

# A Cautionary Note on Estimating Effect Sizes with Confidence Intervals

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## Abstract

An increasingly more popular approach to statistics is to focus on estimation and to forgo hypothesis testing altogether. Through an example, we show that estimates and confidence of effect size are overestimated when ignoring the null when the null is a plausible description of the data. Next, we illustrate how this overestimation can be avoided using Bayesian model averaging. The paper is concluded with a discussion on similarities and differences between estimation based inference and model averaging.

Don: Do we want to explicitly mention that the plant example already appeared in (Berger & Delampady, 1987)?

Your colleague has just conducted an experiment for a registered report. The analysis yields  $p < 0.05$  and your colleague believes that the null hypothesis can be rejected. In line with recommendations both old (e.g., Grant, 1962; Loftus, 1996) and new (e.g., Cumming, 2014, NEJM editorial!) you convince your colleague that it is better to replace the  $p$ -value with an estimate of effect size and a 95% confidence interval (but see Morey, Hoekstra, Rouder, Lee, & Wagenmakers, 2016). You also manage to convince your colleague to plot the data. Instead of simply reporting  $p < .05$ , the statistical analysis in the report is now more informative. The result is shown in Figure 1. In the text of the paper, the result is summarized as Cohen's  $d = 0.35$ ,  $CI = [0.01, 0.69]$ . Given the results shown in Figure 1, what is a reasonable point estimate of effect size? An obvious answer is “0.35”; since the confidence interval does not contain 0, the sample effect size seems to be a reasonable choice. However, your colleague now tells you about the nature of the experiment: plants grow faster when you talk to them. Suddenly, an effect size of “0” also appears plausible.<sup>1</sup>

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<sup>1</sup>Unless you talk out loud, with consumption, and the plant is near.

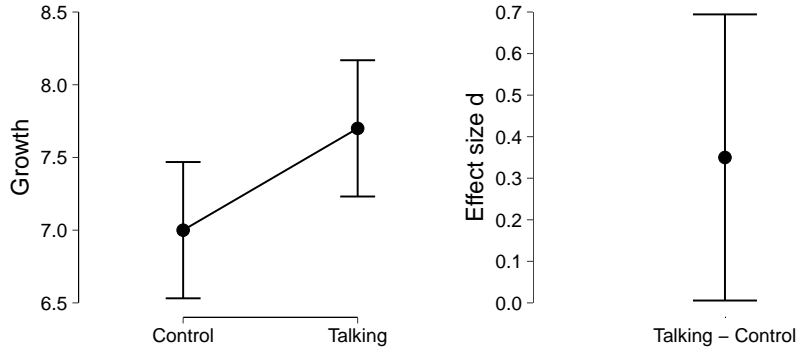


Figure 1: The left panel shows a descriptives plot with the mean and 95% confidence interval of the simulated plant growth. The right panel shows an estimate of the effect size, Cohen's  $d$ , and a 95% confidence interval.

## Why Confidence Intervals Overestimate Effect Size

Point estimates and confidence intervals of effect size based on only the alternative hypothesis tend to overestimate effect size. This overestimation is caused by the unheeded assumption that the null or perinull hypothesis is irrelevant. However, the null or perinull can have high plausibility after seeing the data. This happens when there is a high base-rate of null effects (i.e., high prior odds in favor of a null effect), or when the data are underwhelming. If the null hypothesis has a high posterior plausibility, it is obvious that it cannot be ignored, but that is exactly what is done when estimates are only based on the alternative hypothesis. As a result, the estimates are overconfident and, because the null hypothesis would shrink the estimates towards zero, overestimated.

## A Bayesian Model Averaged Perspective

Here, we illustrate the overestimation and a remedy against it by reanalyzing the simulated data from Figure 1. There are two hypotheses considered: The null hypothesis which states that the two plants grew the same amount no matter whether they were spoken to or not, and the alternative hypothesis which states that there is a difference between these conditions. We test these hypotheses using a paired-samples  $t$ -test.<sup>2</sup> Typically, an estimate of effect size is based on solely the alternative hypothesis, which yields either a frequentist estimate,  $d = 0.35$  (95% CI: [0.01, 0.69]) or a Bayesian one,  $d = 0.25$  (95% CRI: [0.05, 0.44]). However, it has been shown repeatedly that averaging over the models considered provides the best predictive performance (Zellner & Vandaele, 1975, pp. 640–641, as described in Zellner & Siow, 1980, p. 600–601; Haldane, 1932, p. 57, Iverson, Wagenmakers, & Lee, 2010, Rouder, Haaf, & Vandekerckhove, 2018), and conceptually similar ideas date back much further (Wrinch & Jeffreys, 1921, p. 387, Jevons, 1874/1913). Accordingly, our proposed remedy is

<sup>2</sup>Imagine that one week 100 plants were randomly selected and spoken to while the other served as control. The next week, the roles reversed and the previously spoken to plants served as controls while the control plants were now being talked to.

to average the null model and the alternative model, weighted by their posterior model probabilities.<sup>3</sup> Figure 2 contrasts inference based on the alternative hypothesis with inference based on the averaged model by showing intervals and posterior means. The model averaged posterior mean and credible interval are

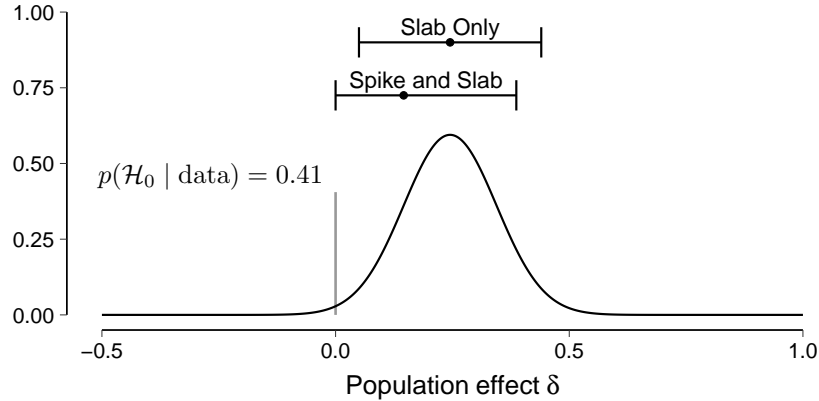


Figure 2: A visualization of model averaging. The black line represents the posterior distribution of effect size given the alternative model (i.e., the slab). The posterior is scaled so that its mode equals the posterior probability of the alternative model. The gray line represent the posterior probability of the null model (i.e., the spike). The error bars and dots above the density show a 95% credible intervals and the posterior mean for both the the slab and the model averaged posterior.

shrunk towards 0 compared to the posterior mean conditional on the alternative hypothesis (0.15 (95% cri: [0.00, 0.39]) vs. 0.25 (95% cri: [0.05, 0.44])). This makes sense as the posterior probability of the null hypothesis (0.41) is non-negligible.

## Discussion

Here, we argued that estimates of effect size based on only the alternative hypothesis tend to overestimate, in particular when a null or perinull hypothesis could also describe the data well. A solution for this overestimation is averaging over the null and alternative hypothesis. Although this idea is not new, the influence of the null is still too often ignored in practice.

This approach somewhat contrasts with the popular estimation mindset, where it is argued that statistical significance should be abandoned in favor of estimation (McShane, Gal, Gelman, Robert, & Tackett, 2019; Cumming, 2014). Some may argue that all null hypotheses are false (Cohen, 1994, ?). However, there are statistical motivations to consider a point null (Berger & Delampady, 1987) and several large-scale replications studies have demonstrated that a near-zero effect size is reasonable in practice (e.g., see the meta-analyses conducted

<sup>3</sup>For effect size, we obtain the following model-averaged posterior distribution:  $p(\delta|\text{data}) = s(\delta)pr(\mathcal{M}_0|\text{data}) + p(\delta|\text{data}, \mathcal{M}_1)pr(\mathcal{M}_1|\text{data})$ . Here,  $s$  is the Dirac delta function which represents the spike under the null,  $pr$  denotes probability, and  $p$  denotes density.

by Klein et al., 2018; Camerer et al., 2018; Nosek & Lakens, 2014).

A key aspect of model averaging is that it does not require model selection; there is no need to commit to a single model to obtain parameter estimates, although multiple models are considered. Therefore, model-averaged predictions and parameter estimates do fit the philosophy behind focusing on estimation. A skeptic might remark that in order to model average, it is necessary to obtain some form of model evidence and transform this into posterior model probabilities (e.g., Bayes factors, information criteria) which reintroduces the importance of testing. This could reintroduce the importance of testing as the method for obtaining model evidence has a large influence on the results. However, the same can be said for traditional estimation based inference, since confidence intervals can be constructed in multiple ways (e.g., bootstrapping, normal approximation).

Don: @EJ maybe we should somewhere quote "those who reject hypothesis testing are doomed to reinvent it?"

### When not to model average

When the sole aim is prediction rather than estimating and interpreting parameters, model averaging will likely be beneficial. However, if the goal is to interpret the parameter estimates, there are several reasons to forgo model averaging. First, if the models are theoretical opposites it makes little sense to model average. The model-averaged parameters may be uninterpretable and thus meaningless. Second, the nature of the problem may be ill-suited for model averaging. For example, imagine we want to maximize patients' quality of life. A new experimental treatment, living at a high altitude for 2 years, will improve patients' quality of life but only if they stay for the entire 2 years. Providing patients with treatment essentially boils down to subsidizing their stay there for two years. For each patient, it is unknown whether they will complete the treatment or not. Averaging over the probability of these states would provide each patient with less than the required amount to complete the treatment, essentially wasting the subsidy.

In sum, we argue that classical estimates overestimate effect size. A remedy for this overestimation is model averaging. Although this idea is not new, model averaging remains underutilized in practice.

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