

Lecture - 23

Field Oriented Control of AC Motor Drives

FOC Introduction

- Until a few years ago, dc motor drives remained the only choice for fast response high performance drives such as steel and paper mill drives, servo drives for machine tool applications, etc., although it was realized that a squirrel cage induction motor enjoys many advantage over a dc motor, such as better power/weight ratio, lower inertia, higher speeds and less maintenance.
- Partly this state of affairs could be attributed to the fact that inverters for AC motor drives were not as rugged and reliable as the line commutated converters needed for dc motor drives.
- More importantly, however, dc motors represent simple control systems.

FOC Introduction

- This is because the dc motor has two separate physical windings, the armature and the field, whose mmfs are always at right angles with respect to one another in space due to the action of the commutator.
- The armature current can be controlled independently of the field current yielding fast response torque and speed control.
- This decoupling between control of torque and control of flux is not available readily in AC motors.
- It was only towards the late 1970s that it was realized that the dynamic behaviour of AC motors could be viewed in a manner analogous to that of the dc motor, provided the machine is modelled in an appropriate manner.

FOC Introduction

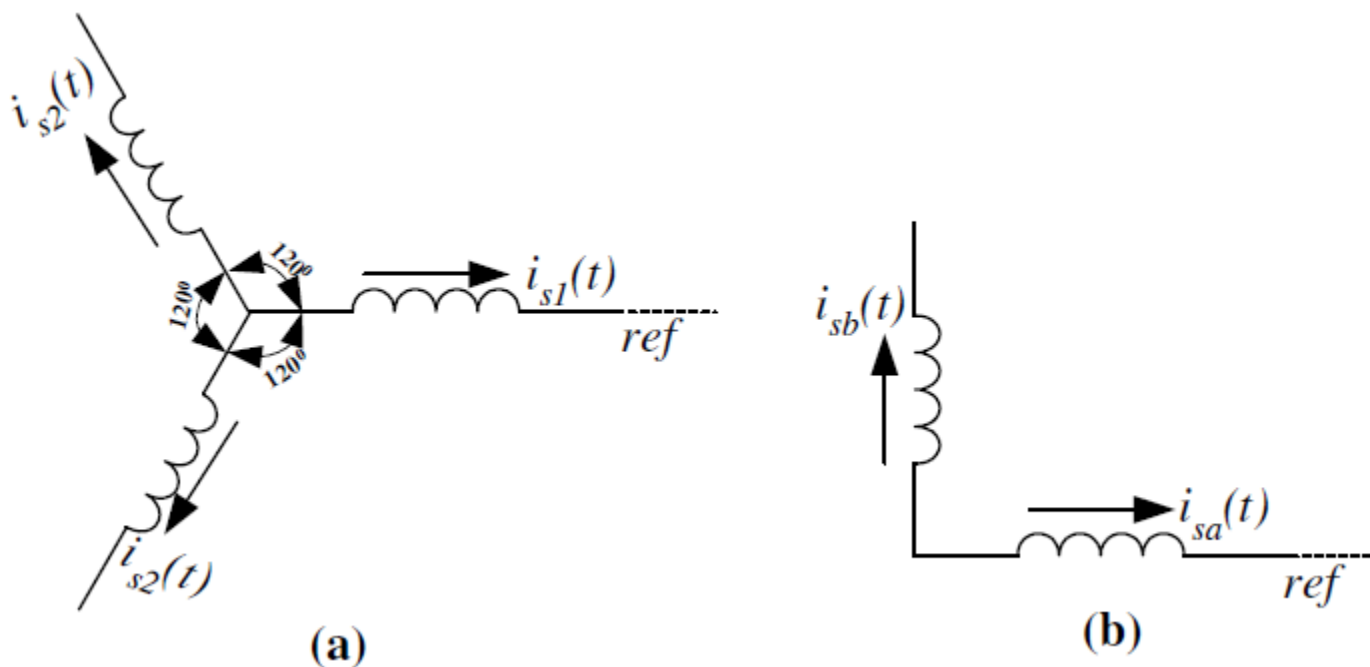
- The conceptual framework for such a model was then established and control methods could then be proposed for AC motor drives, yielding decoupled control of flux and torque with performance equivalent to that of the dc motor drive .
- This technique for decoupled control of torque and ψ in an AC motor has come to be known as field oriented control or vector control of AC motors.
- At the beginning , this technique could not be readily applied because it required complex signal processing.
- With present day advances in microelectronics, this is no longer a serious constraint and field oriented AC drives are emerging as a viable alternative to dc motor drives in high performance applications.

Space Phasors

- The conceptual foundation for field oriented control lies in the so called space phasor modelling of AC machines.
- It is therefore necessary to first develop an appreciation of the concept of space phasors.
- Consider a three phase winding in an AC machine, for example, the stator winding of an induction motor .

Space Phasors

- Figure below shows the schematic diagram of the three phase coils, each of which has N_s turns.
- The diagram shows the spatial orientation of the three coils, the angles being in electrical radians.



Space Phasors

- It is assumed that the spatial distribution of mmf produced by each coil is sinusoidal in nature and also that the neutral is isolated, so that the condition $i_{s1}(t) + i_{s2}(t) + i_{s3}(t) = 0$ holds at all instants of time.
- The currents can have any general variation with respect to time.
- The axis of coil 1 is taken as the reference for spatial orientation.
- At any given instant of time, the net mmf produced by the three coils is given by adding the mmfs due to the individual coils, but with appropriate spatial orientation, i.e. vectorially.

Space Phasors

- The net mmf can therefore in general have components along and perpendicular to the reference direction.
- If these components are denoted by subscripts a and b respectively, their values are given by

$$\text{Mmf}_a = N_s (i_{s1}(t) + i_{s2}(t) \cos \gamma + i_{s3}(t) \cos 2\gamma)$$

$$\text{Mmf}_b = N_s (i_{s2}(t) \sin \gamma + i_{s3}(t) \sin 2\gamma)$$

- where $\gamma = 2\pi/3$ electrical radians.

Space Phasors

- Thus it can be seen that the system of three coils can be replaced by a system of two coils a and b shown in Figure of slide 6, having the same number of turns N_s as the original coils and carrying currents $i_{sa}(t)$ and $i_{sb}(t)$ given by

$$i_{sa}(t) = i_{s1}(t) - \frac{1}{2}i_{s2}(t) - \frac{1}{2}i_{s3}(t)$$

$$i_{sb}(t) = (i_{s2}(t) - i_{s3}(t))\frac{\sqrt{3}}{2}$$

Using the condition, $i_{s1}(t) + i_{s2}(t) + i_{s3}(t) = 0$, the above equations can be rewritten as

$$i_{sa}(t) = \frac{3}{2}i_{s1}(t)$$

$$i_{sb}(t) = (i_{s2}(t) - i_{s3}(t))\frac{\sqrt{3}}{2}$$

Space Phasors

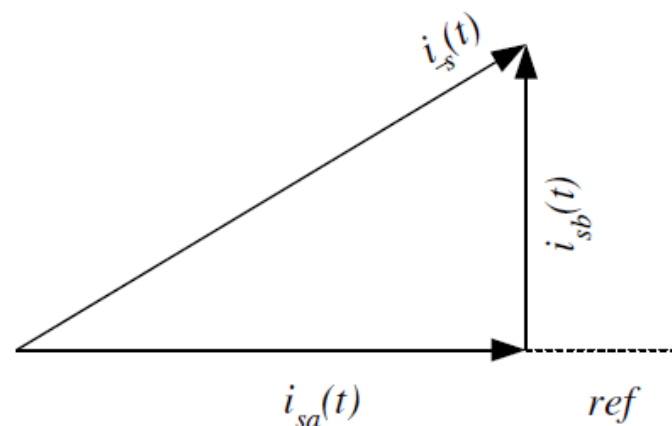
- The net mmf produced by the two systems of coils is identical.
- The net effect of all the currents is thus obtained by adding $i_{sa}(t)$ and $i_{sb}(t)$ with proper spatial orientation.
- Using complex notation, this can be expressed by defining a so called current phasor $\bar{i}_s(t)$ by

$$\bar{i}_s(t) = i_{sa}(t) + ji_{sb}(t)$$

- The current space phasor is thus a complex function of time, whose real and imaginary parts give the components of current along two mutually perpendicular directions in space.

Space Phasors

- Pictorially the space phasor $\bar{i}_s(t)$ can be represented by a vector in a two dimensional plane, the real and imaginary components being $i_{sa}(t)$ and $i_{sb}(t)$.
- This is shown in Figure below.



- The space phasor can also be expressed in polar instead of Cartesian form as follows:

$$\bar{i}_s(t) = i_s(t)e^{j\zeta(t)}$$

Where $i_s(t)$ - instantaneous amplitude of space phasor

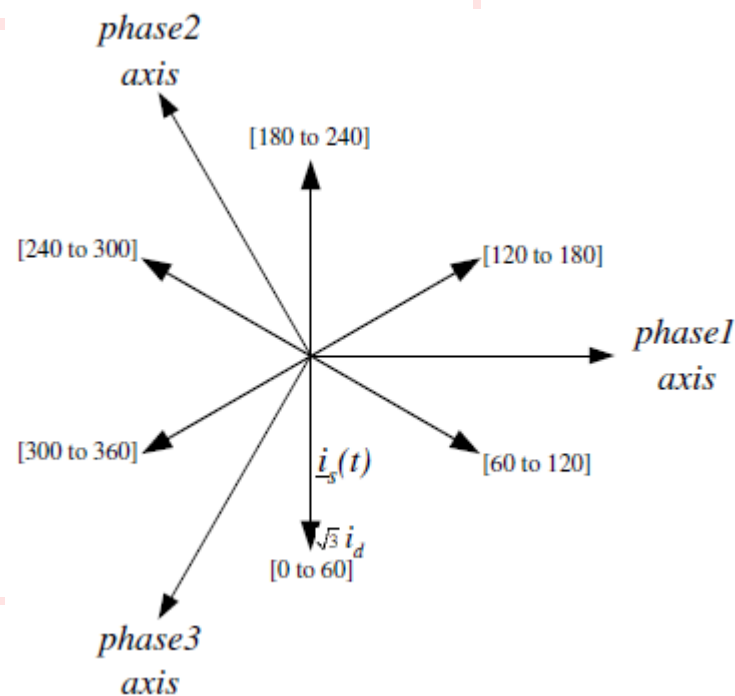
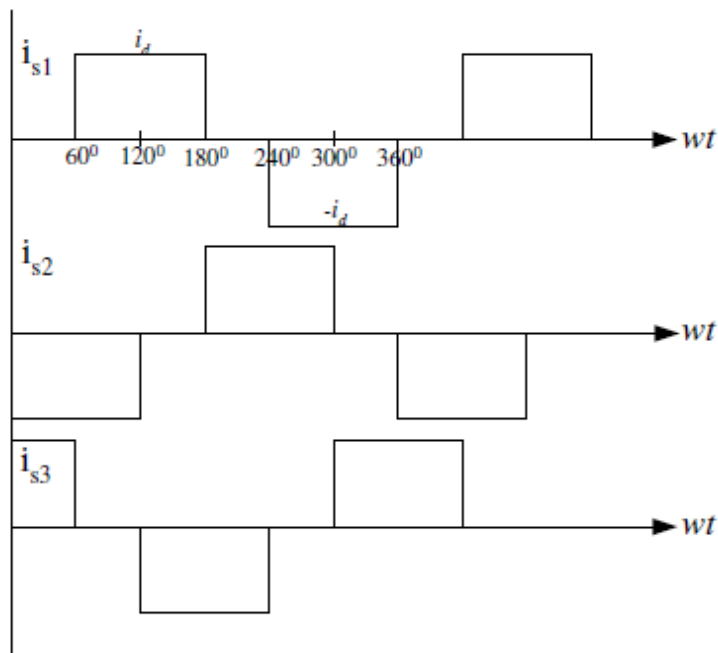
$\zeta(t)$ - instantaneous angle that the space phasor makes with the reference direction.

Space Phasors

- From the study of polyphase windings, it will be seen that if the three individual quantities are balanced three phase sinusoids, then the space phasor will have a constant amplitude and will rotate in space with constant angular velocity.
- But the definition of space phasors is not limited to sinusoidal quantities alone.
- Any general time variation is possible for the three individual variables.
- Thus the concept of a space phasor is a useful tool in the analysis of AC motor drives, because the inverters that drive the motor produce non-sinusoidal voltages.

Space Phasors

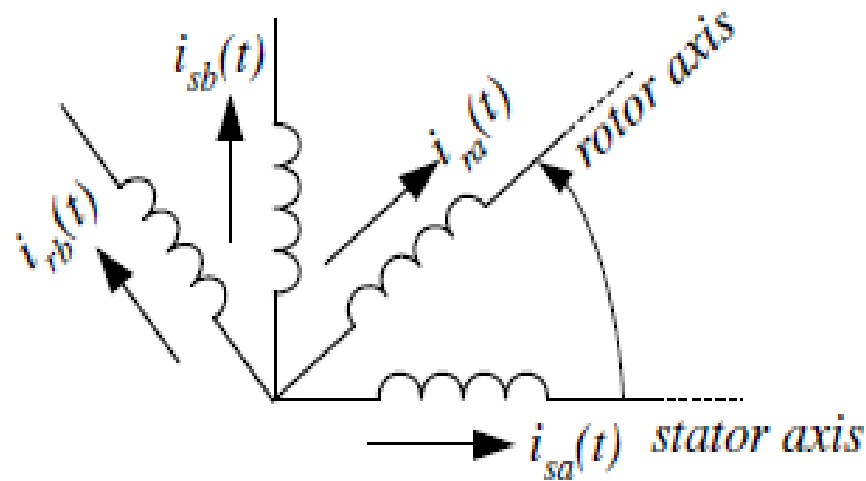
- The currents produced by a current source inverter for example have the waveforms shown in Figure below.



- The corresponding space phasor will therefore occupy a fixed position in space for one sixth of a cycle and jump in position by 60° at every commutation in the inverter.

Induction Machine Equations in Space Phasor Form

- The symmetrical three phase squirrel cage induction motor has a three phase system of coils on the stator and a cage on the rotor which can be considered to be equivalent to a three phase winding.
- The two sets of windings can be represented by two equivalent two phase coils as shown in Figure below.



Induction Machine Equations in Space Phasor Form

- The rotor axis makes an angle $\varepsilon(t)$ with respect to the stator axis.
- Two current space phasors $\bar{i}_s(t)$ and $\bar{i}_r(t)$ can be defined for the stator and rotor current respectively as follows:

$$\bar{i}_s(t) = i_{sa}(t) + ji_{sb}(t)$$

$$\bar{i}_r(t) = i_{ra}(t) + ji_{rb}(t)$$

- Note that the two are defined with respect to different coordinate axes;
 - $\bar{i}_s(t)$ with respect to stator coordinates and
 - $\bar{i}_r(t)$ with respect to rotor coordinates.

Induction Machine Equations in Space Phasor Form

- The flux linkages of the various coils can be written as a first step towards writing down the machine voltage equations

$$\psi_{sa}(t) = L_s i_{sa}(t) + M i_{ra}(t) \cos \varepsilon(t) - M i_{rb}(t) \sin \varepsilon(t)$$

$$\psi_{sb}(t) = L_s i_{sb}(t) + M i_{ra}(t) \sin \varepsilon(t) + M i_{rb}(t) \cos \varepsilon(t)$$

- Where,
 - L_s - self inductance of stator coils and
 - M - maximum value of mutual inductance between stator and rotor coils.
- Combining above two equations to form the stator flux space phasor.

$$\underline{\psi}_s(t) = \psi_{sa}(t) + j\psi_{sb}(t) = L_s \underline{i}_s(t) + M \underline{i}_r(t) e^{j\varepsilon(t)}$$

- Similarly the rotor flux linkage space phasor can be derived as

$$\underline{\psi}_r(t) = \psi_{ra}(t) + j\psi_{rb}(t) = L_r \underline{i}_r(t) + M \underline{i}_s(t) e^{-j\varepsilon(t)}$$

Induction Machine Equations in Space Phasor Form

- The stator flux and rotor flux space phasor equations in previous slide resembles that for a coil with self and mutual inductance except for the expression for current in the second term.
- It must be remembered that stator flux space phasor is with respect to stator coordinate and rotor flux space phasor is with respect to rotor coordinates.
- Therefore, space phasors defined with respect to another coordinate system have to be transformed to the coordinate system of the equation.
- Multiplication by $e^{j\epsilon(t)}$ results in a clockwise rotation of the coordinate system by an angle $\epsilon(t)$, while multiplication by $e^{-j\epsilon(t)}$ results in anticlockwise rotation of the coordinate system by the same angle.

Induction Machine Equations in Space Phasor Form

- The voltage-current equations for the stator and rotor windings can also be written in the space phasor form.
- First the individual coil equations are written as follows:

$$v_{sa}(t) = R_s i_{sa}(t) + \frac{d\psi_{sa}(t)}{dt}$$

$$v_{sb}(t) = R_s i_{sb}(t) + \frac{d\psi_{sb}(t)}{dt}$$

$$v_{ra}(t) = R_r i_{ra}(t) + \frac{d\psi_{ra}(t)}{dt}$$

$$v_{rb}(t) = R_r i_{rb}(t) + \frac{d\psi_{rb}(t)}{dt}$$

- Where R_s - stator resistance, R_r - rotor resistance.

Induction Machine Equations in Space Phasor Form

- Combining equations for $V_{sa}(t)$ and $V_{sb}(t)$, we get:

$$\underline{v}_s(t) = R_s \underline{i}_s(t) + \frac{d\underline{\psi}_s(t)}{dt}$$

- Combining equations for $V_{ra}(t)$ and $V_{rb}(t)$, we get:

$$\underline{v}_r(t) = R_r \underline{i}_r(t) + \frac{d\underline{\psi}_r(t)}{dt}$$

- These equations can be rewritten using the expression for stator and rotor flux space phasor as

$$\underline{v}_s(t) = R_s \underline{i}_s(t) + L_s \frac{d\underline{i}_s(t)}{dt} + M \frac{d}{dt} [\underline{i}_r(t) e^{j\varepsilon(t)}]$$

$$\underline{v}_r(t) = R_r \underline{i}_r(t) + L_r \frac{d\underline{i}_r(t)}{dt} + M \frac{d}{dt} [\underline{i}_s(t) e^{-j\varepsilon(t)}]$$

Induction Machine Equations in Space Phasor Form

- It must be remembered that equation for stator voltage space phasor refers to the stator and is in stator coordinates whereas equation rotor voltage space phasor refers to the rotor and is in rotor coordinates.
- For a squirrel cage induction motor, of course, rotor voltage space phasor is zero.
- Each of the above equations is actually two equations combined into one.
- With these two equations, the electrical behavior of the machine is defined.

Induction Machine Equations in Space Phasor Form

- The torque developed by the machine is given by

$$M_d = \frac{2}{3} \frac{P}{2} M \operatorname{Im}[\underline{i}_s(t) [\underline{i}_r(t) e^{j\varepsilon(t)}]^*]$$

- Where Im stands for imaginary part and $*$ denotes complex conjugate, P is the number of poles.

Induction Machine Equations in Space Phasor Form

- Therefore the complete equations that describe the behavior of the machine are as follows:

$$R_s \underline{i}_s(t) + L_s \frac{d\underline{i}_s(t)}{dt} + M \frac{d}{dt} [\underline{i}_r(t) e^{j\varepsilon(t)}] = \underline{v}_s(t)$$

$$R_r \underline{i}_r(t) + L_r \frac{d\underline{i}_r(t)}{dt} + M \frac{d}{dt} [\underline{i}_s(t) e^{-j\varepsilon(t)}] = 0$$

$$J \frac{d\omega_m}{dt} = \frac{2}{3} M \operatorname{Im} [\underline{i}_s(t) [\underline{i}_r(t) e^{j\varepsilon(t)}]^*] - M_{load}$$

where

$$\frac{P}{2} \omega_m = \omega = \frac{d\varepsilon(t)}{dt}$$

ω_m - rotor speed in mechanical rad/sec

ω - rotor speed in electrical rad/sec

J - moment of inertia

M_{load} - torque load

Sinusoidal Steady State Operation

- It must be remembered that equation for stator voltage space phasor refers to the stator and is in stator coordinates whereas equation rotor voltage space phasor refers to the rotor and is in rotor coordinates.
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