.

9. Find the interval of uncertainty for the following optimization problems using Fibonacci search and golden section search method: Minimize $f(x) = x(x - 5)$ in the interval $[1, 4]$ with reduction ratio less than 0.05.

10. A part-time graduate student in engineering is enrolled in a four-unit mathematics course and a three-unit design course. Since the student has to work for 20 hours a week at a local software company, he can spend a maximum of 40 hours a week to study outside the class. It is known from students who took the courses previously that the numerical grade (g) in each course is related to the study time spent outside the class as $g_m = t_m/6$ and $g_d = t_d/5$, where g indicates the numerical grade ($g = 4$ for A, 3 for B, 2 for C, 1 for D, and 0 for F), t represents the time spent in hours per week to study outside the class, and the subscripts m and d denote the courses, mathematics and design, respectively. The student enjoys design more than mathematics and hence would like to spend at least 75 minutes to study for design for every 60 minutes he spends to study mathematics. Also, as far as possible, the student does not want to spend more time on any course beyond the time required to earn a grade of A on that course. The student wishes to maximize his grade point P, given by $P = 4g_m + 3g_d$, by suitably distributing his study time. Formulate the problem as an linear programming problem.