# Análise de Algoritmos - T1

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17/09/2018

# Multiplicação de N bits

O problema

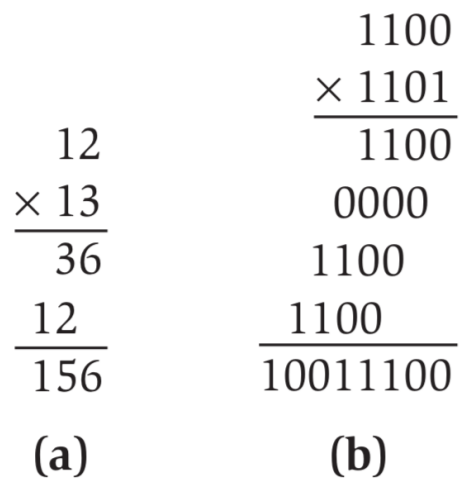


**Figure 5.8** The elementary-school algorithm for multiplying two integers, in (a) decimal and (b) binary representation.

MULTIPLY$(x, y, n)$

IF $(n = 1)$

  RETURN $x \times y$.

ELSE

  $m \leftarrow \lceil n / 2 \rceil$.

  $a \leftarrow \lfloor x / 2^m \rfloor$; $b \leftarrow x \bmod 2^m$.

  $c \leftarrow \lfloor y / 2^m \rfloor$; $d \leftarrow y \bmod 2^m$.

  $\Theta(n)$

  $e \leftarrow$ MULTIPLY$(a, c, m)$.

  $f \leftarrow$ MULTIPLY$(b, d, m)$.

  $g \leftarrow$ MULTIPLY$(b, c, m)$.

  $h \leftarrow$ MULTIPLY$(a, d, m)$.

  $4\,T(\lceil n / 2 \rceil)$

  RETURN $2^{2m} e + 2^m (g + h) + f$.

  $\Theta(n)$

# Divide-and-conquer: Análise

$$T(n) = \begin{cases} \Theta(1) & \text{if } n = 1 \\ 4T(\lceil n/2 \rceil) + \Theta(n) & \text{if } n > 1 \end{cases}$$

# Divide-and-conquer

$$m = \lceil n / 2 \rceil$$

$$a = \lfloor x / 2^m \rfloor \quad b = x \bmod 2^m$$

$$c = \lfloor y / 2^m \rfloor \quad d = y \bmod 2^m$$

use bit shifting to compute 4 terms

$$x\,y = (2^m a + b)(2^m c + d) = \underset{1}{2^{2m} ac} + \underset{2}{2^m (bc} + \underset{3}{ad)} + \underset{4}{bd}$$

Ex. $x = 1\,0\,0\,0\,1\,1\,0\,1 \quad y = 1\,1\,1\,0\,0\,0\,0\,1$

$a \quad b \quad c \quad d$

# Karatsuba

$$m = \lceil n / 2 \rceil$$

$$a = \lfloor x / 2^m \rfloor \quad b = x \bmod 2^m$$

$$c = \lfloor y / 2^m \rfloor \quad d = y \bmod 2^m$$

middle term

$$x\,y = (2^m a + b)(2^m c + d) = 2^{2m} ac + 2^m (bc + ad) + bd$$

$$= 2^{2m} ac + 2^m (ac + bd - (a-b)(c-d)) + bd$$

1 1 3 2 3

$$x = \underbrace{1\,0\,0\,0}_{a}\ \underbrace{1\,1\,0\,1}_{b}$$

$$y = \underbrace{1\,1\,1\,0}_{c}\ \underbrace{0\,0\,0\,1}_{d}$$

KARATSUBA-MULTIPLY$(x, y, n)$

IF $(n = 1)$

RETURN $x \times y$.

ELSE

$m \leftarrow \lceil n / 2 \rceil$.

$a \leftarrow \lfloor x / 2^m \rfloor$; $b \leftarrow x \bmod 2^m$.

$c \leftarrow \lfloor y / 2^m \rfloor$; $d \leftarrow y \bmod 2^m$.

$\leftarrow \Theta(n)$

$e \leftarrow$ KARATSUBA-MULTIPLY$(a, c, m)$.

$f \leftarrow$ KARATSUBA-MULTIPLY$(b, d, m)$.

$g \leftarrow$ KARATSUBA-MULTIPLY$(|a - b|, |c - d|, m)$.

$\leftarrow 3\,T(\lceil n / 2 \rceil)$

Flip sign of $g$ if needed.

RETURN $2^{2m} e + 2^m (e + f - g) + f$. $\leftarrow \Theta(n)$

# Karatsuba: Análise

$$T(n) = \begin{cases} \Theta(1) & \text{if } n = 1 \\ 3T(\lceil n/2 \rceil) + \Theta(n) & \text{if } n > 1 \end{cases}$$

$$\Longrightarrow \quad T(n) = \Theta(n^{\log_2 3}) = O(n^{1.585})$$

# Fonte

**Algorithm Design** - Jon Kleinberg, Eva Tardos - Copyright © 2005 Pearson-Addison Wesley