Linear Programming

Programming ⟶ planning and it refers to a particular plan of action from amongst several alternatives for maximizing profit or minimizing cost etc.

Programming problems ⟶ deal with determining opitimal allocation of limited resources to meet the given objectives, such as least cost, maximum profit, highest margin or least time, when resources have alternative uses.

$$\boxed{\begin{array}{l} \text{optimize } Z = f(x) \\ \text{s.t. constraints } g_i(x) \end{array}}$$

Linear ⟶ all inequalities or equations used and the function to be maximized or minimized are linear.

LPP
- If $f(x)$, $g_i(x)$ $\forall\, i$ are linear ⟶ (LPP)
- If atleast one of $f(x)$ or $g_i(x)$ are non-linear ⟶ (NLPP)

Pb Suppose that a furniture dealer makes two products viz. chairs and tables. Processing of these products is done on two machines A and B. A chair requires 2 hours on machine A and 3 hours on machine B. A table requires 4 hours on machine A and 2 hours on machine B. There are 16 hours of time per day available on machine A and 120 hours on machine B. Case I Profits gained by the manufacturer from a chair and a table are Rs 300 and Rs 200 resp. The manufacturer is willing to know the daily product of each of the two products to maximize his profit.

Item	Chair	Table	Max^n available time
Machine A	2 hrs.	4 hrs.	16 hrs.
Mach. B	3 hrs.	2 hrs.	120 hrs.
Case I → Profit (in Rs)	Rs. 300	Rs 200	

Suppose that the manufacturer produces x chairs and y tables per day.

Machine A
$$2x + 4y \leq 16$$

Machine B
$$3x + 2y \leq 120$$

Since the numbers of chairs and tables are never negative. Therefore $x \geq 0$ and $y \geq 0$.

Total profit in producing x chairs and y tables on a day is given by

$$\text{Profit} = 300x + 200y$$

Thus, we have to maximize

$$\text{Profite} = 300x + 200y$$

Subject to the constraints

$$2x + 4y \leq 16$$
$$3x + 2y \leq 120$$
$$x \geq 0 \;.\; y \geq 0 .$$

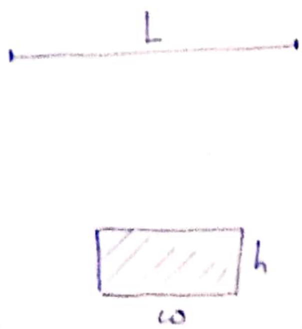
Out of all points (x, y) in the sol^n set of the above linear constraints, the manufacturer has to choose that point, or those points for which the profit $300x + 500y$ has the max^m value.

<u>Case II</u> In the above discussion, if a chair costs Rs 1250 and a table costs Rs 3000 then the total cost of producing x chairs and y ~~cha~~ tables is $1250x + 3000y$. Now the manufacturer will be interested to choose that point, or those points in the sol^n set of the above linear constraints for which the cost $1250x + 3000y$ has the minimum value.

The two situations discussed above give the description of a type of linear programming problems.
In the above discussion, the profit function $= 300x + 500y$ or the cost function $= 1250x + 3000y$ is known as the objective function. The inequations $2x + 4y \le 16$, $3x + 2y \le 12$ are known as the constraints and $x \ge 0$, $y \ge 0$ are known as the non-negativity restrictions.

<u>Pb</u> Consider forming a maximum-area rectangle out of a piece of wire of length L inches. What should be the best width and height of the rectangle?

<u>Sol^n</u>:



$$\text{Max } Z = wh \quad \text{(ob. function)}$$

subject to $\quad 2(w+h) = L \quad$ (constraint)

$$w, h \ge 0 \quad \text{(Non-neg. rest.)}$$

Since, objective function is non-linear
So, it's a problem of NLPP.

$$\text{Max } Z = \frac{L}{4} \times \frac{L}{4} = \frac{L^2}{16}$$

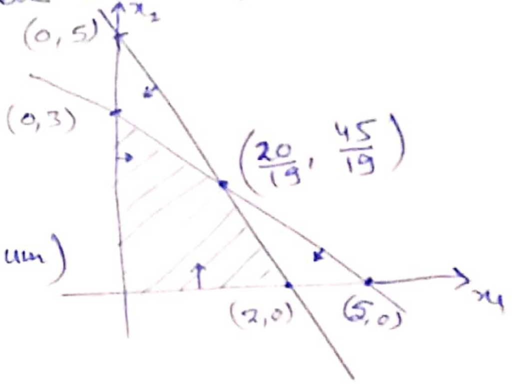
Graphical Method :-

Pb1. Max $Z = 5x_1 + 3x_2$

subject to

$$3x_1 + 5x_2 \le 15$$
$$5x_1 + 2x_2 \le 10$$
$$x_1, x_2 \ge 0$$

Solⁿ First of all we change constraints in equality and find feasible reason.



→ $(0, 3)$ $\quad Z = 9$

→ $\left(\frac{20}{19}, \frac{45}{19}\right)$ $\quad Z = \frac{235}{19}$ (Maximum)

→ $(2, 0)$ $\quad Z = 10$

→ $(0, 0)$ $\quad Z = 0$

Thus $\left(x_1 = \frac{20}{19},\ x_2 = \frac{45}{19}\right)$ is optimal solⁿ for Max Z.

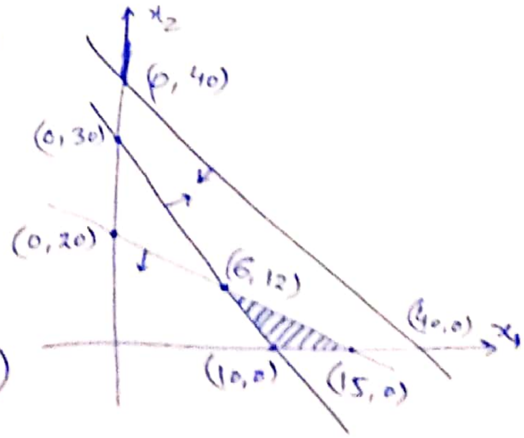
And the optimum value of Z is $\frac{235}{19}$.

Pb2. Min $Z = 20x_1 + 10x_2$

s.t.

$$x_1 + x_2 \le 40$$
$$3x_1 + x_2 \ge 30$$
$$4x_1 + 3x_2 \le 60$$
$$x_1, x_2 \ge 0$$

Solⁿ Plot the constraints using equalities.



at $(6, 12) \longrightarrow Z = 240$

$(10, 0) \longrightarrow Z = 200$ (Min)

$(15, 0) \longrightarrow Z = 300$

Pb 3.

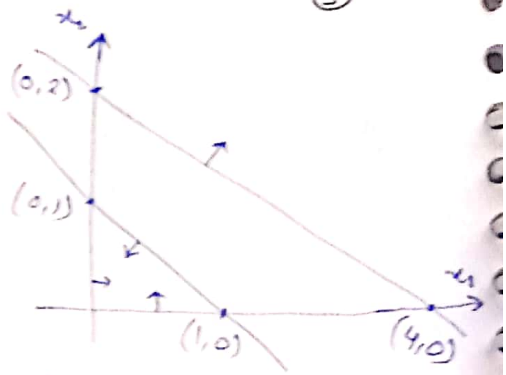
$$\text{Max } Z = 4x_1 + 3x_2$$

s.t.

$$x_1 + x_2 \le 1$$

$$x_1 + 2x_2 \ge 4$$

$$x_1, x_2 \ge 0$$



Sol^n No sol^n as no common region or no feasible region.

Note: If feasible region is unbounded then sol^n may or may not be bounded.

Pb 4.

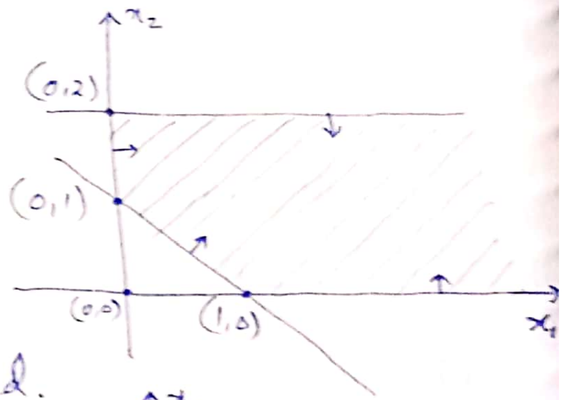
$$\text{Max } Z = x_1 + 4x_2$$

s.t.

$$x_1 + x_2 \ge 1$$

$$x_2 \le 2$$

$$x_1, x_2 \ge 0$$



Sol^n Region is unbounded and sol^n is also unbounded.

Pb 5.

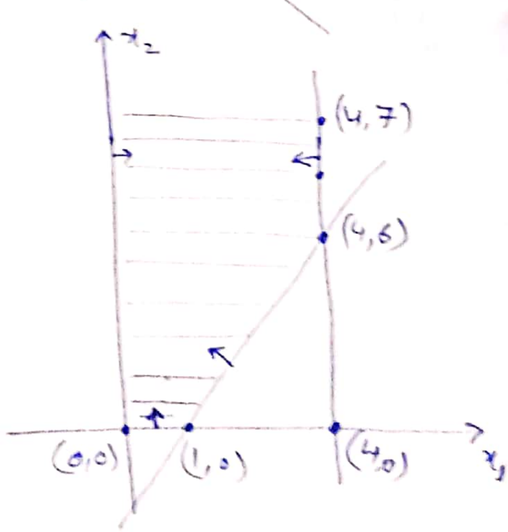
$$\text{Max } Z = 6x_1 - 2x_2$$

s.t.

$$2x_1 - x_2 \le 2$$

$$x_1 \le 4$$

$$x_1, x_2 \ge 0$$



at $(0,0) \longrightarrow Z = 0$

$(1,0) \longrightarrow Z = 6$

$(4,6) \longrightarrow Z = 12$ (Max)

If we take $(4,7) \longrightarrow Z = 10$

$(4,10) \longrightarrow Z$ will start decreasing.

Thus region is unbounded but sol^n is bounded.

Note:

Feasible region	sol^n (optimal)
• Bounded	sol^n always exists.
• No feasible region	No sol^n
• Unbounded region	either sol^n exists or unbounded sol^n

Note: If sol^n is unbounded then region must be unbounded.

Convex Sets & Convex Functions :-

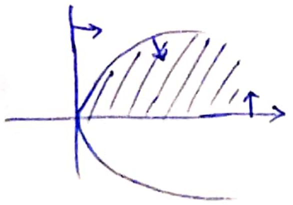
Line :- A line passing through two points x_1 and x_2 in $\mathbb{R}^n$ is the set of points

$$L = \{ x \mid x = \lambda x_1 + (1-\lambda) x_2 , \quad \lambda \in \mathbb{R} \}$$

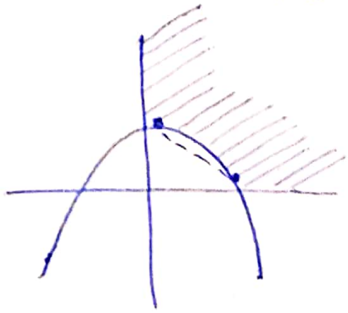
Line segment

$$L = \{ x \mid x = \lambda x_1 + (1-\lambda) x_2 , \quad \lambda \in [0,1] \}$$

4. $S = \{(x,y) \mid y^2 \le 4x , \; x,y \ge 0\}$



3. $S = \{(x_1, x_2) \mid x_2 - 3 \geq -x_1^2 , \; x_1, x_2 \geq 0\}$



Hyperplanes: A hyperplane in $\mathbb{R}^n$ generalizes the notion of a straight line in $\mathbb{R}^2$ and the notion of a plane in $\mathbb{R}^3$.

Def^n: Set of points $(x_1, x_2, \dots, x_n) \in \mathbb{R}^n$ which satisfy $z = c_1x_1 + c_2x_2 + \dots + c_nx_n$ is called hyperplane.

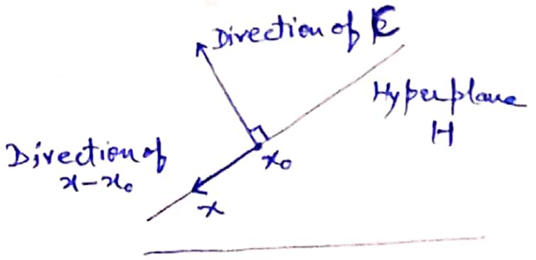
Here, not all $c_i = 0$ and z is scalar.

OR

A hyperplane H in $\mathbb{R}^n$ is a set of the form $\{X \mid CX = Z\}$ where C is a nonzero vector in $\mathbb{R}^n$ and Z is a scalar. Here, C is called the normal or the gradient to the hyperplane.

- The constant k can be eliminated by referring to a fixed point x_0 on the hyperplane.

  If $X_0 \in H$, then $CX_0 = Z$ and for any $X \in H$, we have $CX = Z$. Upon subtraction we get $C(X - X_0) = 0$.



In other words, H can be represented as the collection of points satisfying $C(X - X_0) = 0$, where X_0 is any fixed point in H.

- A hyperplane is a convex set.
- **Half-space**:- A hyperplane divides $\mathbb{R}^n$ into two regions, called half-spaces.
- A hyper plane divides $\mathbb{R}^n$ in two parts.
  Let $H = \{X \mid CX = Z\}$ be the hyper plane

then

Closed Half Space

$H_1 = \{x \mid cx \leq z\}$ and $H_2 = \{x \mid cx > z\}$

OR

$H_3 = \{x \mid cx \geq z\}$ and $H_4 = \{x \mid cx < z\}$

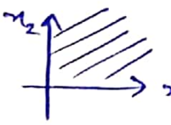
open half space

H_1, H_3 are closed half spaces and
H_2, H_4 are open " " .

- Parallel hyperplane: Two hyperplane $Z_1 = C_1 X$ and $Z_2 = C_2 X$ are called $\|^{rl}$ if $C_1 = \lambda C_2$, for some $0 \neq \lambda \in \mathbb{R}$.

- Polytope: The intersection of a finite number of closed half spaces is called a polytope.

  Ex: $\{(x_1, x_2) \mid x_1 \geq 0\} \cap \{(x_1, x_2) \mid x_2 \geq 0\}$



- Hypersphere: A hyperspere in $\mathbb{R}^n$ with centre at a and radius $\gamma > 0$ is defined to be the set of points

$$S = \{x \mid |x - a| = \gamma\}$$

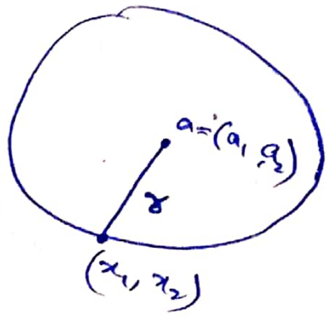
Thus the eq for hypersphere in $\mathbb{R}^n$ is

$$\sum_{i=1}^{n} (x_i - a_i)^2 = \gamma^2 .$$

Ex: for $n = 2$.

$$\sum_{i=1}^{2} (x_i - a_i)^2 = \gamma^2$$

$$\Rightarrow \quad (x_1 - a_1)^2 + (x_2 - a_2)^2 = \gamma^2$$



Convex Sets & Convex Functions :-

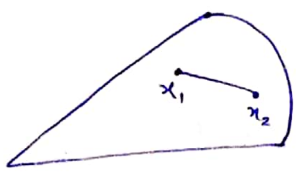
Convex Sets: A set X in R^n is called a convex set if given any two points x_1 and x_2 in X, then $\lambda x_1 + (1-\lambda)x_2 \in X$ for each $\lambda \in [0,1]$.

- Note that $\lambda x_1 + (1-\lambda)x_2$ for λ in the interval $[0,1]$ represents a point on the segment joining x_1 and x_2.
- Any point of the form $\lambda x_1 + (1-\lambda)x_2$ where $0 \le \lambda \le 1$ is called a convex combination or (weighted average) of x_1 and x_2.
- If $\lambda \in (0,1)$, then the convex combination is called strict.

Geometric Interpretation:-

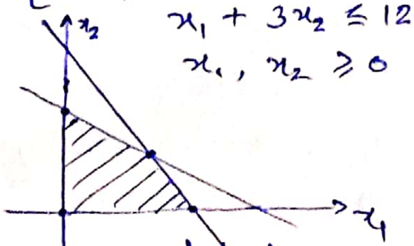
For each pair of points x_1 and x_2 in X, the line segment joining them, or the convex combinations of the two points, must belong to X.

1.



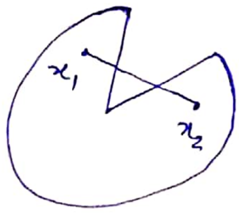
A convex set

2. $S = \left\{ (x_1,x_2) \,\middle|\, \begin{array}{l} 3x_1 + 2x_2 \le 18, \\ x_1 + 3x_2 \le 12 \\ x_1, x_2 \ge 0 \end{array} \right\}$



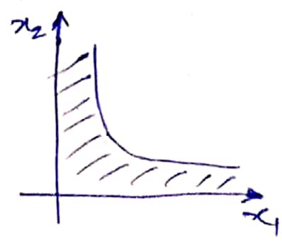
3. In case of polyhedron, all the feasible region is convex.

1.



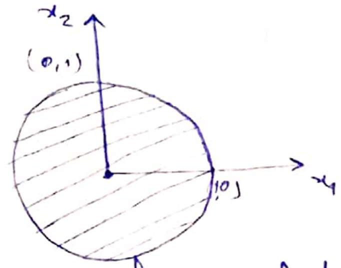
A non-convex set

2. $S = \{ (x_1,x_2) \mid x_1 x_2 \le 1,\ x_1 \ge 0,\ x_2 \ge 0 \}$



Examples:

1. $\{(x_1, x_2) : x_1^2 + x_2^2 \le 1\}$.



2. $X = \{x : Ax = b\}$, where A is an $m \times n$ matrix and b is an m-vector.

Sol^n: Let x_1 and $x_2 \in X$ then $Ax_1 = b$ and $Ax_2 = b$ hold. For $\lambda \in [0,1]$, we have

$$\begin{aligned} A(\lambda x_1 + (1-\lambda)x_2) &= A\lambda x_1 + A(1-\lambda)x_2 \\ &= \lambda A x_1 + (1-\lambda) A x_2 \\ &= \lambda b + (1-\lambda) b \\ &= b \end{aligned}$$

$$\Rightarrow \quad \lambda x_1 + (1-\lambda) x_2 \in X.$$

3. $X = \{x : Ax = b,\ x \ge 0\}$, where A is an $m \times n$ matrix and b is an m-vector.

4. $X = \{x : Ax \le b,\ x \ge 0\}$, " " "
" " "

5. $X = \left\{ x : x = \lambda_1 \underset{v_1}{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}} + \lambda_2 \underset{v_2}{\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}} + \lambda_3 \underset{v_3}{\begin{bmatrix} -1 \\ 2 \\ -3 \end{bmatrix}},\ \lambda_1 + \lambda_2 + \lambda_3 = 1,\ \lambda_1, \lambda_2, \lambda_3 \ge 0 \right\}$

Let $x_1, x_2 \in X$ such that

$$x_1 = \lambda_1 v_1 + \lambda_2 v_2 + \lambda_3 v_3, \quad \lambda_1 + \lambda_2 + \lambda_3 = 1, \quad \lambda_1, \lambda_2, \lambda_3 \ge 0$$

and $x_2 = \lambda_1' v_1 + \lambda_2' v_2 + \lambda_3' v_3, \quad \lambda_1' + \lambda_2' + \lambda_3' = 1, \quad \lambda_1', \lambda_2', \lambda_3' \ge 0$

then for any $\lambda \in [0,1]$

$$\begin{aligned} \lambda x_1 + (1-\lambda) x_2 &= (\lambda\lambda_1 + (1-\lambda)\lambda_1')v_1 + (\lambda\lambda_2 + (1-\lambda)\lambda_2')v_2 \\ &\quad + (\lambda\lambda_3 + (1-\lambda)\lambda_3')v_3 \\ &= \lambda_1'' v_1 + \lambda_2'' v_2 + \lambda_3'' v_3 \quad \text{(say)} \end{aligned}$$

$$x''_1 + x''_2 + x''_3 = \lambda(x_1 + x_2 + x_3) + (1-\lambda)(x'_1 + x'_2 + x'_3)$$
$$= \lambda + (1-\lambda)$$
$$= 1$$

and $\quad x''_i = \underbrace{\lambda x_i}_{\geq 0} + \underbrace{(1-\lambda)x'_i}_{\geq 0} \geq 0 \quad \forall\ i = 1, 2, 3$

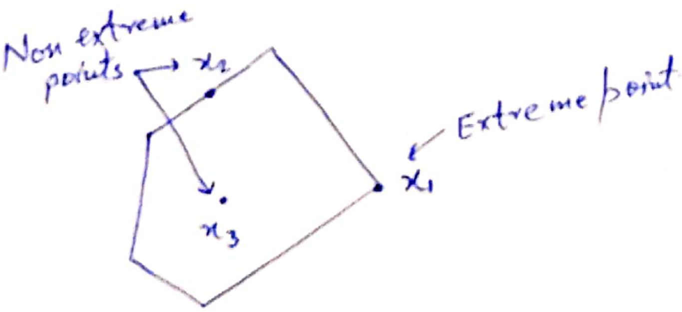
Hence $\quad \lambda x_1 + (1-\lambda) x_2 \in X \quad \forall\ \lambda \in [0,1]$ & $x_1, x_2 \in X$.

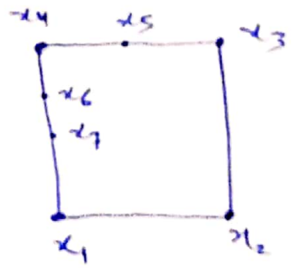
$\Rightarrow$ X is convex set.

Extreme Points :- A point x of a convex set X is called an extreme point if it cannot be expressed as a convex combination of any two other distinct points.

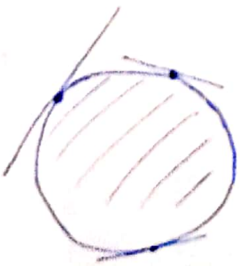
OR

if x can not be represented as a strict convex combination of two distinct points in X. In other words, if $x = \lambda x_1 + (1-\lambda) x_2$ with $\lambda \in (0,1)$ and $x_1, x_2 \in X$, then $x = x_1 = x_2$.





- All four corners are extreme points.
- Apart from all four corners, no points is extreme point.

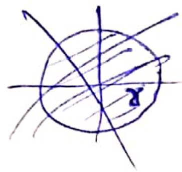


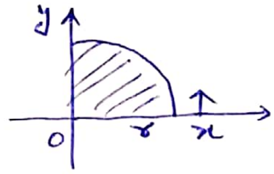
- Boundary of closed disc have infinite extreme points.

- An extreme point can not lie between any other two points of the set.
- Extreme point is a boundary point but converse may not be true.
- Extreme point of the convex set of feasible solⁿ are finite in number.

<u>Note</u>: Closed disc is not taken as a feasible region in LPP. Since it is not linear.

<u>Eg</u>:
$$x^2 + y^2 \le r^2$$
$$x, y \ge 0$$



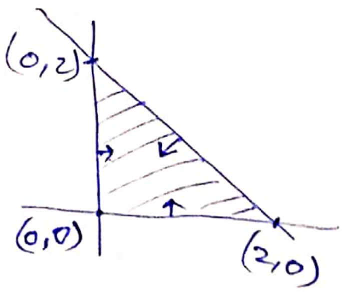


non-linear feasible region

- If we get optimum solution at two extreme points say x_1 and x_2 then we get optimum solⁿ at on those points which can be written as convex combination of these points.

<u>Ex</u>:

$$\text{Max } Z = x_1 + x_2$$
$$x_1 + x_2 \le 2$$
$$x_1, x_2 \ge 0$$



<u>Solⁿ</u>: Clearly, we have two extreme points (0, 2) and (2, 0) where Z have optimum. Therefore convex combination of these two extreme points also gives optimality, i.e., all points lying on this line.

Def: Convex combination of vectors: Given a set of vectors $\{x_1, x_2, \dots, x_k\}$, a linear combination

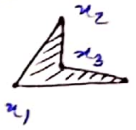
$$x = \lambda_1 x_1 + \lambda_2 x_2 + \dots + \lambda_k x_k$$

is called a convex combination of the given vectors, if $\lambda_1, \lambda_2, \dots, \lambda_k \geq 0$ and $\sum_{i=1}^{k} \lambda_i = 1$.

Ex: A line segment joining x_1 and x_2 is the set of all convex combinations of x_1 and x_2.

$$L = \{x \mid x = \lambda x_1 + (1-\lambda) x_2, \ 0 \leq \lambda \leq 1\}.$$

Thm 1. The set of all convex combinations of a finite number of linearly independent vectors $x_1, x_2, \dots, x_m$ is a convex set.



Thm 2. A hyperplane is a convex set.

Pf: Let $cx = z$ be a hyperplane and let x_1, x_2 be any two points of it.

Then $cx_1 = z$ and $cx_2 = z$

Now, if $0 \leq \lambda \leq 1$, then

$$c(\lambda x_1 + (1-\lambda) x_2) = c(\lambda x_1) + c(1-\lambda) x_2$$
$$= \lambda (cx_1) + (1-\lambda)(cx_2)$$
$$= \lambda z + (1-\lambda) z$$
$$= z$$

Thm 3. The closed half spaces (open half spaces) are convex set.

Thm 4. The intersection of two convex sets is also a convex set. (also true for finite convex set).

Convex polyhedron: The set of all convex combinations of a finite number of linearly independent vectors is called a convex polyhedron.

The convex polyhedron generated by the finite set of linearly independent vectors $x_1, x_2, \ldots, x_m$ is the set

$$\{x \mid x = \sum_{i=1}^{m} \lambda_i x_i, \quad \lambda_i \geq 0, \quad \sum_{i=1}^{m} \lambda_i = 1\}$$

Convex hull :- A convex hull of a given set $S \subseteq \mathbb{R}^n$ is the set of all possible convex combination of points of S.

OR

The smallest convex set containing S is called convex hull of S.
It is denoted by $C(S)$ or $\langle S \rangle$.

OR

The intersection of all the convex set containing S is called convex hull.

Ex: 1. $S = \{x_1, x_2\}$

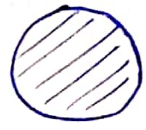
$$\langle S \rangle = \{x \mid x = \lambda x_1 + (1-\lambda) x_2, \quad \lambda \in [0,1]\}$$

2. $S = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = a^2\}$

$$\langle S \rangle = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq a^2\}$$

Remark: Every convex polyhedron is convex hull but converse may not be true.

Convex polyhedron and also convex hull



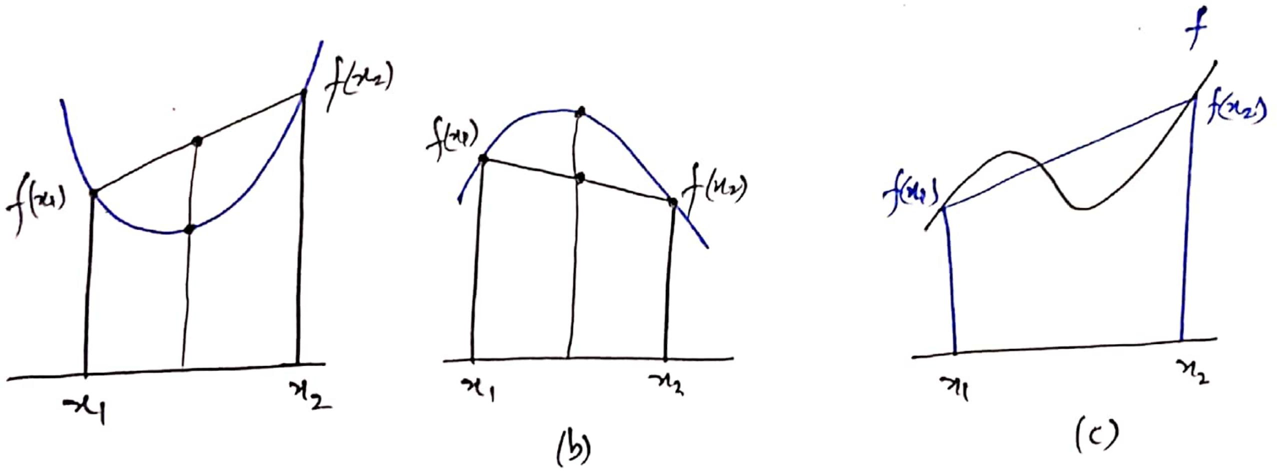
Convex hull but not convex hull.

Convex Function:- Let S be a non-empty convex subset of $\mathbb{R}^n$. A function $f(x)$ on S is said to be convex if for any two vectors x_1 and x_2 in S

$$f(\lambda x_1 + (1-\lambda)x_2) \leq \lambda f(x_1) + (1-\lambda) f(x_2), \quad 0 \leq \lambda \leq 1.$$

Strictly convex function:- Let S be a non-empty convex subset of $\mathbb{R}^n$. A function $f(x)$ on S is said to be strictly convex if for any two different vectors x_1 and x_2 in S,

$$f(\lambda x_1 + (1-\lambda)x_2) < \lambda f(x_1) + (1-\lambda) f(x_2), \quad 0 < \lambda < 1.$$



(a) Convex function

(b) Concave function

(c) Neither convex nor concave

Concave (strictly Concave) function:-

A function $f(x)$ on a non-empty subset S of $\mathbb{R}^n$ is said to be concave (strictly concave) if $-f(x)$ is convex (strictly convex).

General LPP :- We shall now consider the L.P.P in the general context, i.e., when the number of variables is more than two.

Def^n: Let Z be a linear function on $\mathbb{R}^n$ defined by

$$Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n = \sum_{j=1}^{n} c_jx_j \quad \text{——— (a)}$$

where c_j's are constants known as cost coefficients.

Subject to

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &\geq \text{ or } \leq \text{ or } = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &\geq \text{ or } \leq \text{ or } = b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &\geq \text{ or } \leq \text{ or } = b_m \end{aligned}\right\} \text{——— (b)}$$

and finally let $x_1, x_2, \ldots, x_n \geq 0$ ——— (c)

where all a_{ij}'s, b_i's and c_j's are constant and x_j's are variables (are known as decision variables).

The above LPP may be written also in matrix form as follows:

Optimize $$Z = [c_1, c_2, \ldots, c_n]\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

subject to

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \geq \text{ or } \leq \text{ or } = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

and $x_1, x_2, \ldots, x_n \geq 0$.

OR

optimize (Maximize or minimize) $Z = CX$

Subject to $AX \geq \text{ or } \leq \text{ or } = B$

and $X \geq 0$,

where $C = [c_1, c_2, \ldots, c_n]$, $X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$, $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$

Constraint Matrix.

<u>Objective function</u>:- The linear function

$$Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$$

which is to be minimized or maximized is called the objective function of the General L.P.P.

<u>Constraints</u>:- The inequalities (b) are called the constraints of the General LPP.

<u>Non negative restrictions/constraints</u>:- The set of inequations (c) is usually known as the set of non-negative restrictions of the General LPP.

<u>Solution</u>:- A n-tuple $(x_1, x_2, \ldots, x_n)$ of real numbers which satisfies the constraints of a General LPP is called a solⁿ to the General LPP.

<u>Feasible solution</u>:- Any solution to a General LPP which also satisfies the non-negative restrictions of the problem, is called a feasible solution to the General LPP.

Optimum solution :- Any feasible solⁿ which optimizes (maximizes or minimizes) the bijective function of a General LPP is called an optimum solution to the General LPP.

Note: optimal solution is also used for optimum solution

Examples

Diet Problems (For two variable).

Pb1. A dietician wishes to mix two types of food in such a way that the vitamin contents of the mixture contain at least 8 units of Vitamin A and 10 units of vitamin C. Food I contains 2 units per kg of vitamin A and 1 unit per kg of vitamin C while food II contains 1 unit per kg of vitamin A and 2 units per kg of vitamin C. It costs Rs 50 per kg to purchase food I and Rs 70 per kg to produce food II. Formulate the above LPP to minimize the cost of such a mixture.

Solⁿ

Resources	Food I	Food II	Requirements
Vitamin A	2	1	8
Vitamin C	1	2	10
Cost (Rs.)	50	70	

Let the dietician mix x kg of food I and y kg of food II. Clearly $x \geq 0$, $y \geq 0$.

Therefore, total cost (Minimize) $Z = 50x + 70y$

subject to $2x + y \geq 8$

$x + 2y \geq 10$

and $x, y \geq 0$.

Pb2. Example of a general LPP (Diet Problem).

Given the nutrient contents of a number of different foodstuffs and the daily minimum requirement of each nutrient for a diet, determine the balanced diet which satisfied the minimum daily requirements and at the same time has the minimum cost.

Sol^n Mathematical Formulation

Let there are n different types of foodstuffs available and let m different types of nutrients required. Let a_{ij} denote the number of units of nutrient i in one unit of foodstuff j, where $j = 1, 2, 3, \ldots, n$; $i = 1, 2, \ldots, m$.

Let x_j be the number of units of food j in the desired diet.

Let b_i be the minimum number of units of nutrient i required daily.

Let c_j be the cost per unit of food j.

Nutrients ↓	Food stuffs ⟶ 1	2	. . . .	n	Requirements
1	a_{11}	a_{12}	. . . .	a_{1n}	b_1
2	a_{21}	a_{22}	. . . .	a_{2n}	b_2
3	⋮	⋮		⋮	⋮
⋮	⋮	⋮		⋮	⋮
m	a_{m1}	a_{m2}		a_{mn}	b_m
Cost (per unit)	c_1	c_2	. . . .	c_n	

Objective function: Minimize $Z = \sum_{j=1}^{n} c_j x_j$

subject to $\sum_{j=1}^{n} a_{ij} x_j \geq b_i \quad , \quad i = 1, 2, \ldots m$

and $\quad x_j \geq 0 \quad \forall \quad j = 1, 2, \ldots n.$