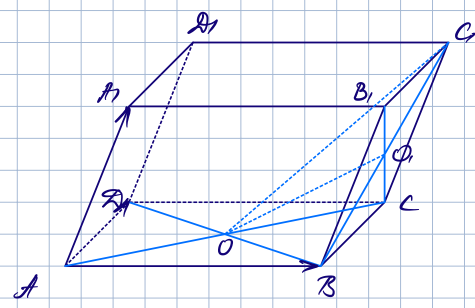
3. Вектори в пространството

1. $ABCDA_1B_1C_1D_1$ е паралелепипед; $AC \cap BD = O$, а $BC_1 \cap CB_1 = O_1$. Да се изразят векторите $\overrightarrow{C_1O}$, $\overrightarrow{C_1O_1}$ и $\overrightarrow{OO_1}$ чрез базата $\overrightarrow{AB}, \overrightarrow{AD}, \overrightarrow{AA_1}$.



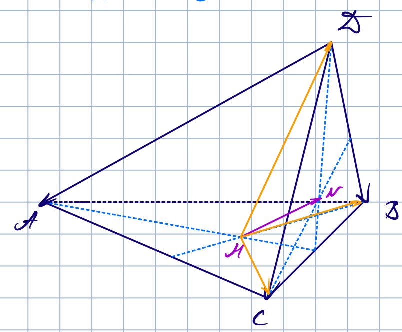
① $\overrightarrow{C_1O} = \underbrace{\overrightarrow{C_1C}}_{\overrightarrow{A_1A}} + \underbrace{\overrightarrow{CO}}_{\frac{1}{2}\overrightarrow{CA}} = \overrightarrow{A_1A} + \frac{1}{2}\overrightarrow{CA} = -\overrightarrow{AA_1} - \frac{1}{2}\overrightarrow{AC} =$

$= -\overrightarrow{AA_1} - \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{AD}) = -\overrightarrow{AA_1} - \frac{1}{2}\overrightarrow{AB} - \frac{1}{2}\overrightarrow{AD}$

② $\overrightarrow{C_1O_1} = \frac{1}{2}\overrightarrow{C_1B} = \frac{1}{2}\overrightarrow{D_1A} = -\frac{1}{2}\overrightarrow{AD_1} = -\frac{1}{2}(\overrightarrow{AA_1} + \overrightarrow{AD}) = -\frac{1}{2}\overrightarrow{AA_1} - \frac{1}{2}\overrightarrow{AD}$

③ $\overrightarrow{OO_1} = \overrightarrow{OC_1} + \overrightarrow{C_1O_1} = \overrightarrow{C_1O_1} - \overrightarrow{C_1O} = -\frac{1}{2}\overrightarrow{AA_1} - \frac{1}{2}\overrightarrow{AD} + \overrightarrow{AA_1} + \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{AD} = \frac{1}{2}\overrightarrow{AA_1} + \frac{1}{2}\overrightarrow{AB}$

2. $ABCD$ е тетраедър. M и N са медицентрове на $\triangle ABC$ и $\triangle BCD$. Да се изрази $\overrightarrow{MN}$ чрез базата $\overrightarrow{DA}, \overrightarrow{DB}, \overrightarrow{DC}$



① $\overrightarrow{MN} = \frac{1}{3}(\overrightarrow{MD} + \overrightarrow{MC} + \overrightarrow{MB}) = ?$

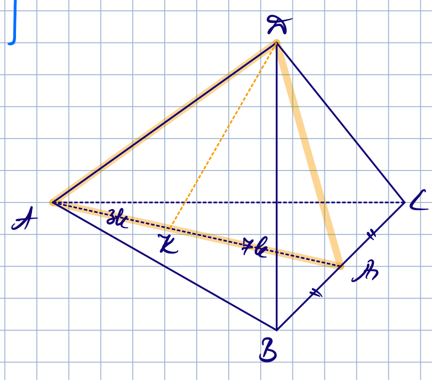
② $\overrightarrow{MN} = \overrightarrow{MD} + \overrightarrow{DN} = \overrightarrow{DN} - \overrightarrow{DM}$;

→ ③ $\overrightarrow{DM} = \frac{1}{3}(\overrightarrow{DA} + \overrightarrow{DB} + \overrightarrow{DC})$

$\overrightarrow{DN} = \frac{1}{3}(\underbrace{\overrightarrow{DD}}_{0} + \overrightarrow{DB} + \overrightarrow{DC}) = \frac{1}{3}(\overrightarrow{DB} + \overrightarrow{DC})$

③ $\overrightarrow{MN} = \overrightarrow{DN} - \overrightarrow{DM} = -\frac{1}{3}\overrightarrow{DA}$

3. $ABCD$ е тетраедър и AA_1 е медиана ($A_1 \in BC$); т. $K \in AA_1$: $AK : KA_1 = 3:7$. Да се изрази $\overrightarrow{DK}$ чрез базата $\overrightarrow{DA}, \overrightarrow{DB}, \overrightarrow{DC}$.

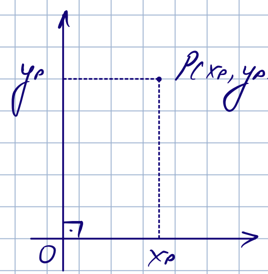


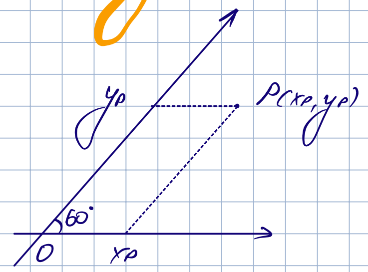
① $\overrightarrow{DK} = \overrightarrow{DA} + \overrightarrow{AK} = \overrightarrow{DA} + \frac{3}{10}\overrightarrow{AA_1} =$

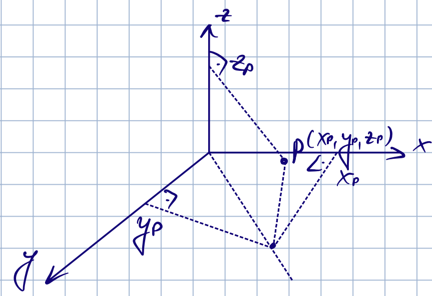
$= \overrightarrow{DA} + \frac{3}{10}(\overrightarrow{DA_1} - \overrightarrow{DA}) = \frac{7}{10}\overrightarrow{DA} + \frac{3}{10}\overrightarrow{DA_1} =$

$= \frac{7}{10}\overrightarrow{DA} + \frac{3}{10} \cdot \frac{1}{2}(\overrightarrow{DB} + \overrightarrow{DC}) = \frac{7}{10}\overrightarrow{DA} + \frac{3}{20}\overrightarrow{DB} + \frac{3}{20}\overrightarrow{DC}$

Координати





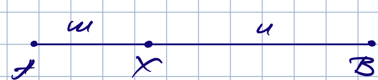


1. Ако т. $A(a_1, a_2, a_3)$ и т. $B(b_1, b_2, b_3)$, то $\overrightarrow{AB}(b_1 - a_1,\ b_2 - a_2,\ b_3 - a_3)$
2. Ако $\vec{a}(a_1, a_2, a_3)$, то $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$
3. Ако т. $A(a_1, a_2, a_3)$ и т. $B(b_1, b_2, b_3)$, то $\overrightarrow{AB} = \sqrt{(b_1 - a_1)^2 + (b_2 - a_2)^2 + (b_3 - a_3)^2}$
4. Ако т. $A(a_1, a_2, a_3)$ и т. $B(b_1, b_2, b_3)$ и т. M е среда на $\overrightarrow{AB}$, то $M\left(\frac{a_1+b_1}{2}, \frac{a_2+b_2}{2}, \frac{a_3+b_3}{2}\right)$
5. Ако т. $A(a_1, a_2, a_3)$, т. $B(b_1, b_2, b_3)$ и т. $C(c_1, c_2, c_3)$ и G е медицентър на $\triangle ABC$, то:

$$G\left(\frac{a_1+b_1+c_1}{2}, \frac{a_2+b_2+c_2}{2}, \frac{a_3+b_3+c_3}{2}\right)$$

6. Ако т. $A(a_1, a_2, a_3)$, т. $B(b_1, b_2, b_3)$ и т. $X \in AB$ и е такава, че $\frac{AX}{BX} = \frac{m}{n}$, то:

$$X\left(\frac{n}{m+n} a_1 + \frac{m}{m+n} b_1,\ \frac{n}{m+n} a_2 + \frac{m}{m+n} b_2,\ \frac{n}{m+n} a_3 + \frac{m}{m+n} b_3\right)$$



Примери: $\boxed{1}$ Да се намерят координатите на $\overrightarrow{AB}$, ако:

а) $A(2, 0, 1)$ и $B(0, 1, -1) \Rightarrow \overrightarrow{AB}(-2, 1, -2)$

б) $A(5, -6)$ и $B(3, 4) \Rightarrow \overrightarrow{AB}(-2, 10)$

$\boxed{2}$ Да се намерят координатите на т. B, ако:

а) $A(2, 0, -1)$ и $\overrightarrow{AB}(1, 1, 1) \Rightarrow B(3, 1, 0)$

б) $A(0, -1, -6)$ и $\overrightarrow{AB}(3, 2, -4) \Rightarrow B(3, 1, -10)$

$\boxed{3}$ Да се намерят координатите на т. C, ако:

$A(0, 0, -1)$, $\overrightarrow{AB}(2, -1, 0)$, $\overrightarrow{CB}(0, 1, -1)$

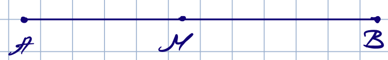
① Намираме т. B първо (координатите ѝ) $\Rightarrow B(2, -1, -1)$

② Намираме координатите на т. $C \Rightarrow C(2, -2, 0)$

4/ Да се намерят координатите на средата M на отсечката AB, ако:

$A(-1,1)$ и $B(-5,15) \Longrightarrow M\left(\frac{-1-5}{2}, \frac{1+15}{2}\right) \Longrightarrow M(-3,8)$

5/ Дадени са точките $A(7,10)$ и $B(-1,2)$. Да се намерят координатите на т. M от отсечката AB, ако $\frac{AM}{BM} = \frac{2}{3}$



$M\left(\frac{2}{5}(-1)+\frac{3}{5}\cdot 7, \frac{2}{5}\cdot 2+\frac{3}{5}\cdot 10\right) \longrightarrow M\left(\frac{19}{5}, \frac{34}{5}\right)$

Скаларно произведение

Дефиниция: На всеки два вектора $\vec{a}$ и $\vec{b}$ съпоставяме реално число, което ще означаваме с $\langle\vec{a},\vec{b}\rangle$ и ще наричаме „скаларно произведение" на векторите $\vec{a}$ и $\vec{b}$

$$\langle\vec{a},\vec{b}\rangle = |\vec{a}||\vec{b}|\cos\sphericalangle(\vec{a},\vec{b})$$

Свойства:

1. $\langle\vec{a},\vec{a}\rangle = |\vec{a}|^2 \geq 0$
2. $\langle\vec{a},\vec{b}\rangle = \langle\vec{b},\vec{a}\rangle$
3. $\langle\vec{a}\pm\vec{b},\vec{c}\rangle = \langle\vec{a},\vec{c}\rangle \pm \langle\vec{b},\vec{c}\rangle$
4. $\langle\lambda\vec{a}, b\rangle = \lambda\langle\vec{a},\vec{b}\rangle$
5. $\langle\lambda\vec{a}\pm\mu\vec{b},\vec{c}\rangle = \lambda\langle\vec{a},\vec{c}\rangle \pm \mu\langle\vec{b},\vec{c}\rangle$
6. Ако $\vec{a}(a_1,a_2,a_3)$ и $b(b_1,b_2,b_3)$, то $\langle\vec{a},\vec{b}\rangle = a_1b_1+a_2b_2+a_3b_3$ (ортонормирана)
7. $\vec{a}\perp\vec{b} \iff \langle\vec{a},\vec{b}\rangle = 0$