Monsoon
2022-2023

EE6303D Dynamics of Electrical Machines (DEM)

Module 1, 2

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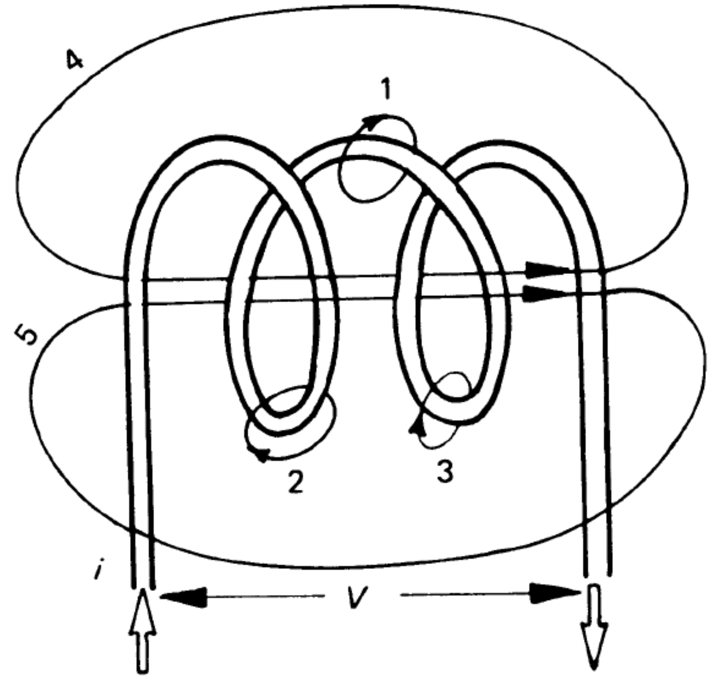
Relevance

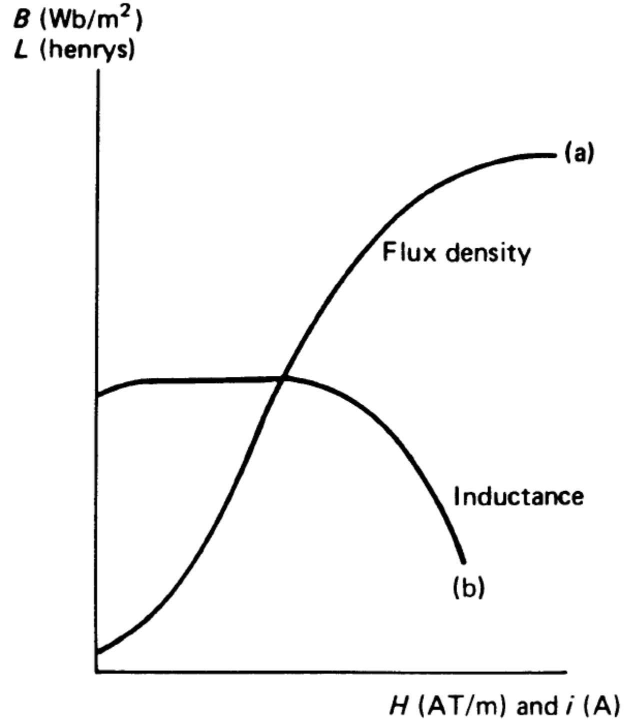
How to run any machine for variable speed application??

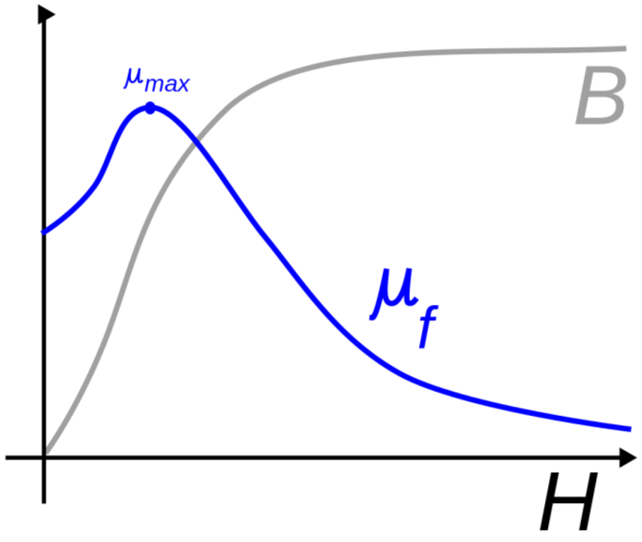
Dr. Gabriel Kron

Electrical machines, despite the differences in their construction and characteristics, have a lot in common. In earlier days, different types of machine were analysed in quite different ways, until in 1934 Dr. Gabriel Kron showed that all machines are basically the same, and can be analysed with the same set of generalised matrix equations. Kron's researches started a trend towards generalised machine concepts in the analysis of all types of machines

Fundamentals: Inductance

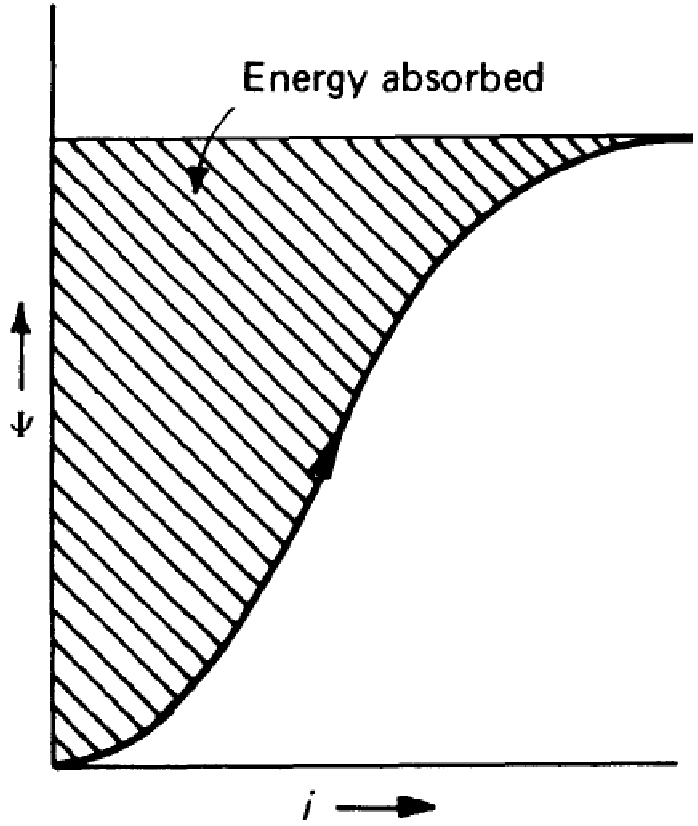


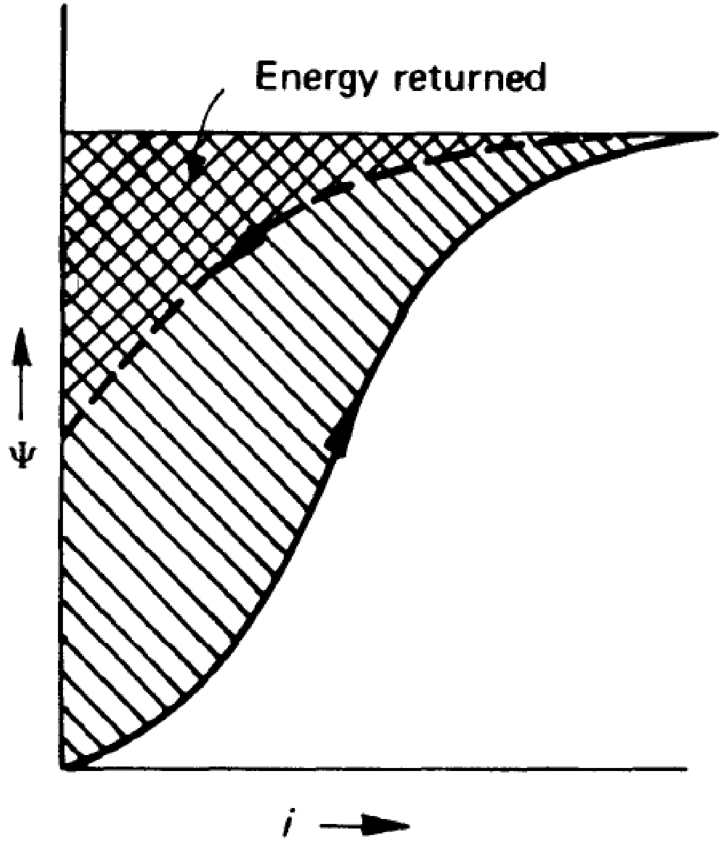


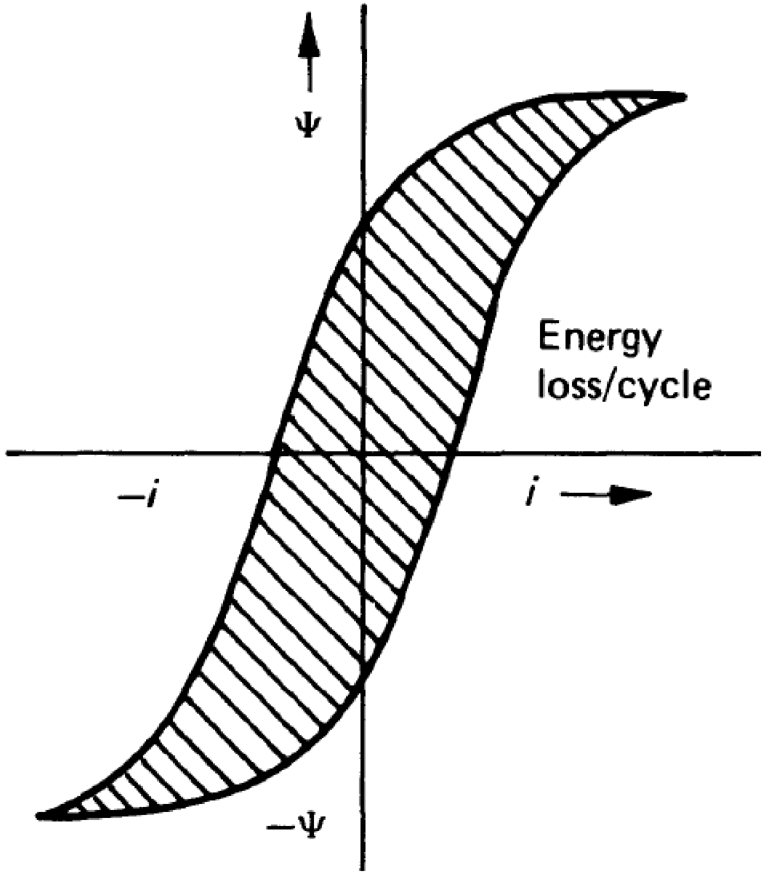


Permeability is the measure of magnetization that a material obtains in response to an applied magnetic field

Fundamentals: Inductance







$$E = \int_0^t V\, i\, \mathrm{d}t = \int_0^{\psi_1} i\, \mathrm{d}\psi$$

Fundamentals: Inductance

$$\Phi = Ni\mathscr{P} \quad \text{(Wb)}$$

$$\mathscr{P} = \mu A/l$$

$$\psi = N\Phi = N^2 i\mathscr{P}$$

Permeance is the property of allowing the passage of lines of magnetic flux

Faraday-Lenz law

$$e = -\frac{d\psi}{dt} = -\frac{d}{dt}(N^2\mathscr{P}i)$$

If the number of turns and the magnetic permeance do not change with time then

$$e = -N^2\mathscr{P}\frac{di}{dt} = -L\frac{di}{dt}$$

Fundamentals: Inductance

If the coils are in motion, this relation is no longer true and we have to write

$$e = -\frac{\mathrm{d}(Li)}{\mathrm{d}t}$$

$$= -L\frac{\mathrm{d}i}{\mathrm{d}t} - i\frac{\mathrm{d}L}{\mathrm{d}t}$$

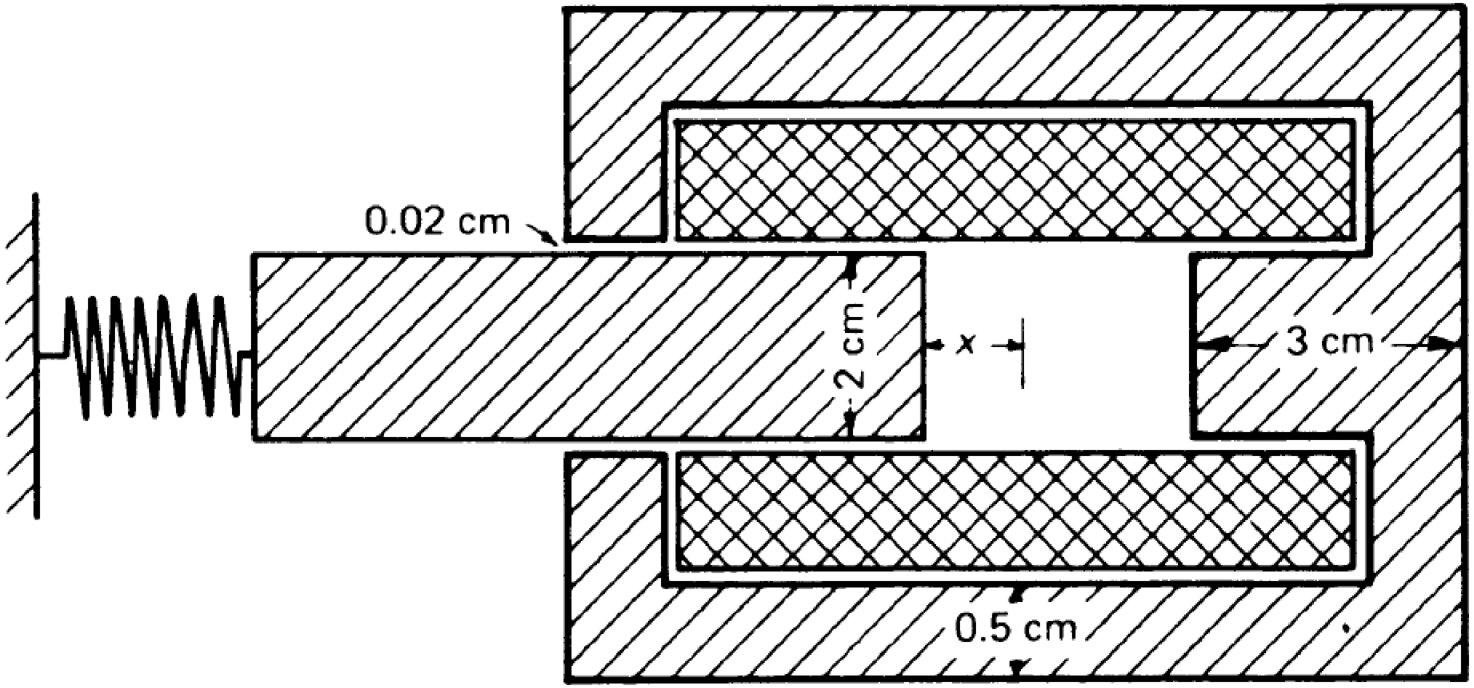
$$= -L\frac{\mathrm{d}i}{\mathrm{d}t} - i\frac{\partial L}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t}$$

if L changes with x.

Electrodynamics

$$V = Ri + \frac{\mathrm{d}}{\mathrm{d}t}(L\,i)$$

$$= Ri + L\frac{\mathrm{d}i}{\mathrm{d}t} + i\frac{\partial L}{\partial x}\cdot\frac{\mathrm{d}x}{\mathrm{d}t}$$



$$Vi = Ri^2 + iL\frac{\mathrm{d}i}{\mathrm{d}t} + i^2\frac{\partial L}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t}$$

$$= Ri^2 + \frac{\mathrm{d}}{\mathrm{d}t}\left(\tfrac{1}{2}Li^2\right) + \tfrac{1}{2}i^2\frac{\partial L}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t}$$

$$P_e = \left(\tfrac{1}{2}i^2\frac{\partial L}{\partial x}\right)\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)$$

First term: power loss due to the resistance of the coil
Second term: rate of change of the stored magnetic energy (i.e. reactive power).
Third term: the power necessary to accelerate the plunger and overcome the tension of the spring. This term represents the mechanical power. Since it has electrical origin it may be called electromechanical power

Electrodynamics

electromechanical force $f_e = \frac{1}{2} i^2 \frac{\partial L}{\partial x}$

K is the stiffness of the spring

$$m \frac{d^2 x}{dt^2} + Kx - \frac{1}{2} i^2 \frac{\partial L}{\partial x} = 0$$

(electrodynamic equation of the plunger)

involves the square of the current, this implies that irrespective of the direction of the currents through the coil the force on the plunger acts only in one direction. Reversing the polarity of the d.c. source, or replacing it by an a.c. source will not alter the direction of force.

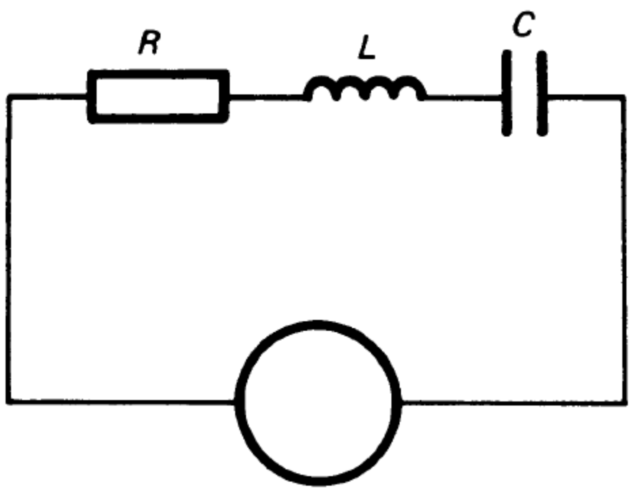
in which direction does the force act?

Electrodynamics

Clearly fe is positive if dL/dx is positive which implies that the plunger will move in such a direction that L increases with x or that the permeance increases with x, i.e., it will tend to move into the coil. This movement will be opposed by the spring and the acceleration will depend on the resultant of these opposing forces

If the plunger overshoots through the coil, the force f will reverse its direction and combine with the restoring force from the spring to pull the plunger back into the coil. It may be generally said that the electromechanical force will act in such a direction as to make the reluctance of the magnetic path a minimum

Electrodynamics



$$\mathscr{E} = N^2 \mathscr{P} \int_0^i i\,\mathrm{d}i = \tfrac{1}{2} L i^2 \quad \mathrm{(J)}$$

$$\text{kinetic energy} = \tfrac{1}{2} m v^2$$
$$= \tfrac{1}{2} J \omega^2$$

$$\mathscr{E}_{\mathrm{c}} = \tfrac{1}{2} C V^2 = \tfrac{1}{2} q^2 / C$$

$$\int_0^x F\,\mathrm{d}x = \int_0^x K x\,\mathrm{d}x = \tfrac{1}{2} K x^2$$

$$L\ddot{q} + R\dot{q} + q/C = V, \qquad \text{electrical network}$$
$$m\ddot{x} + D\dot{x} + Kx = F, \qquad \text{mechanical system}$$

the analogy in general form :
Lagrange's equation

Lagrangian Analysis (lossless system)

Joseph-Louis Lagrange

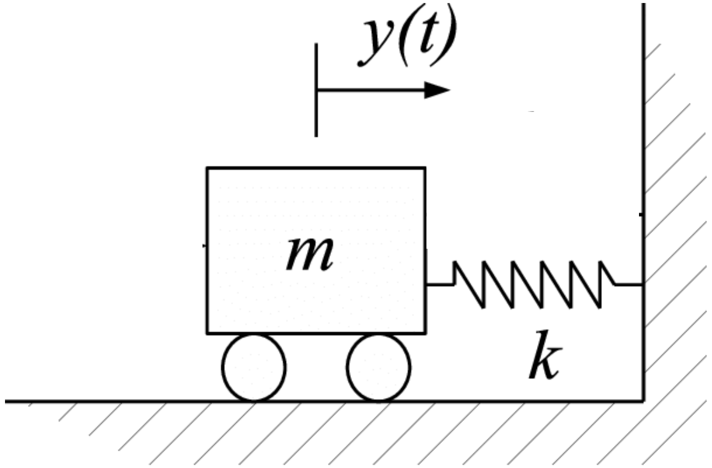
The Lagrangian

$$\mathscr{L} = \mathscr{T} - \mathscr{V}$$

Lagrangian equation in general form for a system with ‘n’ degrees of freedom

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathscr{L}}{\partial \dot{x}_n}\right) - \frac{\partial \mathscr{L}}{\partial x_n} = 0$$

Lagrangian Analysis (Lossless system)



kinetic energy of the mass

$$\mathscr{T} = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m(\dot{x})^2$$

potential energy stored in the spring

$$\mathscr{V} = \int_0^x F\,\mathrm{d}x = \int_0^x Kx\mathrm{d}x = Kx^2/2$$

$$\frac{\partial \mathscr{T}}{\partial \dot{x}} = \frac{\partial}{\partial \dot{x}}\left[\tfrac{1}{2}m(\dot{x})^2\right] = m\dot{x} = mv$$

$$\frac{\partial \mathscr{V}}{\partial x} = \frac{\partial}{\partial x}(\tfrac{1}{2}Kx^2) = Kx = F$$

$$\mathscr{L} = \mathscr{T} - \mathscr{V}$$

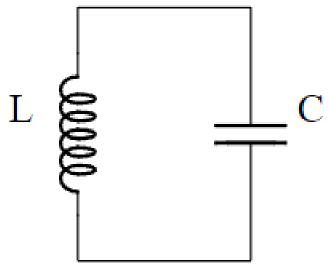
$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathscr{L}}{\partial \dot{x}_n}\right) - \frac{\partial \mathscr{L}}{\partial x_n} = 0$$

Newton's second law of motion

$$\frac{\mathrm{d}}{\mathrm{d}t}(mv) = -F$$

negative sign appears in this case because force is a restoring force

Lagrangian Analysis (Lossless system)



$$\frac{\partial \mathscr{T}}{\partial \dot{x}} = \frac{\partial}{\partial i}\left(\tfrac{1}{2}Li^2\right) = Li$$

$$\frac{\partial \mathscr{V}}{\partial q} = \frac{\partial}{\partial q}\left(\frac{1}{2}\frac{q^2}{C}\right) = \frac{q}{C}$$

$$\mathscr{L} = \mathscr{T} - \mathscr{V}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathscr{L}}{\partial \dot{x}_n}\right) - \frac{\partial \mathscr{L}}{\partial x_n} = 0$$

Substituting these values into Lagrange's equation

$$\frac{\mathrm{d}}{\mathrm{d}t}(Li) + \frac{q}{C} = 0 \qquad L\frac{\mathrm{d}i}{\mathrm{d}t} + \frac{q}{C} = 0$$

Lagrangian Analysis (Lossy system)

The Lagrangian

$$\mathscr{L} = \mathscr{T} - \mathscr{V}$$

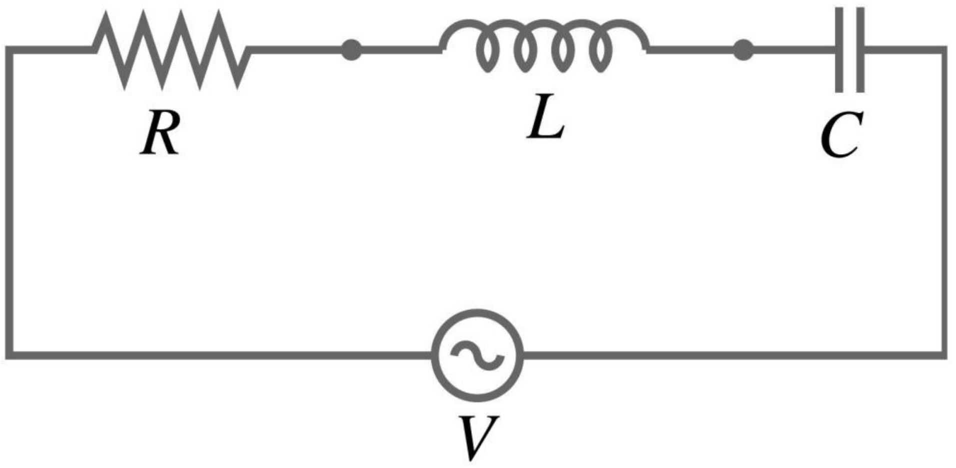
In order to include the effects of loss, a velocity-dependent (or current-dependent) function, the Rayleigh dissipation function is adopted

$$\frac{\partial \mathscr{F}}{\partial \dot{x}_n} = R_n \dot{x}_n \text{ represents force (or voltage)} \qquad \mathscr{F} = \sum_{n=1}^{k} \tfrac{1}{2} R_n (\dot{x}_n)^2$$

Lagrangian equation in general form for a system with ‘n’ degrees of freedom

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathscr{L}}{\partial \dot{x}_n}\right) - \frac{\partial \mathscr{L}}{\partial x_n} + \frac{\partial \mathscr{F}}{\partial \dot{x}_n} = V_n$$

Lagrangian Analysis (Lossy system)



$$\mathscr{L} = \mathscr{T} - \mathscr{V}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathscr{L}}{\partial \dot{x}_n}\right) - \frac{\partial \mathscr{L}}{\partial x_n} + \frac{\partial \mathscr{F}}{\partial \dot{x}_n} = V_n$$

$$\frac{\partial \mathscr{T}}{\partial \dot{x}} = \frac{\partial}{\partial i}\left(\tfrac{1}{2} L i^2\right) = L i$$

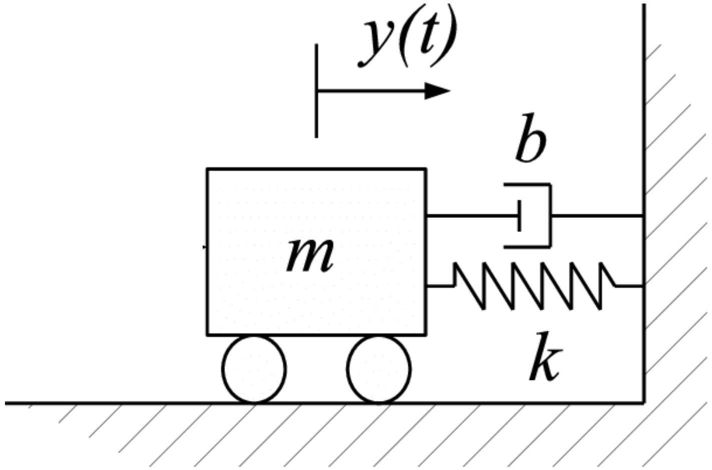
$$\frac{\partial \mathscr{V}}{\partial q} = \frac{\partial}{\partial q}\left(\frac{1}{2}\frac{q^2}{C}\right) = \frac{q}{C}$$

$$\frac{\partial \mathscr{F}}{\partial \dot{x}_n} = R_n \dot{x}_n$$

Substituting these values into Lagrange's equation

$$L\frac{\mathrm{d}^2 q}{\mathrm{d}t^2} + R\frac{\mathrm{d}q}{\mathrm{d}t} + \frac{q}{C} = V$$

Lagrangian Analysis (Lossy system)



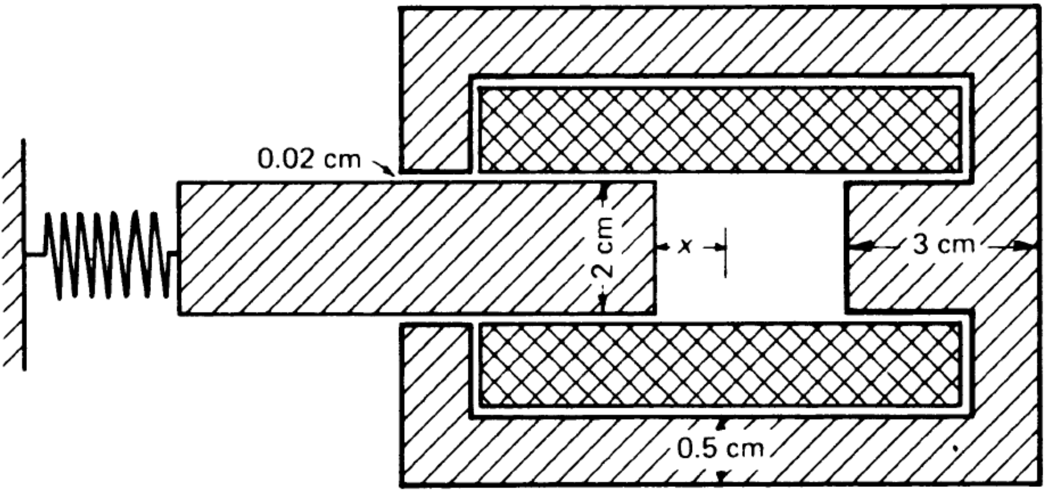
$$\mathscr{L} = \mathscr{T} - \mathscr{V}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathscr{L}}{\partial \dot{x}_n}\right) - \frac{\partial \mathscr{L}}{\partial x_n} + \frac{\partial \mathscr{F}}{\partial \dot{x}_n} = V_n$$

Substituting these values into Lagrange's equation

$$m\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + b\,\frac{\mathrm{d}x}{\mathrm{d}t} + Kx = F$$

Lagrangian Analysis (Lossy system)



Total kinetic energy

$$\mathcal{T} = \tfrac{1}{2}Li^2 + \tfrac{1}{2}mv^2$$

elect mech

Total potential energy

$$\mathcal{V} = \tfrac{1}{2}Kx^2$$

mechanical

Dissipation function

$$\mathcal{F} = \tfrac{1}{2}Ri^2 + \tfrac{1}{2}Dv^2$$

elect mech

$$\mathcal{L} = \mathcal{T} - \mathcal{V} = \tfrac{1}{2}L\dot{x}_1^2 + \tfrac{1}{2}m\dot{x}_2^2 - \tfrac{1}{2}Kx_2^2$$

$$\mathcal{F} = \tfrac{1}{2}R\dot{x}_1^2 + \tfrac{1}{2}D\dot{x}_2^2$$

$$\frac{\partial \mathcal{L}}{\partial \dot{x}_1} = L\dot{x}_1 \qquad \frac{\partial \mathcal{L}}{\partial x_1} = 0 \qquad \frac{\partial \mathcal{F}}{\partial \dot{x}_1} = R_1\dot{x}_1$$

$V_n = V_1 =$ voltage applied to the coil

$$L\frac{di}{dt} + Ri + i\frac{dL}{dt} = V_1$$

(Voltage equation of the coil)

$$\frac{\partial \mathcal{L}}{\partial \dot{x}_2} = m\dot{x}_2 \qquad \frac{\partial \mathcal{L}}{\partial x_2} = -Kx_2 + \tfrac{1}{2}\dot{x}_1^2\frac{\partial L}{\partial x_2}$$

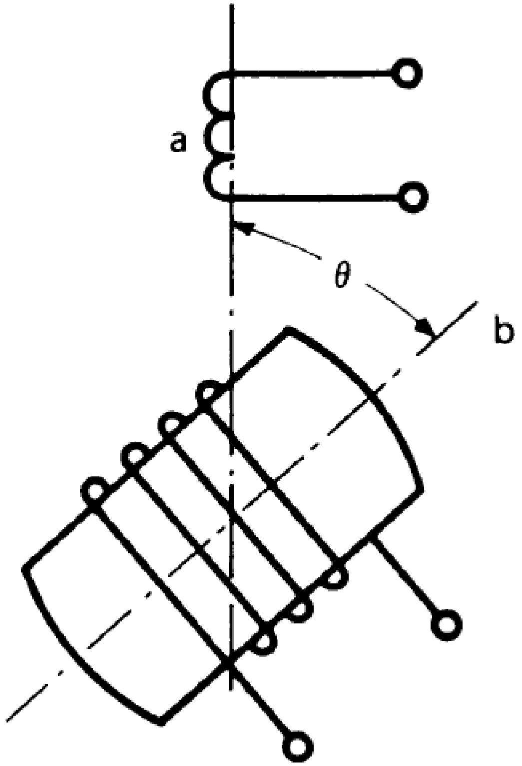
$$\frac{\partial \mathcal{F}}{\partial \dot{x}_2} = D\dot{x}_2 \qquad V_2 = \text{applied force} = F = 0$$

$$m\ddot{x}_2 + D\dot{x}_2 + Kx_2 - \tfrac{1}{2}i^2\frac{\partial L}{\partial x_2} = 0$$

(Dynamic equation of the plunger)

Lagrangian Analysis (Lossy system)

Doubly Excited Coil



x_1 represent the charge q_1 through coil 1
x_2 represent the charge q_2 through coil 2
x_3 represent the displacement θ of the rotor

Total kinetic energy

$$\mathscr{T} = \mathscr{T}_{\mathrm{e}} + \mathscr{T}_{\mathrm{m}} = \tfrac{1}{2}L_{11}i_1^2 + \tfrac{1}{2}L_{22}i_2^2 + i_1 i_2 M_{12} + \tfrac{1}{2}J\omega^2$$
$$= \tfrac{1}{2}L_{11}\dot{x}_1^2 + \tfrac{1}{2}L_{22}\dot{x}_2^2 + \dot{x}_1\dot{x}_2 M_{12} + \tfrac{1}{2}J\dot{x}_3^2$$

Total potential energy

$$\mathscr{V} = 0$$

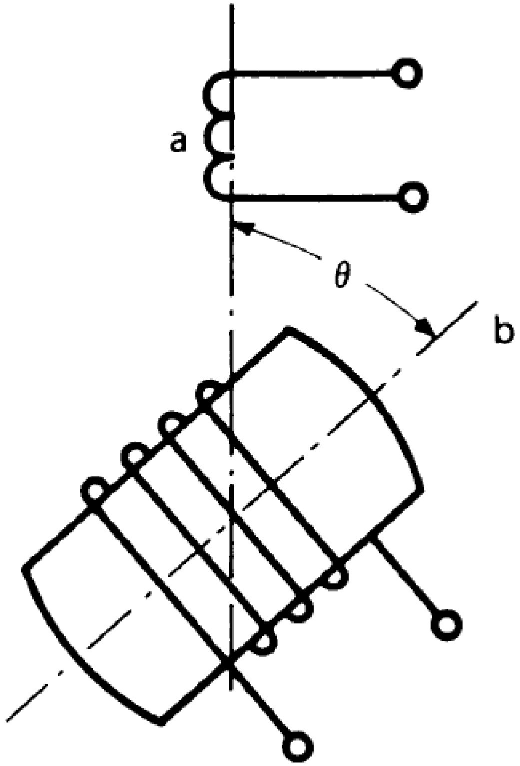
Dissipation function

$$\mathscr{F} = \tfrac{1}{2}R_1 i_1^2 + \tfrac{1}{2}R_2 i_2^2 + \tfrac{1}{2}R_{\mathrm{F}}\omega^2$$
$$= \tfrac{1}{2}R_1\dot{x}_1^2 + \tfrac{1}{2}R_2\dot{x}_2^2 + \tfrac{1}{2}R_{\mathrm{F}}\dot{x}_3^2$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathscr{L}}{\partial \dot{x}_n}\right) - \frac{\partial \mathscr{L}}{\partial x_n} + \frac{\partial \mathscr{F}}{\partial \dot{x}_n} = V_n$$

Lagrangian Analysis (Lossy system)

Doubly Excited Coil



x_1 represent the charge q_1 through coil 1
x_2 represent the charge q_2 through coil 2
x_3 represent the displacement θ of the rotor

$$\frac{\partial \mathscr{L}}{\partial \dot{x}_1} = \frac{\partial \mathscr{L}}{\partial i_1} = \frac{\partial \mathscr{T}}{\partial i_1} = L_{11} i_1 + M_{12} i_2$$

$$\frac{\partial \mathscr{L}}{\partial \dot{x}_2} = \frac{\partial \mathscr{L}}{\partial i_2} = \frac{\partial \mathscr{T}}{\partial i_2} = L_{22} i_2 + M_{12} i_1$$

$$\frac{\partial \mathscr{L}}{\partial \dot{x}_3} = \frac{\partial \mathscr{L}}{\partial \omega} = \frac{\partial \mathscr{T}}{\partial \omega} = J\omega$$

$$\frac{\partial \mathscr{L}}{\partial x_3} = \frac{\partial \mathscr{L}}{\partial \theta} = \tfrac{1}{2} i_1^2 \frac{\partial L_{11}}{\partial \theta} + \tfrac{1}{2} i_2^2 \frac{\partial L_{22}}{\partial \theta} + i_1 i_2 \frac{\partial M_{12}}{\partial \theta}$$

$$\frac{\partial \mathscr{F}}{\partial \dot{x}_1} = \frac{\partial \mathscr{F}}{\partial i_1} = R_1 i_1$$

$$\frac{\partial \mathscr{F}}{\partial \dot{x}_2} = \frac{\partial \mathscr{F}}{\partial i_2} = R_2 i_2$$

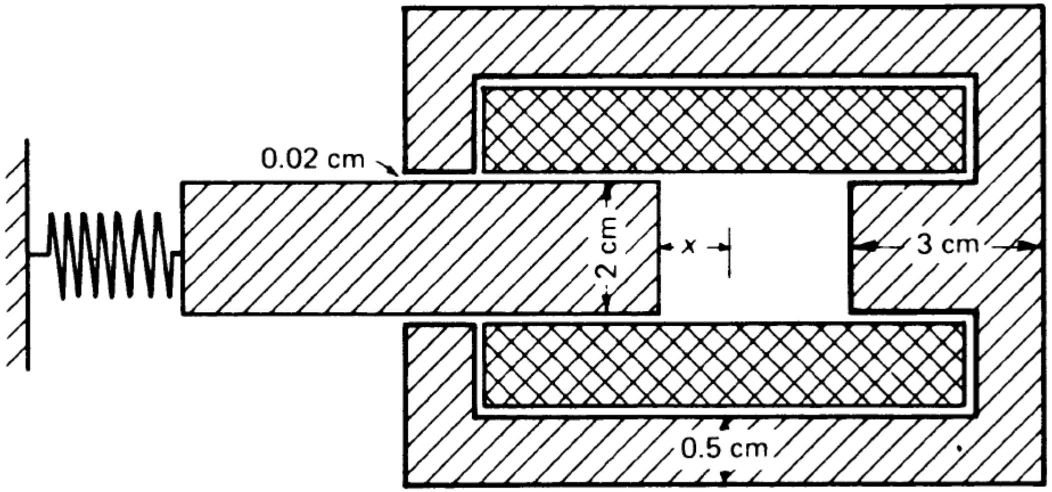
$$\frac{\partial \mathscr{F}}{\partial \dot{x}_3} = \frac{\partial \mathscr{F}}{\partial \omega} = R_F \omega$$

$$V_1 = R_1 i_1 + p[L_{11} i_1 + M_{12} i_2]$$

$$V_2 = R_2 i_2 + p[L_{22} i_2 + M_{12} i_1]$$

$$0 = R_F \omega + J p \omega - \left[\underbrace{\tfrac{1}{2} i_1^2 \frac{\partial L_{11}}{\partial \theta} + \tfrac{1}{2} i_2^2 \frac{\partial L_{22}}{\partial \theta} + i_1 i_2 \frac{\partial M_{12}}{\partial \theta}}_{\text{Mechanical Torque Produced}} \right]$$

Solution!



$$L\frac{\mathrm{d}i}{\mathrm{d}t}+Ri+i\frac{\mathrm{d}L}{\mathrm{d}t}=V_1$$

$$m\ddot{x}_2+D\dot{x}_2+Kx_2-\tfrac{1}{2}i^2\frac{\partial L}{\partial x_2}=0$$

How to know nature of plunger movement or supply current?

The product of the dependent variables, current and velocity in the voltage equations and the square of the current in the dynamical equation, make it difficult to derive a closed form solution.

By considering small perturbations about the operating point these equations can be linearised and a solution obtained. This is standard practice in considering small oscillations in electrical machines. Linearisation of the equations where fairly large displacements are involved leads to considerable errors and in such cases the standard practice is to obtain numerical solutions.

Euler's Method

Runge-Kutta Method

Euler’s Method

$$y' = f(x, y)$$

‘h’ is a small interval defined

$$x_{n+1} - x_n$$

$$y_{n+1} = y_n + h y'_n$$

One of the major disadvantages of Euler's method is that the interval or the step length h must be very small in order to provide correct results. Selecting large step sizes often leads to divergence or numerical instability.

Runge-Kutta Method

Order 2

$$y_{n+1} = y_n + \tfrac{1}{2}(k_1 + k_2)$$

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf(x_n + h \quad , y_n + k_1)$$

Order 4

$$y_{n+1} = y_n + \tfrac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf(x_n + h/2, y_n + k_1)$$

$$k_3 = hf(x_n + h/2, y_n + k_2)$$

$$k_4 = hf(x_n + h, y_n + k_3)$$

$$pi = -\frac{R}{L}i - \frac{1}{L}i\frac{\partial L}{\partial x}v + \frac{V}{L}$$

$$pv = -\frac{D}{m}v - \frac{Kx}{m} + \frac{1}{2m}i^2\frac{\partial L}{\partial x}$$

$$px = v$$

Euler's Method

$$y' = f(x, y)$$

$$y_{n+1} = y_n + h y'_n$$

'h' is a small interval defined

$$x_{n+1} - x_n$$

Runge-Kutta Method

$$y_{n+1} = y_n + \tfrac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

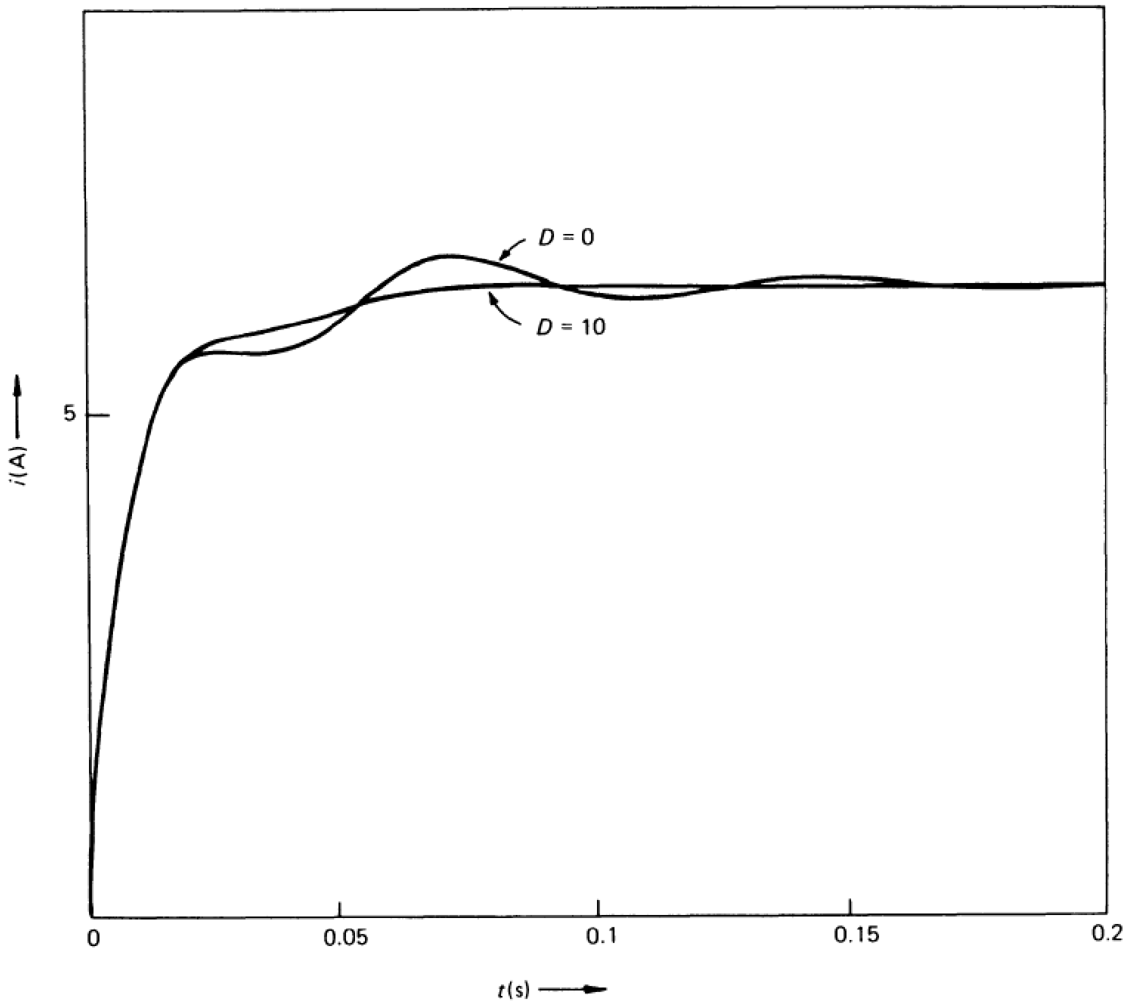
$$k_1 = hf(x_n, y_n)$$

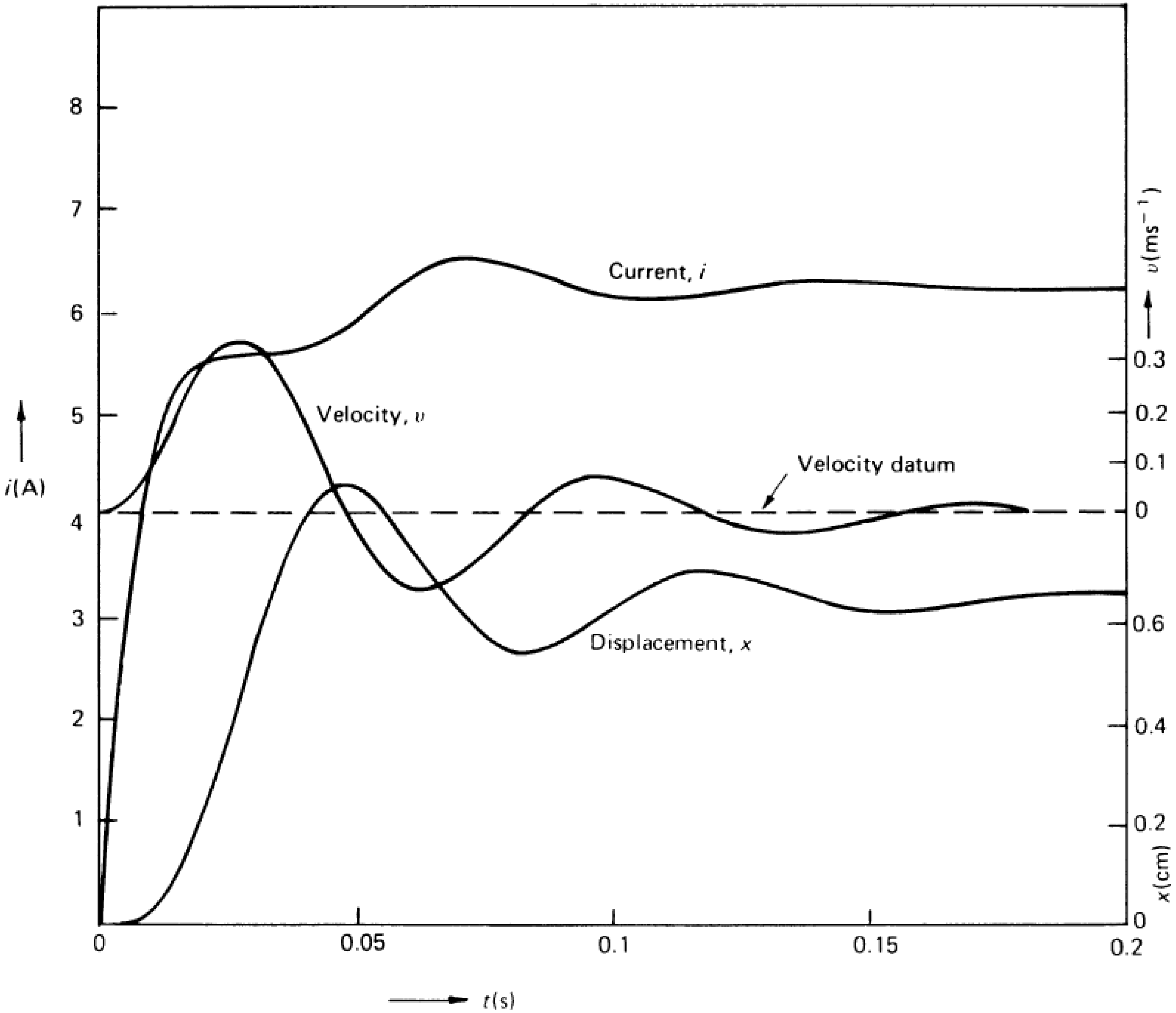
$$k_2 = hf(x_n + h/2,\ y_n + k_1)$$

$$k_3 = hf(x_n + h/2,\ y_n + k_2)$$

$$k_4 = hf(x_n + h,\ y_n + k_3)$$

Solution using RK4





Tutorial Problems #1

Use the Runge-Kutta method with h=0.1 to find approximate values for the solution of the problem at x=0.1, 0.2.

$$y' + 2y = x^3 e^{-2x}, \quad y(0) = 1$$

$$f(x, y) = -2y + x^3 e^{-2x}, \; x_0 = 0, \text{ and } y_0 = 1.$$

x=0.1

$$\begin{aligned}
k_{10} &= f(x_0, y_0) = f(0, 1) = -2, \\
k_{20} &= f(x_0 + h/2, y_0 + hk_{10}/2) = f(.05, 1 + (.05)(-2)) \\
&= f(.05, .9) = -2(.9) + (.05)^3 e^{-.1} = -1.799886895, \\
k_{30} &= f(x_0 + h/2, y_0 + hk_{20}/2) = f(.05, 1 + (.05)(-1.799886895)) \\
&= f(.05, .910005655) = -2(.910005655) + (.05)^3 e^{-.1} = -1.819898206, \\
k_{40} &= f(x_0 + h, y_0 + hk_{30}) = f(.1, 1 + (.1)(-1.819898206)) \\
&= f(.1, .818010179) = -2(.818010179) + (.1)^3 e^{-.2} = -1.635201628, \\
y_1 &= y_0 + \frac{h}{6}(k_{10} + 2k_{20} + 2k_{30} + k_{40}), \\
&= 1 + \frac{.1}{6}(-2 + 2(-1.799886895) + 2(-1.819898206) - 1.635201628) = .818753803,
\end{aligned}$$

Cont...

x=0.2

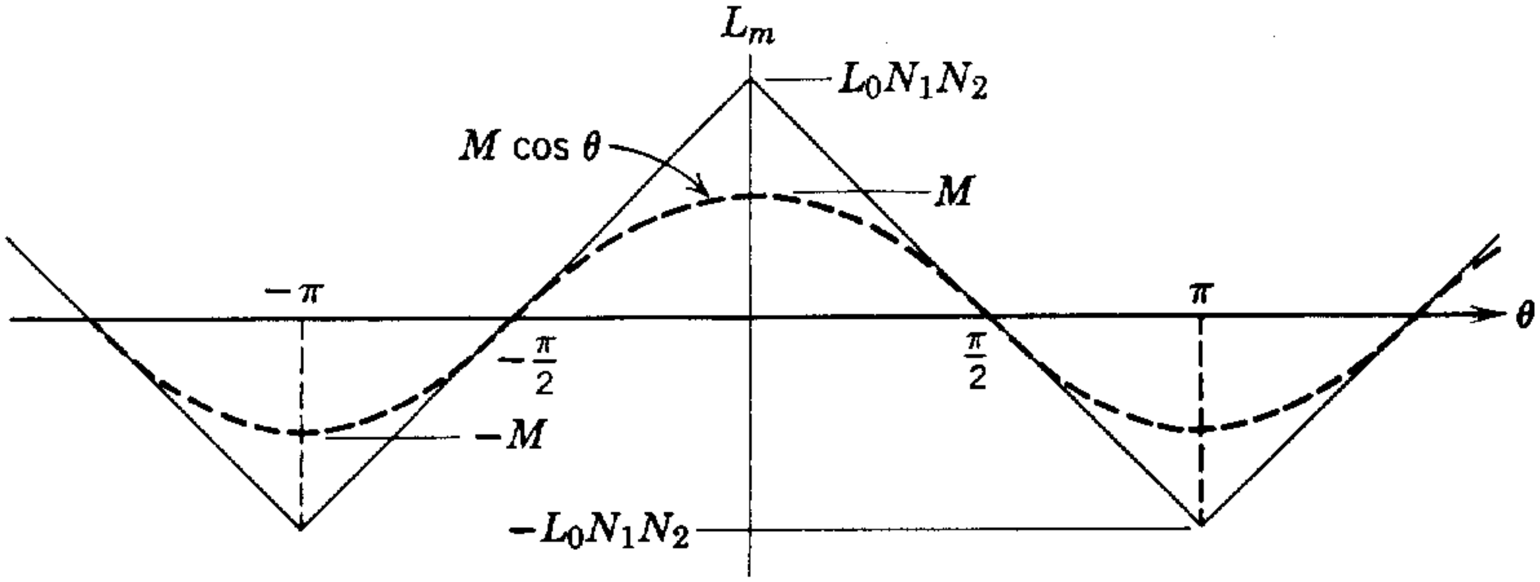
$$
\begin{aligned}
k_{11} &= f(x_1, y_1) = f(.1, .818753803) = -2(.818753803)) + (.1)^3 e^{-.2} = -1.636688875,\\
k_{21} &= f(x_1 + h/2, y_1 + hk_{11}/2) = f(.15, .818753803 + (.05)(-1.636688875))\\
&= f(.15, .736919359) = -2(.736919359) + (.15)^3 e^{-.3} = -1.471338457,\\
k_{31} &= f(x_1 + h/2, y_1 + hk_{21}/2) = f(.15, .818753803 + (.05)(-1.471338457))\\
&= f(.15, .745186880) = -2(.745186880) + (.15)^3 e^{-.3} = -1.487873498,\\
k_{41} &= f(x_1 + h, y_1 + hk_{31}) = f(.2, .818753803 + (.1)(-1.487873498))\\
&= f(.2, .669966453) = -2(.669966453) + (.2)^3 e^{-.4} = -1.334570346,\\
y_2 &= y_1 + \frac{h}{6}(k_{11} + 2k_{21} + 2k_{31} + k_{41}),\\
&= .818753803 + \frac{.1}{6}(-1.636688875 + 2(-1.471338457) + 2(-1.487873498) - 1.334570346)\\
&= .670592417.
\end{aligned}
$$

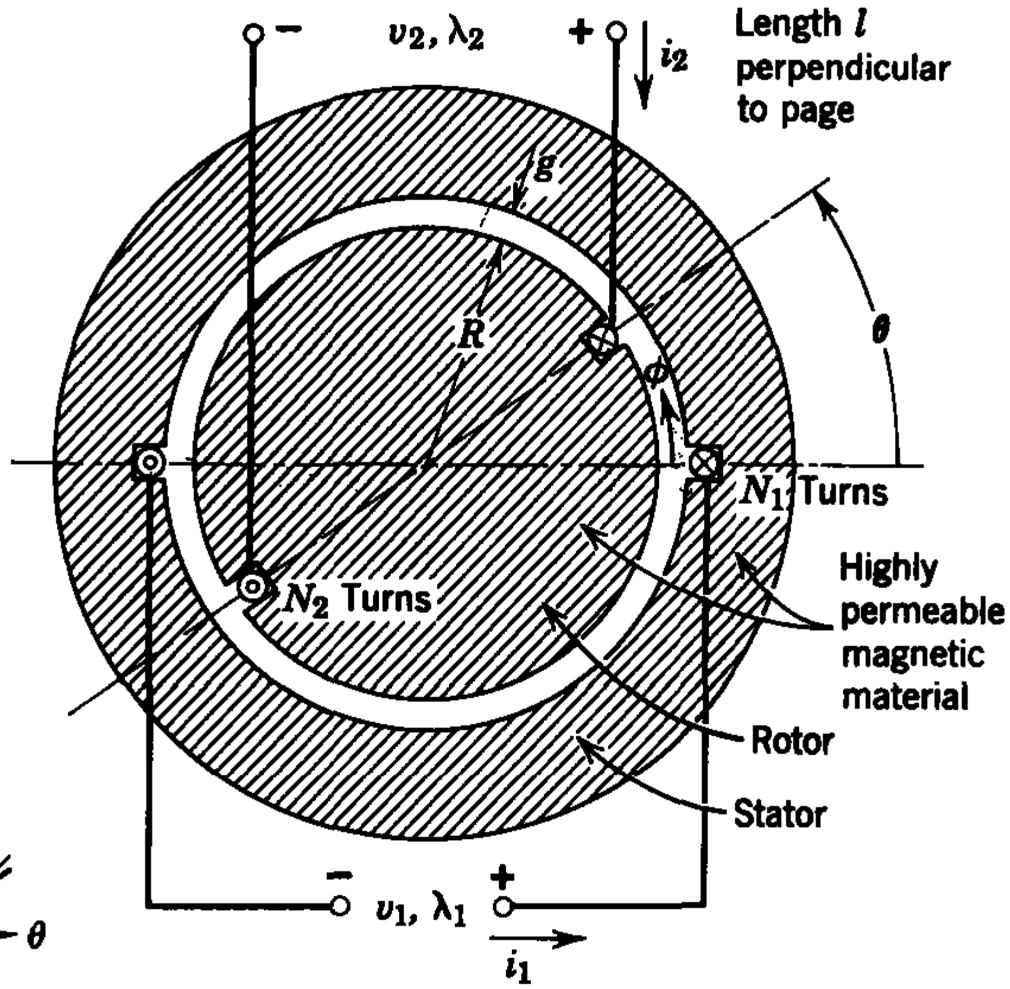
Generalized Machines

- The most numerous and the most widely used electromechanical device in existence is the magnetic field type rotating machine.
- Rotating machines occur in many different types, depending on the nature of the electrical and mechanical systems to be coupled and on the coupling characteristics desired.
- The primary purpose of most rotating machines is to convert energy between electrical and mechanical systems, either for electric power generation or for the production of mechanical power to do useful tasks.

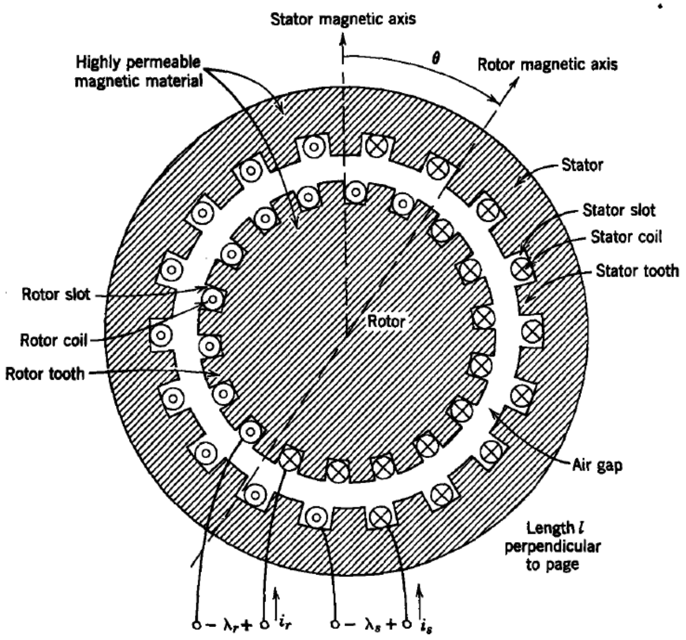
Air gap field, and hence the inductances associated with, play a vital role in the analysis of the electromechanical coupling systems in rotating machines

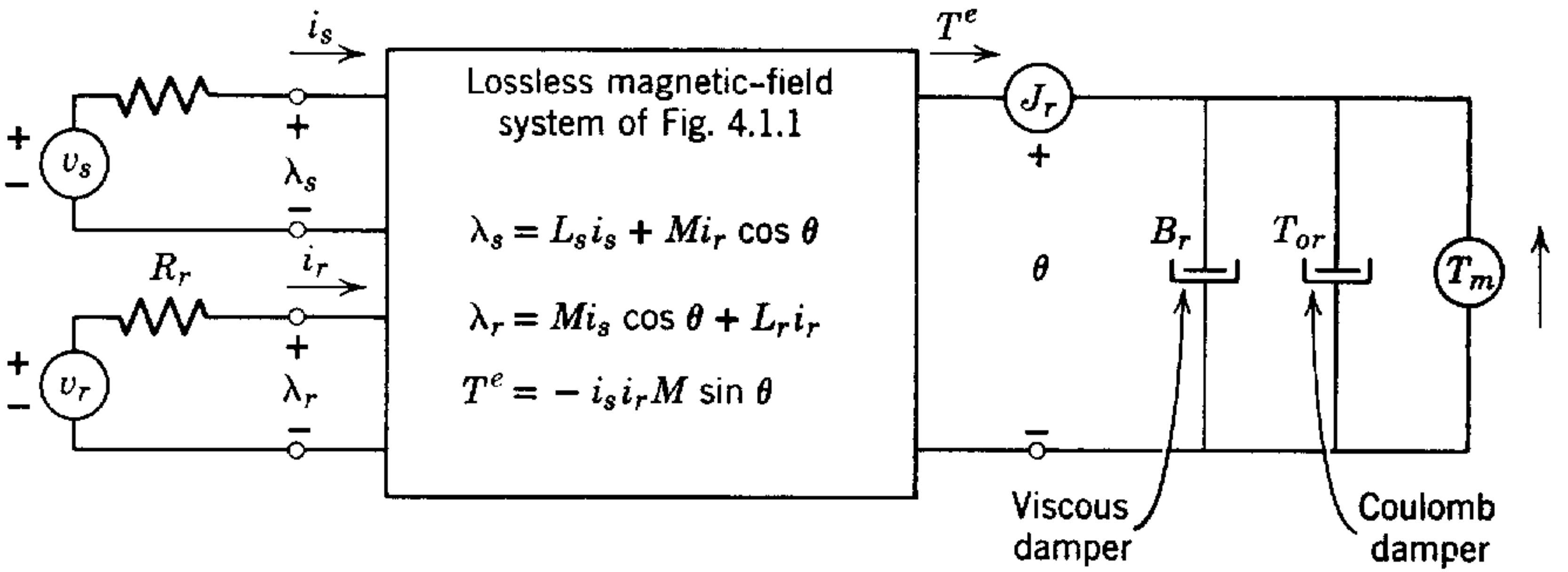
Generalized Machines/ Smooth Airgap Machines





Generalized Machines





$$\lambda_s = L_s i_s + L_{sr}(\theta) i_r,$$
$$\lambda_r = L_{sr}(\theta) i_s + L_r i_r,$$
$$T^e = i_s i_r \frac{dL_{sr}(\theta)}{d\theta},$$

$$L_{sr}(\theta) = M\cos\theta.$$

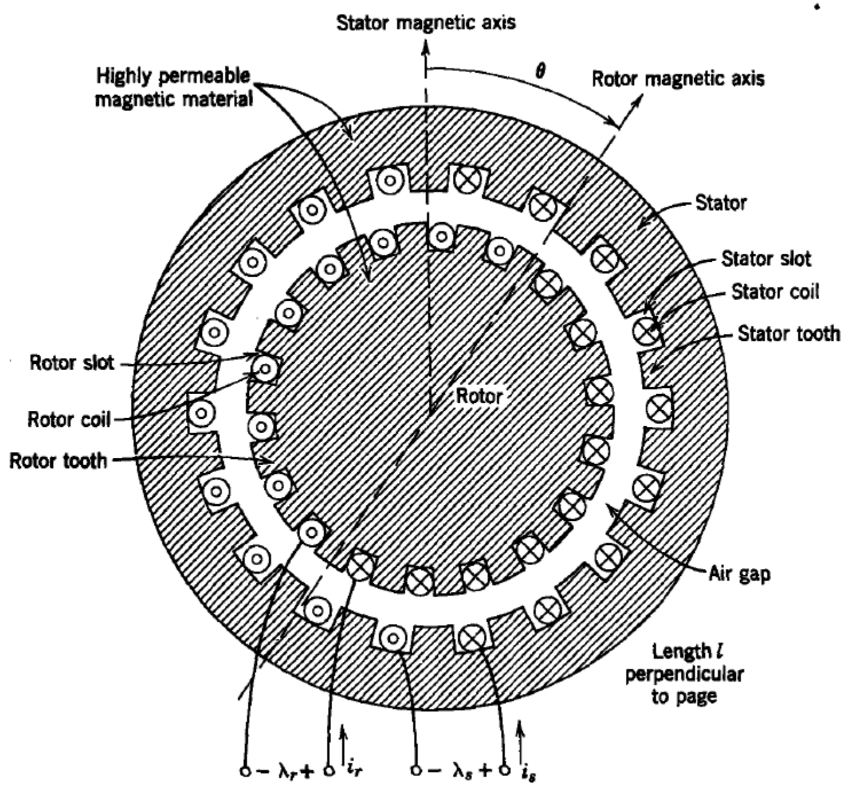
$$\lambda_s = L_s i_s + M i_r \cos\theta,$$
$$\lambda_r = M i_s \cos\theta + L_r i_r,$$
$$T^e = -i_s i_r M \sin\theta.$$

$$v_s = R_s i_s + \frac{d\lambda_s}{dt}$$
$$v_r = R_r i_r + \frac{d\lambda_r}{dt}$$
$$T_m + T^e = J_r \frac{d^2\theta}{dt^2} + B_r \frac{d\theta}{dt} + T_{or}\frac{d\theta/dt}{|d\theta/dt|}.$$

Generalized Machines/ Conditions for Conversion of Average Power

$$i_s(t) = I_s \sin \omega_s t,$$
$$i_r(t) = I_r \sin \omega_r t,$$
$$\theta(t) = \omega_m t + \gamma,$$

the conditions under which the machine with the steady-state excitations can convert average power between the electrical and mechanical systems



instantaneous power p_m, flowing from the coupling system,

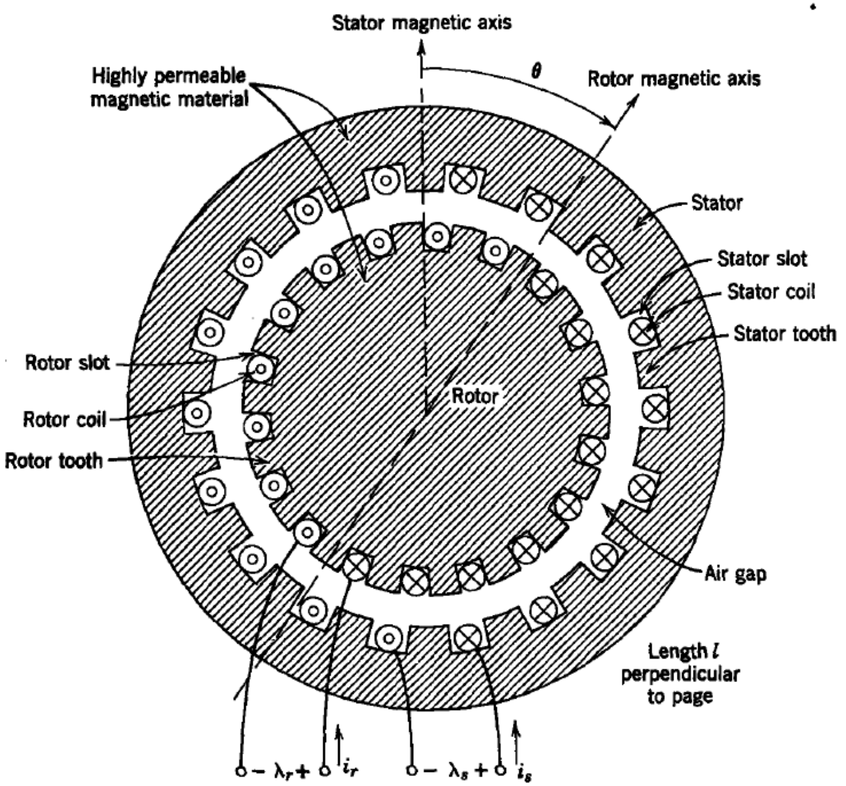
$$p_m = T^e \frac{d\theta}{dt} = T^e \omega_m.$$

$$T^e = -i_s i_r M \sin \theta.$$

$$p_m = -\omega_m I_s I_r M \sin \omega_s t \sin \omega_r t \sin (\omega_m t + \gamma).$$

Generalized Machines/ Conditions for Conversion of Average Power

$$p_m = -\omega_m I_s I_r M \sin \omega_s t \sin \omega_r t \sin(\omega_m t + \gamma).$$



$$\sin x \sin y = \tfrac{1}{2}[\cos(x-y) - \cos(x+y)];$$

$$\sin x \cos y = \tfrac{1}{2}[\sin(x-y) + \sin(x+y)].$$

$$p_m = -\frac{\omega_m I_s I_r M}{4}\{\sin[(\omega_m + \omega_s - \omega_r)t + \gamma] + \sin[(\omega_m - \omega_s + \omega_r)t + \gamma]$$

$$- \sin[(\omega_m + \omega_s + \omega_r)t + \gamma] - \sin[(\omega_m - \omega_s - \omega_r)t + \gamma]\}.$$

Generalized Machines/ Conditions for Conversion of Average Power

$$p_m = -\frac{\omega_m I_s I_r M}{4}\{\sin[(\omega_m + \omega_s - \omega_r)t + \gamma] + \sin[(\omega_m - \omega_s + \omega_r)t + \gamma]$$
$$- \sin[(\omega_m + \omega_s + \omega_r)t + \gamma] - \sin[(\omega_m - \omega_s - \omega_r)t + \gamma]\}.$$

Because a sinusoidal function of time has no average value, can have a time-average value only when one of the coefficients of 't' is zero. These four conditions, which cannot in general be satisfied simultaneously, can be written in the compact form

$$\omega_m = \pm\omega_s \pm \omega_r$$

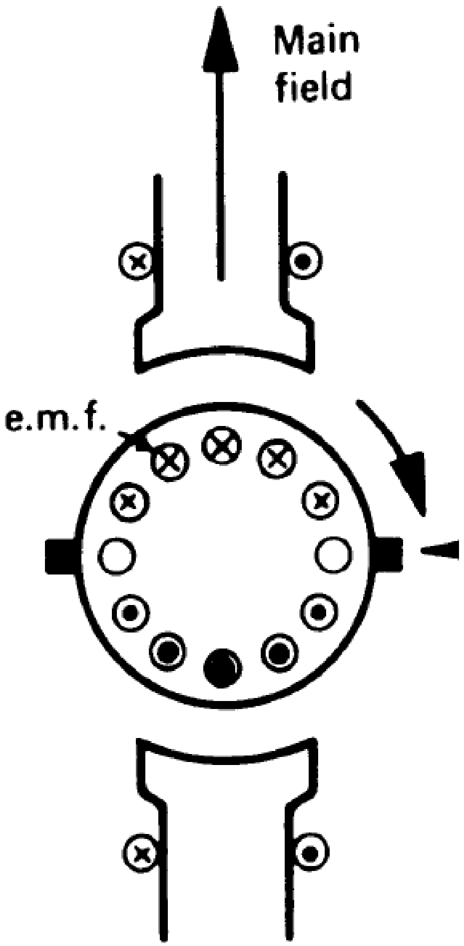
It is evident from these expressions that a necessary condition for average power conversion is the frequency condition. Sufficient conditions for average power conversion are this frequency condition and sin $\gamma \neq 0$

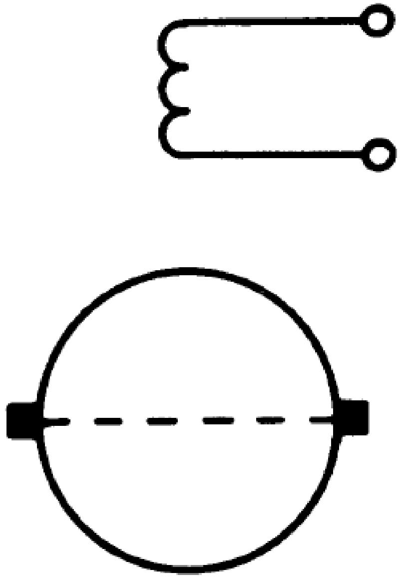
$$\omega_m = -\omega_s + \omega_r,$$
$$p_{m(\mathrm{av})} = -\frac{\omega_m I_s I_r M}{4}\sin\gamma$$

$$\omega_m = \omega_s + \omega_r,$$
$$p_{m(\mathrm{av})} = \frac{\omega_m I_s I_r M}{4}\sin\gamma$$

Primitive Machines

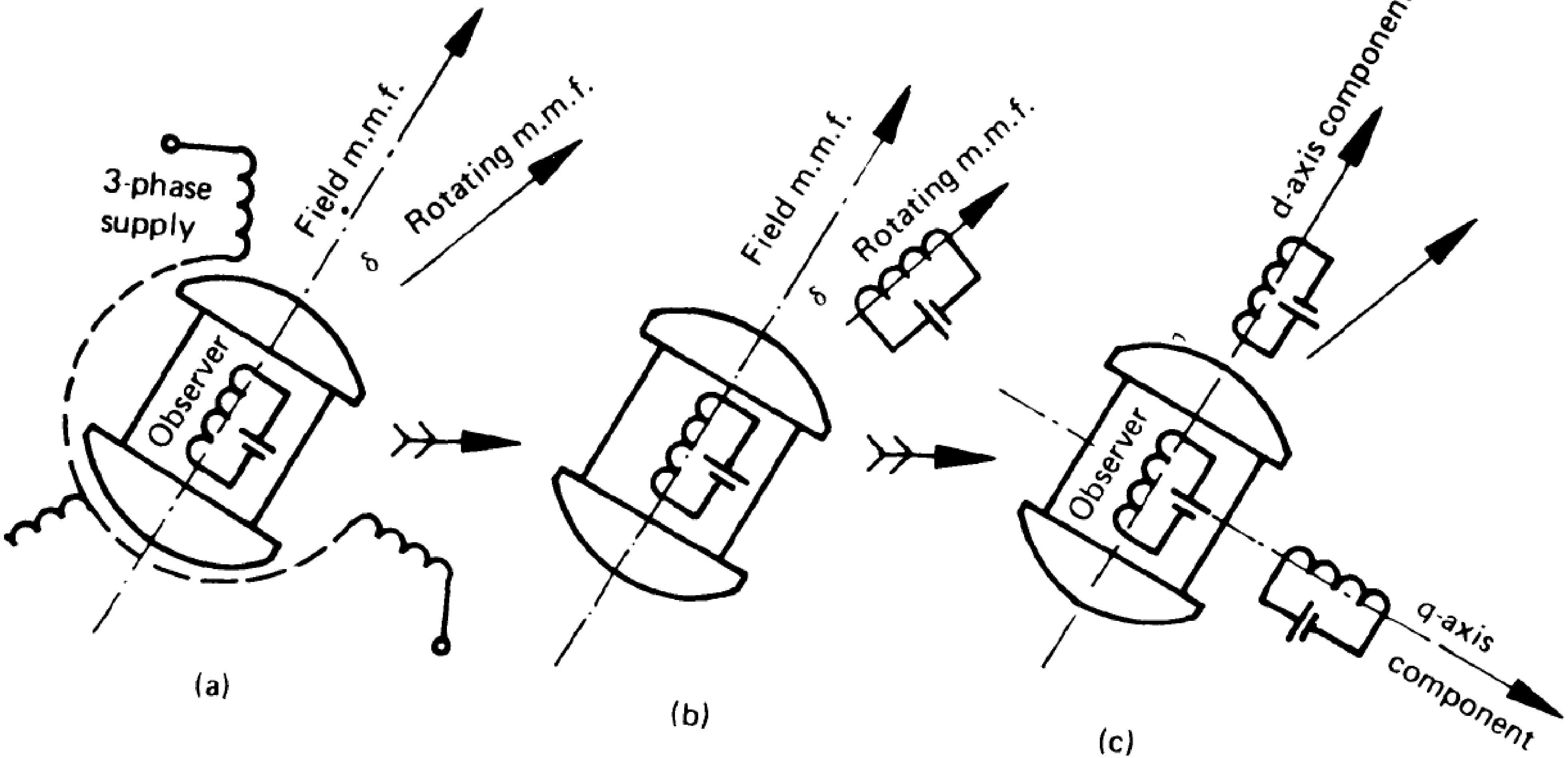
DC machines





Primitive Machines

Synchronous machines

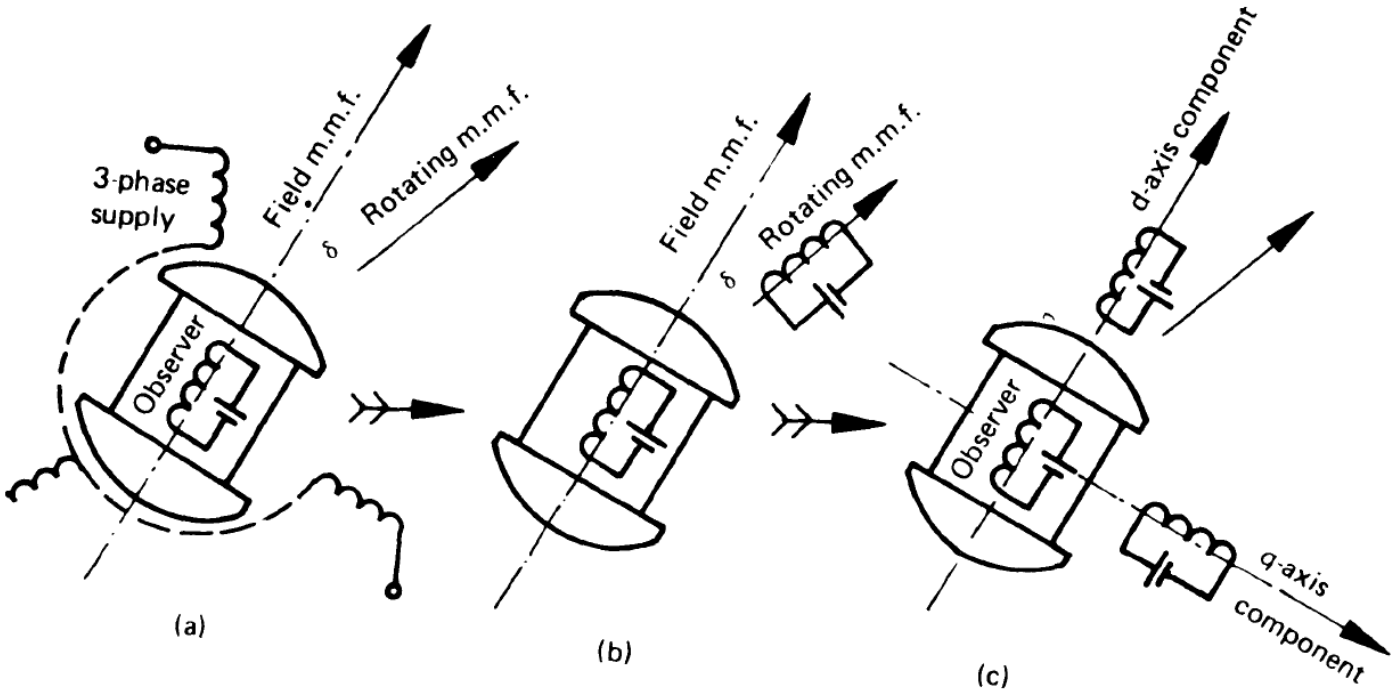


Primitive Machines

Synchronous machines

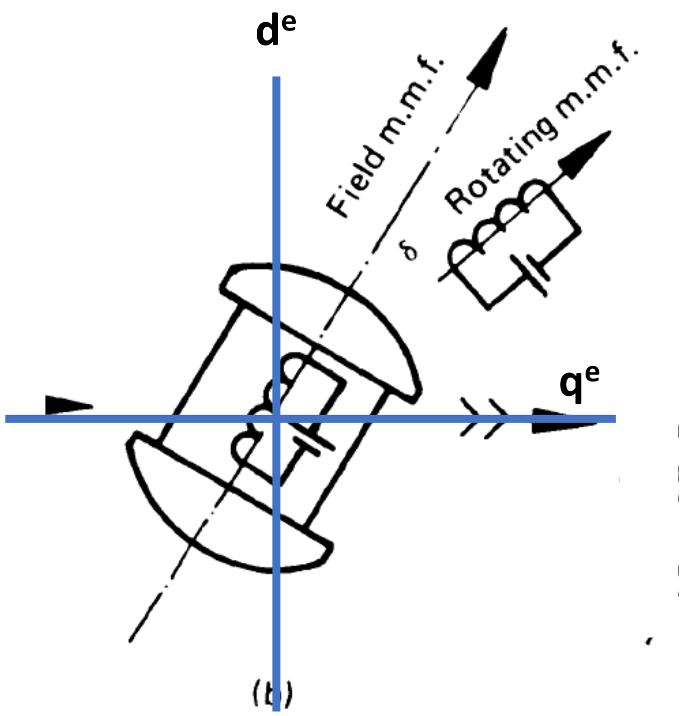
If the observer 'sits on one of the stator windings', the mutual inductance between the stator and the rotor coils changes as the rotor rotates. Also, if the air gap is non-uniform (as is the case with salient-pole machines) the self-inductance varies with varying reluctance of the magnetic path. Hence the impedance becomes a function of the rotor position, which makes the equations rather complicated.

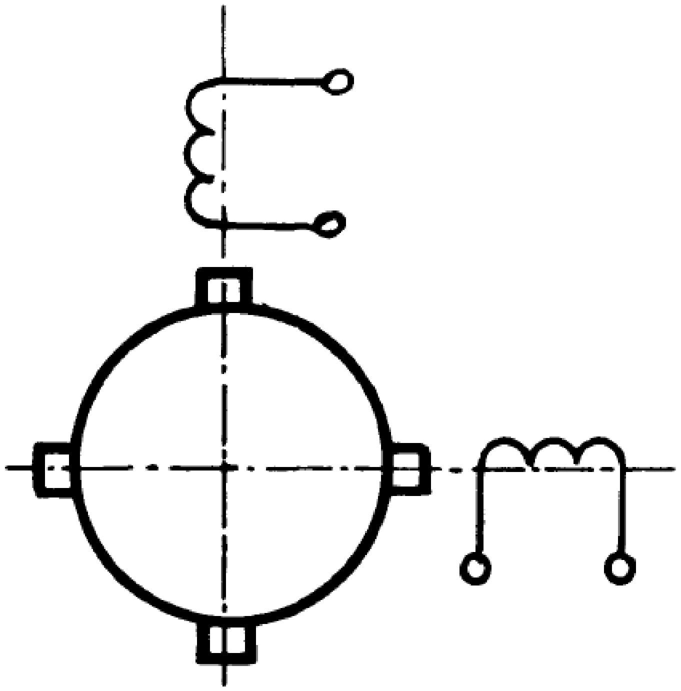
We know that 3-phase voltage fed to the 3-phase winding of the stator produces a magnetic field of constant magnitude, rotating at synchronous speed. An observer 'sitting on the rotor' will see an m.m.f. of constant magnitude, stationary with respect to him, when the rotor also rotates synchronously.



Primitive Machines

Synchronous machines

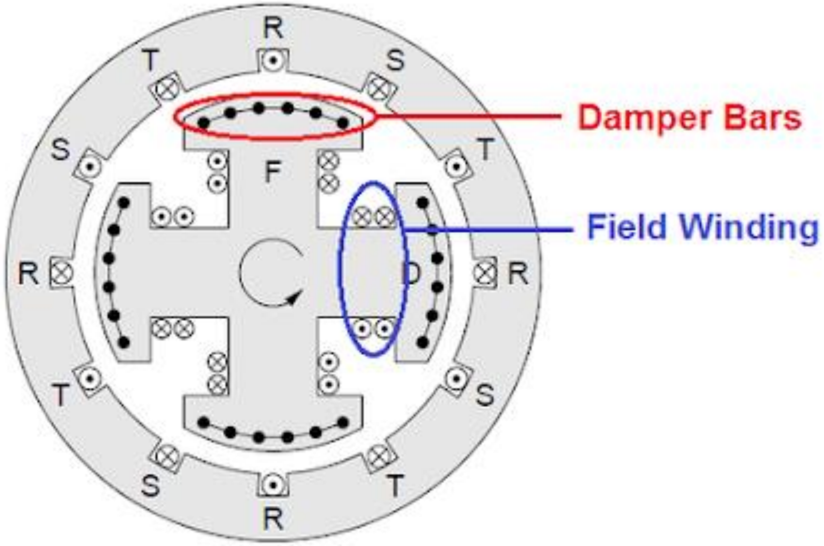


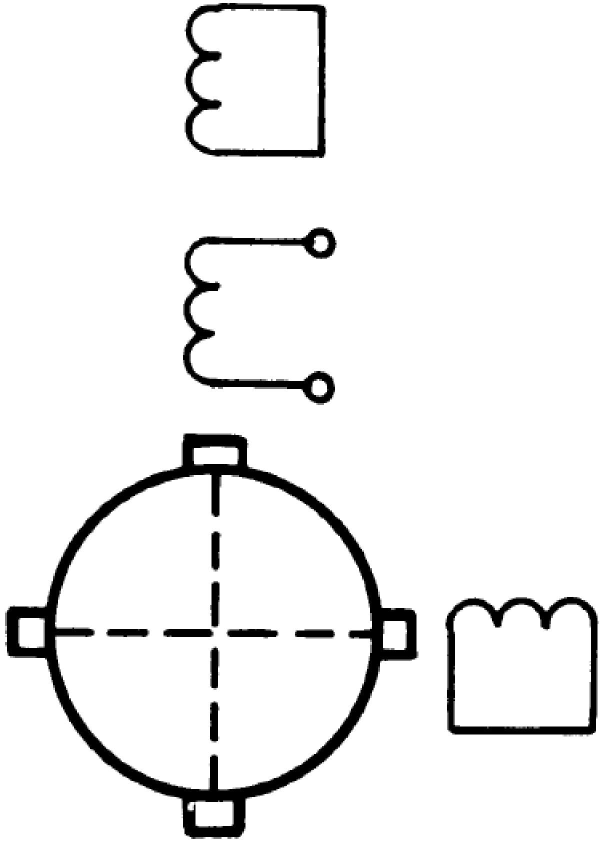


4-coil primitive machine

Primitive Machines

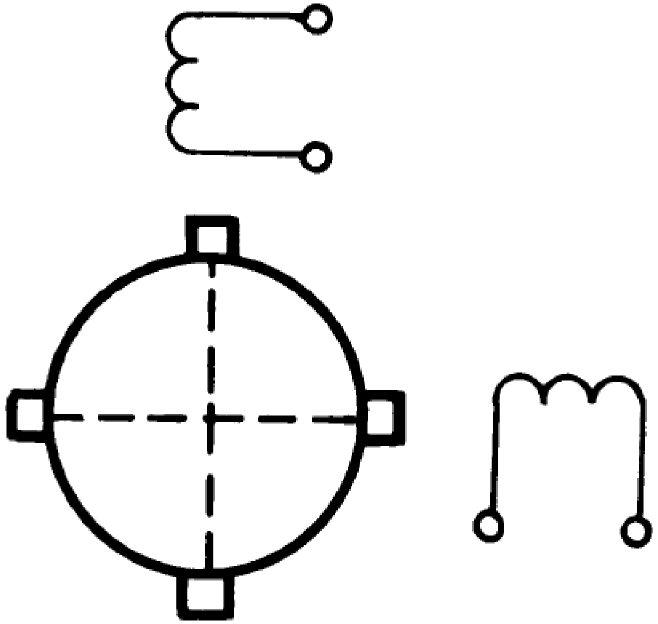
Synchronous machines with amortisseur winding





Primitive Machines

Induction machines



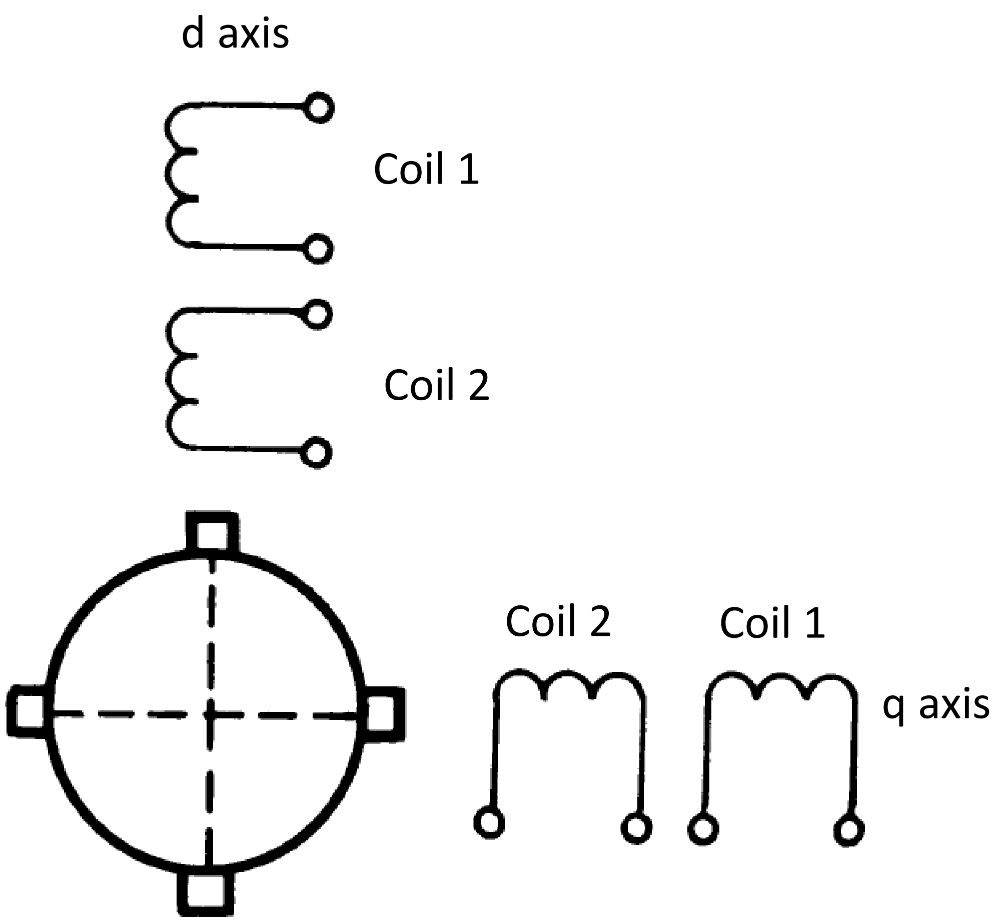
If the observer 'sits on the rotor', the m.m.f. vector produced by the rotor currents rotates at an angular velocity $\omega' = 2\pi sf$ mechanical radian/second with respect to his position. To an observer on the stator, the rotor m.m.f. will appear to rotate at a frequency $\omega' + 2\pi f(l - s) = 2\pi f$.

Since the stator m.m.f. also rotates at an angular frequency $2\pi f$, the stator and rotor m.m.f. waves are again locked and stationary with respect to each other, while rotating synchronously at angular velocity ωs in space.

PRIMITIVE MACHINE EQUATIONS ALONG STATIONARY AXES

FUNDAMENTALS

- Voltage induced in a coil by its own flux
- Voltage induced in a coil by external flux



PRIMITIVE MACHINE EQUATIONS ALONG STATIONARY AXES

voltage induced in a stator coil $V = Ri + \mathrm{d}\psi/\mathrm{d}t$

(transformer voltage)

equation for the applied voltage in a rotor coil will contain the terms

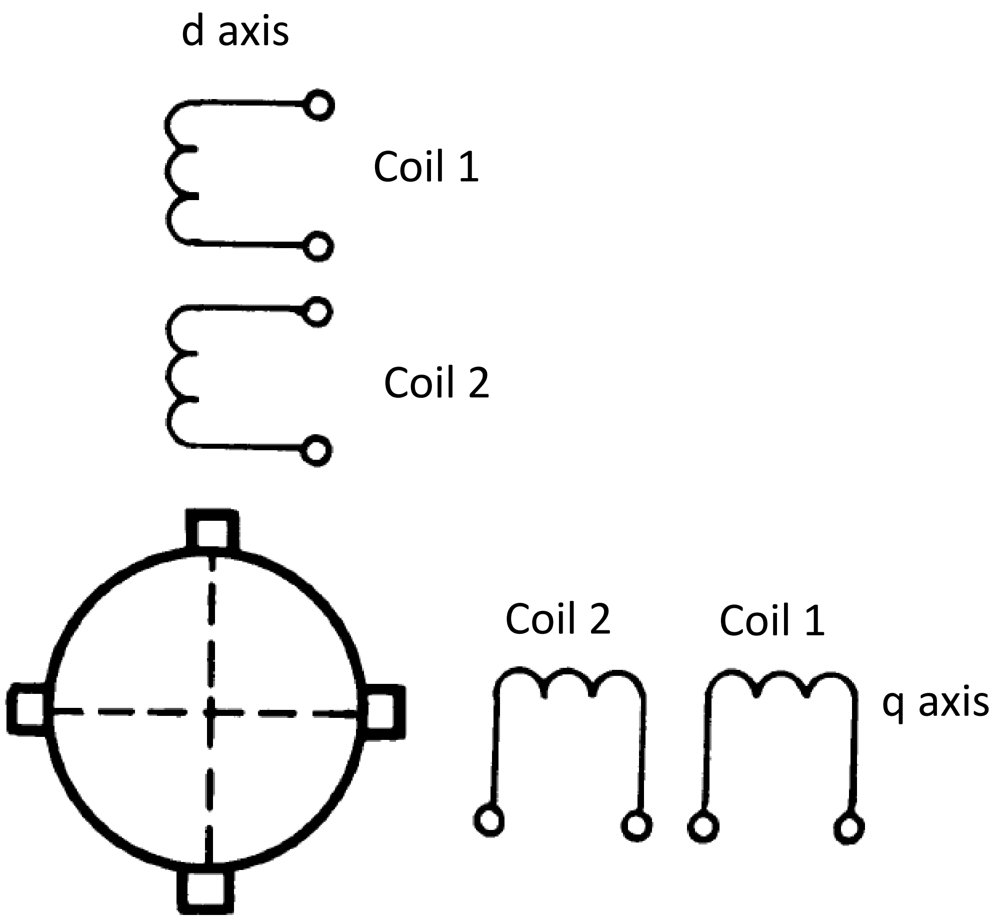
$$\begin{array}{lllll} \mathbf{V} & = \mathbf{Ri} & + \mathrm{d}\psi/\mathrm{d}t & + \mathbf{B}p\theta \\ \text{app.voltage} & = \text{res.} & + \text{transformer} & + \text{generated} \\ & \text{drop} & \text{voltage} & \text{voltage} \end{array}$$

Here $Bp\theta$ is the generated voltage, which depends on the speed of the rotor and the flux density B of the magnetic field in which the coils rotate.

While calculating the transformer voltage, we assume that the conductors are stationary and the flux is changing. We then consider the flux to be constant and the conductors rotating with respect to it, in order to obtain the generated voltage.

PRIMITIVE MACHINE EQUATIONS ALONG STATIONARY AXES

Flux linkages



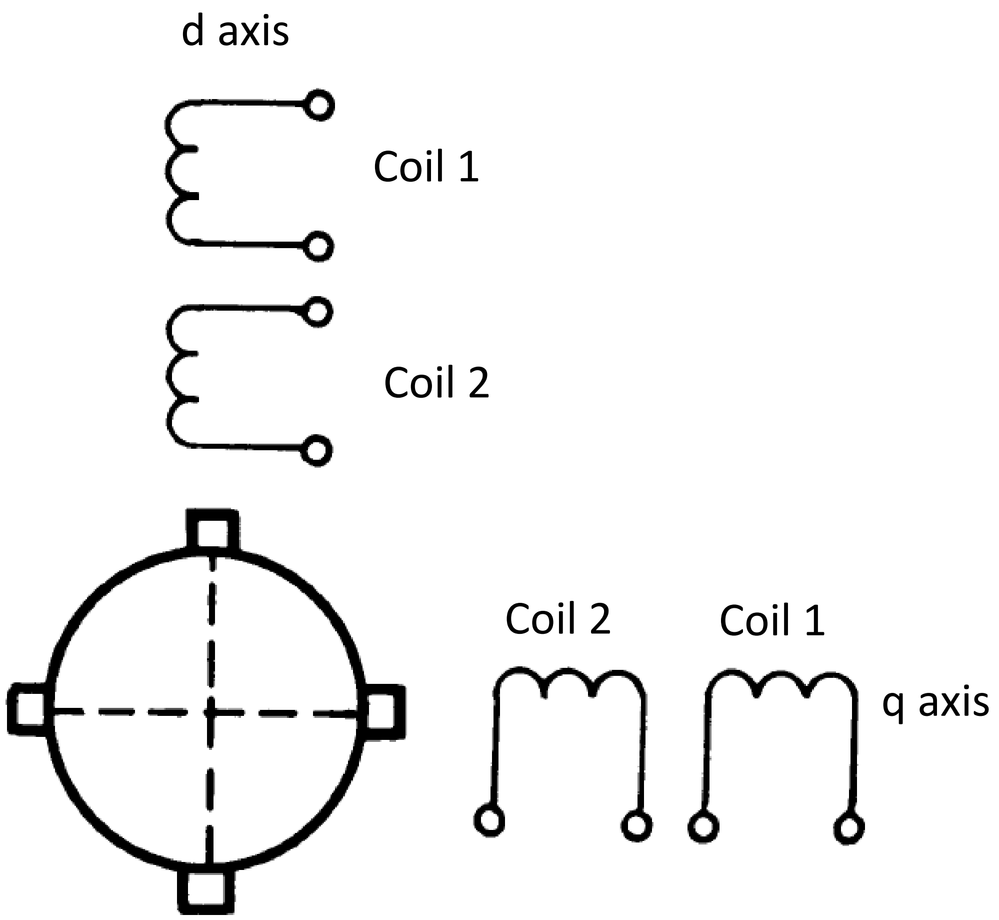
All these coils may not be necessary for a particular machine, in which case the appropriate coils are simply omitted and the equations are modified by deleting the corresponding rows and columns in the matrix equations.

Nomenclature

- The letters d and q obviously stand for direct and quadrature axes
- s for stator coils, r for rotor coils
- M_{d1} is the mutual inductance between coil 1 on the stator and the rotor coil along the direct axis
- M_{d12} is the mutual inductance between coils 1 and 2 in the stator along the direct axis

PRIMITIVE MACHINE EQUATIONS ALONG STATIONARY AXES

Flux linkages



Nomenclature

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- M_{d1} is the mutual inductance between coil 1 on the stator and the rotor coil along the direct axis
- M_{d12} is the mutual inductance between coils 1 and 2 in the stator along the direct axis

$$\psi_{dr} = L_{dr} i_{dr} + M_{d1} i_{ds1} + M_{d2} i_{ds2}$$

$$\psi_{qr} = L_{qr} i_{qr} + M_{q1} i_{qs1} + M_{q2} i_{qs2}$$

$$\psi_{ds1} = L_{ds1} i_{ds1} + M_{d1} i_{dr} + M_{d12} i_{ds2}$$

$$\psi_{qs1} = L_{qs1} i_{qs1} + M_{q1} i_{qr} + M_{q12} i_{qs2}$$

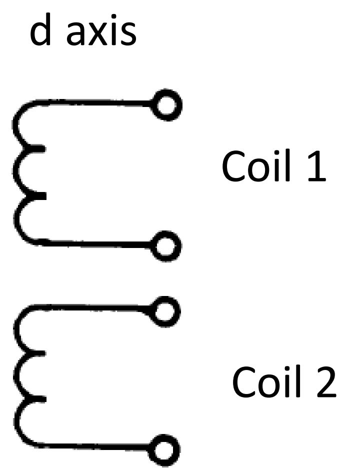
$$\psi_{ds2} = L_{ds2} i_{ds2} + M_{d2} i_{dr} + M_{d12} i_{ds1}$$

$$\psi_{qs2} = L_{qs2} i_{qs2} + M_{q2} i_{qr} + M_{q12} i_{qs1}$$

$L_{dr} \neq L_{qr}$ For salient pole machines

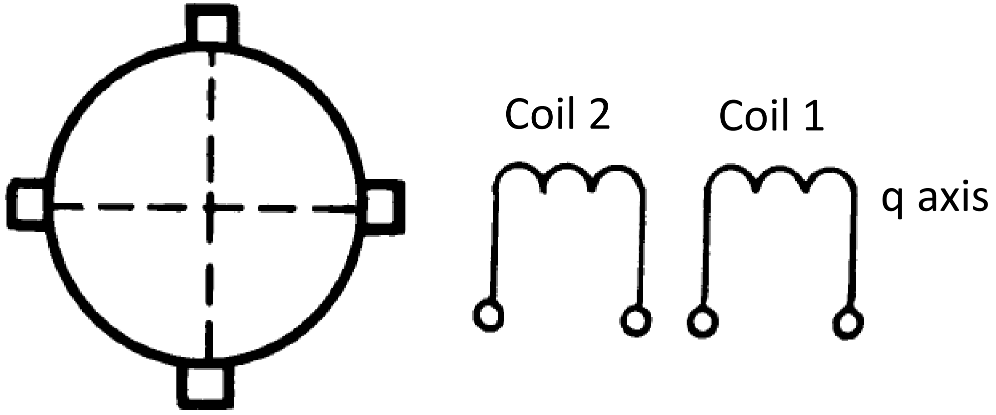
PRIMITIVE MACHINE EQUATIONS ALONG STATIONARY AXES

Stator Voltages



$$V_{ds1} = R_{ds1} i_{ds1} + \frac{d\psi_{ds1}}{dt}$$

$$V_{ds1} = R_{ds1} i_{ds1} + L_{ds1} p i_{ds1} + M_{d1} p i_{dr} + M_{d12} p i_{ds2}$$



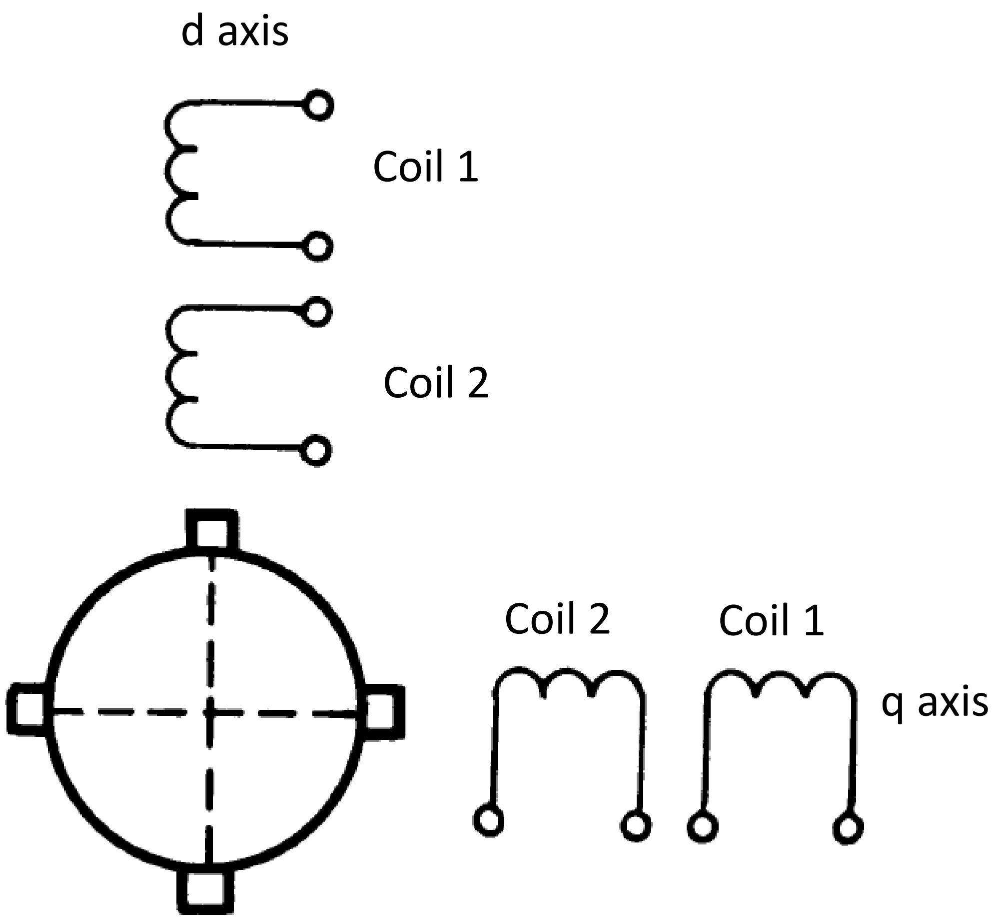
$$V_{ds2} = R_{ds2} i_{ds2} + L_{ds2} p i_{ds2} + M_{d2} p i_{dr} + M_{d12} p i_{ds1}$$

$$V_{qs1} = R_{qs1} i_{qs1} + L_{qs1} p i_{qs1} + M_{q1} p i_{qr} + M_{q12} p i_{qs2}$$

$$V_{qs2} = R_{qs2} i_{qs2} + L_{qs2} p i_{qs2} + M_{q2} p i_{qr} + M_{q12} p i_{qs1}$$

PRIMITIVE MACHINE EQUATIONS ALONG STATIONARY AXES

Rotor Voltages



$$V_{dr} = R_{dr} i_{dr} + \frac{d\psi_{dr}}{dt} + B_{qr} p\theta$$

$$V_{qr} = R_{qr} i_{qr} + \frac{d\psi_{qr}}{dt} - B_{dr} p\theta$$

<u>flux density terms</u>

$$B_{qr} = L'_{qr} i_{qr} + M'_{q1} i_{qs1} + M'_{q2} i_{qs2}$$

$$B_{dr} = L'_{dr} i_{dr} + M'_{d1} i_{ds1} + M'_{d2} i_{ds2}$$

$$V_{dr} = R_{dr} i_{dr} + L_{dr} p i_{dr} + M_{d1} p i_{ds1} + M_{d2} p i_{ds2}$$
$$+ L'_{qr} i_{qr} p\theta + M'_{q1} i_{qs1} p\theta + M'_{q2} i_{qs2} p\theta$$

$$V_{qr} = R_{qr} i_{qr} + L_{qr} p i_{q2} + M_{q1} p i_{qs1} + M_{q2} p i_{qs2}$$
$$- L'_{dr} i_{dr} p\theta - M'_{d1} i_{ds1} p\theta - M'_{d2} i_{ds2} p\theta$$

PRIMITIVE MACHINE EQUATIONS ALONG STATIONARY AXES

$$[\mathbf{V}] = [\mathbf{Z}][\mathbf{i}]$$

$$[\mathbf{V}] = [\mathbf{R}][\mathbf{i}] + [\mathbf{L}]p[\mathbf{i}] + [\mathbf{G}][\mathbf{i}]p\theta$$

$$\mathbf{V} = \mathbf{R}\mathbf{i} + \mathbf{L}p\mathbf{i} + \mathbf{G}\mathbf{i}p\theta$$

$$\mathbf{R} = \begin{array}{c|cccccc} & \mathrm{ds1} & \mathrm{ds2} & \mathrm{dr} & \mathrm{qr} & \mathrm{qs1} & \mathrm{qs2} \\ \hline \mathrm{ds1} & R_{ds1} & & & & & \\ \mathrm{ds2} & & R_{ds2} & & & & \\ \mathrm{dr} & & & R_r & & & \\ \mathrm{qr} & & & & R_r & & \\ \mathrm{qs1} & & & & & R_{qs1} & \\ \mathrm{qs2} & & & & & & R_{qs2} \end{array}$$

$$\mathbf{L} = \begin{array}{c|cccccc} & \mathrm{ds1} & \mathrm{ds2} & \mathrm{dr} & \mathrm{qr} & \mathrm{qs1} & \mathrm{qs2} \\ \hline \mathrm{ds1} & L_{ds1} & M_{d12} & M_{d1} & & & \\ \mathrm{ds2} & M_{d12} & L_{ds2} & M_{d2} & & & \\ \mathrm{dr} & M_{d1} & M_{d2} & L_{dr} & & & \\ \mathrm{qr} & & & & L_{qr} & M_{q1} & M_{q2} \\ \mathrm{qs1} & & & & M_{q1} & L_{qs1} & M_{q12} \\ \mathrm{qs2} & & & & M_{q2} & M_{q12} & L_{qs2} \end{array}$$

$$\mathbf{G} = \begin{array}{c|cccccc} & \mathrm{ds1} & \mathrm{ds2} & \mathrm{dr} & \mathrm{qr} & \mathrm{qs1} & \mathrm{qs2} \\ \hline \mathrm{ds1} & & & & & & \\ \mathrm{ds2} & & & & & & \\ \mathrm{dr} & & & & L'_{qr} & M'_{q1} & M'_{q2} \\ \mathrm{qr} & -M'_{d1} & -M'_{d2} & -L'_{dr} & & & \\ \mathrm{qs1} & & & & & & \\ \mathrm{qs2} & & & & & & \end{array}$$

$$\begin{bmatrix} V_{ds1} \\ V_{ds2} \\ V_{dr} \\ V_{qr} \\ V_{qs1} \\ V_{qs2} \end{bmatrix} = \begin{array}{c|cccccc} & \mathrm{ds1} & \mathrm{ds2} & \mathrm{dr} & \mathrm{qr} & \mathrm{qs1} & \mathrm{qs2} \\ \hline \mathrm{ds1} & R_{ds1}+L_{ds1}p & M_{d12}p & M_{d1}p & & & \\ \mathrm{ds2} & M_{d12}p & R_{ds2}+L_{ds2}p & M_{d2}p & & & \\ \mathrm{dr} & M_{d1}p & M_{d2}p & R_r+L_{dr}p & L'_{qr}p\theta & M'_{q1}p\theta & M'_{q2}p\theta \\ \mathrm{qr} & -M'_{d1}p\theta & -M'_{d2}p\theta & -L'_{dr}p\theta & R_r+L_{qr}p & M_{q1}p & M_{q2}p \\ \mathrm{qs1} & & & & M_{q1}p & R_{qs}+L_{qs1}p & M_{q12}p \\ \mathrm{qs2} & & & & M_{q2}p & M_{q12}p & R_{qs2}+L_{qs2}p \end{array} \cdot \begin{bmatrix} i_{ds1} \\ i_{ds2} \\ i_{dr} \\ i_{qr} \\ i_{qs1} \\ i_{qs2} \end{bmatrix}$$

PRIMITIVE MACHINE EQUATIONS ALONG STATIONARY AXES

$$[\mathbf{V}] = [\mathbf{Z}][\mathbf{i}]$$

$$[\mathbf{V}] = [\mathbf{R}][\mathbf{i}] + [\mathbf{L}]p[\mathbf{i}] + [\mathbf{G}][\mathbf{i}]p\theta$$

$$\mathbf{V} = \mathbf{Ri} + \mathbf{L}p\mathbf{i} + \mathbf{Gi}p\theta$$

power input to the motor $\quad \mathbf{i}_t^*\mathbf{V} = \mathbf{i}_t^*\mathbf{Ri} + \mathbf{i}_t^*\mathbf{L}p\mathbf{i} + \mathbf{i}_t^*\mathbf{Gi}p\theta$

dissipated as copper loss

rate of change of stored magnetic energy $\quad \dfrac{\mathrm{d}}{\mathrm{d}t}(\tfrac{1}{2}\mathbf{Li}^2) = \mathbf{iL}p\mathbf{i}$

mechanical power at the output shaft

$$P_\mathrm{m} = \omega T = Tp\theta$$

$$T_\mathrm{o} = \mathbf{i}_t^*\mathbf{Gi} = \mathbf{i}_t^*\mathbf{B}$$

G is the torque matrix.
B represents flux density in the air gap

PRIMITIVE MACHINE EQUATIONS ALONG STATIONARY AXES

$$T_i = Jp^2\theta + R_F p\theta - \mathbf{i}_t^* \mathbf{G}\mathbf{i}$$

T_i represents the mechanical input torque

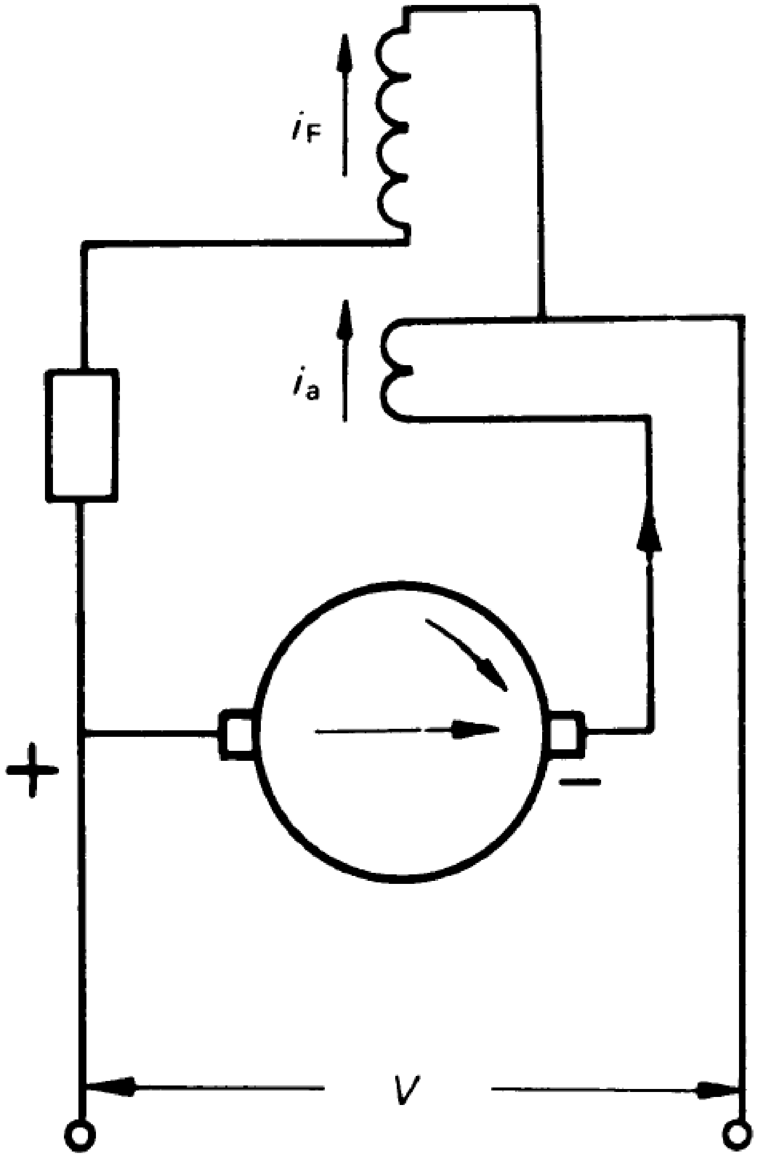
R_F is a mechanical coefficient representing dissipation due to friction and windage

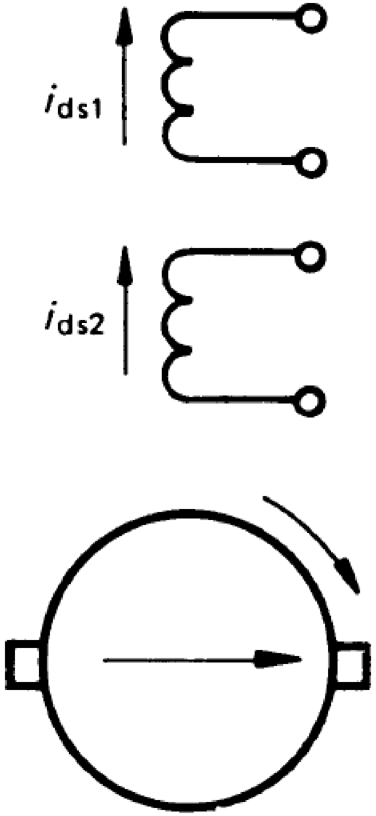
J is the moment of inertia of the rotating mass (kg m^2)

$$\begin{array}{cc} \text{elect.} \\ \text{mech.} \end{array} \begin{bmatrix} \mathbf{V} \\ T_i \end{bmatrix} = \begin{array}{c} \begin{array}{cc} \text{elect.} & \text{mech.} \end{array} \\ \begin{bmatrix} \mathbf{R} + \mathbf{L}p & \mathbf{G}\mathbf{i} \\ -\mathbf{i}_t^* \mathbf{G} & Jp + R_F \end{bmatrix} \end{array} \cdot \begin{bmatrix} \mathbf{i} \\ \omega \end{bmatrix}$$

$$\begin{bmatrix} V_{ds1} \\ V_{dr} \\ V_{qr} \\ V_{qs1} \\ T_i \end{bmatrix} = \begin{array}{c|ccccc} & ds1 & dr & qr & qs1 & s \\ \hline ds1 & R_{ds1} + L_{ds1}p & M_{d1}p & & & \\ dr & M_{d1}p & R_r + L_{dr}p & & & L'_{qr} i_{qr} + M'_{q1} i_{qs1} \\ qr & & & R_r + L_{qr}p & M_{q1}p & -M'_{d1} i_{ds1} - L'_{dr} i_{dr} \\ qs1 & & & M_{q1}p & R_r + L_{qs1}p & \\ s & i_{qr} M'_{d1} & -i_{qr} L'_{dr} & i_{dr} L'_{qr} & i_{dr} M'_{q1} & Jp + R_F \end{array} \cdot \begin{bmatrix} i_{ds1} \\ i_{dr} \\ i_{qr} \\ i_{qs1} \\ p\theta \end{bmatrix}$$

ANALYSIS OF DC MACHINE USING PRIMITIVE MACHINE EQUATIONS





Connection Matrix

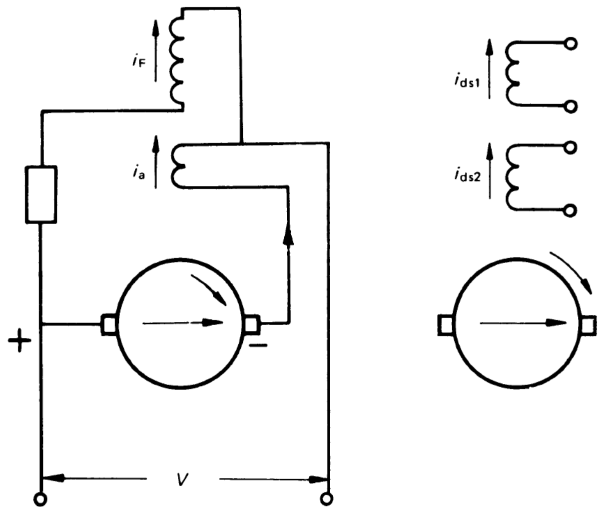
$$\begin{bmatrix} i_{ds1} \\ i_{ds2} \\ i_{qr} \end{bmatrix} = \begin{array}{c|cc} & f & a \\ \hline ds1 & 1 & \\ ds2 & & 1 \\ qr & & 1 \end{array} \cdot \begin{bmatrix} i_f \\ i_a \end{bmatrix}$$

$$\mathbf{C}_t = \begin{array}{c|ccc} & ds1 & ds2 & qr \\ \hline f & 1 & & \\ a & & 1 & 1 \end{array}$$

Impedance Transformation: $\mathbf{Z}' = \mathbf{C}_t \mathbf{Z} \mathbf{C}$

Voltage Equation: $\mathbf{C}_t \mathbf{V}$

ANALYSIS OF DC MACHINE USING PRIMITIVE MACHINE EQUATIONS



Impedance Transformation:

$$\mathbf{Z}' = \mathbf{C}_t \mathbf{Z} \mathbf{C}$$

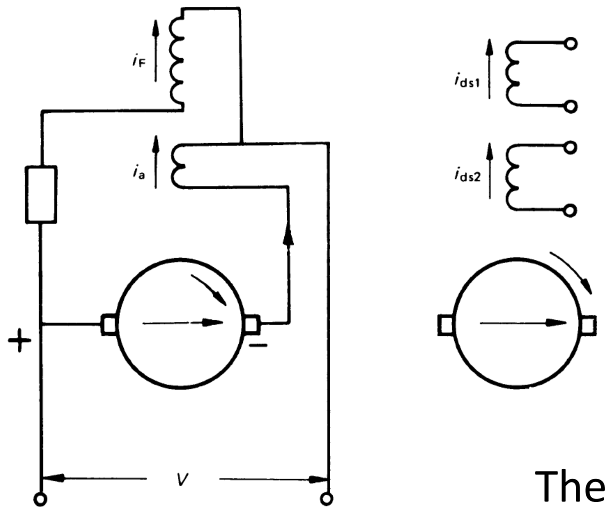
$$\begin{array}{c} \\ \mathrm{f} \\ \mathrm{a} \end{array}\!\!\begin{array}{c} \begin{array}{ccc} \mathrm{ds1} & \mathrm{ds2} & \mathrm{qr} \end{array} \\ \begin{bmatrix} 1 & & \\ & 1 & 1 \end{bmatrix} \end{array}
\quad
\begin{array}{c|ccc} & \mathrm{ds1} & \mathrm{ds2} & \mathrm{qr} \\ \hline \mathrm{ds1} & R_x + R_{ds1} + L_{ds1}p & M_{d12}p & \\ \mathrm{ds2} & M_{d12}p & R_{ds2} + L_{ds2}p & \\ \mathrm{qr} & -M'_{d1}p\theta & -M'_{d2}p\theta & R_r + L_{qr}p \end{array}
\; . \;
\begin{array}{c|cc} & \mathrm{f} & \mathrm{a} \\ \hline \mathrm{ds1} & 1 & \\ \mathrm{ds2} & & 1 \\ \mathrm{qr} & & 1 \end{array}$$

Rx represents a control resistance in series with the field winding.

Final impedance matrix

$$\mathbf{Z} = \begin{array}{c|cc} & \mathrm{f} & \mathrm{a} \\ \hline \mathrm{f} & R_x + R_{ds1} + L_{ds1}p & M_{d12}p \\ \mathrm{a} & M_{d12}p - M'_{d1}p\theta & (R_r + R_{ds2}) + (L_{qr} + L_{ds2})p - M'_{d2}p\theta \end{array}$$

ANALYSIS OF DC MACHINE USING PRIMITIVE MACHINE EQUATIONS



Voltage Equation: $\mathbf{C_t V}$

$$
\begin{array}{c|ccc|}
 & ds1 & ds2 & qr \\
\hline
f & 1 & & \\
a & & 1 & 1 \\
\hline
\end{array}
\cdot
\begin{array}{c|c|}
\hline
ds1 & V_{ds1} \\
ds2 & V_{ds2} \\
qr & V_{qr} \\
\hline
\end{array}
$$

The values of V_{ds1} and (V_{ds2} + V_{qr}) are known. These are equal to the applied voltage V. Hence the final voltage equation

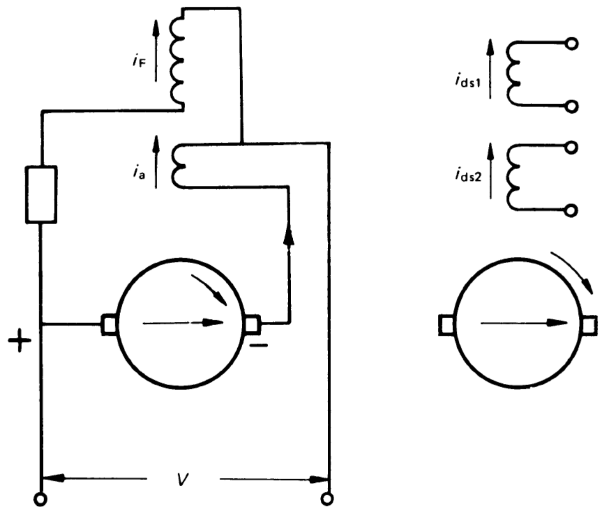
$$
= \begin{array}{c|c|}
\hline
f & V_{ds1} \\
a & V_{ds2} + V_{qr} \\
\hline
\end{array}
$$

Final Voltage Equation

$$
\begin{bmatrix} V \\ V \end{bmatrix}
=
\begin{array}{c|cc|}
 & f & a \\
\hline
f & R_x + R_{ds1} + L_{ds1}p & M_{d12}p \\
a & M_{d12}p - M'_{d1}p\theta & (R_r + R_{ds2}) + (L_{qr} + L_{ds2})p - M'_{d2}p\theta \\
\hline
\end{array}
\cdot
\begin{bmatrix} i_f \\ i_a \end{bmatrix}
$$

For steady-state operation the rate of change of current in this machine is zero, and the operator to current p = d/dt = 0.

ANALYSIS OF DC MACHINE USING PRIMITIVE MACHINE EQUATIONS



Final Voltage Equation

$$\begin{array}{c} \\ \\ \end{array}\begin{bmatrix} V \\ V \end{bmatrix} = \begin{array}{c|cc} & \text{f} & \text{a} \\ \hline \text{f} & R_x + R_{ds1} + L_{ds1}p & M_{d12}p \\ \text{a} & M_{d12}p - M'_{d1}p\theta & (R_r + R_{ds2}) + (L_{qr} + L_{ds2})p - M'_{d2}p\theta \end{array} \cdot \begin{bmatrix} i_f \\ i_a \end{bmatrix}$$

For steady-state operation the rate of change of current in this machine is zero, and the operator to current p = d/dt = 0.

$-M'_{d1}p\theta i_f$: Generated EMF due to the main field. In a motor this is the back EMF.

When the machine runs as a generator, i_a becomes negative

$\omega = p\theta$: Angular velocity

$-M'_{d2}p\theta i_a$:Series compounding

SQUIRREL CAGE INDUCTION MOTOR MODEL FROM PRIMITIVE MACHINE EQUATIONS

Step 1

Remove coil 2 and corresponding rows and column

Replace the operator p by $j\omega_1$

$p\theta = \omega_r$

$$\omega_r = \omega_1(1-s)$$

$$\begin{array}{c} \\ \begin{bmatrix} V_{ds1} \\ V_{dr} \\ V_{qr} \\ V_{qs1} \end{bmatrix} \end{array} = \begin{array}{c} \\ ds1 \\ dr \\ qr \\ qs1 \end{array} \begin{array}{c} \begin{array}{cccc} ds1 & dr & qr & qs1 \end{array} \\ \begin{bmatrix} R_{ds1} + j\omega_1 L_{ds1} & j\omega_1 M_{d1} & & \\ j\omega_1 M_{d1} & R_r + j\omega_1 L_{dr} & \omega_1(1-s)M_{q1} & \omega_1(1-s)M_{q1} \\ -\omega_1(1-s)M_{d1} & -\omega_1(1-s)L_{dr} & R_r + j\omega_1 L_{qr} & j\omega_1 M_{q1} \\ & & j\omega_1 M_{q1} & R_{qs1} + j\omega_1 L_{qs1} \end{bmatrix} \end{array} \begin{array}{c} \\ \begin{bmatrix} i_{ds1} \\ i_{dr} \\ i_{qr} \\ i_{qs1} \end{bmatrix} \end{array}$$

SQUIRREL CAGE INDUCTION MOTOR MODEL FROM PRIMITIVE MACHINE EQUATIONS

Step 2

$$R_{ds1} = R_{qs1} = R_1, \qquad R_r = R_2$$

$$L_{ds1} = L_{qs1} = L_1, \qquad L_{dr} = L_{qr} = L_2$$

$$M_{d1} = M_{q1} = M$$

$$M'_{d1} = M_{d1} = M$$

$$M'_{q1} = M_{q1} = M$$

$$\begin{bmatrix} V_{ds1} \\ V_{dr} \\ V_{qr} \\ V_{qs1} \end{bmatrix} = \begin{array}{c|cccc} & ds1 & dr & qr & qs1 \\ \hline ds1 & R_1 + jX_1 & jX_m & & \\ dr & jX_m & R_2 + jX_2 & (1-s)X_2 & (1-s)X_m \\ qr & -(1-s)X_m & -(1-s)X_2 & R_2 + jX_2 & jX_m \\ qs1 & & & jX_m & R_1 + jX_1 \end{array} \begin{bmatrix} i_{ds1} \\ i_{dr} \\ i_{qr} \\ i_{qs1} \end{bmatrix}$$

stator coils which are symmetrically distributed

air gap is uniform

flux wave is sinusoidally distributed in space and hence the coefficients of mutual inductance for transformer and generated voltages are the same

SQUIRREL CAGE INDUCTION MOTOR MODEL FROM PRIMITIVE MACHINE EQUATIONS

Step 3

$$
\begin{array}{c c c c c c c c}
 & & \mathrm{ds1} & \mathrm{dr} & \mathrm{qr} & \mathrm{qs1} & & \\
\left[\begin{array}{c} V_1 \\ 0 \\ 0 \\ -jV_1 \end{array}\right] & \begin{array}{r} \mathrm{ds1} \\ = \mathrm{dr} \\ \mathrm{qr} \\ \mathrm{qs1} \end{array} & \multicolumn{4}{c}{\left[\begin{array}{cccc} R_1 + jX_1 & jX_m & & \\ jX_m & R_2 + jX_2 & (1-s)X_2 & (1-s)X_m \\ -(1-s)X_m & -(1-s)X_2 & R_2 + jX_2 & jX_m \\ & & jX_m & R_1 + jX_1 \end{array}\right]} & \cdot & \left[\begin{array}{c} I_1 \\ I_2 \\ -jI_2 \\ -jI_1 \end{array}\right]
\end{array}
$$

net m.m.f. they produce rotates at synchronous speed

During balanced operation

$$\bar{N}_1 (I_1^2 \sin^2 \omega_1 t + I_1^2 \cos \omega_1 t)^{\frac{1}{2}}$$

rotor voltages are zero since the rotor coils in an induction motor are short-circuited

SQUIRREL CAGE INDUCTION MOTOR MODEL FROM PRIMITIVE MACHINE EQUATIONS

dr

$$0 = jX_m I_1 - jI_1(1-s)X_m + (R_2 + jX_2)I_2 - j(1-s)X_2 I_2$$

$$= jI_1 sX_m + (R_2 + jsX_2)I_2$$

$$= jI_1 X_m + \left[\frac{R_2}{s} + jX_2\right]I_2$$

qr

$$0 = jI_1 X_m + \left[\frac{R_2}{s} + jX_2\right]I_2$$

$$\begin{bmatrix} V_1 \\ 0 \end{bmatrix} = \begin{array}{c} \\ \text{ds1} \\ \text{dr} \end{array} \begin{array}{cc} \text{ds1} & \text{dr} \\ \left[\begin{matrix} R_1 + jX_1 & jX_m \\ jX_m & \frac{R_2}{s} + jX_2 \end{matrix}\right. & \left.\right] \end{array} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ 0 \end{bmatrix} = \begin{array}{c} \\ \text{ds1} \\ \text{dr} \end{array} \begin{array}{cc} \text{ds1} & \text{dr} \\ \left[\begin{matrix} R_1 + jX_1 & jX_m \\ jX_m & \frac{(R_2 + jX_2) + R_2(1-s)}{s} \end{matrix}\right. & \left.\right] \end{array} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Clarification

$$
\begin{array}{c|c|c}
 & & \begin{array}{cccc} ds1 & dr & qr & qs1 \end{array} \\
\begin{bmatrix} V_1 \\ 0 \\ 0 \\ -jV_1 \end{bmatrix} & = \begin{array}{c} ds1 \\ dr \\ qr \\ qs1 \end{array} & \begin{bmatrix} R_1+jX_1 & jX_m & & \\ jX_m & R_2+jX_2 & (1-s)X_2 & (1-s)X_m \\ -(1-s)X_m & -(1-s)X_2 & R_2+jX_2 & jX_m \\ & & jX_m & R_1+jX_1 \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \\ -jI_2 \\ -jI_1 \end{bmatrix}
\end{array}
$$

dr

$$0 = jX_m I_1 - jI_1(1-s)X_m + (R_2+jX_2)I_2 - j(1-s)X_2 I_2$$

$$= jI_1 sX_m + (R_2 + jsX_2)I_2$$

$$= jI_1 X_m + \left[\frac{R_2}{s} + jX_2\right] I_2$$

$$
\begin{array}{ccc}
 & & \begin{array}{cc} ds1 & dr \end{array} \\
\begin{bmatrix} V_1 \\ 0 \end{bmatrix} & = \begin{array}{c} ds1 \\ dr \end{array} & \begin{bmatrix} R_1+jX_1 & jX_m \\ jX_m & \frac{R_2}{s}+jX_2 \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}
\end{array}
$$

qr

$$0 = jI_1 X_m + \left[\frac{R_2}{s} + jX_2\right] I_2$$

$$
\begin{array}{ccc}
 & & \begin{array}{cc} ds1 & dr \end{array} \\
\begin{bmatrix} V_1 \\ 0 \end{bmatrix} & = \begin{array}{c} ds1 \\ dr \end{array} & \begin{bmatrix} R_1+jX_1 & jX_m \\ jX_m & \frac{(R_2+jX_2)+R_2(1-s)}{s} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}
\end{array}
$$

SQUIRREL CAGE INDUCTION MOTOR MODEL FROM PRIMITIVE MACHINE EQUATIONS

Part of Assignment 1

Generalized Torque Equation

$$T = -\mathbf{i}^{*}\mathbf{G}\mathbf{i}$$

Expand

Assume Balanced condition. Terms containing idr and iqr get cancels (substitute as currents in equal magnitudes, 90deg phase lag)

$$= -i_{qr}^{*} M_{d} i_{ds} + i_{dr}^{*} M_{q} i_{qs}$$

$$\begin{array}{ccccc} & & & \text{elect.} & \text{mech.} \\ \text{elect.} & \mathbf{V} & & \mathbf{R}+\mathbf{L}p & \mathbf{G}\mathbf{i} \\ & & = & & & \cdot & \begin{array}{c}\mathbf{i} \\ \omega\end{array} \\ \text{mech.} & T_{i} & & -\mathbf{i}_{t}^{*}\mathbf{G} & Jp+R_{F} \end{array}$$

$$\mathbf{G} = \begin{array}{c} \\ \text{ds1} \\ \text{ds2} \\ \text{dr} \\ \text{qr} \\ \text{qs1} \\ \text{qs2} \end{array} \begin{array}{cccccc} \text{ds1} & \text{ds2} & \text{dr} & \text{qr} & \text{qs1} & \text{qs2} \\ \hline & & & & & \\ & & & & & \\ & & & L'_{qr} & M'_{q1} & M'_{q2} \\ -M'_{d1} & & -L'_{dr} & & & \\ & & & & & \\ & & & & & \end{array}$$

SQUIRREL CAGE INDUCTION MOTOR MODEL FROM PRIMITIVE MACHINE EQUATIONS

Part of Assignment 1

$$\begin{bmatrix} V_1 \\ 0 \end{bmatrix} = \begin{array}{c|cc} & \mathrm{ds1} & \mathrm{dr} \\ \hline \mathrm{ds1} & R_1 + jX_1 & jX_m \\ \mathrm{dr} & jX_m & \frac{R_2}{s} + jX_2 \end{array} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} V \\ 0 \end{bmatrix} = \begin{array}{c|cc} & \mathrm{ds} & \mathrm{dr} \\ \hline \mathrm{ds} & R_{ds} + jX_{ds} & jX_m \\ \mathrm{dr} & jsX_m & R_{dr} + jsX_{dr} \end{array} \cdot \begin{bmatrix} i_{ds} \\ i_{dr} \end{bmatrix}$$

$$\begin{bmatrix} i_{ds} \\ i_{dr} \end{bmatrix} = \frac{1}{D} \begin{array}{c|cc} & \mathrm{ds} & \mathrm{dr} \\ \hline \mathrm{ds} & Z_{dr} & -jX_m \\ \mathrm{dr} & -jsX_m & Z_{ds} \end{array} \cdot \begin{bmatrix} V \\ 0 \end{bmatrix}$$

D is the determinant

$$i_{ds} = \frac{1}{D}(Z_{dr} V)$$

$$i_{dr} = jsX_m V/D$$

SQUIRREL CAGE INDUCTION MOTOR MODEL FROM PRIMITIVE MACHINE EQUATIONS

Part of Assignment 1

$$i_{ds} = \frac{1}{D}(Z_{dr} V) \qquad i_{dr} = jsX_m V/D$$

$$T = -\mathbf{i}^* \mathbf{G} \mathbf{i}$$

$$= -i_{qr}^* M_d i_{ds} + i_{dr}^* M_q i_{qs}$$

ωT= Synchronous Power in Watts

$\omega M = X_m$

Per phase synchronous power $T_{ph} = -i_{ds} j X_m i_{dr}^* \quad \text{(synchronous W)}$

SQUIRREL CAGE INDUCTION MOTOR MODEL FROM PRIMITIVE MACHINE EQUATIONS

Part of Assignment 1

Per phase synchronous power $T_{ph} = -i_{ds} jX_m i_{dr}^* \quad \text{(synchronous W)}$

Substitute for i_{ds} in terms of V
Substitute for V in terms of i_{dr}
$Z_{dr}=R_r$

$$i_{ds} = \frac{1}{D}(Z_{dr}V)$$

$$i_{dr} = jsX_m V/D$$

Generated Shaft torque in Sync. Watts/phase = Per phase synchronous power - Per phase rotor copper loss

Per phase rotor copper loss= $i_{dr}^2 R_r$

$$= i_{dr}^2 R_r\left(\frac{1}{s} - 1\right)$$

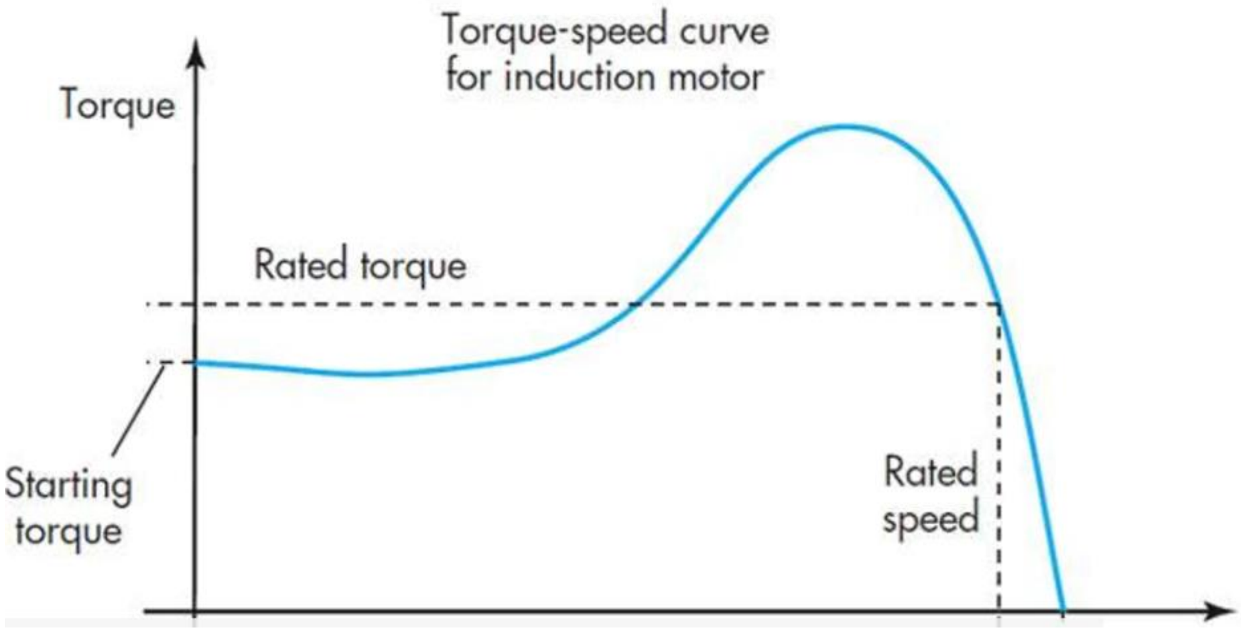
This part to be submitted on or before 01.12.2021

Submission folder available at Eduserver. All answer scripts as part of Assignment 1 to be submitted in this folder

Inference

$$T = -\mathbf{i}^* \mathbf{G} \mathbf{i}$$

$$= -i_{qr}^* M_d i_{ds} + i_{dr}^* M_q i_{qs}$$



SQUIRREL CAGE INDUCTION MOTOR MODEL FROM PRIMITIVE MACHINE EQUATIONS

Power Invariance

Power input to the stator of the primitive machine is

$$P_s = V_{ds1}^* I_{ds1} + V_{qs1}^* I_{qs1}$$

For balanced operation of the induction motor

$$P_s = 2 V_1^* I_1$$

Power input to the stator of a 3-phase induction motor in terms of phase voltage and phase current

$$P_s = 3 V_p^* I_p$$

power is invariant $\quad 2 V_1^* I_1 = 3 V_p^* I_p$

equate the copper-loss in the stator windings in the 3-phase machine and its 'commutator-type' equivalent

$$3 I_p^2 R_1 = 2 I_1^2 R_1$$

assuming that the resistance $R_{ds1} = R_1$

$$I_1 = \sqrt{\frac{3}{2}} I_p \qquad V_1 = \sqrt{\frac{3}{2}} V_p$$

Squirrel Cage Induction Motor: Transient Stability Analysis

$$[\mathbf{V}] = [\mathbf{Z}][\mathbf{i}]$$

$$[\mathbf{V}] = [\mathbf{R}][\mathbf{i}] + [\mathbf{L}]p[\mathbf{i}] + [\mathbf{G}][\mathbf{i}]p\theta$$

$$\mathbf{V} = \mathbf{R}\mathbf{i} + \mathbf{L}p\mathbf{i} + \mathbf{G}\mathbf{i}p\theta$$

$$\mathbf{i} = \mathbf{Z}^{-1}\mathbf{V}$$

Solution is difficult to achieve as Z contains *p*-terms large number of variables is involved

$$\mathbf{L}p\mathbf{i} = \mathbf{V} - \mathbf{R}\mathbf{i} - \mathbf{G}\mathbf{i}p\theta$$

$$p\mathbf{i} = \mathbf{L}^{-1}\mathbf{V} - \mathbf{L}^{-1}(\mathbf{R} + \mathbf{G}p\theta)\mathbf{i}$$

Squirrel Cage Induction Motor: Transient Stability Analysis

Part of Assignment 1

This part to be submitted on or before 13.12.2021

A three coil is considered here for simplicity of calculation

$$p\begin{bmatrix} i_{ds} \\ i_{dr} \\ i_{qr} \end{bmatrix} = \begin{bmatrix} L_{ds1} & M_d & \\ M_d & L_{dr} & \\ & & L_{qr} \end{bmatrix}^{-1} \begin{bmatrix} V_{ds1} \\ V_{dr} \\ V_{qr} \end{bmatrix} - \begin{bmatrix} L_{ds1} & M_d & \\ M_d & L_{dr} & \\ & & L_{qr} \end{bmatrix}^{-1} \begin{bmatrix} R_{ds1} & & \\ & R_r & L_{qr}\omega \\ -\omega M_d & -\omega L_{dr} & R_r \end{bmatrix} \begin{bmatrix} L_{ds} \\ i_{dr} \\ i_{qr} \end{bmatrix}$$

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

$$\dot{\mathbf{x}} = p\mathbf{i}$$

$$\mathbf{A} = -(\mathbf{L}^{-1}\mathbf{R} + \mathbf{L}^{-1}\mathbf{G}p\theta)$$

$$\mathbf{B}\mathbf{u} = \mathbf{L}^{-1}\mathbf{V}$$

In this case the matrices A and B have constant terms if the angular velocity pθ does not change when currents and voltages change under sudden transient conditions. This assumption is not true in general but it is valid in many cases. For correct analysis, it is necessary to take into account changes in the rotor velocity, and we have to consider the torque equation also.

Squirrel Cage Induction Motor: Transient Stability Analysis

$$\begin{bmatrix} \mathbf{V} \\ T_i \end{bmatrix} = \begin{bmatrix} \mathbf{R}+\mathbf{L}p & \mathbf{B} \\ -\mathbf{B} & Jp+R_F \end{bmatrix} \cdot \begin{bmatrix} \mathbf{i} \\ \omega \end{bmatrix}$$

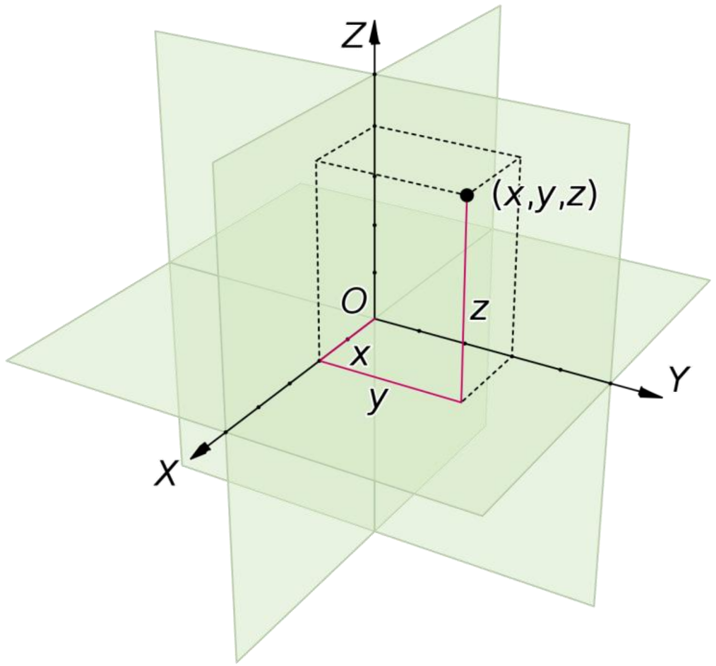
$$\begin{bmatrix} \mathbf{V} \\ T_i \end{bmatrix} = \begin{bmatrix} \mathbf{R} & \\ & R_F \end{bmatrix} \cdot \begin{bmatrix} \mathbf{i} \\ \omega \end{bmatrix} + \begin{bmatrix} \mathbf{L} & \\ & J \end{bmatrix} p \begin{bmatrix} \mathbf{i} \\ \omega \end{bmatrix} + \begin{bmatrix} & \mathbf{B} \\ -\mathbf{B} & \end{bmatrix} \cdot \begin{bmatrix} \mathbf{i} \\ \omega \end{bmatrix}$$

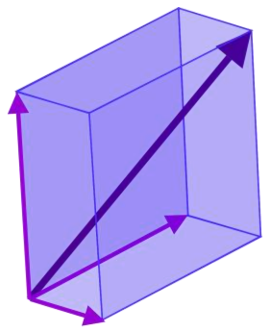
$$p \begin{bmatrix} \mathbf{i} \\ \omega \end{bmatrix} = \underbrace{-\begin{bmatrix} \mathbf{L} & \\ & J \end{bmatrix}^{-1} \cdot \begin{bmatrix} \mathbf{R} & \mathbf{B} \\ -\mathbf{B} & R_F \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} \mathbf{i} \\ \omega \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} \mathbf{L} & \\ & J \end{bmatrix}^{-1}}_{B} \cdot \begin{bmatrix} \mathbf{V} \\ T_i \end{bmatrix}$$

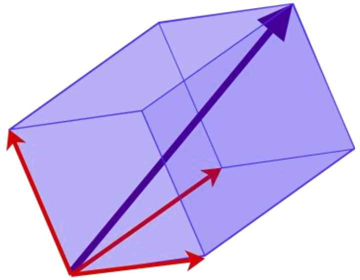
The matrix A is time-dependent, and a function of current. The solution must therefore be obtained numerically.

Fundamentals

- State Space
- State Vector
- Eigen Values
- Eigen Vector
- State Trajectory
- Vector space, Basis







Squirrel Cage Induction Motor: Stability Analysis

$$\begin{bmatrix} \mathbf{V} \\ T_i \end{bmatrix} = \begin{bmatrix} \mathbf{R} + \mathbf{L}p & \mathbf{B} \\ -\mathbf{B} & Jp + R_F \end{bmatrix} \cdot \begin{bmatrix} \mathbf{i} \\ \omega \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{V} \\ T_i \end{bmatrix} = \begin{bmatrix} \mathbf{R} & \\ & R_F \end{bmatrix} \cdot \begin{bmatrix} \mathbf{i} \\ \omega \end{bmatrix} + \begin{bmatrix} \mathbf{L} & \\ & J \end{bmatrix} p \begin{bmatrix} \mathbf{i} \\ \omega \end{bmatrix} + \begin{bmatrix} & \mathbf{B} \\ -\mathbf{B} & \end{bmatrix} \cdot \begin{bmatrix} \mathbf{i} \\ \omega \end{bmatrix}$$

$$p \begin{bmatrix} \mathbf{i} \\ \omega \end{bmatrix} = - \begin{bmatrix} \mathbf{L} & \\ & J \end{bmatrix}^{-1} \cdot \begin{bmatrix} \mathbf{R} & \mathbf{B} \\ -\mathbf{B} & R_F \end{bmatrix} \cdot \begin{bmatrix} \mathbf{i} \\ \omega \end{bmatrix} + \begin{bmatrix} \mathbf{L} & \\ & J \end{bmatrix}^{-1} \cdot \begin{bmatrix} \mathbf{V} \\ T_i \end{bmatrix}$$

Squirrel Cage Induction Motor: Stability Analysis

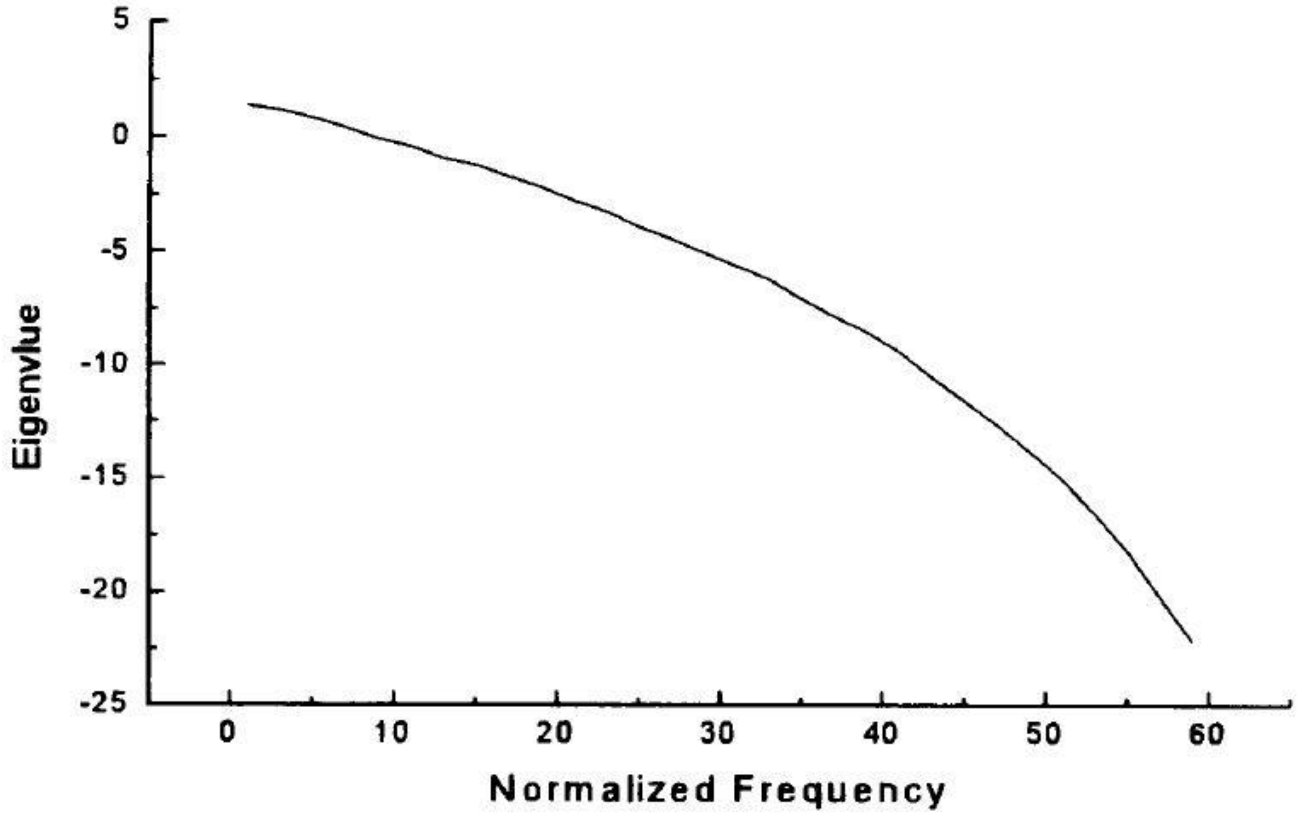
Case Study

Machines Parameters	Value	Per Unit Value
Horse Power (Hp)	50 hp	-
Voltage (V_L)	460 V	-
Frequency (Hz)	60 Hz	-
Stator Resistance (r_s)	0.087 Ω	0.015336
Stator Reactance (X_{ls})	0.302 Ω	0.053235
Mutual Reactance (X_M)	13.08 Ω	2.30569
Equivalent Rotor Resistance (r'_r)	0.302 Ω	0.040191
Equivalent Rotor Reactance (X'_{lr})	0.228 Ω	0.053235
Moment of Inertia (J)	1.662 Ω	-

M. B. Uddin, M. N. Pramanik and S. A. Reza, "Low frequency stability study of a three-phase induction motor," *2007 7th International Conference on Power Electronics*, 2007, pp. 1115-1120

Squirrel Cage Induction Motor: Stability Analysis

Inference



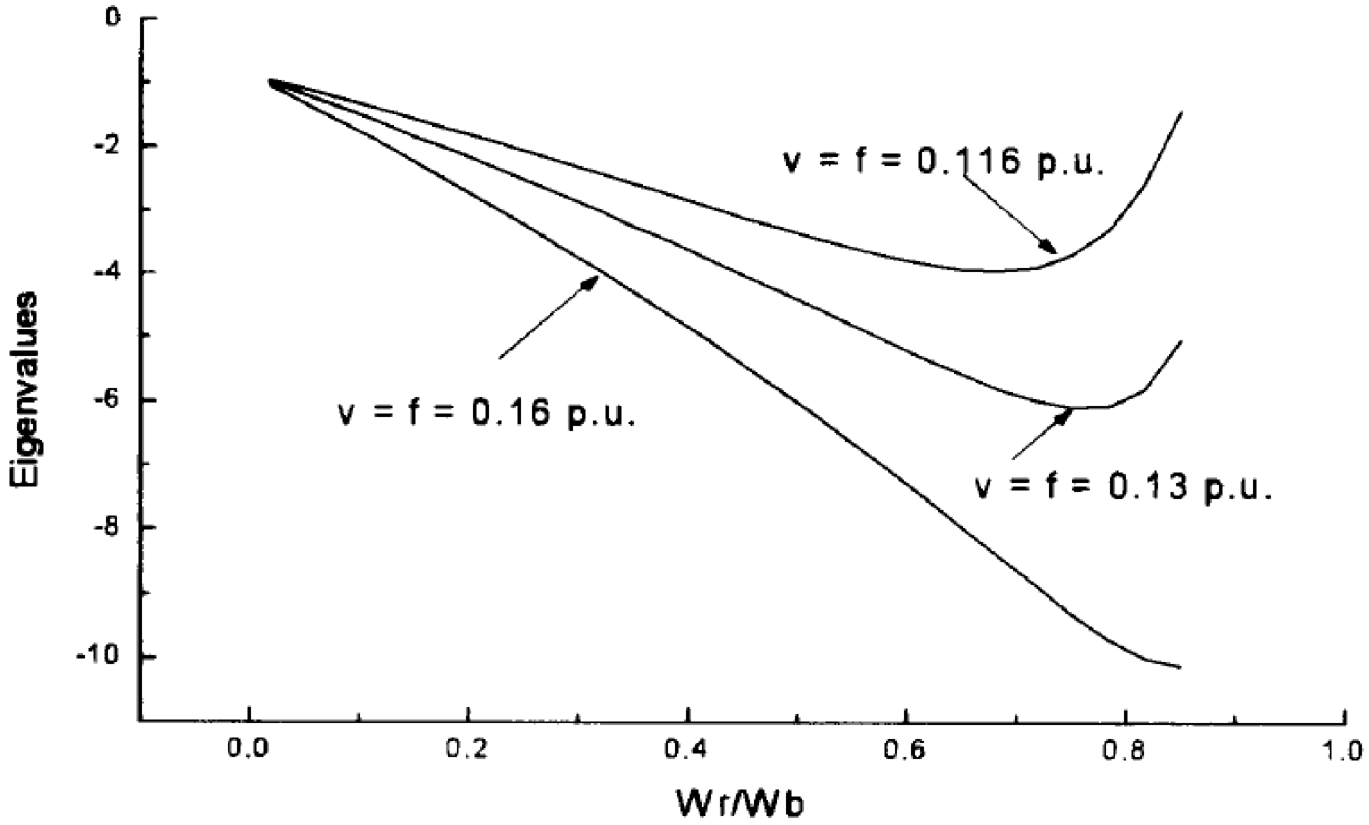
M. B. Uddin, M. N. Pramanik and S. A. Reza, "Low frequency stability study of a three-phase induction motor," *2007 7th International Conference on Power Electronics*, 2007, pp. 1115-1120

The excursion of eigen values for the rated operating condition of induction motor are depicted in Fig. Which indicate that operation becomes unstable at lower frequency. In each step of computation the applied voltage is decreased linearly with frequency. The eigen values are found to cross the boundary at low frequency of 0.116 p. u. (7 Hz) while the corresponding stator voltage is also 0.116 p. u. indicating unstable operation below this frequency.

The straight line parallel to the x-axis passing through zero value of the ordinate forms the boundary between the stable and unstable region of operation. The lower part of the boundary represents the stable region and the upper part signifies the unstable region of operation.

Squirrel Cage Induction Motor: Stability Analysis

Inference

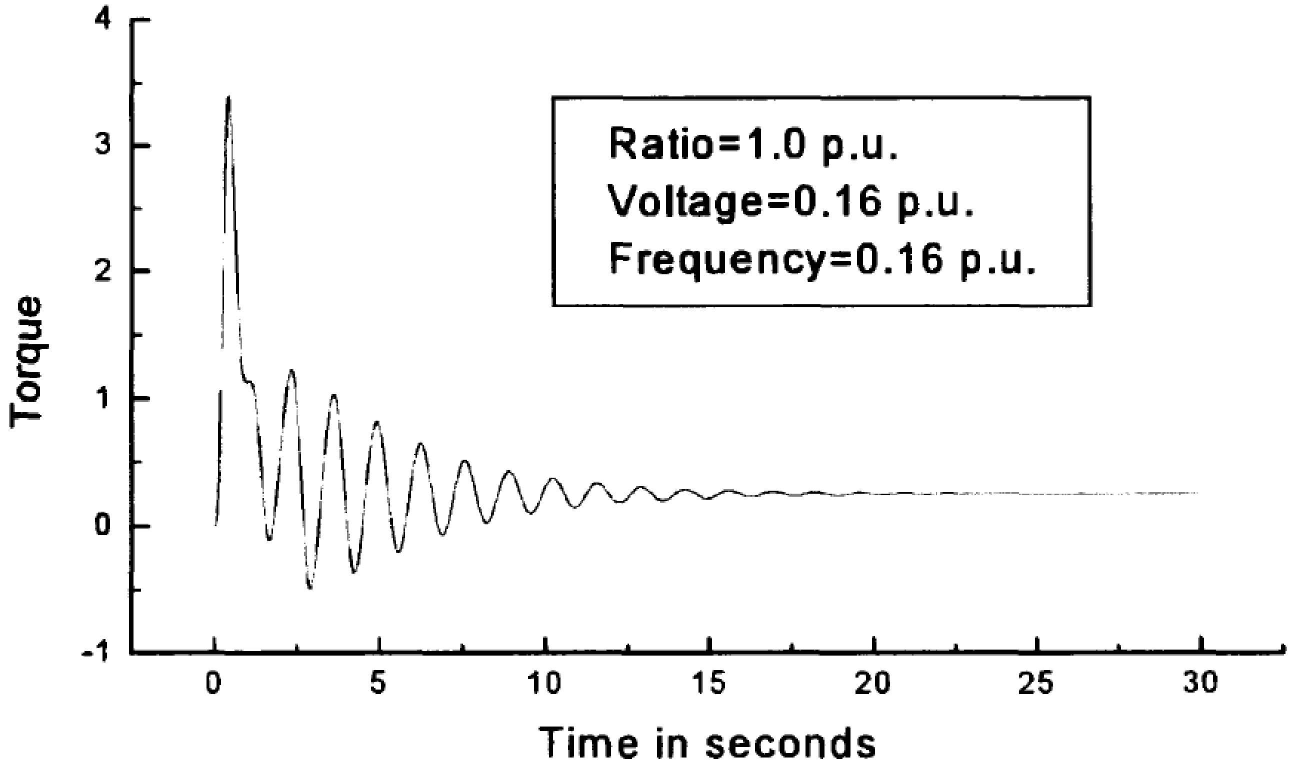


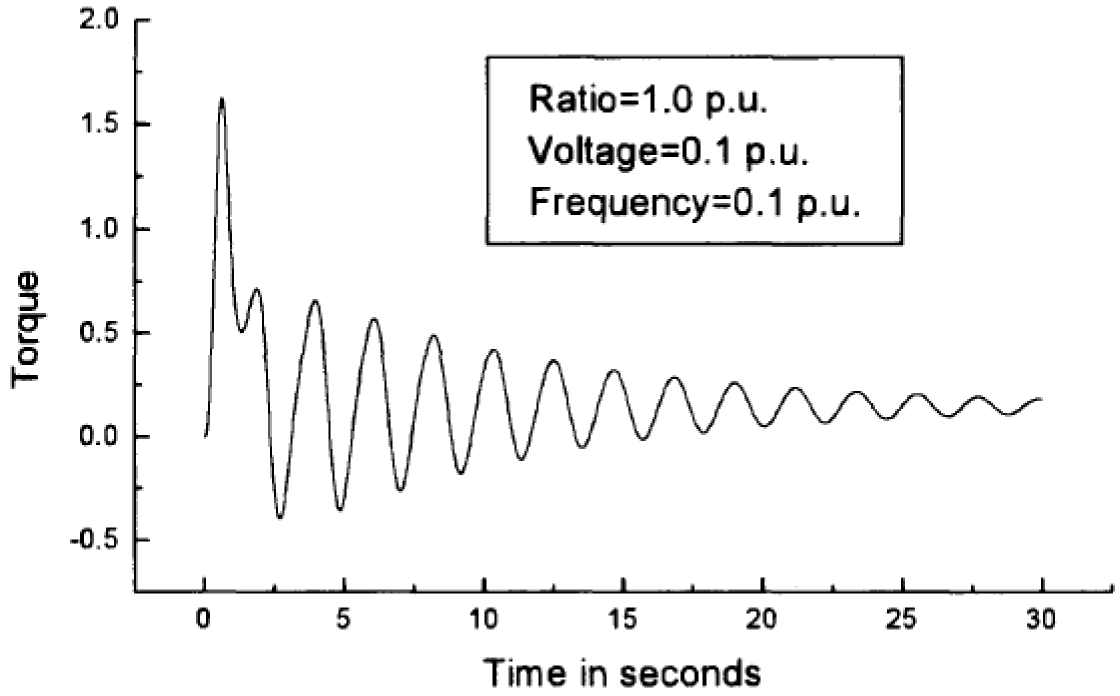
M. B. Uddin, M. N. Pramanik and S. A. Reza, "Low frequency stability study of a three-phase induction motor," *2007 7th International Conference on Power Electronics*, 2007, pp. 1115-1120

The effect of amplitude of the stator voltage on stability is illustrated in Fig. These curves are obtained by decreasing the stator voltage and frequency keeping volt/Hz ratio constant. The reduction of voltage and frequency simultaneously is indicating the induction motor trends to be unstable at lower frequency

Squirrel Cage Induction Motor: Stability Analysis

Inference





M. B. Uddin, M. N. Pramanik and S. A. Reza, "Low frequency stability study of a three-phase induction motor," *2007 7th International Conference on Power Electronics*, 2007, pp. 1115-1120

Squirrel Cage Induction Motor: Stability Analysis

Last Part of Assignment 1

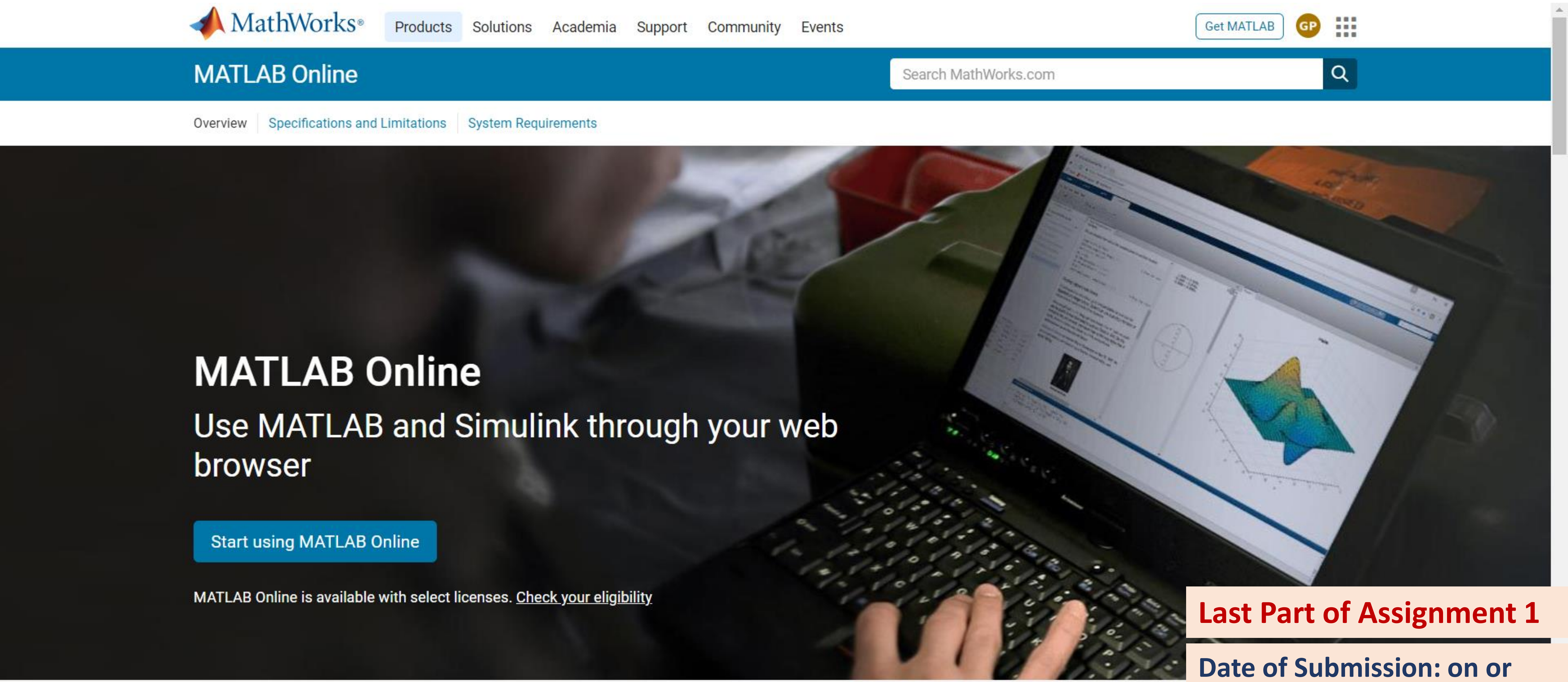
Date of Submission: on or before 27.12.2021

Case Study

Machines Parameters	Value	Per Unit Value
Horse Power (Hp)	50 hp	-
Voltage (V_L)	460 V	-
Frequency (Hz)	60 Hz	-
Stator Resistance (r_s)	0.087 Ω	0.015336
Stator Reactance (X_{ls})	0.302 Ω	0.053235
Mutual Reactance (X_M)	13.08 Ω	2.30569
Equivalent Rotor Resistance (r'_r)	0.302 Ω	0.040191
Equivalent Rotor Reactance (X'_{lr})	0.228 Ω	0.053235
Moment of Inertia (J)	1.662 Ω	-

M. B. Uddin, M. N. Pramanik and S. A. Reza, "Low frequency stability study of a three-phase induction motor," *2007 7th International Conference on Power Electronics*, 2007, pp. 1115-1120

Squirrel Cage Induction Motor: Stability Analysis



Last Part of Assignment 1

Date of Submission: on or before 27.12.2021

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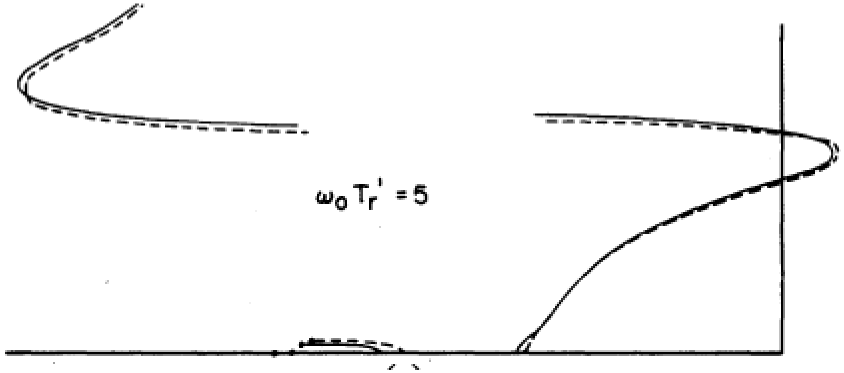
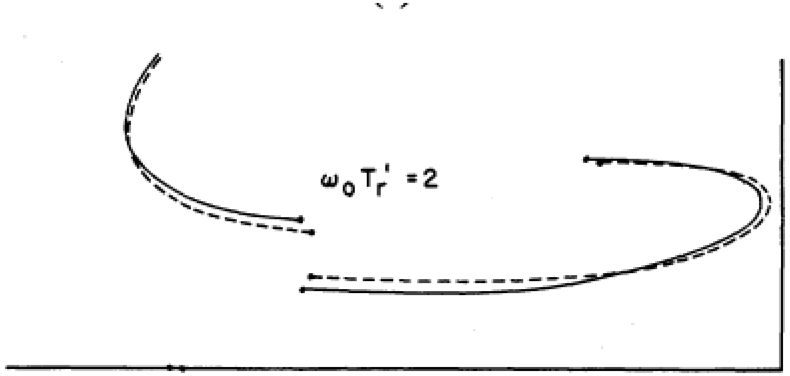
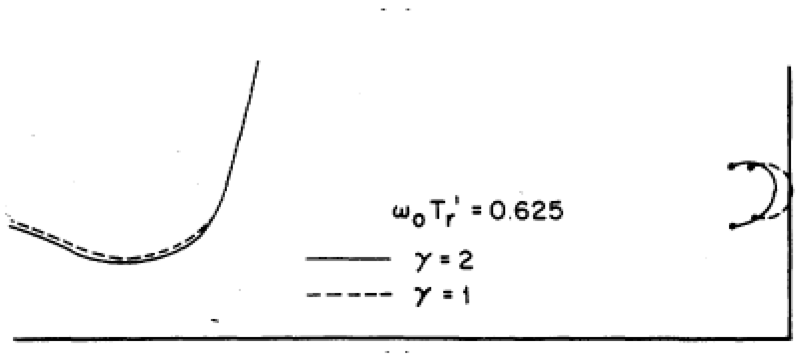
Squirrel Cage Induction Motor: Stability Analysis







The leftward shift of the overall root locus in the region of low damping is perhaps the most significant effect of transient saturation.

Modern high efficiency machines which have increased pu magnetizing reactance and are designed to operate at reduced flux levels are likely to have larger and stronger regions of instability and to exhibit poorer damped transient response than conventional machines.

Squirrel Cage Induction Motor: Stability Analysis

Case Study

Machines Parameters	Value	Per Unit Value
Horse Power (Hp)	50 hp	-
Voltage (V_L)	460 V	-
Frequency (Hz)	60 Hz	-
Stator Resistance (r_s)	0.087 Ω	0.015336
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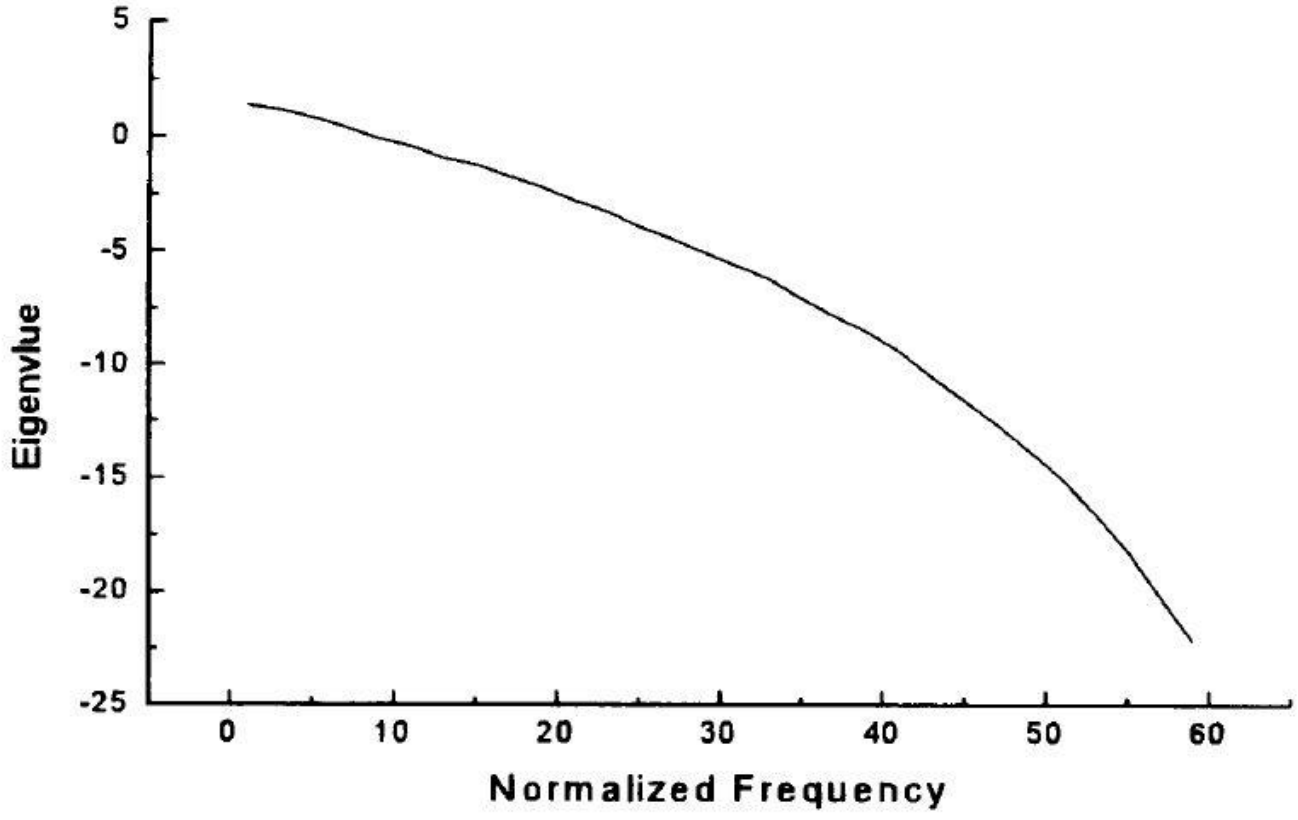
Squirrel Cage Induction Motor: Stability Analysis

Inference



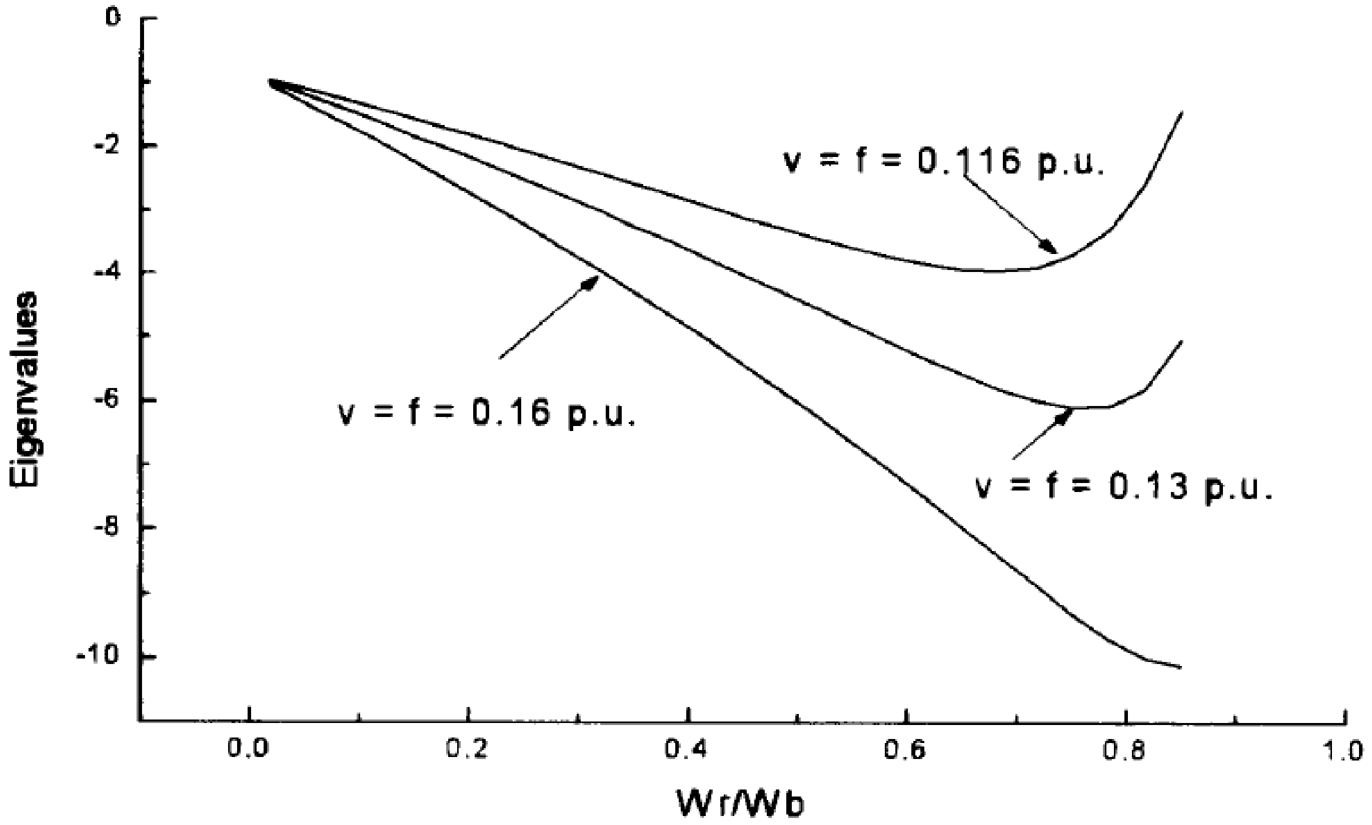
M. B. Uddin, M. N. Pramanik and S. A. Reza, "Low frequency stability study of a three-phase induction motor," *2007 7th International Conference on Power Electronics*, 2007, pp. 1115-1120

The excursion of eigen values for the rated operating condition of induction motor are depicted in Fig. Which indicate that operation becomes unstable at lower frequency. In each step of computation the applied voltage is decreased linearly with frequency. The eigen values are found to cross the boundary at low frequency of 0.116 p. u. (7 Hz) while the corresponding stator voltage is also 0.116 p. u. indicating unstable operation below this frequency.

The straight line parallel to the x-axis passing through zero value of the ordinate forms the boundary between the stable and unstable region of operation. The lower part of the boundary represents the stable region and the upper part signifies the unstable region of operation.

Squirrel Cage Induction Motor: Stability Analysis

Inference

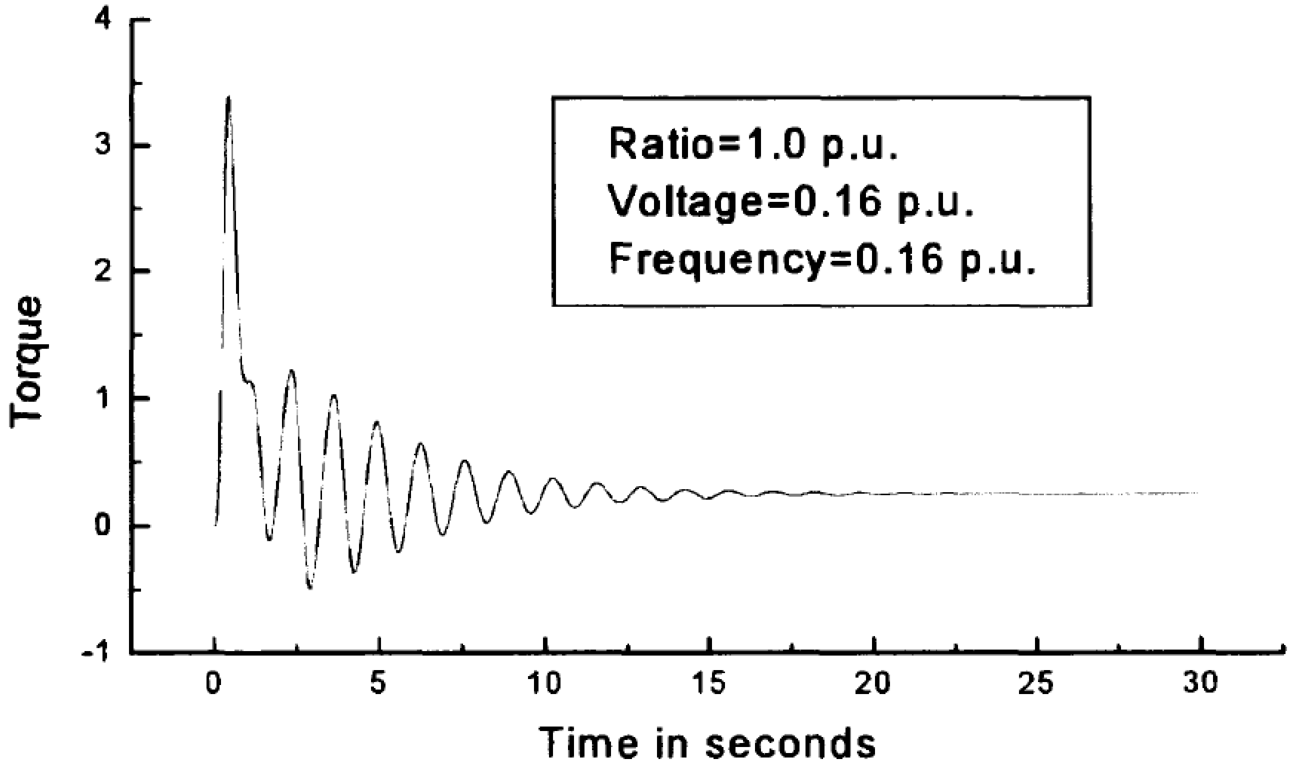


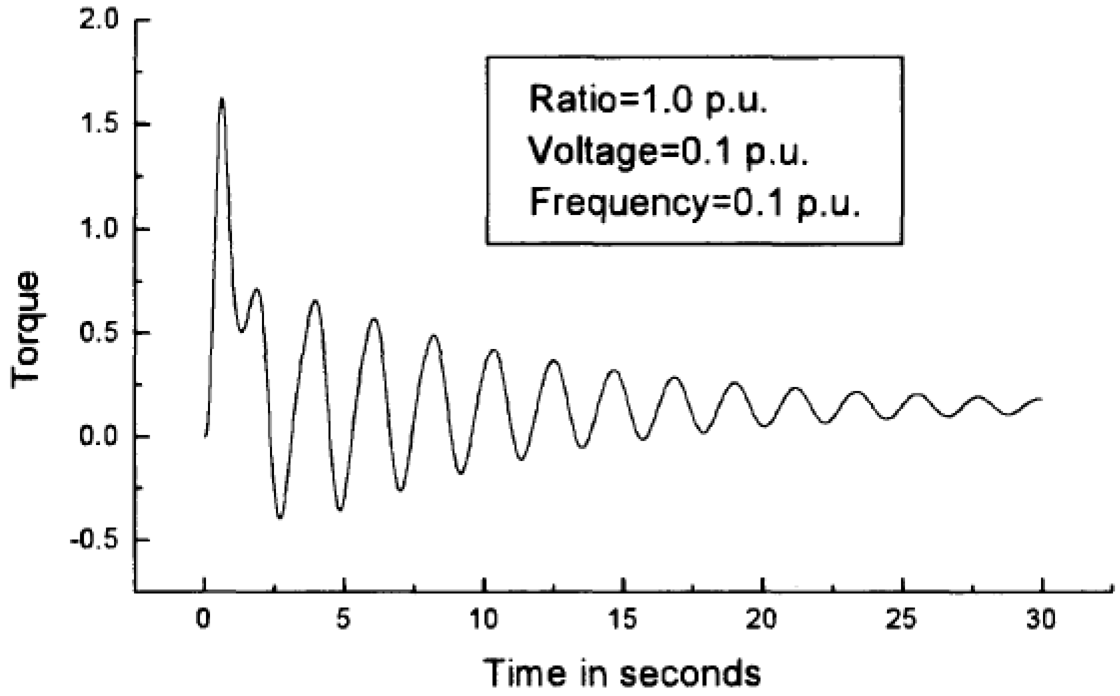
M. B. Uddin, M. N. Pramanik and S. A. Reza, "Low frequency stability study of a three-phase induction motor," *2007 7th International Conference on Power Electronics*, 2007, pp. 1115-1120

The effect of amplitude of the stator voltage on stability is illustrated in Fig. These curves are obtained by decreasing the stator voltage and frequency keeping volt/Hz ratio constant. The reduction of voltage and frequency simultaneously is indicating the induction motor trends to be unstable at lower frequency

Squirrel Cage Induction Motor: Stability Analysis

Inference





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Squirrel Cage Induction Motor: Stability Analysis

Last Part of Assignment 1

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Case Study

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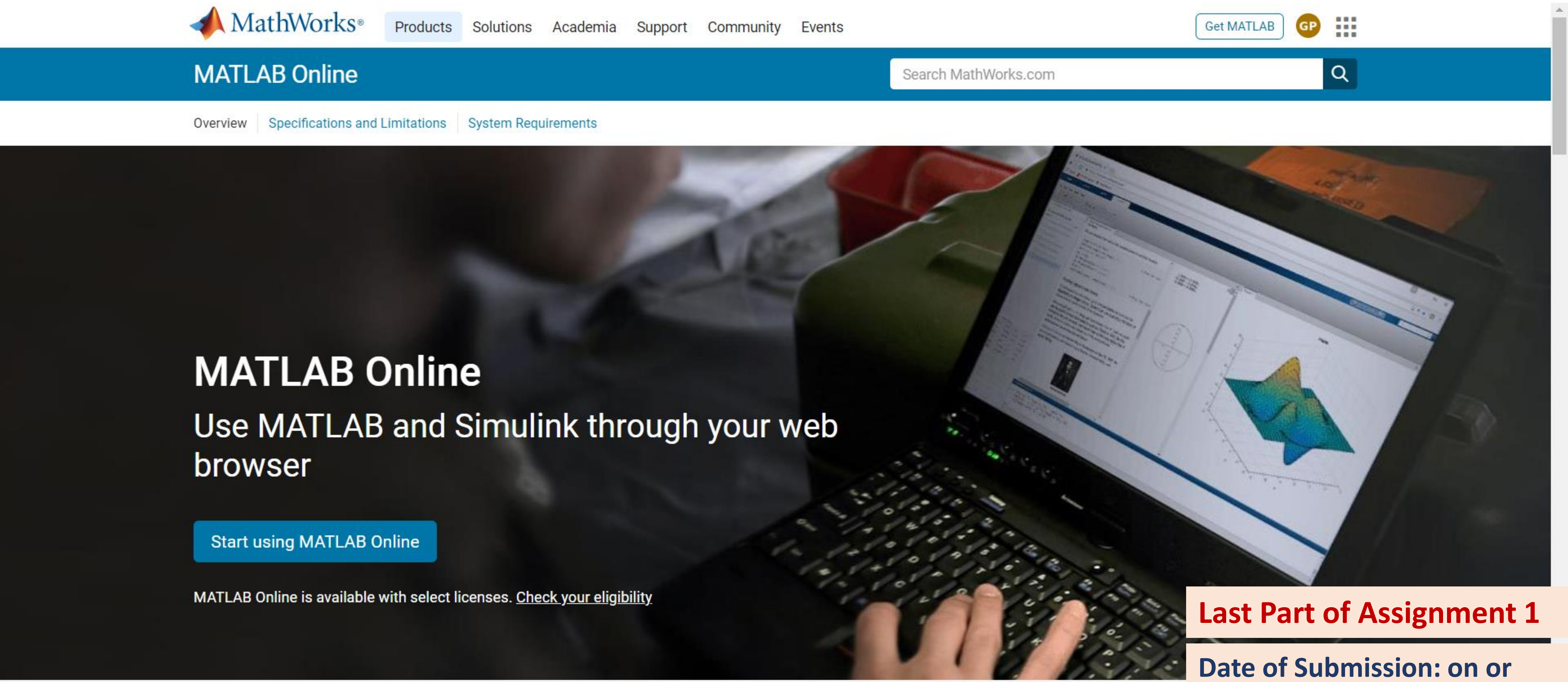
Squirrel Cage Induction Motor: Stability Analysis



Last Part of Assignment 1

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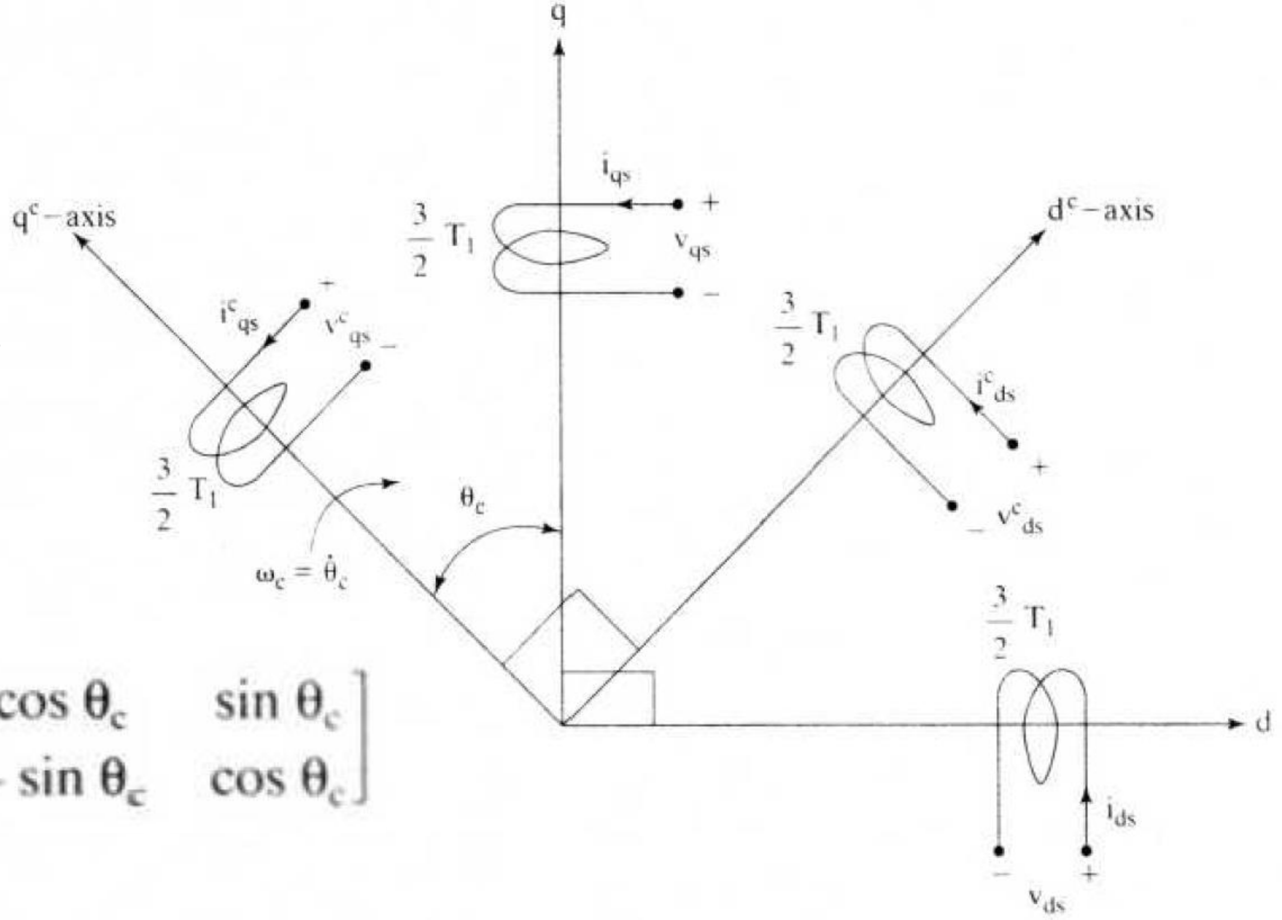
Machine Model at Various Reference Frames

Induction Motor

Approach 1

Develop new primitive machine according to arbitrary reference frame

Approach 2



$$T^c = \begin{bmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{bmatrix}$$

$$\begin{bmatrix} v^c_{qs} \\ v^c_{ds} \\ v^c_{qr} \\ v^c_{dr} \end{bmatrix} = \begin{bmatrix} R_s + L_s p & \omega_c L_s & L_m p & \omega_c L_m \\ -\omega_c L_s & R_s + L_s p & -\omega_c L_m & L_m p \\ L_m p & (\omega_c - \omega_r)L_m & R_r + L_r p & (\omega_c - \omega_r)L_r \\ -(\omega_c - \omega_r)L_m & L_m p & -(\omega_c - \omega_r)L_r & R_r + L_r p \end{bmatrix} \begin{bmatrix} i^c_{qs} \\ i^c_{ds} \\ i^c_{qr} \\ i^c_{dr} \end{bmatrix}$$

Oscillations

Induction Machines

Synchronous Machines

1. Isolated synchronous generator to a load
2. Synchronous generator connected to infinite bus
3. Synchronous Motor

The problem of small oscillations is encountered mainly with machines in which there is some form of feedback. This may consist of signals generated within the machine inherently, due to the configuration of windings, or externally in the form of a control system, or a voltage regulator on a d.c. generator.

Self-excited oscillations are fairly common in power systems. Usually two kinds of self-excited oscillations are encountered. One is a high-frequency electrical oscillation, to which the machine rotors, because of their high inertia, cannot respond. The analysis of the complete system using eigenvalue techniques will indicate whether or not such frequencies will be present. We are primarily concerned here with low-frequency oscillations (a few hertz) to which the rotors can respond. These are electromechanical in nature.

Synchronous Machine Equation During Small Oscillations

$$Jp^2\Delta\theta + T_{de}p\Delta\theta + R_F p\Delta\theta + T_S\Delta\theta = \Delta T_i$$

where $T_S\Delta\theta$ = synchronising torque

$T_{de}p\Delta\theta$ = electrical damping torque

$R_F p\Delta\theta$ = mechanical damping torque

$Jp^2\Delta\theta$ = rotational torque due to the inertia of the rotor

ΔT_i = change in input/output torque

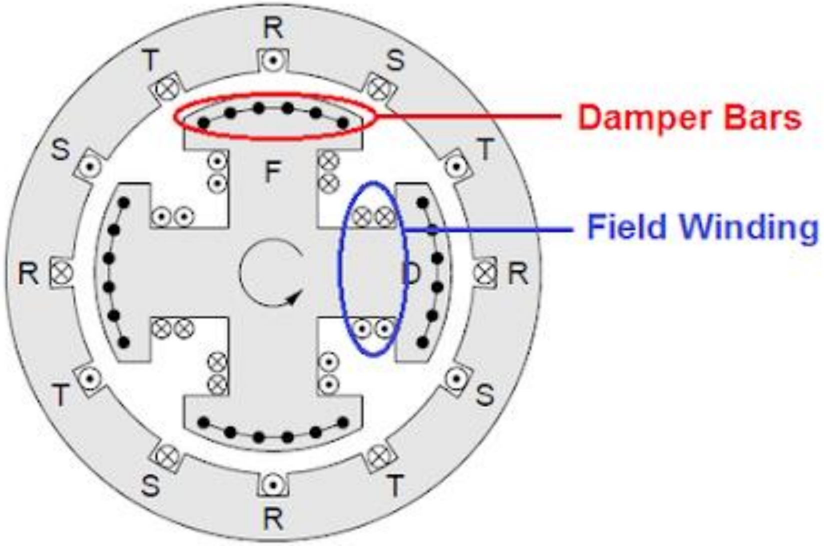
When the oscillations are self-excited ΔTi= 0 and hence

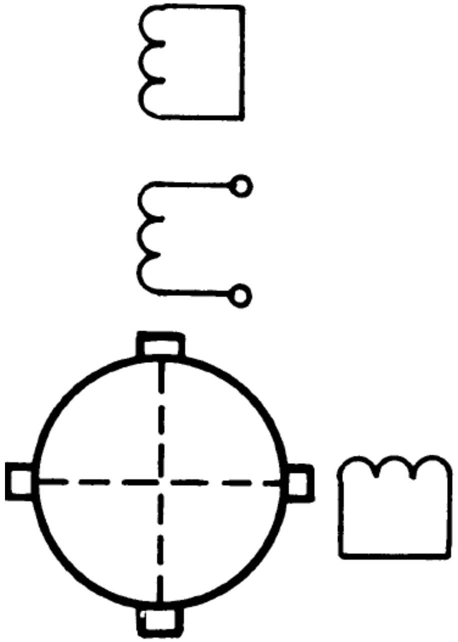
$$Jp^2\Delta\theta + (T_{de} + R_F)p\Delta\theta + T_S\Delta\theta = 0$$

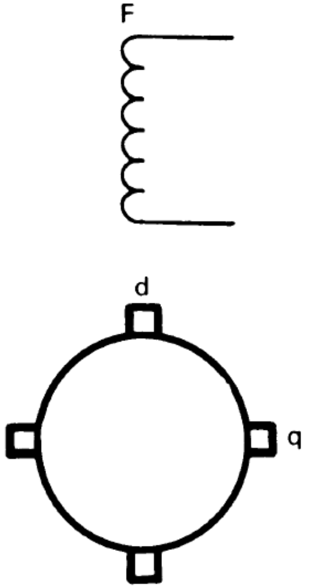
The damping and synchronising torque coefficients not only indicate whether or not a system is stable, they clearly indicate the extent of the stability /instability region and provide physical concepts which are not obtained by other methods.

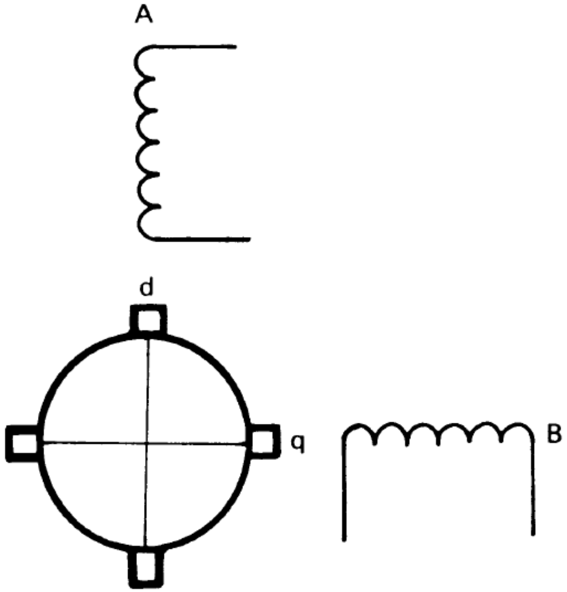
Primitive Machines

Synchronous machines with amortisseur winding









Synchronous Machine Equation During Small Oscillations

$$Jp^2\Delta\theta + T_{de}p\Delta\theta + R_F p\Delta\theta + T_S\Delta\theta = \Delta T_i$$

where $T_S\Delta\theta$ = synchronising torque

$T_{de}p\Delta\theta$ = electrical damping torque

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ΔT_i = change in input/output torque

When the oscillations are self-excited $\Delta Ti = 0$ and hence

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The damping and synchronising torque coefficients not only indicate whether or not a system is stable, they clearly indicate the extent of the stability /instability region and provide physical concepts which are not obtained by other methods.

Synchronous Machine Equation During Small Oscillations

$$Jp^2\Delta\theta + (T_{de} + R_F)p\Delta\theta + T_S\Delta\theta = 0$$

(a) if $T_S = 0$, the machine does not oscillate, but loses synchronism
(b) if $(T_{de} + R_F) = 0$ oscillations begin and are self-sustained

(c) $(T_{de} + R_F) < 0$ oscillations build up in magnitude and the machine loses synchronism
(d) $(T_{de} + R_F) > 0$ there may or may not be oscillations but in either case the system will be stable if $T_S > 0$

Synchronous Machine Equation During Small Oscillations

Oscillations of small amplitude, the system equations may be linearised about the operating points. This makes it fairly easy to evolve criteria which will tell us whether the machines are dynamically stable or not, using methods such as the following:

(1) the Nyquist criterion

(2) Routh-Hurwitz criterion

(3) computation of the eigenvalues

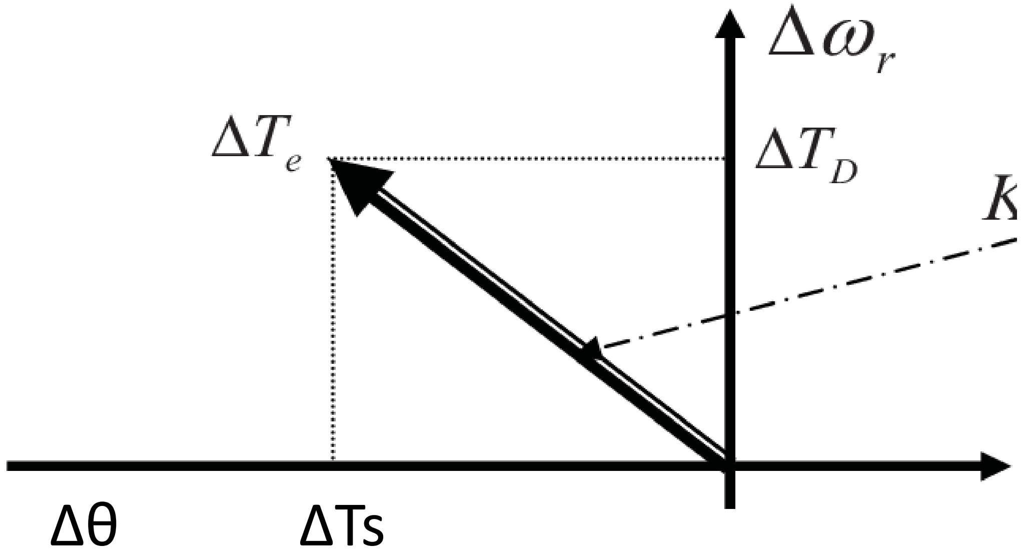
(4) computation of the synchronising and damping torque coefficients

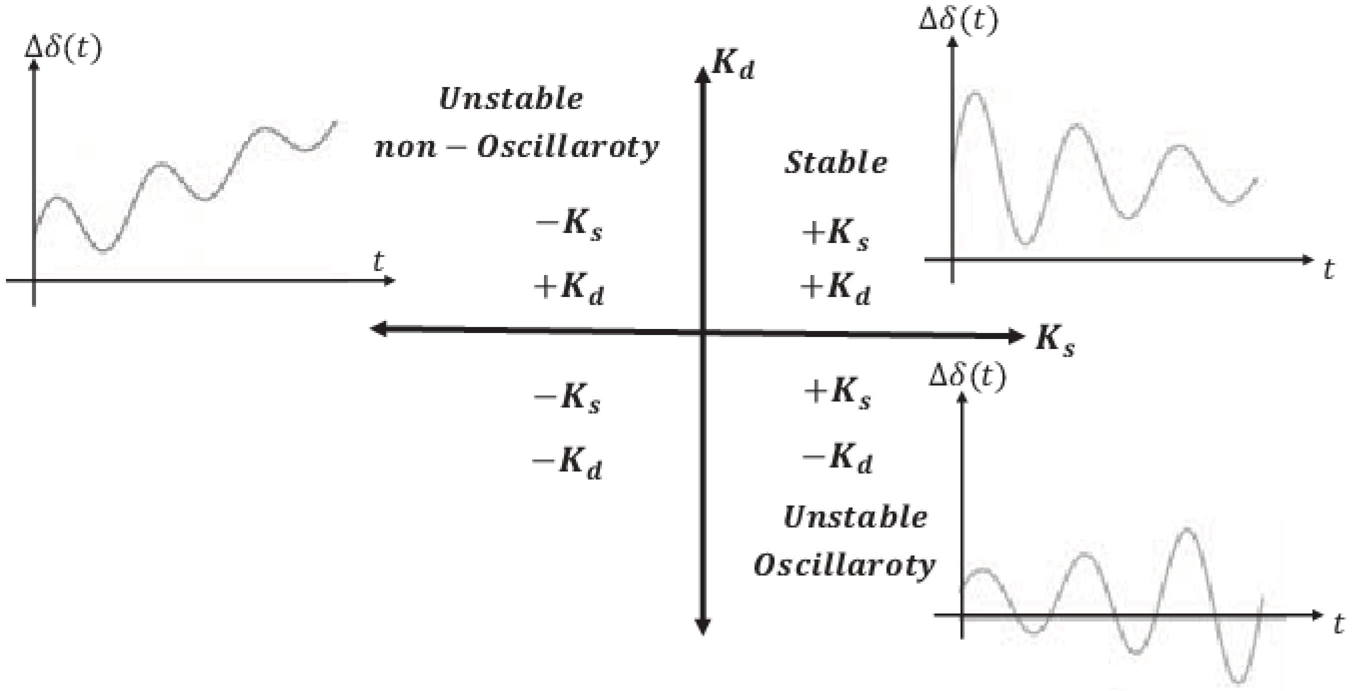
(5) D-partition techniques

(6) computation of rotor swing with time-the so-called swing curves

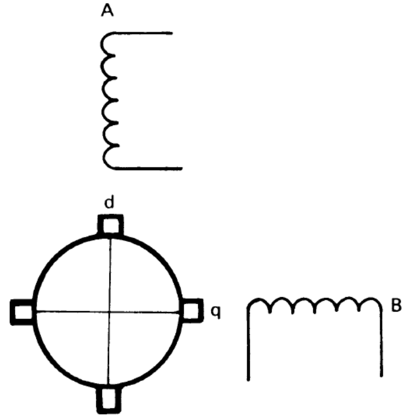
In the case of transient (nonlinear) instability, the standard approach is to compute the swing curves.

Synchronous Machine Equation During Small Oscillations





Synchronous Machine Equation During Small Oscillations



$$
\begin{bmatrix} V_{ds} \\ V_{dr} \\ V_{qr} \\ V_{qs} \\ T \end{bmatrix}
=
\begin{array}{c}
 \\ ds \\ dr \\ qr \\ qs \\ s
\end{array}
\begin{array}{ccccc}
ds & ar & qr & qs & s \\
\hline
R_{ds} + L_{ds} p & M_d p & & & \\
M_d p & R_{dr} + L_{dr} p & & & L_{qr} i_{qr} + M_q i_{qs} \\
 & & R_{qr} + L_{qr} p & M_q p & -M_d i_{ds} - L_{dr} i_{dr} \\
 & & M_q p & R_{qs} + L_{qs} p & \\
-M_d i_{qr} & -L_{dr} i_{qr} & L_{qr} i_{dr} & M_q i_{dr} & J p + R_F
\end{array}
\cdot
\begin{bmatrix} i_{ds} \\ i_{dr} \\ i_{qr} \\ i_{qs} \\ \omega_r \end{bmatrix}
$$

Synchronous Machine Equation During Small Oscillations

transient equation for torque

$$Jp^2\theta + R_F p\theta - \mathbf{i}_t^* \mathbf{G}\mathbf{i} = T_{\text{input}}$$

For small oscillations, we take increments in θ, i, and Ti;

$$Jp^2\Delta\theta + R_F p\Delta\theta - \mathbf{i}_t^* \mathbf{G}\Delta\mathbf{i} - \Delta\mathbf{i}_t^* \mathbf{G}\mathbf{i} - \mathbf{i}_t^* \frac{\partial \mathbf{G}}{\partial \theta}\Delta\theta\, \mathbf{i} = \Delta T_i$$

$$Jp^2\Delta\theta + R_F p\Delta\theta - \mathbf{i}_t^* (\mathbf{G} + \mathbf{G}_t)\Delta\mathbf{i} = \Delta T_i$$

$$\mathbf{V} = \mathbf{R}\mathbf{i} + \mathbf{L}p\mathbf{i} + \mathbf{G}\mathbf{i}p\theta$$

$$\mathbf{V} + \Delta\mathbf{V} = \mathbf{R}(\mathbf{i} + \Delta\mathbf{i}) + \mathbf{L}p(\mathbf{i} + \Delta\mathbf{i}) + \mathbf{G}(\mathbf{i} + \Delta\mathbf{i})p\theta + \mathbf{G}\mathbf{i}p(\Delta\theta)$$

$$\Delta\mathbf{V} = \mathbf{R}\Delta\mathbf{i} + \mathbf{L}p\Delta\mathbf{i} + \mathbf{G}p\theta\Delta\mathbf{i} + \mathbf{G}\mathbf{i}p(\Delta\theta)$$

Synchronous Machine Equation During Small Oscillations

$$
\begin{bmatrix} \Delta V_{ds} \\ \Delta V_{dr} \\ \Delta V_{qr} \\ \Delta V_{qs} \\ \Delta T_{i} \end{bmatrix}
=
\begin{array}{cc}
 & \begin{array}{ccccc} \text{ds} & \text{dr} & \text{qr} & \text{qs} & \text{s} \end{array} \\
\begin{array}{c} \text{ds} \\ \text{dr} \\ \text{qr} \\ \text{qs} \\ \text{s} \end{array} &
\begin{bmatrix}
R_{ds} + L_{ds} p & M_{d} p & & & \\
M_{d} p & R_{r} + L_{dr} p & L_{qr} p\theta & M_{q} p\theta & L_{qr} i_{qr} + M_{q} i_{qs} \\
- M_{d} p\theta & - L_{dr} p\theta & R_{r} + L_{qr} p & M_{q} p & - M_{d} i_{ds} - L_{dr} i_{dr} \\
 & & M_{q} p & R_{qs} + L_{qs} p & \\
- i_{qr}^{*} M_{d} & - i_{qs}^{*} M_{q} - (L_{qr} - L_{dr}) i_{qr}^{*} & - i_{ds}^{*} M_{d} - i_{dr}^{*} (L_{qr} - L_{dr}) & - i_{dr}^{*} M_{q} & J p + R_{F}
\end{bmatrix}
\end{array}
\cdot
\begin{bmatrix} \Delta i_{ds} \\ \Delta i_{dr} \\ \Delta i_{qr} \\ \Delta i_{qs} \\ p\Delta\theta \end{bmatrix}
$$

Synchronous Machine Equation During Small Oscillations

$$\begin{array}{|c|}\hline \Delta \mathbf{V} \\ \hline \Delta T_{\mathrm{i}} \\ \hline \end{array} = \begin{array}{|c|c|}\hline \overline{\mathbf{L}} & \\ \hline & J \\ \hline \end{array} \; p \begin{array}{|c|}\hline \Delta \mathbf{i} \\ \hline \Delta \omega \\ \hline \end{array} + \begin{array}{|c|c|}\hline \mathbf{R} + \mathbf{G} p \theta_0 & \mathbf{G}\mathbf{i}_0 \\ \hline -\mathbf{i}_0^*(\mathbf{G}+\mathbf{G}_{\mathrm{t}}) & R_{\mathrm{F}} \\ \hline \end{array} \cdot \begin{array}{|c|}\hline \Delta \mathbf{i} \\ \hline \Delta \omega \\ \hline \end{array}$$

$$\Delta \mathbf{x} = \begin{array}{|c|}\hline \Delta \mathbf{i} \\ \hline \Delta \omega \\ \hline \end{array}$$

$$\Delta \mathbf{f} = \begin{array}{|c|}\hline \Delta \mathbf{V} \\ \hline \Delta T_{\mathrm{i}} \\ \hline \end{array}$$

$$\mathbf{L} = \begin{array}{|c|c|}\hline \overline{\mathbf{L}} & \\ \hline & J \\ \hline \end{array}$$

$$p\Delta \mathbf{x} = -\mathbf{L}^{-1}(\overline{\mathbf{A}})\Delta \mathbf{x} + (\mathbf{L})^{-1}\Delta \mathbf{f}$$

$$\overline{\mathbf{A}} = \begin{array}{|c|c|}\hline \mathbf{R} + \mathbf{G} p \theta_0 & \mathbf{G}\mathbf{i}_0 \\ \hline -\mathbf{i}_0^*(\mathbf{G}+\mathbf{G}_{\mathrm{t}}) & R_{\mathrm{F}} \\ \hline \end{array}$$

$$\Delta \dot{\mathbf{x}} = \mathbf{A}\Delta \mathbf{x} + \mathbf{B}\Delta \mathbf{u}$$

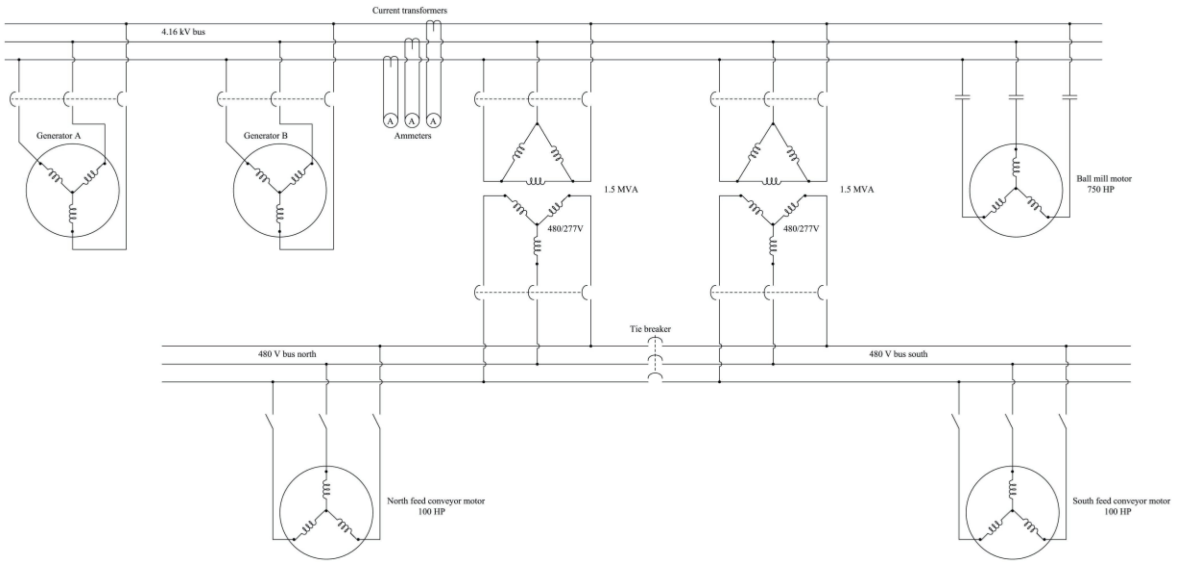
The characteristic equation is

$$\det\ (s\mathbf{I} - \mathbf{A}) = 0$$

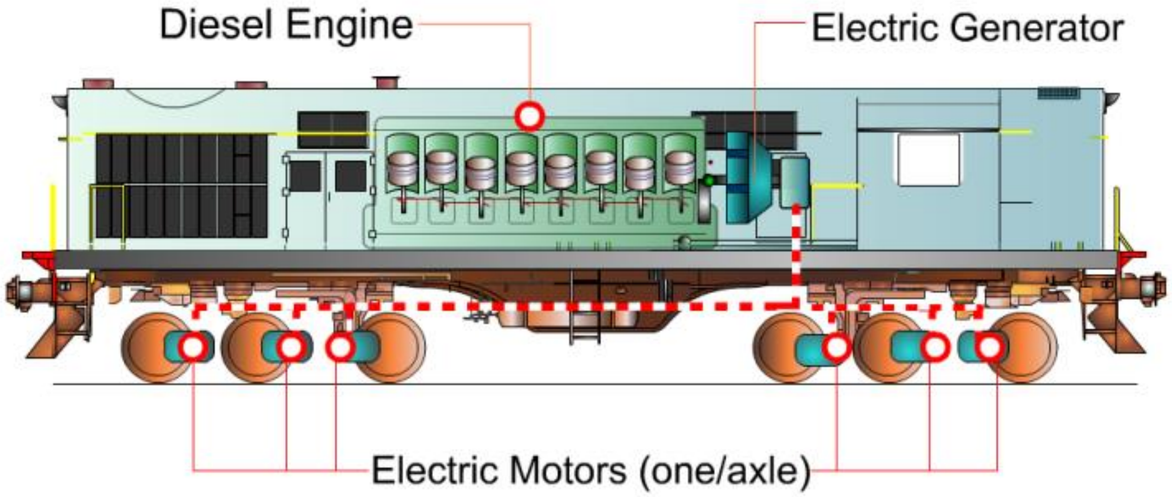
Synchronous Machine Equation During Small Oscillations

Having obtained the characteristic equation of a machine, the roots, or eigenvalues, can be computed by standard procedures. If the roots have no positive real parts, the system is stable. However, when the roots are obtained it may be found that one of the roots has a small negative real part and a low value of imaginary part. This root will predominantly influence the rotor dynamics. This will indicate that the machine will oscillate at an angular frequency determined by the imaginary part of the root and the rate of decay of the oscillations will depend on the magnitude of the real part. If a voltage regulator is present, roots with relatively small imaginary parts will be present which will also affect the dynamics of the rotor. The effect of the other roots, with faster decay rates (higher negative real values) will be apparent only at the initial stage and these will not affect the small oscillation behavior over the longer period given by the dominant root. Roots with large values of imaginary part will not affect the rotor dynamics because the rotor, due to its inertia, will not respond to high frequency oscillations. Hence we can often assume that there will be virtually one dominant root which will determine the machine kinetics during small oscillation

Interconnected Machines



In industrial processes electrical machines may be used singly, in groups of identical machines, or in groups of different machines interconnected to achieve some required overall operating characteristics. A simple example of this is the use of a pilot exciter and main exciter to provide controlled and stable field current for a large alternator in a power station, all machine rotors being on one shaft. In steel production, groups of machines may operate in parallel or in controlled sequence.



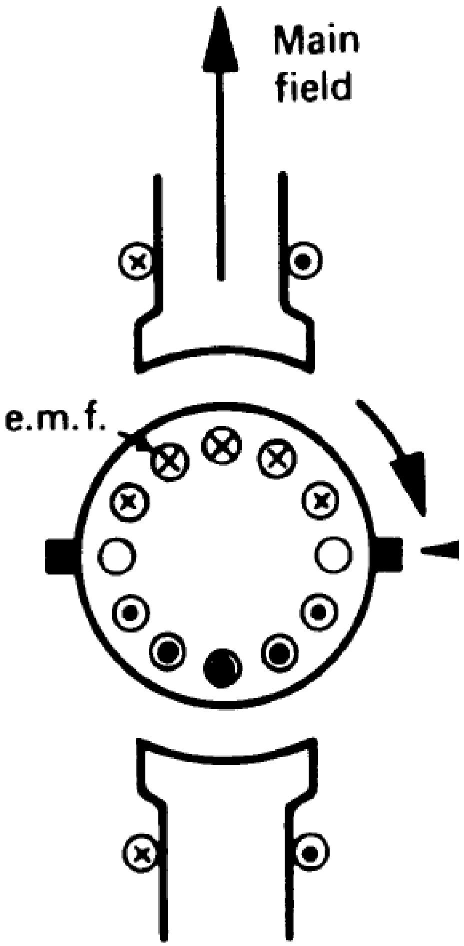
Interconnected Machines

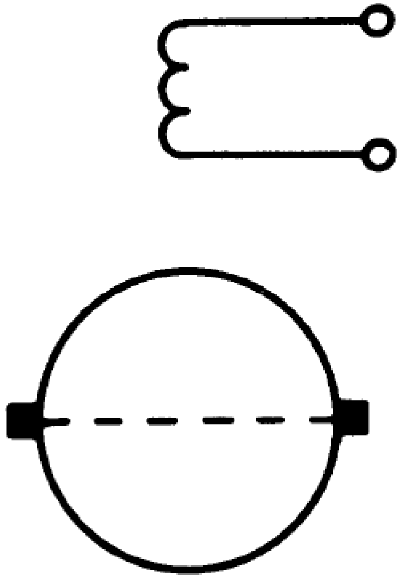
In the machines which we have investigated in the foregoing analysis of torque equations, the choice of d and q axes was obvious. However, where machines of different types are interconnected, we must consider carefully the choice of reference axes. For example, the armature voltage and torque equations of synchronous machines are usually expressed along Park's axes, namely in a direct axis along the rotor field pole and a quadrature axis lying between the field poles. In the ideal synchronous machine in steady state operation, all values of voltage, current, and torque along these axes are constant with respect to time, and the operator p(= d/dt) in the equations becomes zero. With the induction motor the usual reference frame is fixed to the stator with the direct axis along one phase and the quadrature axis orthogonal to this position. The rotor axes are also fixed in this position in space. Along these axes, in steady state operation, the p operator in the equations becomes jω, where ω = 2πf. Obviously if these machines are operating together, it is essential that we select reference axes such that the operator p has the same steady-state and transient significance for all time dependent behaviour. A reference frame for the induction motor, which rotates uniformly with the synchronous flux wave gives equations of the same form as those of Park for the synchronous machine and a complex arrangement of such machines may then be treated mathematically as a single coupled dynamical system. In simple cases of a few coupled machines it may be possible in a computer programme to treat each machine separately with its own frame of reference, the computer making the adjustments in values when transformations between the reference axes become necessary as the machines interact.

For many purposes it is desirable to programme the overall dynamical behaviour of interconnected machines, using the **combined equations of performance**; for example when using the state A-matrices and in the study of stability conditions and eigenvalues.

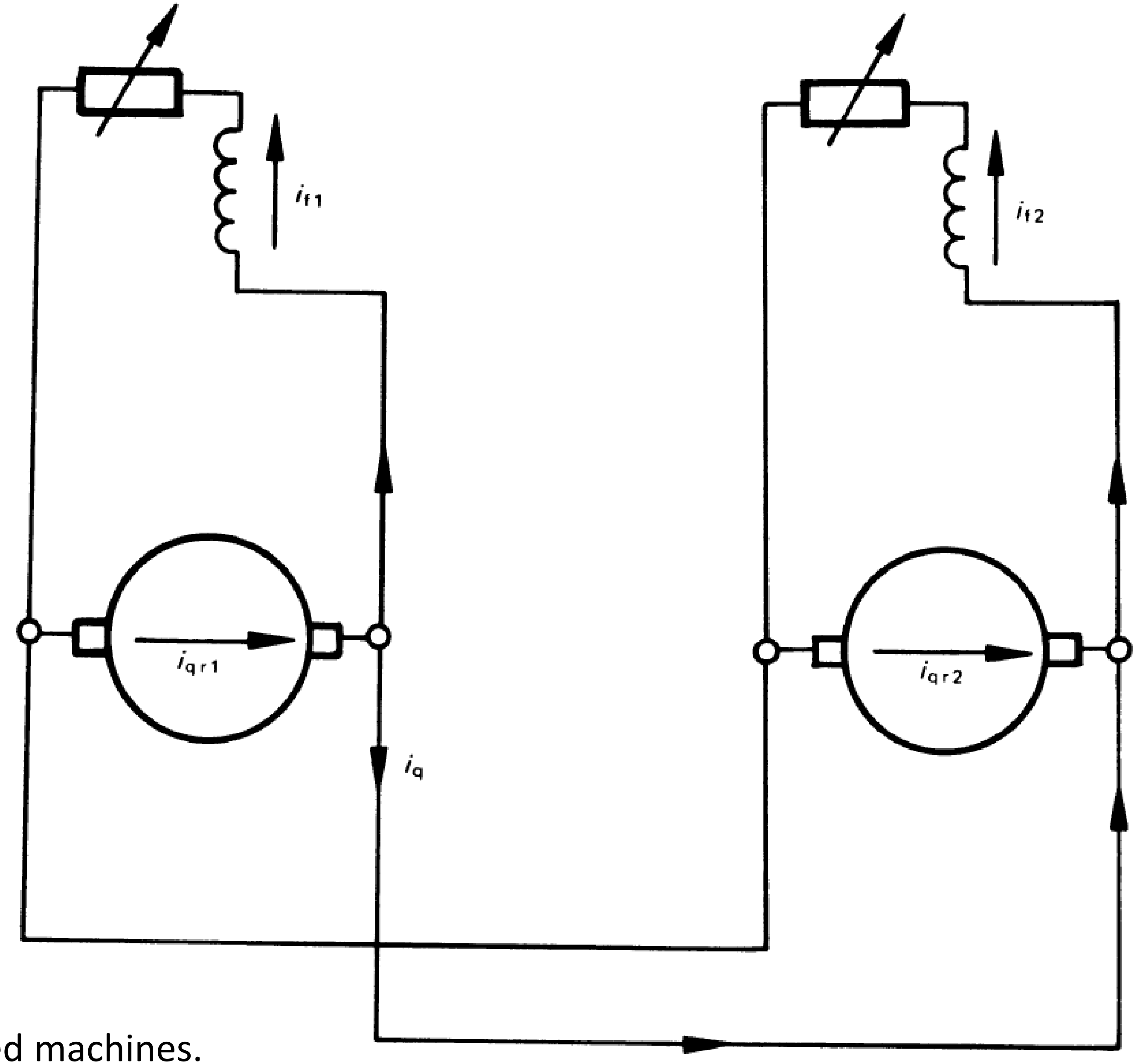
Primitive Machines

DC machines





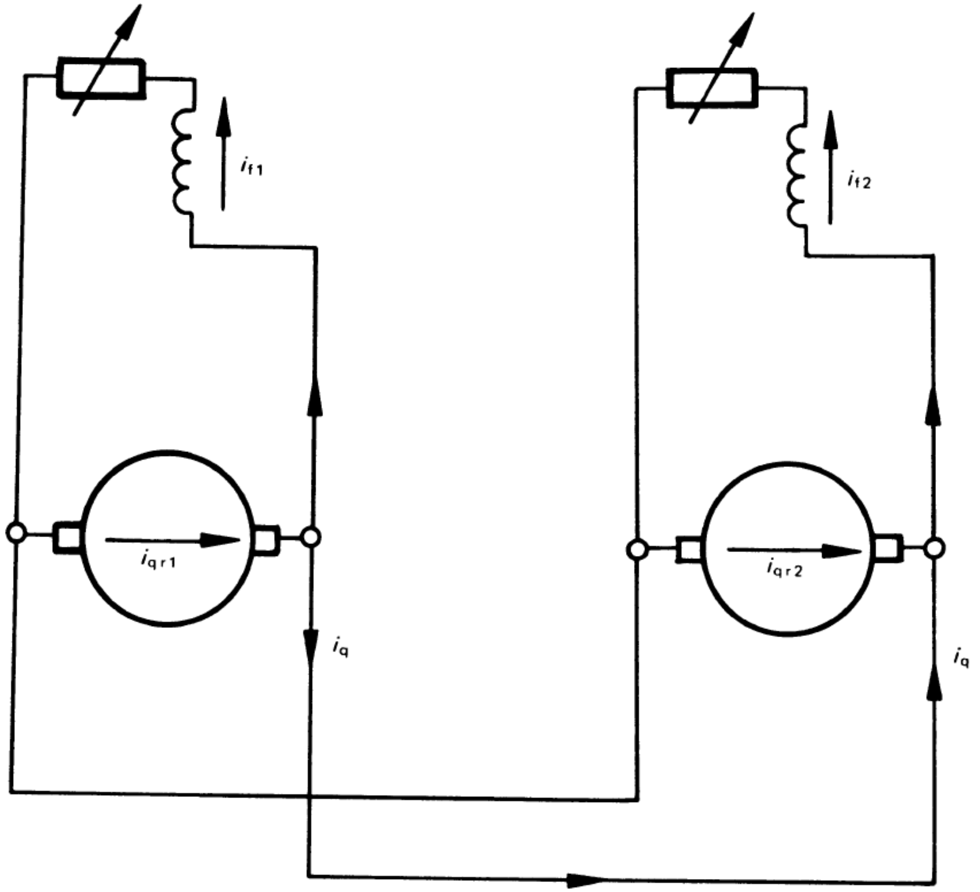
Interconnected Machines



assume that the prime mover driving the shunt generator is very large and runs at constant speed whatever the load on the generator

electrically coupled machines.

Interconnected Machines



Considering impressed voltages on each machine, in accordance with Kron's convention, the equations for each machine are identical in form

generator

$$\begin{bmatrix} V_{f1} \\ V_{a1} \end{bmatrix} = \begin{array}{c|cc} & ds1 & qr1 \\ \hline ds1 & R_{f1} + L_{f1}p & \\ qr1 & -M_{d1}p\theta & R_{a1} + L_{a1}p \end{array} \cdot \begin{bmatrix} i_{f1} \\ i_{a1} \end{bmatrix}$$

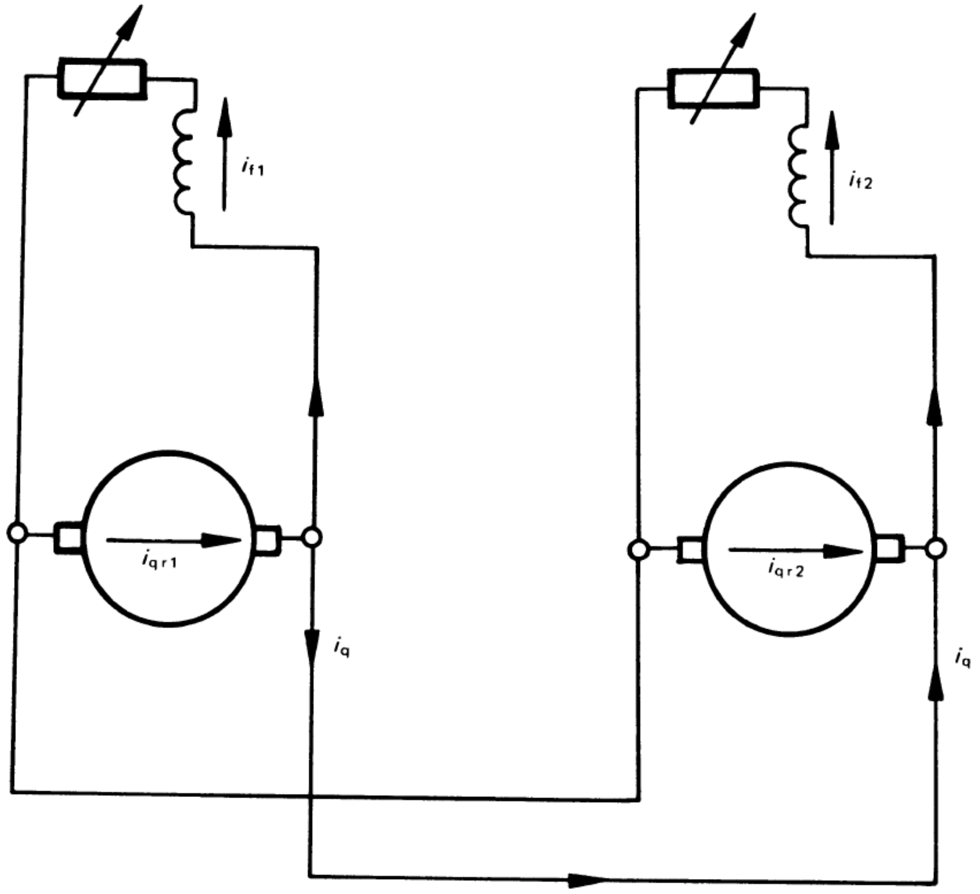
$$V_1 = Z_1 i_1$$

motor

$$\begin{bmatrix} V_{f2} \\ V_{a2} \end{bmatrix} = \begin{array}{c|cc} & ds2 & qr2 \\ \hline ds2 & R_{f2} + L_{f2}p & \\ qr2 & -M_{d2}p\theta & R_{a2} + L_{a2}p \end{array} \cdot \begin{bmatrix} i_{f2} \\ i_{a2} \end{bmatrix}$$

$$V_2 = Z_2 i_2$$

Interconnected Machines



$$i_{ds1} = i_{f1}$$
$$i_{qr1} = i_{f1} + i_q$$
$$i_{ds2} = i_{f2}$$
$$i_{qr2} = i_{f2} - i_q$$

$$\mathbf{C} = \begin{array}{c|ccc} & f1 & f2 & q \\ \hline ds1 & 1 & & \\ qr1 & 1 & & 1 \\ ds2 & & 1 & \\ qr2 & & +1 & -1 \end{array}$$

combined matrix is given by

$$\mathbf{Z}' = \mathbf{C}_t \mathbf{Z} \mathbf{C}$$

$$\mathbf{Z} = \begin{array}{c|cccc} & ds1 & qr1 & ds2 & qr2 \\ \hline ds1 & R_{f1} + L_{f1}p & & & \\ qr1 & -M_{d1}p\theta & R_{a1} + L_{a1}p & & \\ ds2 & & & R_{f2} + L_{f2}p & \\ qr2 & & & -M_{d2}p\theta & R_{a2} + L_{a2}p \end{array}$$

Interconnected Machines

$$
\mathbf{Z}' = \begin{array}{c|c|c|c|}
 & \text{f1} & \text{f2} & \text{q} \\
\hline
\text{f1} & R_{f1} + L_{f1}p - M_{d1}p\theta + R_{a1} + L_{a1}p & & R_{a1} + L_{a1}p \\
\hline
\text{f2} & & R_{f2} + L_{f2}p - M_{d2}p\theta + R_{a2} + L_{a2}p & -R_{a2} - L_{a2}p \\
\hline
\text{q} & -M_{d1}p\theta + R_{a1} + L_{a1}p & M_{d2}p\theta - R_{a2} - L_{a2}p & R_{a1} + L_{a1}p + R_{a2} + L_{a2}p \\
\hline
\end{array}
$$

In the steady state, terms containing the operator p become zero

$$
\mathbf{Z}'_{\text{s-state}} = \begin{array}{c|c|c|c|}
 & \text{f1} & \text{f2} & \text{q} \\
\hline
\text{f1} & R_{f1} + R_{a1} - M_{d1}p\theta & & R_{a1} \\
\hline
\text{f2} & & R_{f2} + R_{a2} - M_{d2}p\theta & -R_{a2} \\
\hline
\text{q} & -M_{d1}p\theta + R_{a1} & M_{d2}p\theta - R_{a2} & R_{a1} + R_{a2} \\
\hline
\end{array}
$$

Interconnected Machines

$$
\begin{array}{cc}
 & \begin{array}{ccc} f1 & f2 & q \end{array} \\
\mathbf{G} = \begin{array}{c} f1 \\ f2 \\ q \end{array} & \begin{bmatrix} -M_{d1} & & \\ & -M_{d2} & \\ -M_{d1} & M_{d2} & \end{bmatrix}
\end{array}
$$

Torque generated in each machine is given by iGi where, from matrix

generated torque

$$T = -i_{f1}M_{d1}(i_q + i_{f1}) + i_{f2}M_{d2}(i_q - i_{f2})$$

The first term is the torque generated by current in the generator, to oppose the torque impressed upon the shaft by the prime mover. The second term gives the torque generated by the motor to meet the load torque impressed upon the motor shaft.

Interconnected Machines

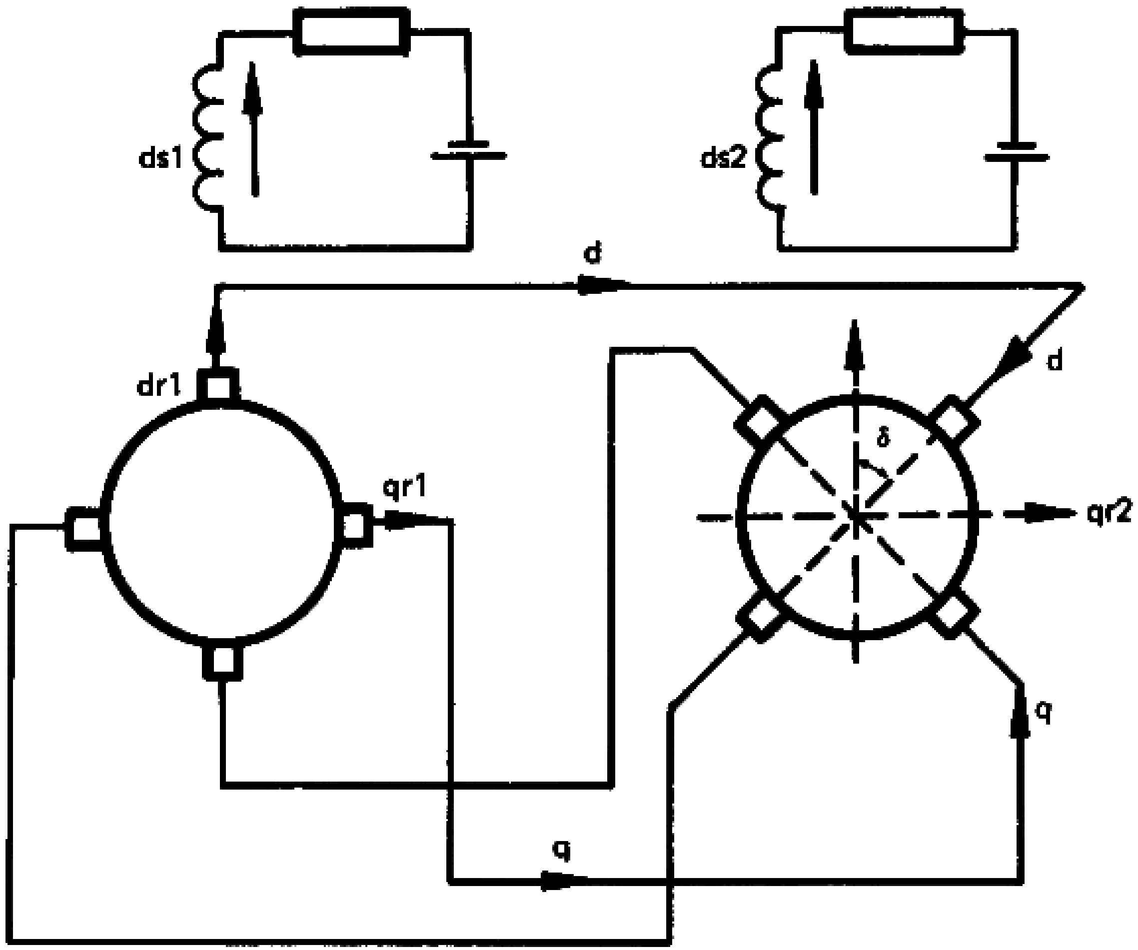
The new voltage vector is given by $\mathbf{V}' = \mathbf{C}_t \mathbf{V}$

$$\mathbf{V} = \begin{array}{c} \text{ds1} \\ \text{qr1} \\ \text{ds2} \\ \text{qr2} \end{array} \begin{bmatrix} 0 \\ V_{qr1} \\ 0 \\ V_{qr2} \end{bmatrix}$$

$$\mathbf{V}' = \begin{array}{c} \text{f1} \\ \text{f2} \\ \text{q} \end{array} \begin{bmatrix} V_{qr1} \\ V_{qr2} \\ V_{qr1} - V_{qr2} \end{bmatrix} = \begin{array}{c} \text{f1} \\ \text{f2} \\ \text{q} \end{array} \begin{bmatrix} V_{f1} \\ V_{f2} \\ V_{q} \end{bmatrix}$$

Interconnected Machines

Case II: alternator supplying a synchronous motor



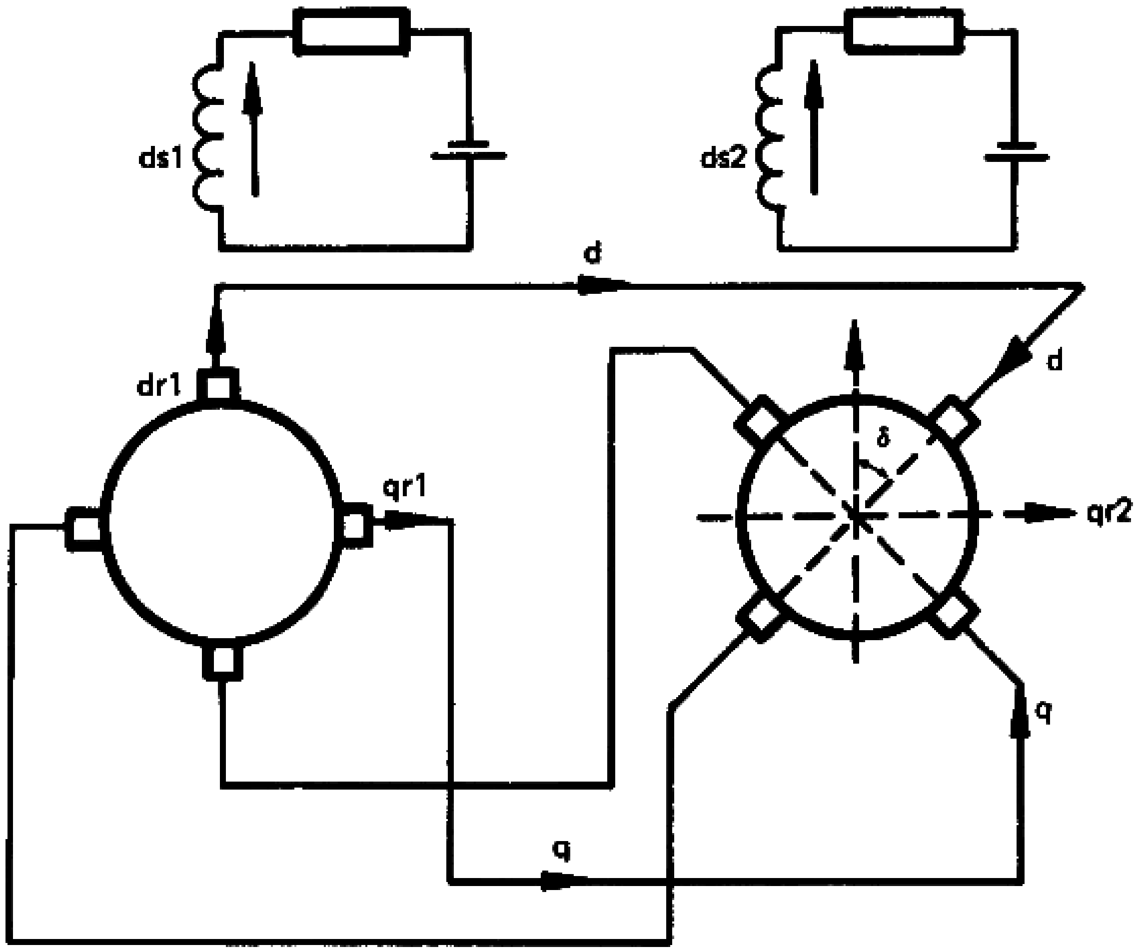
C =

	ds1	d′	q′	ds2
ds1	1			
dr1		1		
qr1			1	
ds2				1
dr2		$-\cos\delta$	$\sin\delta$	
qr2		$-\sin\delta$	$-\cos\delta$	

Due to the shaft load on the motor, it operates with the armature direct and quadrature axes lagging those of the alternator by the load angle δ. Our assumption of synchronous operation implies that pδ is zero.

Interconnected Machines

Case II: alternator supplying a synchronous motor



$$\begin{bmatrix} V_{ds} \\ V_{dr} \\ V_{qr} \end{bmatrix} = \begin{array}{cc} & \begin{array}{ccc} ds & dr & qr \end{array} \\ \begin{array}{c} ds \\ dr \\ qr \end{array} & \begin{bmatrix} R_{ds}+L_{ds}p & M_d p & \\ M_d p & R_{dr}+L_{dr}p & L_{qr}p\theta \\ -M_d p\theta & -L_{dr}p\theta & R_{qr}+L_{qr}p \end{bmatrix} \end{array} \cdot \begin{bmatrix} i_{ds} \\ i_{dr} \\ i_{qr} \end{bmatrix}$$

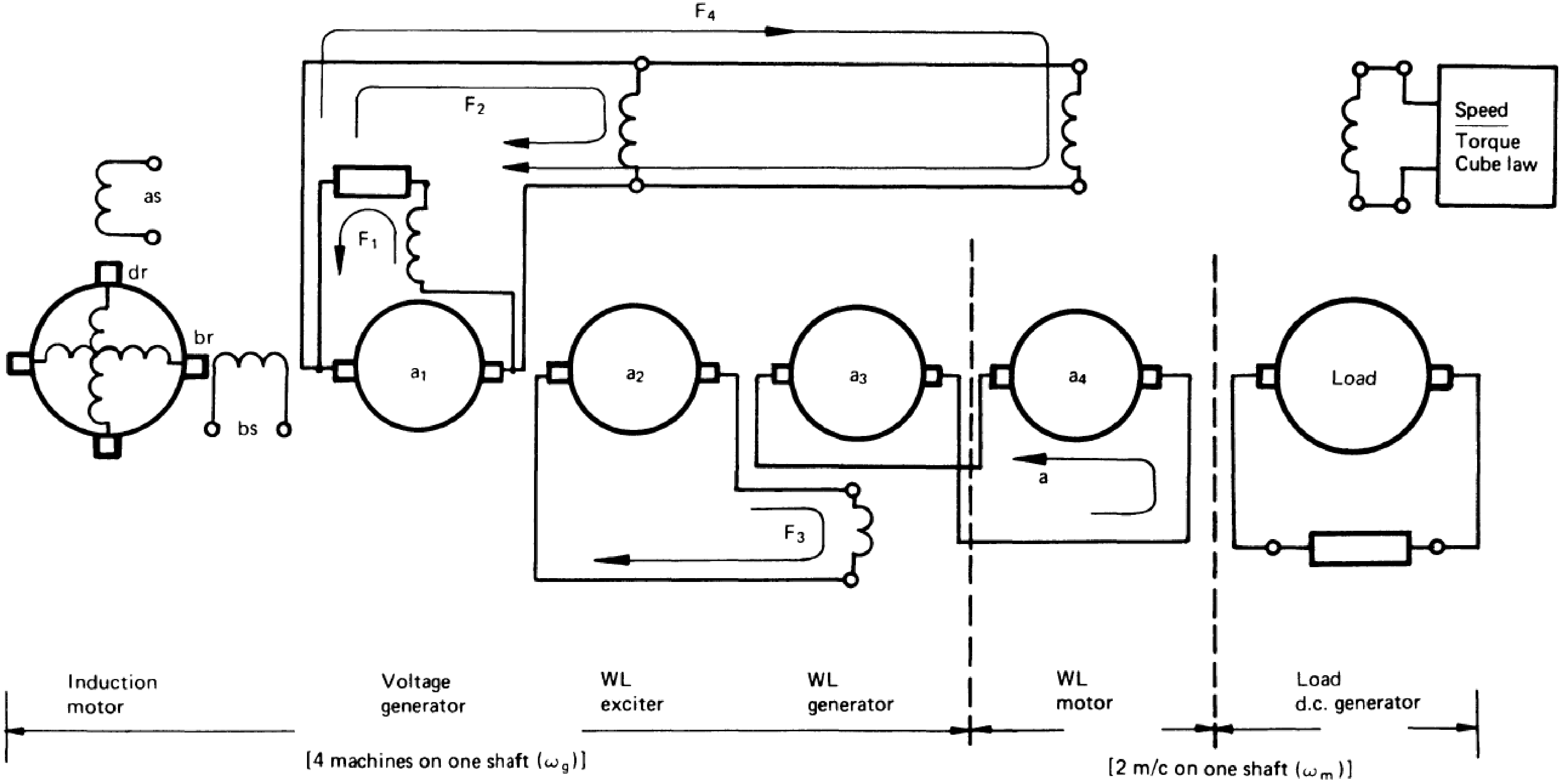
$$\mathbf{Z} = \begin{bmatrix} \mathbf{Z}_{alt} & \\ & \mathbf{Z}_{motor} \end{bmatrix} \equiv \begin{bmatrix} \mathbf{Z}_1 & \\ & \mathbf{Z}_2 \end{bmatrix}$$

$$\mathbf{Z}' = \mathbf{C}_t \mathbf{Z} \mathbf{C}$$

Estimate Z', Steady state impedance matrix, Torque equation

Interconnected Machines

Case III: Ward-Leonard five-machine system



Interconnected Machines

Case III: Ward-Leonard five-machine system

$$
\mathbf{Z} = \begin{array}{c|c|c|c|c|c|}
 & \text{ds} & \text{dr} & \text{qr} & \text{qs} & \text{s} \\
\hline
\text{ds} & R_s + L_s p & Mp & & & \\
\hline
\text{dr} & Mp & R_r + L_r p & L_r p\theta & Mp\theta & \\
\hline
\text{qr} & -Mp\theta & -L_r p\theta & R_r + L_r p & Mp & \\
\hline
\text{qs} & & & Mp & R_s + L_s p & \\
\hline
\text{s} & Mi_{qr} & L_r i_{qr} & -L_r i_{dr} & -Mi_{dr} & Jp + R_F \\
\hline
\end{array}
$$

induction motor

DC machine

$$
\mathbf{Z} = \begin{array}{c|c|c|c|}
 & \text{F} & \text{a} & \text{s} \\
\hline
\text{F} & R_f + L_f p & & \\
\hline
\text{a} & -Mp\theta & R_a + L_a p & \\
\hline
\text{s} & Mi_a & & Jp + R_F \\
\hline
\end{array}
$$

Interconnected Machines

Case III: Ward-Leonard five-machine system

$\mathbf{C} =$

		F_1	F_2	F_3	F_4	a	as	ar	br	bs	sA	sB
	F_1	1										
VG	a1	1	1		1							
	s1										−1	
	F_2		1									
WLE	a2			1								
	s2										−1	
	F_3			1								
WLG	a3					1						
	s3										−1	
	F_4				1							
WLM	a4					−1						
	s4											1
	as						1					
	ar							1				
IM	br								1			
	bs									1		
	s5										1	

Interconnected Machines

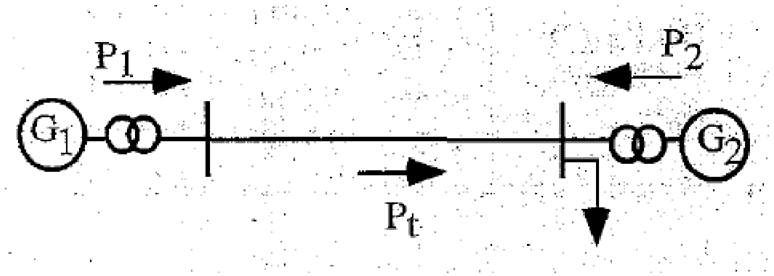
Hunting analysis of interconnected machines

C. Jing, J. D. McCalley and M. Kommareddy, "An energy approach to analysis of interarea oscillations in power systems," in IEEE Transactions on Power Systems, vol. 11, no. 2, pp. 734-740, May 1996

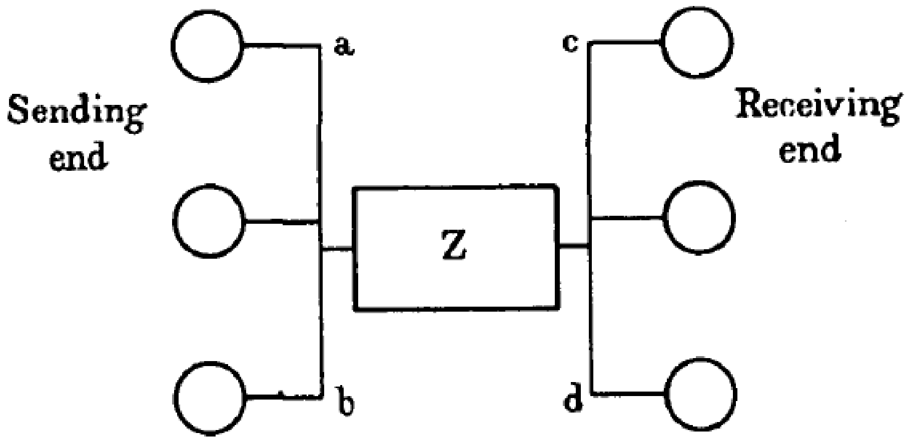
Y. Yu, S. Grijalva, J. J. Thomas, L. Xiong, P. Ju and Y. Min, "Oscillation Energy Analysis of Inter-Area Low-Frequency Oscillations in Power Systems," in IEEE Transactions on Power Systems, vol. 31, no. 2, pp. 1195-1203, March 2016

Interconnected Machines

Hunting analysis of interconnected machines



- Modes, Modal analysis
- Motion modes
- Energy modes
- Mode shapes
- Participation factor



There are many more “energy modes” than “motion modes”