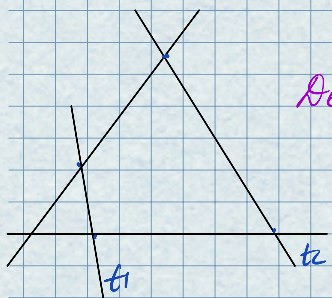
16. Трансверзала, ос-отсечка, разстояние между две прави



Дефиниция: Нека $\underline{a}$ и $\underline{b}$ са две кръстосани прави. Трансверзала на правите $\underline{a}$ и $\underline{b}$ се нарича $\forall$ права от $\underline{a}$ до b

$\boxed{1}$ Да се докаже, че правите $g: \begin{cases} x = 1+2\lambda \\ y = 4+4\lambda \\ z = 4+\lambda \end{cases}$ и $h: \begin{cases} x = -1 \\ y = -1-5\mu \\ z = 1+3\mu \end{cases}$ са кръстосани.

Да се намери тяхната трансверзала t, която:

а) минава през т. $A(1,1,1)$

б) е успоредна на правата $a: \begin{vmatrix} x+5y+4z-3=0 \\ 2x-5y-4z+1=0 \end{vmatrix}$

в) лежи в равнината

$\alpha: 2x+y-3z+6=0$

① Ще докажем, че $\underline{g}$ и $\underline{h}$ са кръстосани $\Longleftrightarrow$ g и h не са успоредни и не се пресичат

② $\vec{p}(2,4,1) \parallel g$ и $\vec{q}(0,-5,3) \parallel h$ и допускаме, че

$g \parallel h \Rightarrow \vec{p} \parallel \vec{q} \Rightarrow \vec{p} \times \vec{q} = 0$, но

$$\vec{p} \times \vec{q} = \left(\begin{vmatrix} 4 & 1 \\ -5 & 3 \end{vmatrix}, \begin{vmatrix} 1 & 2 \\ 3 & 0 \end{vmatrix}, \begin{vmatrix} 2 & 4 \\ 0 & -5 \end{vmatrix} \right) = (17,-6,-10) \neq (0,0,0),$$ противоречие

$\Rightarrow g \nparallel h$

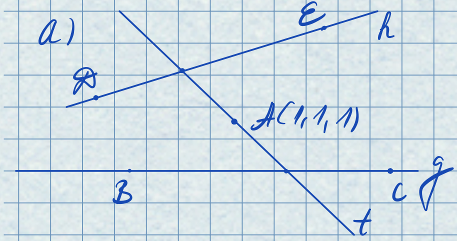
③ Да допуснем, че $g \cap h = X(x_1, x_2, x_3)$

$\Rightarrow X \in g \begin{vmatrix} x_1 = 1+2\lambda \\ x_2 = 4+4\lambda \\ x_3 = 4+\lambda \end{vmatrix}$

$X \in h \begin{vmatrix} x_1 = -1 \\ x_2 = -1-5\mu \\ x_3 = 1+3\mu \end{vmatrix}$

$\begin{vmatrix} 1+2\lambda = -1 \\ 4+4\lambda = -1-5\mu \\ 4+\lambda = 1+3\mu \end{vmatrix}$ $\left.\begin{vmatrix} \lambda = -1 \\ \mu = -1/5 \\ \text{невъзможно} \end{vmatrix}\right\}$ с-мата н.р. $\Rightarrow g \not\cap h$

а)



① $t \cap g \Rightarrow$ лежат в една равнина β

② $t \cap h \Rightarrow$ лежат в една равнина γ

$\Rightarrow t$ е пресечна права на β и γ

③ $\lambda = 0: \ B(1,4,4) \in \beta$

$\lambda = 1: \ C(3,8,5) \in \beta$, но т.к. $A \in \beta \Rightarrow \vec{AB}(0,3,3) \in \beta$ и $\vec{AC}(2,7,4) \in \beta$

$$\rightarrow \beta: \begin{vmatrix} x-1 & 0 & 2 \\ y-1 & 3 & 7 \\ z-1 & 3 & 4 \end{vmatrix} = 0 \Rightarrow$$

$\beta: 12(x-1)+6(y-1)-6(z-1)-21(x-1)=0$

$\beta: -9x+6y-6z+9=0 \quad | : (-3)$

$\beta: 3x-2y+2z-3=0$

④ $\mu = 0: \ D(-1,-1,1) \in \gamma$

$\mu = 1: \ E(-1,-6,4) \in \gamma$, но т.к. $A \in \gamma \Rightarrow \vec{AD}(-2,-2,0)$ и $\vec{AE}(-2,-7,3) \in \gamma$

$$\rightarrow \gamma: \begin{vmatrix} x-1 & -2 & -2 \\ y-1 & -2 & -7 \\ z-1 & 0 & 3 \end{vmatrix} = 0 \Rightarrow$$

$\gamma: -6(x-1)+14(z-1)-4(z-1)+6(y-1)=0$

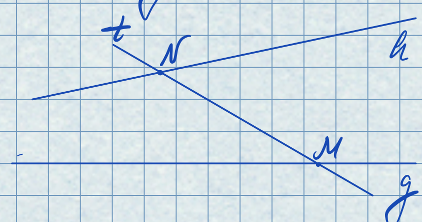
$\gamma: -6x+10z+6y-10=0 \quad | : (-2)$

$\gamma: 3x-3y-5z+5=0$

⑤ $\Rightarrow$

$$t: \begin{cases} 3x-2y+2z-3=0 \\ 3x-3y-5z+5=0 \end{cases}$$

б)



① Нека пресечната точка $t \cap g = M$,

$M \in g \Rightarrow M(1+\lambda_0, 4+4\lambda_0, 4+\lambda_0)$ за някое $\lambda_0 \in \mathbb{R}$

$\rightarrow M \in t \in \alpha \Rightarrow 2(1+2\lambda_0)+(4+4\lambda_0)-3(4+\lambda_0)+6=0$; $5\lambda_0 = 0 \rightarrow \lambda_0 = 0$

$\Rightarrow M(1,4,4)$

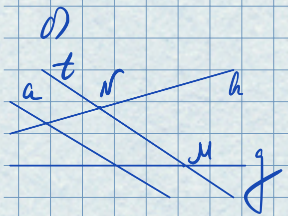
② Нека пресеч. $t \cap h = N$; $N \in h \Rightarrow N(-1, -1-5\mu_0, 1+3\mu_0)$ за някое $\mu_0 \in \mathbb{R}$

$\rightarrow N \in t \in \alpha \Rightarrow 2(-1)+(-1-5\mu_0)-3(1+3\mu_0)+6=0$; $-14\mu_0 = 0 \rightarrow \mu_0 = 0$

$\Rightarrow N(-1,-1,1)$

③ $\vec{NM}(2,5,3) \Rightarrow t \equiv MN: \begin{cases} x = 1+2a \\ y = 4+5a \\ z = 4+3a \end{cases}$

б)



① Нека $g \cap t = M \Rightarrow M \in g$

$\rightarrow M(1+2\mu_0,\ 4+4\lambda_0,\ 4+\lambda_0)$

② Нека $h \cap t = N \Rightarrow N \in h$

$\rightarrow N(-1,\ -1-5\mu_0,\ 1+3\mu_0)$

$$\Bigg\}\ \vec{NM}(2+2\lambda_0,\ 5+4\lambda_0-5\mu_0,\ 3+\lambda_0-3\mu_0)$$

③ $z=0$ $\left|\begin{array}{l} x+5y=0 \\ 2x-5y+1=0 \end{array}\right\}\ x=\frac{2}{3},\ y=\frac{7}{15}\ \Big\}$ т. $A\left(\frac{2}{3}, \frac{7}{15}, 0\right) \in a$

④ $z=1$ $\left|\begin{array}{l} x+5y+1=0 \\ 2x-5y-3=0 \end{array}\right\}\ x=\frac{2}{3},\ y=-\frac{1}{3}\ \Big\}$ т. $B\left(\frac{2}{3}, \frac{-1}{3}, 1\right) \in a$

$\rightarrow \vec{AB}\left(0, -\frac{4}{5}, 1\right) \parallel a \Rightarrow \vec{AB} \parallel a \parallel t \parallel \vec{NM} \Rightarrow \vec{AB} \parallel \vec{NM} \rightarrow \vec{AB} \times \vec{NM} = 0$

⑤ $\vec{NM}(2+2\lambda_0,\ 5+4\lambda_0+5\mu_0,\ 3+\lambda_0-3\mu_0)$

$\vec{AB}\left(0,\ \frac{-4}{5},\ 1\right)$

$$\vec{AB} \times \vec{NM} = \left(\begin{vmatrix} 5+4\lambda_0+5\mu_0 & 3+\lambda_0-3\mu_0 \\ \frac{-4}{5} & 1 \end{vmatrix}, \begin{vmatrix} 3+\lambda_0-3\mu_0 & 2+2\lambda_0 \\ 1 & 0 \end{vmatrix}, \begin{vmatrix} 2+2\lambda_0 & 5+4\lambda_0+5\mu_0 \\ 0 & \frac{-4}{5} \end{vmatrix} \right)$$

$= \left(\frac{37}{5} + \frac{24}{5}\lambda_0 + \frac{13}{5}\mu_0,\ -2-2\lambda_0,\ -\frac{8}{5} - \frac{8}{5}\lambda_0\right)$, но $\vec{NM} \parallel \vec{AB} \Rightarrow \vec{NM} \times \vec{AB} = \vec{\theta}$

$\Rightarrow \vec{AB} \times \vec{NM} = (0,0,0)$

$$\rightarrow \left|\begin{array}{ll} \frac{37}{5} + \frac{24}{5}\lambda_0 + \frac{13}{5}\mu_0 = 0 & \rightarrow \mu_0 = -1 \\ -2-2\lambda_0 = 0 & \rightarrow \lambda_0 = -1 \\ -\frac{8}{5} - \frac{8}{5}\lambda_0 = 0 & \rightarrow \lambda_0 = -1 \end{array}\right\}$$

$M(-1, 0, 3)$ $N(-1, 4, -2)$

$\vec{MN}(0, 4, -5) \Rightarrow t \equiv \vec{MN}$

$$\rightarrow t: \begin{cases} x = -1 \\ y = -4-4b \\ z = -2-5b \end{cases}$$

Дефиниция: Ако a и b са две кръстосани прави, а на тези прави пресичаща правата трансверзала, която е перпендикулярна