2nd Simulation Project

Control Engineering

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General Parameters.

Date of Birth: May 10, 1999.

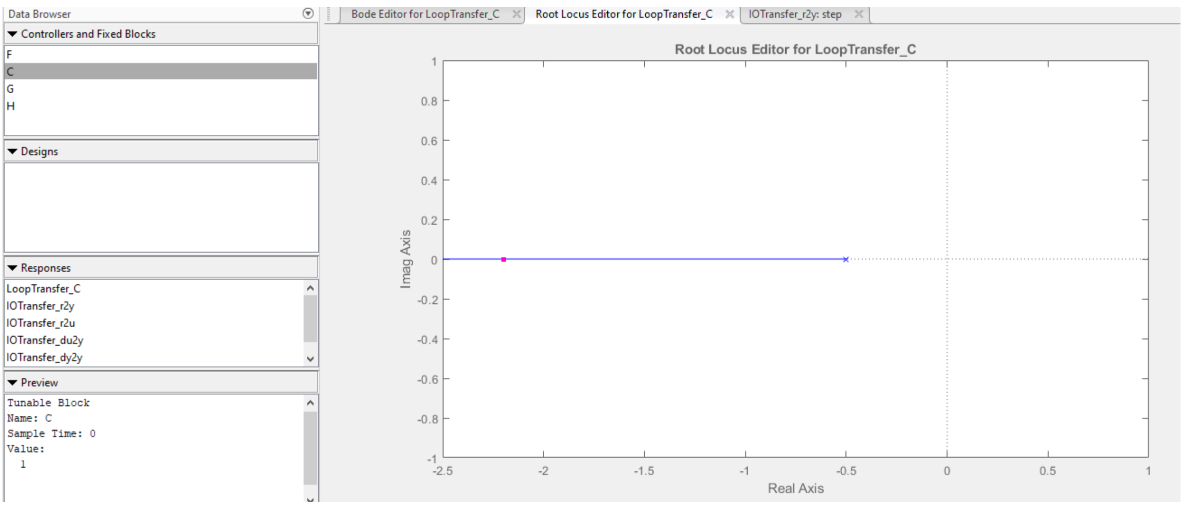
```
%General
a=10;
b=5;
c=17;
ut=a/2;
rt=a;
k=b;
```

System 1.

$$G_1(s) = \frac{c}{as+b} =$$

```
   17
--------
10 s + 5
```

$$K_{max} = \infty$$



1. Proportional Control Design.

```
>> rt=b

rt =

     5

>> ess=0.05

ess =

    0.0500
```

```
>> yss=rt-ess

yss =

    4.9500
```

$$G_{1CL} = \frac{17k}{10s+5+17k}$$

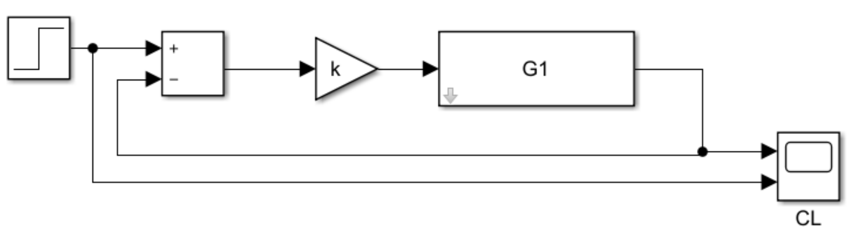
$$y_{ss} = \lim_{s\to 0} r(t)G_{1CL}(s) = 5(\frac{17k}{10(0)+5+17k})$$

$$y_{ss} = \frac{85k}{5+17k}$$

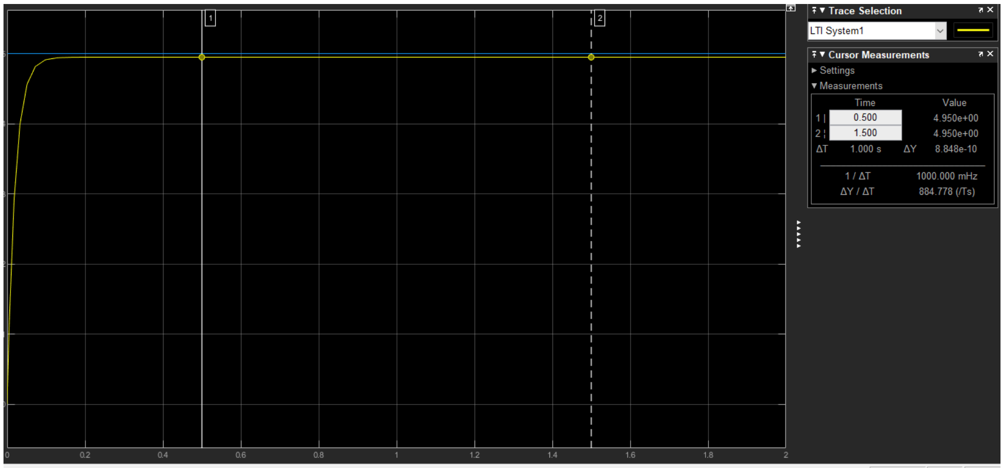
$$4.95 = \frac{85k}{5+17k}$$

Despejando para k:

```
>> k=(5*yss)/(rt*17-yss*17)

k =

   29.1176
```



Final value: 4.95



2. **PID Design: Sustained Oscillations Method.**

   This method cannot be done in this system. In order to do this method in any of the systems it must be at least 2nd order and have branches of its root locus diagram that crosses through the right side of the plane.

3. **PID Design: Damped Oscillations Method.**

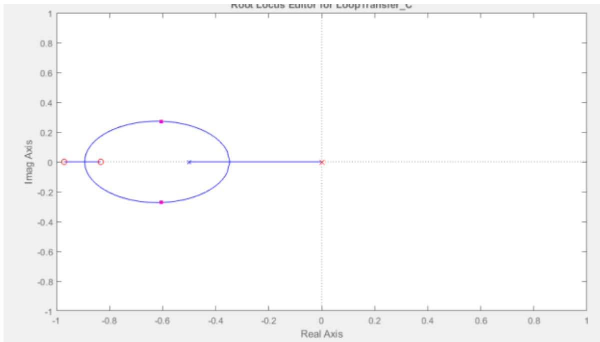
   This method cannot be done in this system. In order to perform this method, the system must show oscillations, and, in this case, it does not show them. It also must have a damping ratio greater than 0.21 in Open Loop.

4. **PID Design: Algebraic Method.**

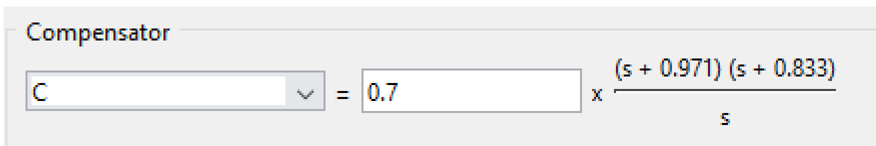
   $r(t) = b = 5$

   1 Pole at zero and adding 2 zeros at -0.971 and -0.833.

Real Zero	-0.971
Real Zero	-0.833
Integrator	0



$$\frac{k(s+z_a)(s+z_b)}{s} = \frac{k_d s^2 + k_c s + k_i}{s} = \frac{ks^2 + k(z_a+z_b) + kz_a z_b}{s}$$



Gain = 0.7

z_a = 0.971 z_b = 0.833

k = 0.7

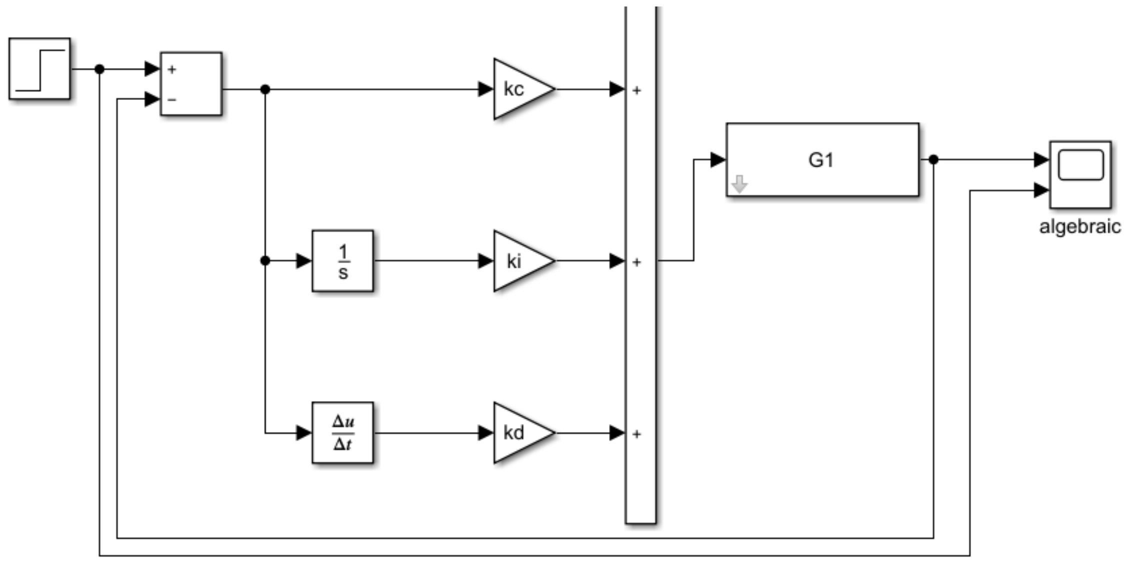
$k_d = k = 0.7$

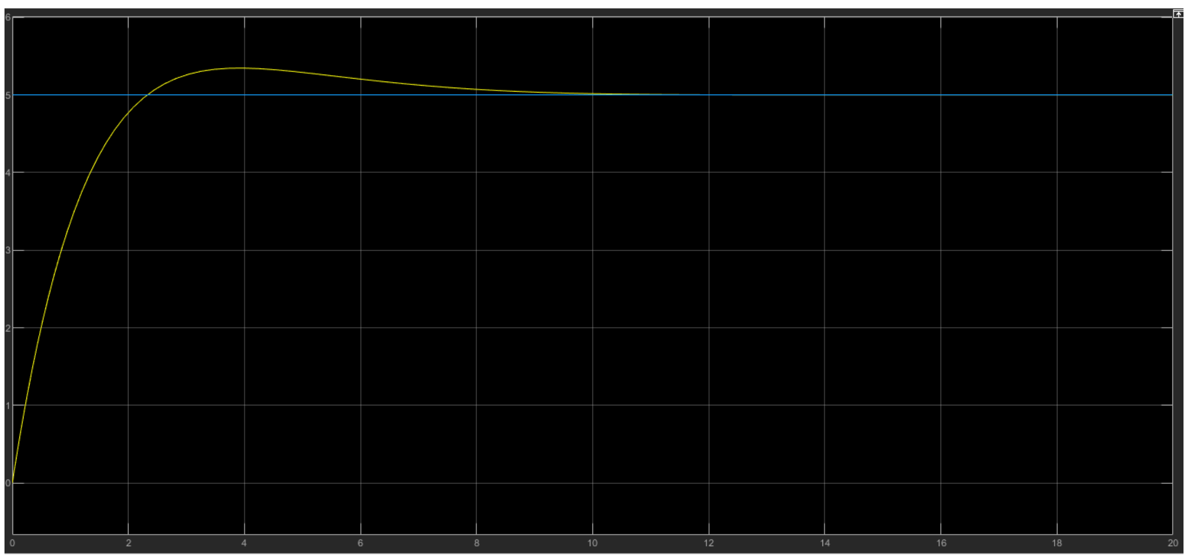
$k_c = k(z_a + z_b)$

```
>> kc=k*(za+zb)

kc =

    1.2628
```

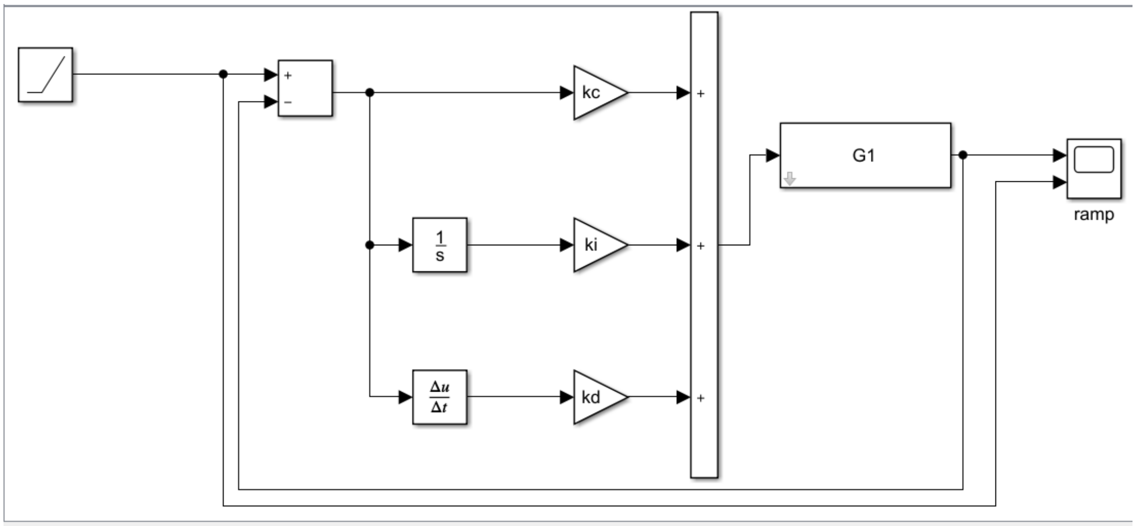
$k_i = k * z_a * z_b$

```
>> ki=k*za*zb

ki =

    0.5662
```

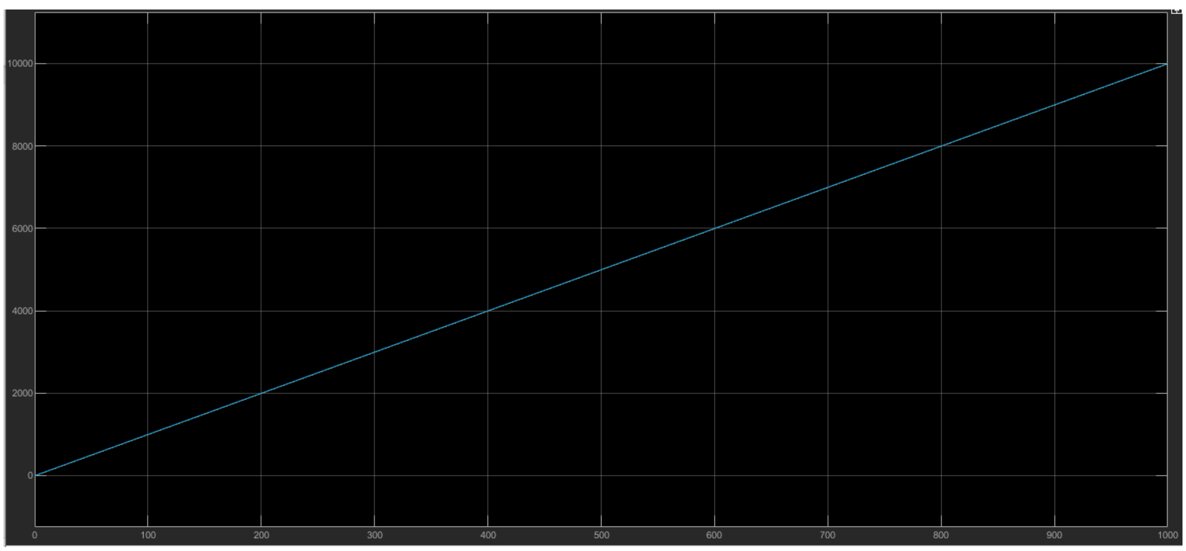

kc
1
s
ki
Δu
Δt
kd
G1
algebraic



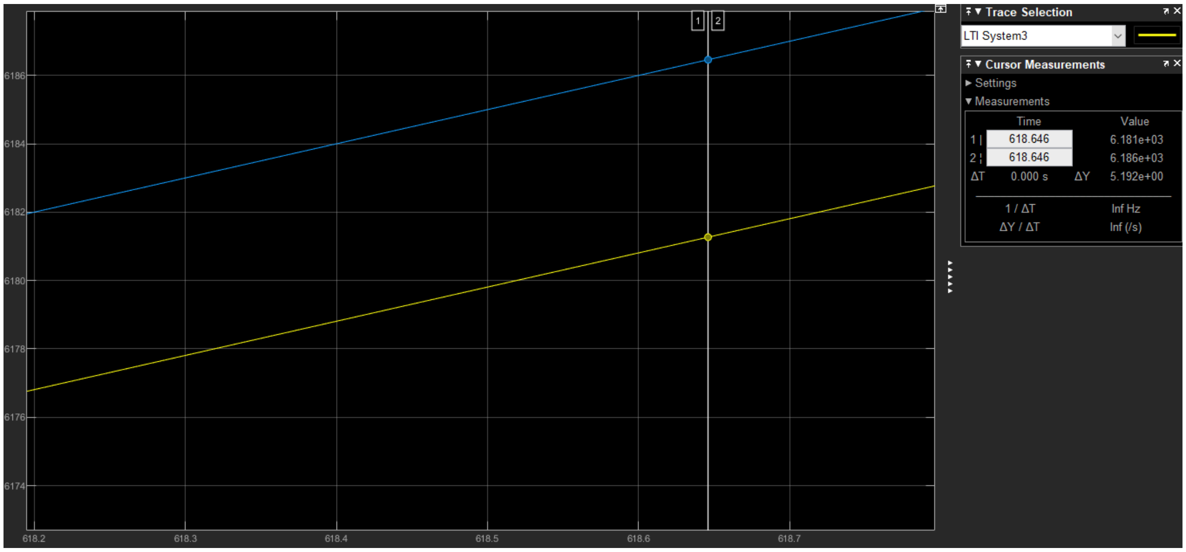
5. **PID Design: Ramp.**



Then we obtain the following graphs:



After zoom in it:

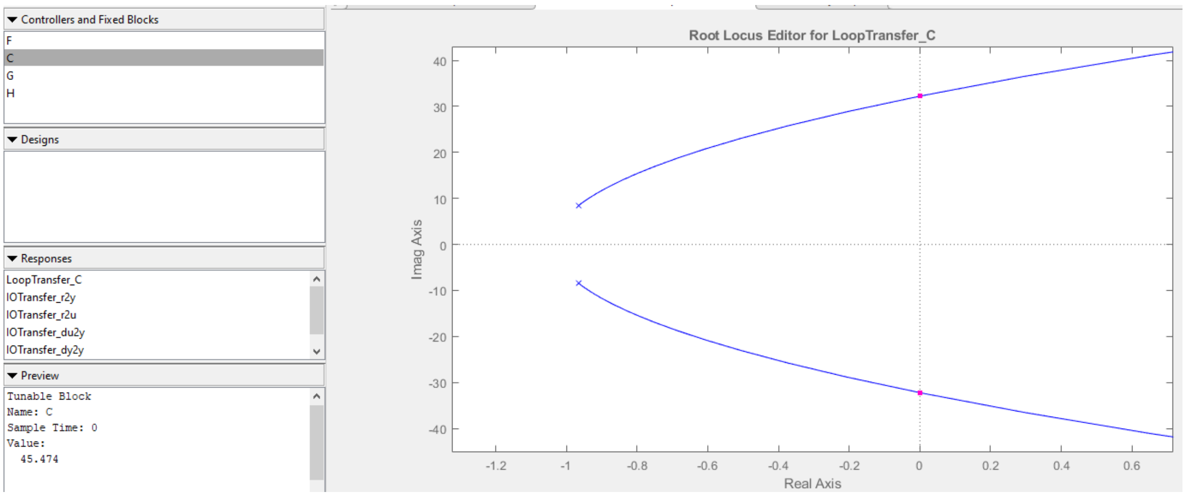


We obtain a difference of 5.192 from the reference and our system, therefore:

$$e_{ss} = \frac{5.192 * 100}{6.186e + 03} = 0.08\%$$

System 2.

```
G2 =

      -0.17 s + 85
  ---------------------
  4 s^2 + 7.727 s + 289

Continuous-time transfer function.
```



As we can see, maximum gain is 45.474.

1. Proportional Control Design.

```
>> rt=b

rt =

     5

>> ess=0.05

ess =

    0.0500
```

```
>> yss=rt-ess

yss =

    4.9500
```

$$G_{2CL} = \frac{(-0.17s+85)k}{4s^2+7.727s+289+(-0.17s+85)k}$$

$$y_{ss} = \lim_{s\to 0} r(t)G_{2CL}(s) = 5\left(\frac{-0.17(0)k+85k}{4(0)+7.727(0)+289-0.17(0)k+85k}\right)$$

$$y_{ss} = \frac{425k}{289+85k}$$

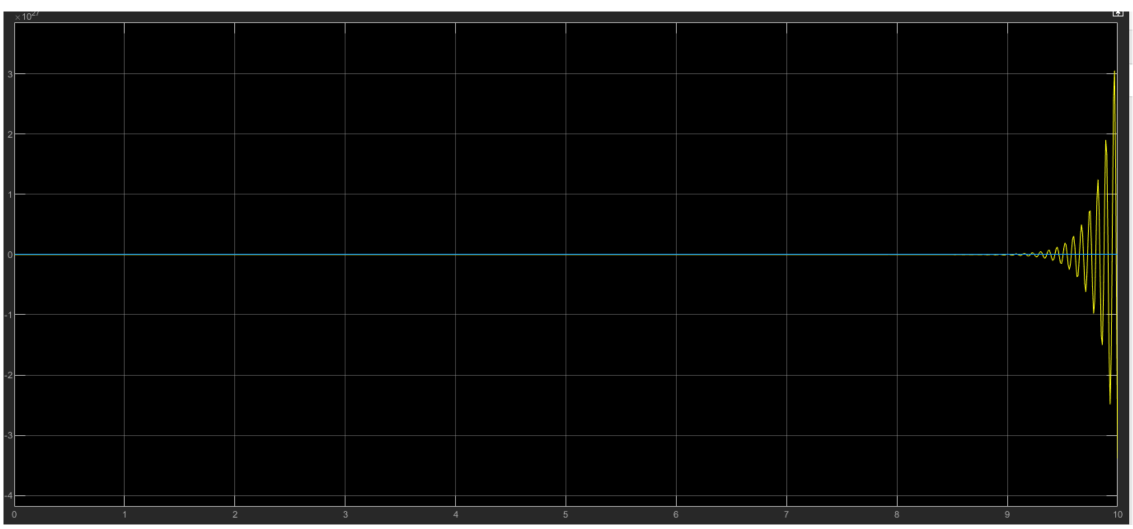
$$4.95 = \frac{425k}{289+85k}$$

Despejando para k:

$$4.95(289 + 85k) = 425k$$

$$4.95 * 289 = (425 - 4.95 * 85)k$$

$$k = \frac{1430.55}{4.25} = 336.6$$



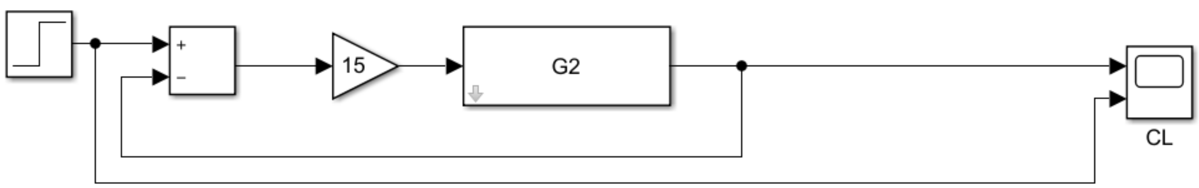
This system is unstable; therefore I picked another gain to make it stable.

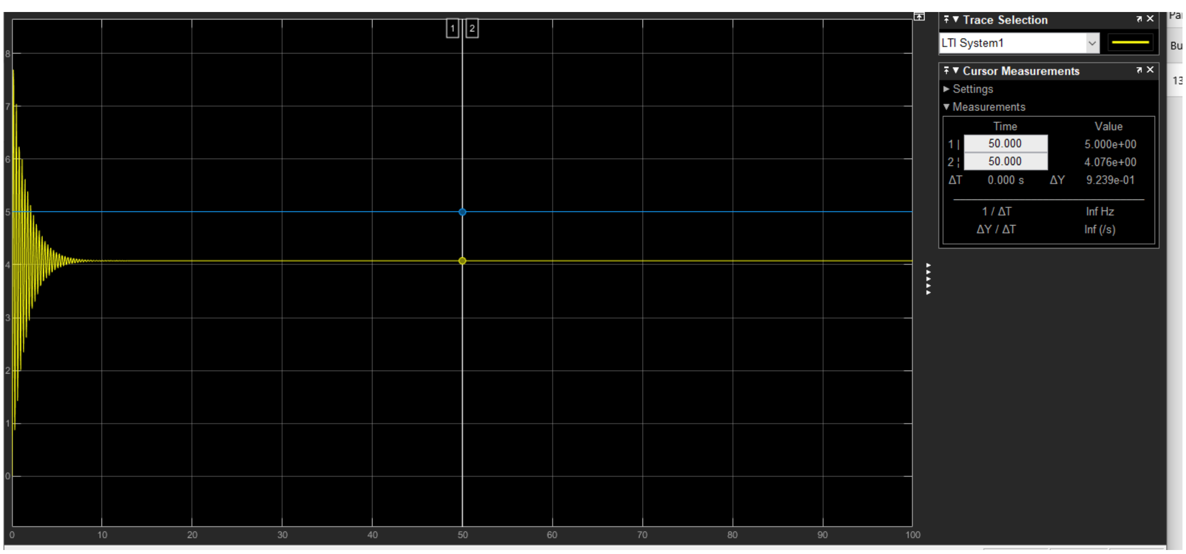
$$y_{ss} = \frac{425k}{289+85k}$$

```
>> yss=(425*15)/(289+85*15)

yss =

    4.0761
```

$$e_{ss} = \frac{100 * (r(t) - y_{ss})}{r(t)}$$

$$e_{ss} = 18.478\%$$





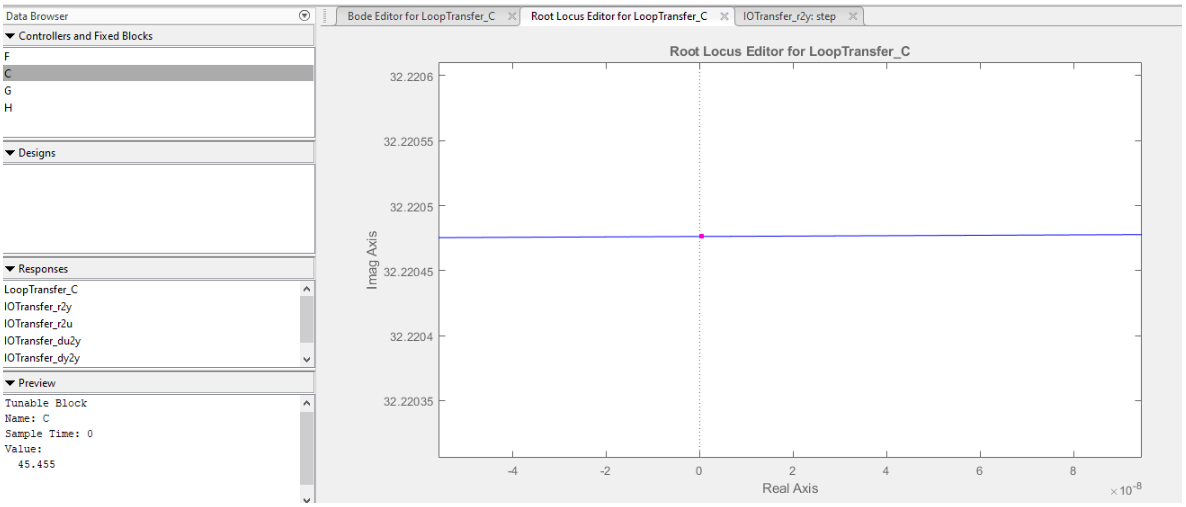
As we can see from the graph, the difference between my system and the step is 0.9239.

$$e_{ss} = \frac{100 * (0.9239)}{r(t)} = 18.478\%$$

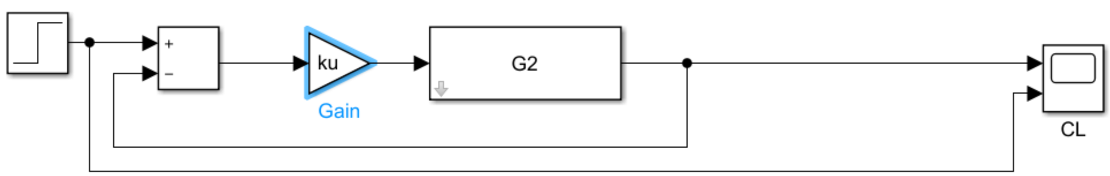
2. PID: Design: Sustained Oscillations.

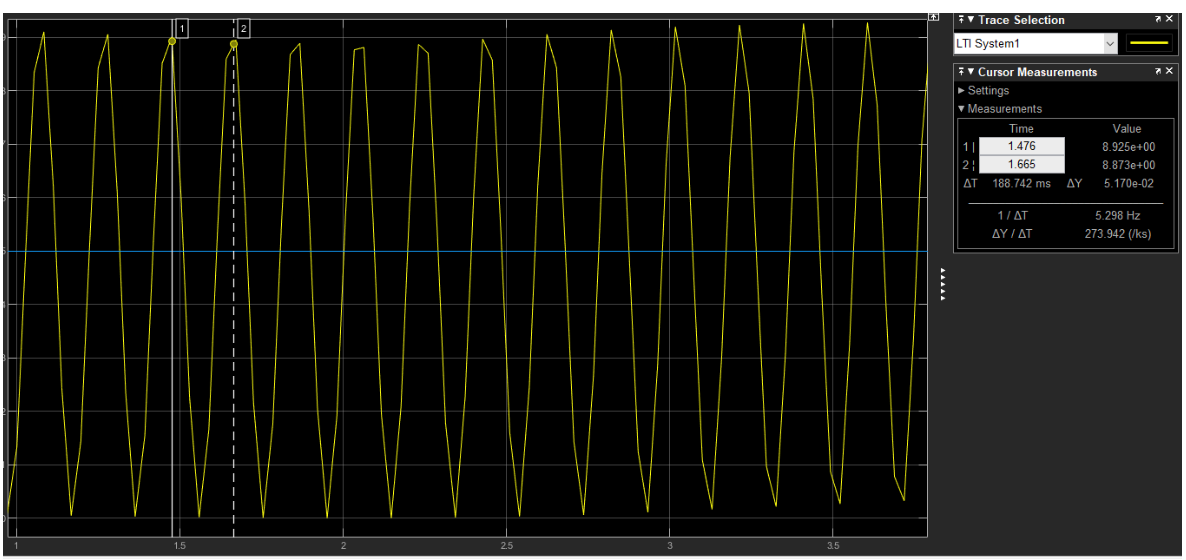
$r(t) = b = 5$

In order to place the pole closer to 0, zoom in and check the critical gain.



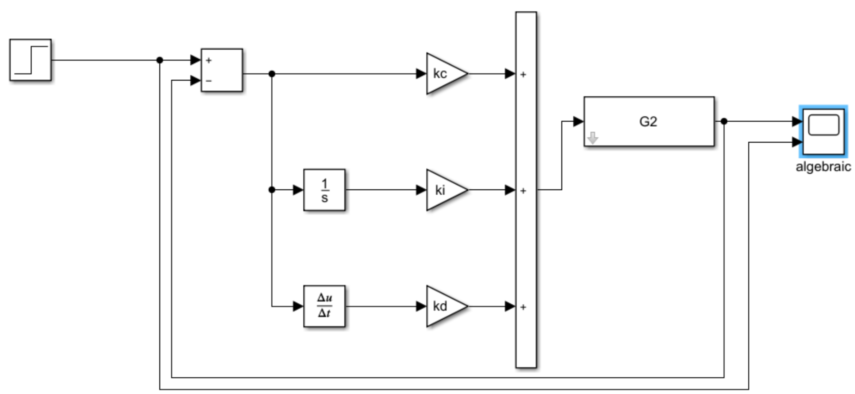
$k_u = 45.455$





$t_u = 188.742ms = 0.189$

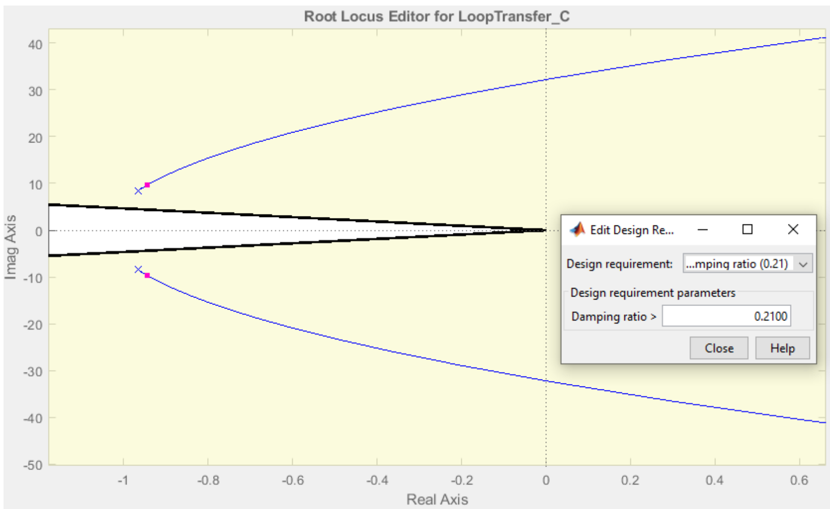
```
>> kc=ku/1.7

kc =

   26.7382

>> tu=0.189

tu =

    0.1890

>> tu=0.188742

tu =

    0.1887

>> ti=tu/2

ti =

    0.0944

>> td=tu/8

td =

    0.0236

>> ki=kc/ti

ki =

  283.3311

>> kd=kc*td

kd =

    0.6308
```



3. PID: Damped Oscillations.

$r(t) = b = 5$

In order to perform this method, the system must have a damping ratio greater or equal than 0.21 in Open Loop.

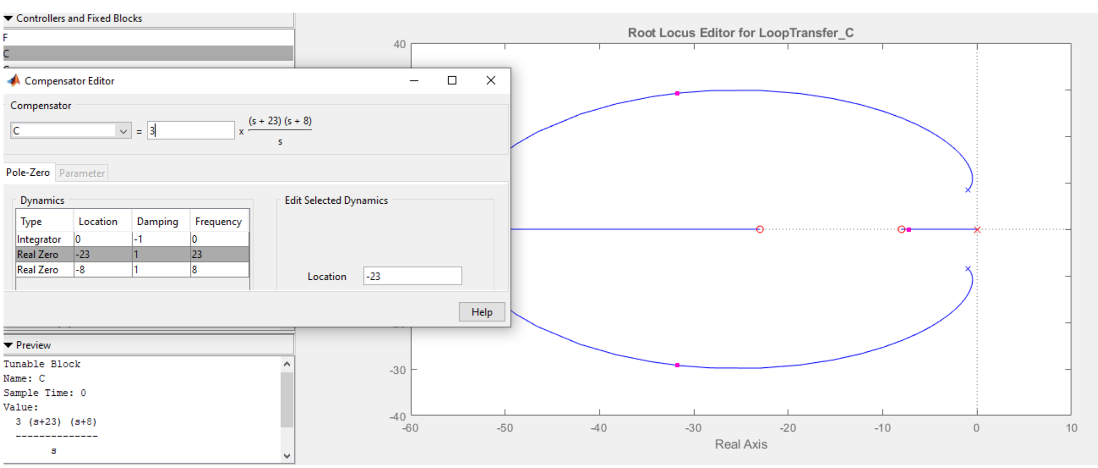


Since the branches are not inside the damping radio, this method cannot be done.

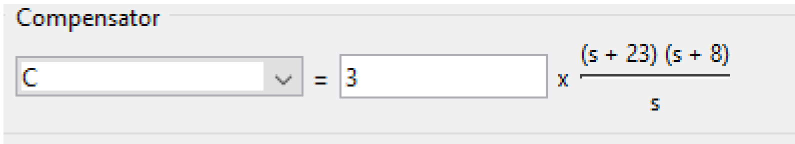
4. PID: Algebraic.

$r(t) = b = 5$

Add 1 Pole at zero and adding 2 zeros at -23 and -8.



$$\frac{k(s+z_a)(s+z_b)}{s} = \frac{k_d s^2 + k_c s + k_i}{s} = \frac{ks^2 + k(z_a+z_b) + kz_a z_b}{s}$$



Gain = 3

z_a = 23 z_b = 8

k = 3

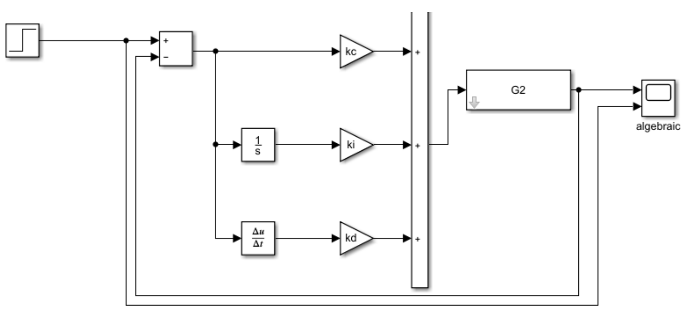
$k_d = k = 3$

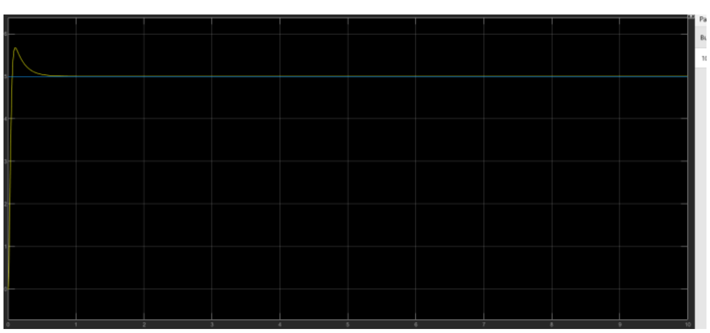
```
>> kc=k*(za+zb)

kc =

    93
```

$k_c = k(z_a + z_b)$

```
>> ki=k*(za*zb)

ki =

   552
```

$k_i = k * z_a * z_b$





5. **PID: Ramp.**

$r(t) = a = 10$

$$G_{2R}(s) = C * G_2(s) = \frac{k(s+z_a)(s+z_b)}{s} * \frac{-0.17s + 85}{4s^2 + 7.727s + 289}$$

$$k_v = \lim_{s \to 0} sG_{2R}(s) = \lim_{s \to 0}[\frac{s}{s}(k)(s+z_a)(s+z_b)(\frac{-0.17s + 85}{4s^2 + 7.727s + 289)}$$

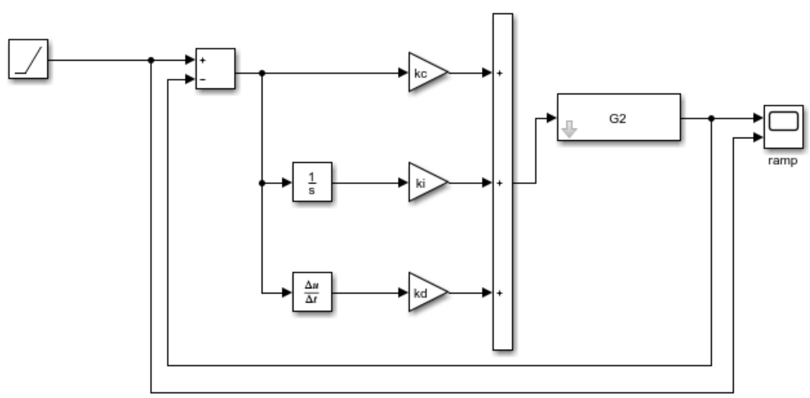
```
>> kv=k*za*zb*85/289

kv =

  162.3529
```

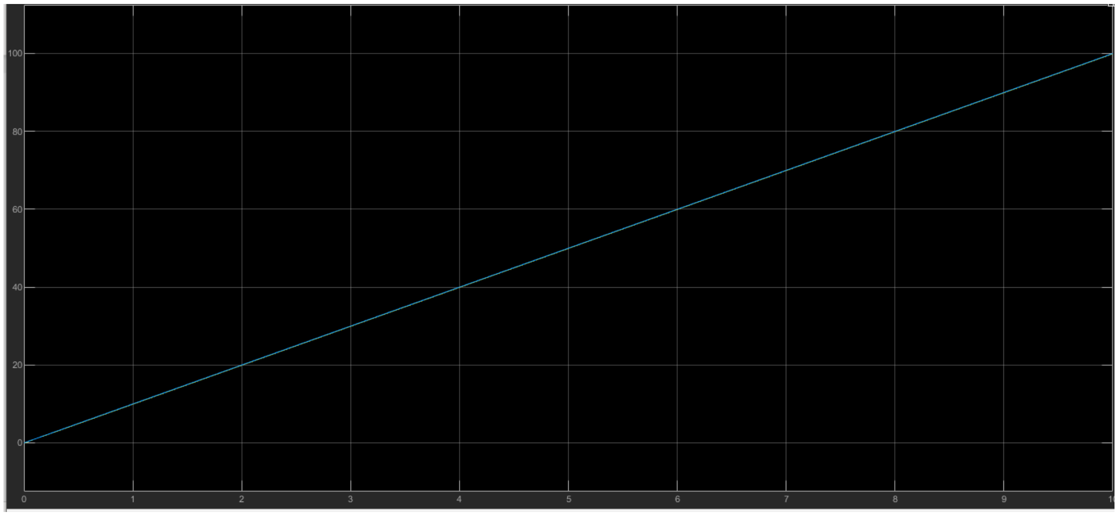
$$k_v = k(z_a)(z_b)(\frac{85}{289})$$

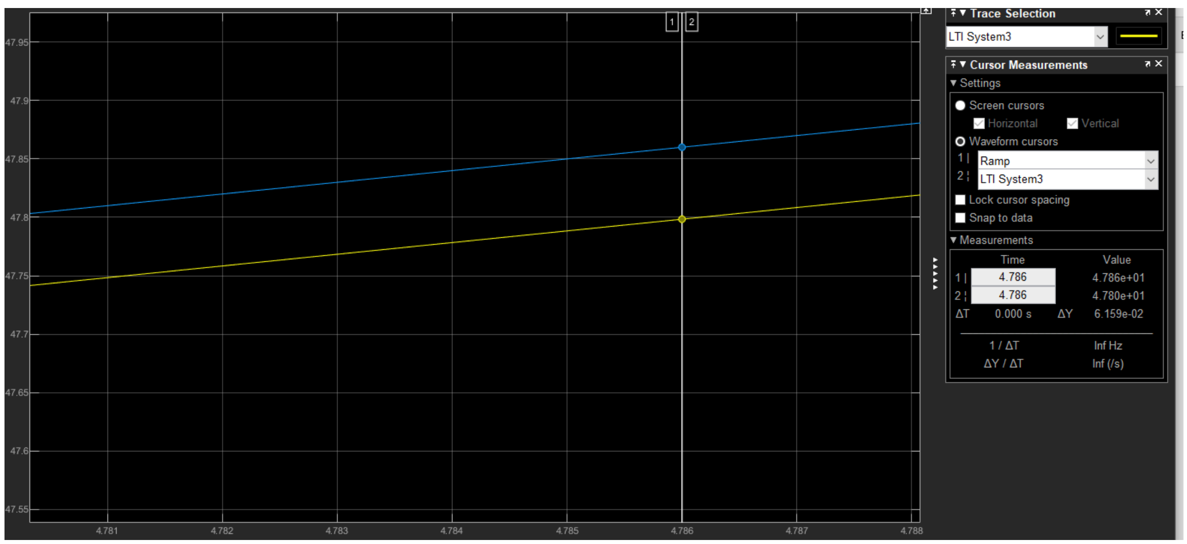
```
>> ess=rt/kv

ess =

    0.0616
```

$$e_{ss} = \frac{r(t)}{k_v}$$

$$e_{ss} = 6.16\%$$



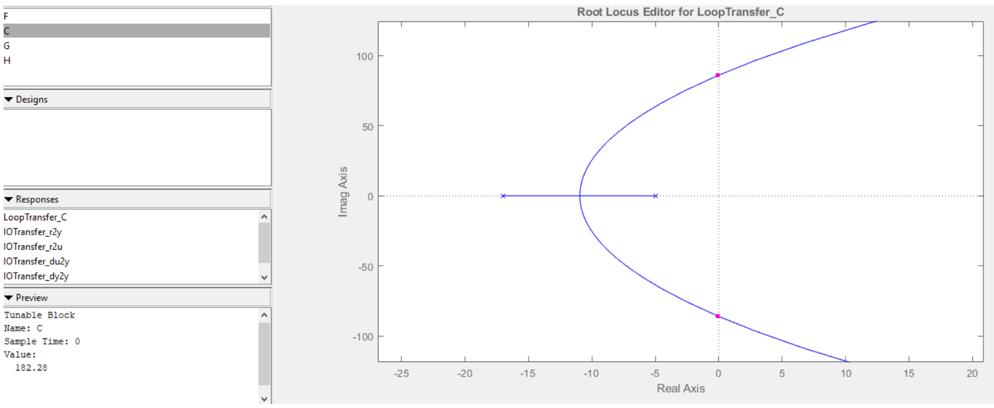




The difference from my system and the references is 0.00616 which is 6.16% of error.

System 3.

```
G3 =

   -0.12 s + 40
  ---------------
  s^2 + 22 s + 85
```



Maximum Gain: 182.28

1. Proportional Control Design.

```
>> rt=b

rt =

     5

>> ess=0.05

ess =

    0.0500
```

```
>> yss=rt-ess

yss =

    4.9500
```

$$G_{3CL} = \frac{(-0.12s+40)k}{s^2+22s+85+(-0.12s+40)k}$$

$$y_{ss} = \lim_{s\to 0} r(t)G_{3CL}(s) = 5\left(\frac{-0.12(0)k+40k}{(0)+22(0)+85-0.12(0)k+40k}\right)$$

$$y_{ss} = \frac{5*40k}{85+40k}$$

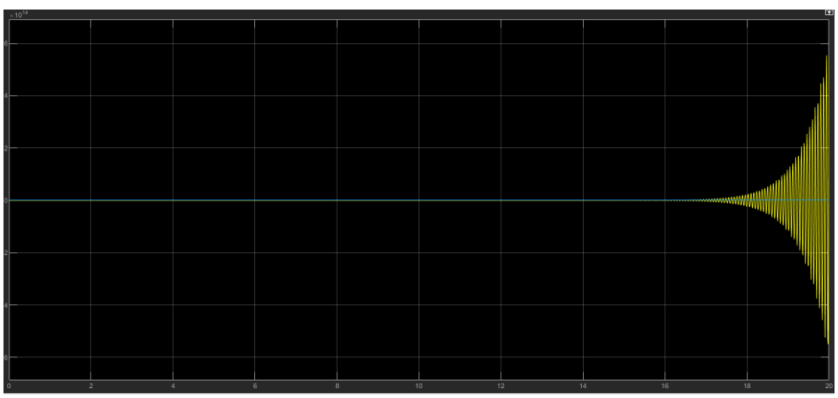
$$4.95 = \frac{200k}{85+40k}$$

Despejando para k:

$$4.95(85 + 40k) = 200k$$

$$4.95 * 85 = (200 - 4.95 * 40)k$$

$$k = \frac{420.75}{2} = 210.375$$



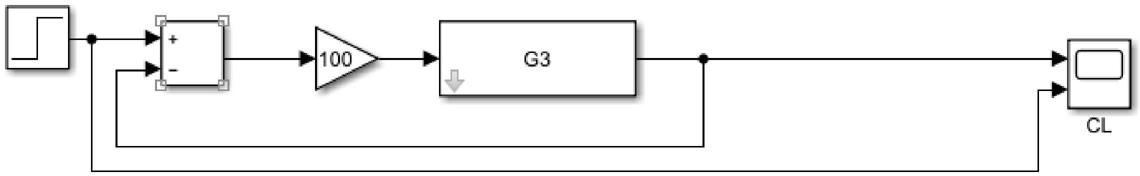
This system is unstable; therefore, I picked another gain to make it stable.

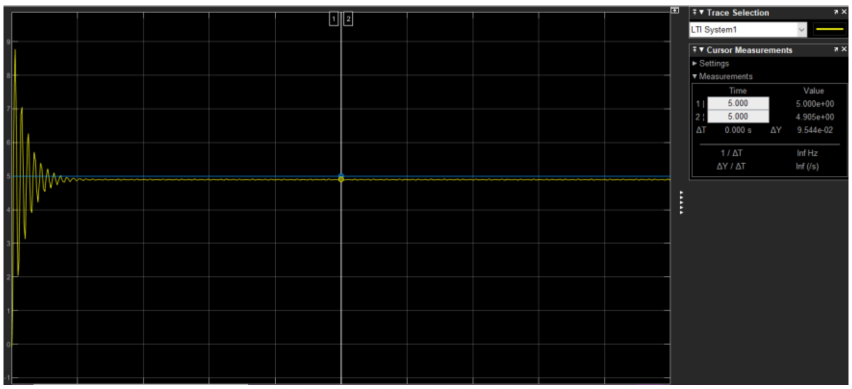
```
>> yss=200*(100)/(85+40*100)

yss =

    4.8960
```

$$y_{ss} = \frac{200k}{85+40k}$$

```
>> ess=100*(rt-yss)/rt

ess =

    2.0808
```

$$e_{ss} = (r(t) - y_{ss}) * 100/r(t)$$



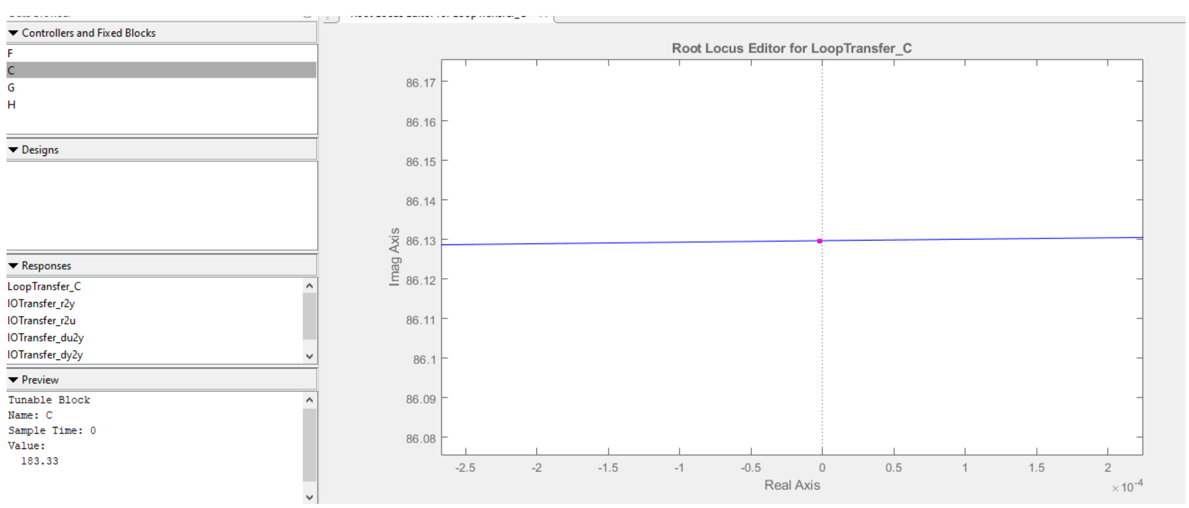


$$e_{ss} = (0.09544) * \frac{100}{r(t)} = 1.908\%$$

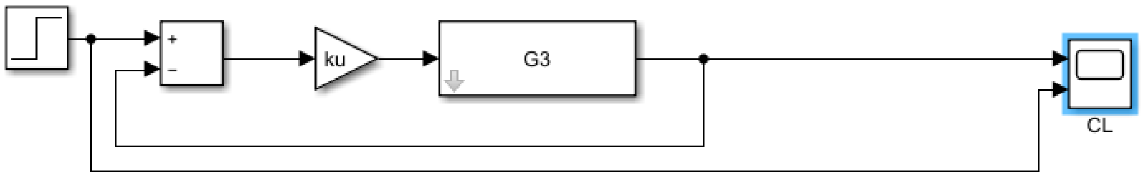
2. PID: Design: Sustained Oscillations.

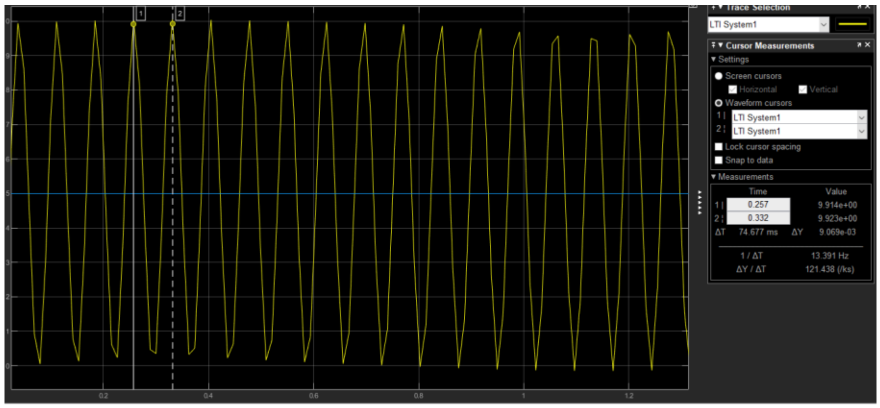
$$r(t) = b = 5$$

In order to place the pole closer to 0, zoom in and check the critical gain.



$$k_u = 183.33$$

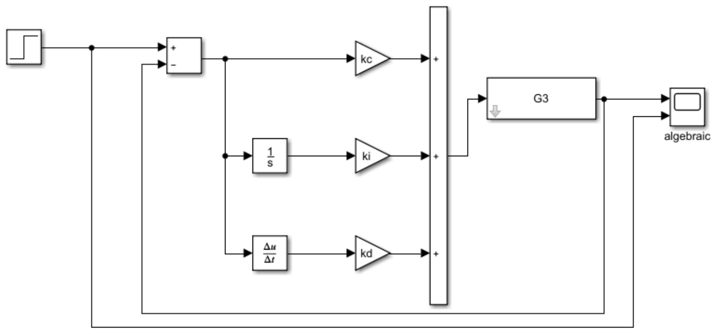


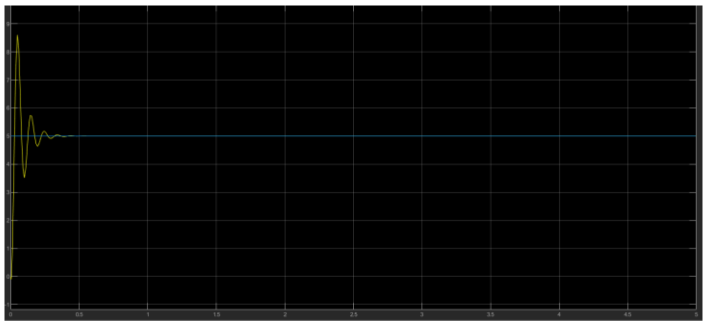


$$t_u = 76.677ms = 0.076677$$

```
>> kc=ku/1.7

kc =

  107.8412

>> ti=tu/2

ti =

    0.0383

>> td=tu/8

td =

    0.0096

>> ki=kc/ti

ki =

   2.8129e+03
```

```
>> kd=kc*td

kd =

    1.0336
```

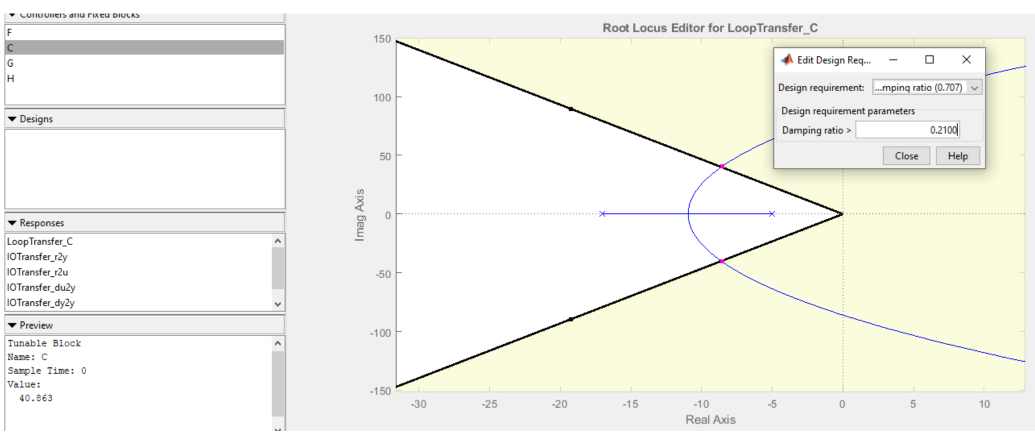


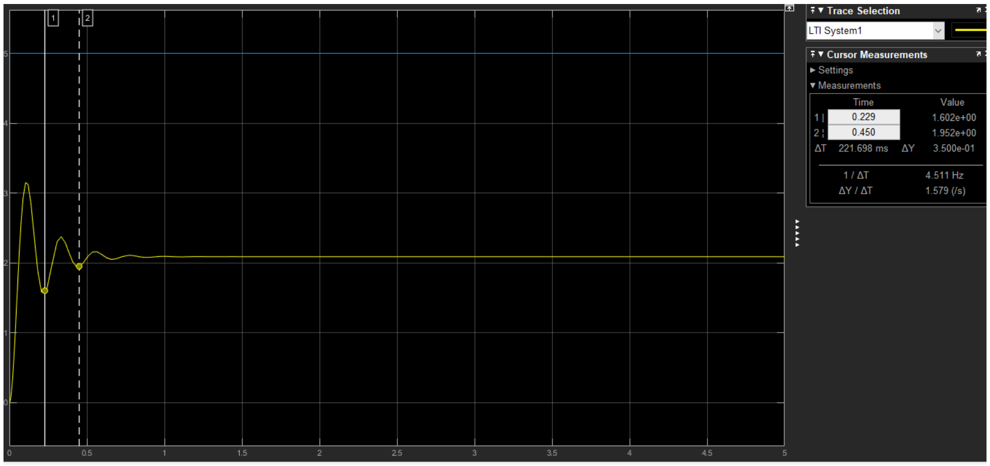


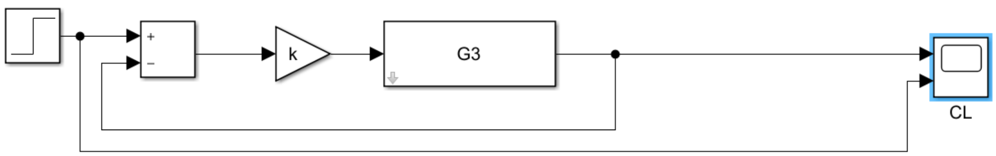
3. **PID: Damped Oscillations.**

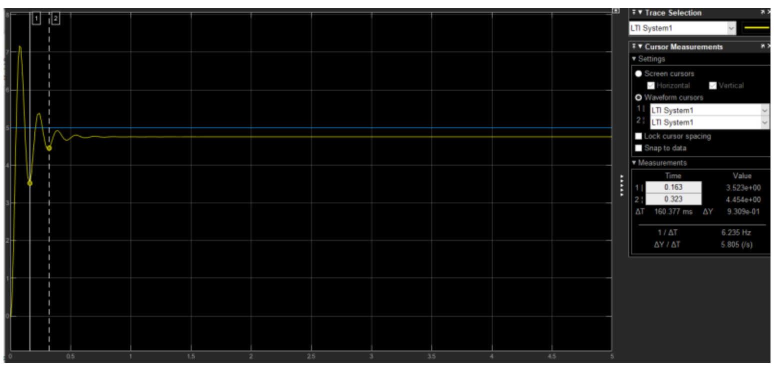
$r(t) = b = 5$

In order to perform this method, the system must have a damping ratio greater or equal than 0.21 in Open Loop.







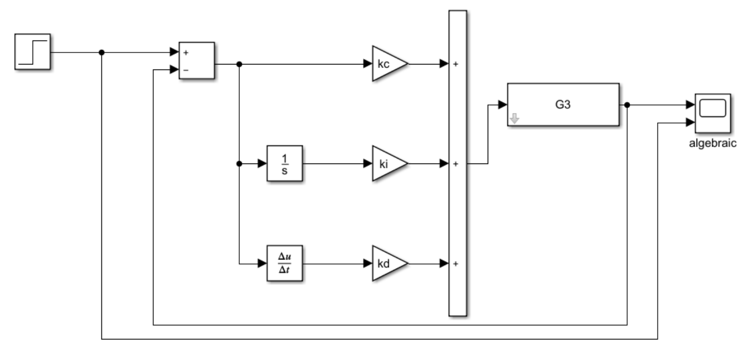


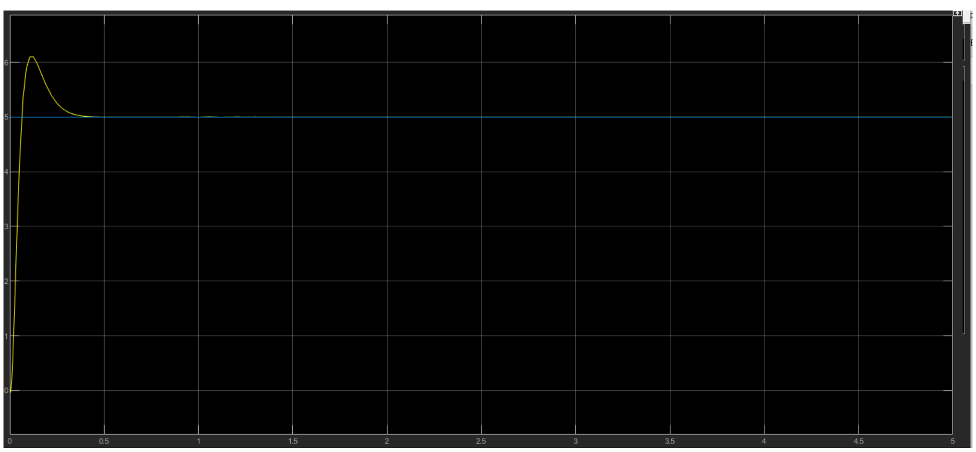
$k_0 = 40.863$

$t_0 = 160.377ms = 0.160377s$

```
>> kc=ko

kc =

   40.8630

>> ti=to/1.5

ti =

    0.1069

>> td=to/6

td =

    0.0267
```

```
>> ki=kc/ti

ki =

  382.1901

>> kd=kc*td

kd =

    1.0922
```

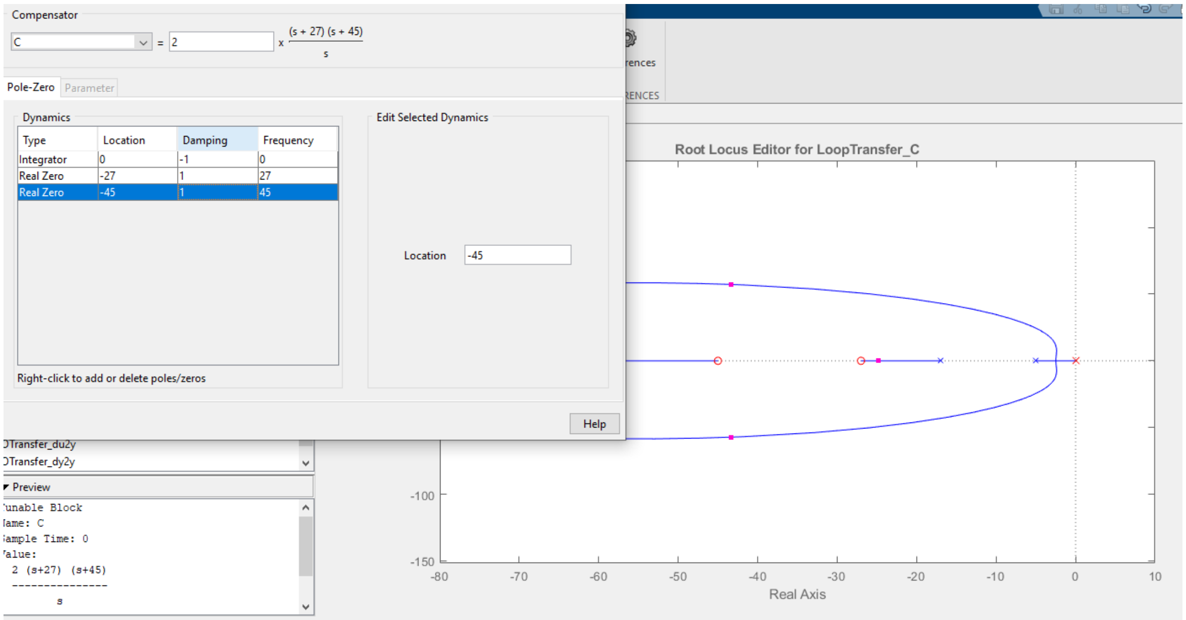




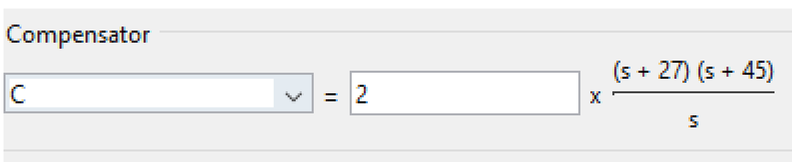
4. PID: Algebraic.

$r(t) = b = 5$

Add 1 Pole at zero and adding 2 zeros at -23 and -8.



$$\frac{k(s+z_a)(s+z_b)}{s} = \frac{k_d s^2 + k_c s + k_i}{s} = \frac{ks^2 + k(z_a+z_b) + kz_a z_b}{s}$$



Gain = 2

$z_a = 27$ $\quad z_b = 45$

k = 2

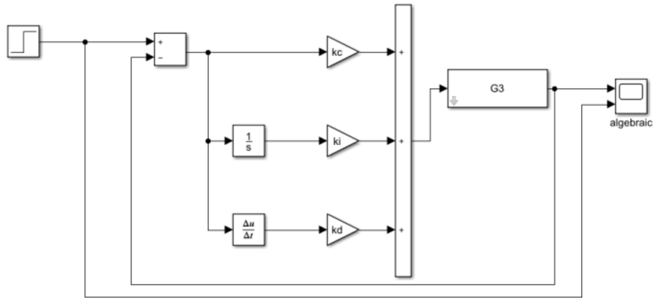
$k_d = k = 2$

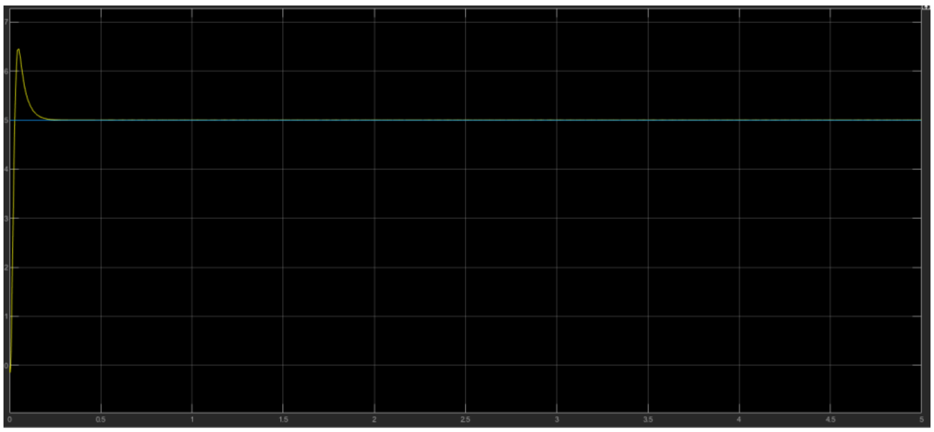
```
>> kc=k*(za+zb)

kc =

   144
```

$k_c = k(z_a + z_b)$

```
>> ki=k*(za*zb)

ki =

        2430
```

$k_i = k * z_a * z_b$





5. **PID: Ramp.**

$$r(t) = a = 10$$

$$G_{3R}(s) = \ C * G_3(s) = \frac{k(s+z_a)(s+z_b)}{s} * \frac{-0.12s+40}{s^2+22s+85}$$

$$k_v = \lim_{s\to 0} sG_{2R}(s) = \lim_{s\to 0}[\frac{s}{s}(k)(s+z_a)(s+z_b)(\frac{-0.12s+40}{s^2+22s+85})$$

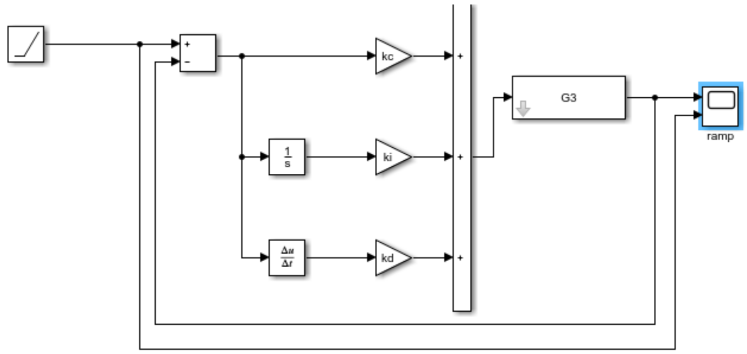
```
>> kv=k*za*zb*40/85

kv =

   1.1435e+03
```

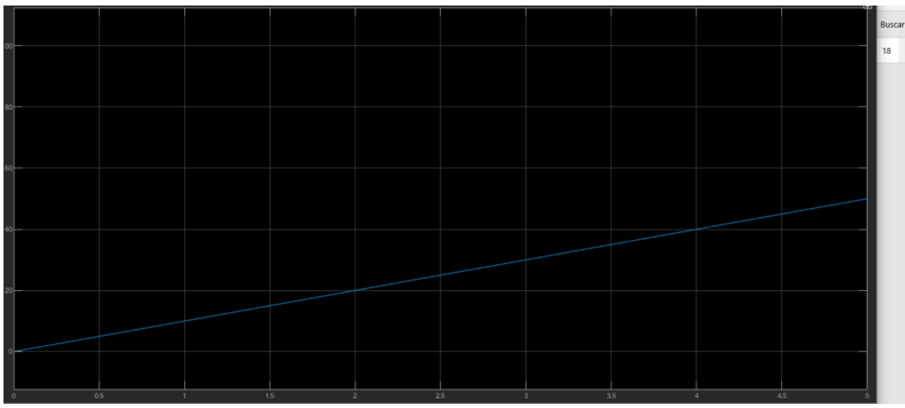
$$k_v = k(z_a)(z_b)(\frac{40}{85})$$

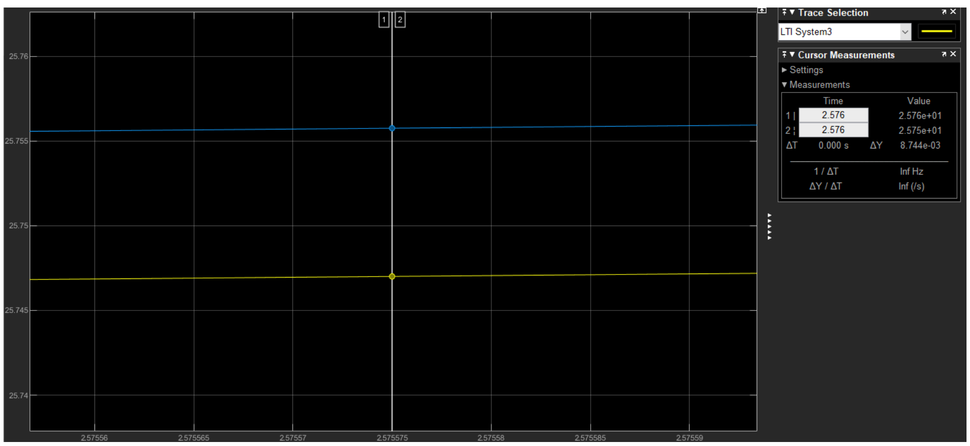
```
>> ess=rt/kv

ess =

    0.0087
```

$$e_{ss} = \frac{r(t)}{k_v}$$

$$e_{ss} = 0.8745\%$$



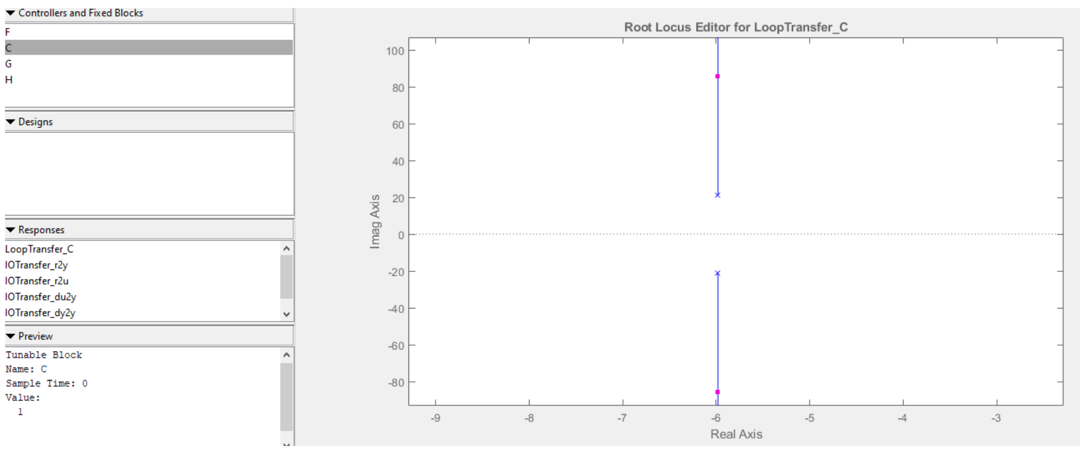




The difference from my system and the references is 0.008744 which is 0.874% of error.

System 4.

```
G =

           6869
  ---------------------
  s^2 + 11.96 s + 490.6

Continuous-time transfer function.
```



Since there are not branches crossing the imaginary axis, the maximum gain is infinity.

1. Proportional Control Design.

```
>> rt=b

rt =

     5

>> ess=0.05

ess =

    0.0500
```

```
>> yss=rt-ess

yss =

    4.9500
```

$$G_{3CL} = \frac{(6869)k}{s^2 + 11.96s + 490.6 + (6869)k}$$

$$y_{ss} = \lim_{s \to 0} r(t)G_{3CL}(s) = 5\left(\frac{6869k}{(0)+11.96(0)+490.6+6869k}\right)$$

$$y_{ss} = \frac{5*6869k}{490.6+6869k}$$

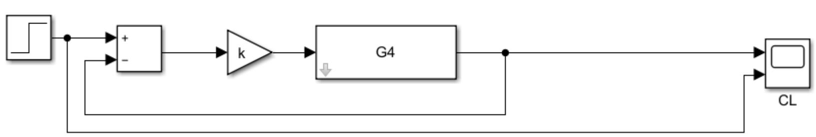
$$4.95 = \frac{34345k}{490.6+6869k}$$

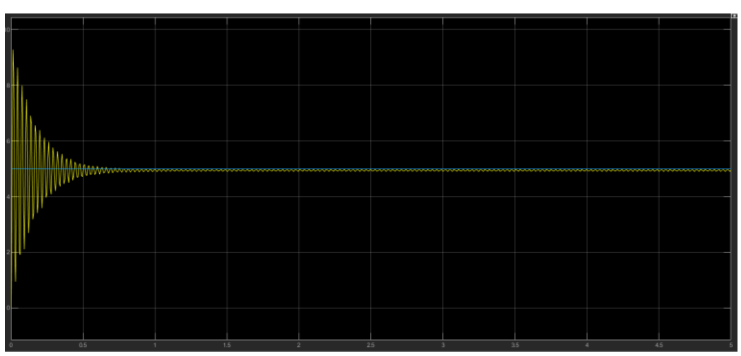
Despejando para k:

$$4.95(490.6 + 6869k) = 34345k$$

$$4.95 * 490.6 = (34345 - 4.95 * 6869)k$$

$$k = \frac{2050.71}{343.45} = 5.97091$$





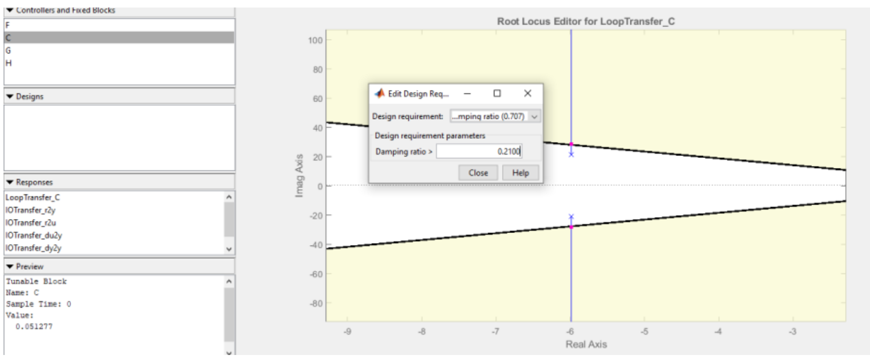
2. **PID: Design: Sustained Oscillations.**

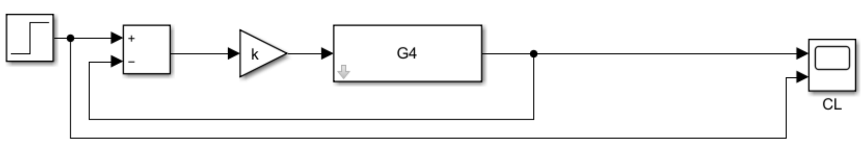
   This method cannot be done in this system, because in order to do it the system must have branches of its root locus diagram that crosses through the Right-Hand Plane.

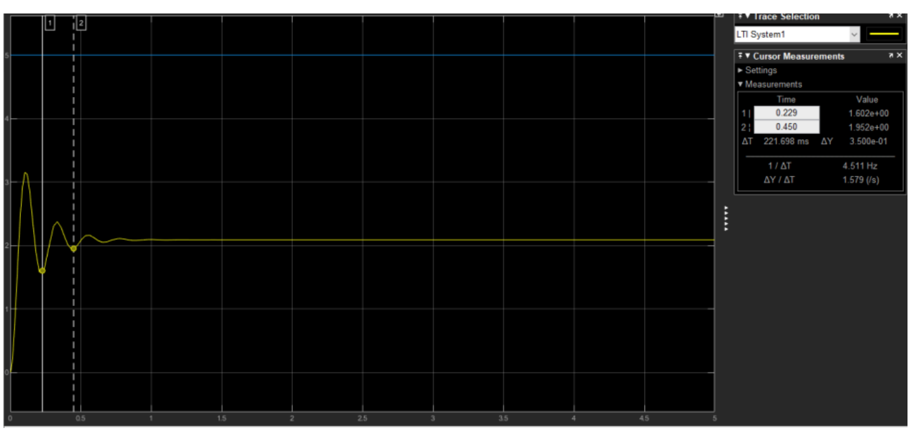
3. **PID: Damped Oscillations.**

   $r(t) = b = 5$

   In order to perform this method, the system must have a damping ratio greater or equal than 0.21 in Open Loop.



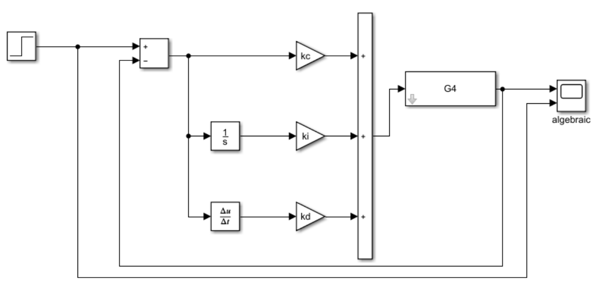


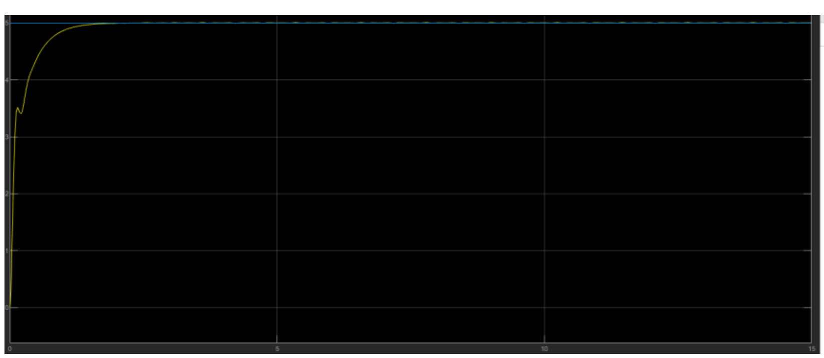


$k_0 = 0.051277$

$t_0 = 221.698ms = 0.221698s$

```
kc =

    0.0513

>> ti=to/1.5

ti =

    0.1478

>> td=to/6

td =

    0.0369

>> ki=kc/ti

ki =

    0.3469

>> kd=kc*td

kd =

    0.0019
```

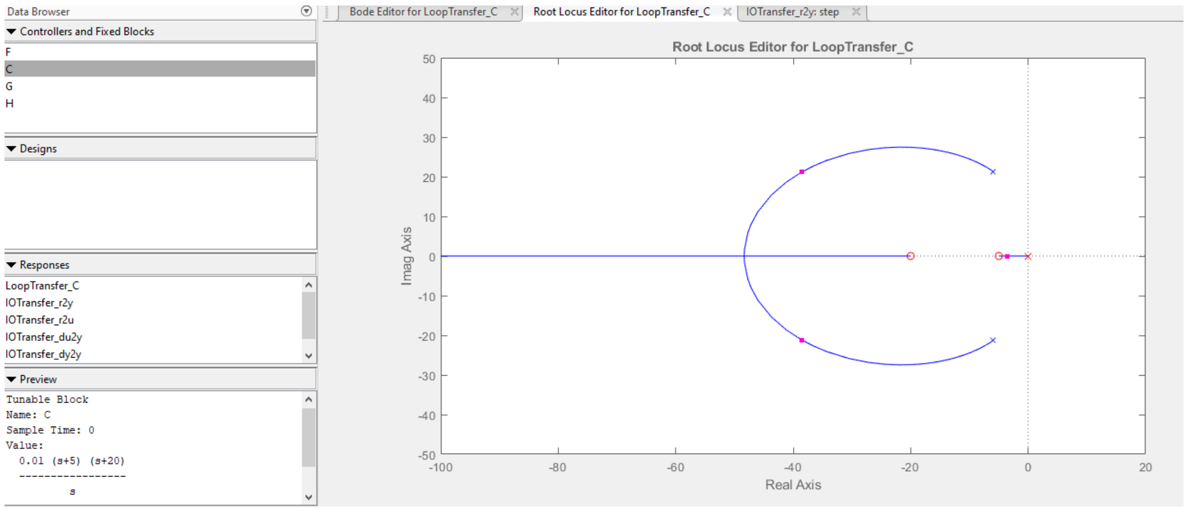




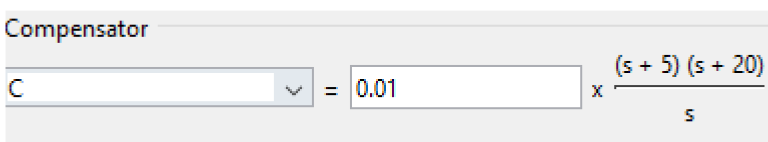
4. PID: Algebraic.

$r(t) = b = 5$

Add 1 Pole at zero and adding 2 zeros at -23 and -8.



$$\frac{k(s+z_a)(s+z_b)}{s} = \frac{k_d s^2 + k_c s + k_i}{s} = \frac{ks^2 + k(z_a + z_b) + k z_a z_b}{s}$$



Gain = 0.01

$z_a = 5$ $\quad z_b = 20$

k = 0.01

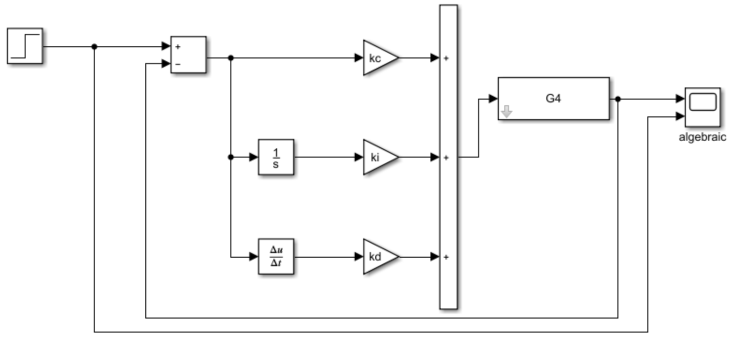
$k_d = k = 0.01$

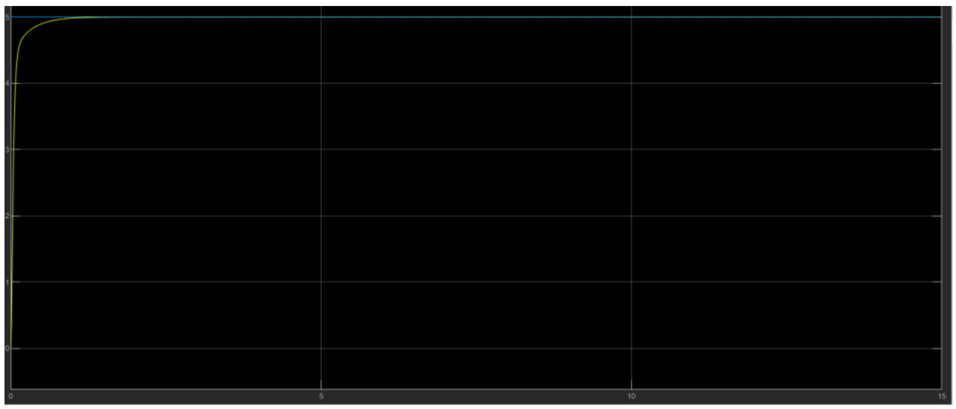
```
>> kc=k*(za+zb)

kc =

    0.2500
```

$k_c = k(z_a + z_b)$

```
>> ki=k*(za*zb)

ki =

     1
```

$k_i = k * z_a * z_b$





5. PID: Ramp.

$$r(t) = a = 10$$

$$G_{4R}(s) = C * G_4(s) = \frac{k(s+z_a)(s+z_b)}{s} * \frac{6869}{s^2+11.96s+490.6}$$

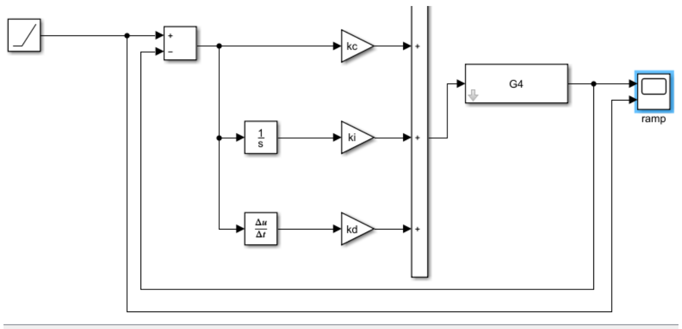
$$k_v = \lim_{s\to 0} sG_{2R}(s) = \lim_{s\to 0}[\frac{s}{s}(k)(s+z_a)(s+z_b)(\frac{6869}{s^2+11.96s+490.6})$$

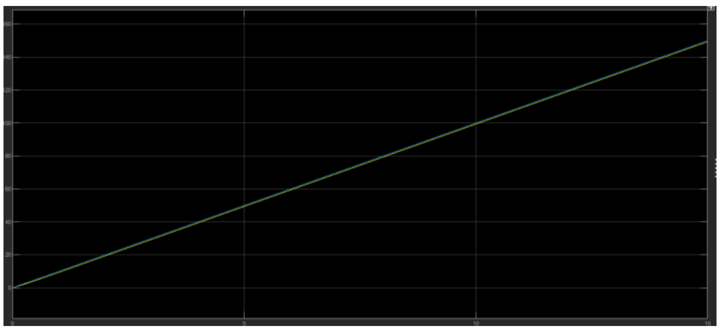
$$k_v = k(z_a)(z_b)(\frac{6869}{490.6})$$

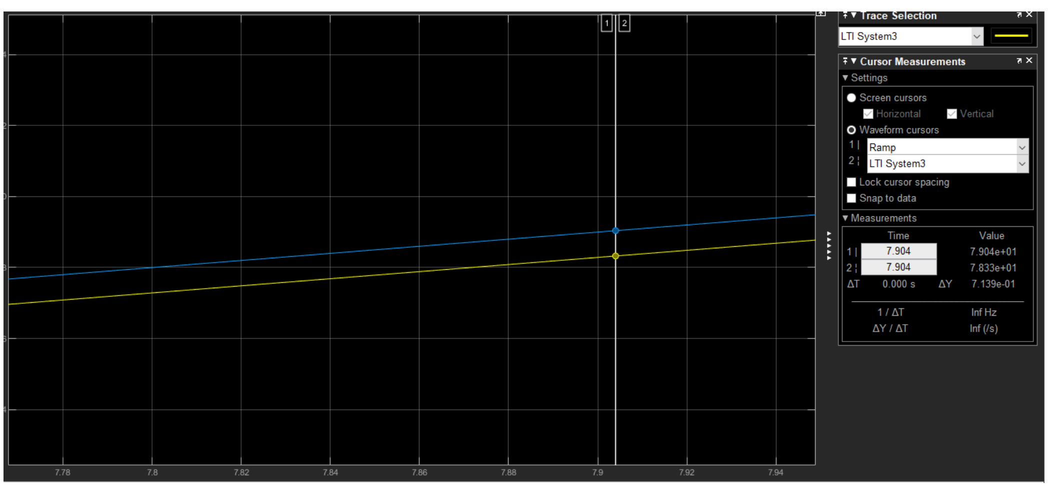
```
>> kv=k*za*zb*6869/490.6

kv =

   14.0012
```

$$e_{ss} = \frac{r(t)}{k_v}$$

```
>> ess=rt/kv

ess =

    0.7142
```







The difference from my system and the references is 0.7139 which is the same as calculated.

System 5.

```
>> R1=Ref_1(2);
>> R2=Ref_2(2);
>> R1

R1 =

    7.1250

>> R2

R2 =

    9.0250
```

1.
```
G1 =

     1.7
  ----------
  66.3 s + 1

Continuous-time transfer function.
```

2.
```
>> G2

G2 =

                4
  ---------------------------
  1.724e04 s^2 + 326.3 s + 1

Continuous-time transfer function.
```

3. **Design a proportional controller for H1 with the assigned reference Ref_1.**

   The steady state error should be less than 10%. Compute and simulate.

   $e_{ss} < 10\%$ $\quad$ $e_{ss} = 0.1$

   $H_{1CL} = \frac{1.7k}{66.3s+1+1.7k}$ $\quad$ $r(t) = R1 = 7.125$

   $y_{ss} = \lim_{s\to 0}[r(t)H_{1CL}(s)] = 7.125\left(\frac{1.7k}{1+1.7k}\right) = \frac{12.1125k}{1+1.7k}$

   $y_{ss} = r(t)$- $e_{ss} = 7.125 - 0.1 = 7.025$

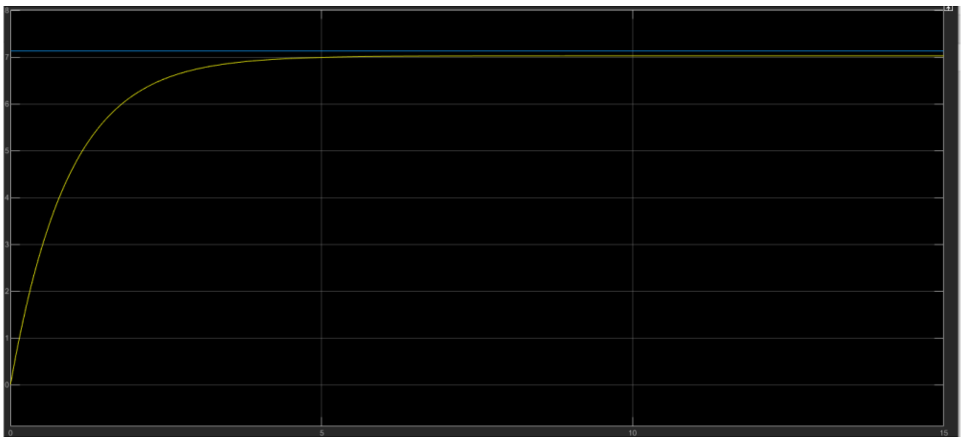
$7.025 = \frac{12.1125k}{1+1.7k}$

Despejando para k:

$7.025(1 + 1.7k) = 12.1125k$

$7.025 = (12.1125 - 7.025 * 1.7)k$

$k = \frac{7.025}{0.17} = 41.3235$



4. **Design a proportional controller for H2 with the assigned reference Ref_2. The steady state error should be less than 10%. Compute and simulate.**

$e_{ss} < 10\% \qquad e_{ss} = 0.1$

$H_{2CL} = \frac{4k}{1.724e04s^2+326.3s+1+4k} \qquad r(t) = R2 = 9.025$

$y_{ss} = \lim_{s \to 0}[r(t)H_{1CL}(s)] = 9.025\left(\frac{4k}{1+4k}\right) = \frac{36.1k}{1+4k}$

$y_{ss} = r(t)\text{-}\ e_{ss} = 9.025 - 0.1 = 8.925$

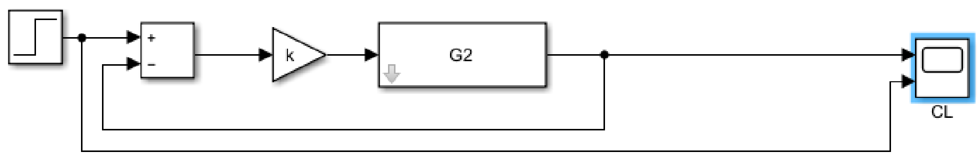
$8.925 = \frac{36.1k}{1+4k}$

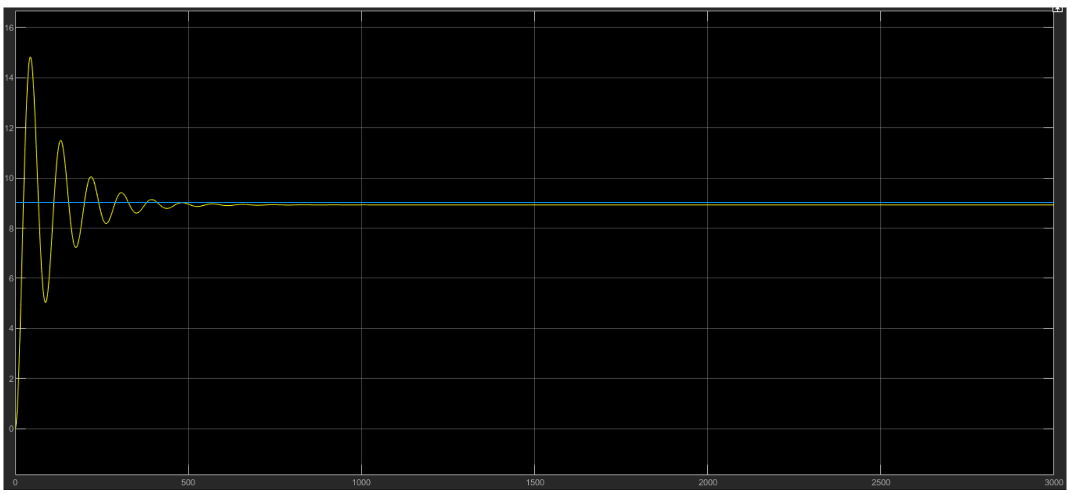
Despejando para k:

$8.925(1 + 4k) = 36.1k$

$8.925 = (36.1 - 8.925 * 4)k$

$k = \frac{8.925}{0.4} = 22.3125$



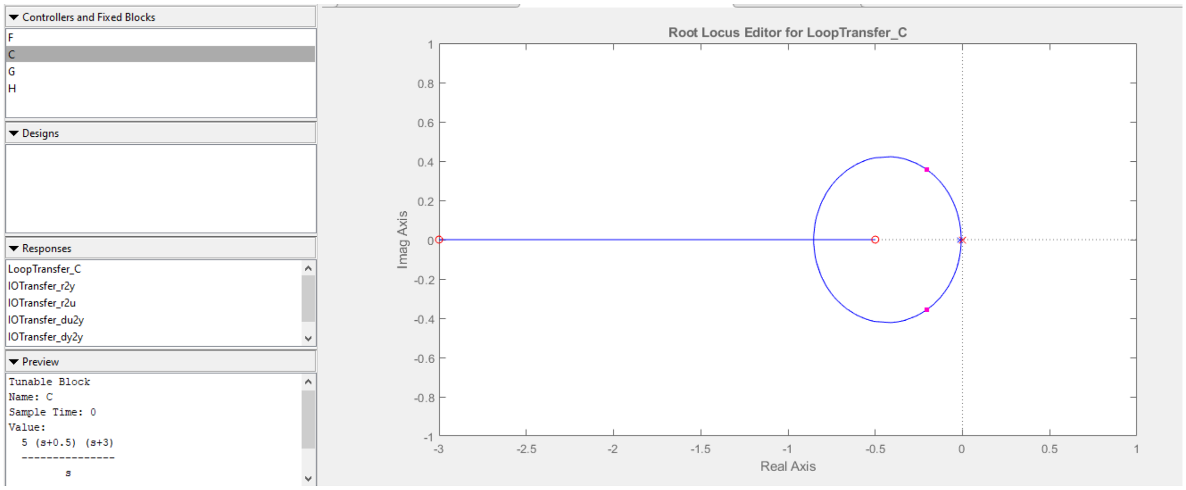


5. Design a PID controller for H1 with the assigned reference Ref_1. Compute and simulate.

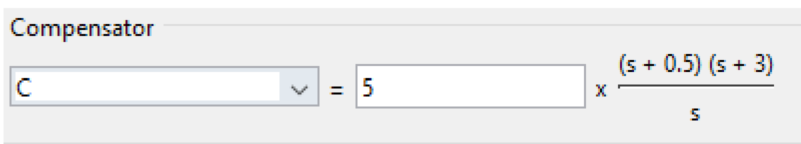
Using the algebraic method.

$r(t) = R1 = 7.125$

Add 1 Pole at zero and adding 2 zeros at -3 and -0.5.



$$\frac{k(s+z_a)(s+z_b)}{s} = \frac{k_d s^2+k_c s+k_i}{s} = \frac{ks^2+k(z_a+z_b)+kz_a z_b}{s}$$



Gain = 5

$z_a = 0.5$ $z_b = 3$

k = 5

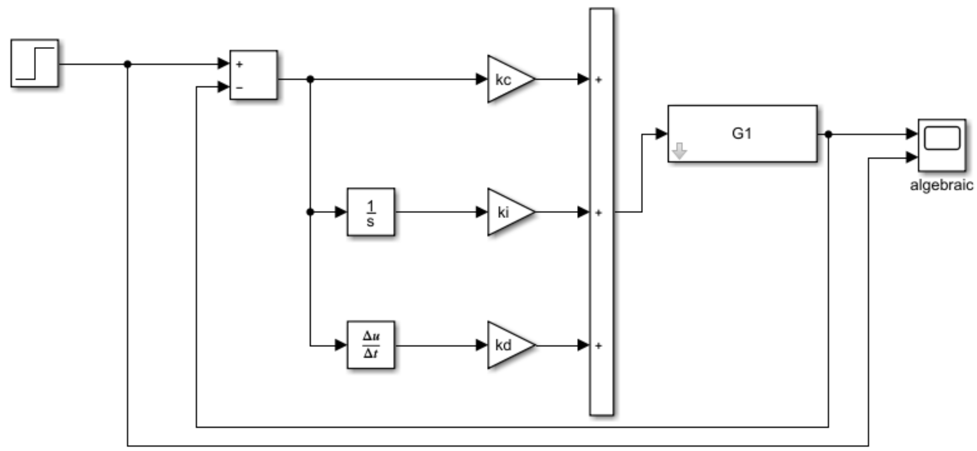
$k_d = k = 5$

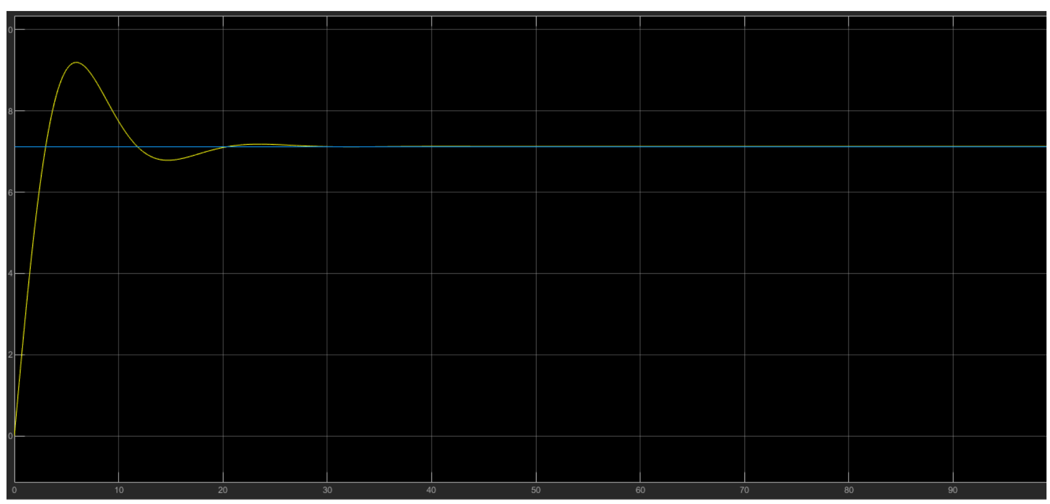
$k_c = k(z_a + z_b)$

```
>> kc=k*(za+zb)

kc =

   17.5000
```

$k_i = k * z_a * z_b$

```
>> ki=k*za*zb

ki =

    7.5000
```


kc
ki
kd
G1
algebraic

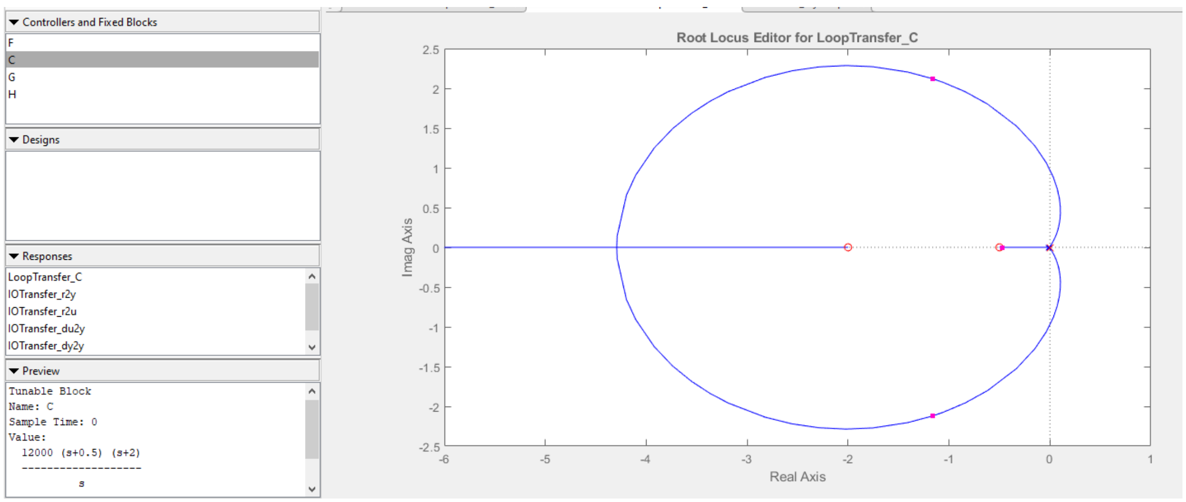


6. **Design a PID controller for H2 with the assigned reference Ref_2. Compute and simulate.**

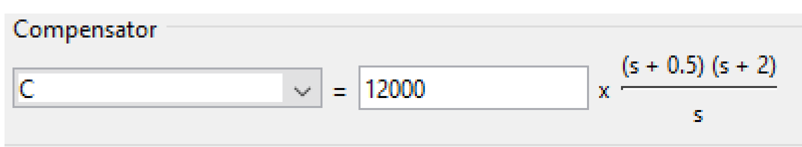
Using the algebraic method.

$r(t) = R2 = 9.025$

Add 1 Pole at zero and adding 2 zeros at -0.5 and 2.



$$\frac{k(s+z_a)(s+z_b)}{s} = \frac{k_d s^2 + k_c s + k_i}{s} = \frac{ks^2 + k(z_a + z_b) + kz_a z_b}{s}$$



Gain = 12 000

$z_a = 0.5 \qquad z_b = 2$

k = 12 000

$k_d = k = 12\ 000$

```
>> kc=k*(za+zb)

kc =

    30000
```

$k_c = k(z_a + z_b)$

```
>> ki=k*za*zb

ki =

       12000
```

$$k_i = k * z_a * z_b$$