COS457
GROUP WORK : (50 POINTS)
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1. For each of the following triplets of points, find the normal vector to the plane (if it exists) that passes through the triplets :

a. P1 = {1,1,1}
P2 = {1 , 2 , 1}
P3 = {3 , 0, 4}
P3 - P1 = (2, -1, -3)
P2 - P1 = (0, 1, 0)
Normal_Vector = {-3, 0, 2}

```
P1 = {1,1,1}
P2 = {1 , 2 , 1}
P3 = {3 , 0, 4}
```

```
P3 - P1
P2 - P1
```

{2, -1, 3}

{0, 1, 0}

```
Cross[{2,-1,3},{0,1,0}]
```

{-3, 0, 2}

b. P1B = {8 , 9 , 7}
P2B = {-8 , -9 , -7}
P3B = {1 , 2 , 1}
P3 - P1 = (-7, -7, -6)
P2 - P1 = (-16, -18, -14)
Norma_Vector = { -10, -2, 14}

```
P1B = {8 , 9 , 7}
P2B = {-8 , -9 , -7}
P3B = {1 , 2 , 1}
```

```
P3B - P1B
P2B - P1B
```

{-7, -7, -6}

{-16, -18, -14}

```
Cross[{-7,-7,-6},{-16,-18,-14}]
```

{-10, -2, 14}

c. P1C = {6, 3, -4}
P2C = {0, 0, 0}
P3C = {2, 1, -1}
P3 - P1 = {-4, -2, 3}
P2 - P1 = {-6, -3, 4}
Normal_Vector = {1,-2,0}

```
P1C = {6, 3, -4}
P2C = {0, 0, 0}
P3C = {2, 1, -1}
```

```
P3C - P1C
P2C - P1C
```

{-4, -2, 3}

{-6, -3, 4}

```
Cross[{-4,-2,3},{-6,-3,4}]
```

{1, -2, 0}

d. Switch the sequence of the points in part c to be P3, P2,P1
P1D = {2,1,-1}
P2D = {0,0,0}
P3D = {6,3,-4}
P3 - P1 = {4, 2, -3}
P2 - P1 = {-2, -1, 1}
Normal_Vector = {-1,2,0}

```
P1D = {2,1,-1}
P2D = {0,0,0}
P3D = {6,3,-4}
```

```
P3D - P1D
P2D - P1D
```

{4, 2, -3}

{-2, -1, 1}

```
Cross[{4,2,-3},{-2,-1,1}]
```

{-1, 2, 0}

e. Provide the normal form for the plane defined by the points in Part c.
Pn = {1, 2, 0}

P1 = {6, 3, -4}
1(x - x0) - 2(y - y0) + 0(z - z0) = 0
1(x - 6) - 2(y - 3) + 0(z + 4) = 0
x - 2y = 0

f. Provide the parametric form for the plane defined by the points in Part c.
P1 = {6, 3, -4}
P2 = {0, 0, 0}
P3 = {2, 1, -1}

```
P1 = {6, 3, -4}
P2 = {0, 0, 0}
P3 = {2, 1, -1}
```

```
P3 -P1
P2 - P1
```

{-4, -2, 3}

{-6, -3, 4}

(x - y - z) = (6 , 3 , -4) + s(-4 , -2 , 3) + t(-6 , -3 , 4)
x = 6 - 4s - 6t
∴ y = 3 - 2s - 3t
z = -4 + 3s + 4t

2. This question will have you do things the long way and then the short way :

a. Provide a matrix that rotates a point in 3D space 45 degrees about the y axis where the fixed point is the origin (0, 0, 0).

```
Ry =MatrixForm[{{Cos [45Degree],0,Sin[ 45Degree],0},{0, 1, 0, 0},{-Sin[ 45 Degree],0 , Co
```

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

b. Transform the point (1, 1, 1) by the matrix specified in part a.

$$\text{MatrixForm}\left[\begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.\{1,1,1,1\}\right]$$

$$\begin{pmatrix} \sqrt{2} \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

c. Provide a matrix that translates a point by (1,2,3)

```
Tx = MatrixForm[{{1,0,0,1},{0,1,0,2},{0,0,1,3},{0,0,0,1}}]
```

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

d. Transform the point from Part b by the matrix in part c.

$$\text{MatrixForm}\left[\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}.\{1,1,1,1\}\right]$$

$$\begin{pmatrix} 2 \\ 3 \\ 4 \\ 1 \end{pmatrix}$$

e. Provide a matrix that rotates a point in 3D space by 45 degrees about the z axis.

```
Rx=MatrixForm[{{Cos [45Degree],-Sin[ 45Degree],0,0},{Sin[ 45Degree], Cos [45Degree], 0, 0
```

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

f. Transform the point from part d by the matrix in part e.

$$\text{MatrixForm}\left[\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.\begin{pmatrix} 2 \\ 3 \\ 4 \\ 1 \end{pmatrix}\right]$$

$$\begin{pmatrix} -\frac{3}{\sqrt{2}} + \sqrt{2} \\ \frac{3}{\sqrt{2}} + \sqrt{2} \\ 4 \\ 1 \end{pmatrix}$$

g. Now provide a single matrix to rotate 45 degrees, about the y axis, followed by a translation

of (1,2,3), and then a rotation of 45 degrees about the z axis.

$$\text{MatrixForm}\left[\begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\right]$$

$$\begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} & 2\sqrt{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 2 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} & \sqrt{2} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

h. Transform the point (1,1,1) by the matrix in Part g.

$$\text{MatrixForm}\left[\begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} & 2\sqrt{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 2 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} & \sqrt{2} \\ 0 & 0 & 0 & 1 \end{pmatrix}.\{1,1,1,1\}\right]$$

$$\begin{pmatrix} \frac{1}{\sqrt{2}} + 2\sqrt{2} \\ 2 + \sqrt{2} \\ \frac{1}{\sqrt{2}} + \sqrt{2} \\ 1 \end{pmatrix}$$

3. Suppose the point (1, 1, 1) is first rotated about the x-axis 45 degrees and then translated in the direction of (1, 2, 1). What is the resulting transformation matrix and the value of the transformed point ?

$$\textbf{Q3} = \begin{pmatrix} 1 \\ 0 \\ \sqrt{2} \\ 2+\sqrt{2} \end{pmatrix}$$

```
Rx = MatrixForm[{{{{1,0,0,0},{0, Cos [45Degree],-Sin[ 45 Degree], 0},{0,Sin[ 45 Degree], (
```

$$\left(\begin{pmatrix} 1 \\ 0 \\ \sqrt{2} \\ 1 \end{pmatrix}\right)$$

```
Tx = MatrixForm[{{1,0,0,1},{0,1,0,2},{0,0,1,1},{0,0,0,1}}]
```

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
Q3 = MatrixForm[{{1},{0},{√2},{1}} . {{1,0,0,1},{0,1,0,2},{0,0,1,1},{0,0,0,1}}]
```

$$\left(\begin{pmatrix} 1 \\ 0 \\ \sqrt{2} \\ 2+\sqrt{2} \end{pmatrix}\right)$$

4. Suppose the point (1, 1, 1) is first translated in the direction of (1, 2, 1) and then rotated about the x-axis 45 degrees . What is the resulting transformation matrix and the result? Is this result identical to the
one you obtained in the previous problem? Why or why not ?

$$\mathbf{Q4} = \begin{pmatrix} 2 \\ 0 \\ 2\sqrt{2} \\ 1 \end{pmatrix}$$

Both of the answer are not the same, since the previous one is rotated first, then it is translated. Question 4 is translated first, and then it is rotated.

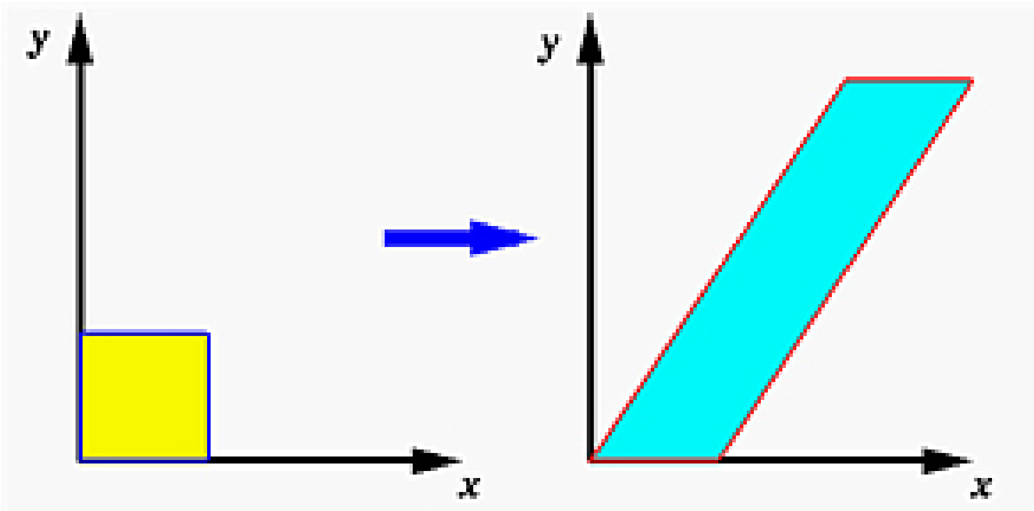
```
Tx = MatrixForm[{{1,0,0,1},{0,1,0,1},{0,0,1,1},{0,0,0,1}}.{1,1,1,1}]
```

$$\begin{pmatrix} 2 \\ 2 \\ 2 \\ 1 \end{pmatrix}$$

```
Q4 =MatrixForm[{{1,0,0,0},{0, Cos [45Degree],-Sin[ 45 Degree], 0},{0,Sin[ 45 Degree], Cos
```

$$\begin{pmatrix} 2 \\ 0 \\ 2\sqrt{2} \\ 1 \end{pmatrix}$$

5. An application needs to transform a unit square whose vertices are (0,0), (1,0), (1,1) and (0,1) to a parallelogram whose vertices are (0,0), (1,0), (3,3) and (2,3) as show below. Explain how you will
achieve this. Write down each individual transformation and its transformation matrix, compute the composite transformation, and verify your result. Note that the answer is not unique.



$c^2 = a^2 + b^2$
$c^2 = 13$
$c = \sqrt{13}$

$\sin\theta = \dfrac{3}{\sqrt{13}}$

$\cos\theta = \dfrac{2}{\sqrt{13}}$

$\cot\theta = \dfrac{2}{3}$

```
Hx = MatrixForm[{{1, 2/3,0,0},{0,1,0,0},{0,0,1,0},{0,0,0,1}}]
```

$$\begin{pmatrix} 1 & \frac{2}{3} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Scaling in 3 in y direction

$$\text{Sy} = \text{MatrixForm}\left[\begin{pmatrix} 1 & \frac{2}{3} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.\{\{1,0,0,0\},\{0,3,0,0\},\{0,0,1,0\},\{0,0,0,1\}\}\right]$$

$$\begin{pmatrix} 1 & 2 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.\{0,0,0,1\}$$

{0, 0, 0, 1}

{0, 0, 0, 1}

$$\begin{pmatrix} 1 & 2 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.\{1,0,0,1\}$$

{1, 0, 0, 1}

$$\begin{pmatrix} 1 & 2 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.\{1,1,0,1\}$$

{3, 3, 0, 1}

$$\begin{pmatrix} 1 & 2 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.\{0,1,0,1\}$$

{2, 3, 0, 1}

The new points are (0 , 0), (1 , 0), (3 , 3), (2 , 3)