3rd Simulation Project

Control Engineering

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Zapopan, Jalisco, México.

May 12, 2020.

General Parameters.

Date of Birth: May 10, 1999.

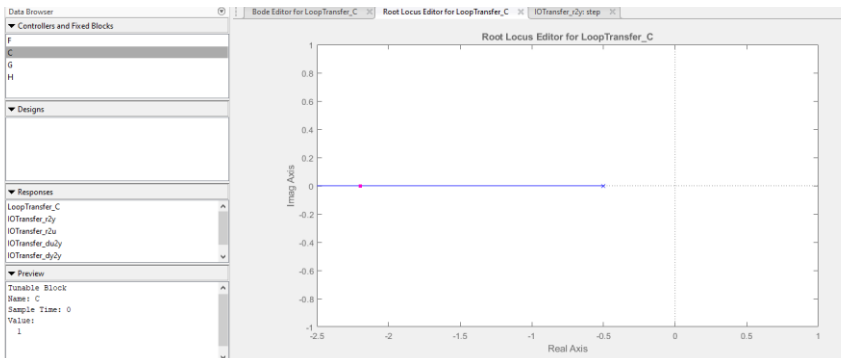
```
%General
a=10;
b=5;
c=17;
ut=a/2;
rt=a;
k=b;
```

System 1.

$$G_1(s) = \frac{c}{as+b} =$$

```
     17
--------
10 s + 5
```

$$K_{max} = \infty$$



1. **Lead Compensator. Design a Lead compensator with e(t)=10%. Reference r(t)=a.**

   $C(s) = \frac{k(s+a)}{s+b}$ Therefore G(s)C(s) is type 0 and unit step.

   $$e_{ss} = 0.1 = \frac{1}{1+k_p}$$

   $$k_p = 9 = \lim_{s\to 0} C(s)G(s) = \frac{k(a)}{b}\left(\frac{17}{5}\right)$$

   $$k = \frac{9*5*b}{17*a} = \frac{45b}{17a}$$

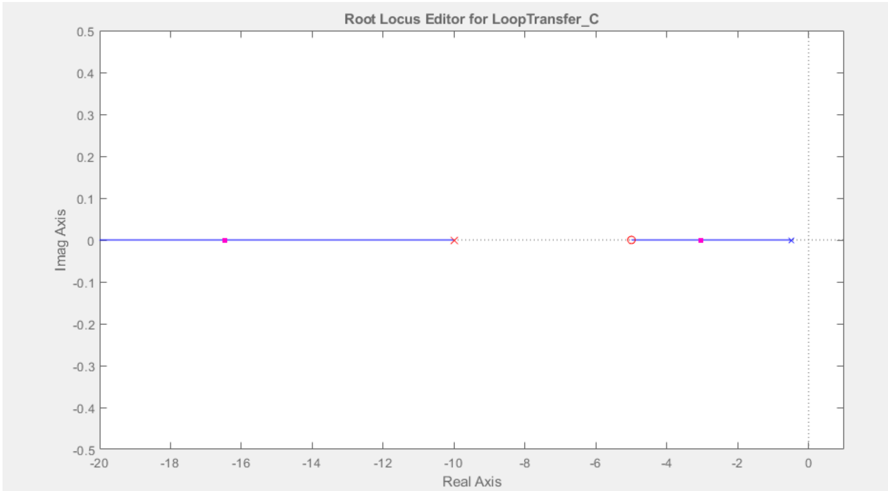
   $$|a| < |b|$$

   From the diagram of root locus, I decided that $a = -5$ y $b = -10$, therefore:

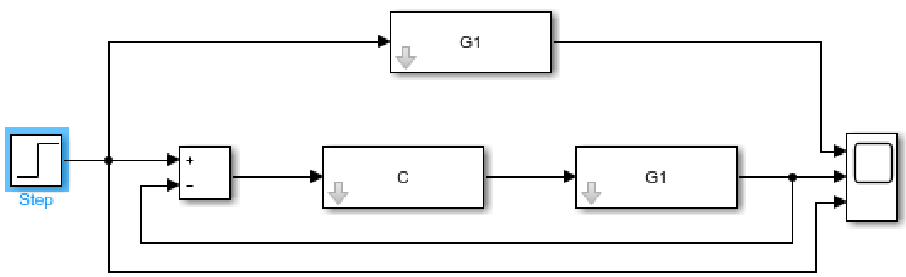
   $$k = \frac{45*10}{17*5} \approx 5.294$$

   $$C_1(s) = \frac{5.294(s+5)}{s+10}$$

Root Locus diagram:



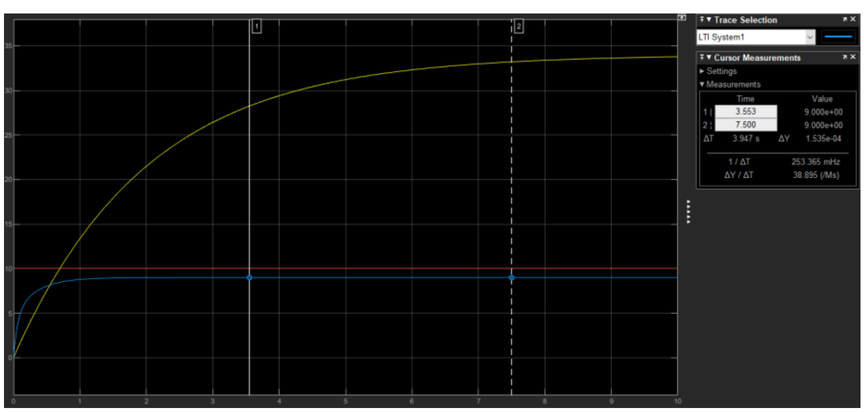
Simulink Diagram:



Simulink response:

Yellow = transfer function. Orange = step function Blue= transfer function with controller.

Final Value = 9 $e_{ss} = \frac{10-9}{10} = .1$



2. **Lead-Lag Compensator. Design a Lead Lag compensator with e(t)=5%. Reference r(t)=b. Make assumptions about the maximum static gain on the Lead and improve it.**

To obtain this compensator I am using $C_1(s)$, as the lead, and making $C_2(s)$ as the lag and both together make the Lead-Lag Compensator.

$C(s) = C_1(s)\, C_2(s)$ Therefore G(s)C(s) is type 0 and unit step.

$$C_2(s) = \frac{k_2(s+a_2)}{s+b_2}$$

$$e_{ss} = 0.05 = \frac{1}{1+k_p}$$

$$k_p = 19 = \lim_{s\to 0} C(s)G(s) = (\frac{5.294(5)}{10})(\frac{k_2(a_2)}{b_2})(\frac{17}{5})$$

$$k = \frac{19 * 10 * b_2}{17 * 5.294 * a_2} = \frac{19b}{9a}$$

$$|a| > |b|$$

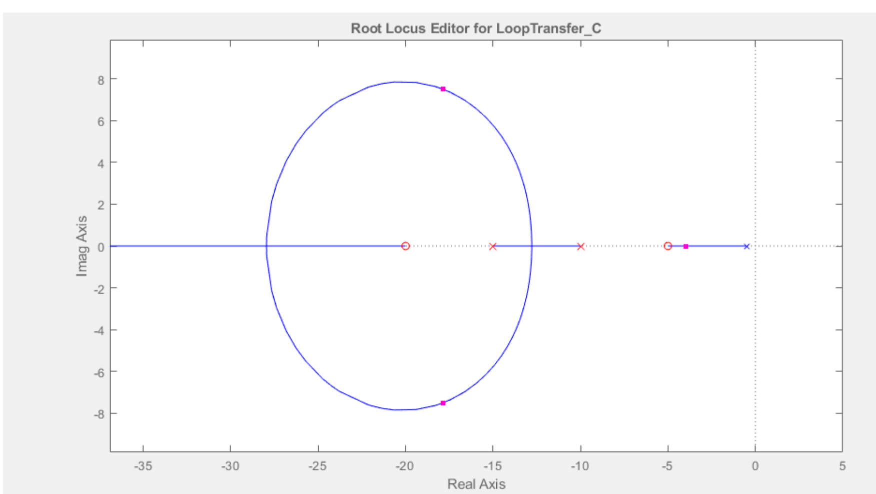
From the diagram of root locus, I decided that $a = -20$ y $b = -15$, therefore:

$$k = \frac{19 * 15}{9 * 20} \approx 1.5833$$

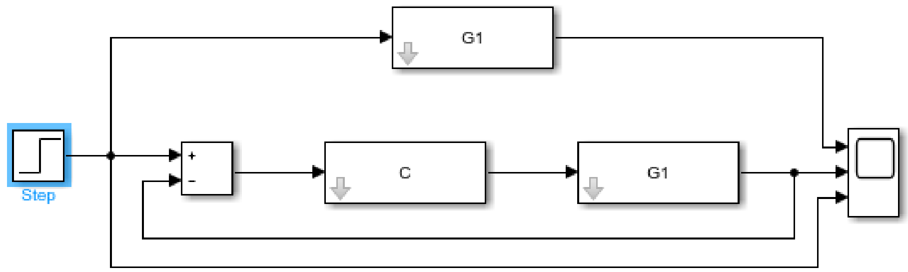
$$C_2(s) = \frac{1.5833(s+15)}{s+2}$$

$$C(s) = C_1(s)\, C_2(s) = (\frac{3.971(s+5)}{s+10})(\frac{0.281(s+20)}{s+15})$$

Root Locus Diagram:



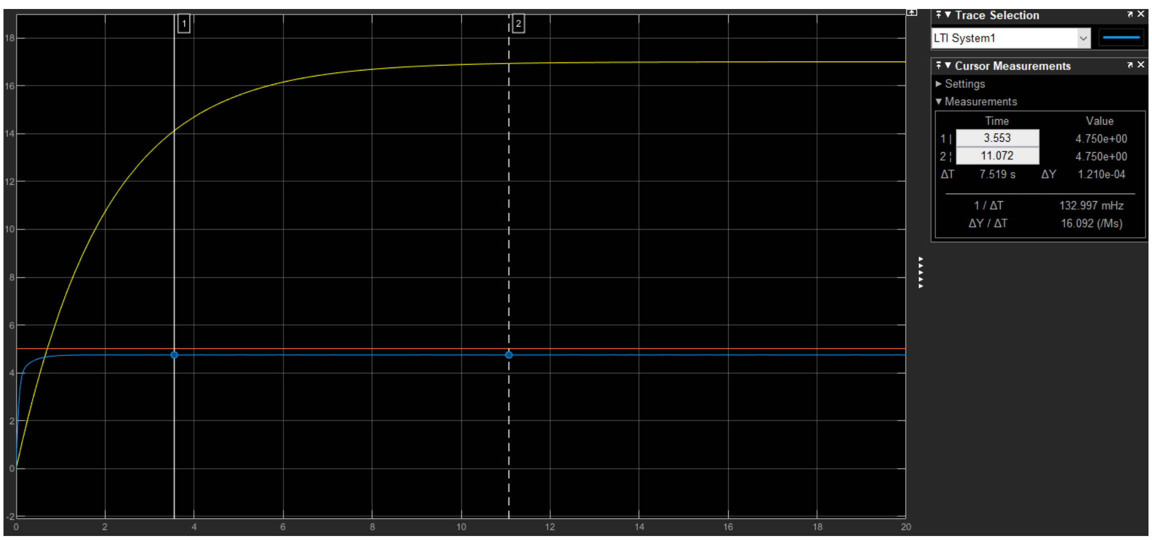
Simulink Diagram:



Simulink response:

Yellow = transfer function.   Orange = step function   Blue= transfer function with controller.

Final Value theorical = $b * 0.95 = 5 * .95 = 4.75$ Actual Final Value = 4.75

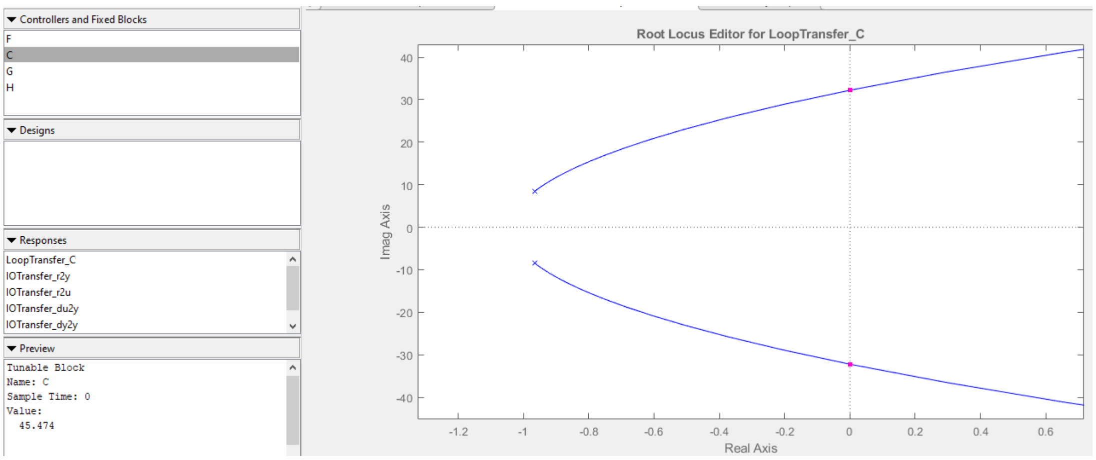


System 2

```
G2 =

     -0.17 s + 85
  ---------------------
  4 s^2 + 7.727 s + 289

Continuous-time transfer function.
```



As we can see, maximum gain is 45.474.

1. **Lead Compensator. Design a Lead compensator with e(t)=10%. Reference r(t)=a.**

$C(s) = \frac{k(s+a)}{s+b}$ Therefore G(s)C(s) is type 0 and unit step.

$e_{ss} = 0.1 = \frac{1}{1+k_p}$

$k_p = 9 = \lim_{s \to 0} C(s)G(s) = \frac{k(a)}{b}\left(\frac{85}{289}\right)$

$k = \frac{9*289*b}{85*a} = \frac{153b}{5a}$

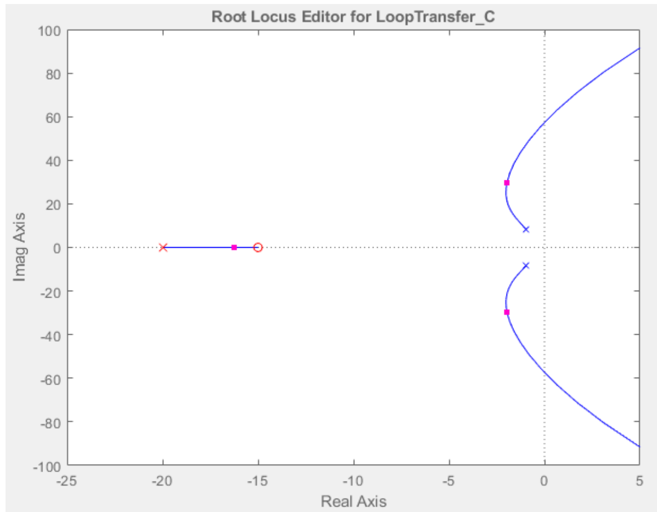
$|a| < |b|$

From the diagram of root locus, I decided that $a = -15$ y $b = -20$, therefore:

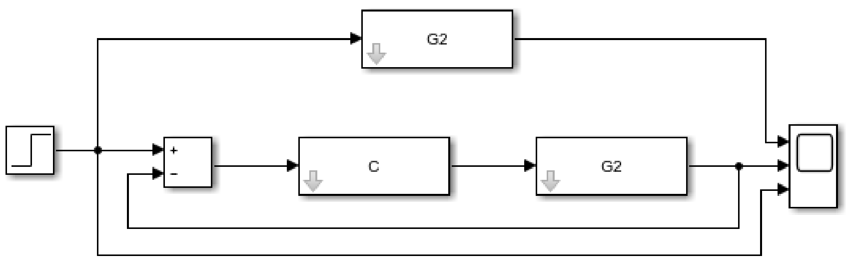
$$k = \frac{153 * 20}{5 * 15} \approx 40.8$$

$C_1(s) = \frac{40.8(s+15)}{s+20}$

Root Locus diagram:



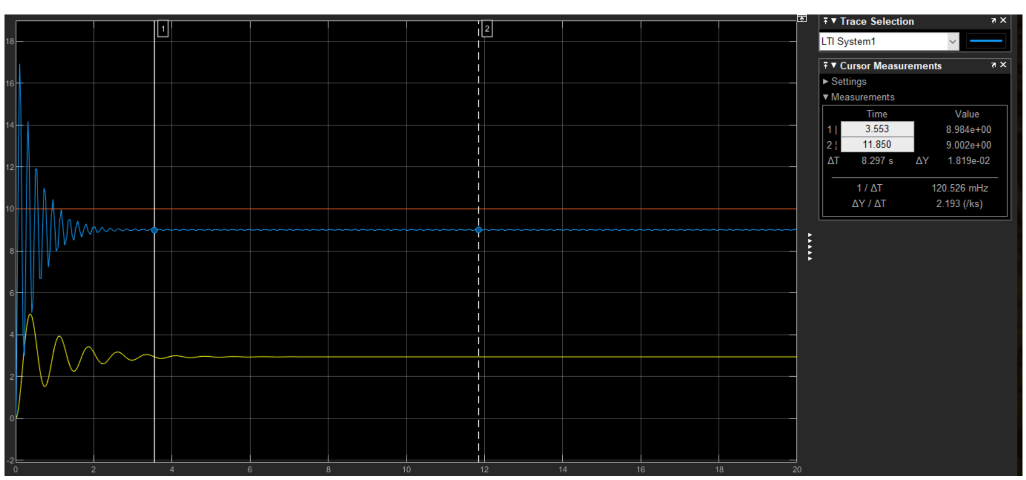
Simulink Diagram:



Simulink response:

Yellow = transfer function. Orange = step function Blue= transfer function with controller.

Final Value = 9 $e_{ss} = \frac{10-9}{10} = .1$



2. **Lead-Lag Compensator. Design a Lead Lag compensator with e(t)=5%. Reference r(t)=b. Make assumptions about the maximum static gain on the Lead and improve it.**

To obtain this compensator I am using $C_1(s)$, as the lead, and making $C_2(s)$ as the lag and both together make the Lead-Lag Compensator.

$C(s) = C_1(s)\,C_2(s)$ Therefore G(s)C(s) is type 0 and unit step.

$$C_2(s) = \frac{k_2(s+a_2)}{s+b_2}$$

$$e_{ss} = 0.05 = \frac{1}{1+k_p}$$

$$k_p = 19 = \lim_{s\to 0} C(s)G(s) = (\frac{40.8(15)}{20})(\frac{k_2(a_2)}{b_2})(\frac{85}{289})$$

$$k = \frac{19b_2}{9 * a_2}$$

$$|a| > |b|$$

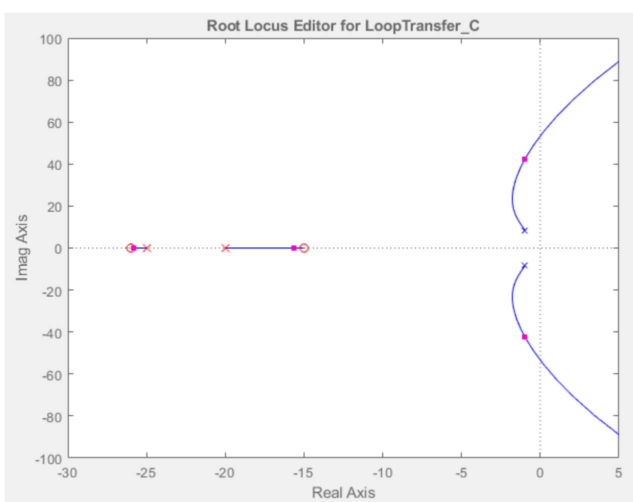
From the diagram of root locus, I decided that $a = -26$ y $b = -25$, therefore:

$$k = \frac{19 * 25}{9 * 26} = \frac{475}{234}$$

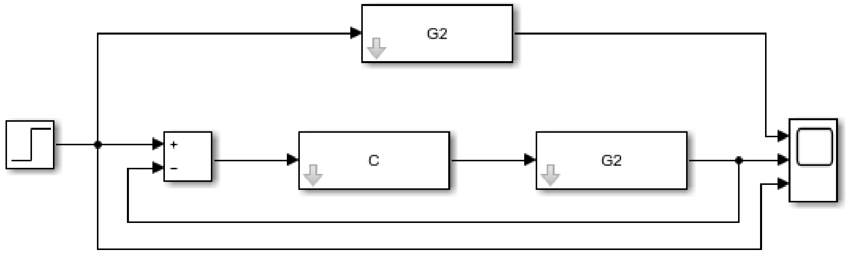
$$C_2(s) = \frac{475/234(s+26)}{s+25}$$

$$C(s) = C_1(s)\,C_2(s) = (\frac{40.8(s+15)}{s+20})(\frac{2.029(s+26)}{s+25})$$

Root Locus Diagram:



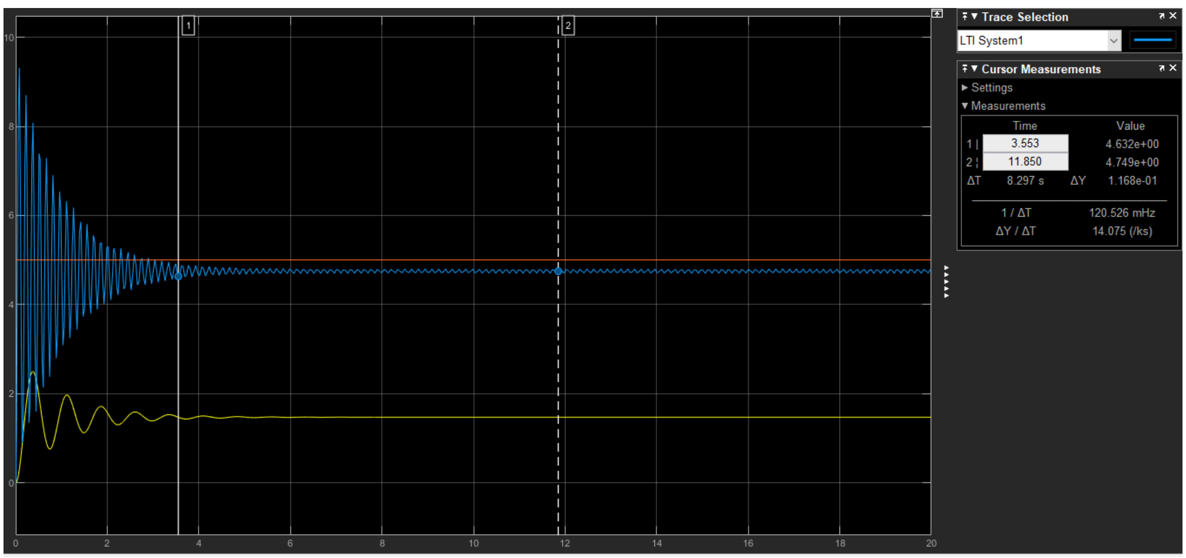
Simulink Diagram:



Simulink response:

Yellow = transfer function. Orange = step function Blue= transfer function with controller.

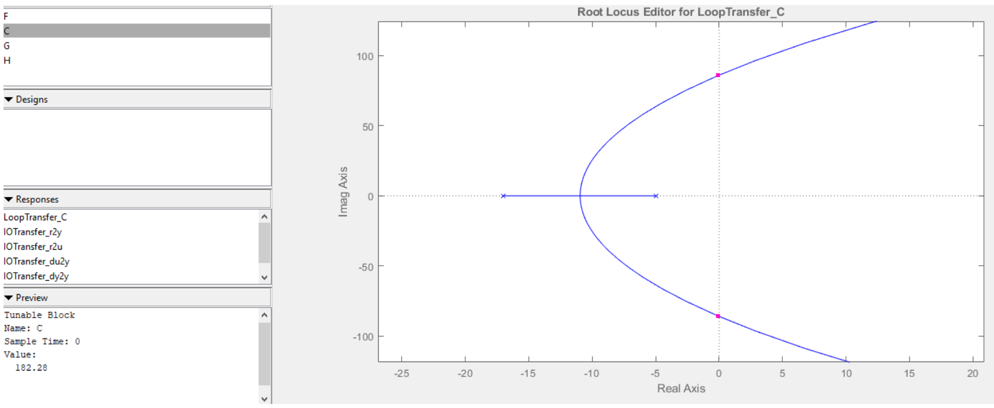
Final Value theorical = $b * 0.95 = 5 * .95 = 4.75$ Actual Final Value = 4.75



System 3

```
G3 =

   -0.12 s + 40
  ---------------
  s^2 + 22 s + 85
```



Maximum Gain: 182.28

1. **Lead Compensator. Design a Lead compensator with e(t)=10%. Reference r(t)=a.**

$C(s) = \frac{k(s+a)}{s+b}$ Therefore G(s)C(s) is type 0 and unit step.

$e_{ss} = 0.1 = \frac{1}{1+k_p}$

$k_p = 9 = \lim_{s\to 0} C(s)G(s) = \frac{k(a)}{b}(\frac{40}{85})$

$k = \frac{9*85*b}{40*a} = \frac{153b}{8a}$

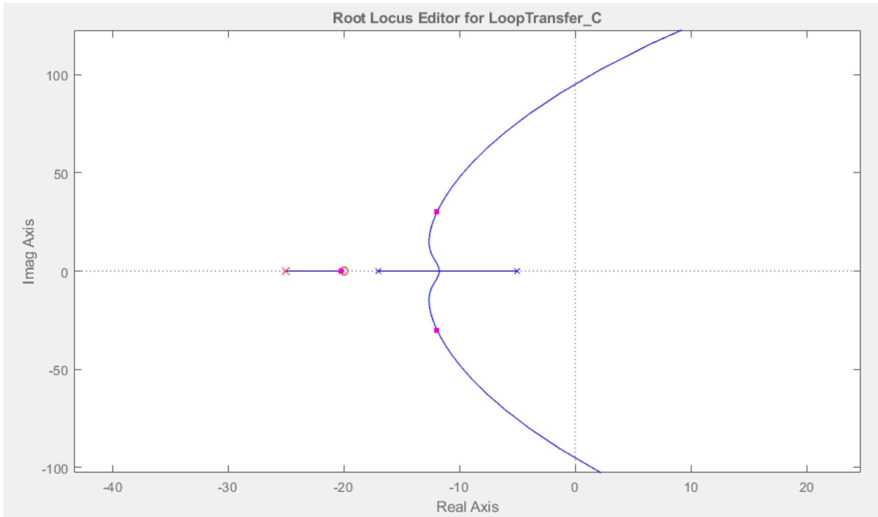
$|a| < |b|$

From the diagram of root locus, I decided that $a = -20$ y $b = -25$, therefore:

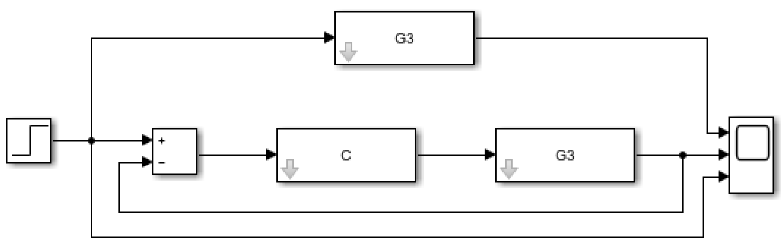
$$k = \frac{153 * 25}{8 * 20} \approx 23.906$$

$C_1(s) = \frac{23.906(s+20)}{s+25}$

Root Locus diagram:



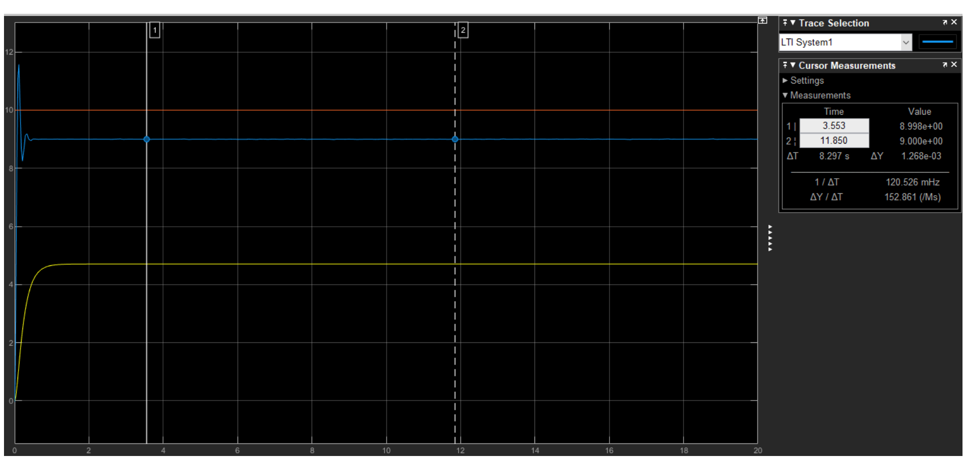
Simulink Diagram:



Simulink response:

Yellow = transfer function. Orange = step function Blue= transfer function with controller.

Final Value = 9 $e_{ss} = \frac{10-9}{10} = .1$



2. **Lead-Lag Compensator. Design a Lead Lag compensator with e(t)=5%. Reference r(t)=b. Make assumptions about the maximum static gain on the Lead and improve it.**

To obtain this compensator I am using $C_1(s)$, as the lead, and making $C_2(s)$ as the lag and both together make the Lead-Lag Compensator.

$C(s) = C_1(s)\, C_2(s)$ Therefore G(s)C(s) is type 0 and unit step.

$C_2(s) = \frac{k_2(s+a_2)}{s+b_2}$

$e_{ss} = 0.05 = \frac{1}{1+k_p}$

$k_p = 19 = \lim_{s\to 0} C(s)G(s) = (\frac{23.906(20)}{25})(\frac{k_2(a_2)}{b_2})(\frac{40}{85})$

$k = \frac{19b_2}{9 * a_2}$

$|a| > |b|$

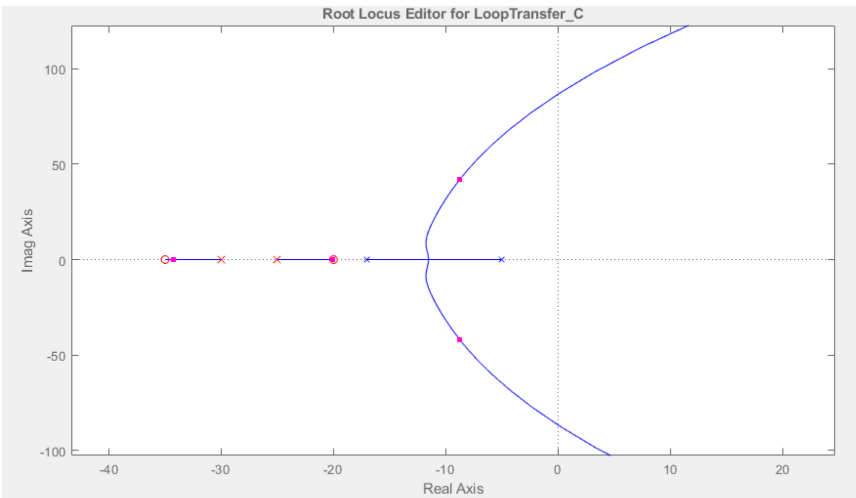
From the diagram of root locus, I decided that $a = -30$ y $b = -35$, therefore:

$$k = \frac{19 * 30}{9 * 35} = \frac{38}{21}$$

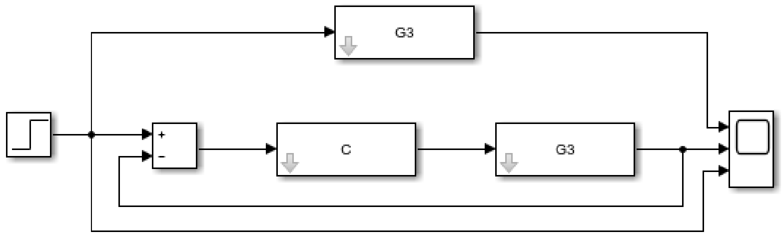
$C_2(s) = \frac{38/21(s+30)}{s+35}$

$C(s) = C_1(s)\, C_2(s) = (\frac{23.906(s+20)}{s+25})(\frac{38/21(s+30)}{s+35})$

Root Locus Diagram:



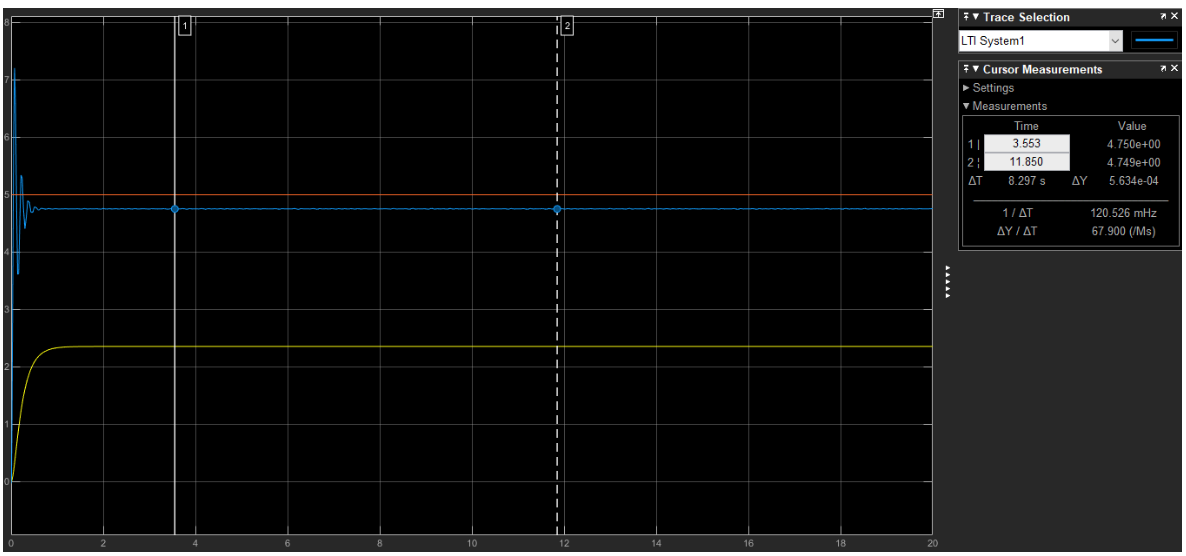
Simulink Diagram:



Simulink response:

Yellow = transfer function.   Orange = step function   Blue= transfer function with controller.

Final Value theorical = $b * 0.95 = 5 * .95 = 4.75$ Actual Final Value = 4.75

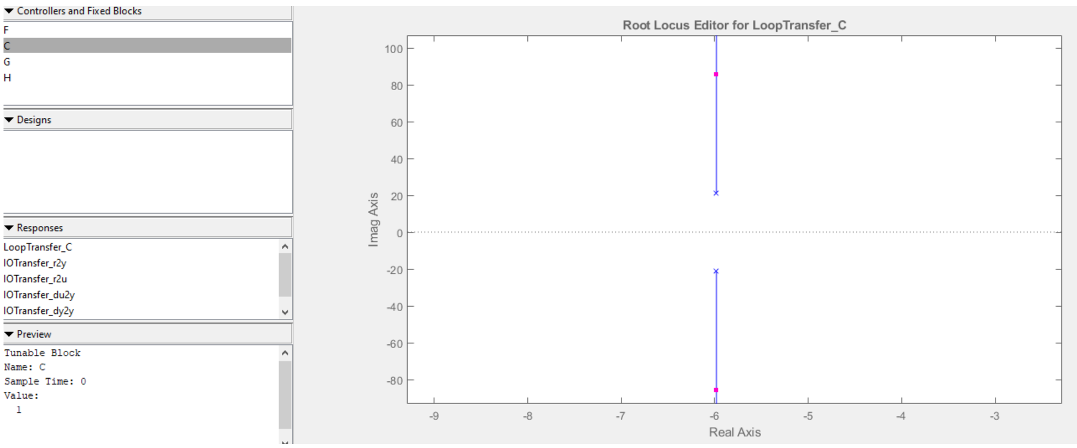


System 4

```
G =

          6869
  ---------------------
  s^2 + 11.96 s + 490.6

Continuous-time transfer function.
```



Since there are not branches crossing the imaginary axis, the maximum gain is infinity.

1. **Lead Compensator. Design a Lead compensator with e(t)=10%. Reference r(t)=a.**

$C(s) = \frac{k(s+a)}{s+b}$ Therefore G(s)C(s) is type 0 and unit step.

$e_{ss} = 0.1 = \frac{1}{1+k_p}$

$k_p = 9 = \lim_{s \to 0} C(s)G(s) = \frac{k(a)}{b}\left(\frac{6869}{490.6415}\right)$

$k = \frac{9*490.6415*b}{6869*a} = 0.6428\frac{b}{a}$

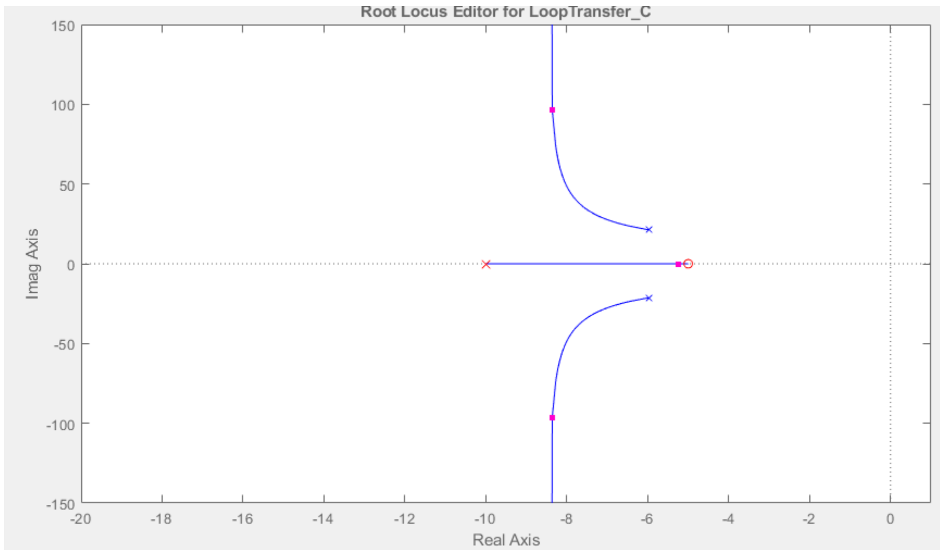
$|a| < |b|$

From the diagram of root locus, I decided that $a = -10$ y $b = -5$, therefore:

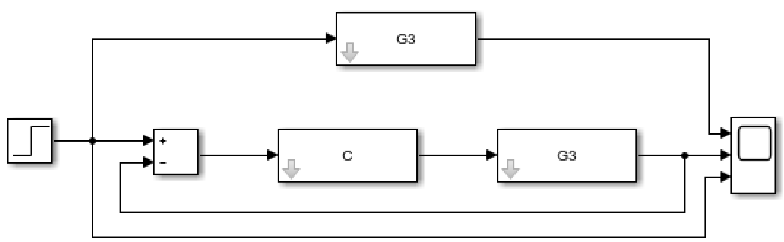
$$k = 0.6428\frac{10}{5} \approx 1.2857$$

$C_1(s) = \frac{1.2857(s+5)}{s+10}$

Root Locus diagram:



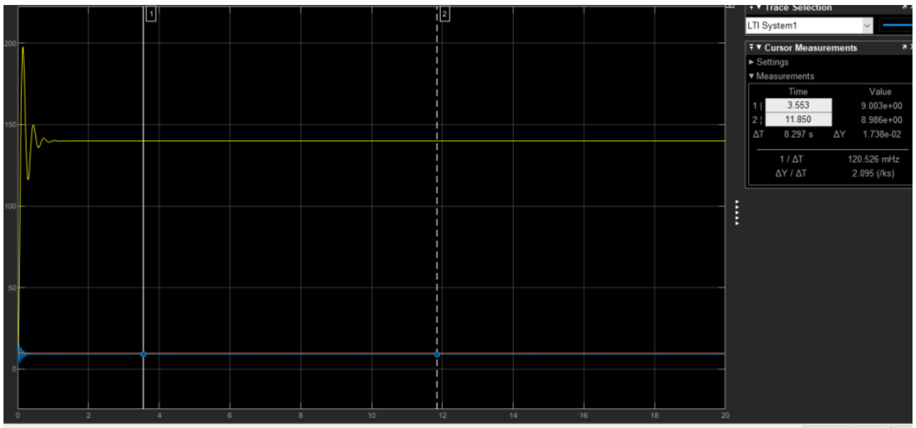
Simulink Diagram:

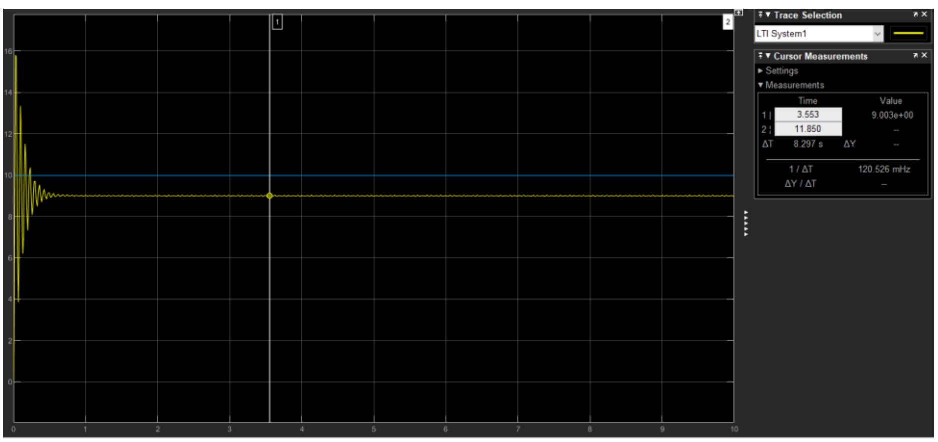


Simulink response:

Yellow = transfer function.   Orange = step function   Blue= transfer function with controller.

Final Value = 9 $e_{ss} = \frac{10-9}{10} = .1$





2. **Lead-Lag Compensator. Design a Lead Lag compensator with e(t)=5%. Reference r(t)=b. Make assumptions about the maximum static gain on the Lead and improve it.**

To obtain this compensator I am using $C_1(s)$, as the lead, and making $C_2(s)$ as the lag and both together make the Lead-Lag Compensator.

$C(s) = C_1(s)\, C_2(s)$ Therefore G(s)C(s) is type 0 and unit step.

$$C_2(s) = \frac{k_2(s+a_2)}{s+b_2}$$

$$e_{ss} = 0.05 = \frac{1}{1+k_p}$$

$$k_p = 19 = \lim_{s\to 0} C(s)G(s) = \left(\frac{1.2857(5)}{10}\right)\left(\frac{k_2(a_2)}{b_2}\right)\left(\frac{6869}{490.6415}\right)$$

$$k = \frac{19 * b_2}{9 * a_2}$$

$$|a| > |b|$$

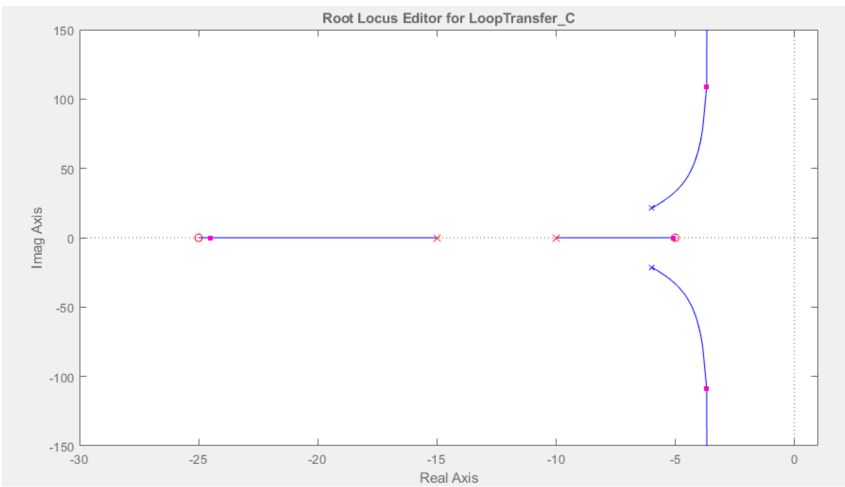
From the diagram of root locus, I decided that $a = -25$ y $b = -15$, therefore:

$$k = \frac{19 * 15}{9 * 25} = \frac{19}{15}$$

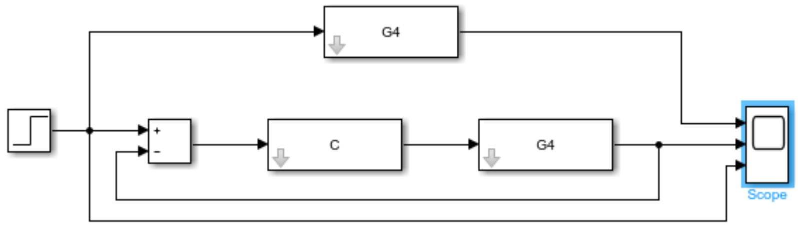
$$C_2(s) = \frac{19/15(s+25)}{s+15}$$

$$C(s) = C_1(s)\, C_2(s) = \left(\frac{1.2857(s+5)}{s+10}\right)\left(\frac{1.2667(s+25)}{s+15}\right)$$

Root Locus Diagram:



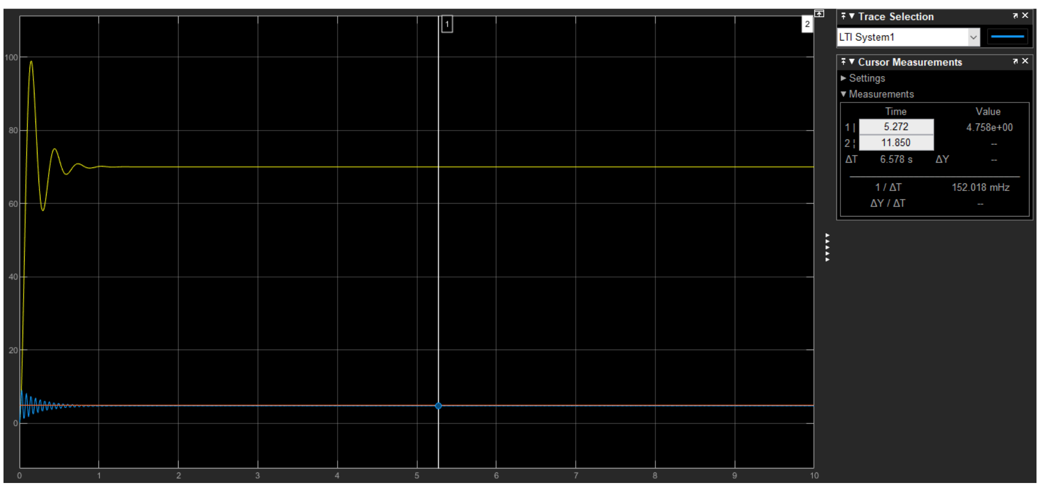
Simulink Diagram:



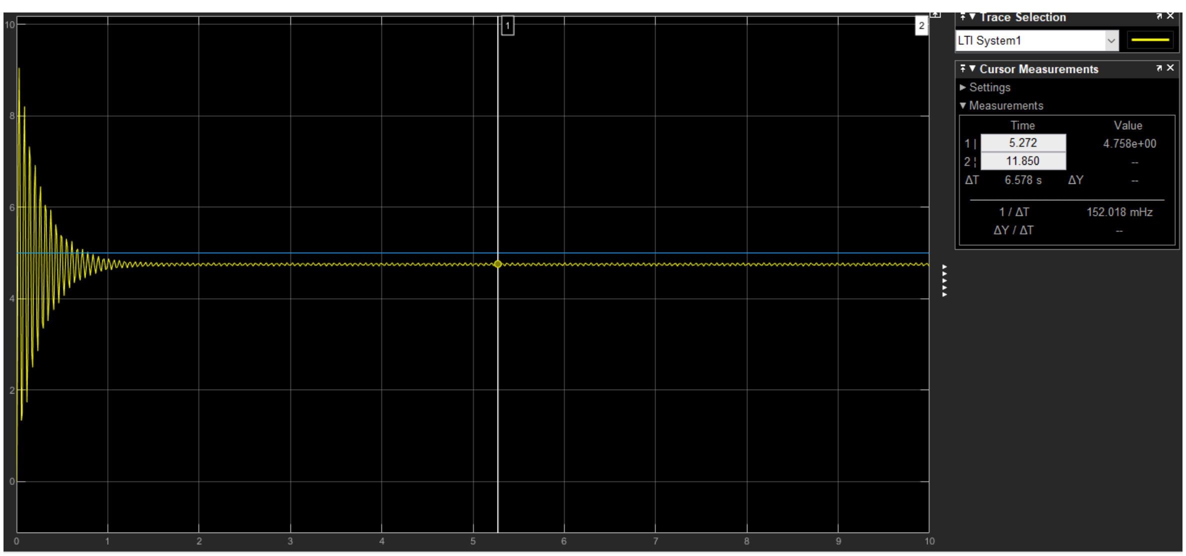
Simulink response:

Yellow = transfer function. Orange = step function Blue= transfer function with controller.

Final Value theorical = $b * 0.95 = 5 * .95 = 4.75$ Actual Final Value = 4.75



Blue = step function Yellow = transfer function with controller



System 5

1. **DONE**

2. **a.**

```
G1 =

     1.7
  ----------
  66.3 s + 1

Continuous-time transfer function.
```

b.

```
>> G2

G2 =

                4
  --------------------------
  1.724e04 s^2 + 326.3 s + 1

Continuous-time transfer function.
```

3. **For H1(s)/Qin(s) = G1 Design a Lead Compensator. Target: 10% error**

$C(s) = \frac{k(s+a)}{s+b}$ Therefore G(s)C(s) is type 0 and unit step.

$e_{ss} = 0.1 = \frac{1}{1+k_p}$

$k_p = 9 = \lim_{s \to 0} C(s)G_1(s) = \frac{k(a)}{b}(1.7)$

$k = \frac{9b}{1.7a} = 5.2941\frac{b}{a}$

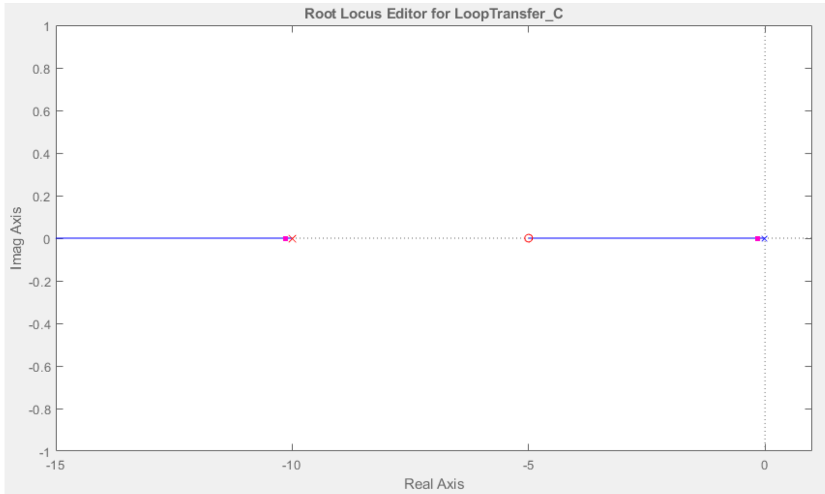
$|a| < |b|$

From the diagram of root locus, I decided that $a = -5$ y $b = -10$, therefore:

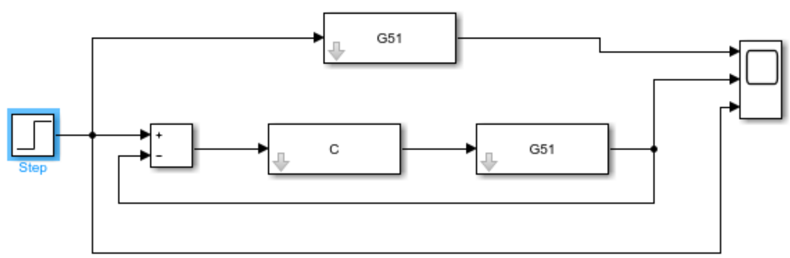
$$k = 5.2941\frac{10}{5} \approx 10.5882$$

$$C_1(s) = \frac{10.5882(s+5)}{s+10}$$

Root Locus diagram:



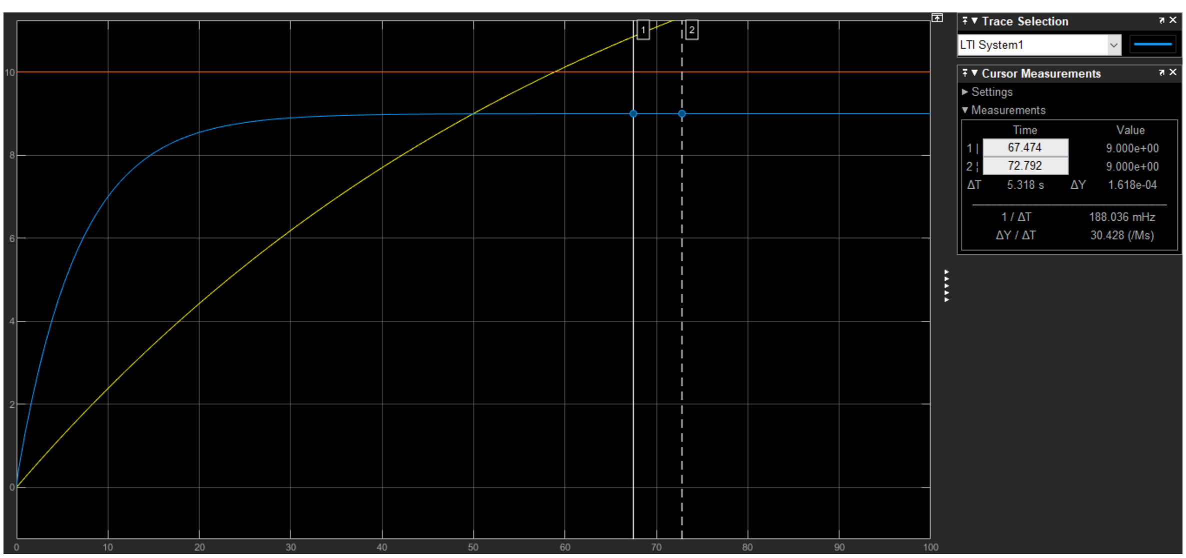
Simulink Diagram:



Simulink response:

Yellow = transfer function. Orange = step function Blue= transfer function with controller.

Final Value = 9 $e_{ss} = \frac{10-9}{10} = .1$



4. **For H2(s)/Qin(s) Design a Lead-Lag Compensator. Target: 3% error and a pre-defined maximum input (selected by you).**

To obtain this compensator I am using $C_1(s)$, as the lead, and making $C_2(s)$ as the lag and both together make the Lead-Lag Compensator.

$C(s) = C_1(s)\, C_2(s)$ Therefore G(s)C(s) is type 0 and unit step.

$e_{ss} = 0.03 = \frac{1}{1+k_p}$

$k_p = \frac{97}{3} = \lim_{s\to 0} C(s)G_2(s) = (\frac{k_1(a_1)}{b_1})(\frac{k_2(a_2)}{b_2})(4)$

$|a| < |b|$

For the Lead Compensator, I decided that $a_1 = -5$, $b_1 = -10$ and $k_1 = 2$.

$\frac{97}{3} = (\frac{2(5)}{10})(\frac{k_2(a_2)}{b_2})(4)$

$C_1(s) = \frac{2(s+5)}{s+10}$

Then for the Lag Compensator:

$$k_2 = \frac{97 * 10 * b_2}{3 * 2 * 5 * 4 * a_2} = \frac{97b_2}{12a_2}$$

$|a| < |b|$

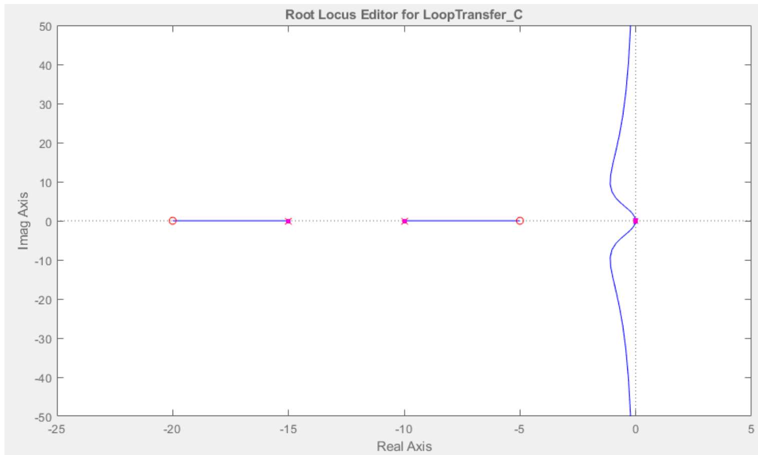
From the diagram of root locus, I decided that $a = -20$ y $b = -15$, therefore:

$$k = \frac{97 * 15}{12 * 20} = \frac{97}{16}$$

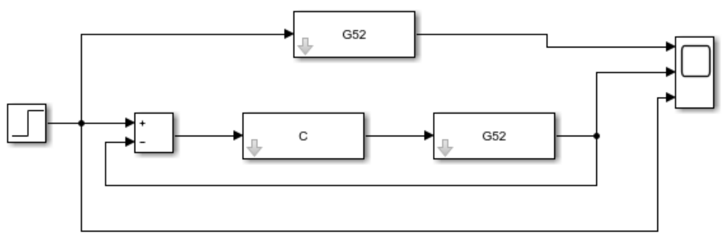
$C_2(s) = \frac{97/16(s+20)}{s+15}$

$C(s) = C_1(s)\, C_2(s) = (\frac{2(s+5)}{s+10})(\frac{97/16(s+20)}{s+15})$

Root Locus Diagram:



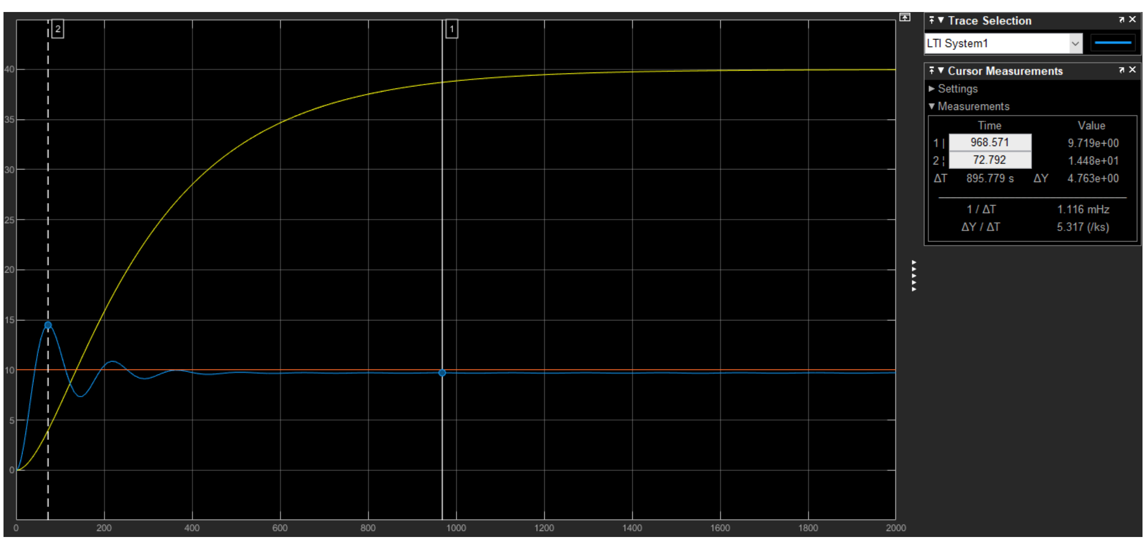
Simulink Diagram:

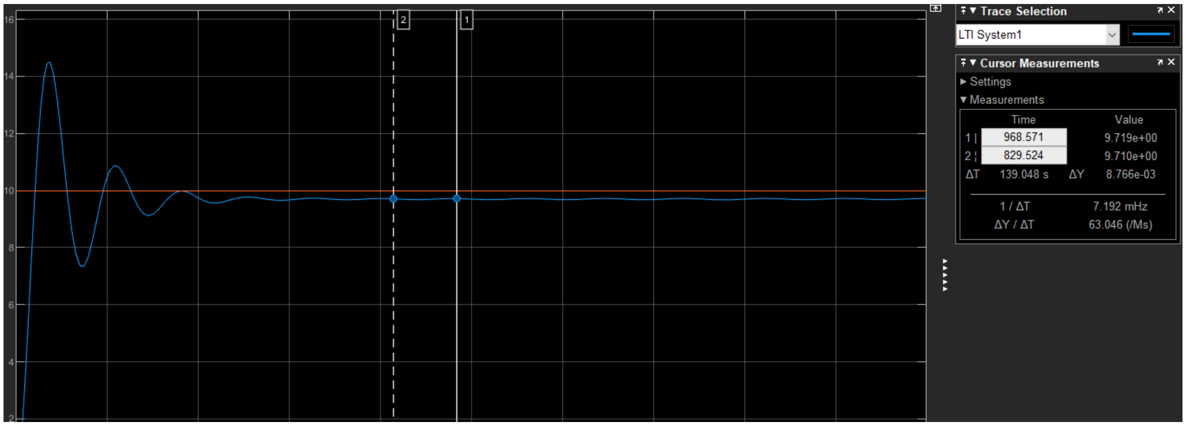


Simulink response:

Yellow = transfer function. Orange = step function Blue= transfer function with controller.

Final Value theorical = $a * 0.95 = 10 * .97 = 9.7$ Actual Final Value = 9.7





16
14
12
10
8
6
4
2
2
1
Trace Selection
LTI System1
Cursor Measurements
Settings
Measurements
Time
Value
1
968.571
9.719e+00
2
829.524
9.710e+00
ΔT
139.048 s
ΔY
8.766e-03
1 / ΔT
7.192 mHz
ΔY / ΔT
63.046 (/Ms)