Poincare Sphere Coupled Mode Theory

Multi- Electrode Directional Coupler with pre and post bias electrodes

Standard Coupled mode theory definitions:

A1 is defined as the complex coefficient representing the amplitude and phase of the optical field in guide "1"
A2 is defined as the complex coefficient representing the amplitude and phase of the optical field in guide "2"

From this two mode basis set a basis transformation is made to a symmetric optical eigenmode an antisymmetric optical eigenmode through the following definition :

$$a_s(z) = e^{j\beta_{av}z}\left[a_{s0}\cos(bz) + j\sin(bz) \times \left(\frac{a_{s0}\Delta\beta}{2b} + \frac{a_{a0}\chi}{b}\right)\right], \tag{1}$$

$$a_a(z) = e^{j\beta_{av}z}\left[a_{a0}\cos(bz) - j\sin(bz) \times \left(\frac{a_{a0}\Delta\beta}{2b} - \frac{a_{s0}\chi}{b}\right)\right],$$

where β_s and β_a are the propagation constants, $\Delta\beta = \beta_s - \beta_a$, $\chi = \chi(V)$ is a function of the applied voltage, $b = [(\Delta\beta/2)^2 + \chi^2]^{1/2}$ and $\beta_{av} = (\beta_s + \beta_a)/2$. $a_s(a_a)$ is the complex amplitude of the symmetric (antisymmetric) mode. For a passive, lossless directional coupler, $a_s(z) = a_{s0}e^{i(\beta_s z + \phi_0)}$ and $a_a(z) = a_{a0}e^{i\beta_a z}$, where a_{s0} and a_{a0} are real, and ϕ_0 is the phase difference at $z=0$ which may be nonzero due to the taper region. We assume adiabatic mode conversion in the taper regions for the remainder of this letter; this does not qualitatively change the results. The Poincaré coordinates [3] are defined by

```
S2 =. ; S1 =. ; S3 =. ; A1 =. ; A2 =. ;
X1 =. ; X2 =. ; X3 =. ; b1 =. ; b2 =. ; b3 =. ;
dbeta =. ; As =. ; Aa =. ; rr =. ; L1 =. ; L2 =. ;
L3 =. ; Signal =. ; tl =. ; xx1 =. ; P11 =. ; rr =.
prebias =.;
postbias =.;

(* Define the symetric and antisymetric wave equations*)
```

$$\text{Aso} = \frac{\text{A1} + \text{A2}}{\sqrt{2}};$$

$$\text{Aao} = \frac{\text{A1} - \text{A2}}{\sqrt{2}};$$

```
(*Initial electrode relative length *)
L1 = rr * (Pi / dbeta); L2 = .75 * (Pi / dbeta); L3 = rr * (Pi / dbeta);
(* Effective χn(Vapplied) *)
X1 =  dbeta * ( Pi / 7 ) * prebias;
X2 = dbeta * (Signal + .4);
X3 = -dbeta * postbias;

b1 = Sqrt[dbeta ^ 2 / 4 + X1 ^ 2];
b2 = Sqrt[dbeta ^ 2 / 4 + X2 ^ 2];
b3 = Sqrt[dbeta ^ 2 / 4 + X3 ^ 2];
Print["Initial Conditions \nDc Pre-Bias E1 = " b1, "\n",
 "Drive Voltage Center Electrode = ", b2, "\n Post-Bias E3 = ", b3]
(*normalize Delta Beta to 1 *)
dbeta = 1;
rr := .55;

As1 = Aso * Cos[b1 L1 ] + ((Aso * dbeta) / (2 b1) + (Aao X1 / b1)) * I Sin[b1 L1];
Aa1 = Aao * Cos[b1 L1 ] - ((Aao * dbeta) / (2 b1 ) - (Aso X1 / b1)) * I Sin[b1 L1];

As2 = As1 * Cos[b2 L2] + ((As1 * dbeta) / (2 b2) + (Aa1 X2 / b2)) * I Sin[b2 L2];
Aa2 = Aa1 * Cos[b2 L2 ] - ((Aa1 * dbeta) / (2 b2 ) - (As1 X2 / b2)) * I Sin[b2 L2];

As3 = As2 * Cos[b3 L3 ] + ((As2 * dbeta) / (2 b3) + (Aa2 X3 / b3)) * I Sin[b3 L3];
Aa3 = Aa2 * Cos[b3 L3 ] - ((Aa2 * dbeta) / (2 b3) - (As2 X3 / b3)) * I Sin[b3 L3];


S1[Signal_ , prebias_, postbias_] := Evaluate[ Abs[As3] ^ 2 - Abs[Aa3] ^ 2];
S2[Signal_ , prebias_, postbias_ ] :=
  Evaluate[2 * Abs[As3 * Aa3] Cos[Arg[As3] - Arg[Aa3]]];
S3[Signal_ , prebias_, postbias_] :=
  Evaluate[ 2 * Abs[As3 * Aa3] Sin[Arg[As3] - Arg[Aa3]]];
(*Inital Waveguide mode*)
A1 := .5; A2 := .5
prebias := 1; postbias := 1;
Plot[S1[Signal, prebias, postbias], {Signal, -Pi, Pi},
 Frame → True, FrameLabel → {"time", "Signal"}, PlotLabel →
  "50:50 input Optical Power Modulation: Symmetric to Anti-Symmetric  mode
 Abs[As3]^2-Abs[Aa3]^2"]

A1 := 0; A2 := 1;
prebias := 1; postbias := 1;
Plot[S1[Signal, prebias, postbias], {Signal, -Pi, Pi},
 Frame → True, FrameLabel → {"time", "Signal"}, PlotLabel →
```

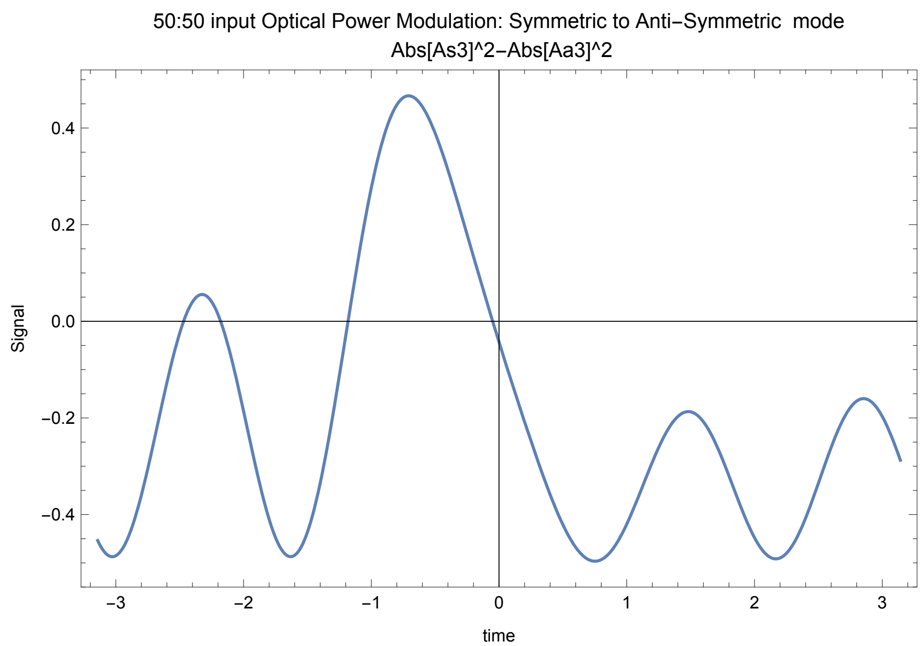
```
  "Guide2 in Only Optical Power Modulation: Symmetric to Anti-Symmetric  mode
  Abs[As3]^2-Abs[Aa3]^2]"]
A1 := 1; A2 := 0;
prebias := 1; postbias := 1;
Plot[S1[Signal, prebias, postbias], {Signal, -Pi, Pi},
 Frame → True, FrameLabel → {"time", "Signal"}, PlotLabel →
  "Guide1 in Only Optical Power Modulation: Symmetric to Anti-Symmetric  mode
 ( Abs[As3]^2-Abs[Aa3]^2)"]
```

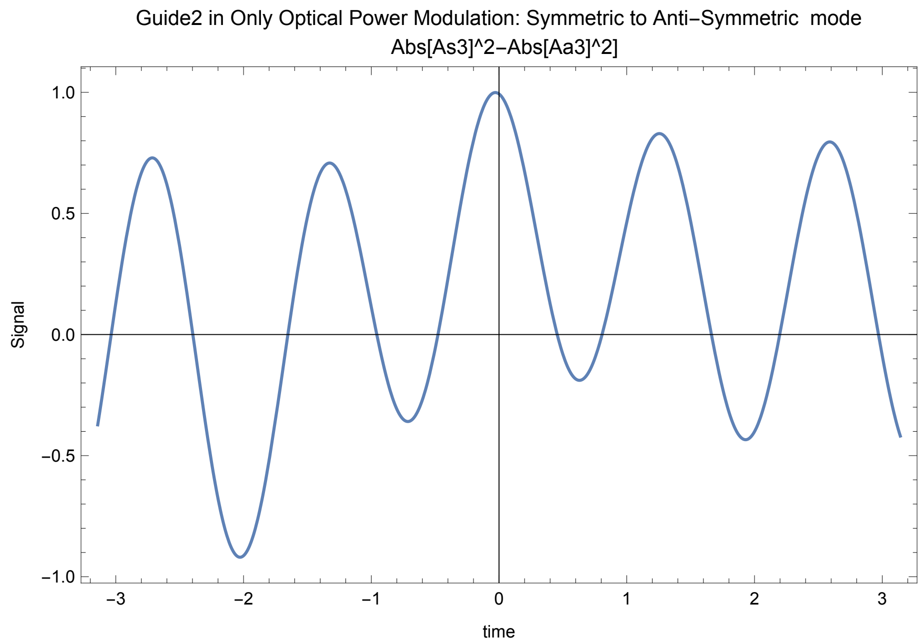
Initial Conditions

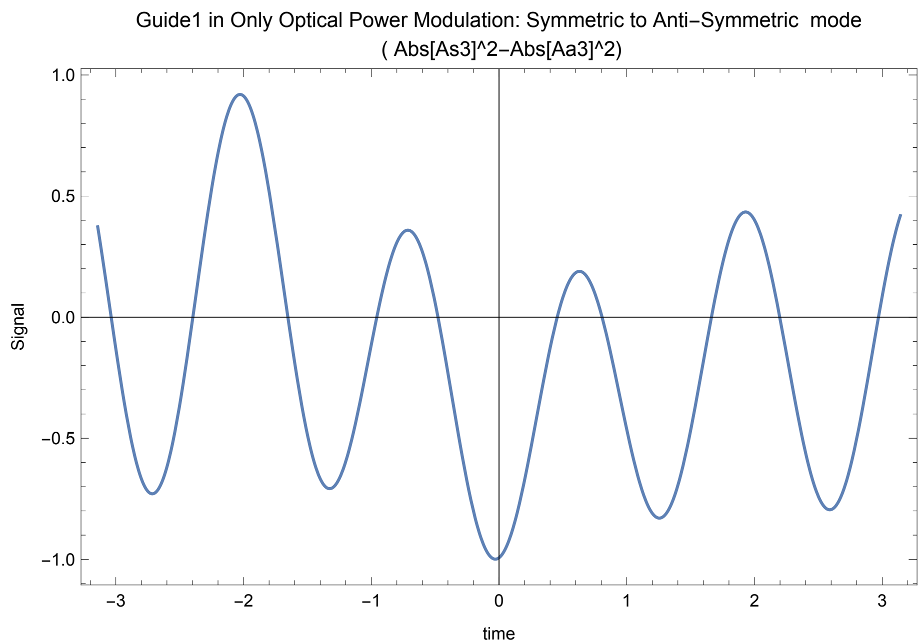
Dc Pre-Bias E1 $= \sqrt{\frac{\text{dbeta}^2}{4} + \frac{1}{49}\,\text{dbeta}^2\,\pi^2\,\text{prebias}^2}$

Drive Voltage Center Electrode $= \sqrt{\frac{\text{dbeta}^2}{4} + \text{dbeta}^2\,(0.4 + \text{Signal})^2}$

Post-Bias E3 $= \sqrt{\frac{\text{dbeta}^2}{4} + \text{dbeta}^2\,\text{postbias}^2}$





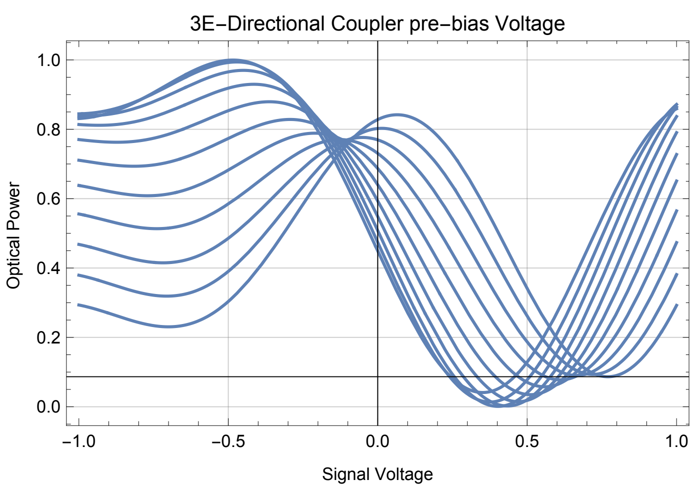


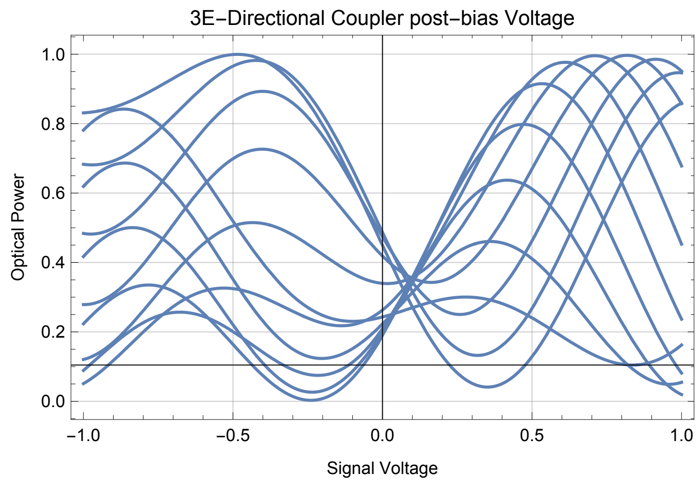
Optical Power Output Waveguide

```
A1 := 0; A2 := 1;

preb =.
postb =.
P11[signal_, preb_, postb_] := Evaluate[(1 + S2[signal, preb, postb]) / 2];
P22[signal_, preb_, postb_] := Evaluate[(1 - S2[signal, preb, postb]) / 2];
preb := 1
postb := 1
Simplify[P11[sig, preb, postb]]
prebMax = 1;
prebMin = -1;
postbMax = prebMax;
postbMin = prebMin;
biastable1 :=
 Table[Plot[Evaluate[P11[signal, preb, postb]], {signal, -1, 1}, PlotRange → All,
   Frame → True, PlotLabel → "3E-Directional Coupler pre-bias Voltage",
   GridLines → Automatic, FrameLabel → {"Signal Voltage", "Optical Power "}],
  {preb, prebMin, prebMax, .2}]
Show[Table[biastable1[[zzz]], {zzz, 1, Length[biastable1]}]]
biastable1 :=
 Table[Plot[Evaluate[P11[signal, preb, postb]], {signal, -1, 1}, PlotRange → All,
   Frame → True, PlotLabel → "3E-Directional Coupler post-bias Voltage",
   GridLines → Automatic, FrameLabel → {"Signal Voltage", "Optical Power"}],
  {postb, postbMin, postbMax, .2}]
Show[Table[biastable1[[zzz]], {zzz, 1, Length[biastable1]}]]
```

$$
\begin{aligned}
&\frac{1}{2} + 0.499067 \\
&\text{Abs}\Big[\frac{1}{0.41 + 0.8\,\text{sig} + \text{sig}^2}\Big(1.\,\sqrt{0.41 + 0.8\,\text{sig} + \text{sig}^2}\ \text{Cos}\big[2.35619\sqrt{0.41 + 0.8\,\text{sig} + \text{sig}^2}\big] + \\
&\quad\big((-0.0790788 + 0.0370458\,i) - (0.758488 - 0.672999\,i)\,\text{sig}\big) \\
&\quad\text{Sin}\big[2.35619\sqrt{0.41 + 0.8\,\text{sig} + \text{sig}^2}\big]\Big) \\
&\quad\Big(1.\,\sqrt{0.41 + 0.8\,\text{sig} + \text{sig}^2}\ \text{Cos}\big[2.35619\sqrt{0.41 + 0.8\,\text{sig} + \text{sig}^2}\big] + \\
&\quad\big((0.0699781 - 0.872451\,i) + (0.671197 - 0.724232\,i)\,\text{sig}\big) \\
&\quad\text{Sin}\big[2.35619\sqrt{0.41 + 0.8\,\text{sig} + \text{sig}^2}\big]\Big)\Big] \\
&\text{Cos}\Big[\text{Arg}\Big[(0.524061 - 0.441398\,i)\ \text{Cos}\big[2.35619\sqrt{0.41 + 0.8\,\text{sig} + \text{sig}^2}\big] + \\
&\quad\Big(\big((-0.0250902 + 0.0543195\,i) - (0.100433 - 0.687487\,i)\,\text{sig}\big) \\
&\quad\quad\text{Sin}\big[2.35619\sqrt{0.41 + 0.8\,\text{sig} + \text{sig}^2}\big]\Big)\Big/\Big(\sqrt{0.41 + 0.8\,\text{sig} + \text{sig}^2}\Big)\Big] - \\
&\quad\text{Arg}\Big[(0.646132 + 0.336216\,i)\ \text{Cos}\big[2.35619\sqrt{0.41 + 0.8\,\text{sig} + \text{sig}^2}\big] + \\
&\quad\Big(\big((0.338547 - 0.540191\,i) + (0.67718 - 0.242283\,i)\,\text{sig}\big) \\
&\quad\quad\text{Sin}\big[2.35619\sqrt{0.41 + 0.8\,\text{sig} + \text{sig}^2}\big]\Big)\Big/\Big(\sqrt{0.41 + 0.8\,\text{sig} + \text{sig}^2}\Big)\Big]\Big]
\end{aligned}
$$






```
A1 := 1; A2 := 0;

preb =.
postb =.
P11[signal_, preb_, postb_] := Evaluate[(1 + S2[signal, preb, postb]) / 2];
P22[signal_, preb_, postb_] := Evaluate[(1 - S2[signal, preb, postb]) / 2];
preb := 1
postb := 1
Simplify[P11[sig, preb, postb]]
```

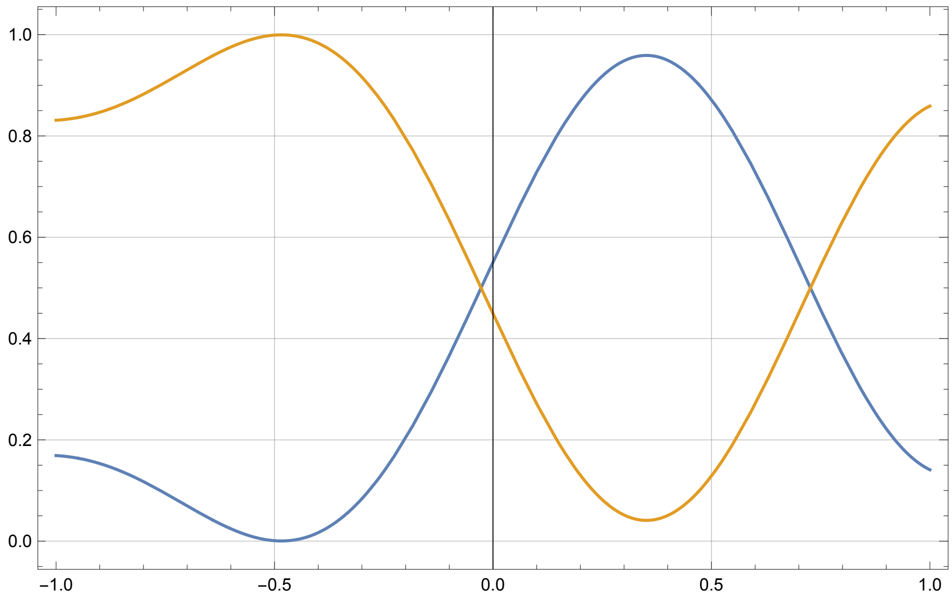
$$\frac{1}{2} + 0.499067\, \mathrm{Abs}\Big[\frac{1}{0.41 + 0.8\,\mathrm{sig} + \mathrm{sig}^2} \Big(1.\,\sqrt{0.41 + 0.8\,\mathrm{sig} + \mathrm{sig}^2}\,\mathrm{Cos}\big[2.35619\sqrt{0.41 + 0.8\,\mathrm{sig} + \mathrm{sig}^2}\big] + \big((-0.0790788 - 0.0370458\,i) - (0.758488 + 0.672999\,i)\,\mathrm{sig}\big)\,\mathrm{Sin}\big[2.35619\sqrt{0.41 + 0.8\,\mathrm{sig} + \mathrm{sig}^2}\big]\Big)$$

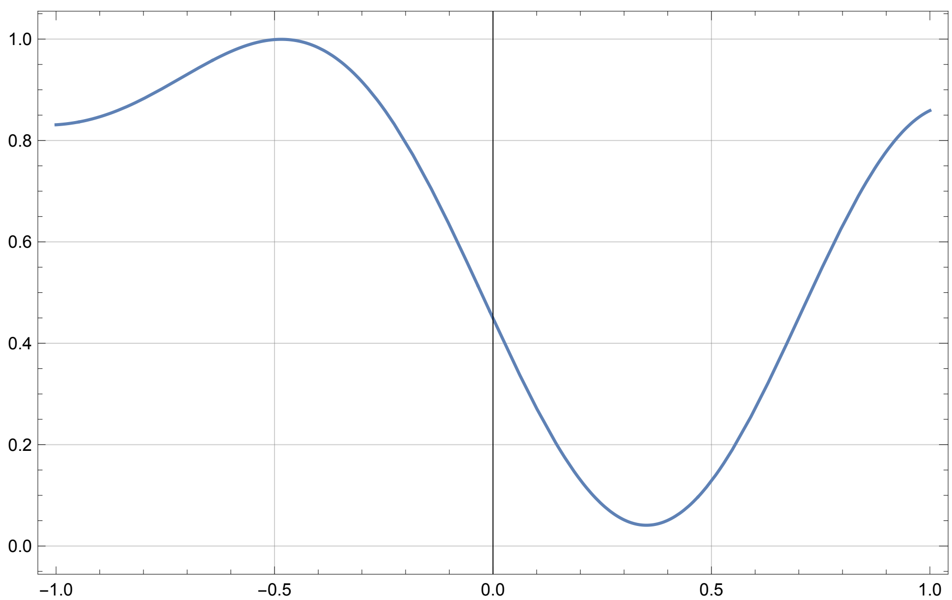
$$\Big(1.\,\sqrt{0.41 + 0.8\,\mathrm{sig} + \mathrm{sig}^2}\,\mathrm{Cos}\big[2.35619\sqrt{0.41 + 0.8\,\mathrm{sig} + \mathrm{sig}^2}\big] + \big((0.0699781 + 0.872451\,i) + (0.671197 + 0.724232\,i)\,\mathrm{sig}\big)\,\mathrm{Sin}\big[2.35619\sqrt{0.41 + 0.8\,\mathrm{sig} + \mathrm{sig}^2}\big]\Big)\Big]$$

$$\mathrm{Cos}\Big[\mathrm{Arg}\Big[(-0.524061 - 0.441398\,i)\,\mathrm{Cos}\big[2.35619\sqrt{0.41 + 0.8\,\mathrm{sig} + \mathrm{sig}^2}\big] + \Big(\big((0.0250902 + 0.0543195\,i) + (0.100433 + 0.687487\,i)\,\mathrm{sig}\big)\,\mathrm{Sin}\big[2.35619\sqrt{0.41 + 0.8\,\mathrm{sig} + \mathrm{sig}^2}\big]\Big) \Big/ \Big(\sqrt{0.41 + 0.8\,\mathrm{sig} + \mathrm{sig}^2}\Big)\Big] -$$

$$\mathrm{Arg}\Big[(0.646132 - 0.336216\,i)\,\mathrm{Cos}\big[2.35619\sqrt{0.41 + 0.8\,\mathrm{sig} + \mathrm{sig}^2}\big] + \Big(\big((0.338547 + 0.540191\,i) + (0.67718 + 0.242283\,i)\,\mathrm{sig}\big)\,\mathrm{Sin}\big[2.35619\sqrt{0.41 + 0.8\,\mathrm{sig} + \mathrm{sig}^2}\big]\Big) \Big/ \Big(\sqrt{0.41 + 0.8\,\mathrm{sig} + \mathrm{sig}^2}\Big)\Big]\Big]$$

```
Plot[{P11[sig, preb, postb], P22[sig, preb, postb]},
 {sig, -1, 1}, Frame → True, GridLines → Automatic]
Plot[P22[sig, preb, postb], {sig, -1, 1}, Frame → True, GridLines → Automatic]
Print[preb, postb]
```



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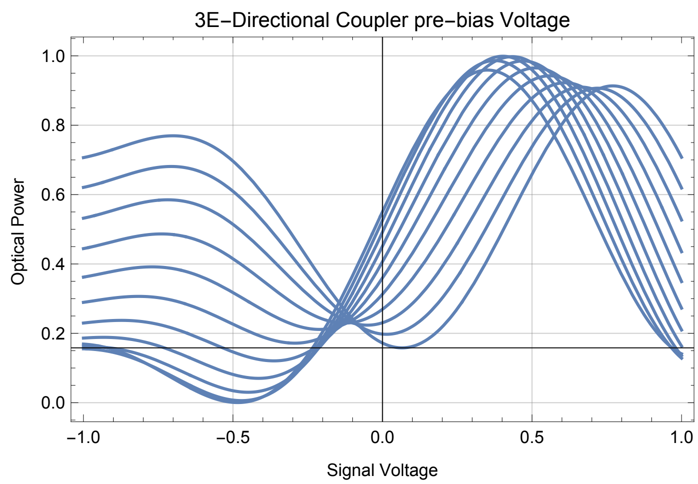
```

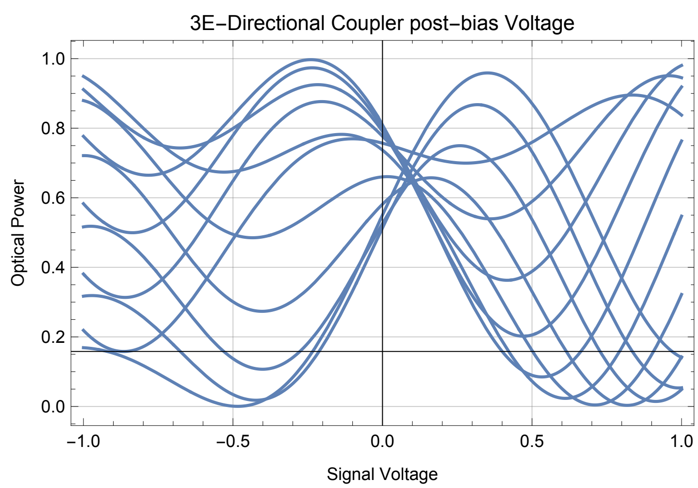
```
Plot3D[P11[signal, 0, 1], {signal, -2, 2}, PlotPoints → 50, FaceGrids → All,
 PlotLabel → "Single Electrode Directional Coupler Modulator Response",
 AxesLabel → {"Device Length (coupling length)",
   "Switching Voltage (Vpi)", "Optical Power (Normalized)"}]
```

```
- SurfaceGraphics -
```

Two parameters of the directional coupler modulator are coupling length, (tl) ,and the strength of the coupling factor between the two optical guides as a function of the applied voltage, S2[voltage applied]. The following three dimensional plot shows the Optical power in guide one as function of these two variables:

```
prebMax=1;
prebMin=-1;
postbMax=prebMax;
postbMin=prebMin;
biastable1:=Table[Plot[Evaluate[P11[signal,preb,postb]],{signal,-1,1},PlotRange->All
 "3E-Directional Coupler pre-bias Voltage",GridLines->Automatic,
 FrameLabel->{"Signal Voltage","Optical Power "}],{preb,prebMin,prebMax,.2}]
Show[Table[biastable1[[zzz]],{zzz,1,Length[biastable1]}]]
biastable1:=Table[Plot[Evaluate[P11[signal,preb,postb]],{signal,-1,1},PlotRange->All
 "3E-Directional Coupler post-bias Voltage",
 GridLines->Automatic,
 FrameLabel->{"Signal Voltage","Optical Power"}],{postb,postbMin,postbMax,.2}]
Show[Table[biastable1[[zzz]],{zzz,1,Length[biastable1]}]]
```





Fixing the coupling length (tl) such that with zero bias or drive voltage all of the optical power is transferred to the other optical waveguide;
 Delta beta * length =Pi

Then the with initial bias points, signal is applied to the central electrode

```
tl:=1
preb:=1
Plot[P11[signal,-Pi/7,1],{signal ,-.4,.4},PlotRange->All,Frame->True,
PlotLabel→ "Three Electrode Directional Coupler Modulator Response Plus Bias",
FrameLabel->{"Signal Voltage","Optical Power "},GridLines->Automatic]
```