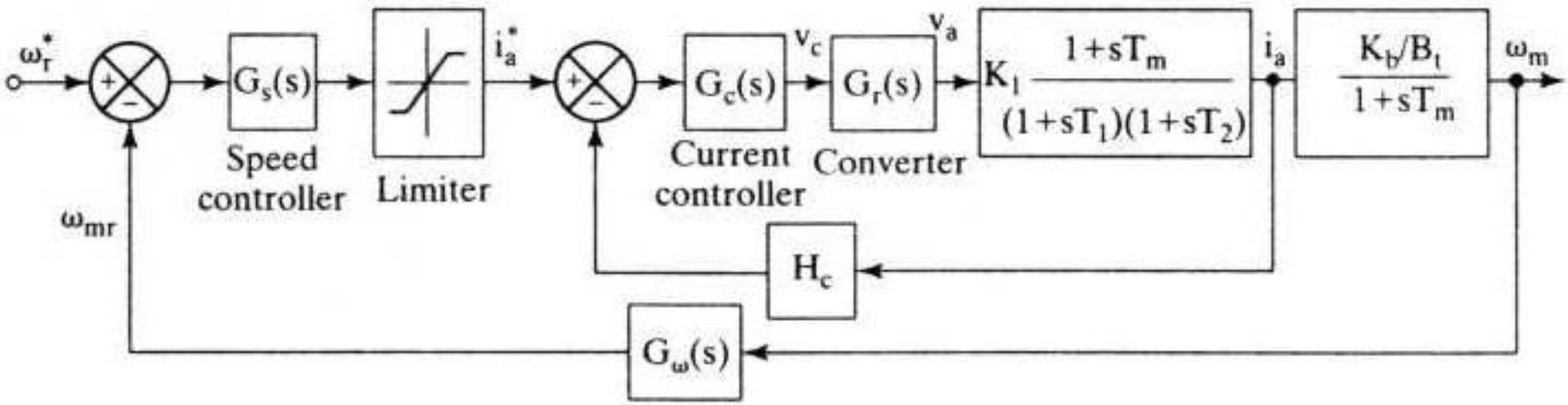
Electrical Drives

Lecture 12 (09-02-2024)

Design of Controllers

The overall closed-loop system is shown in Figure



It is seen that the current loop does not contain the inner induced-emf loop.

Design of Controllers

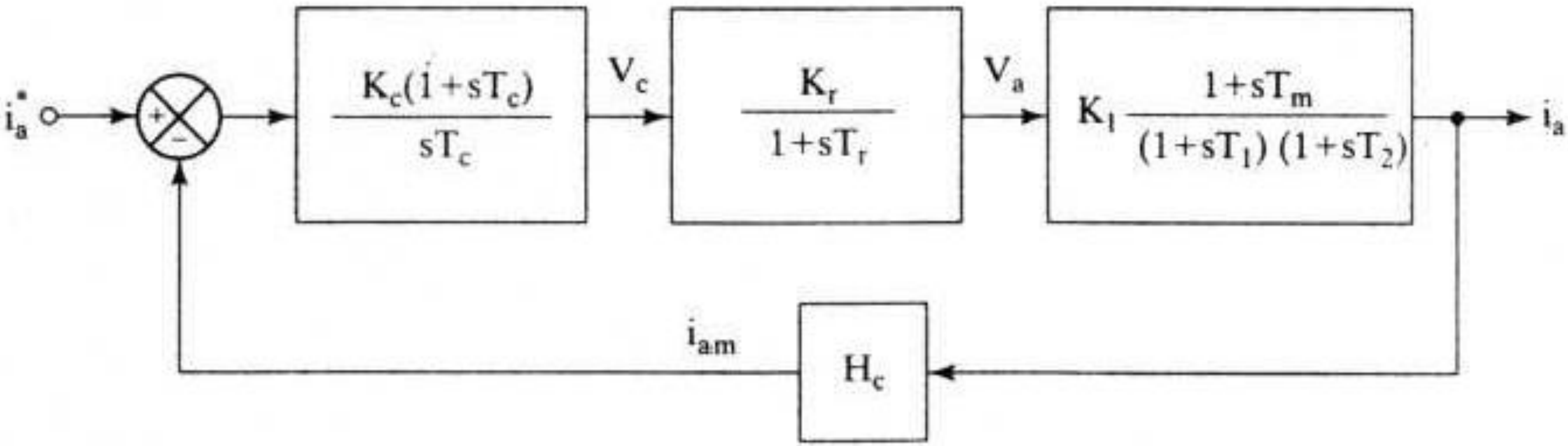
The design of control loops starts from the innermost (fastest) loop and proceeds to the slowest loop, which in this case is the outer speed loop.

The reason to proceed from the inner to the outer loop in the design process is that the gain and time constants of only one controller at a time are solved, instead of solving for the gain and time constants of all the controllers simultaneously. Not only is that logical; it also has a practical implication.

- Every motor drive need not be speed controlled, but may be torque-controlled. Eg: Traction
- But current loops is essential and exists regardless of speed or torque is controlled
- The performance of outer loop is dependent on the inner loop, therefore the tuning of inner loop has to precede the design and tuning of outer loop
- Therefore, the dynamics of inner loop can be simplified and the impact of the outer loop on its performance could be minimized.
- The design of current and speed controllers are considered now……

Design of Controllers – Current Controller

The current-control loop is shown in Figure



- The loop gain of the current control loop is given by

$$GH_i(s) = \left\{\frac{K_1 K_c K_r H_c}{T_c}\right\} \cdot \frac{(1 + sT_c)(1 + sT_m)}{s(1 + sT_1)(1 + sT_2)(1 + sT_r)}$$

Design of Controllers – Current Controller

This is a fourth-order system, and simplification is necessary to synthesize a controller without resorting to a computer. Noting that T_m is on the order of a second and in the vicinity of the gain crossover frequency, we see that the following approximation is valid:

$$(1 + sT_m) \cong sT_m$$

which reduces the loop gain function to

$$GH_i(s) \cong \frac{K(1 + sT_c)}{(1 + sT_1)(1 + sT_2)(1 + sT_r)}$$

where

$$K = \frac{K_1 K_c K_r H_c T_m}{T_c}$$

Design of Controllers – Current Controller

The time constants in the denominator are seen to have the relationship

$$T_r < T_2 < T_1$$

The equation (3.75) can be reduced to second order, to facilitate a simple controller synthesis, by judiciously selecting

$$T_c = T_2$$

Then the loop function is

$$GH_i(s) \cong \frac{K}{(1 + sT_1)(1 + sT_r)}$$

Design of Controllers – Current Controller

The characteristic equation or denominator of the transfer function between the armature current and its command is

$$(1 + sT_1)(1 + sT_r) + K$$

This equation is expressed in standard form as

$$T_1T_r\left\{s^2 + s\left(\frac{T_1 + T_r}{T_1T_r}\right) + \frac{K + 1}{T_1T_r}\right\}$$

from which the natural frequency and damping ratio are obtained as

$$\omega_n^2 = \frac{K + 1}{T_1T_r} \qquad \zeta = \frac{\left(\frac{T_1 + T_r}{T_1T_r}\right)}{2\sqrt{\frac{K + 1}{T_1T_r}}}$$

where ω_n and ζ are the natural frequency and damping ratio, respectively. For good dynamic performance, it is an accepted practice to have a damping ratio of 0.707.

Design of Controllers – Current Controller

Hence, equating the damping ratio to 0.707 in equation, we get

$$K + 1 = \frac{\left(\frac{T_1 + T_r}{T_1 T_r}\right)^2}{\left(\frac{2}{T_1 T_r}\right)}$$

Realizing that

$$K >> 1 \qquad T_1 >> T_r$$

K is approximated as $K \cong \frac{T_1^2}{2T_1 T_r} \cong \frac{T_1}{2T_r}$

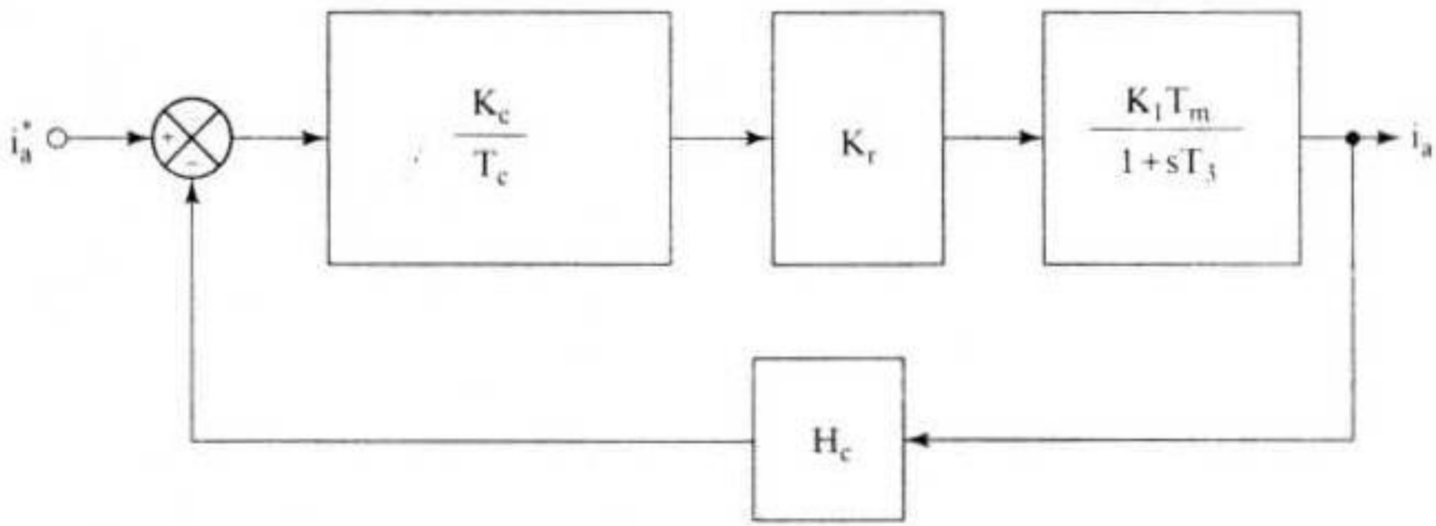
the current-controller gain is evaluated as

$$K_c = \frac{1}{2} \cdot \frac{T_1 T_c}{T_r} \cdot \left(\frac{1}{K_1 K_r H_c T_m}\right)$$

Design of Controllers – Current Controller

First-Order Approximation of Inner Current Loop

To design the speed loop, the second-order model of the current loop is replaced with an approximate first-order model. This helps to reduce the order of the overall speed-loop gain function. The current loop is approximated by adding the time delay in the converter block to T_1 of the motor; because of the cancellation of one motor pole by a zero of the current controller, the resulting current loop can be shown in Figure 3.31. The transfer function of the current and its commanded value is



Design of Controllers – Current Controller

First-Order Approximation of Inner Current Loop

$$\frac{I_a(s)}{I_a^*(s)} = \frac{\dfrac{K_c K_r T_1 T_m}{T_c} \cdot \dfrac{1}{(1 + sT_3)}}{1 + \dfrac{K_1 K_c K_r H_c T_m}{T_c} \cdot \dfrac{1}{(1 + sT_3)}}$$

where $T_3 = T_1 + T_r$. The transfer function can be arranged simply as

$$\frac{I_a(s)}{I_a^*(s)} = \frac{K_i}{(1 + sT_i)}$$

where

$$T_i = \frac{T_3}{1 + K_{fi}} \qquad K_i = \frac{K_{fi}}{H_c} \cdot \frac{1}{(1 + K_{fi})}$$

$$K_{fi} = \frac{K_c K_r K_1 T_m H_c}{T_c}$$

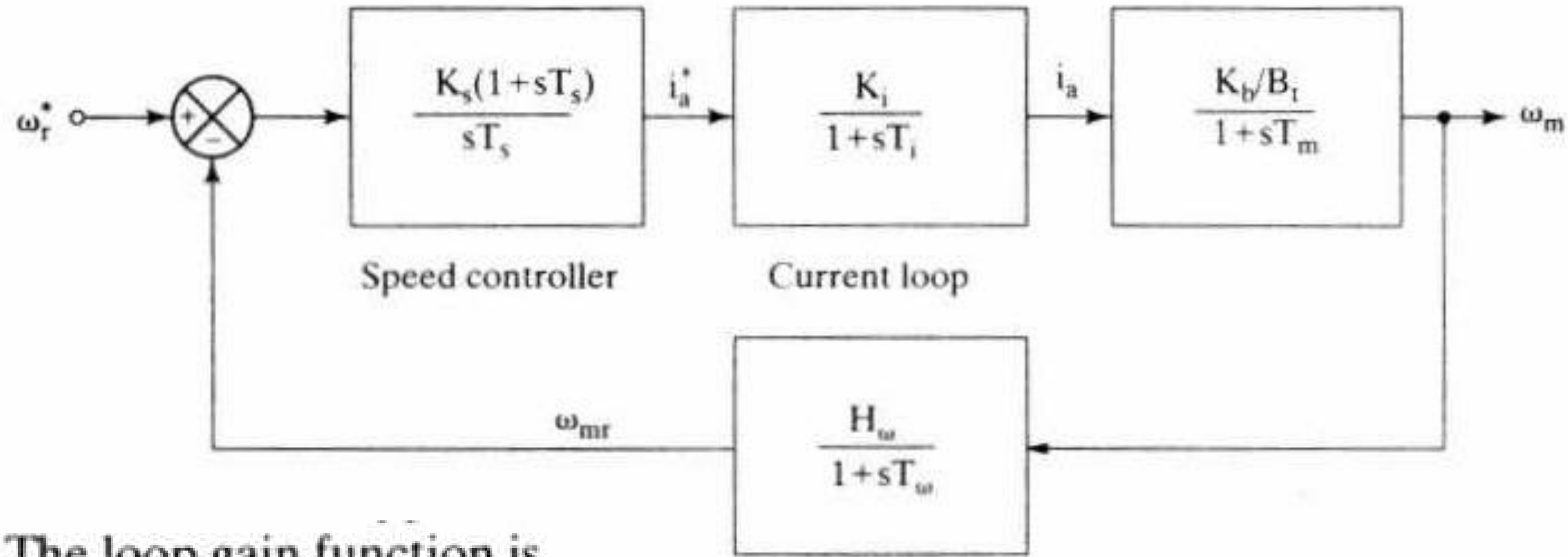
Design of Controllers – Current Controller

First-Order Approximation of Inner Current Loop

The resulting model of the current loop is a first-order system, suitable for use in the design of a speed loop. The gain and delay of the current loop can also be found experimentally in a motor-drive system. That would be more accurate for the speed-controller design.

Design of Controllers – Speed Controller

The speed loop with the first-order approximation of the current-control loop is shown in Figure :



The loop gain function is

$$GH_s(s) = \left\{\frac{K_sK_iK_bH_\omega}{B_tT_s}\right\} \cdot \frac{(1 + sT_s)}{s(1 + sT_i)(1 + sT_m)(1 + sT_\omega)}$$

Design of Controllers – Speed Controller

This is a fourth-order system. To reduce the order of the system for analytical design of the speed controller, approximation serves. In the vicinity of the gain crossover frequency, the following is valid:

$$(1 + sT_m) \cong sT_m$$

The next approximation is to build the equivalent time delay of the speed feedback filter and current loop. Their sum is very much less than the integrator time constant, T_s, and hence the equivalent time delay, T_4, can be considered the sum of the two delays, T_i and T_ω. This step is very similar to the equivalent time delay introduced in the simplification of the current-loop transfer function. Hence, the approximate gain function of the speed loop is

$$GH_s(s) \cong K_2 \cdot \frac{K_s}{T_s} \cdot \frac{(1 + sT_s)}{s^2(1 + sT_4)}$$

where

$$T_4 = T_i + T_\omega$$

$$K_2 = \frac{K_i K_b H_\omega}{B_t T_m}$$

Design of Controllers – Speed Controller

The closed-loop transfer function of the speed to its command is

$$\frac{\omega_m(s)}{\omega_r^*(s)} = \frac{1}{H_\omega}\left[\frac{\frac{K_2K_s}{T_s}(1 + sT_s)}{s^3T_4 + s^2 + sK_2K_s + \frac{K_2K_s}{T_s}}\right]$$

$$= \frac{1}{H_\omega}\frac{(a_0 + a_1s)}{(a_0 + a_1s + a_2s^2 + a_3s^3)}$$

where

$$a_0 = K_2K_s/T_s$$

$$a_1 = K_2K_s$$

$$a_2 = 1$$

$$a_3 = T_4$$

Design of Controllers – Speed Controller

This transfer function is optimized to have a wider bandwidth and a magnitude of one over a wide frequency range by looking at its frequency response. Its magnitude is given by

$$\left|\frac{\omega_m(j\omega)}{\omega_r^*(j\omega)}\right| = \frac{1}{H_\omega}\sqrt{\frac{a_0^2 + \omega^2 a_1^2}{\{a_0^2 + \omega^2(a_1^2 - 2a_0a_2) + \omega^4(a_2^2 - 2a_1a_3) + \omega^6 a_3^2\}}}$$

This is optimized by making the coefficients of ω^2 and ω^4 equal zero, to yield the following conditions:

$$a_1^2 = 2a_0a_2$$

$$a_2^2 = 2a_1a_3$$

Substituting these conditions in terms of the motor and controller parameters given in (3.100) into (3.103) yields

$$T_s^2 = \frac{2T_s}{K_sK_2}$$

resulting in

$$T_sK_s = \frac{2}{K_2}$$

Design of Controllers – Speed Controller

Similarly,

$$\frac{T_s^2}{K_s^2 K_2^2} = \frac{2T_s^2 T_4}{K_s K_2}$$

which, after simplification, gives the speed-controller gain as

$$K_s = \frac{1}{2K_2 T_4}$$

Substituting equation (3.110) into equation (3.108) gives the time constant of the speed controller as

$$T_s = 4T_4$$

Substituting for K_s and T_s into (3.99) gives the closed-loop transfer function of the speed to its command as

$$\frac{\omega_m(s)}{\omega_r^*(s)} = \frac{1}{H_\omega}\left[\frac{1 + 4T_4 s}{1 + 4T_4 s + 8T_4^2 s^2 + 8T_4^3 s^3}\right]$$

Design of Controllers – Speed Controller

It is easy to prove that for the open-loop gain function the corner points are $1/4T_4$ and $1/T_4$, with the gain crossover frequency being $1/2T_4$. In the vicinity of the gain crossover frequency, the slope of the magnitude response is −20 dB/decade, which is the most desirable characteristic for good dynamic behavior. Because of its symmetry at the gain crossover frequency, this transfer function is known as a symmetric optimum function. Further, this transfer function has the following features:

(i) Approximate time constant of the system is $4T_4$.

(ii) The step response is given by

$$\omega_r(t) = \frac{1}{H_\omega}\left(1 + e^{-t/2T_4} - 2e^{-t/4T_4}\cos(\sqrt{3}t/4T_4)\right) \tag{3.113}$$

with a rise time of $3.1T_4$, a maximum overshoot of 43.4%, and a settling time of $16.5T_4$.

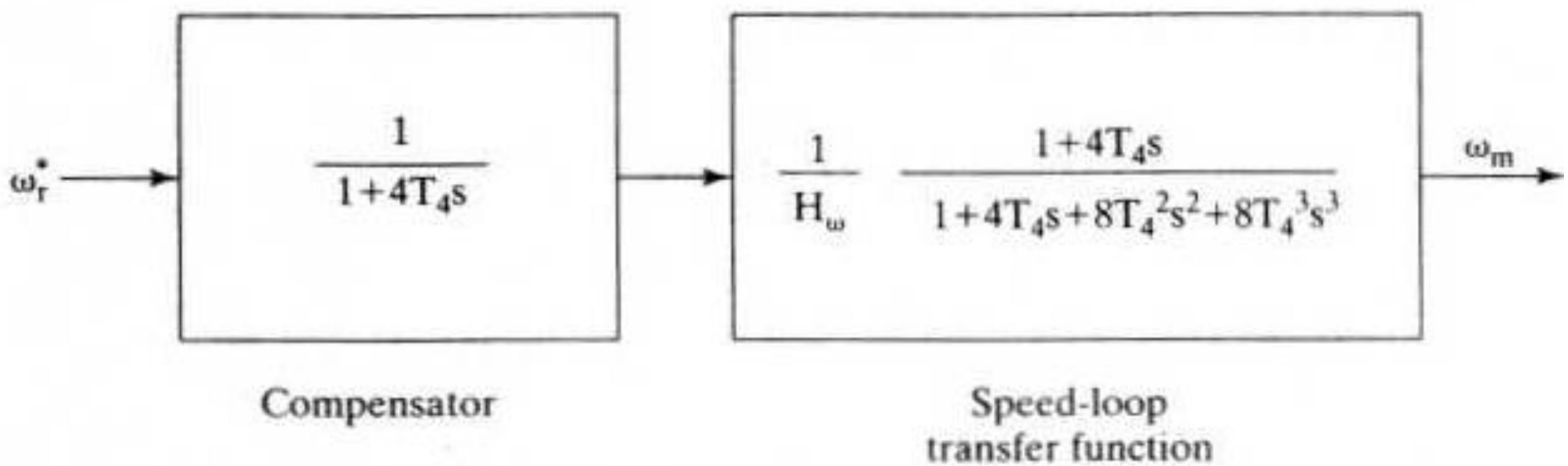
Design of Controllers – Speed Controller

(iii) Since the overshoot is high, it can be reduced by compensating for its cause, i.e., the zero of a pole in the speed command path, as shown in Figure 3.33. The resulting transfer function of the speed to its command is

$$\frac{\omega_m(s)}{\omega_r^*(s)} = \frac{1}{H_\omega}\left[\frac{1}{1 + 4T_4s + 8T_4^2s^2 + 8T_4^3s^3}\right] \tag{3.114}$$

whose step response is

$$\omega_r(t) = \frac{1}{H_\omega}\left(1 - e^{-t/4T_4} - \frac{2}{\sqrt{3}}e^{-t/4T_4}\sin(\sqrt{3}t/4T_4)\right) \tag{3.115}$$



Design of Controllers – Speed Controller

with a rise time of $7.6T_4$, a maximum overshoot of 8.1%, and a settling time of $13.3T_4$. Even though the rise time has increased, the overshoot has been reduced to approximately 20% of its previous value, and the settling time has come down by 19%.

(iv) The poles of the closed-loop transfer function are

$$s = -\frac{1}{2T_4}; -\frac{1}{4T_4} \pm j\frac{\sqrt{3}}{4T_4} \tag{3.116}$$

The real parts of the poles are negative, and there are no repeated poles at the origin, so the system is asymptotically stable. Hence, in the symmetric optimum design, the system stability is guaranteed, and there is no need to check for it in the design process. Whether this is true for the original system without approximation will be explored in the following example.

(v) Symmetric optimum eliminates the effects due to the disturbance very rapidly compared to other optimum techniques employed in practical systems, such as linear or modulus optimum. This approach indicates one of the possible methods to synthesize the speed controller. That the judicious choice of approximation is based on the physical constants of the motor, on the converter and transducer gains, and on time delays is to be emphasized here.

Design of Controllers – Speed Controller

That the speed-loop transfer function is expressed in terms of T_4 is significant in that it clearly links the dynamic performance to the speed-feedback and current-loop time constants. That a faster current loop with a smaller speed-filter time constant accelerates the speed response is evident from this. Expressing T_4 in terms of the motor, the converter and transducer gains, and the time delays by using expressions

$$T_i = \frac{T_3}{1 + K_{fi}} \qquad T_4 = T_i + T_\omega$$

$$T_4 = T_i + T_\omega = \frac{T_3}{1 + K_{fi}} + T_\omega = \frac{T_1 + T_r}{1 + K_{fi}} + T_\omega$$

Since $K_{fi} >> 1$, T_4 is found approximately after substituting for K_{fi} from equation

$$K_{fi} = \frac{K_c K_r K_1 T_m H_c}{T_c}$$

$$T_4 \approx \frac{(T_1 + T_r)T_2}{T_m} \cdot \frac{1}{K_1 K_c K_r H_c} + T_\omega$$

Design of Controllers – Speed Controller

This clearly shows the influence of the subsystem parameters on the system dynamics. A clear understanding of this would help the proper selection of the subsystems

to obtain the required dynamic performance of the speed-controlled motor-drive system. Further, this derivation demonstrates that the system behavior to a large degree depends on the subsystem parameters rather than only on the current and speed-controller parameters or on the sophistication of their design.