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Control Engineering

Homework 1: Laplace

$$\mathcal{L}\{\sin(\omega t+\phi)\} = \frac{s\cdot\sin(\phi)+\omega\cos(\phi)}{s^2+\omega^2} \qquad \mathcal{L}[e^{\alpha t}] = \frac{1}{s-\alpha}$$

$$\mathcal{L}[t^n e^{-at}] = \frac{n!}{(s+\alpha)^{n+1}}$$

Homework: Laplace Transform

- For the following functions, assume that $f(t)=0$, for all $t<0$. Compute the Laplace transform of the following functions:

① $f(t)=t^2e^{-3t}$

$$\mathcal{L}\{t^2e^{-3t}\} = \frac{2!}{(s+3)^{2+1}} = \frac{2}{(s+3)^3} \,//$$

$$\underline{\mathcal{L}\{t^2e^{-3t}\} = \frac{2}{(s+3)^3}} \,//$$

② $f(t)=\sin(\omega t+\phi),\ t\geq 0$

$$\mathcal{L}\{\sin(\omega t+\phi)\} = \frac{s\cdot\sin(\phi)+\omega\cos(\phi)}{s^2+\omega^2}$$

$$\underline{\mathcal{L}\{\sin(\omega t+\phi)\} = \frac{s\cdot\sin(\phi)+\omega\cos(\phi)}{s^2+\omega^2}} \,//$$

③ $f(t)=e^{-\alpha t}\sin(\omega t+\phi),\ t\geq 0$ si $\mathcal{L}\{f(t)\}=F(s)$

$\mathcal{L}\{e^{\alpha t}f(t)\}=F(s-\alpha)$

$f(t)=\sin(\omega t+\phi)$

$$F(s)=\frac{s\cdot\sin(\phi)+\omega\cos(\phi)}{s^2+\omega^2}$$

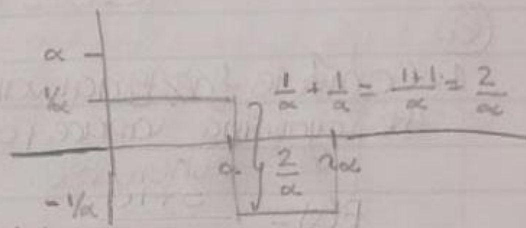
$$F(s+\alpha)^{(-\alpha)}=\frac{(s+\alpha)\sin(\phi)+\omega\cos(\phi)}{(s+\alpha)^2+\omega^2}$$

$$\underline{\mathcal{L}\{e^{-\alpha t}\sin(\omega t+\phi)\} = \frac{(s+\alpha)\sin(\phi)+\omega\cos(\phi)}{(s+\alpha)^2+\omega^2}} \,//$$

$\mathcal{L}[u_a(t)f(t)] = e^{-\alpha s}\mathcal{L}[f(t+\alpha)]$

$\mathcal{L}[u_a(t)] = \frac{1}{s}e^{-as}$

④ $f(t) = \begin{cases} 1/\alpha & 0 \le t < \alpha \\ -1/\alpha & \alpha \le t < 2\alpha \\ 0 & \text{otherwise} \end{cases}$



$f(t) = \frac{1}{\alpha} - \frac{2}{\alpha}u_\alpha(t) + \frac{1}{\alpha}u_{2\alpha}(t)$

$f(t) = \frac{1}{\alpha} - \frac{2}{\alpha}H(t-\alpha) + \frac{1}{\alpha}H(t-2\alpha)$

$\mathcal{L}\{f(t)\} = \mathcal{L}\left[\frac{1}{\alpha} - \frac{2}{\alpha}u_\alpha(t) + \frac{1}{\alpha}u_{2\alpha}(t)\right]$

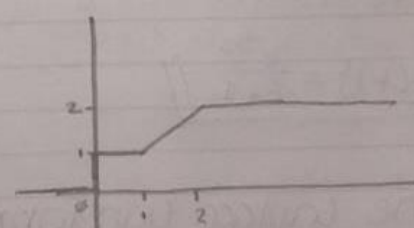
$= \mathcal{L}\{\frac{1}{\alpha}\} - \frac{2}{\alpha}\mathcal{L}\{u_\alpha(t)\} + \frac{1}{\alpha}\{u_{2\alpha}(t)\}$

$F(s) = \frac{1}{\alpha s} - \frac{2}{\alpha}\left[\frac{1}{s}e^{-\alpha s}\right] + \frac{1}{\alpha}\left[\frac{1}{s}e^{-2\alpha s}\right]$

$F(s) = \frac{1}{\alpha s} - \frac{2e^{-\alpha s}}{\alpha s} + \frac{e^{-2\alpha s}}{\alpha s}$ //

$F(s) = \frac{1}{\alpha s}\left[1 - 2e^{-\alpha s} + e^{-2\alpha s}\right]$ //

⑤ $f(t) = \begin{cases} 1 & 0 \le t < 1 \\ t & 1 \le t < 2 \\ 2 & t \geq 2 \\ 0 & \text{otherwise} \end{cases}$



$f(t) = 1 + t(u_1(t) - u_2(t)) + 2u_2(t)$

$\mathcal{L}[t+1] = \frac{1}{s^2} + \frac{1}{s} = \frac{1}{s}(1+\frac{1}{s})$

$f(t) = 1 + u_1(t)t - u_2(t)t + 2u_2(t)$

$\mathcal{L}[t+2] = \frac{1}{s^2} + \frac{2}{s} = \frac{1}{s}(2+\frac{1}{s})$ //

$f(t) = 1 + u_1(t)t + u_2(t)t$

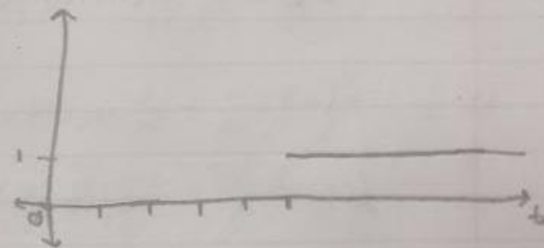
$\mathcal{L}(f(t)) = \mathcal{L}\{1\} + \mathcal{L}\{tu_1(t)\} + \mathcal{L}\{tu_2(t)\}$

$= \frac{1}{s} + \frac{e^{-s}}{s}(1+\frac{1}{s}) + \frac{e^{-2s}}{s}(2+\frac{1}{s})$

$\mathcal{L}\{f(t)\} = \frac{1}{s}(1 + e^{-s}(1+\frac{1}{s}) + e^{-2s}(2+\frac{1}{s}))$

(6)

$$f(t) = \begin{cases} 1 & t \geq 5 \\ \varnothing & \text{otherwise} \end{cases}$$



$\mathcal{L}[u_\alpha t] = \frac{1}{s} e^{-\alpha s}$

$f(t) = 1\, u_5(t)$

$\mathcal{L}(f(t)) = \mathcal{L}(u_5(t)) = \frac{e^{-5s}}{s}$

$\underline{\mathcal{L}\{f(t)\} = \frac{e^{-5s}}{s}}$ //

(7)

$$f(t) = \begin{cases} \sin(t-5) & t \geq 5 \\ \varnothing & \text{otherwise} \end{cases}$$

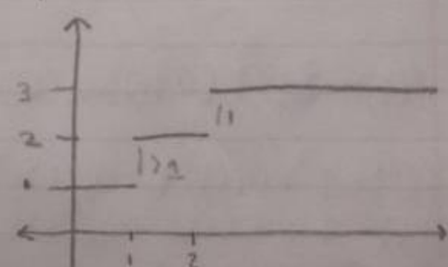
$\mathcal{L}\{\sin(t-5+5)\} = \mathcal{L}\{\sin(t)\} = \frac{1}{s^2+1}$

$f(t) = U_5(t)\sin(t-5) = e^{-5s}\left(\frac{1}{s^2+1}\right) = \frac{e^{-5s}}{s^2+1}$

$\underline{\mathcal{L}\{f(t)\} = \frac{e^{-5s}}{s^2+1}}$ //

Obtain the laplace transform of

$$f(t) = \begin{cases} 1 & \varnothing \leq t < 1 \\ 2 & 1 \leq t < 2 \\ 3 & t \geq 2 \end{cases}$$



$f(t) = 1 + u_1(t) + u_2(t)$

$\mathcal{L}\{f(t)\} = \mathcal{L}\{1\} + \mathcal{L}\{u_1(t)\} + \mathcal{L}\{u_2(t)\}$

$\mathcal{L}\{f(t)\} = \frac{1}{s} + \frac{e^{-s}}{s} + \frac{e^{-2s}}{s} = \frac{1}{s}(1 + e^{-s} + e^{-2s})$

$\underline{\mathcal{L}\{f(t)\} = \frac{1}{s}(1 + e^{-s} + e^{-2s})}$ //

$$\mathcal{L}\{e^{-\alpha t}\} = \frac{1}{s+\alpha} \qquad \mathcal{L}\{te^{-\alpha t}\} = \frac{1}{(s+\alpha)^2}$$

Compute the inverse Laplace Transform of the following functions

* $F(s) = \frac{5}{s+3}$

$$\mathcal{L}\{e^{-\alpha t}\} = \frac{1}{s+\alpha}$$

$$5\mathcal{L}\{e^{-\alpha t}\} = \frac{5}{s+\alpha} \qquad \alpha = 3$$

$\underline{f(t) = 5e^{-3t}}$ //

* $F(s) = \frac{s-2}{(s+3)(s+2)^2} = \frac{A}{s+3} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$

$$s-2 = A(s+2)^2 + B(s+3)(s+2) + C(s+3)$$

$$s-2 = A(s^2+4s+4) + B(s^2+5s+6) + Cs + 3C$$

$$s-2 = \cancel{As^2} + 4As + 4A + \cancel{Bs^2} + 5Bs + 6B + C\cancel{s} + 3C$$

$A + B = \emptyset \qquad \underline{B = -A}$ / $\qquad \underline{C = 1 + A}$ //

$4A + 5B + C = 1 \qquad 4A + 5(-A) + C = 1 \qquad -A + C = 1$

$4A + 6B + 3C = -2$

$4A + 6(-A) + 3(1+A) = -2$

$4A - 6A + 3 + 3A = -2$

$\underline{A = -5}$ // $\quad \underline{B = 5}$ // $\quad \underline{C = -4}$ //

$$\mathcal{L}^{-1}\left\{-\frac{5}{s+3} + \frac{5}{s+2} - \frac{4}{(s+2)^2}\right\} = -5\mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\} + 5\mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\} - 4\mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2}\right\}$$

$$= -5e^{-3t} + 5e^{-2t} - 4te^{-2t}$$

$\underline{f(t) = -5e^{-3t} + 5e^{-2t} - 4te^{-26}}$ //

* $F(s) = \frac{e^{-3s}(s+1)}{(s+2)(s-3)} \qquad \alpha = 3 \qquad F(s) = \frac{(s+1)}{(s+2)(s-3)} = \frac{A}{s+2} + \frac{B}{s-3}$

$$\mathcal{L}^{-1}\{F(s)\} = \frac{1}{5}\mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\} + \frac{4}{5}\mathcal{L}^{-1}\left\{\frac{1}{s-3}\right\}$$

$$f(t) = \frac{e^{-2t}}{5} + \frac{4e^{3t}}{5}$$

$$\mathcal{L}^{-1}\{e^{-\alpha s}F(s)\} = H(t-\alpha)f(t-\alpha)$$

$\underline{f(t) = H(t-3)\left(\frac{e^{-2(t-3)}}{5} + \frac{4e^{3(t-3)}}{5}\right)}$ //

$s + 1 = A(s-3) + B(s+2)$

$= As - 3A + Bs + 2B$

$A + B = 1 \qquad A = 1 - B$

$-3A + 2B = 1$

$-3(1-B) + 2B = 1 \qquad \underline{B = \frac{4}{5}}$ //

$-3 + 3B + 2B = 1 \qquad \underline{A = \frac{1}{5}}$ //

$5B = 4$

$$\mathcal{L}\{e^{\alpha t}\cos\omega t\} = \frac{s+\alpha}{(s+\alpha)^2+\omega^2}$$

$$\mathcal{L}\{\sin\omega t\} = \frac{\omega}{s^2+\omega^2}$$

$$s-\frac{1}{2}-\frac{5}{2}$$

Determine $c(\infty) = \lim_{t\to\infty} c(t)$ for $C(s) = \dfrac{(s-3)}{s^2-s+1-\frac{3}{4}+\frac{3}{4}}$

$$(s-\tfrac{1}{2})^2+\tfrac{3}{4}$$

and $C(s) = \dfrac{2}{s^2+2s+1}$

① $c(s) = \dfrac{(s-\frac{1}{2})-\frac{5}{2}}{(s-\frac{1}{2})^2+\frac{3}{4}} = \dfrac{s-\frac{1}{2}}{(s-\frac{1}{2})^2+\frac{3}{4}} - \dfrac{5}{2}\cdot\dfrac{1}{(s-\frac{1}{2})^2+\frac{3}{4}}$

$$\mathcal{L}^{-1}(c(s)) = \mathcal{L}^{-1}\left(\frac{s-\frac{1}{2}}{(s-\frac{1}{2})^2+\frac{3}{4}}\right) - \frac{5}{2}\mathcal{L}^{-1}\left\{\frac{1}{(s-\frac{1}{2})^2+\frac{3}{4}}\right\} \quad \mathcal{L}^{-1}\{F(s-a)\} = e^{at}f(t)$$

$$= e^{\frac{t}{2}}\cos\left(\tfrac{\sqrt{3}}{2}t\right) - \frac{5e^{\frac{t}{2}}}{\cancel{2}}\mathcal{L}^{-1}\left\{\frac{1\cdot\frac{\sqrt{3}}{2}}{s^2+(\frac{\sqrt{3}}{2})^2}\right\}\frac{\cancel{2}}{\sqrt{3}}$$

$$c(t) = e^{t/2}\cos\left(\tfrac{\sqrt{3}}{2}t\right) - \frac{5e^{t/2}}{\sqrt{3}}\sin\left(\tfrac{\sqrt{3}}{2}t\right)$$

$\lim_{t\to\infty} c(t)$ = divergent since it has cos and sin //

$$\mathcal{L}\{te^{\alpha t}\} = \frac{1}{(s-\alpha)^2}$$

② $$C(s) = \frac{2}{s^2+2s+1} = \frac{2}{(s+1)^2}$$

$$\mathcal{L}^{-1}\{C(s)\} = 2\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2}\right\}$$

$$\underline{c(t) = 2te^{-t}}$$

$$\lim_{t\to\infty} c(t) = \lim_{t\to\infty}(2te^{-t}) = 2\lim_{t\to\infty}(te^{-t}) = 2\lim_{t\to\infty}\left(\frac{t}{\frac{1}{e^t}}\right)$$

Aplicamos Lopital

$$= 2\lim_{t\to\infty}\left(\frac{1}{e^t}\right) = 2\,\frac{\lim_{t\to\infty}(1)}{\lim_{t\to\infty}(e^t)} = \frac{2(1)}{\infty} = \frac{2}{\infty}$$

$$\underline{= 0}$$

$$\underline{c(\infty) = 0}$$

• Describe a closed-loop control system in your field (electronics), describe the plant, the sensor and transducers, and the actuators required.

An inverter AC where the sensor measure the air temperature at that moment, the plant is the AC itself the actuators change the speed in the AC.