

17. Кръстованки прави

Разстоянието между две кръстованки прави е дължината на α -отсечка

[1] Дадени са кръстованките прави

да се намери отсечка a на g и h и разстоянието между g и h

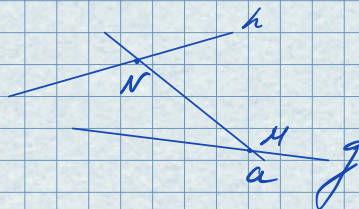
$$g: \begin{cases} x = -1 - 2t \\ y = -2 - 3t \\ z = 10 + 7t \end{cases} \quad \text{и} \quad h: \begin{cases} x = -1 + 2s \\ y = 11 - s \\ z = 5 + s \end{cases}$$

① Нека $a \perp g = M$ и $a \perp h = N$

$$= \begin{cases} M(-1 - 2t_0, -2 - 3t_0, 10 + 7t_0) \\ N(-1 + 2s_0, 11 - s_0, 5 + s_0) \end{cases}$$

$$\rightarrow \vec{MN}(2t_0 + 2s_0, 13 - t_0 + 3s_0, -5 + t_0 - 7s_0)$$

$$\vec{p}(2, -1, 1) \parallel g \text{ и } \vec{q}(-2, -3, 7) \parallel h$$



② $a \perp g, h \Rightarrow \vec{MN} \perp g, h$

$$\Rightarrow \vec{MN} \perp \vec{p}, \vec{q} \Rightarrow \langle \vec{MN}, \vec{p} \rangle = 0 \text{ и } \langle \vec{MN}, \vec{q} \rangle = 0$$

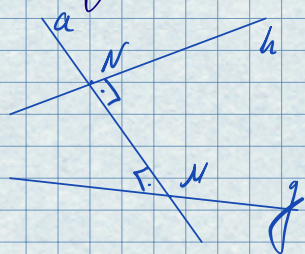
[...]

[2] Да се намери разстоянието между кръстованки прави:

$$g: \begin{cases} 2x + 7y - 13 = 0 \\ 8y - 2z - 1 = 0 \end{cases}$$

$$h: \begin{cases} x = 3 - \lambda \\ y = 1 \\ z = 16 - \lambda \end{cases}$$

① Нека a е отсечка на g и h . Нека $a \perp g = M$ и $a \perp h = N$. Понараме $t = \lambda'$



$$\begin{cases} 2x + 7y - 13 = 0 \\ 3y - 2\lambda' - 1 = 0 \end{cases} \Rightarrow y = \frac{2\lambda' + 1}{3}$$

$$2x + 14\lambda' + 7 - 13 = 0$$

$$x = \frac{3x - 14\lambda' - 7}{3} = \frac{16 - 7\lambda'}{3}$$

② $\begin{cases} x = \frac{16}{3} - \frac{7}{3}\lambda' \\ y = \frac{1}{3} + \frac{2}{3}\lambda' \\ z = \lambda' \end{cases}, \lambda' \neq 1 \text{ е параметър}$

$$\vec{MN}\left(\frac{8}{3} - \frac{7s}{3} - 10, -\frac{1}{3} - \frac{2s}{3} + 10, 16 - s - 10\right)$$

③ $M \in g:$

$$\Rightarrow M\left(\frac{16 - 7s}{3}, \frac{1}{3} + \frac{2s}{3}, s\right)$$

$N \in h:$

$$\Rightarrow N(8 - 10, 10, 16 - 10)$$

④ $\vec{p}\left(-\frac{7}{3}, \frac{2}{3}, 1\right) \parallel g \Rightarrow 9\vec{p}(-21, 6, 9) \parallel \vec{p} \parallel g$

$$\vec{q}(-1, 1, -1) \parallel h \Rightarrow 3\vec{q} \parallel \vec{q} \parallel h \parallel g$$

$$\textcircled{5} \vec{g_P}, \vec{g_Q} \perp \vec{MN} : \begin{cases} \langle \vec{g_P}, \vec{MN} \rangle = 0 \\ \langle \vec{g_Q}, \vec{MN} \rangle = 0 \end{cases}$$

$$-21\left(\frac{8}{3} + \frac{7}{3}s - 10\right) + 6\left(-\frac{1}{3} + \frac{2}{3}s + 10\right) + 9(16 - s - 10) = 0$$

$$-3\left(\frac{8}{3} - \frac{7}{3}s - 10\right) + 5\left(-\frac{1}{3} + \frac{2}{3}s + 10\right) - 3(16 - s - 10) = 0$$

$$-56 - 49s + 210 - 2 + 4s + 60 + 144 - 9s - 90 = 0$$

$$-8 + 7s - 30 - 1 + 2s + 30 - 48 + 3s + 30 = 0$$

$$90 - 8s + 43 = 0$$

$$0 \cdot 10 + 31s - 43 - 68 - 57 = 0$$

$$15s = 100, s = 4$$

$$90 - 6s - 57 = 0$$

$$90 - 6s - 57 = 0$$

$$10 = 9$$

$$\Rightarrow M(-4, 3, 4) \text{ u } N(-1, 9, 7) \Rightarrow \vec{MN}(3, 6, 3)$$