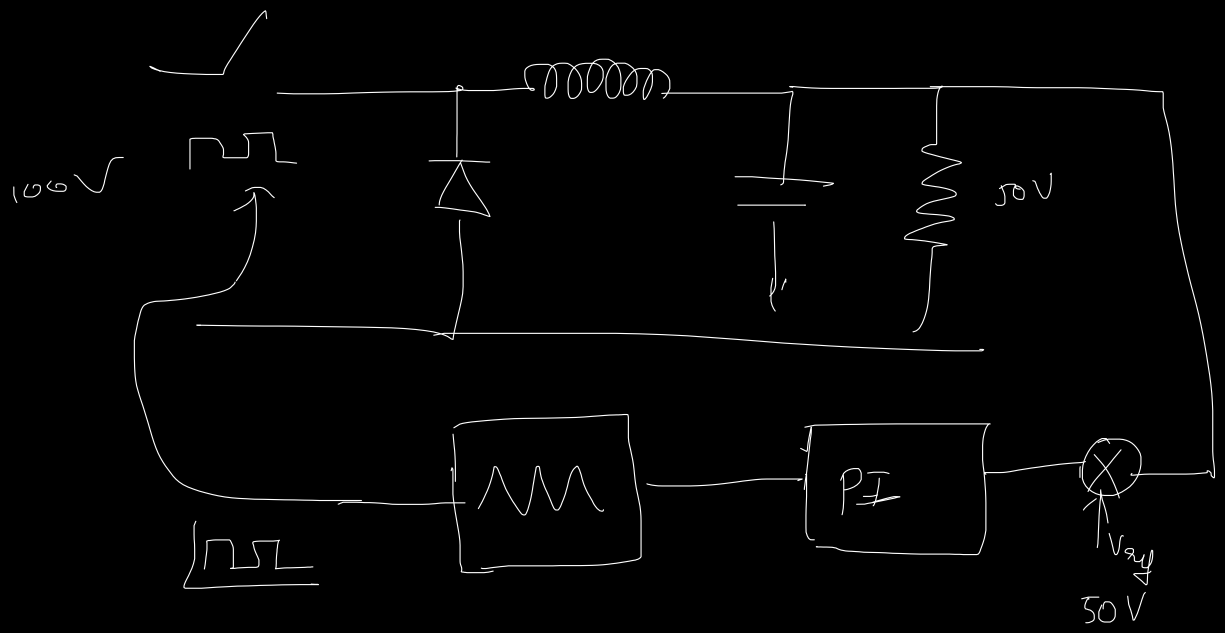
DIGITAL CONTROL OF POWER ELECTRONIC CIRCUITS

C1 - Slot

F28379D
TI Launch Pad

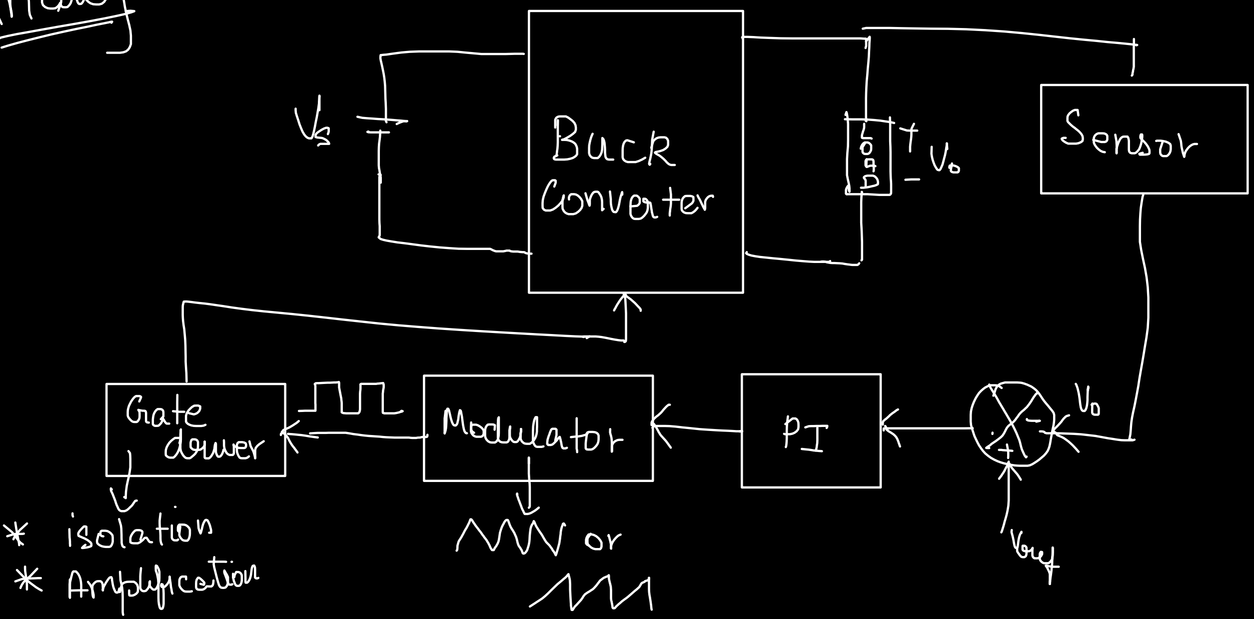
Tentative

End Sem Theory	30
End Sem Lab	20
Midsem Theory	20
Lab session	20
Theory Assignment	10



One class missed

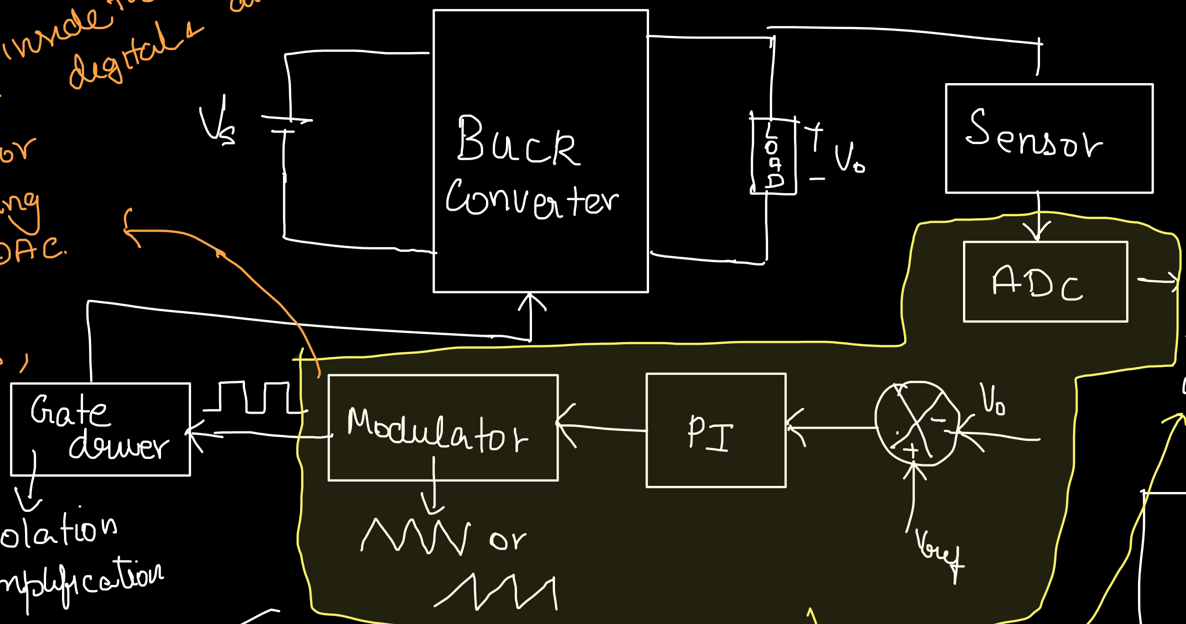
Analog



Digital

inside the processor digital & discrete

Modulator is behaving as an DAC. it gives continuous, digital signal



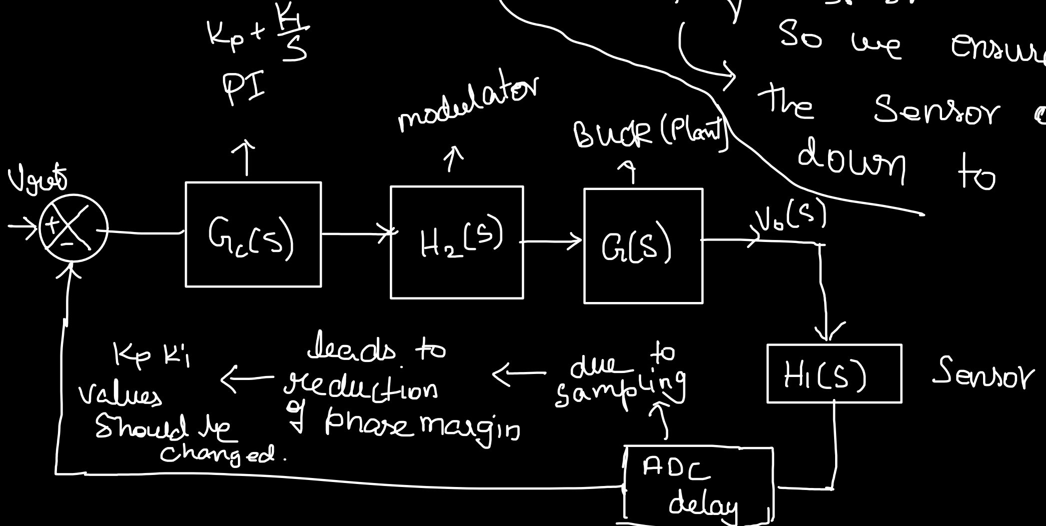
for DSP It will be integrated within the chip

FPGA
↓
we need to add all those peripherals
↓
the code is hardwired

DSP ← Can take i/p of 0-3.3V & give o/p of 0-3.33V

So we ensure that the Sensor o/p is scaled down to 3.3V.
↓
Will this affect the values of control loop. because we decide to design based on

eq Control loop (Approx)



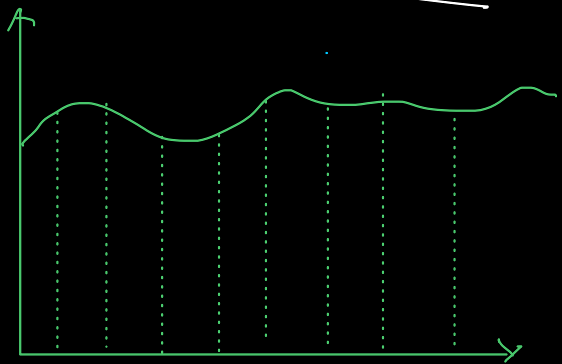
ADC → Sampling
↳ values @ diff time
→ Quantisation
→ Amplitude Quant } rounded to the nearest level
↓
depends on resolution
→ Time Quant

introduces non linear effects

∴

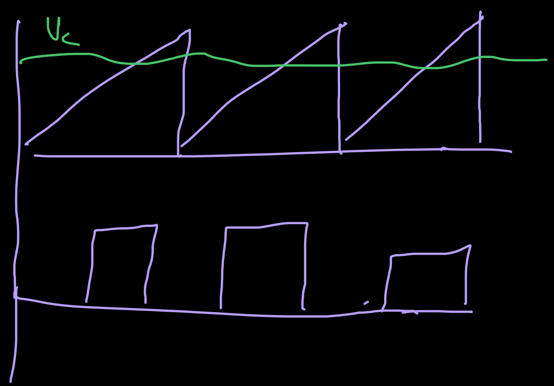
We can just scale it up in the code to the designed values.

buck op voltage before step down to 3.3 V.
(back to buck voltage from 3.3V)



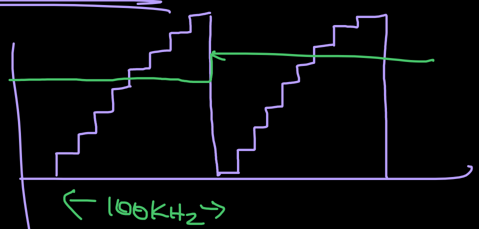
We have to consider all the effects of Digital Control (ADC, Sensors ---etc) while designing Control loop.

Analog PWM

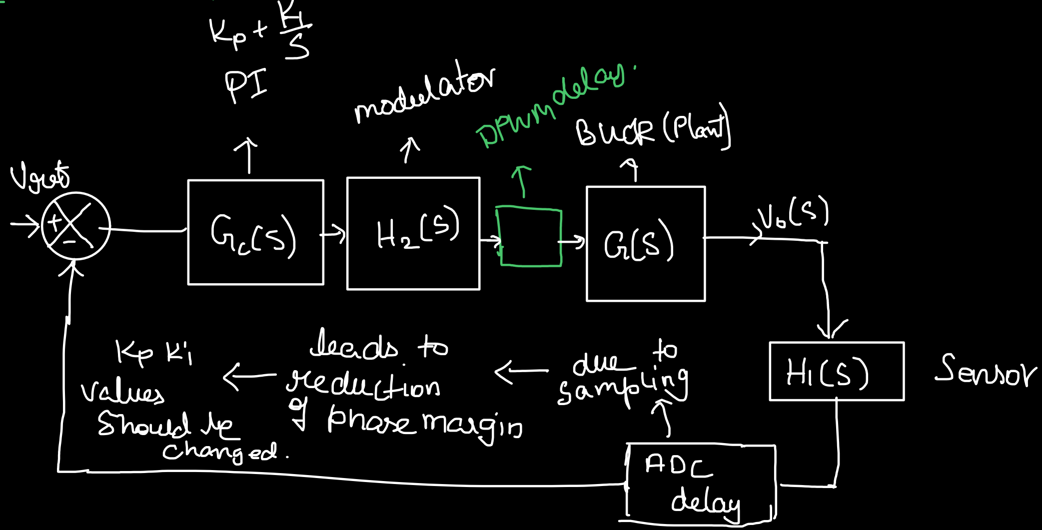


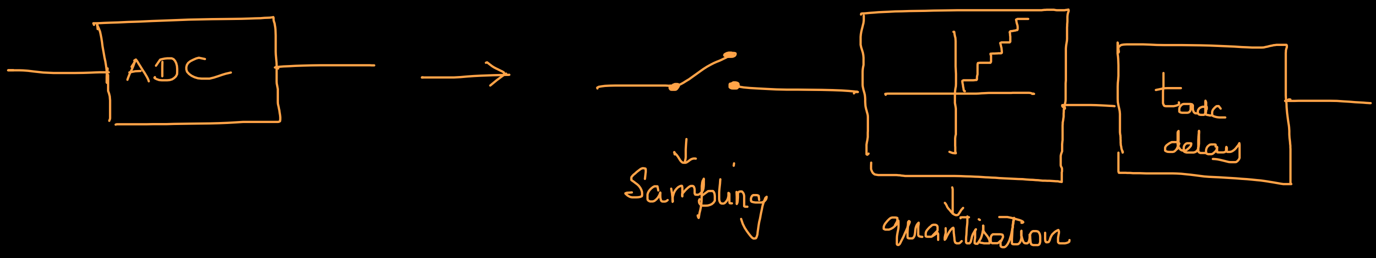
here the variation in V_c is reflected instantenously, So they are accounted

Digital PWM (essentially a counter)



Since 100KHz, the disturbance is only updated once in $\frac{1}{100K}$ s, so there is another delay in the control loop.



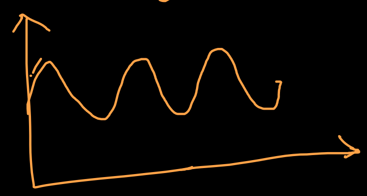


Switching Frequency choice

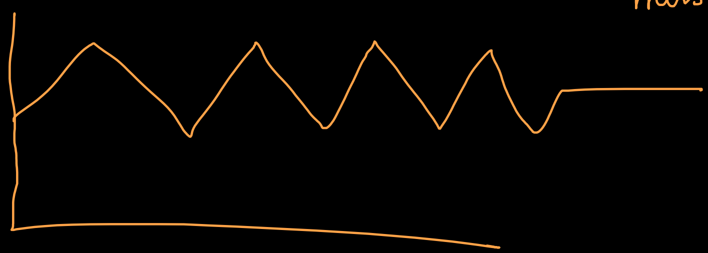
- the instance of sampling
- no of samples per cycle

} matters

only if we sample @ peak we miss peak.

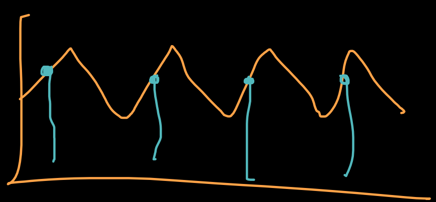


The buck converter has output ripple.



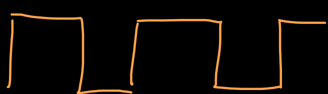
we need to eliminate the ripple because it will affect the control loop design

we can eliminate by sample @ f_s so that we get a DC value



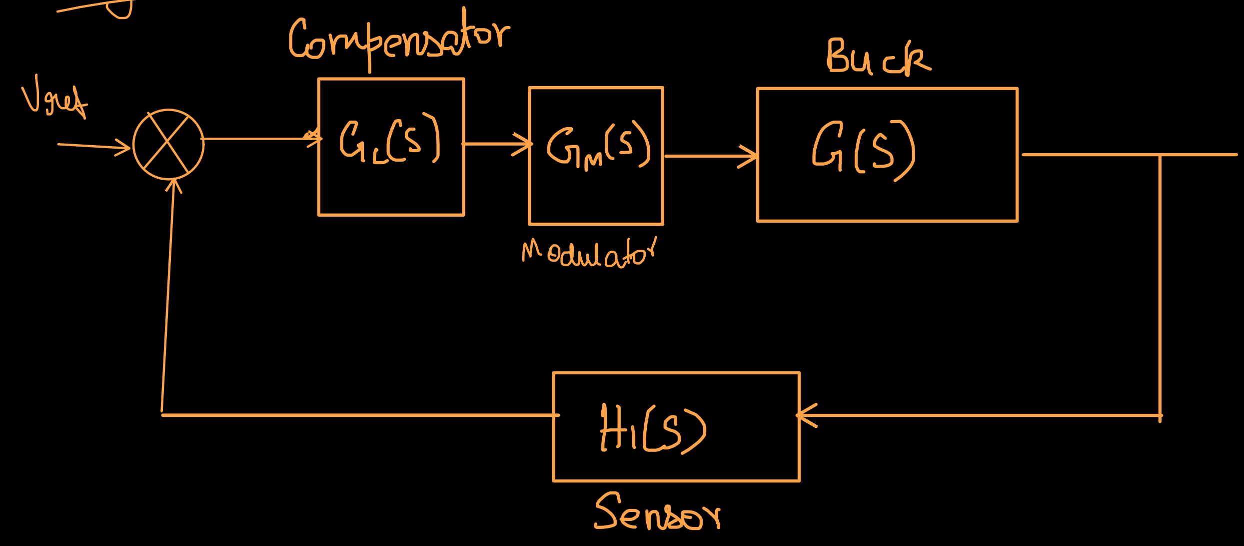
But to get the DC value we need to sample at the midpoint of the signals. ie we need to sample @ midpoint of rising or falling slope. But finding midpoint is difficult.

But we know duty ratio



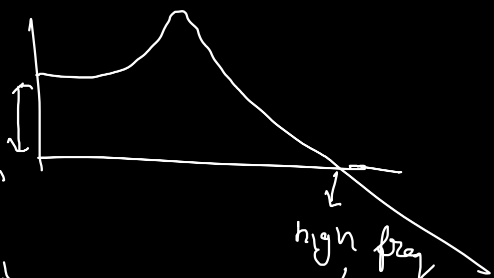
So we can sample @ midpoint of on time or off time which will correspond to the mid of risetime.

Analog Control loop



Uncompensated loop gain = $G(S)\,G_M(S)\,H_1(S)$

Draw bode plot →



low DC gain → allows DC signal. So we need to increase

high freq → need to reduce because to reduce switching ripple.

Compensated loop gain $= G_c(S)\,G(S)\,G_M(S)\,H_1(S)$

high DC gain ←

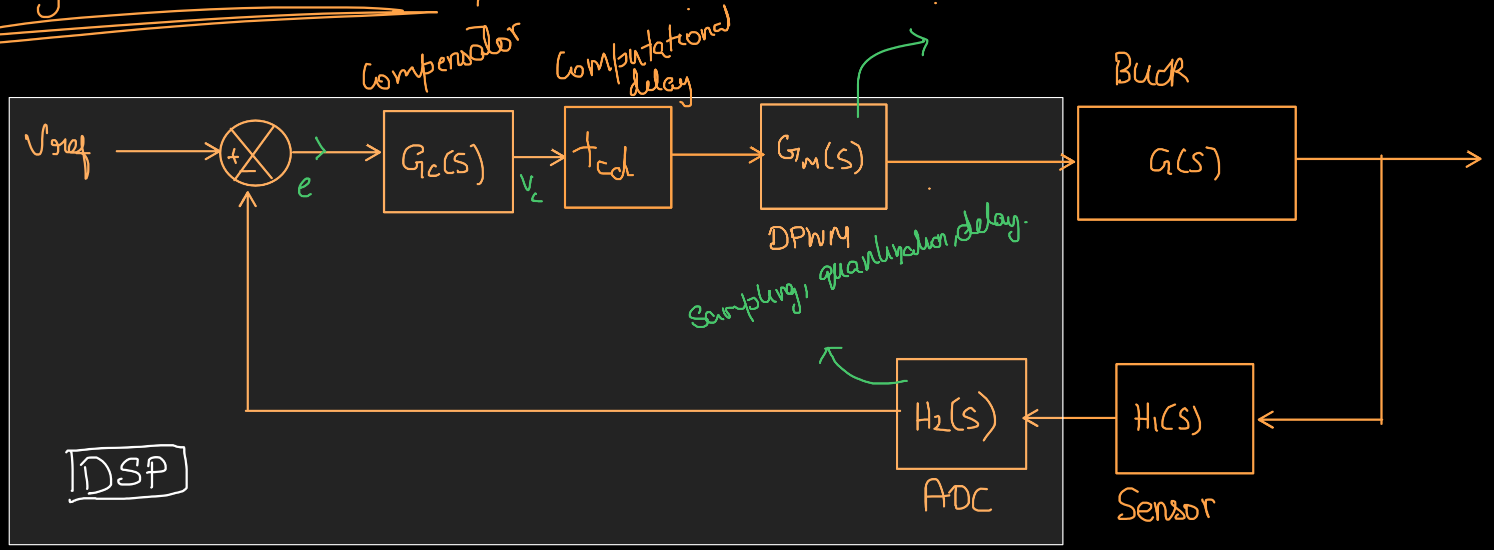
generally $\frac{1}{10}^{th}$ of f_s

low freq ⇒ less switching ripple.

We need to to get ↑ this desired shape

So our $G_c(S)$ should be such that we need to get the desired shape.

Digital Control loop

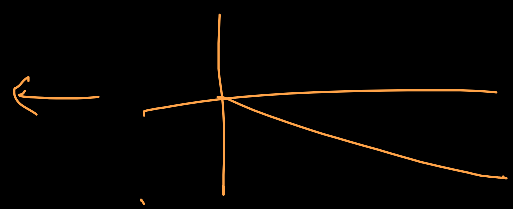


Value of $G_c(s)$ found in analog has to be redesigned because new components

$t_{cd} \rightarrow$ delay $\Rightarrow$ can be represented as e^{-st}

$\downarrow$

phase plot has a decreasing slope



So @ higher frequencies phase margin reduces $\rightarrow$ might be in verge of instability

$\rightarrow$ So when we directly apply analog design the o/p will have sustained oscillations or even shootup.

So changes with time $\uparrow$ due to switching $\hookrightarrow$ 2 configs exist

PEC $\rightarrow$ Non Linear time Variant System

How come we use TF?

$\hookrightarrow$ we can linearise the system

we can make it time invariant by writing 2 sets of equations (for on & off) and average it $\rightarrow$ gives non linear time invariant $\}$ then linearise

we can take i/p o/p is around a small band then, we can assume within that band, it is linear to implement the $G_c(s)$ in code

we need TF to be in time domain

generaly it will look like

$$V_c[n] = V_c[n-1] + a\, e[n] + b\, e[n-1]$$

So we use z transform, and we convert

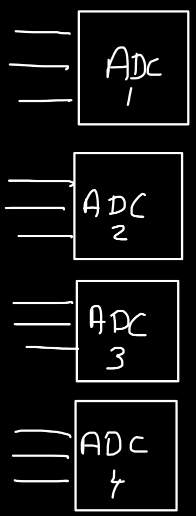
to Z-domain and get difference equation.

---

difference between fixed point vs floating point representation

Fixed no of digits after decimal

↳ Variable no of digits after decimal



4 ADC

↳ 4 Channels each

So we can Sample 16 Signals.

but only 4 Signals at once can be sampled

MODULE – II

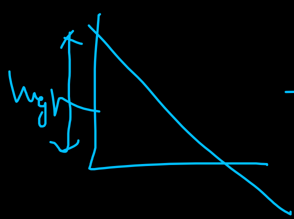
* System Variable → Controlled Variable (duty cycle, Switching Sequence)
* Control Variable → Controls
  - ↳ usually one variable

Consider DC-DC Converter
↳ freq components of V_o → DC
→ f_s
→ multiples of f_s

when there is a small disturbance there will also be a low freq component for a while
↳ The time it takes to Settle to SS is called Settling time

Our Consideration

* Stability → poles → LH Side or +ve GM & PM
* Steady State error →



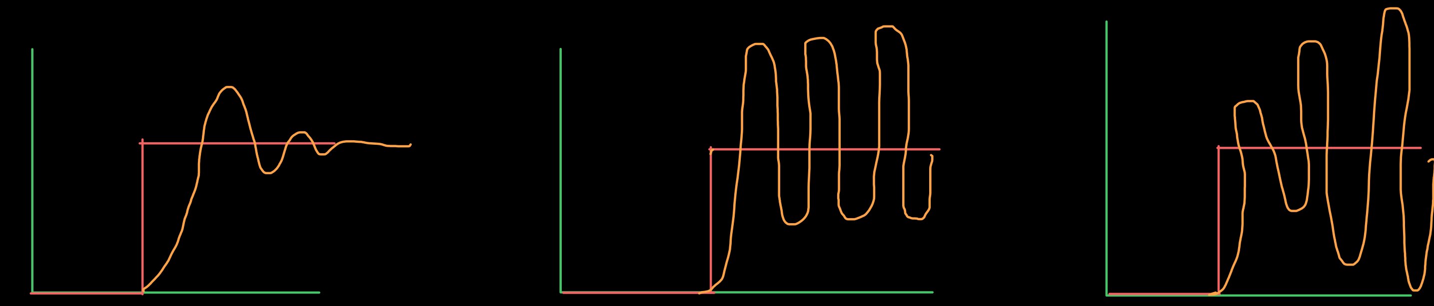
→ allows only DC to pass through
↳ we use Compensator

* Transient Response →

Standard test inputs → They are extreme condition to test our converters so that it works properly.

Usually only Step is used to test

There may be 3 possible outcomes



(a) Stable (b) Sustained oscillation (c) Unstable

* In root locus, poles on RHS will result in e^{t} (exponentially increasing) so it will cause unstability. So when poles are in LHS e^{-t} (exponentially decaying) so it will die down so it is stable.

* In bode, we saw gain & phase margin is positive for stability.

  * we have a -ve feedback system
  * We calculate gm & pm @ phase & gain crossover freq.
  * If it goes below, i.e magnitude becomes -ve, so the -ve feedback becomes +ve so it will amplify the error.

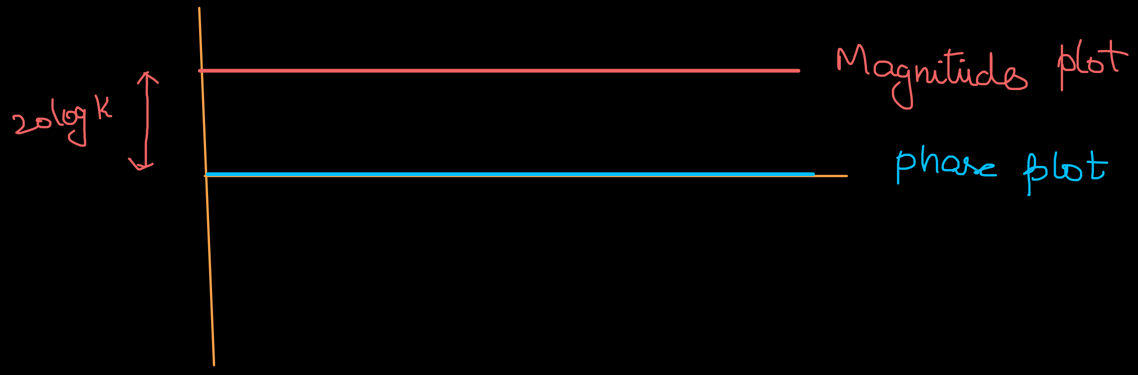
Systems that have right half zeros are called non-minimum phase system.

eg boost converter
buck boost converter

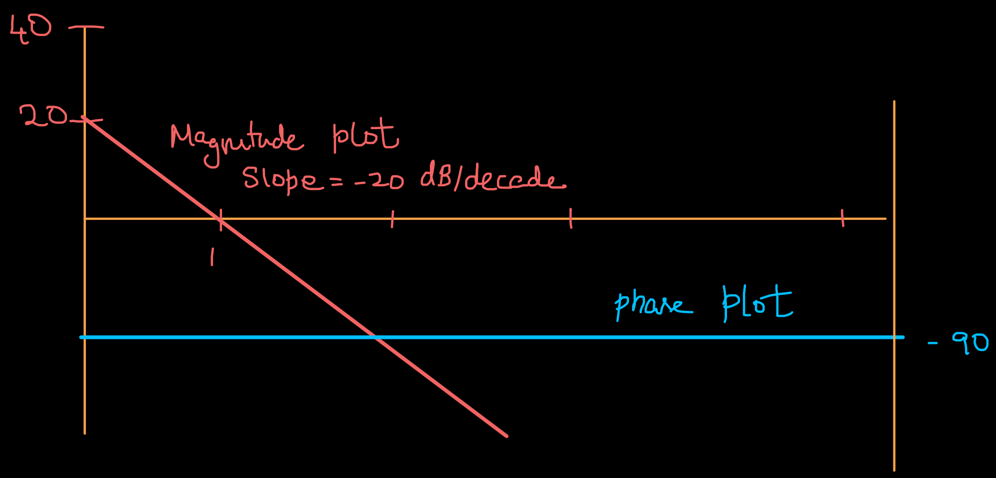
In boost converter first energy is stored in inductor, then after that transfered to load

Bode plot

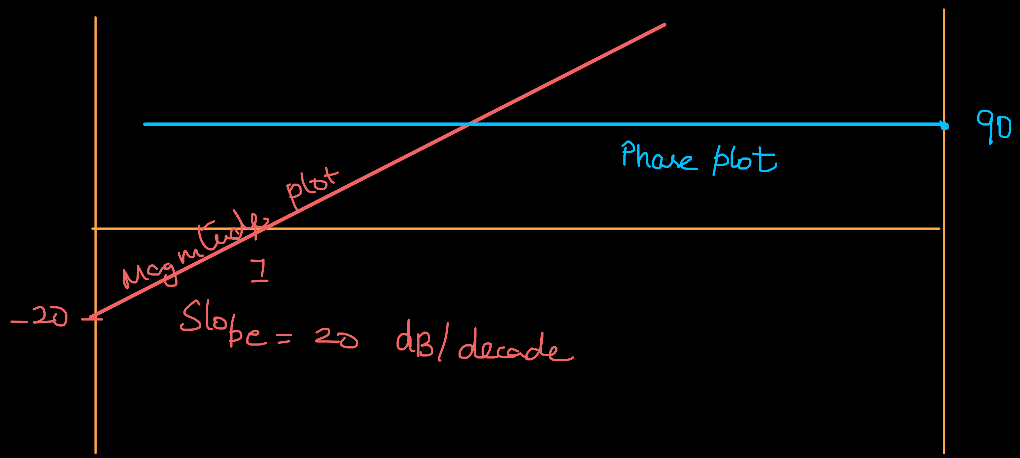
① K



② $\frac{1}{s}$

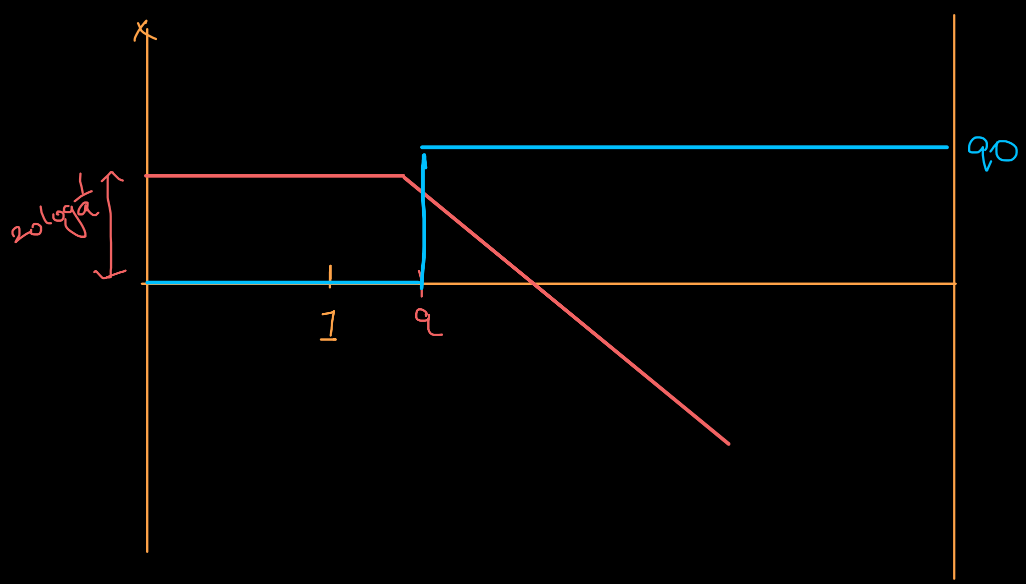


③ s

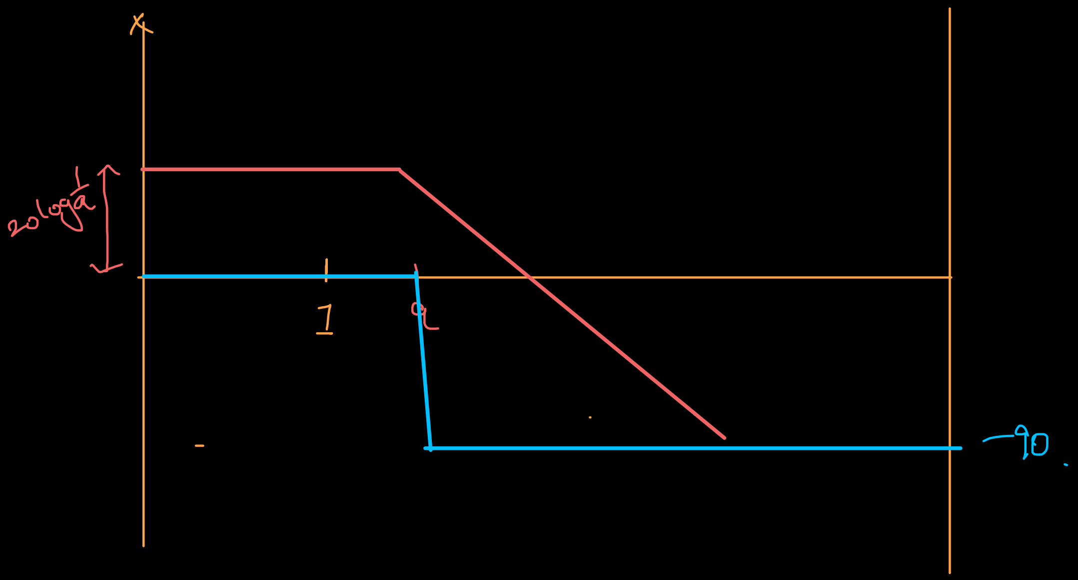


$\frac{1}{s+a}$

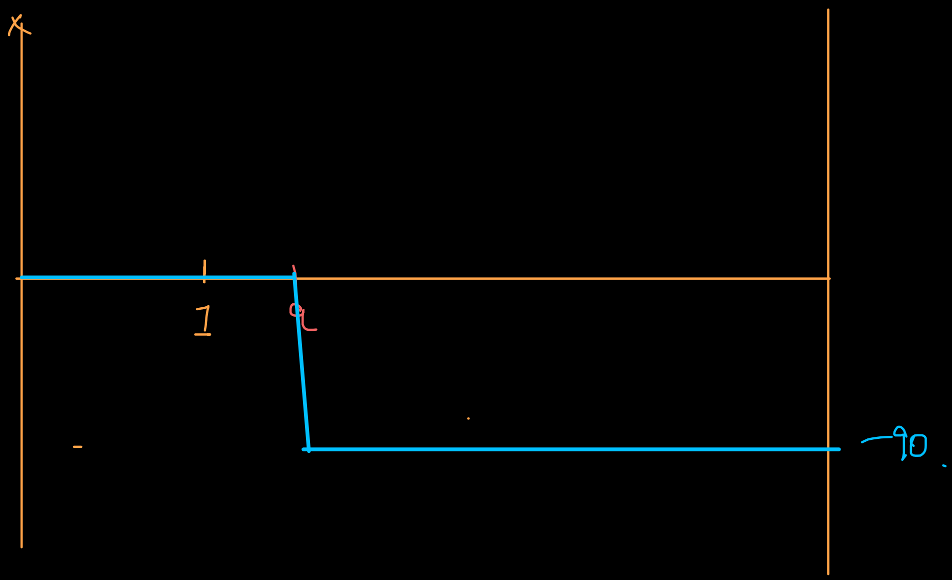
$= \frac{1}{a\left(1+\frac{s}{a}\right)}$

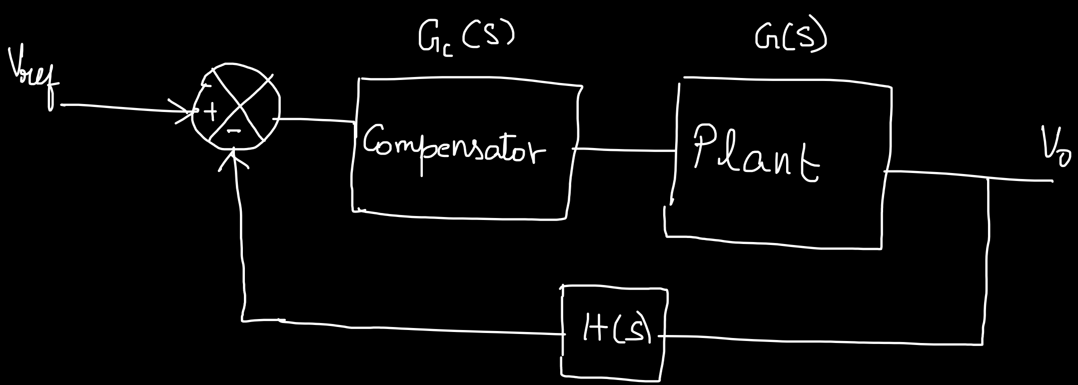


(5) $\frac{1}{S-a}$



(6) $S+a$



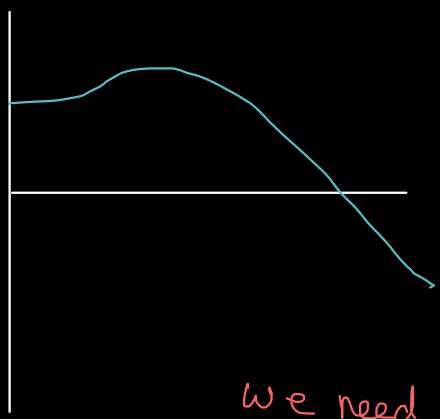


uncompensated loop Gain $= G(S)\,H(S) \qquad = G_1(S)$

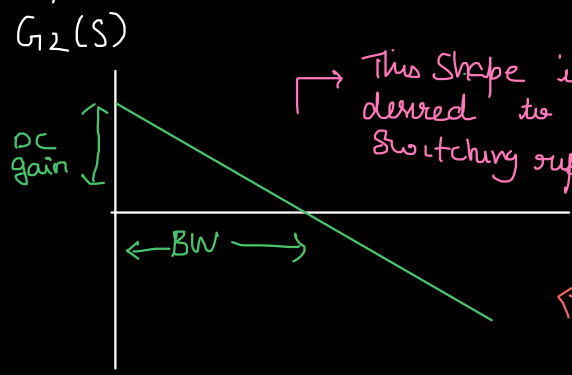
Compensated loop Gain $= G_c(S)\,G(S)\,H(S) = G_2(S)$

Say,

$G_1(S)$



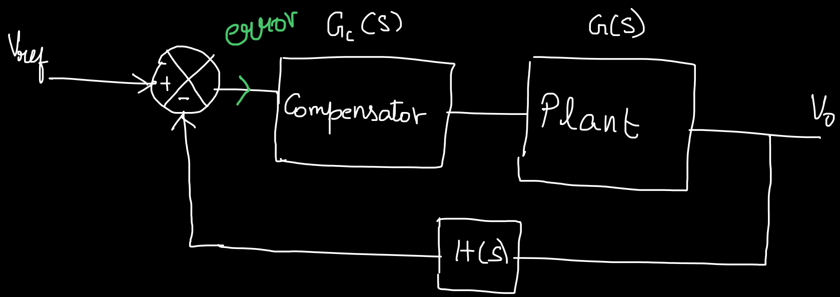
desired bode plot



This shape is desired to eliminate switching ripple & noise

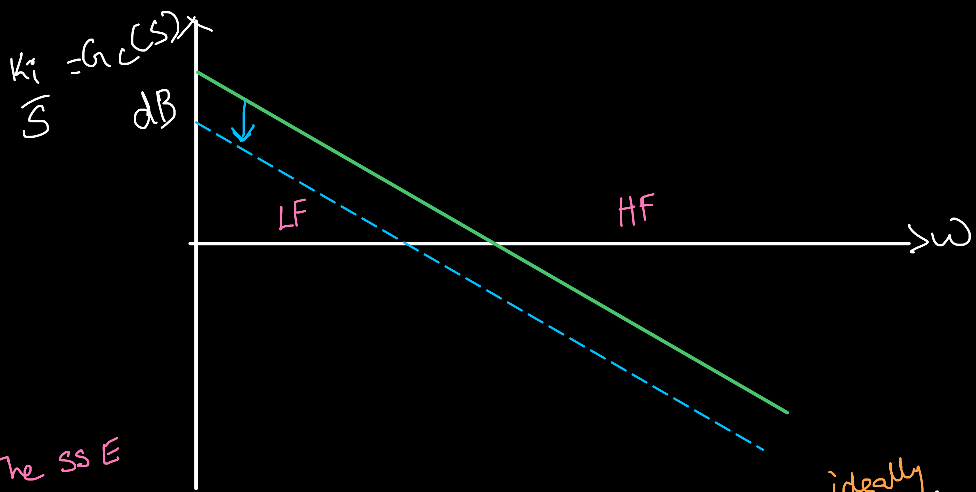
We need to design $G_c(S)$ such that the bode plot becomes like that

We can place poles and zeros to get the desired shape.

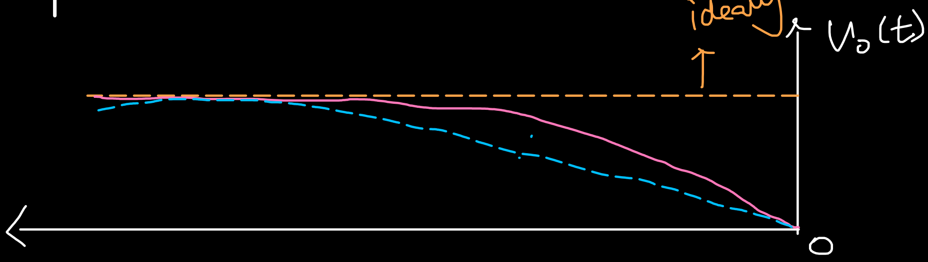


Ideally we need infinite gain to make the error 0.

So that more effort is put in the corrective action



time response to unit step input



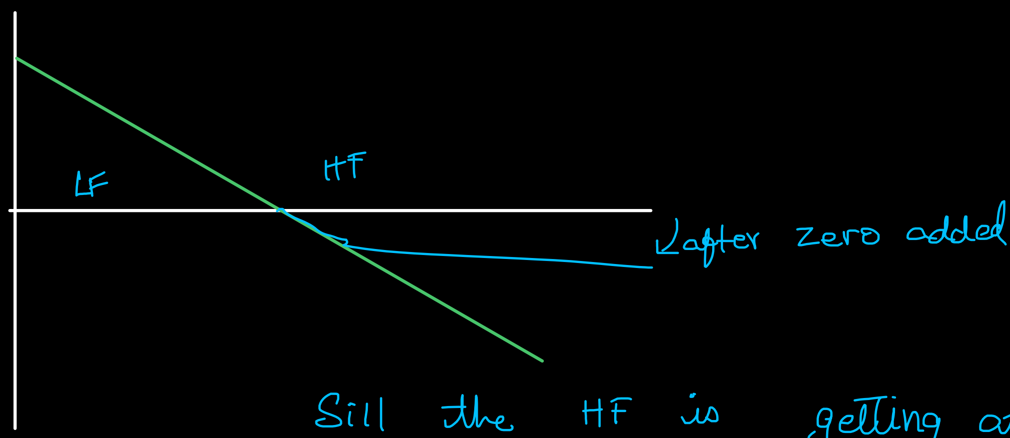
The ss E is 0 because we have high dc gain

when we shift it down the response becomes slower

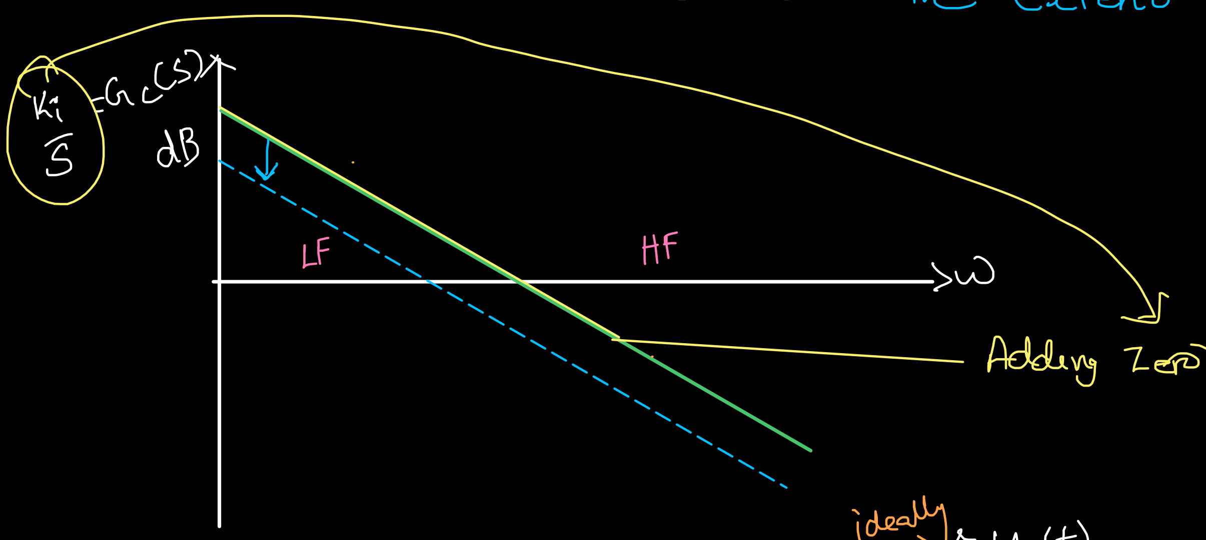
The blue curve is filtering more frequency. The DC gain is anyway ∞ @ 0. So it affects the BW.

The BW is low so response has become slower.

So to improve the transient response, we can add a zero in the High frequency region.



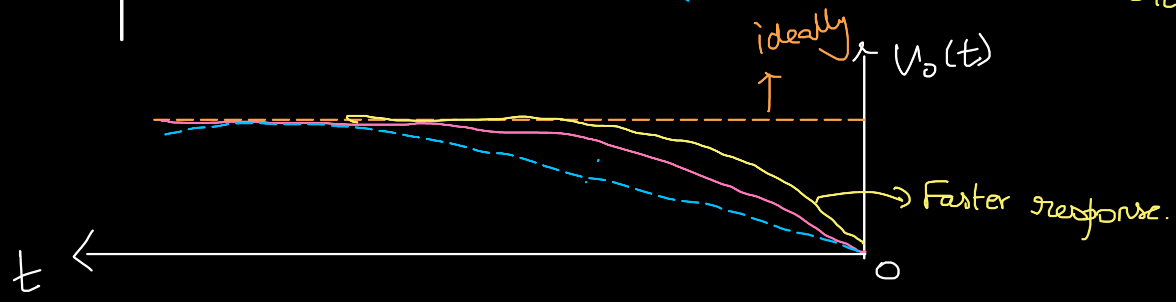
Still the HF is getting attenuated but not to the extent of before.



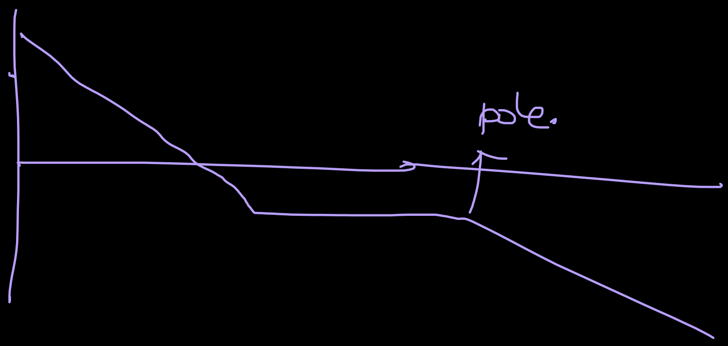
$$G_c(S) = K_p + \frac{K_i}{S}$$

$$= \frac{SK_p + K}{S}$$

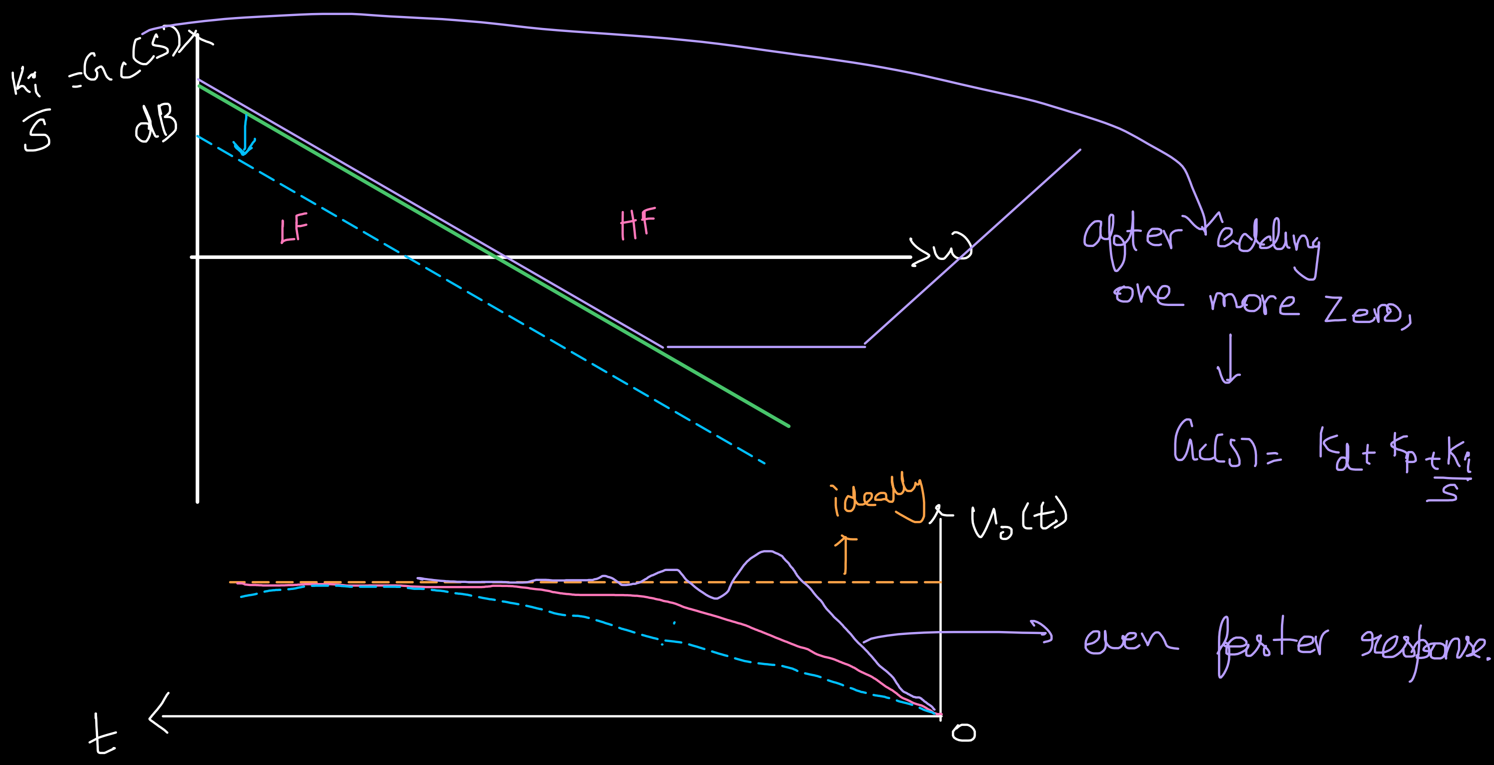
new Zero



To remove high frequency noise, we can put a pole somewhere further



To get even faster response, we can add another zero.



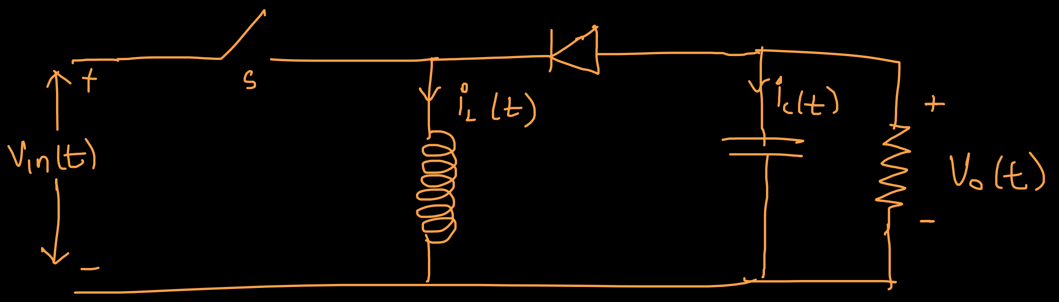
So in Conclusion,

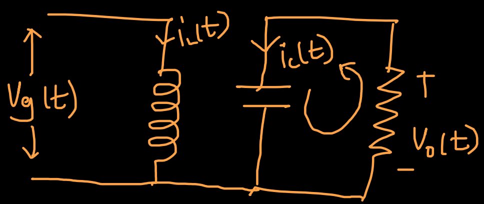
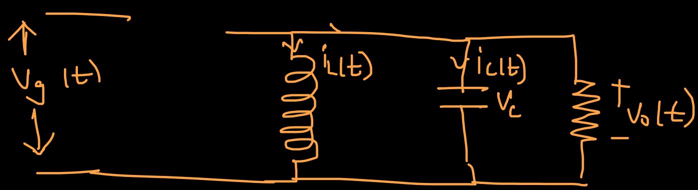
* SS response is affected by DC gain
* TS response is affected by B.W.

* P, PI, ... lag, lead .- are all linear controllers.

To track fast changing waves like high frequency Sine we need non linear controllers.

Modelling of Converter



DT_s	$(1-D)T_s$
	

$$V_g(t) = L\frac{di_L(t)}{dt} \quad — (1)$$

$$i_c(t) = -\frac{V_o(t)}{R}$$

$$C\frac{dV_o}{dt} = -\frac{V_o(t)}{R} \quad — (2)$$

$$V_o(t) = \frac{L\,di_L(t)}{dt} \quad — (3)$$

$$\frac{C\,dV_o}{dt} = \frac{-V_o}{R} - i_L(t) \quad — (4)$$

Now we average them to reduce to 1 set of eqn.

$$\left[(1) \times DT_s + (2) \times (1-D)T_s \right] \div T_s$$

$$\frac{V_g(t)\,DT_s + V_o(t)(1-D)T_s}{T_s} = DT_s\,\frac{L\,di_L(t)}{dt} + (1-D)T_s\,\frac{L\,di_L(t)}{dt}$$

$D \to$ also function of time

$D \to D(t)$

$$d(t)\, V_g(t) + (1-d(t))\, V_o(t) = d(t)\, L\frac{di_L(t)}{dt} + (1-d(t))\, L\frac{di_L(t)}{dt}$$

$$d(t)\, V_g(t) + (1-d(t))\, V_o(t) = L\frac{di_L(t)}{dt} \quad \text{— (5)}$$

lly

$$\left[(2)\, d(t)\, T_s + (4)\, (1-d(t))\, T_s\right] \div T_s$$

$$d(t)\, C\frac{dv_o(t)}{dt} + [1-d(t)]\, C\frac{dv_o(t)}{dt} = -\frac{V_o(t)}{R}\, d(t) - (1-d(t))\frac{V_o(t)}{R} - (1-d(t))\, i_L(t)$$

$$C\frac{dV_o(t)}{dt} = \frac{-V_o(t)}{R} - (1-d(t))\, i_L(t) \quad \text{— (6)}$$

$\underbrace{(5)\ \&\ (6)}_{\text{Called as "Time averaged model"}}$ $\Rightarrow$ has averaged the dynamics of 2 modes of ckt

So we have made it time invariant in one switching cycle

But still is non-linear

↓ done by linearization.

* Consider a small operating region.
* We can assume it is linear in a small region around the operating point.

So the controller will be able to handle only small variations within the eqn we will derive will become invalid.

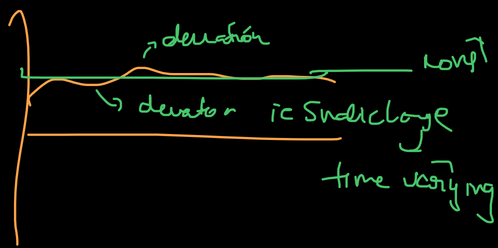
This is called 'small signal model'

$$d(t) = \underset{\text{const}}{D} + \overset{\text{small chang } \Delta d}{\hat{d}}$$

$$v_g(t) = V_g + \hat{v}_g$$

$$v_o(t) = V_o + \hat{v}_o$$

$$i_L(t) = I_L + \hat{i}_L$$



$$\therefore (5) \Rightarrow L\frac{d}{dt}(I_L + \hat{i}_L) = (D+\hat{d})(V_g + \hat{v}_g) + (1 - D - \hat{d})(V_o + \hat{v}_o)$$

$$L\frac{dI_L}{dt} + L\frac{d\hat{i}_L}{dt} = DV_g + D\hat{v}_g + \hat{d}V_g + \hat{d}\hat{v}_g + V_o - DV_o - \hat{d}V_o + \hat{v}_o - D\hat{v}_o - \hat{d}\hat{v}_o$$

$$= DV_g + V_o - DV_o + D\hat{v}_g + \hat{d}V_g - \hat{d}V_o - \hat{v}_o D + \hat{v}_o + \hat{d}\hat{v}_g - \hat{d}\hat{v}_o$$

$$\overset{\approx 0}{L\left(\frac{dI_L}{dt}\right)} + \frac{d\hat{i}_L}{dt} = \underbrace{\left[DV_g + (1-D)V_o\right]}_{\text{DC terms}} + \underbrace{\left[D\hat{v}_g + (1-D)\hat{v}_o + (V_g - V_o)\hat{d}\right]}_{\text{1st order ac terms}} + \underbrace{\left[\hat{d}(\hat{v}_g - \hat{v}_o)\right]}_{\text{2nd order ac terms}}$$

2nd order ac terms → Can be neglected because very small.

$$\therefore \quad L\frac{d\hat{i}_L}{dt} = D\hat{v}_g + (1-D)\hat{v}_o + (V_g - V_o)\hat{d} \quad \text{— (7)}$$

$$(6) \Rightarrow \quad C\frac{d}{dt}\left(V_o + \hat{v}_o\right) = -\frac{V_o}{R} - \frac{\hat{v}_o}{R} - (1-D-\hat{d})(I_L + \hat{i}_L)$$

$$C\frac{dV_o}{dt} + C\frac{d\hat{v}_o}{dt} = -\frac{V_o}{R} - \frac{\hat{v}_o}{R} - I_L - \hat{i}_L + DI_L + D\hat{i}_L + \hat{d}I_L + \hat{d}\hat{i}_L$$

$$= \frac{-V_o}{R} - I_L + DI_L - \frac{\hat{v}_o}{R} - \hat{i}_L + D\hat{i}_L + \hat{d}I_L + \hat{d}\hat{i}_L$$

$$\underbrace{C\left(\frac{dV_o}{dt}\right.}_{\text{Amp-Sec balance.}} + \left.\frac{d\hat{v}_o}{dt}\right) = \underbrace{\left[-(1-D)I_L - \frac{V_o}{R}\right]}_{\substack{\text{DC term} \\ = 0}} + \left[-(1-D)\hat{i}_L - \frac{\hat{v}_o}{R} + I_L\hat{d}\right] + \underbrace{\boxed{\hat{d}\,\hat{i}_L}}_{\substack{2^{nd}\text{ order} \\ \text{terms} \\ \text{neglected} \\ \text{because small}}}$$

$$\therefore \quad C\frac{d\hat{v}_o}{dt} = -(1-D)\hat{i}_L - \frac{\hat{v}_o}{R} + I\hat{d} \quad \text{— (8)}$$

(7) & (8) $\rightarrow$ Small Signal forms.

We are using d to control V_o. So we need the transfer function $\left.\dfrac{\hat{v}_o(s)}{\hat{d}(s)}\right|_{v_g(s)=0}$

Laplace (7) & (8)

$$(7) \Rightarrow \quad L\,s\,\hat{I}_L(s) = 0 + (1-D)\,\hat{V}_o(s) + (V_g - V_o)\,\hat{D}(s)$$

$$(1-D)\,\hat{V}_o(s) = L\,s\,\hat{I}_L(s) - (V_g - V_o)\,\hat{D}(s)$$

$$(8) \Rightarrow C s\,\hat{V}_o(s) = (1-D)\,\hat{I}_L(s) - \frac{\hat{V}_o(s)}{R} - I_L\,\hat{D}(s)$$

$$\left(Cs + \frac{1}{R}\right)\hat{V}_o(s) = (1-D)\,\hat{I}_L(s) - I_L\,\hat{D}(s)$$

$$\hat{I}_L(s) = \frac{\left(Cs + \frac{1}{R}\right)\hat{V}_o(s) + I_L\,\hat{D}(s)}{Cs + 1/R}$$

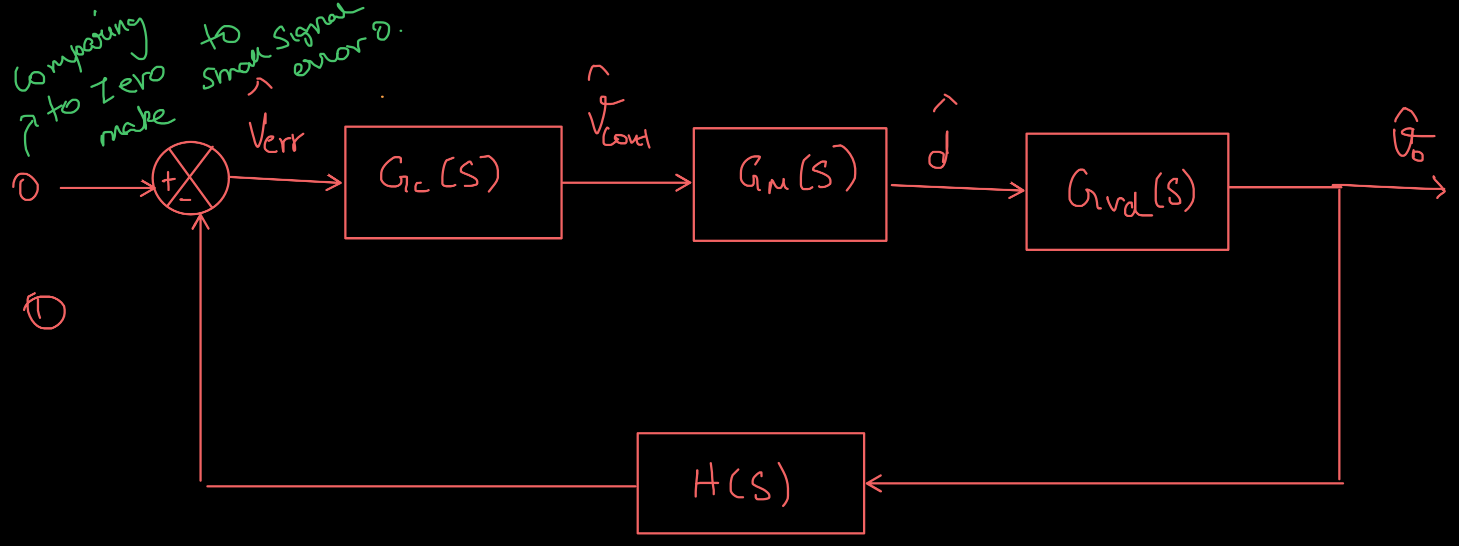
$$=$$

$I_L \rightarrow$ in terms of I_o

$I_o = \frac{V_o}{R}$

$$G_{vd}(s) = \frac{\hat{V}_o(s)}{\hat{d}(s)} = \frac{G_{do}\left(1 - \frac{s}{\omega_z}\right)}{1 + \frac{s}{Q\,\omega_o} + \frac{s^2}{\omega_o^2}}$$

	G_{do}	ω_o	Q	ω_z
Buck	V_o/D	$1/\sqrt{LC}$	$R\sqrt{\frac{C}{L}}$	∞
Boost	$V_o/1-D$	$\frac{1-D}{\sqrt{LC}}$	$(1-D)R\sqrt{\frac{C}{L}}$	$\frac{(1-D)^2 R}{L}$
Buck-Boost	$\frac{V_o}{D(1-D)}$	$\frac{(1-D)}{\sqrt{LC}}$	$(1-D)R\sqrt{\frac{C}{L}}$	$\frac{(1-D)^2 R}{DL}$



There are 2 approaches to closed loop control

① We can take the small signal value. ie deviation from average value. and find $\hat{d}$ which is chand in duty - So we need to add $D + \hat{d}$ and give to converter - And while sensing V_o we need to extract $\hat{v}_o$ from the signal by subtracting with average value.

② We can directly sense the $V_o(t)$ and comput $d(t)$.

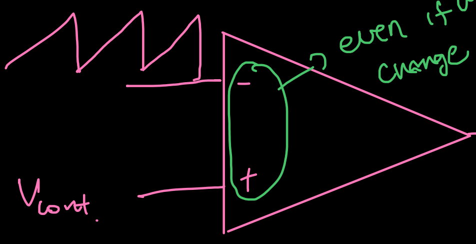
ie) * in ① we are trying to make $\hat{v}_o$ zero

* in ② we are trying to make $v(t)$ equal to average value

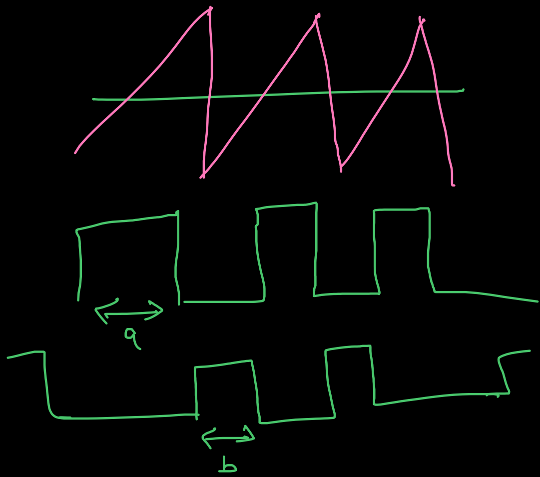
$\hat{V}_o$ is very small.

The time taken by integrator to settle bigger number is more.

So ② will have sluggish response

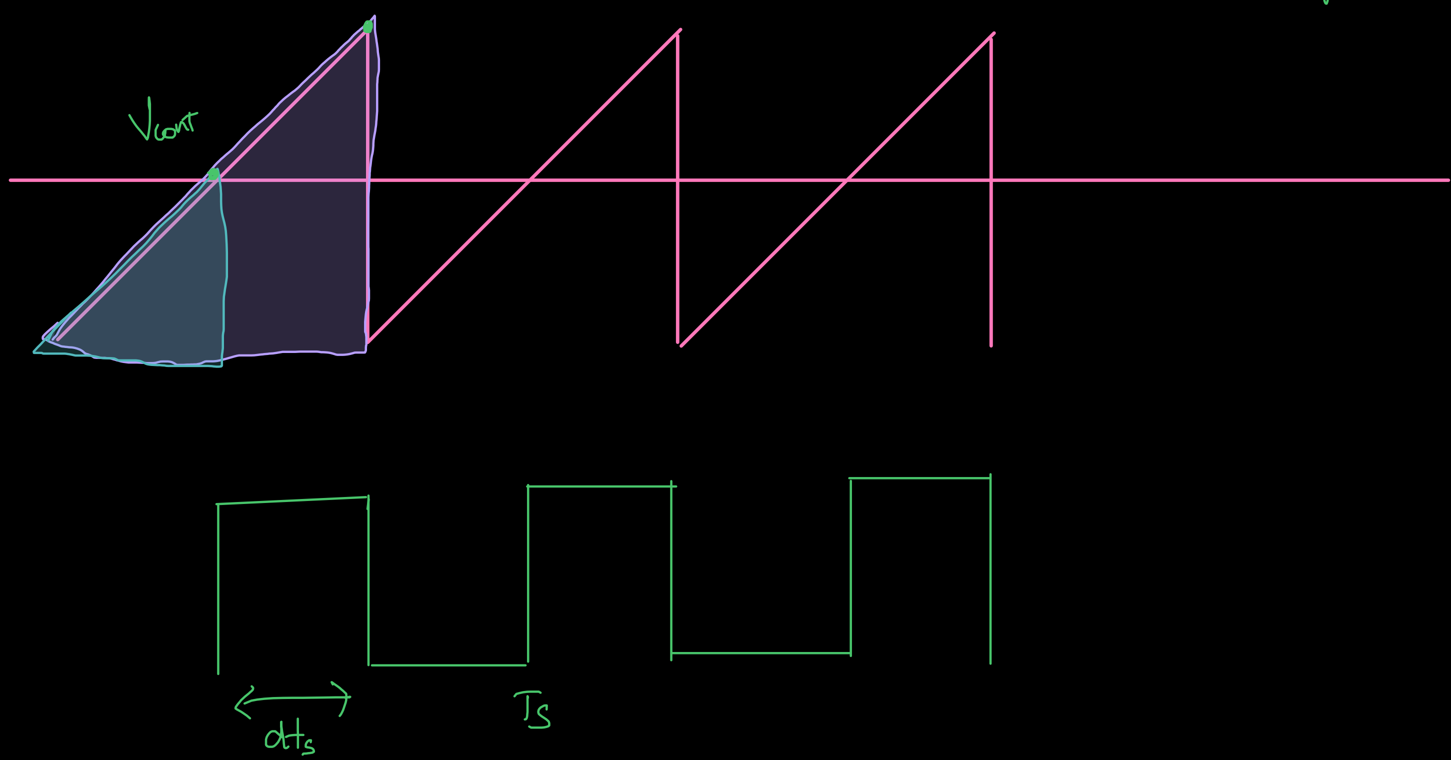


even if we change this the output will eventually settle @ desired op due to the closed loop.



$a \neq b$

but due to closed loop eventually $a \to b$ due to closed loop. feedback.



$$V_m \propto T_s$$

$$V_{cont} \propto d(t) T_s$$

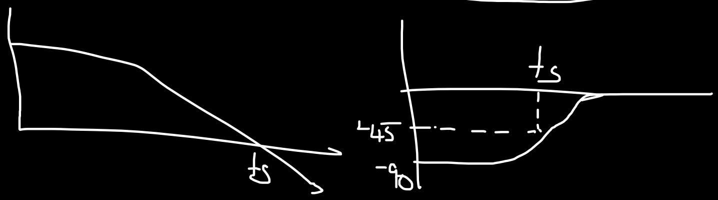
$$\frac{V_m}{V_{cont}} = \frac{T_s}{d(t) T_s}$$

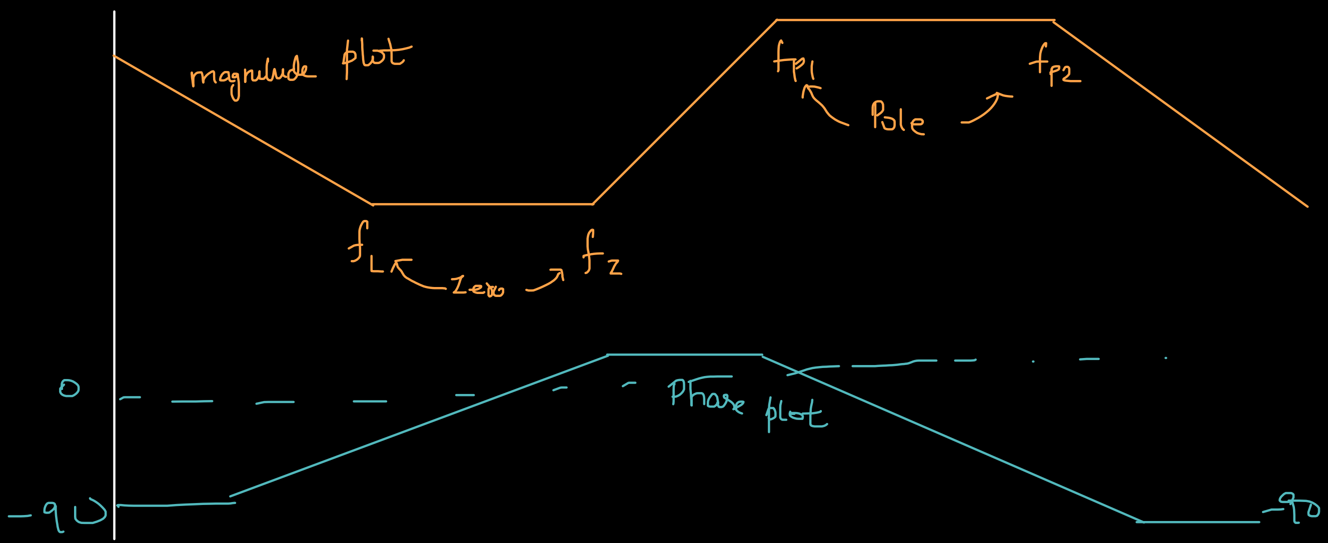
$$d_t(t) = \frac{V_{con}(t)}{V_m}$$

$$\Rightarrow \boxed{\frac{d(t)}{V_{cont}(t)} = \frac{1}{V_m}}$$

Lab - Session 1

1. Lag Compensator $\longrightarrow$ PI
2. Lag lead Compensator $\longrightarrow$ PID





$$G(s) = \frac{G_0\left(1+\frac{\omega_L}{s}\right)\left(1+\frac{s}{\omega_z}\right)}{\left(1+\frac{s}{\omega_{P_1}}\right)\left(1+\frac{s}{\omega_{P_2}}\right)}$$

Buck Converter

$V_g = 28\,V$

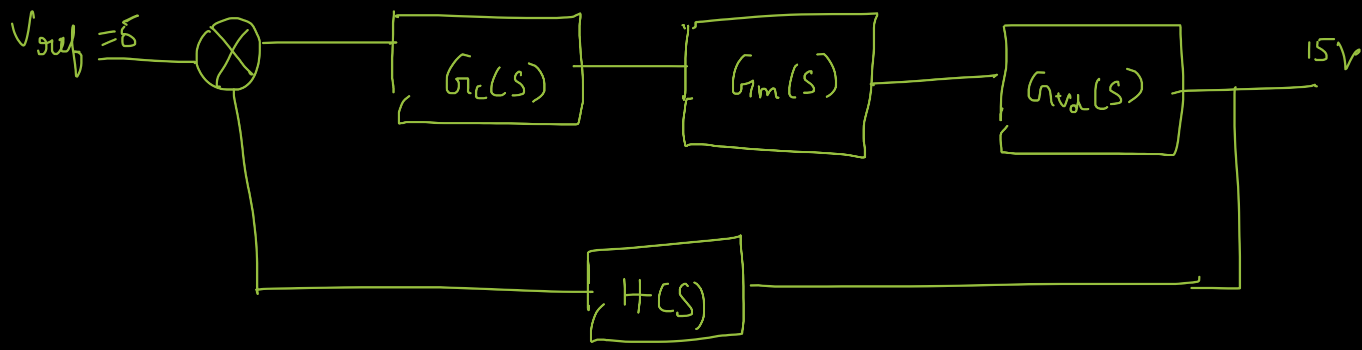
$V_o = 15\,V$

$I_o = 5\,A$

$R = 3\,\Omega$

$V_{ref} = 5\,V$

$V_{in} = 4\,V$



$$H(S) = \frac{5}{15} = \frac{1}{3}$$

$$G_m(S) = \frac{1}{V_m} = \frac{1}{4}$$

$$G_{v_d}(S) = \frac{\hat{V}_o(S)}{\hat{d}(S)} = \frac{G_{do}\left(1 - \frac{S}{\omega_z}\right)}{1 + \frac{S}{Q\omega_o} + \frac{S^2}{\omega_o^2}}$$

$$G_{do} = \frac{V_o}{D} = \frac{15}{0.5357} = 28$$

$$Q = R\sqrt{\frac{C}{L}} = 3\sqrt{\frac{500 \times 10^{-6}}{50 \times 10^{-6}}} = 9.4868$$

$$GV_d(S) = \frac{28\left(1 - \frac{S}{\infty}\right)}{1 + \frac{S}{9.4868(6324.555)} + \frac{S^2}{(6324.555)^2}}$$

$$\omega_o = \frac{1}{\sqrt{LC}} = 6324.555$$

$$= \frac{28}{2.5\times10^{-8}s^2 + 1.6\times10^{-5}s + 1}$$

Uncompensated loop gain $= G_{vd}(s)\, G_m(s)\, H(s)$

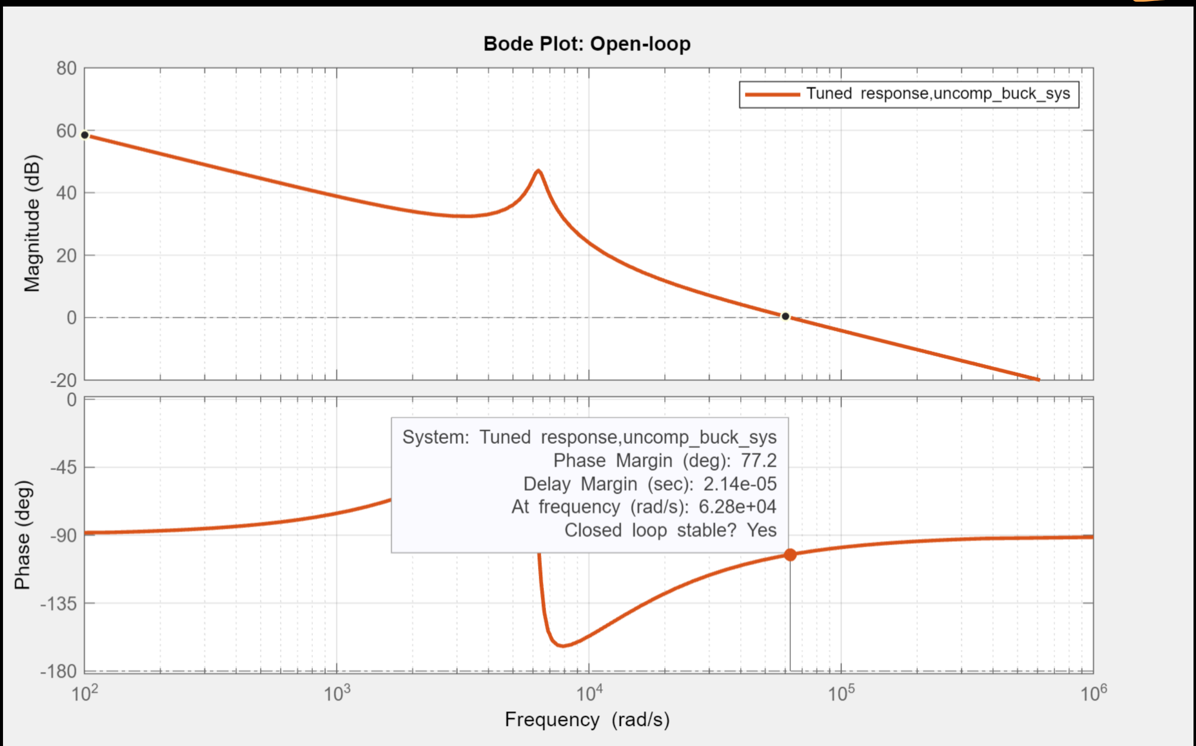
$$= \frac{2.33}{2.5\times10^{-8}s^2 + 1.6\times10^{-5}s + 1}$$

Desired B.W $= 10$ kHz

PM $= 60^\circ$

DC gain $= \infty$

Compensated TF $= T_c(s) = T_u(s)\, \underbrace{G_c(s)}_{}$ $\uparrow$ P.D



	Tuned
Kp	9.7454
Ki	36071.3877
Kd	0.00065823
Tf	n/a

```
PID =

             1
  Kp + Ki * --- + Kd * s
             s

  with Kp = 9.75, Ki = 3.61e+04, Kd = 0.000658
```

Now convert the $G_c(s)$ to $G_c(z)$

- Forward Euler Method
- Backward Euler Method
- Tustin Method

(bilinear Transformation)

$$S = \frac{2}{T_s}\left(\frac{Z-1}{Z+1}\right)$$

$$\therefore G_c(s) = 9.75 + \frac{36071.3877}{s} + 0.00065823\, s$$

c2d → matlab function

$$= 9.75 + \frac{36071.3877}{2(100\times10^{3})\left(\frac{Z-1}{Z+1}\right)} + 0.00065823\; 2(100\times10^{3})\left(\frac{Z-1}{Z+1}\right)$$

$$= 9.75 + 0.18035\left(\frac{z+1}{z-1}\right) + 131.646\,\frac{z-1}{z+1}$$

$$= \frac{(z-1)(z+1) + 0.18035\,(z+1)^2 + 131.646\,(z-1)^2}{(z+1)\,(z-1)}$$

$$= 9.75z^2 - 9.75 + 0.18035z^2 + 0.18035 + 0.36071z + 131.646z^2 + 131.646 - 263.292z$$

~~263.6527~~

$$= 142.57635z^2 + -262.9313 + \frac{120}{}$$

Backward Euler	Forward Euler	Tustin (Bilinear Transformat)
$S = \frac{z-1}{z\,T_{samp}}$	$S = \frac{z-1}{T_{samp}}$	$S = \frac{2}{T_{samp}}\left(\frac{z-1}{z+1}\right)$

$$G_c(s) = 9.75 + \frac{36071.3877}{s} + 0.00065823\,s$$

$$G_c(z) = 9.75 + \frac{36071.3877\,z\,(100\times10^{3})}{(z-1)} + \frac{0.00065823\,(z-1)}{z\,(100\times10^{3})}$$

$$= 9.75 + \frac{3.61\times10^{9}\,z}{z-1} + \frac{6.6\times10^{-9}\,(z-1)}{z}$$

$$= \frac{9.75\,(z-1)\,z + 3.61\times10^{9}\,z^{2} + 6.6\times10^{-9}\,(z-1)^{2}}{z\,(z-1)}$$

$$= 9.75\,z^{2} - 9.75\,z + 3.61\times10^{9}\,z^{2} + 6.6\times10^{-9}\,z^{2}$$

$$- 13\times10^{-9}\,z + 6.6\times10^{-9}$$

$$= \frac{3.6\times10^{9} - 13\times10^{-9}\,z + 6.6\times10^{-9}}{z\,(z-1)}$$