Tecnológico de Monterrey
Escuela de Ingeniería y Ciencias

Final Exam

Control Engineering

Enrique Aguayo Lara

Perla Vanessa Jaime Gaytán

ITE

A00344428

Instituto Tecnológico de Estudios Superiores de Monterrey Campus Guadalajara.

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a = 10

b = 5

c = 8

1. (15) For the following system, design a state feedback control such that the following system becomes stable. Obtain any state space representation for the system in order to design the controller.

$$G1(s) = \frac{4s + 45}{(s + 2a)(s - b)(s - 3c)}$$

Make sure that the initial conditions of the system are different than 0 to secure the transient on the control.
Minimum expected plots: Output in open loop, Open in closed loop, input and state variables in closed loop.
Minimum expected screen shots: State space model, Simulink Model, Command to obtain K and its value.

Problem 1:

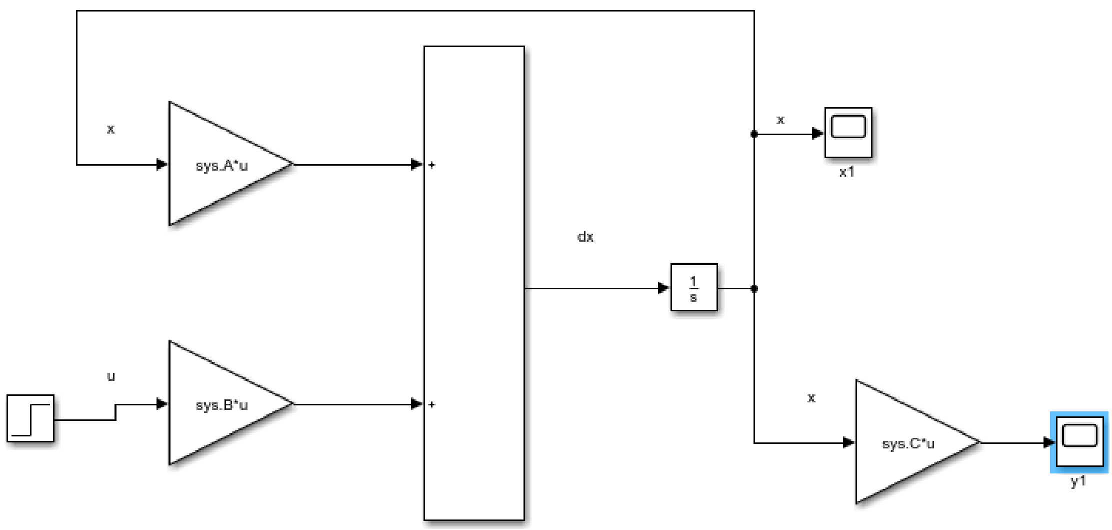
$$G1(s) = \frac{2s + 54}{(s + 3a)(s - b)(s - 2c)}$$

$$(s^2 + s(3a - b) - 3ab)(s - 2c) = s^3 - 2cs^2 + s^2(3a - b) - 2sc(3a - b) - 3abs - 6abc$$
$$= s^3 + s^2(-2c + 3a + b) + s(-6ac + 2bc - 3ab) - 6abc =$$
$$s^3 + s^2(19) + s(-550) - 2400$$

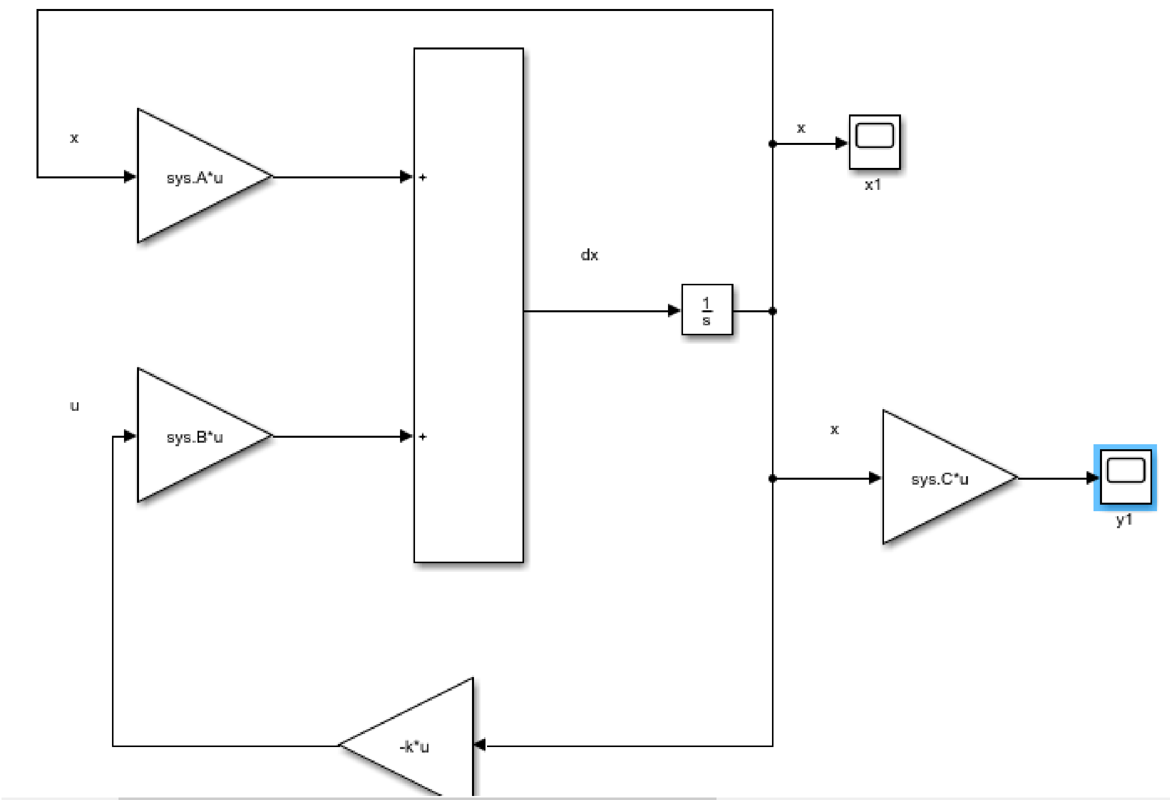
```
>> G=tf([2 54],[1 19 -550 -2400])

G =

          2 s + 54
  ---------------------------
  s^3 + 19 s^2 - 550 s - 2400

Continuous-time transfer function.

>> sys = ss(G)

sys =

  A =
          x1      x2      x3
   x1    -19   17.19   9.375
   x2     32       0       0
   x3      0       8       0

  B =
        u1
   x1  0.5
   x2    0
   x3    0
```

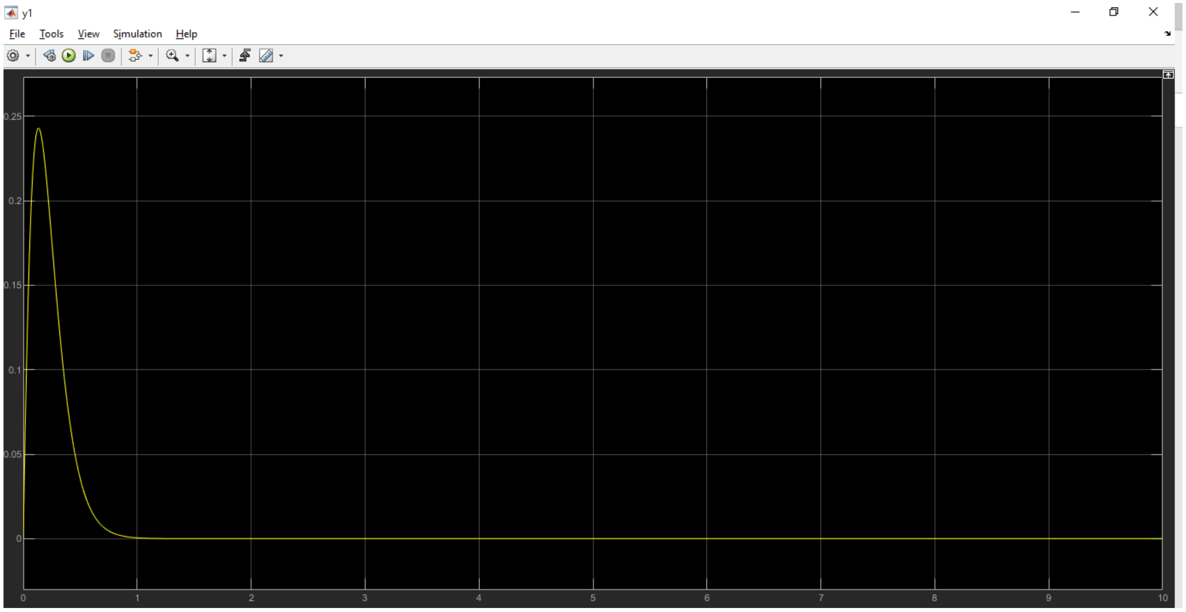
```
>> cont = ctrb(sys.A,sys.B);
>> rank(cont)

ans =

     3
```

It is controlable.



```
>> cont = ctrb(sys.A,sys.B);
>> rank(cont)

ans =

     3

>> k = acker(sys.A,sys.B, [-10 -11 -13])

k =

   30.0000   58.3125   29.9219
```




y1
File Tools View Simulation Help
0.25
0.2
0.15
0.1
0.05
0
0
1
2
3
4
5
6
7
8
9
10

2. (25) For the following system, design a control action u(t)=-K x(t)+v to follow a constant reference r(t)=2. Obtain any state space representation for the system

$$G2(s) = \frac{2s - 27}{(s - a/2)(s + b)(s - 2c)}$$

Minimum expected plots: Output in open loop, Output with reference comparison in closed loop, input and state variables in closed loop.
Minimum expected screen shots: State space model, Simulink Model, Command to obtain K and its value, computation of the value for v.

Make sure that the initial conditions of the observer are different than the ones of the system. If the plot of the plant states and the observed states is not included, the problem will not be graded.

Problem 2:

$$G2(s) = \frac{4s - 37}{(s - a/3)(s + b/2)(s - 2c)}$$

$$\left(s^2 - \frac{sa}{3} + \frac{sb}{2} - \frac{ab}{6}\right)(s - 2c) = \left(s^2 + s\left(-\frac{a}{3} + \frac{b}{2}\right) - \frac{ab}{6}\right)(s - 2c)$$

$$\left(s^3 + s^2\left(-\frac{a}{3} + \frac{b}{2}\right) - s\frac{ab}{6} - 2cs^2 - 2cs\left(-\frac{a}{3} + \frac{b}{2}\right) + \frac{abc}{3}\right) =$$

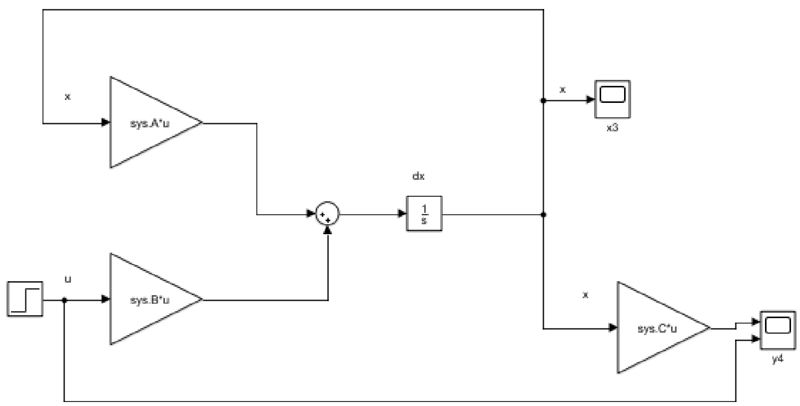
$$\left(s^3 + s^2\left(-\frac{a}{3} + \frac{b}{2} - 2c\right) + s\left(-\frac{ab}{6} - \frac{ac}{3} + \frac{bc}{2}\right) + \frac{abc}{3}\right) =$$

$$(s^3 + s^2(-16.833) + s(-15) + 133.333) =$$

```
K>> G = tf ([4 -37],[1 -16.833 -15 133.333])

G =

              4 s - 37
  ------------------------------
  s^3 - 16.83 s^2 - 15 s + 133.3

Continuous-time transfer function.

K>> sys = ss(G)

sys =

  A =
            x1      x2      x3
   x1    16.83    3.75  -4.167
   x2        4       0       0
   x3        0       8       0

  B =
       u1
   x1   2
```

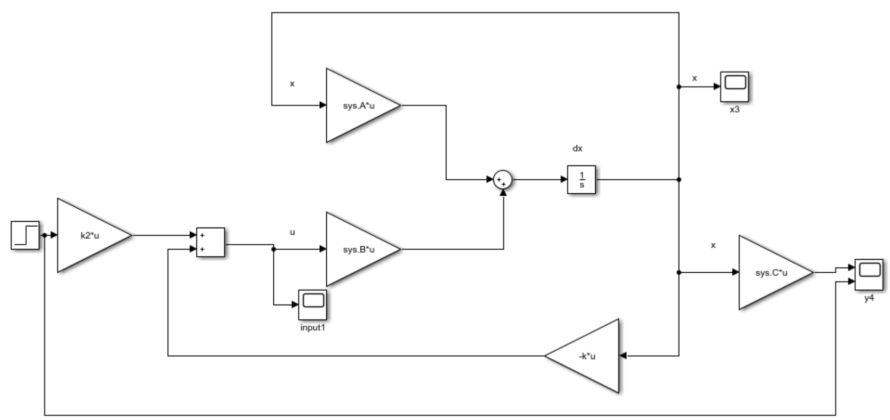
```
K>> cont = ctrb(sys.A,sys.B);
K>> rank(cont)

ans =

     3
```

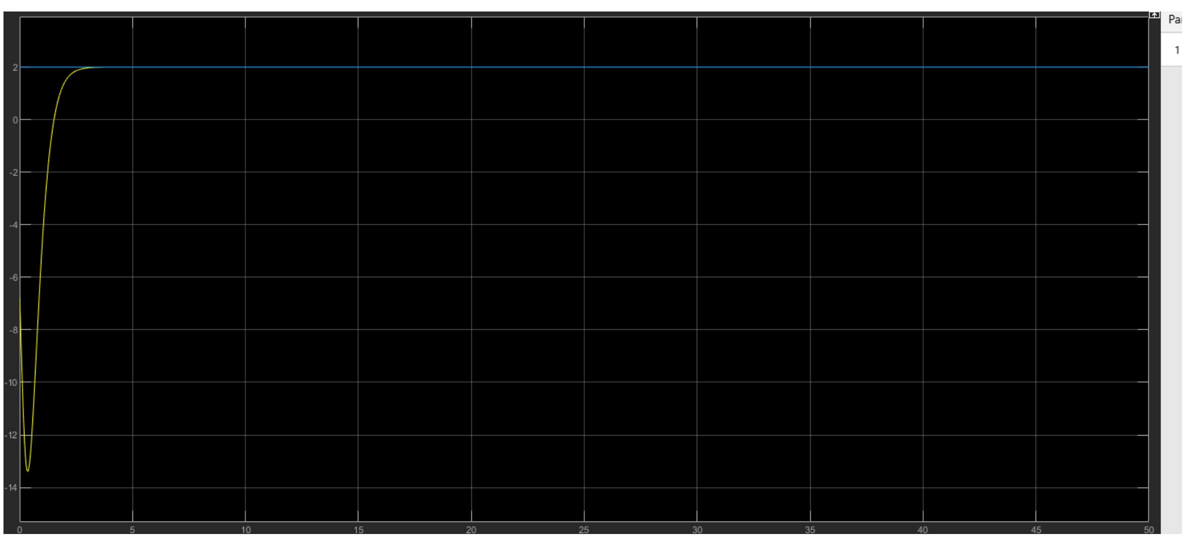
It is controlable




```
>> k = acker(sys.A,sys.B, [-3 -4 -5])

k =

   14.4165    7.7500   -1.1458

>> A2 = sys.a-sys.b*k

A2 =

  -12.0000  -11.7500   -1.8750
    4.0000         0         0

>> systemCL =ss (A2 , sys.B,sys.C,0)

systemCL =

  A =
            x1      x2      x3
   x1      -12  -11.75  -1.875
   x2        4       0       0
   x3        0       8       0

  B =
       u1
   x1   2
   x2   0
   x3   0

  C =
            x1       x2       x3
   y1        0      0.5  -0.5781

  D =
       u1
   y1   0
```

```
>> tf(systemCL)

ans =

          4 s - 37
  ------------------------
  s^3 + 12 s^2 + 47 s + 60

Continuous-time transfer function.

>> k2 = 60/-37

k2 =

   -1.6216
```





3. (5p) For the following system, design a control action u(t)=-K o(t)+v to follow a constant reference **r(t)=3**.
   Assume that the state variables are not available and o(t) are the observer's state variables.
   The poles in closed loop should be in [-4.5 -7.3 -6.2].
   The input to the system should be **bounded** and limited to a maximum value of 3 units.
   The value for variable x_2 should never be negative and the maximum value that x_3 can have is 2.

$$A = \begin{bmatrix} a+c & 0 & 0 \\ -1 & a & 3 \\ 2 & 0 & b \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 7 \\ 0 \end{bmatrix} \quad C = [2 \quad 3 \quad 0]$$

Minimum expected plots: Output in open loop, Output with reference comparison, input and a plot with the observed and the real state variable for each state variable. Include separate plots for u, x_2 and x_3 to show that the listed transient conditions are met***. Make sure that the initial conditions of the observer are different than the ones of the system.*** If the plot of the plant states and the observed states is not included, the problem will not be graded.

Minimum expected screen shots: State space model, Simulink Model, Value of gain K, Value of gain L, computation of the value for v. Make sure to use the observer's state variables for the state feedback simulation.

Problem 3:

$$A = \begin{bmatrix} a+c & 0 & 0 \\ -1 & a & 3 \\ 2 & 0 & b \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 7 \\ 0 \end{bmatrix} \quad C = [2 \quad 3 \quad 0]$$

4. (30) For the following system, design a control action u(t)=-K o(t)+v to follow a constant reference r(t)=4. Assume that the state variables are not available and o(t) are the observer's state variables. Obtain any state space representation for the system.

$$G2(s) = \frac{43.7}{(s-2a)(s-2b)(s-2c)}$$

Minimum expected plots: Output in open loop, Output with reference comparison, input and a plot with the observed and the real state variable for each state variable.

Make sure that the initial conditions of the observer are different than the ones of the system. If the plot of the plant states and the observed states is not included, the problem will not be graded.

Minimum expected screen shots: State space model, Simulink Model, Command to obtain K and its value, Value of gain L, computation of the value for v. Make sure to use the observer's state variables for the state feedback simulation.

Problem 4:

$$G2(s) = \frac{53.7}{(s-3a)(s-3b)(s-3c)}$$

$$(s^2 + s(-3a-3b) + 9ab)(s-3c) =$$

$$(s^3 - 3s^2c + s^2(-3a-3b) - 3cs(-3a-3b) + 9abs - 27abc) =$$

$$s^3 + s^2(-3c-3a-3b) + s(9ac+9bc+9ab) - 27abc =$$

$$s^3 + s^2(-69) + s(1530) - 10800$$

```
>> G = tf (53.7,[1 -69 1530 10800])

G =

             53.7
  -----------------------------
  s^3 - 69 s^2 + 1530 s + 10800

Continuous-time transfer function.

>> sys = ss(G)
```

```
>> obs =obsv(sys.A,sys.C)

obs =

          0          0     0.4195
          0     6.7125          0
   214.8000          0          0

>> rank(obs)

ans =

     3
```

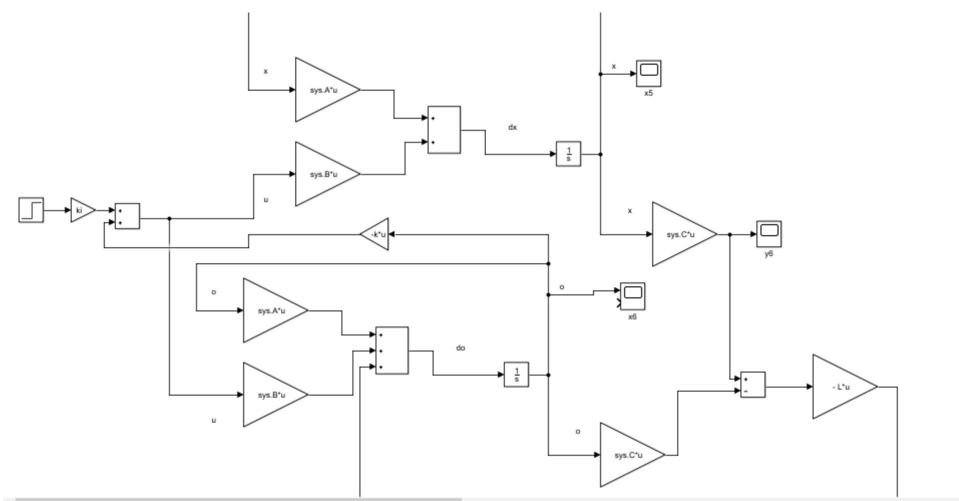
It is observable.

```
>> k = acker(sys.A,sys.B, [-5 -6 -7])

k =

  348.0000 -177.8750  -82.7344

>> roots([1 -69 1530 10800])

ans =

  37.2775 +23.5488i
  37.2775 -23.5488i
  -5.5551 + 0.0000i

>> eig(sys.A)

ans =

  37.2775 +23.5488i
  37.2775 -23.5488i
  -5.5551 + 0.0000i

>> L=acker(sys.A',sys.c',[-19 -20 -21])

L =
```

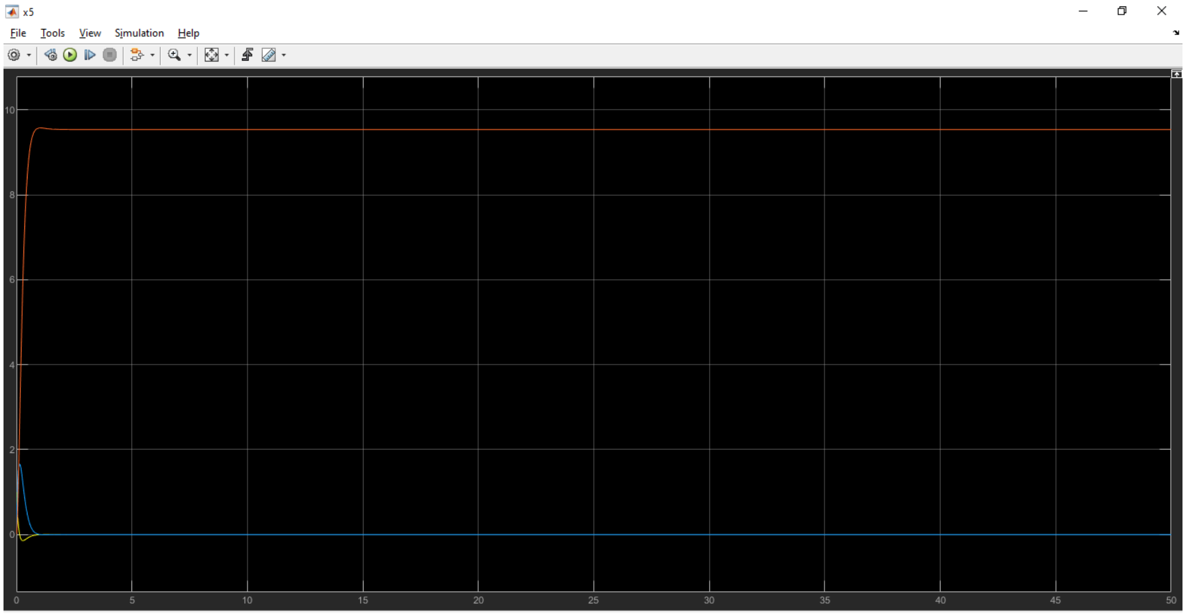
```
>> L=acker(sys.A',sys.c',[-19 -20 -21])'

L =

   1.0e+03 *

    1.8209
    1.2767
    0.3075
```

```
k =

  348.0000 -177.8750  -82.7344

>> A2 = sys.A-sys.B*k

A2 =

  -18.0000   -3.3438   -0.4102
   32.0000         0         0
         0   16.0000         0

>> systemCL=ss(A2, sys.b,sys.c,0)
```

```
>> tf(systemCL)

ans =

              53.7
  ----------------------------
  s^3 + 18 s^2 + 107 s + 210

Continuous-time transfer function.

>> ki=210/53.7

ki =

    3.9106
```



x=


x5
File Tools View Simulation Help
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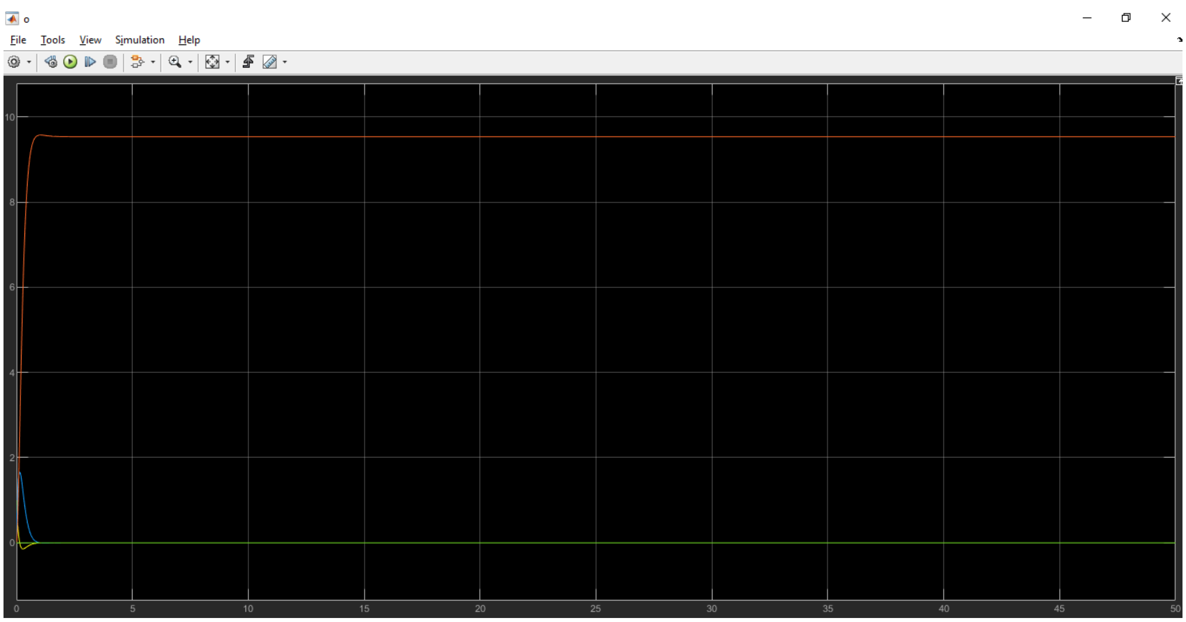
35
40
45
50

o=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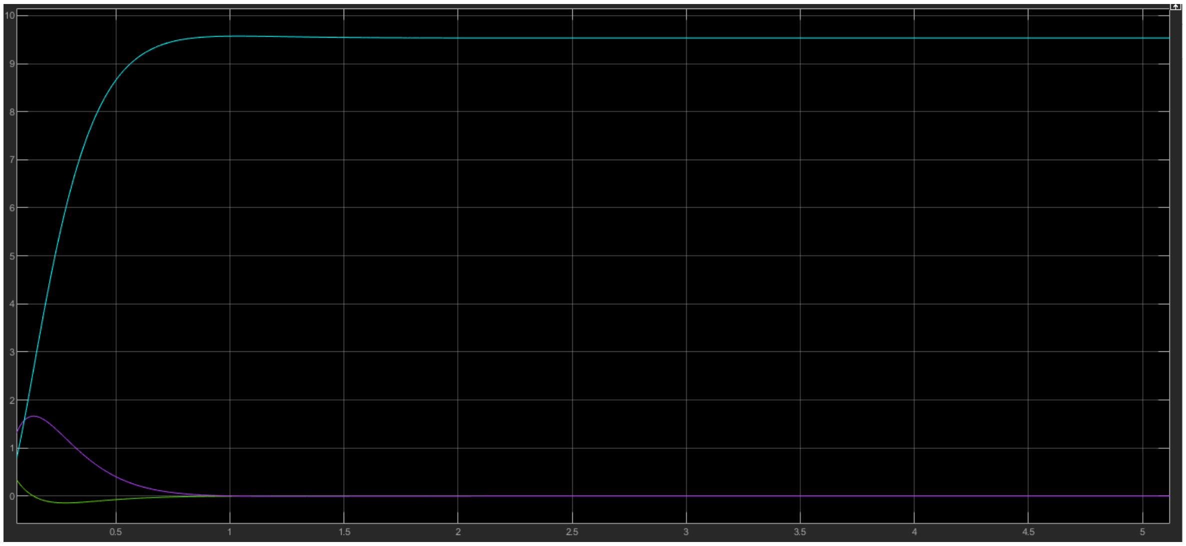
x and o



5. (25) For the following system, design **independently** a PID controller and a Compensator to follow a Reference r(t)=7.
   The maximum error allowed in the compensator is 5%.
   Make sure that the input to the system from the compensator is lower than the input from the PID.
   Settling time is not an issue for this problem.

$$G2(s) = \frac{47\, e^{-0.003s}}{s^2 + as + 150}$$

Minimum expected plots: Output in open loop, Output with reference comparison, PID signal (Input to the plant from the PID), Compensator signal (Input to the plant with the compensator), Input comparison among PID and Compensator.

Minimum expected screen shots: State space model with both controllers, Simulink Model, Design for PID (as many screenshots as needed), Design of the Compensator.

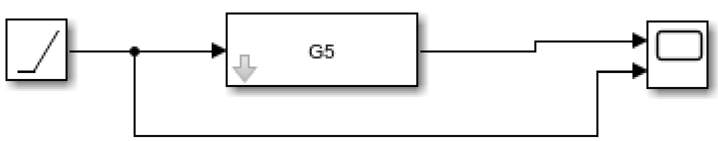
Problem 5:

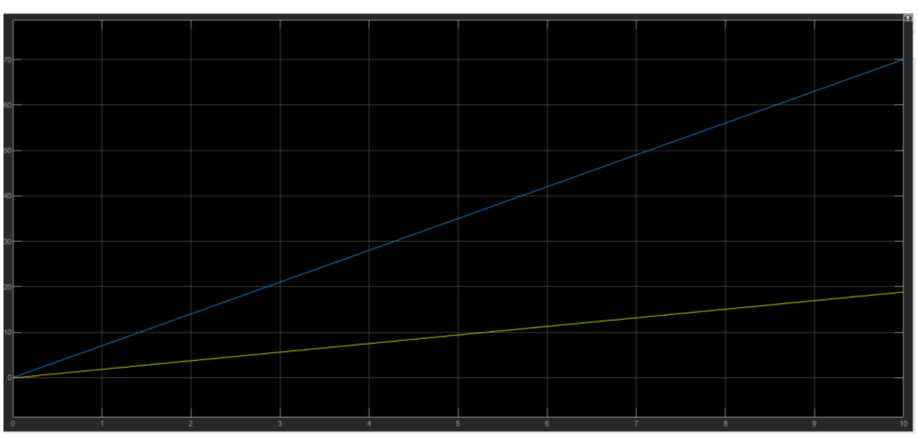
$$G2(s) = \frac{54\, e^{-0.005s}}{s^2 + cs + 200}$$

e^-0.005 = (-0.005s/2 + 1) = -0.0025s +1

54 e^-0.005= 54(-0.0025s +1) = -0.135 s + 54

```
>> G5 = tf([-0.135 54],[1 c 200])

G5 =

   -0.135 s + 54
  ---------------
  s^2 + 8 s + 200

Continuous-time transfer function.
```

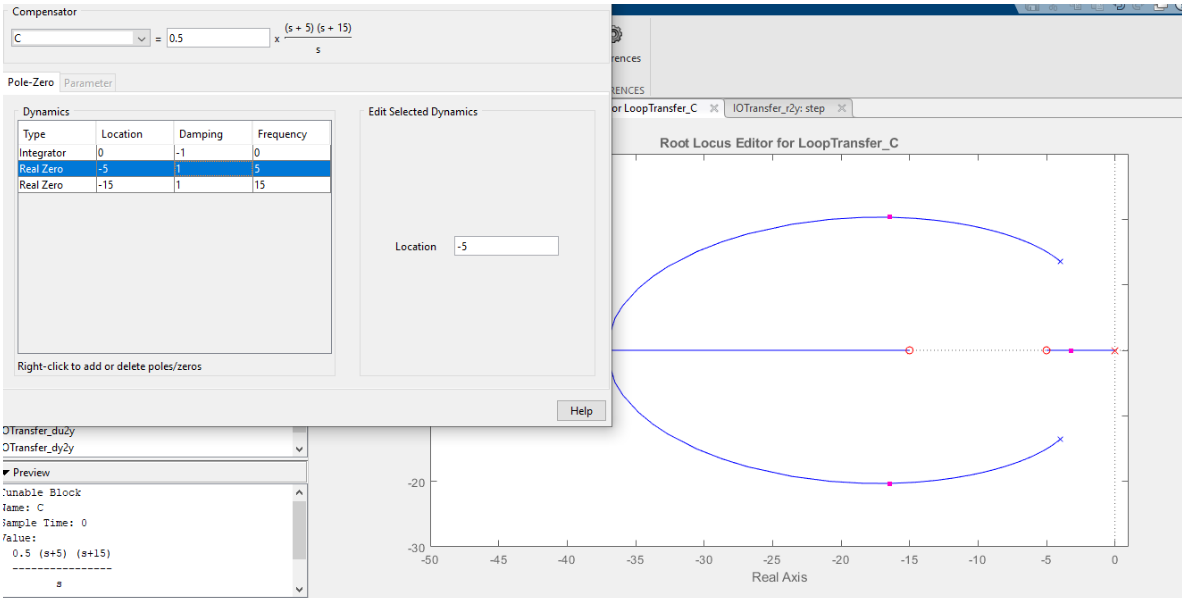




r(t) = 7

PID :

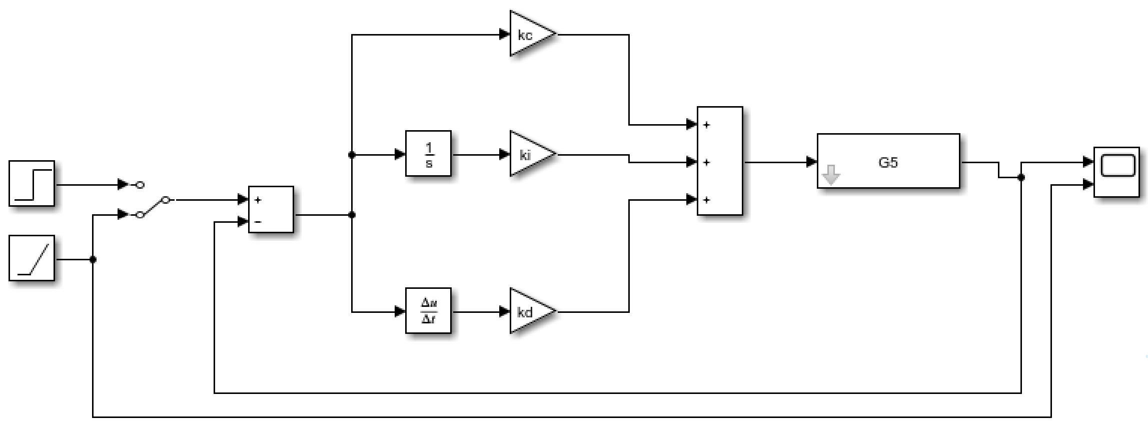
Add 1 Pole at zero and adding 2 zeros at -5 and -15

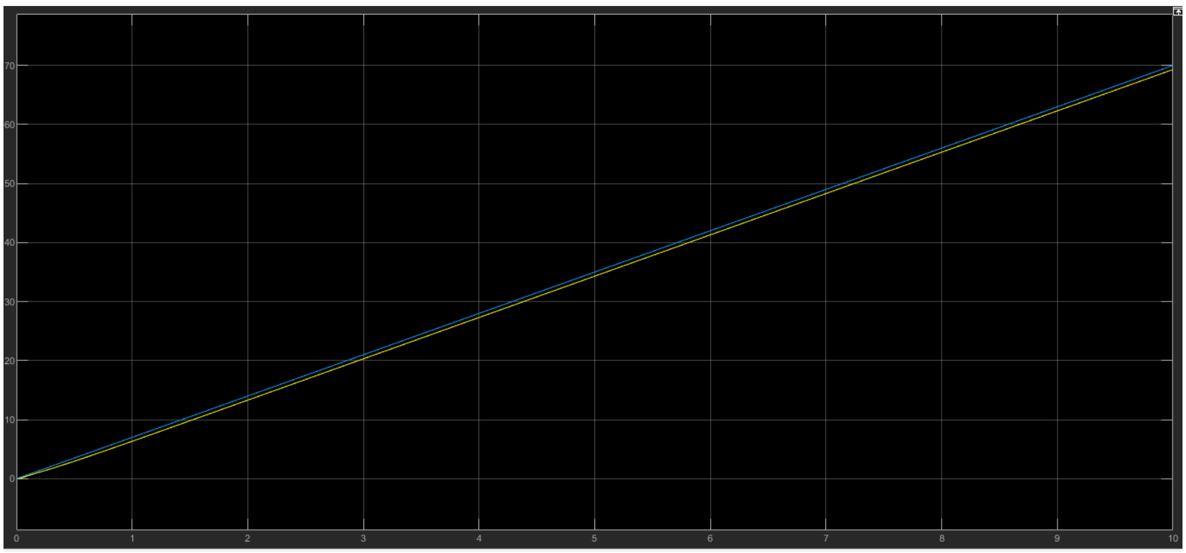


Gain = 0.5

za = 5 zb =15   k = 0.5

```
>> k=0.5

k =

    0.5000

>> kd = k

kd =

    0.5000

>> kc = k(za +zb)
Index exceeds the number of array elements (1).

>> kc = k*(za +zb)

kc =

    10

>> ki = k*za *zb

ki =
```





Compensator:

C(s) = k (s+a)/(s+b) therefore G(s)C(s) is type 0 unit ramp.
Since the error of this problem tend to infinity, it is not possible make a maximum error as 5%, this problem cannot have a compensator.

Extra points:

1. (30) Design a control action to follow a reference r(t)=sin(**3**t) for the following system.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a/2 & b & -c \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

Minimum expected plots: Output in open loop, Output with reference comparison, input and state variables on independent scopes.
Minimum expected screen shots: State space model, Simulink Model, Computation for the controller (as many screenshots as needed).

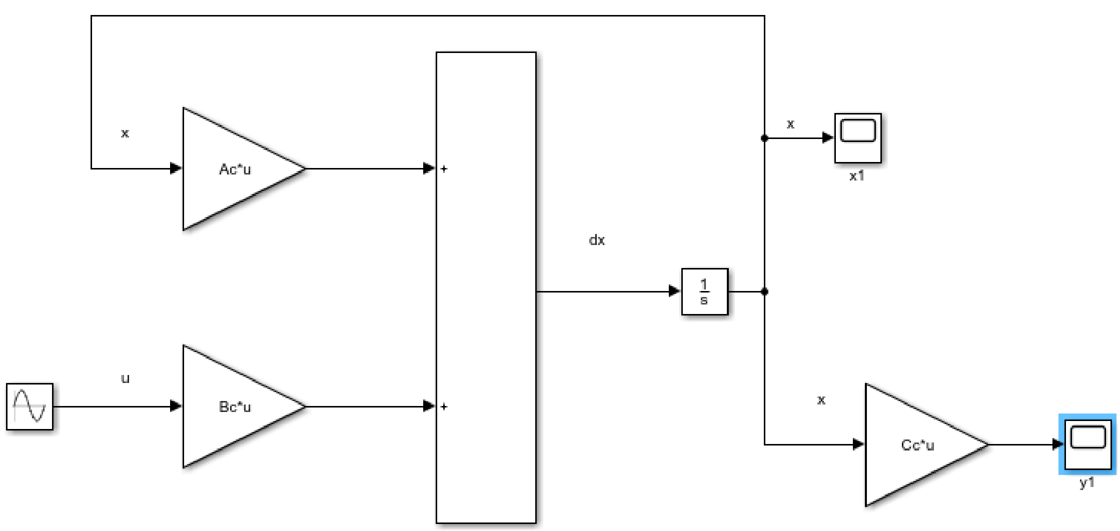
Extra:

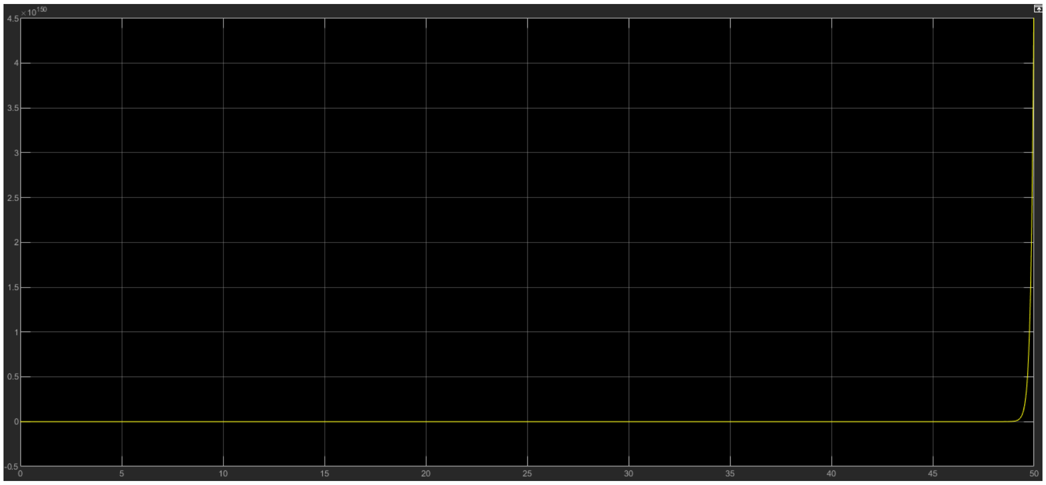
$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a & -b & c \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

```
>> cont = ctrb(Ac,Bc);
>> rank(cont)

ans =

     3
```

It is controllable.

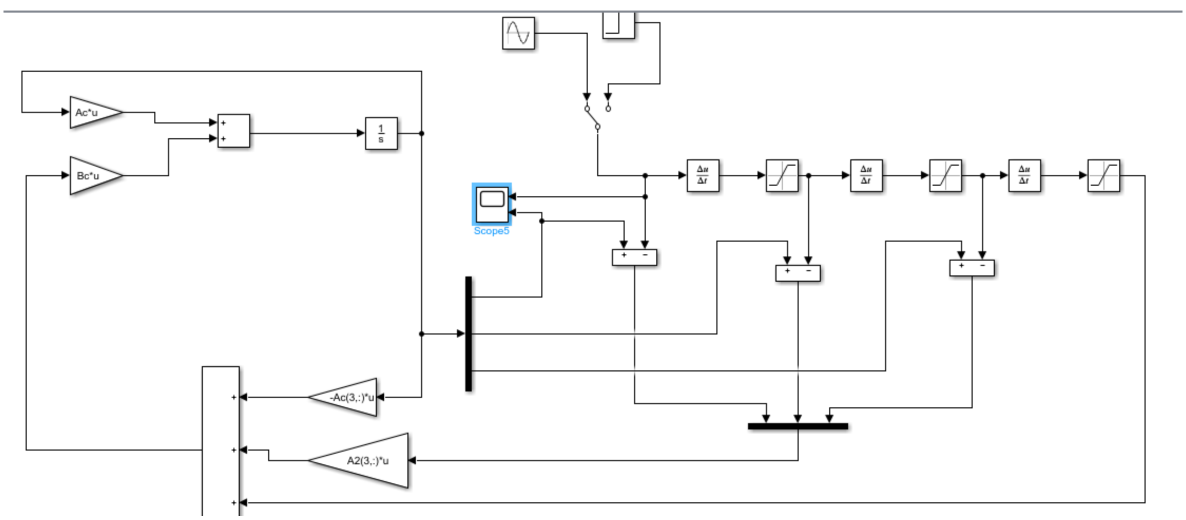
OL:

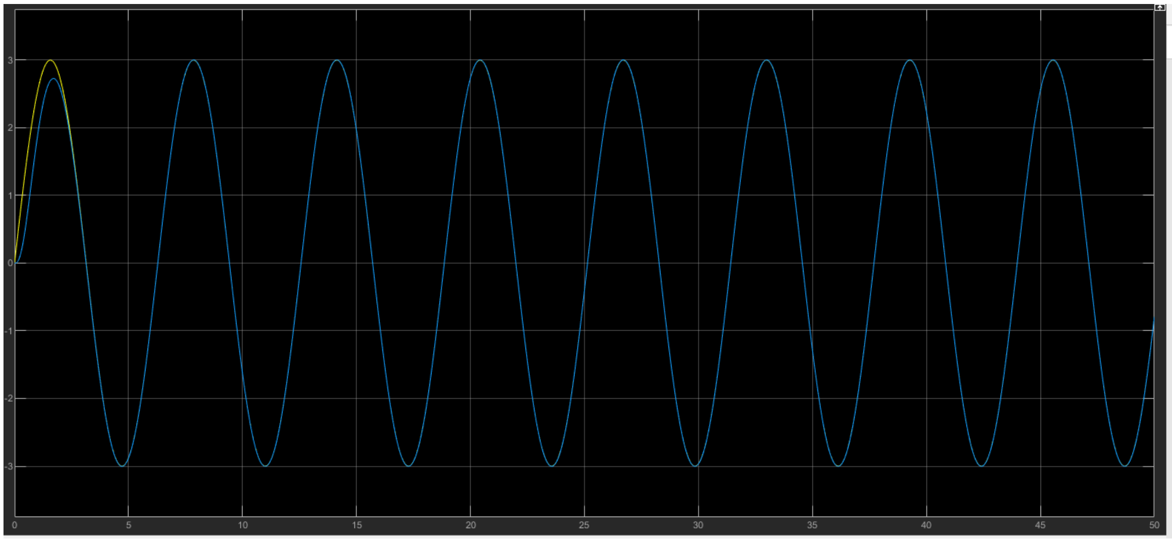




Calculations for controller:

```
>> k = acker(Ac,Bc, [-2 -3 -4])

k =

    14    21    17

>> A2= Ac-Bc*k

A2 =

     0     1     0
     0     0     1
   -24   -26    -9

>>
```





3
2
1
0
-1
-2
-3
0
5
10
15
20
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50