

Examen Parcial 2.

①



$$G(s) = \frac{s+28}{s+6}$$

$$G_{cl}(s) = \frac{K(s+28)}{s+6+K(s+28)} = \frac{\dots}{s+16}$$

$$s(1+K) + 6 + K(28) = s + 16$$

$$s + \frac{6+28K}{1+K} = s + 16$$

$$\frac{6+28K}{1+K} = 16$$

$$6+28K = 16 + 16K$$

$$(28-16)K = 16-6$$

$$K = 10/12$$

$$K = 0.833 //$$

② $-4.32 \pm 26.82j$

$$\omega_n = \sqrt{(-4.32)^2 + 26.82^2}$$

$$\omega_n = 27.166 //$$

② $e_{ss} = ?$

$$a=16 \quad b=17 \quad c=17$$

$$p=19 \quad I=6 \quad D=2$$

$$G(s) = \frac{a}{(s+b)(s+c)}$$

Type 1, unit ramp $e_{ss} = \frac{1}{K_v}$

$$C(s) = \frac{Ps + I + Ds^2}{s}$$

$$K_v = \lim_{s \rightarrow 0} sG(s)C(s) = \left(\frac{16}{(17)(17)} \right) (6)$$

$$r(t) = t$$

$$K_v = \frac{96}{289}$$

$$e_{ss} = \frac{289}{96} = 3.010 //$$

④

$$\begin{array}{cc} \times & \times \\ -3a & -a \\ -33 & -11 \end{array}$$

$$a=11 \\ N(s)=10$$

poles CL $s = -2(11) \pm 10j$

$$G(s) = \frac{N(s)}{(s+3a)(s+a)} = \frac{10}{(s+33)(s+11)} = \frac{10}{s^2 + 44s + 363}$$

$$G_{CL}(s) = \frac{10K}{s^2 + 44s + 363 + 10K} \quad s = -22 \pm 10j \\ (s+22)^2 - (10j)^2 = s^2 + 44s + 484 + 100$$

$$\cancel{s^2 + 44s + 363} + 10K = \cancel{s^2 + 44s + 363} + 221$$

$$10K = 221$$

$$K = 22.1 //$$

⑤ $a=9.92 \quad b=24.96$

$$-9.92 \pm 24.96j$$

$$t_p = ?$$

$$t_p = \pi / \omega_d$$

$$\omega_d = \omega_n \sqrt{1 - \zeta^2}$$

$$\omega_n = \sqrt{(-9.92)^2 + (24.96)^2} = 26.859$$

$$\theta = \tan^{-1} \left(\frac{24.96}{-9.92} \right) = -1.1925$$

$$\zeta = \cos \theta = 0.3693$$

$$\omega_d = 24.96$$

$$t_p = 0.126 //$$

⑥ $e_{ss} = ?$

$$G(s) = \frac{a}{s(s+b)(s+c)}$$

Type 2, unit ramp

$$C(s) = \frac{Ps + I + Ds^2}{s}$$

$$e_{ss} = 0 //$$

$$r(t) = t$$

⑦

$\begin{array}{c} \times \\ -3a \\ -18 \end{array} \quad \begin{array}{c} \times \\ -a \\ -6 \end{array}$

$N(s) = 20$ poles in CL $s = -4 \pm 10j$

$$G(s) = \frac{N(s)}{(s+3a)(s+a)} = \frac{20}{(s+18)(s+6)} = \frac{20}{s^2 + 24s + 108}$$

$$s = -4 \pm 10j \quad G_{cl}(s) = \frac{20K}{s^2 + 24s + 108 + 20K}$$

$$(s+4)^2 - (10j)^2 = s^2 + 8s + 16 + 100 = s^2 + 24s + 108 + 20K$$

$$-16s + 8 = 20K \quad \times$$

Not such gain

⑧

$a=11 \quad b=36$ Is it possible to design a PID using the damped oscillations method?

$-11 \pm 36j$

$\zeta > 0.21 \quad \zeta = \cos(\tan^{-1}(\frac{36}{-11})) = 0.29 \checkmark \checkmark$

TRUE!!

⑨ $a=7 \quad b=14 \quad N(s)=55$

$C(s) = \frac{5(s+z)}{s+p}$ $z=20 \quad K_p=?$
 $p=48 \quad e_{ss}=7\%$

$-7 \pm 14j \quad \omega_n = \sqrt{7^2 + 14^2} = 15.62 \quad \zeta_{ss} = 0.07$

$\zeta = \cos(\tan^{-1}(\frac{14}{-7})) = \frac{15.62}{5} \quad \zeta_{ss} = \frac{1}{1+K_p}$

$G(s) = \frac{N(s)}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{55}{s^2 + 14s + 245}$

$C(s) = \frac{5(s+7)}{s+p} = \frac{5(s+20)}{s+48}$

$0.07 = \frac{1}{1 + (\frac{55}{245})(\frac{5(20)}{48})K_p} \quad 1 + \frac{275}{588}K_p = \frac{1}{0.07}$

$K_p = \frac{588}{275}(13.286) \quad K_p = 28.407 \quad \frac{275}{588}K_p = 14.286 - 1$

⑩ $e_{ss} = ?$

$$G(s) = \frac{a}{s(s+b)(s+c)}$$

$$C(s) = K$$

$$r(t) = 1$$

Type = 1 unit step

$$e_{ss} = 0 //$$

⑪ $G(s) = \frac{s-8}{s^6+7s^5+4s^4+5s^3+9s^2+15s+22}$

$$K = 7.45$$

$$G_{cl}(s) = \frac{K(s-8)}{s^6+7s^5+4s^4+5s^3+9s^2+15s+22+K(s-8)}$$

$$\begin{aligned} s^6+7s^5+4s^4+5s^3+9s^2+15s+22 &= s^6+7s^5+4s^4+5s^3+9s^2+22.45s \\ &\quad +7.45s-59.6 \end{aligned}$$

$$-37.6$$

*matlab unstable

⑫ $e_{ss} = ?$ $a=15$ $b=10$ $c=18$ $K=13$

$$G(s) = \frac{a}{s(s+b)(s+c)}$$

Type 1, unit ramp

$$e_{ss} = 1/K_v$$

$$C(s) = K$$

$$r(t) = t$$

$$K_v = \lim_{s \rightarrow 0} s G(s) C(s) = \left(\frac{15}{10(18)} \right) (13)$$

$$K_v = 13/12 \quad e_{ss} = \frac{12}{13} \approx 0.923 //$$

⑬ $a=14$ $b=26$

Damped oscillations method?

$$-14 \pm 26j$$

$$\zeta > 0.21$$

$$\zeta = \cos(\tan^{-1}(\frac{26}{-14})) = 0.474$$

TRUE //

14) $a=4.24$ $b=25.15$
 $\omega_d = ?$

$-4.24 \pm 25.15j$

$\omega_d = \omega_n \sqrt{1 - \zeta^2}$

$\omega_d = 25.15 //$

$\omega_n = \sqrt{4.24^2 + 25.15^2}$

$\omega_n = 25.505$

$\zeta = \cos(\tan^{-1}(\frac{25.15}{-4.24})) = 0.166$

15) $G(s) = \frac{a}{(s+b)(s+c)}$

$C(s) = \frac{K(s+z_1)(s+z_2)}{(s+p_1)(s+p_2)}$

$r(t) = 1$

$a=15$ $b=10$ $c=15$ $z_1=39$ $z_2=0.017$
 $p_1=57$ $p_2=0.004$ $K=13$

Type 0, unit step

$e_{ss} = \frac{1}{1+K_p}$

$K_p = \lim_{s \rightarrow 0} G(s)C(s) = \left(\frac{15}{10(15)}\right) \left(\frac{13(39)(0.017)}{57(0.004)}\right)$

$K_p = 3.7802$

$e_{ss} = \frac{1}{1+K_p} = 0.209 //$

16) $-a \pm bj$ $a=4.62$ $b=27.06$

$t_{ss} = ?$ 2%

$t_{ss} = \frac{4}{\omega_n \zeta}$

$-4.62 \pm 27.06j$

$t_{ss} = 0.866 //$

$\omega_n = \sqrt{(-4.62)^2 + 27.06^2} = 27.4516$

$\zeta = \cos(\tan^{-1}(\frac{27.06}{-4.62})) = 0.168$

$$(17) \quad G(s) = \frac{a}{(s+b)(s+c)}$$

$$C(s) = \frac{Ps + I + Ds^2}{s}$$

$$r(t) = 1$$

Type 1, unit step

$$e_{ss} = 0 //$$

$$(18) \quad G(s) = \frac{a}{s(s+b)(s+c)}$$

$$C(s) = \frac{K(s+z)}{s+p}$$

$$r(t) = t$$

$$e_{ss} = 4.913 //$$

Type 1, unit ramp

$$a=10 \quad b=19 \quad c=16 \quad K=11$$

$$z=27 \quad p=48$$

$$e_{ss} = \frac{1}{K_v}$$

$$K_v = \lim_{s \rightarrow 0} sG(s)C(s)$$

$$= \left(\frac{10}{19(16)} \right) \left(\frac{11(27)}{48} \right)$$

$$K_v = \frac{495}{2432}$$

$$(19) \quad a=7 \quad b=7 \quad N(s)=60$$

$$C(s) = \frac{7(s+z)}{s+p}$$

$$z=15$$

$$p=47$$

$$e_{ss} = ?$$

$$r(t) = 1$$

$$e_{ss} = \frac{1}{1+K_p}$$

$$G(s) = \frac{N(s)}{s^2 + 2\omega_n \zeta s + \omega_n^2}$$

$$\omega_n = \sqrt{(-7)^2 + 7^2} = 7\sqrt{2}$$

$$\zeta = \cos(\tan^{-1}(\frac{-7}{7})) = \frac{\sqrt{2}}{2}$$

$$e_{ss} = \frac{329}{779} \approx 0.422 //$$

$$G(s) = \frac{60}{s^2 + 14s + 98}$$

$$C(s) = \frac{7(s+15)}{s+47}$$

$$K_p = \lim_{s \rightarrow 0} G(s)C(s)$$

$$K_p = \left(\frac{60}{98} \right) \left(\frac{7(15)}{47} \right)$$

$$K_p = 450/329$$

21

$$G(s) = \frac{s-8}{s^6 + 7s^5 + 4s^4 + 5s^3 + 9s^2 + 15s + 22}$$

$$C(s) = \frac{4(s+z)}{(s+p)} \quad z=21 \quad p=14$$

$$C(s) = \frac{4(s+21)}{(s+14)} = \frac{4s+84}{s+14}$$

$$s^6 + 7s^5 + 4s^4 + 5s^3 + 9s^2 + 19s + 106 = 0$$

*matlab roots

unstable

22

$$G(s) = \frac{a}{(s+b)(s+c)} \quad C(s) = K \quad r(t) = t$$

$$a=10 \quad b=17 \quad c=10 \quad K=10$$

$e_{ss} = ?$ Type 0, unit ramp ($e_{ss} = \infty$)

23

$$a=3.5 \quad b=17.31$$

$$-a \pm bj$$

$$\zeta = ?$$

$$\zeta = \cos\left(\tan^{-1}\left(\frac{17.31}{-3.5}\right)\right) = 0.198 //$$

(24) $r(t) = 1$

$$G(s) = \frac{a}{s(s+b)(s+c)}$$

Unit step, Type 1

$$C(s) = \frac{K(s+z_1)(s+z_2)}{(s+p_1)(s+p_2)}$$

$$e_{ss} = 0 //$$

(25)

$$G(s) = \frac{a}{(s+b)(s+c)}$$

Type 0, unit step

$$C(s) = K$$

$$e_{ss} = \frac{1}{1+K_p}$$

$$r(t) = 1$$

$$a=10 \quad b=16 \quad c=14 \quad K=14$$

$$e_{ss} = 0.13 = 0.615 //$$

$$K_p = \lim_{s \rightarrow 0} G(s)C(s) = \left(\frac{10}{16(14)} \right) (14)$$

$$K_p = \frac{10}{16}$$

(26)

$$G(s) = \frac{a}{(s+b)(s+c)}$$

$$C(s) = \frac{K(s+z_1)(s+z_2)}{(s+p_1)(s+p_2)} \quad r(t) = t$$

Unit ramp, type 0 $e_{ss} = \infty$

(27)

$$G(s) = \frac{a}{(s+b)(s+c)}$$

$$C(s) = \frac{K(s+z)}{s+p}$$

$$r(t) = 1$$

$$a=18 \quad b=11 \quad c=12 \quad z=25 \quad p=71 \quad K=14$$

Unit step Type 0

$$e_{ss} = \frac{1}{1+K_p} = 0.598 //$$

$$K_p = \lim_{s \rightarrow 0} G(s)C(s) = \left(\frac{18}{11(12)} \right) \left(\frac{14(25)}{71} \right) = 0.672$$

(28) $e_{ss} = ?$

$$G(s) = \frac{a}{s(s+b)(s+c)}$$

$$C(s) = \frac{K(s+z)}{s+p}$$

$$r(t) = 1$$

Unit step Type 1

$$e_{ss} = 0 //$$

(29) Type 2, unit step

$$e_{ss} = 0 //$$

(30) $G(s) = \frac{a}{s(s+b)(s+c)}$

Type 1, unit ramp

$$C(s) = \frac{K(s+z_1)(s+z_2)}{(s+p_1)(s+p_2)}$$

$$e_{ss} = \frac{1}{K_v}$$

$$r(t) = t$$

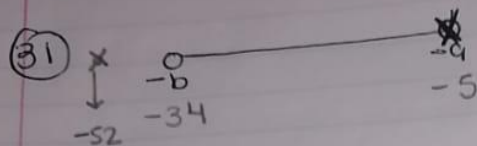
$$a = 11 \quad b = 19 \quad c = 18$$

$$K = 10 \quad z_1 = 29 \quad z_2 = 0.019$$

$$p_1 = 48 \quad p_2 = 0.001$$

$$K_v = \lim_{s \rightarrow 0} s G(s) C(s) = \left(\frac{11}{19(18)} \right) \left(\frac{10(29)(0.019)}{48(0.001)} \right)$$

$$K_v = 3.692 \quad e_{ss} = 0.271 //$$



$$a = 5 \quad b = 34$$

$$s = -52$$

(32)

$$G(s) = \frac{a}{(s+b)(s+c)}$$

Unit ramp
type 0

$$C(s) = \frac{K(s+z)}{s+p}$$

$$e_{ss} = \infty //$$

$$r(t) = t$$