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Kata 11

*Resuelve el siguiente sistema usando Gauss-Seidel

$$\begin{pmatrix} 7 & -2 & 1 & 2 \\ 2 & 8 & 3 & 1 \\ -1 & 0 & 5 & 2 \\ 0 & 2 & -1 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 5 \\ 4 \end{pmatrix}$$

$$7x_1 - 2x_2 + x_3 + 2x_4 = 3$$
$$2x_1 + 8x_2 + 3x_3 + x_4 = -2$$
$$-x_1 + 5x_3 + 2x_4 = 5$$
$$2x_2 - x_3 + 4x_4 = 4$$

$$x_1 = \frac{3 + 2x_2 - x_3 - 2x_4}{7}$$

$$x_2 = \frac{-(2 + 2x_1 + 3x_3 + x_4)}{8}$$

$$x_3 = \frac{5 + x_1 - 2x_4}{5}$$

$$x_4 = \frac{4 - 2x_2 + x_3}{4}$$

① Suponemos que $x_4 = 0$ $x_2 = 0$ $x_3 \neq 0$

$$x_1 = 3/7$$

$$x_2 = \frac{-(2 + 2(3/7))}{8} = -5/14$$

$$x_3 = \frac{5 + 3/7 - 0}{5} = 38/35$$

$$x_4 = \frac{4 - 2(-5/14) + 38/35}{4} = 29/20$$

② $x_2 = -5/14$ $x_3 = 38/35$ $x_4 = 29/20$

$$x_1 = \frac{3 + 2(-5/14) - 38/35 - 2(29/20)}{7} = -17/70$$

$$x_2 = \frac{-(2 + 2(-17/70) + 3(38/35) + 29/20)}{8} = -871/1120$$

$$x_3 = \frac{5 + (-17/70) - 2(29/20)}{5} = 13/35$$

$$x_4 = \frac{4 - 2(-871/1120) + 13/35}{4} = 3319/2240$$

③ $x_2 = -871/1120 \quad x_3 = 13/35 \quad x_4 = 3319/2240$

$$x_1 = \frac{3 + 2(-871/1120) - 13/35 - 2(3319/2240)}{7} = \frac{-2117}{7840}$$

$$x_2 = \frac{-(2 + 2(-2117/7840) + 3(13/35) + 3319/2240)}{8} = \frac{-63597}{125440}$$

$$x_3 = \frac{5 + (-2117/7840) - 2(3319/2240)}{5} = 277/784$$

$$x_4 = \frac{4 + 277/784 - 2(-63597/125440)}{4} = 292317/250880$$

④ $x_2 = -63597/125440 \quad x_3 = 277/784 \quad x_4 = 292317/250880$

$$x_1 = \frac{3 + 2(-63597/125440) - 277/784 - 2(292317/250880)}{7} = -87511/878080$$

$$x_2 = \frac{-(2 + 2(-87511/878080) + 3(277/784) + 292317/250880)}{8} = -1413987/2809856$$

$$x_3 = \frac{5 + (-87511/878080) - 2(292317/250880)}{5} = \frac{225667}{439040}$$

$$x_4 = \frac{4 + 225667/439040 - 2(-1413987/2809856)}{4} = 5539881/40140800$$

⑤ $x_2 = -1413987/2809856 \quad x_3 = 225667/439040 \quad x_4 = 5539881/40140800$

$$x_1 = \frac{3 + 2(-1413987/2809856) - 225667/439040 - 2(5539881/40140800)}{7}$$

$$x_1 = -2570363/14049280$$

$$x_2 = \frac{-(2 + 2(-2570363/14049280) + 3(225667/439040) + 5539881/40140800)}{8}$$

$$x_2 = -128672899/224788480$$

$$X_3 = \frac{5 + \left(-2570363/14049280\right) - 2\left(5539881/4014080\right)}{5} = \frac{2889687}{7024640}$$

$$X_4 = \frac{4 + 2889687/7024640 - 2\left(-128027899/224788480\right)}{4}$$

$$X_4 = \frac{623834851}{449576960}$$

Errores

$$|X_1^5 - X_1^4| = 0.083$$
$$|X_2^5 - X_2^4| = 0.066$$
$$|X_3^5 - X_3^4| = 0.103$$
$$|X_4^5 - X_4^4| = 0.0075$$