4442 PSA Demo

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##Preamble

#Data Information CONTEXT - The Sloan Digital Sky Survey or SDSS is a major multi-spectral imaging and spectroscopic redshift survey using a dedicated 2.5-m wide-angle optical telescope at Apache Point Observatory in New Mexico, United States. SDSS has gathered image and spectroscopic data of millions of celestial objects spanning the entire night sky. Each object is classified in one of the eight categories by help of their optical and spectroscopic properties. Data Source - This public domain data was downloaded February 2024 and appears to have been available on this site for about a year. Link - <https://www.kaggle.com/datasets/hari31416/celestialclassify?resource=download>

Description: The listed data source training data included 4,000,00 observations across 52 variables. The variable explanations are as followed:

objID: Unique SDSS identifier composed from [skyVersion,rerun,run,camcol,field,obj].

run: Run number

camcol: Camera column

field: Field number

type: Type classification of the object (star, galaxy, cosmic ray, etc.)

rowv: Row-component of object’s velocity (deg/day)

colv: Column-component of object’s velocity (deg/day)

psfMag\_u: PSF magnitude (mag)

psfMag\_g: PSF magnitude (mag)

psfMag\_r: PSF magnitude (mag)

psfMag\_i: PSF magnitude (mag)

psfMag\_z: PSF magnitude (mag)

petroRad\_u: Petrosian radius (arcsec)

petroRad\_g: Petrosian radius (arcsec)

petroRad\_r: Petrosian radius (arcsec)

petroRad\_i: Petrosian radius (arcsec)

petroRad\_z: Petrosian radius (arcsec)

q\_u: Stokes Q parameter

q\_g: Stokes Q parameter

q\_r: Stokes Q parameter

q\_i: Stokes Q parameter

q\_z: Stokes Q parameter

u\_u: Stokes U parameter

u\_g: Stokes U parameter

u\_r: Stokes U parameter

u\_i: Stokes U parameter

u\_z: Stokes U parameter

expRad\_u: Exponential fit scale radius, here defined to be the same as the half-light radius, also called the effective radius (arcsec)

expRad\_g: Exponential fit scale radius, here defined to be the same as the half-light radius, also called the effective radius (arcsec)

expRad\_r: Exponential fit scale radius, here defined to be the same as the half-light radius, also called the effective radius (arcsec)

expRad\_i: Exponential fit scale radius, here defined to be the same as the half-light radius, also called the effective radius (arcsec)

expRad\_z: Exponential fit scale radius, here defined to be the same as the half-light radius, also called the effective radius (arcsec)

expAB\_u: Exponential fit b/a

expAB\_g: Exponential fit b/a

expAB\_r: Exponential fit b/a

expAB\_i: Exponential fit b/a

expAB\_z: Exponential fit b/a

modelFlux\_u: better of DeV/Exp flux fit (nanomaggies)

modelFlux\_g: better of DeV/Exp flux fit (nanomaggies)

modelFlux\_r: better of DeV/Exp flux fit (nanomaggies)

modelFlux\_i: better of DeV/Exp flux fit (nanomaggies)

modelFlux\_z: better of DeV/Exp flux fit (nanomaggies)

ra: J2000 Right Ascension (r-band) (deg)

dec: J2000 Declination (r-band) (deg)

b: Galactic latitude (deg)

l: Galactic longitude (deg)

u: Shorthand alias for modelMag (mag)

g: Shorthand alias for modelMag (mag)

r: Shorthand alias for modelMag (mag)

i: Shorthand alias for modelMag (mag)

z: Shorthand alias for modelMag (mag)

The various measurements are taken by the SDSS camera which has five filters, which together span the optical window. These filters are labeled u, g, r, i and z and are indicated with the corresponding letter at the end of each variable name.

Loading the data and working with this large of a data frame results in long processing time in R. For this demonstration, the training set was randomly sampled to collect 10,000 observations. The sample file is included with this demo.

#Research Question Which variables have the largest loadings in the principal components that explain the most variance?

#Reading in the data

dat<-read.csv("samp\_celestial.csv", header = TRUE)  
head(dat)

## X objID run camcol field type rowv colv  
## 1 2000074 1.23765e+18 756 2 140 galaxy -0.000930131 -0.006476541  
## 2 435309 1.23765e+18 307 2 144 star 0.016467360 0.077337340  
## 3 3351757 1.23765e+18 756 4 262 galaxy 0.001354466 -0.001599089  
## 4 1159773 1.23765e+18 259 5 556 star 0.002178201 -0.003557452  
## 5 1524786 1.23765e+18 259 6 528 star 0.000862903 -0.001686370  
## 6 2262816 1.23765e+18 1037 3 170 galaxy 0.007129668 -0.010157660  
## u g r i z psfMag\_u psfMag\_g psfMag\_r  
## 1 22.48308 22.10694 22.08556 21.36183 21.47369 24.15477 22.92056 23.49476  
## 2 25.57899 23.83201 21.96375 21.21920 20.40985 25.54693 23.96713 22.08007  
## 3 22.21263 20.79209 19.74672 19.21204 18.81962 22.56821 21.27234 20.28003  
## 4 24.06112 21.14494 20.14259 19.64180 19.28249 23.73955 21.15279 20.14617  
## 5 20.13653 17.41362 15.97394 15.32602 14.95725 20.11522 17.40752 15.98047  
## 6 21.56130 20.24093 19.59859 19.14935 18.88804 21.70344 20.52193 19.90851  
## psfMag\_i psfMag\_z modelFlux\_u modelFlux\_g modelFlux\_r modelFlux\_i modelFlux\_z  
## 1 22.59898 22.46111 0.9964053 1.4305850 1.454960 2.841428 2.360715  
## 2 21.29351 20.57317 -0.2754116 0.2655965 1.629906 3.243322 6.775950  
## 3 19.76347 19.37591 1.2879690 4.8196180 12.626240 20.661030 29.640230  
## 4 19.64844 19.28457 0.1548919 3.4812560 8.767623 13.906220 19.336120  
## 5 15.29243 14.95755 8.8161270 108.2805000 407.778900 740.615800 1040.156000  
## 6 19.46402 19.19187 2.3657310 8.0088880 14.472130 21.889150 27.827750  
## petroRad\_u petroRad\_g petroRad\_r petroRad\_i petroRad\_z expRad\_u  
## 1 1.931593 2.969071 4.583549 2.9694010 1.678484 7.14430600  
## 2 2.968868 2.969270 1.475165 1.0304300 1.143815 2.07600900  
## 3 1.336241 1.858368 2.098733 1.7315410 2.871861 0.47197950  
## 4 18.018270 1.196026 1.050479 0.9782889 1.121818 59.33182000  
## 5 1.310031 1.093868 1.184593 1.1481790 1.106220 0.02737134  
## 6 2.212703 2.391112 2.470691 2.4232590 3.429187 0.34579950  
## expRad\_g expRad\_r expRad\_i expRad\_z q\_u q\_g  
## 1 0.816067600 2.73787500 2.734939000 2.19760000 0.989256500 1.564843000  
## 2 0.304393400 0.19934220 0.001270269 1.06581900 -0.285744400 1.425631000  
## 3 0.487858100 0.63848700 0.531515700 0.49684520 -0.182387600 0.037860990  
## 4 0.126910300 0.01701023 0.002986188 0.12605090 -0.026834420 -0.118920900  
## 5 0.000145485 0.03653416 0.000025700 0.02248107 -0.004962464 -0.050370020  
## 6 0.570596300 0.55526330 0.683810800 0.62502280 -0.133133600 -0.005938655  
## q\_r q\_i q\_z u\_u u\_g u\_r  
## 1 -0.82165850 0.105111400 0.42180240 1.599925000 -3.565000000 -0.697574500  
## 2 -0.02563342 0.008531981 0.13087630 -0.167288000 0.034708480 -0.135619200  
## 3 0.07709192 0.082707380 0.07893183 -0.100521100 0.001144267 -0.032854570  
## 4 -0.05711155 -0.058382220 -0.17658310 0.668298200 -0.025581470 -0.011957620  
## 5 -0.08155926 -0.157007500 -0.14572050 -0.005393851 -0.022152720 -0.087659910  
## 6 -0.02037297 -0.053612460 -0.19134810 0.442278600 0.024043000 0.008015963  
## u\_i u\_z expAB\_u expAB\_g expAB\_r expAB\_i expAB\_z  
## 1 -0.995616500 0.32320940 0.05000000 0.21227350 0.1461744 0.0500000 0.0500000  
## 2 0.036505010 -0.39774130 0.15246540 0.05204362 0.7764918 0.2312458 0.0500000  
## 3 0.003224926 0.03733748 0.43732690 0.72818700 0.5609612 0.5776812 0.6139212  
## 4 -0.019552880 0.02043905 0.05000000 0.39984380 0.9996263 0.7388217 0.9999557  
## 5 -0.111938200 0.01700760 0.42127480 0.62434260 0.8740420 0.9082988 0.9997272  
## 6 -0.066575180 0.01668781 0.09446066 0.85623280 0.8754274 0.6167579 0.2584458  
## ra dec b l  
## 1 136.83248 -0.5333474 29.541990 230.5332  
## 2 97.56097 -0.4516757 -4.920701 210.9136  
## 3 155.05753 0.2788040 44.863503 243.0776  
## 4 89.88940 0.6394364 -11.224206 206.3596  
## 5 85.72856 1.0649460 -14.692453 203.9612  
## 6 65.13949 -5.3108075 -35.758121 198.9988

#Cleaning the data

Assess variable types to ensure they are compliant with data requirements for PCA. PCA does not have an outcome variable and although performing PCA on non-continuous data is possible, it’s best to have continuous variables.

Almost all of the variables are continuous with the exception of the character variable for denoting a galaxy or star. Before any principal component analysis, we will want to ensure no data is missing. The necessity of the values will also be evaluated. For this data set, the index value, unique identifier, run number, camera column, field, and celestial object type (galaxy or star) will be removed. These variables will not be needed for PCA.

# View data types  
str(dat)

## 'data.frame': 10000 obs. of 52 variables:  
## $ X : int 2000074 435309 3351757 1159773 1524786 2262816 762171 2032273 1847078 1266558 ...  
## $ objID : num 1.24e+18 1.24e+18 1.24e+18 1.24e+18 1.24e+18 ...  
## $ run : int 756 307 756 259 259 1037 307 308 1000 259 ...  
## $ camcol : int 2 2 4 5 6 3 3 4 5 4 ...  
## $ field : int 140 144 262 556 528 170 180 11 109 456 ...  
## $ type : chr "galaxy" "star" "galaxy" "star" ...  
## $ rowv : num -0.00093 0.016467 0.001354 0.002178 0.000863 ...  
## $ colv : num -0.00648 0.07734 -0.0016 -0.00356 -0.00169 ...  
## $ u : num 22.5 25.6 22.2 24.1 20.1 ...  
## $ g : num 22.1 23.8 20.8 21.1 17.4 ...  
## $ r : num 22.1 22 19.7 20.1 16 ...  
## $ i : num 21.4 21.2 19.2 19.6 15.3 ...  
## $ z : num 21.5 20.4 18.8 19.3 15 ...  
## $ psfMag\_u : num 24.2 25.5 22.6 23.7 20.1 ...  
## $ psfMag\_g : num 22.9 24 21.3 21.2 17.4 ...  
## $ psfMag\_r : num 23.5 22.1 20.3 20.1 16 ...  
## $ psfMag\_i : num 22.6 21.3 19.8 19.6 15.3 ...  
## $ psfMag\_z : num 22.5 20.6 19.4 19.3 15 ...  
## $ modelFlux\_u: num 0.996 -0.275 1.288 0.155 8.816 ...  
## $ modelFlux\_g: num 1.431 0.266 4.82 3.481 108.281 ...  
## $ modelFlux\_r: num 1.45 1.63 12.63 8.77 407.78 ...  
## $ modelFlux\_i: num 2.84 3.24 20.66 13.91 740.62 ...  
## $ modelFlux\_z: num 2.36 6.78 29.64 19.34 1040.16 ...  
## $ petroRad\_u : num 1.93 2.97 1.34 18.02 1.31 ...  
## $ petroRad\_g : num 2.97 2.97 1.86 1.2 1.09 ...  
## $ petroRad\_r : num 4.58 1.48 2.1 1.05 1.18 ...  
## $ petroRad\_i : num 2.969 1.03 1.732 0.978 1.148 ...  
## $ petroRad\_z : num 1.68 1.14 2.87 1.12 1.11 ...  
## $ expRad\_u : num 7.1443 2.076 0.472 59.3318 0.0274 ...  
## $ expRad\_g : num 0.816068 0.304393 0.487858 0.12691 0.000145 ...  
## $ expRad\_r : num 2.7379 0.1993 0.6385 0.017 0.0365 ...  
## $ expRad\_i : num 2.73 1.27e-03 5.32e-01 2.99e-03 2.57e-05 ...  
## $ expRad\_z : num 2.1976 1.0658 0.4968 0.1261 0.0225 ...  
## $ q\_u : num 0.98926 -0.28574 -0.18239 -0.02683 -0.00496 ...  
## $ q\_g : num 1.5648 1.4256 0.0379 -0.1189 -0.0504 ...  
## $ q\_r : num -0.8217 -0.0256 0.0771 -0.0571 -0.0816 ...  
## $ q\_i : num 0.10511 0.00853 0.08271 -0.05838 -0.15701 ...  
## $ q\_z : num 0.4218 0.1309 0.0789 -0.1766 -0.1457 ...  
## $ u\_u : num 1.59993 -0.16729 -0.10052 0.6683 -0.00539 ...  
## $ u\_g : num -3.565 0.03471 0.00114 -0.02558 -0.02215 ...  
## $ u\_r : num -0.6976 -0.1356 -0.0329 -0.012 -0.0877 ...  
## $ u\_i : num -0.99562 0.03651 0.00322 -0.01955 -0.11194 ...  
## $ u\_z : num 0.3232 -0.3977 0.0373 0.0204 0.017 ...  
## $ expAB\_u : num 0.05 0.152 0.437 0.05 0.421 ...  
## $ expAB\_g : num 0.212 0.052 0.728 0.4 0.624 ...  
## $ expAB\_r : num 0.146 0.776 0.561 1 0.874 ...  
## $ expAB\_i : num 0.05 0.231 0.578 0.739 0.908 ...  
## $ expAB\_z : num 0.05 0.05 0.614 1 1 ...  
## $ ra : num 136.8 97.6 155.1 89.9 85.7 ...  
## $ dec : num -0.533 -0.452 0.279 0.639 1.065 ...  
## $ b : num 29.54 -4.92 44.86 -11.22 -14.69 ...  
## $ l : num 231 211 243 206 204 ...

# Removing index, object ID, run, camcol, field, and type variables. Run, camcol, field, and type variables are composites of object ID and are not qualities of the celestial objects worth analyzing.  
dat.sub<-dat[ , -c(1,2,3,4,5,6)]  
  
# Check for NA values, PCA should not be performed on data with missing values.  
sum(is.na(dat))

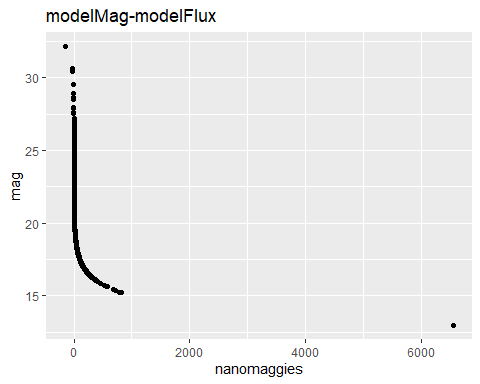
## [1] 0

Primary Component Analysis (PCA) is a a non-parametric method best used on large datasets with many continuous features that are correlated.

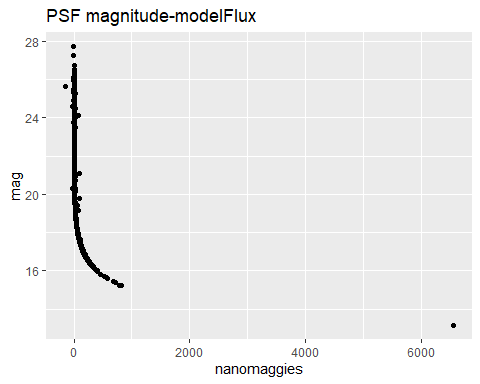
The following plots demonstrate the relationship between various variables in the data set. Some relevant information is that astronomers use two scales to measure the brightness of astronomical sources: the magnitude scale and the flux scale. Magnitude and flux are logarithmically related.

# Including Plots

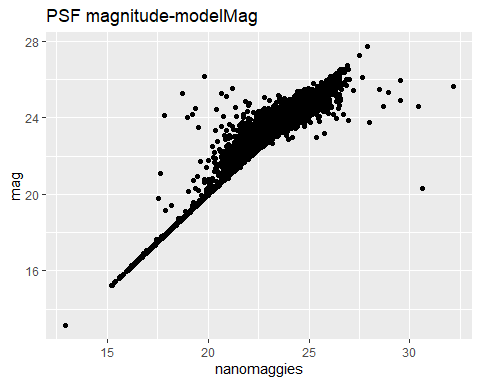
# Many of these variables are closely related as they are measuring the same/similar celestial qualities. These graphs demonstrate the relationship between magnitude and flux measurements.  
ggplot(dat.sub, aes(x=modelFlux\_u, y=u)) +   
 geom\_point()+ggtitle("modelMag-modelFlux")+xlab("nanomaggies")+ylab("mag")



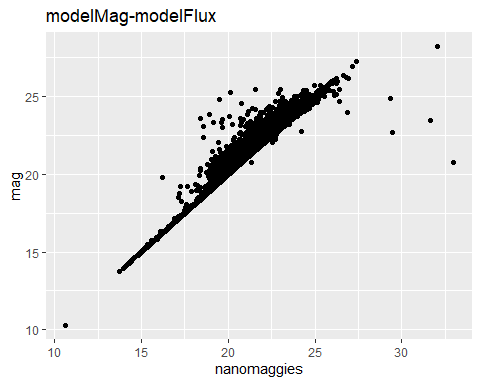
ggplot(dat.sub, aes(x=modelFlux\_u, y=psfMag\_u)) +   
 geom\_point()+ggtitle("PSF magnitude-modelFlux")+xlab("nanomaggies")+ylab("mag")



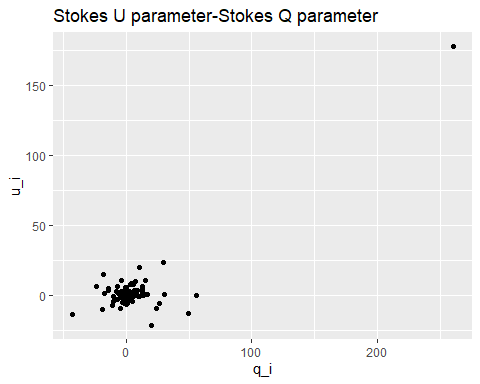
# Linearly correlated variables. Both these variables are measurements of magnitude.  
ggplot(dat.sub, aes(x=u, y=psfMag\_u)) +   
 geom\_point()+ggtitle("PSF magnitude-modelMag")+xlab("nanomaggies")+ylab("mag")



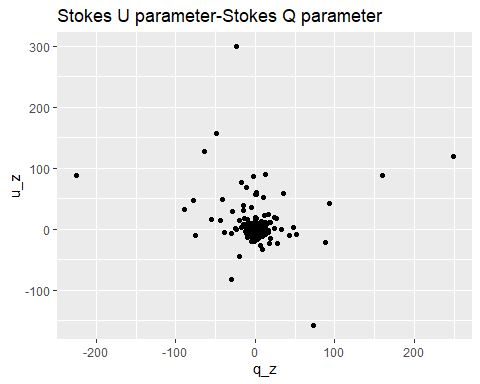
ggplot(dat.sub, aes(x=g, y=psfMag\_g)) +   
 geom\_point()+ggtitle("modelMag-modelFlux")+xlab("nanomaggies")+ylab("mag")



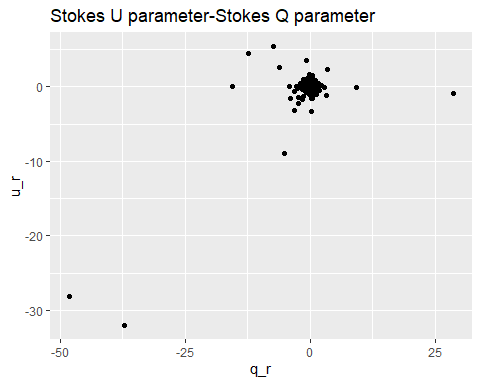
# There is a relationship between Stokes parameters and polarization ellipse parameters.  
ggplot(dat.sub, aes(x=q\_i, y=u\_i)) +   
 geom\_point()+ggtitle("Stokes U parameter-Stokes Q parameter ")+xlab("q\_i")+ylab("u\_i")



ggplot(dat.sub, aes(x=q\_z, y=u\_z)) +   
 geom\_point()+ggtitle("Stokes U parameter-Stokes Q parameter ")+xlab("q\_z")+ylab("u\_z")



ggplot(dat.sub, aes(x=q\_r, y=u\_r)) +   
 geom\_point()+ggtitle("Stokes U parameter-Stokes Q parameter ")+xlab("q\_r")+ylab("u\_r")



## PCA Method

PCA is a dimensionality reduction technique that makes large data sets more manageable by constructing new variables that are linear combinations of the initial variables (principal components) that best retain the characteristics of the original data set.

There are general requirements for PCA, however you don’t need to vigorously check these assumptions prior to performing PCA. Generally, your data should be linearly related, variables should be continuous, there shouldn’t be any outliers, and you should have many variables and observations.

The first step in performing PCA is to standardize the range of the continuous variables. This is done because PCA is sensitive to differences in variances among initial variables. Next, compute the covariance matrix. Essentially, the covariance matrix summarizes the correlation between all variables. The next step is to compute the eigenvectors and eigenvalues of the covariance matrix. The eigenvectors of the covariance matrix are the principal components, and the eigenvalues describe the amount of variance carried in each principal component. Using the prcomp() function, R accomplishes these steps.

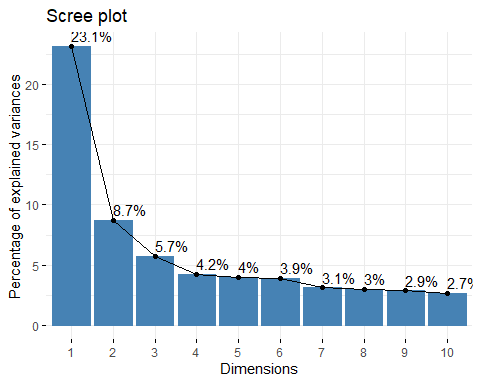
# Use prcomp to perform PCA on data, set scale = TRUE to standardize each variable to have a mean of 0 and a standard deviation of 1.  
pca\_result <- prcomp(dat.sub, scale = TRUE)  
  
# View eigenvalues  
get\_eigenvalue(pca\_result)

## eigenvalue variance.percent cumulative.variance.percent  
## Dim.1 10.646765903 23.145143267 23.14514  
## Dim.2 3.997611214 8.690459161 31.83560  
## Dim.3 2.630508374 5.718496466 37.55410  
## Dim.4 1.939766436 4.216883556 41.77098  
## Dim.5 1.832295160 3.983250349 45.75423  
## Dim.6 1.796734542 3.905944656 49.66018  
## Dim.7 1.438376241 3.126904872 52.78708  
## Dim.8 1.388213880 3.017856261 55.80494  
## Dim.9 1.324946070 2.880317544 58.68526  
## Dim.10 1.236982588 2.689092583 61.37435  
## Dim.11 1.227446082 2.668361049 64.04271  
## Dim.12 1.170905633 2.545447028 66.58816  
## Dim.13 1.137409325 2.472628966 69.06079  
## Dim.14 1.026119271 2.230694068 71.29148  
## Dim.15 0.993878865 2.160606229 73.45209  
## Dim.16 0.944778004 2.053865226 75.50595  
## Dim.17 0.915880132 1.991043764 77.49700  
## Dim.18 0.859094223 1.867596136 79.36459  
## Dim.19 0.839330910 1.824632414 81.18922  
## Dim.20 0.802440247 1.744435319 82.93366  
## Dim.21 0.791000816 1.719566990 84.65323  
## Dim.22 0.744789883 1.619108441 86.27233  
## Dim.23 0.718131160 1.561154695 87.83349  
## Dim.24 0.676717678 1.471125386 89.30461  
## Dim.25 0.648664940 1.410141175 90.71476  
## Dim.26 0.630073244 1.369724443 92.08448  
## Dim.27 0.615810333 1.338718116 93.42320  
## Dim.28 0.593694879 1.290641042 94.71384  
## Dim.29 0.573389240 1.246498348 95.96034  
## Dim.30 0.536379627 1.166042666 97.12638  
## Dim.31 0.265070861 0.576241002 97.70262  
## Dim.32 0.220984173 0.480400377 98.18302  
## Dim.33 0.183393959 0.398682519 98.58170  
## Dim.34 0.157103218 0.341528735 98.92323  
## Dim.35 0.130534695 0.283771075 99.20700  
## Dim.36 0.082577303 0.179515877 99.38652  
## Dim.37 0.072625622 0.157881788 99.54440  
## Dim.38 0.059160069 0.128608845 99.67301  
## Dim.39 0.054781636 0.119090513 99.79210  
## Dim.40 0.042206238 0.091752692 99.88385  
## Dim.41 0.019850569 0.043153411 99.92701  
## Dim.42 0.017068070 0.037104500 99.96411  
## Dim.43 0.007037872 0.015299722 99.97941  
## Dim.44 0.004446359 0.009665998 99.98908  
## Dim.45 0.003013740 0.006551609 99.99563  
## Dim.46 0.002010716 0.004371121 100.00000

# The summary function on the result object gives us standard deviation, proportion of variance explained by each principal component, and the cumulative proportion of variance explained.  
summary(pca\_result)

## Importance of components:  
## PC1 PC2 PC3 PC4 PC5 PC6 PC7  
## Standard deviation 3.2629 1.9994 1.62188 1.39275 1.35362 1.34042 1.19932  
## Proportion of Variance 0.2314 0.0869 0.05718 0.04217 0.03983 0.03906 0.03127  
## Cumulative Proportion 0.2314 0.3184 0.37554 0.41771 0.45754 0.49660 0.52787  
## PC8 PC9 PC10 PC11 PC12 PC13 PC14  
## Standard deviation 1.17822 1.1511 1.11220 1.10790 1.08208 1.06649 1.01298  
## Proportion of Variance 0.03018 0.0288 0.02689 0.02668 0.02545 0.02473 0.02231  
## Cumulative Proportion 0.55805 0.5868 0.61374 0.64043 0.66588 0.69061 0.71291  
## PC15 PC16 PC17 PC18 PC19 PC20 PC21  
## Standard deviation 0.99693 0.97200 0.95702 0.92687 0.91615 0.89579 0.8894  
## Proportion of Variance 0.02161 0.02054 0.01991 0.01868 0.01825 0.01744 0.0172  
## Cumulative Proportion 0.73452 0.75506 0.77497 0.79365 0.81189 0.82934 0.8465  
## PC22 PC23 PC24 PC25 PC26 PC27 PC28  
## Standard deviation 0.86301 0.84743 0.82263 0.8054 0.7938 0.78474 0.77052  
## Proportion of Variance 0.01619 0.01561 0.01471 0.0141 0.0137 0.01339 0.01291  
## Cumulative Proportion 0.86272 0.87833 0.89305 0.9072 0.9208 0.93423 0.94714  
## PC29 PC30 PC31 PC32 PC33 PC34 PC35  
## Standard deviation 0.75722 0.73238 0.51485 0.4701 0.42825 0.39636 0.36130  
## Proportion of Variance 0.01246 0.01166 0.00576 0.0048 0.00399 0.00342 0.00284  
## Cumulative Proportion 0.95960 0.97126 0.97703 0.9818 0.98582 0.98923 0.99207  
## PC36 PC37 PC38 PC39 PC40 PC41 PC42  
## Standard deviation 0.2874 0.26949 0.24323 0.23405 0.20544 0.14089 0.13064  
## Proportion of Variance 0.0018 0.00158 0.00129 0.00119 0.00092 0.00043 0.00037  
## Cumulative Proportion 0.9939 0.99544 0.99673 0.99792 0.99884 0.99927 0.99964  
## PC43 PC44 PC45 PC46  
## Standard deviation 0.08389 0.06668 0.05490 0.04484  
## Proportion of Variance 0.00015 0.00010 0.00007 0.00004  
## Cumulative Proportion 0.99979 0.99989 0.99996 1.00000

# Extract and visualize the eigenvalues/variances of dimensions: "Scree plot" (factoextra).  
fviz\_eig(pca\_result, addlabels = TRUE)

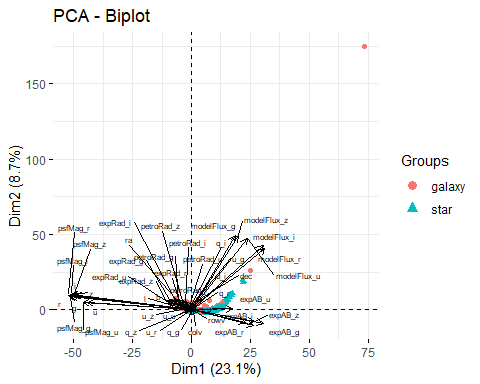


# Display the principal components (the eigenvectors of the covariance matrix), in the original coordinate system. These are the loadings.  
options(max.print=10000) # increase printed variables  
pca\_result$rotation

## PC1 PC2 PC3 PC4 PC5  
## rowv 0.002831092 -0.0015467455 -0.017955920 0.0018309479 -0.032557137  
## colv 0.004073531 -0.0044520799 0.002135490 -0.0271682062 -0.026745528  
## u -0.247852358 0.0427099620 0.057344276 -0.0194671131 -0.108060305  
## g -0.284979782 0.0783672224 0.047218599 -0.0164859141 -0.059464520  
## r -0.291341310 0.0782703976 0.094934468 -0.0401991998 -0.010905966  
## i -0.285434314 0.0923255520 0.025368142 -0.0017409674 0.021659553  
## z -0.276609007 0.0886156959 -0.004110059 0.0018239032 0.041975441  
## psfMag\_u -0.257869805 0.0440007976 0.028860260 -0.0096341709 -0.101371510  
## psfMag\_g -0.288763593 0.0805678502 0.011678132 -0.0010126601 -0.048040227  
## psfMag\_r -0.293187066 0.0869841299 0.011043144 0.0002922390 0.002017012  
## psfMag\_i -0.286775939 0.0929076519 -0.011201182 0.0090067290 0.033000490  
## psfMag\_z -0.278444274 0.0872146500 -0.005492572 0.0021806923 0.049285047  
## modelFlux\_u 0.132569096 0.4329142394 0.060198046 -0.0135409819 -0.013454886  
## modelFlux\_g 0.103770939 0.4489490606 0.066195884 -0.0166020831 -0.024259351  
## modelFlux\_r 0.175000131 0.3698845961 0.028293957 -0.0001716544 -0.003301670  
## modelFlux\_i 0.170792253 0.3895584143 0.040890830 -0.0067398885 -0.010659533  
## modelFlux\_z 0.111075260 0.4480875192 0.063017949 -0.0160921424 -0.026862448  
## petroRad\_u -0.006635412 0.0124185309 -0.160279380 0.0917450896 -0.021828340  
## petroRad\_g -0.029951876 0.0421174755 -0.201678091 0.1342157213 -0.045896832  
## petroRad\_r -0.012025956 0.0482681022 -0.469483711 0.1455349386 -0.033836913  
## petroRad\_i -0.013853651 0.0306864410 -0.285024569 -0.1441941554 -0.076473586  
## petroRad\_z -0.018799748 0.0240941245 -0.092723221 0.0343081767 0.011373945  
## expRad\_u -0.052228411 0.0412097839 -0.191174479 0.0921370693 -0.049840600  
## expRad\_g -0.049644116 0.0548584615 -0.196050337 0.1259858478 -0.045653584  
## expRad\_r -0.028711258 0.0582268725 -0.457717529 0.2165008301 0.018150214  
## expRad\_i -0.039053203 0.0585904262 -0.306130927 0.1133885160 0.038678123  
## expRad\_z -0.059446470 0.0565816225 -0.134880378 0.0540142691 0.022057922  
## q\_u 0.002248388 0.0001758336 -0.018823013 -0.0211427885 0.026444025  
## q\_g 0.001845261 -0.0118955522 0.190181872 0.2148750047 0.100108366  
## q\_r 0.002384565 0.0044404842 -0.140603978 -0.4706888674 -0.155571373  
## q\_i -0.004962080 0.0091333272 -0.125047903 -0.4140589281 0.239089128  
## q\_z -0.001092060 0.0001886882 0.016415899 0.0629518140 0.038060398  
## u\_u -0.003504366 0.0013088190 0.021152751 0.0279793094 -0.029309387  
## u\_g -0.002135834 0.0107957939 -0.194383990 -0.2572312588 -0.113954098  
## u\_r 0.001323358 -0.0006730103 -0.060778783 -0.4005770027 -0.103820663  
## u\_i -0.004778028 0.0089685124 -0.108106050 -0.3977609464 0.253238683  
## u\_z -0.007442806 0.0090232280 -0.039288906 -0.0607669088 0.004419915  
## expAB\_u 0.098189472 0.0062222266 -0.058329928 0.0401973063 0.008493791  
## expAB\_g 0.171707639 -0.0833634604 -0.059942136 0.0249291646 0.081648171  
## expAB\_r 0.148179893 -0.0957643727 -0.048936024 0.0220479344 0.011791945  
## expAB\_i 0.128949234 -0.0760158727 -0.043263593 0.0171483965 0.005532880  
## expAB\_z 0.150036632 -0.0741414354 -0.040387254 0.0039161458 0.042824235  
## ra -0.034607415 0.0600470650 -0.085882379 0.0179067240 0.557666370  
## dec 0.022907059 0.0680070340 -0.218258318 0.0362664809 -0.137034011  
## b -0.055378025 0.0456389113 -0.021385584 0.0244700153 0.632773933  
## l -0.022916769 0.0130029421 0.007314486 0.0359502546 0.158526289  
## PC6 PC7 PC8 PC9 PC10  
## rowv 0.028620213 -0.1286178841 6.816228e-01 -0.054221240 0.020721300  
## colv 0.002426734 -0.1275905015 6.882772e-01 -0.067025172 0.011948869  
## u 0.032774338 0.0026568772 1.227098e-02 0.175690297 0.079458076  
## g 0.015693799 -0.0025901452 7.775972e-03 0.080771727 0.030382950  
## r -0.007201816 -0.0175958289 -1.238769e-03 0.006125761 0.020409442  
## i -0.009838837 0.0052752063 -2.021639e-03 -0.035589242 -0.031129822  
## z -0.005947366 0.0124496807 5.962623e-03 -0.057572854 -0.056735512  
## psfMag\_u 0.037923118 0.0059709229 1.656291e-02 0.147667850 0.085683936  
## psfMag\_g 0.018110830 0.0048704947 5.445187e-03 0.039522652 0.050733639  
## psfMag\_r 0.007835652 0.0034955048 5.579761e-03 -0.015701774 0.016832483  
## psfMag\_i -0.005574372 0.0029152792 4.752397e-05 -0.065379302 -0.020130238  
## psfMag\_z -0.015701199 0.0103477935 1.048967e-03 -0.103436339 -0.032716582  
## modelFlux\_u 0.005951296 -0.0007138115 1.842857e-03 -0.002582656 0.007657603  
## modelFlux\_g 0.008530241 -0.0029684141 6.277574e-03 0.019537335 0.019077853  
## modelFlux\_r 0.003213222 0.0038886318 -2.453363e-03 -0.016230610 -0.011444816  
## modelFlux\_i 0.004576887 0.0007034933 -6.823556e-04 0.002432727 -0.001307246  
## modelFlux\_z 0.008319560 -0.0026956596 6.771226e-03 0.027040685 0.019342442  
## petroRad\_u 0.020386998 0.1303290457 3.427805e-03 -0.035756712 0.279510483  
## petroRad\_g 0.030082215 0.1784182088 6.498363e-03 -0.099362092 0.210667791  
## petroRad\_r 0.029439374 0.0487332907 5.331663e-02 0.174864037 -0.152130793  
## petroRad\_i -0.139129947 -0.3115936671 -7.553850e-02 -0.004486594 0.096054470  
## petroRad\_z -0.028164513 -0.0450853854 -4.356555e-03 -0.510210065 -0.053497713  
## expRad\_u 0.010987274 0.1024293935 1.952964e-02 0.044160593 0.155204710  
## expRad\_g 0.020061358 0.1553990368 -6.559323e-02 -0.160725385 0.341378827  
## expRad\_r 0.115308972 0.1294803474 5.644174e-02 0.134825189 -0.176736467  
## expRad\_i 0.064855298 0.0102822938 -2.429133e-02 -0.012980504 -0.195923006  
## expRad\_z -0.027298079 -0.0516848934 -3.617049e-02 -0.557576268 -0.149625424  
## q\_u 0.019337254 -0.0121347312 -1.525405e-02 0.081167516 -0.525686487  
## q\_g 0.184205271 0.4033984028 1.029494e-01 0.045583054 -0.039465617  
## q\_r -0.349799339 0.3081498408 4.931946e-02 -0.008507339 -0.029499319  
## q\_i 0.494204456 -0.0289456269 -1.884591e-02 -0.021801086 0.028989206  
## q\_z 0.050813069 0.1284168672 -1.067354e-02 0.063566203 -0.016232150  
## u\_u -0.053083575 -0.0144441660 -8.906893e-03 0.068103301 -0.491166342  
## u\_g -0.188355769 -0.4052780024 -9.152385e-02 0.098944776 0.080004977  
## u\_r -0.254405394 0.5339540985 9.662588e-02 -0.041976520 -0.084144106  
## u\_i 0.508427308 0.0281440917 -7.557007e-03 -0.024223501 0.019031032  
## u\_z -0.075642030 -0.1587911657 -4.033490e-02 -0.113316576 -0.004011342  
## expAB\_u 0.015232207 0.0732201224 -3.489359e-02 -0.034380825 0.005493520  
## expAB\_g -0.037509049 0.0241207527 1.938661e-04 -0.087393124 0.011443511  
## expAB\_r -0.015487683 0.0256636056 5.605079e-03 0.008222998 0.086405022  
## expAB\_i -0.011955553 0.0220714723 4.733735e-02 0.065841991 0.136666935  
## expAB\_z 0.016637264 0.0200108731 1.162647e-02 -0.013405460 0.100230296  
## ra -0.285102844 -0.0237474875 5.746781e-02 0.237365021 0.073775443  
## dec 0.012943563 -0.0449577983 2.229779e-02 0.186767150 -0.017522765  
## b -0.305244017 0.0030664470 3.035849e-02 0.034655793 0.052918124  
## l -0.081250637 0.0336716359 -3.218773e-02 -0.320621512 -0.068726221  
## PC11 PC12 PC13 PC14 PC15  
## rowv 0.031578676 -0.0486953700 0.0652016937 0.032328195 -0.0526115120  
## colv 0.031465963 -0.0147875316 0.0287887858 0.025550237 -0.0028437723  
## u -0.035937348 -0.0691055018 -0.1360718440 -0.009811297 0.0463740036  
## g 0.007126535 -0.0269885910 -0.0785482789 -0.014157051 -0.0106515838  
## r 0.012429927 -0.0333844998 -0.0556721592 -0.016298348 -0.0176547207  
## i 0.020106565 0.0167791576 -0.0267375613 -0.036448879 -0.0490190889  
## z 0.025267427 0.0370138843 -0.0398997262 -0.036965271 -0.0605613567  
## psfMag\_u -0.036312793 -0.0886139964 -0.1552382088 -0.015468438 0.0443467220  
## psfMag\_g 0.007396191 -0.0559808893 -0.0914092826 -0.016866966 -0.0368694409  
## psfMag\_r 0.029458905 -0.0388817825 -0.0853183759 -0.023082387 -0.0330902909  
## psfMag\_i 0.020815842 -0.0039839981 -0.0671359795 -0.030440138 -0.0562862204  
## psfMag\_z 0.008188456 0.0041630028 -0.0650869432 -0.034737518 -0.0627473558  
## modelFlux\_u 0.026289188 -0.0251206207 -0.0404252750 -0.008996876 -0.0098310799  
## modelFlux\_g 0.012126117 -0.0399891839 -0.0837152785 -0.020588047 -0.0117707626  
## modelFlux\_r 0.012206307 0.0127538185 0.0284855395 0.012584376 0.0115481679  
## modelFlux\_i 0.014621219 -0.0004345447 0.0091401616 0.006262929 0.0082189014  
## modelFlux\_z 0.005538690 -0.0368606898 -0.0805292997 -0.019602362 -0.0071794981  
## petroRad\_u -0.075917180 -0.3813120491 0.1569793229 0.022474959 0.4197797164  
## petroRad\_g -0.107071597 -0.2170764347 0.3060752340 -0.064553640 -0.3713739307  
## petroRad\_r 0.164950393 0.1234769218 -0.0595042242 -0.034243843 0.1200929848  
## petroRad\_i 0.006510617 -0.0779641667 0.0012522002 0.093403413 -0.2052129910  
## petroRad\_z -0.374847570 0.0444638157 -0.1825748734 0.116968057 0.0984077179  
## expRad\_u -0.047101761 -0.2767730837 -0.0350864364 0.058895656 0.5466946677  
## expRad\_g 0.016547381 -0.1663613905 0.2304565483 0.004010439 -0.4114041529  
## expRad\_r 0.176961834 0.1633847585 -0.0792813918 -0.052773851 -0.0030409119  
## expRad\_i 0.127397538 0.1647361251 -0.1278577420 0.007479637 -0.0685371328  
## expRad\_z -0.221009644 0.1029241646 -0.0818378194 0.023567220 0.0750025687  
## q\_u -0.117247219 -0.4230407720 0.0809981008 -0.014043051 -0.0998992711  
## q\_g -0.046679202 0.0688080316 0.0006833652 -0.182238769 -0.0897515961  
## q\_r 0.017278461 0.0346631393 -0.0070875552 0.000130359 -0.0003773150  
## q\_i -0.022032044 -0.0499184535 0.0207309942 0.002598367 0.0006621362  
## q\_z -0.054112007 -0.0030100423 -0.1116870388 0.689159597 -0.0400236168  
## u\_u -0.117628893 -0.4604391976 0.1152346660 -0.010805426 -0.0752387869  
## u\_g 0.127862276 -0.0770856719 0.0308537326 0.108052506 -0.0491288817  
## u\_r -0.011765521 0.0664670575 -0.0283299178 -0.024143350 0.0108133891  
## u\_i -0.027728918 -0.0363568932 0.0227769642 -0.014909102 0.0061006123  
## u\_z 0.016309614 0.0174707194 0.1239323539 -0.612908301 0.1700799130  
## expAB\_u 0.164425977 0.0517022241 0.1715190824 0.047059361 0.0055190791  
## expAB\_g 0.009590296 -0.0785819040 -0.2543986794 -0.123184490 -0.1116514169  
## expAB\_r -0.004821095 -0.1792256330 -0.4195095105 -0.117369323 -0.1293040937  
## expAB\_i -0.030529519 -0.2171902269 -0.5010438673 -0.124238470 -0.1564085414  
## expAB\_z -0.006157787 -0.1389848294 -0.2904267925 -0.056695975 -0.0407022014  
## ra -0.298129740 0.0658452810 0.0539235196 -0.010881337 -0.0059727149  
## dec -0.479188389 0.1880130176 0.0458483032 -0.049393834 -0.0199261897  
## b 0.060497105 -0.0555468423 0.0188139451 0.024791908 0.0137352852  
## l 0.553486145 -0.1634267505 -0.0121385973 0.077715226 0.0707083117  
## PC16 PC17 PC18 PC19  
## rowv -0.0213002581 -0.052304960 0.0292397854 0.0198345218  
## colv -0.0119059998 0.003351638 0.0011059321 0.0002381232  
## u 0.0812287006 -0.077670560 -0.2809029714 0.0576093000  
## g 0.0322003194 -0.053227483 -0.0867116408 -0.0044861713  
## r 0.0073227154 -0.040661370 0.0395663962 -0.0116563736  
## i -0.0123487994 -0.024972047 0.1349894526 -0.0505041352  
## z -0.0376132693 -0.030673232 0.2081705424 -0.0118172311  
## psfMag\_u 0.0655107112 -0.060213135 -0.2495391280 0.0417379152  
## psfMag\_g 0.0221816354 -0.047215206 -0.0588415789 0.0109352161  
## psfMag\_r 0.0007030575 -0.041557710 0.0571589302 -0.0074158823  
## psfMag\_i -0.0216013818 -0.025453328 0.1692042132 0.0010865402  
## psfMag\_z -0.0377608377 -0.024048007 0.2271849931 -0.0104638208  
## modelFlux\_u -0.0044227419 -0.005961773 0.0233110743 -0.0170622509  
## modelFlux\_g -0.0074035198 0.007156814 0.0073375028 -0.0064171521  
## modelFlux\_r 0.0106426916 -0.015545119 -0.0117574051 -0.0068211087  
## modelFlux\_i 0.0026441150 -0.001331844 -0.0159178383 -0.0032963878  
## modelFlux\_z -0.0069750131 0.013939379 -0.0072678936 -0.0018222302  
## petroRad\_u 0.0733878957 -0.129769328 0.3349448682 0.3272715324  
## petroRad\_g -0.0845115722 0.146569866 -0.0660005882 -0.3183569095  
## petroRad\_r 0.0363912548 -0.015710570 -0.1331304957 -0.1997568381  
## petroRad\_i -0.0647460869 -0.016313612 -0.0096290848 0.3897806813  
## petroRad\_z 0.1511146193 -0.086369370 -0.3565817129 0.1477635082  
## expRad\_u -0.0711171955 0.186222839 -0.0152927029 -0.2237193959  
## expRad\_g -0.0010620390 -0.028624384 -0.1147014520 0.0515752492  
## expRad\_r 0.0189904349 -0.018788118 -0.1066082817 -0.0472894823  
## expRad\_i -0.0317620760 0.036237017 0.1953039579 0.2265345660  
## expRad\_z 0.0410073587 -0.028784354 0.1024552612 -0.2486858873  
## q\_u 0.0303234457 -0.082863316 0.0141562260 0.1543769614  
## q\_g -0.0641270441 -0.005182692 -0.1048895434 0.3177734099  
## q\_r -0.0061055949 0.001662152 -0.0004167314 0.0092381839  
## q\_i -0.0015479957 0.016863651 -0.0134997242 -0.0197957960  
## q\_z -0.6389288204 -0.241555105 -0.0115110490 -0.0247000811  
## u\_u 0.0065722837 -0.005566247 -0.0711945842 -0.1292443521  
## u\_g 0.0311988442 -0.020366753 -0.0276175402 -0.0823457593  
## u\_r -0.0098729307 -0.002195082 -0.0067506204 0.0150215664  
## u\_i 0.0012369232 0.014081045 -0.0114348427 -0.0117322223  
## u\_z -0.6359555123 -0.290056699 -0.1500759238 0.0209987009  
## expAB\_u 0.3127311103 -0.771545161 0.0784651100 0.0181230241  
## expAB\_g -0.0807200712 0.031039059 0.2970488136 -0.0479062262  
## expAB\_r -0.0258346381 -0.005350517 0.2389656190 0.0012971983  
## expAB\_i 0.0192248408 -0.084378354 0.0245855217 -0.0420801113  
## expAB\_z 0.0359661411 -0.165246498 -0.3436996469 -0.0277967498  
## ra 0.0437995401 -0.040648261 -0.0255404933 -0.0211308363  
## dec -0.0514225362 0.171940703 0.0366109460 0.3636947222  
## b 0.0381673670 0.014861896 -0.0791163802 -0.0132593517  
## l -0.0521146400 0.281248021 -0.2141760527 0.3354621555  
## PC20 PC21 PC22 PC23  
## rowv 0.0357099983 0.0719202978 -0.0534220812 -0.1225846316  
## colv 0.0057446330 -0.0206291065 0.0698710210 0.1303437595  
## u 0.0037728395 -0.0109633814 0.1126787983 0.0861741810  
## g -0.0103125323 -0.0170618596 0.0181720895 0.0322554862  
## r -0.0135453983 -0.0119960090 -0.0063209485 0.0081938994  
## i -0.0626226652 -0.0523275759 0.0001393158 -0.0182751636  
## z -0.0584761602 -0.0283415573 -0.0241070430 -0.0552074571  
## psfMag\_u 0.0224064563 0.0134325729 0.1157826347 0.0830050054  
## psfMag\_g 0.0093013952 0.0124885384 0.0115679470 0.0257783626  
## psfMag\_r -0.0091314060 0.0009105681 -0.0159053863 -0.0041986896  
## psfMag\_i -0.0255365132 0.0110800066 -0.0017469000 -0.0126195834  
## psfMag\_z -0.0394096630 0.0003912223 0.0002182991 -0.0406864446  
## modelFlux\_u 0.0031601355 -0.0073948498 -0.0178006602 0.0049937348  
## modelFlux\_g -0.0036683107 -0.0228095728 -0.0166070328 0.0219124563  
## modelFlux\_r -0.0002711444 0.0060176460 -0.0021789706 -0.0267193273  
## modelFlux\_i 0.0101278360 0.0080309320 -0.0047470185 -0.0063676783  
## modelFlux\_z -0.0035507309 -0.0233943799 -0.0111112485 0.0218901108  
## petroRad\_u -0.1372875253 -0.2643693521 -0.2473714836 0.3229160081  
## petroRad\_g -0.0493052632 -0.1903234848 0.3474618988 0.2185410473  
## petroRad\_r -0.1670533356 -0.2758898164 -0.1056691383 -0.0691333487  
## petroRad\_i 0.0128816528 0.1657131600 -0.1760867005 -0.1869574877  
## petroRad\_z -0.0876610466 -0.1083644037 -0.1296925174 -0.1334580407  
## expRad\_u 0.2452835234 0.3862559436 0.1856502376 -0.3601736206  
## expRad\_g 0.1694414128 0.1976646085 -0.2372690946 -0.1972463976  
## expRad\_r -0.0682174587 -0.1107303656 -0.1952805085 -0.0808564081  
## expRad\_i 0.3405582406 0.4147106004 0.0764652071 0.4142801438  
## expRad\_z 0.0654732465 0.0134129117 0.0636847151 0.0662063005  
## q\_u 0.5330659957 -0.3776432487 0.1184721577 -0.1505829961  
## q\_g -0.0630685078 0.0482563781 -0.1383909150 -0.2162311655  
## q\_r -0.0045012923 -0.0113489633 -0.0044308414 -0.0051682016  
## q\_i -0.0596569045 0.0126644978 0.0130105306 -0.0118742188  
## q\_z -0.0532194558 -0.0641564015 0.0276949272 0.0193808880  
## u\_u -0.5002737866 0.4317283887 -0.1642538443 0.1101357173  
## u\_g -0.0540983929 -0.1063759387 -0.0173266707 -0.0341696380  
## u\_r 0.0344668931 0.0358645531 -0.0261719223 0.0322478342  
## u\_i -0.0453053743 0.0295998693 0.0172507872 -0.0180277048  
## u\_z 0.0123769499 -0.0009839398 0.0171073605 -0.0010551228  
## expAB\_u -0.1045065831 0.1509353649 0.3837278188 -0.1362841065  
## expAB\_g -0.1106698893 -0.0387961927 -0.0155951520 0.0131700247  
## expAB\_r -0.0567544192 0.0119889736 0.1728029473 -0.1640053696  
## expAB\_i -0.0970718599 -0.0736800465 0.0491361920 -0.1458529266  
## expAB\_z 0.1669454723 0.0950038084 -0.0502956094 0.4820837986  
## ra 0.0137390359 0.0300047504 0.0122129764 0.0106445127  
## dec -0.2380974656 0.0167442797 0.4545000022 0.0002130323  
## b 0.0116032381 0.0013502413 0.0091839987 -0.0037389088  
## l -0.2129770760 -0.0722071428 0.3640951159 -0.0098122092  
## PC24 PC25 PC26 PC27  
## rowv -0.1108423789 0.2562896052 0.2019531943 0.3241409281  
## colv 0.0959453950 -0.2242144073 -0.1823303009 -0.3533381772  
## u -0.1142399200 -0.0797457589 -0.0020074391 -0.0483882290  
## g -0.0260450979 0.0065560584 0.0129310353 -0.0026380811  
## r 0.0344363749 0.0325999265 -0.0046063687 0.0224014827  
## i 0.0965207762 0.0810801885 -0.0225610532 0.0334246370  
## z 0.0951997687 0.0840399702 -0.0527095371 0.0411886018  
## psfMag\_u -0.0903673766 -0.0712704828 0.0088703061 -0.0434713113  
## psfMag\_g -0.0187888360 0.0125928437 0.0086552542 -0.0026767336  
## psfMag\_r 0.0376602285 0.0374494906 -0.0066052573 0.0188088428  
## psfMag\_i 0.0990796803 0.0731153415 -0.0065351739 0.0335883831  
## psfMag\_z 0.1064177776 0.0898443791 -0.0323669468 0.0358712949  
## modelFlux\_u 0.0205799613 0.0047967590 -0.0034726835 0.0076855120  
## modelFlux\_g 0.0108085339 0.0097201520 -0.0006064413 0.0043587936  
## modelFlux\_r 0.0025067880 -0.0198214591 0.0093957297 0.0003727051  
## modelFlux\_i -0.0133878856 -0.0157712364 -0.0040024591 -0.0094028848  
## modelFlux\_z -0.0030568747 0.0032613780 0.0020197966 -0.0025261846  
## petroRad\_u -0.0839915169 -0.0443884858 0.0374700832 -0.0495994111  
## petroRad\_g 0.2303430566 -0.3031499699 0.0199149200 0.1916123432  
## petroRad\_r -0.0048995127 0.0741375183 -0.0194829544 -0.0197875523  
## petroRad\_i 0.0876367136 -0.2894529550 -0.5026842352 0.2232447004  
## petroRad\_z 0.2478728509 -0.0939618272 0.3849291289 0.1415332372  
## expRad\_u 0.2064696661 0.0093550397 -0.0688945718 0.0568484080  
## expRad\_g -0.2649617167 0.3260642214 0.1311410376 -0.2457526434  
## expRad\_r -0.0463960431 0.0011596844 -0.0842330662 -0.0044682958  
## expRad\_i 0.0176858473 -0.2346460494 0.3727062502 0.0132887708  
## expRad\_z -0.2840804540 -0.0380917627 -0.3598390025 -0.2866558524  
## q\_u -0.0004930136 0.0744067122 -0.0252779040 -0.0094748979  
## q\_g 0.3902515444 -0.1811355473 -0.1036103689 -0.2716220675  
## q\_r 0.0130043157 0.0041523923 0.0116792965 0.0034048887  
## q\_i -0.0079522652 0.0002639816 0.0106778319 -0.0122273743  
## q\_z -0.0191367517 0.0097491859 0.0142944921 -0.0387912311  
## u\_u -0.0436597531 -0.0109666023 0.0371660681 -0.0484472545  
## u\_g 0.3502649769 0.0292906936 0.2777375587 -0.3970905554  
## u\_r -0.0440668602 -0.0093509017 0.0004507223 0.0148610313  
## u\_i -0.0276830009 -0.0010986832 0.0014999755 -0.0032499726  
## u\_z -0.0669057890 -0.0279837779 0.0921124389 -0.0068927927  
## expAB\_u 0.0773801666 -0.0064235969 -0.0381827109 0.0424131116  
## expAB\_g 0.3185004396 0.2534734067 -0.0434279854 0.2344561746  
## expAB\_r -0.0031001037 0.0222035268 0.1268668460 -0.4121526294  
## expAB\_i -0.3423348441 -0.3060764670 0.0706340300 0.1937385296  
## expAB\_z 0.2446973462 0.4130261664 -0.3079786171 0.0251785463  
## ra -0.0324193466 0.0044564900 0.0127087917 -0.0339237689  
## dec -0.0834708974 0.3189686154 -0.0420742378 -0.0457139011  
## b -0.0451197375 -0.0443632828 0.0167619475 -0.0161161065  
## l -0.0943681006 0.1188464685 -0.0167154170 0.0016805278  
## PC28 PC29 PC30 PC31 PC32  
## rowv -0.434212378 0.2415006912 0.042630066 -0.0097008111 0.0035306819  
## colv 0.452354054 -0.2362407500 -0.017301566 0.0063468449 -0.0014031232  
## u 0.002464370 0.1383468510 0.360791084 -0.0387672395 0.0140986879  
## g 0.011149139 -0.0050184598 0.025988532 -0.1306778717 -0.0119783552  
## r 0.006394109 -0.0289708661 -0.043394557 -0.0280057196 0.0182103643  
## i 0.016324154 -0.0363871553 -0.119830583 -0.0251693377 0.0048939953  
## z 0.021844845 -0.0531602450 -0.151408033 -0.0146089195 -0.0072988158  
## psfMag\_u -0.005161199 0.1314199595 0.328113623 -0.0048508843 0.0004769258  
## psfMag\_g 0.010676733 -0.0012936907 0.025031736 -0.1066423630 -0.0267566048  
## psfMag\_r 0.010190445 -0.0198952668 -0.052183821 -0.0185686938 -0.0093963829  
## psfMag\_i 0.021429649 -0.0406200443 -0.120912999 -0.0159575362 -0.0079593612  
## psfMag\_z 0.001424985 -0.0568146724 -0.154796543 0.0088428809 0.0091706397  
## modelFlux\_u 0.003207297 0.0065484321 -0.019403377 0.2475948697 0.0069816499  
## modelFlux\_g 0.008955091 0.0294251308 0.030737499 0.4363860127 0.0122034294  
## modelFlux\_r 0.001907900 -0.0176605699 -0.035626640 -0.6364960471 -0.0263607706  
## modelFlux\_i -0.005829837 -0.0006264115 0.003450084 -0.4667663330 -0.0142036143  
## modelFlux\_z 0.006121199 0.0322119306 0.049284804 0.2766995360 0.0164906943  
## petroRad\_u -0.070994235 0.0201731359 0.002897824 -0.0192660766 -0.0031199527  
## petroRad\_g -0.089882949 0.1341137100 -0.044586987 -0.0007091230 -0.0117012864  
## petroRad\_r 0.019268806 0.0503739314 -0.018832618 0.0006734551 0.1244993137  
## petroRad\_i 0.002665001 0.2027076893 -0.003376991 -0.0007473796 -0.0636120950  
## petroRad\_z 0.205714285 0.0483568834 -0.077545785 0.0173392457 -0.0064182667  
## expRad\_u 0.015243475 -0.0964057260 -0.039510918 0.0168880410 -0.0053775875  
## expRad\_g 0.186747954 -0.1652374037 0.084184615 0.0202872411 0.0322580652  
## expRad\_r 0.054447318 0.0367097242 -0.002285834 0.0170335975 -0.1303246554  
## expRad\_i -0.042187453 -0.0406007761 0.044927676 0.0007798224 0.0636583503  
## expRad\_z -0.356480010 -0.0029727978 0.217584268 0.0031174860 0.0283237882  
## q\_u 0.032922003 -0.0474607449 0.018699521 -0.0017936800 0.0039166361  
## q\_g -0.391973298 -0.0928405567 0.082530012 0.0021948272 0.1149566984  
## q\_r -0.005969257 0.0037157176 -0.002963172 -0.0271277858 0.6918574176  
## q\_i -0.006103199 0.0015241142 0.015663200 0.0069419074 -0.0371618335  
## q\_z -0.007535787 -0.0103890426 0.025039253 -0.0007821850 0.0184259957  
## u\_u 0.020366417 0.0017188732 -0.002971978 0.0038704071 0.0109268798  
## u\_g -0.367739742 -0.2342373341 0.079258353 0.0104095575 -0.1709316776  
## u\_r -0.007998056 0.0164611558 0.002685724 0.0257374299 -0.6513600173  
## u\_i 0.021628859 0.0010891678 0.006102409 -0.0092811833 0.0705437299  
## u\_z 0.046253274 0.0121384463 -0.001271824 0.0021041926 -0.0216592279  
## expAB\_u 0.032691179 -0.0287145718 0.019528829 0.0570647269 -0.0027339748  
## expAB\_g 0.198461295 -0.1238446808 0.663857526 -0.0750440932 -0.0131967728  
## expAB\_r 0.096962023 0.6047215042 -0.178954613 -0.0554267644 -0.0116183629  
## expAB\_i -0.162598634 -0.5039004081 -0.158352916 -0.0085722923 0.0070816878  
## expAB\_z -0.107046257 -0.0381134687 -0.296084725 -0.0248528484 0.0128536969  
## ra -0.002360822 0.0078879723 0.026797439 0.0013324547 -0.0103828275  
## dec -0.018107505 -0.1581545363 -0.068187938 0.0130974569 -0.0278839833  
## b -0.015065880 0.0404144392 0.035380333 0.0097372851 0.0062655258  
## l -0.028529107 -0.0522848245 -0.002376309 0.0050299573 -0.0121111048  
## PC33 PC34 PC35 PC36 PC37  
## rowv 0.0036159901 -0.012575595 -0.0007413231 -0.011520956 0.0111587881  
## colv -0.0016233833 0.014167749 0.0035591718 0.007602739 -0.0004045487  
## u 0.3343198956 0.119851736 0.0702924692 -0.123157526 0.1309111889  
## g -0.4427241712 -0.119966718 0.0115683755 -0.480839226 -0.0310007682  
## r -0.2095453186 0.017445893 0.0493599350 0.021908558 0.0713752383  
## i 0.0889258908 0.054527273 0.0959180819 -0.066714527 0.6853479423  
## z 0.3982759270 0.107379760 0.0576206212 -0.478704749 -0.2796190552  
## psfMag\_u 0.2372395305 0.043196142 -0.0195456284 0.244969814 -0.0896246256  
## psfMag\_g -0.4203821801 -0.187145184 -0.1228660142 -0.012472575 -0.2107778353  
## psfMag\_r -0.1921316941 -0.096300669 -0.1005923076 0.380191653 -0.1083834520  
## psfMag\_i 0.0377060692 0.002109560 -0.0118426449 0.369829734 0.2652543737  
## psfMag\_z 0.2719070471 0.102748310 -0.0078186258 0.122824858 -0.4272299355  
## modelFlux\_u -0.0317261645 -0.007900542 -0.0074148625 -0.087415091 0.1362667111  
## modelFlux\_g -0.0266101935 -0.014267601 0.0035866362 -0.039904539 -0.0087430128  
## modelFlux\_r 0.0825917771 0.030227547 0.0049579109 0.020853010 0.1483183510  
## modelFlux\_i 0.0385268393 0.012771688 -0.0078634963 0.067027269 -0.1916282079  
## modelFlux\_z -0.0262045093 -0.009690318 0.0077241298 0.049661196 -0.0881127843  
## petroRad\_u 0.0078601499 -0.023699545 -0.0113487826 -0.019871959 0.0114995960  
## petroRad\_g 0.0072779637 -0.034234340 -0.0188893311 -0.016696334 -0.0159590193  
## petroRad\_r -0.2157832136 0.474538057 0.3757365324 0.075169484 -0.0759337080  
## petroRad\_i -0.0499437552 0.059802061 0.0810070235 -0.004152152 0.0185198380  
## petroRad\_z 0.0117814059 -0.007292856 -0.0010682825 -0.017271436 -0.0027581233  
## expRad\_u 0.0087495201 -0.009732786 -0.0101813247 -0.037472511 0.0201138032  
## expRad\_g 0.0193224505 0.087160738 0.0556342769 -0.011550850 0.0189326098  
## expRad\_r 0.2029463866 -0.502656215 -0.3993008519 -0.042486562 0.0716324492  
## expRad\_i -0.0404813259 0.089385204 0.0665825469 -0.038291154 0.0201372340  
## expRad\_z -0.0522596850 -0.014804340 0.0016762353 -0.026566048 0.0223840655  
## q\_u -0.0074888350 0.002830265 -0.0007821829 0.001244373 0.0037602829  
## q\_g -0.0552462447 0.065583509 0.0268924352 0.003721790 0.0024499791  
## q\_r 0.0331701761 -0.096391373 -0.1455853823 -0.013971011 0.0154017727  
## q\_i -0.0737080987 0.436789899 -0.5446731273 -0.053510941 0.0465971135  
## q\_z -0.0039667466 -0.004775994 0.0010736792 0.003312205 0.0087742623  
## u\_u 0.0056032934 0.002047716 0.0027809975 0.002975216 -0.0039408802  
## u\_g 0.0590445157 -0.094495297 -0.0022127872 -0.004017145 -0.0059805218  
## u\_r -0.0331826407 0.078066319 0.1059110932 0.020525866 -0.0049783064  
## u\_i 0.0592976899 -0.420447781 0.5544949789 0.048067151 -0.0406606854  
## u\_z 0.0005930653 -0.008843784 -0.0057704156 0.001706002 0.0023346812  
## expAB\_u -0.0301257588 -0.007977711 -0.0101315670 0.001634280 -0.0073879675  
## expAB\_g -0.0786347921 -0.022979114 0.0002481644 -0.031448296 0.0068164875  
## expAB\_r -0.0334123805 -0.016816425 -0.0027322523 -0.036424356 0.0211514309  
## expAB\_i 0.0352159461 0.024536479 0.0158606729 -0.003937534 0.0280854508  
## expAB\_z 0.0422376784 0.009346924 0.0050149480 -0.009713218 0.0038998158  
## ra -0.0146800293 -0.014196053 0.0103987610 -0.232961353 0.0559804302  
## dec -0.0328087567 -0.023825075 -0.0205215476 0.082681642 -0.0103909279  
## b -0.0090810069 0.007031137 -0.0194270915 0.232555908 -0.0617556311  
## l -0.0085317554 -0.011323910 0.0054600100 -0.102618673 0.0249012848  
## PC38 PC39 PC40 PC41  
## rowv 0.0011757833 -0.0029437185 -0.0026145165 -7.062654e-04  
## colv -0.0021252034 -0.0047680923 0.0005708223 7.435507e-05  
## u -0.0372597404 -0.0533741348 -0.1662781705 -1.364727e-01  
## g -0.0833339945 -0.1904587965 0.1601755992 -1.105735e-02  
## r 0.1194471255 0.0379782252 -0.6808751990 1.099027e-01  
## i 0.1120326571 -0.1884136320 0.1274316841 -7.171970e-02  
## z -0.0527817015 -0.1206418651 -0.1132858087 7.291125e-02  
## psfMag\_u -0.0309803844 0.1229110275 0.1904238881 1.114218e-01  
## psfMag\_g -0.1048044345 0.0333343262 0.3233992794 -2.257172e-02  
## psfMag\_r 0.0663770025 0.2030852216 -0.3518679383 5.414951e-02  
## psfMag\_i 0.0504572313 0.0590622806 0.3200274818 -1.536004e-01  
## psfMag\_z -0.0799310004 0.1398513527 0.1810537162 4.725122e-03  
## modelFlux\_u -0.5919291162 0.3403450389 -0.0947634234 -2.949031e-01  
## modelFlux\_g 0.0426954570 -0.0441259741 0.0298011173 5.312288e-02  
## modelFlux\_r -0.2171164365 0.1285124785 0.0322968757 5.513843e-01  
## modelFlux\_i 0.3119608241 -0.1503931836 -0.0695948378 -6.399362e-01  
## modelFlux\_z 0.4355153863 -0.2557290266 0.1031492764 3.394984e-01  
## petroRad\_u -0.0033984996 -0.0074637449 -0.0000687011 -1.129367e-03  
## petroRad\_g 0.0047805130 -0.0019806041 -0.0284957558 2.165588e-03  
## petroRad\_r -0.0163998294 0.0414746935 0.0291055347 -3.668205e-03  
## petroRad\_i 0.0026020978 -0.0041505514 0.0036343297 2.552467e-03  
## petroRad\_z 0.0052469183 -0.0128547851 -0.0132802127 -1.357670e-02  
## expRad\_u 0.0094743867 -0.0209128336 -0.0132397726 -1.526955e-03  
## expRad\_g 0.0069932856 -0.0037696899 -0.0117534690 3.098548e-03  
## expRad\_r 0.0232053725 -0.0252666124 -0.0876918515 2.410617e-03  
## expRad\_i 0.0062395084 -0.0118076073 -0.0283687002 6.767120e-03  
## expRad\_z 0.0042588612 0.0015555818 -0.0119297608 5.615860e-03  
## q\_u 0.0001343778 0.0002566142 0.0009019757 -2.511544e-04  
## q\_g -0.0017138999 -0.0003530082 0.0056567241 3.581794e-04  
## q\_r 0.0009918960 -0.0004152925 0.0027973404 1.860457e-04  
## q\_i 0.0169075382 -0.0165718877 0.0011128632 -3.628242e-04  
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## u\_u -0.0017354521 -0.0019687958 0.0001270731 2.574536e-04  
## u\_g -0.0010788840 -0.0066950647 -0.0067794839 -2.635883e-05  
## u\_r -0.0003170816 0.0001863691 -0.0037517519 -7.367918e-04  
## u\_i -0.0180806272 0.0126950016 0.0034061721 -1.422545e-03  
## u\_z -0.0015580829 -0.0012005441 -0.0001060159 -1.013872e-03  
## expAB\_u 0.0233889506 -0.0126372386 -0.0023076692 4.940808e-03  
## expAB\_g -0.0129843417 0.0024178354 -0.0194280492 -2.662676e-04  
## expAB\_r 0.0034140522 0.0006794662 -0.0280269252 -1.955352e-03  
## expAB\_i 0.0095619455 -0.0010165164 -0.0002907659 -9.392911e-03  
## expAB\_z 0.0031827000 -0.0112886054 -0.0092422890 3.500589e-03  
## ra 0.3203814527 0.5076257786 0.0742857442 -9.470144e-03  
## dec -0.1058857579 -0.1492815018 -0.0619610064 2.596803e-03  
## b -0.3448134455 -0.5364861826 -0.0757698675 1.064840e-02  
## l 0.1231236914 0.1864122710 0.0224479082 -1.456041e-03  
## PC42 PC43 PC44 PC45  
## rowv -1.944740e-03 0.0002168547 1.466657e-03 0.0013511113  
## colv 5.316184e-04 -0.0051844051 -1.058419e-03 -0.0013438930  
## u -5.927860e-01 -0.0498981044 -2.445294e-03 0.0134690975  
## g 6.819649e-02 0.0745564487 5.205432e-01 -0.2265638079  
## r 1.750396e-01 -0.1234258837 1.449715e-01 0.4949692608  
## i 1.845099e-01 -0.3740801874 -2.380290e-01 -0.2101700706  
## z 1.255870e-01 0.4139841759 -3.057761e-01 -0.0021228774  
## psfMag\_u 6.523956e-01 0.0571052191 2.012755e-02 -0.0148759312  
## psfMag\_g -1.518911e-01 -0.1326116543 -5.821401e-01 0.2810512941  
## psfMag\_r -1.275911e-01 0.1571273933 -1.538886e-01 -0.6304487852  
## psfMag\_i -1.896431e-01 0.5377435347 2.694411e-01 0.3013651827  
## psfMag\_z -1.336272e-01 -0.5646710128 3.270151e-01 -0.0044954653  
## modelFlux\_u 9.199670e-02 -0.0244248674 -3.353045e-02 0.1019955863  
## modelFlux\_g -4.234476e-02 0.0634089289 5.865422e-02 -0.2204366497  
## modelFlux\_r -9.387344e-02 0.0206765560 2.842636e-02 -0.0228901637  
## modelFlux\_i 1.321675e-01 -0.0165823594 -1.635397e-02 -0.0259826969  
## modelFlux\_z -8.149969e-02 -0.0422684688 -3.531558e-02 0.1655926566  
## petroRad\_u 7.259528e-03 -0.0037350505 2.849694e-03 0.0004951591  
## petroRad\_g -8.817902e-03 0.0113997250 1.333152e-02 0.0080838128  
## petroRad\_r -6.815060e-03 0.0220152119 -3.764159e-02 -0.0003092263  
## petroRad\_i 1.025444e-02 -0.0131788661 4.025574e-03 -0.0087503562  
## petroRad\_z 6.805053e-03 -0.0014840167 -3.873437e-03 0.0011568924  
## expRad\_u -1.198469e-02 -0.0047564676 -2.950457e-04 0.0014880047  
## expRad\_g 1.231787e-02 -0.0019477579 1.327878e-02 -0.0044209807  
## expRad\_r 1.171104e-02 -0.0561266062 7.875487e-02 0.0617021913  
## expRad\_i 8.813835e-03 -0.0202420216 -1.669012e-02 -0.0117510112  
## expRad\_z 4.871400e-04 0.0130288063 6.404341e-04 0.0032144104  
## q\_u 1.955345e-04 -0.0005362306 1.315936e-03 -0.0011295525  
## q\_g 2.343103e-03 0.0026763734 -4.188200e-03 -0.0023330130  
## q\_r 1.838714e-03 0.0072098150 1.741969e-03 -0.0001289348  
## q\_i 4.040560e-03 0.0021382209 6.963921e-03 0.0009814365  
## q\_z 1.799091e-03 -0.0022485549 -1.559131e-04 0.0009962358  
## u\_u -7.396492e-04 0.0001858095 -9.540236e-05 -0.0005451002  
## u\_g -4.803808e-03 0.0017703729 2.007403e-03 0.0023070161  
## u\_r -1.611734e-03 -0.0029369059 -2.438249e-03 -0.0010988882  
## u\_i -6.564948e-03 -0.0047799059 -6.466875e-03 -0.0010949758  
## u\_z -2.411459e-03 -0.0011305636 1.713493e-03 0.0006990739  
## expAB\_u 6.090436e-03 -0.0015826926 -1.563697e-03 0.0028760940  
## expAB\_g 4.613875e-03 -0.0033022228 1.978272e-03 -0.0010344175  
## expAB\_r 3.635938e-03 -0.0043819183 2.298316e-03 0.0048452463  
## expAB\_i 4.514763e-03 0.0009797904 -2.495400e-03 0.0016543171  
## expAB\_z -1.342768e-03 0.0025664454 -4.176888e-04 0.0003297285  
## ra -1.124021e-02 0.0010725009 -9.615566e-04 0.0004844331  
## dec 9.620255e-03 -0.0010519639 -3.923952e-03 -0.0019362855  
## b 1.642179e-02 -0.0002519306 -8.773466e-04 0.0001772168  
## l -9.180548e-05 0.0001289179 9.750074e-04 0.0014347257  
## PC46  
## rowv 2.101806e-04  
## colv 5.057577e-05  
## u -1.757186e-02  
## g -1.235520e-01  
## r 1.627939e-01  
## i -1.574539e-02  
## z -2.671751e-02  
## psfMag\_u 8.282421e-03  
## psfMag\_g 1.485141e-01  
## psfMag\_r -1.871916e-01  
## psfMag\_i 2.647030e-02  
## psfMag\_z 1.438599e-02  
## modelFlux\_u -3.377934e-01  
## modelFlux\_g 7.142359e-01  
## modelFlux\_r 1.059797e-01  
## modelFlux\_i 4.178930e-02  
## modelFlux\_z -5.112947e-01  
## petroRad\_u -7.716125e-04  
## petroRad\_g 3.002557e-04  
## petroRad\_r 5.234913e-03  
## petroRad\_i -2.103416e-03  
## petroRad\_z -3.786762e-03  
## expRad\_u -3.946741e-05  
## expRad\_g -2.168757e-03  
## expRad\_r 1.290583e-02  
## expRad\_i 3.522131e-04  
## expRad\_z 2.106076e-03  
## q\_u -1.397122e-04  
## q\_g 1.012298e-03  
## q\_r 3.264364e-03  
## q\_i -3.782989e-04  
## q\_z 8.252825e-04  
## u\_u -3.453073e-04  
## u\_g -8.703762e-04  
## u\_r -2.678735e-03  
## u\_i 6.358570e-04  
## u\_z 1.657579e-04  
## expAB\_u 6.218622e-04  
## expAB\_g -8.220759e-04  
## expAB\_r 1.393989e-03  
## expAB\_i 4.337529e-04  
## expAB\_z -2.029409e-03  
## ra 1.464242e-04  
## dec 5.705542e-05  
## b 2.527899e-04  
## l 5.643335e-04

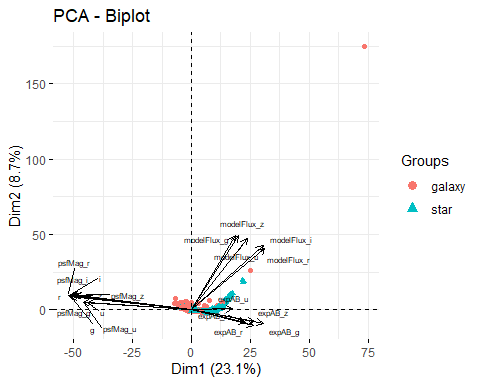
# Biplot with all variables  
fviz\_pca(pca\_result, label = "var", labelsize = 2, habillage = dat$type, col.var = "black", jitter = "point")

## Warning: argument jitter is deprecated; please use repel instead.  
  
## Warning: argument jitter is deprecated; please use repel instead.



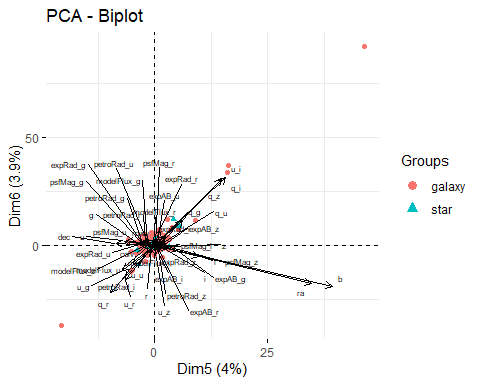
# Biplot of variables with largest PC1 and PC2 loadings  
fviz\_pca(pca\_result, label = "var", labelsize = 2, habillage = dat$type, col.var = "black", jitter = "point", select.var = list(name = c("expAB\_u", "expAB\_g", "expAB\_r", "expAB\_i", "expAB\_z", "modelFlux\_g", "modelFlux\_u", "modelFlux\_r","modelFlux\_i","modelFlux\_z", "psfMag\_u", "psfMag\_g", "psfMag\_r", "psfMag\_i", "psfMag\_z", "u", "g", "r", "i", "z")))

## Warning: argument jitter is deprecated; please use repel instead.  
  
## Warning: argument jitter is deprecated; please use repel instead.



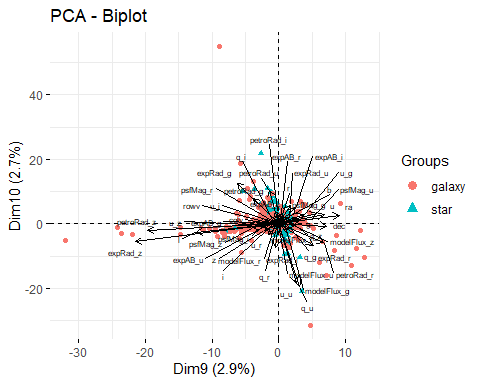
# Viewing biplots of different PCs. This is done by changing the axes (axes must be of length 2)  
fviz\_pca(pca\_result, axes = c(5, 6), label = "var", labelsize = 2, habillage = dat$type, col.var = "black", jitter = "point")

## Warning: argument jitter is deprecated; please use repel instead.  
  
## Warning: argument jitter is deprecated; please use repel instead.

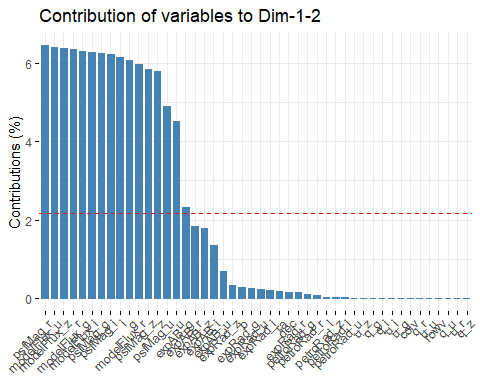


fviz\_pca(pca\_result, axes = c(9, 10), label = "var", labelsize = 2, habillage = dat$type, col.var = "black", jitter = "point")

## Warning: argument jitter is deprecated; please use repel instead.  
  
## Warning: argument jitter is deprecated; please use repel instead.



# This shows the relative contribution of original variables to the PC1 and PC2. Change axes to view different PCs (no limit on axes length).  
fviz\_contrib(pca\_result, choice ="var",axes=1:2)



# Display your data set projected on the principal components ("rotated")  
options(max.print=1000)  
pca\_result$x

## PC1 PC2 PC3 PC4 PC5  
## [1,] -2.602927e+00 7.875572e-01 -1.654099e-01 1.944012e+00 6.818667e-01  
## [2,] -2.119988e+00 2.072444e-01 8.605213e-01 -1.510643e-01 -1.145725e+00  
## [3,] 2.056557e+00 -8.356414e-01 -2.043331e-01 3.488097e-02 1.007090e+00  
## [4,] 1.435995e+00 -8.656009e-01 -9.358456e-01 5.291304e-01 -1.148274e+00  
## [5,] 9.049818e+00 -1.270400e+00 -5.509593e-02 1.543476e-01 -5.811640e-01  
## [6,] 2.714929e+00 -1.249562e+00 4.733394e-03 -1.859244e-02 -1.110924e+00  
## [7,] 3.451899e+00 -1.338393e+00 3.110204e-01 -1.461671e-01 -2.687058e-01  
## [8,] -9.239667e-01 -5.419498e-02 1.204490e+00 2.505819e-01 -1.345397e+00  
## [9,] 5.315681e+00 -4.502025e-02 -2.469412e+00 4.761751e-01 1.171447e-01  
## [10,] 7.136015e-01 -4.803365e-01 5.731751e-01 -1.633191e-01 -1.049771e+00  
## [11,] 6.290891e+00 -1.674565e+00 -3.311399e-03 -2.822434e-02 1.366193e-01  
## [12,] 1.116508e+01 1.658886e+00 1.879987e-01 1.972500e-01 -2.143232e-01  
## [13,] -1.695490e-01 -2.579954e-01 6.760896e-01 -2.517693e-01 -9.622326e-01  
## [14,] -1.094535e+00 3.454698e-01 -1.580461e+00 7.091007e-01 1.268241e+00  
## [15,] -8.021572e-01 -7.624031e-03 4.608484e-01 -2.303717e-01 -1.336316e+00  
## [16,] 1.367595e+00 -9.154108e-01 6.802298e-01 4.954126e-03 -1.213506e+00  
## [17,] -2.726119e+00 5.887560e-01 9.217948e-01 -1.420065e-01 -3.351012e-01  
## [18,] 1.073018e-01 -4.815521e-01 5.884925e-01 -1.879603e-01 -4.508918e-01  
## [19,] 1.460574e+00 -9.432604e-01 7.333170e-02 1.308306e-01 -8.441108e-01  
## [20,] -1.316483e+00 2.480839e-01 1.195074e-01 -1.670039e-02 1.528359e+00  
## [21,] -2.740473e+00 7.819968e-01 5.580223e-01 -1.814187e-01 1.515560e+00  
## PC6 PC7 PC8 PC9 PC10  
## [1,] 8.035022e-01 -4.842317e-01 -1.873902e-01 -3.289079e-01 -8.977076e-01  
## [2,] 6.032072e-01 -3.075761e-01 1.504057e+00 2.683369e-01 1.471911e-01  
## [3,] -4.259429e-01 1.388493e-01 2.106106e-02 -2.313984e-01 4.595724e-02  
## [4,] 5.300991e-01 8.416558e-01 1.588272e-01 4.685484e-01 1.783707e+00  
## [5,] 3.825087e-01 -1.779369e-02 -1.011023e-01 -4.378706e-02 2.289191e-01  
## [6,] 5.393917e-01 7.285268e-02 -1.743069e-01 -7.948694e-01 -2.397102e-01  
## [7,] 1.126548e-01 4.874598e-03 -1.097373e-01 -1.101558e-01 -1.336476e-01  
## [8,] -5.781918e-01 -1.041296e-01 -9.111890e-02 -1.952530e-01 3.493605e+00  
## [9,] -5.276625e-01 -6.385868e-01 4.374600e-01 2.444676e+00 -2.142256e-01  
## [10,] 5.112497e-01 -7.291589e-02 -2.893697e-02 -6.337386e-01 -5.423580e-01  
## [11,] 5.791558e-02 6.401404e-02 -1.280139e-01 -4.205964e-01 -1.577688e-01  
## [12,] 3.210984e-01 -1.103720e-01 -3.854057e-01 -7.389592e-01 -7.762862e-01  
## [13,] 4.281528e-01 5.190098e-02 -7.220222e-02 2.487383e-01 -7.525537e-02  
## [14,] -5.354867e-01 -3.448220e-01 2.602433e+00 -4.444547e-01 -5.743777e-01  
## [15,] 4.654328e-01 -1.708379e-01 -1.049255e-02 -3.553193e-01 -6.503029e-01  
## [16,] 5.510904e-01 -2.643543e-01 1.401525e-02 7.875261e-01 3.441242e-01  
## [17,] 2.036783e-01 2.408591e-01 -1.648631e+00 2.298077e-01 -3.308282e-01  
## [18,] 2.833923e-01 -1.143865e-01 3.442479e-02 2.954022e-01 -1.694196e-02  
## [19,] 4.254641e-01 5.553764e-02 -5.819446e-03 4.348451e-02 6.791730e-01  
## [20,] -7.895330e-01 1.615671e-02 7.196531e-01 -2.782856e-01 -1.039163e-01  
## [21,] -7.350249e-01 -1.003255e-01 -2.121319e-01 -8.303219e-02 -5.839934e-01  
## PC11 PC12 PC13 PC14 PC15  
## [1,] 5.824748e-01 6.479918e-01 4.586120e-01 3.885778e-02 1.322888e-01  
## [2,] 3.518031e-01 -3.067066e-01 -4.283767e-01 6.894016e-03 -9.525447e-02  
## [3,] 4.683746e-01 -2.142293e-01 -7.473097e-01 -1.553887e-01 -4.035921e-01  
## [4,] -3.020056e-01 -3.143975e+00 -2.224313e+00 -1.886992e-01 3.355163e+00  
## [5,] 2.237797e-01 -3.782512e-01 -5.899733e-01 -6.365521e-02 -6.809789e-02  
## [6,] 5.555022e-01 -3.154418e-01 -1.133932e+00 -3.660131e-01 -6.335440e-01  
## [7,] 2.269629e-01 -8.867828e-02 -4.002722e-01 -1.778595e-01 -2.195146e-01  
## [8,] 9.587051e-01 2.207393e+00 -1.280659e-01 1.665815e-01 9.020344e-01  
## [9,] -5.452769e+00 2.623226e+00 3.284139e+00 4.136793e-01 9.345767e-01  
## [10,] 2.262359e-01 4.697685e-01 4.657057e-01 1.117653e-01 4.152825e-03  
## [11,] 4.431844e-01 1.782952e-01 5.596614e-01 1.595274e-01 8.187938e-02  
## [12,] 7.866157e-01 1.301242e+00 3.492197e+00 1.030918e+00 8.573039e-01  
## [13,] 3.462705e-01 4.025294e-02 3.619867e-01 1.046855e-01 3.998663e-01  
## [14,] 9.272199e-01 5.612795e-02 -2.610491e-01 3.495058e-01 3.893578e-01  
## [15,] 2.720705e-01 6.493492e-01 5.068751e-01 2.424856e-01 1.018528e-01  
## [16,] 4.270994e-02 -4.109478e-01 -1.138015e+00 -1.784177e-01 -9.968194e-02  
## [17,] 6.399145e-02 2.868228e-01 1.017515e-01 8.170564e-01 -2.030681e-01  
## [18,] 1.992836e-01 -2.917017e-01 -9.196626e-01 -2.226527e-01 -1.786898e-01  
## [19,] -2.756210e-01 -1.118949e+00 -2.113160e+00 -2.953158e-01 6.025325e-01  
## [20,] 7.890484e-01 -1.941829e-01 -3.086400e-01 -2.155578e-02 -4.196743e-01  
## [21,] 7.624433e-01 5.659118e-01 9.266791e-01 3.065089e-01 -4.205182e-02  
## PC16 PC17 PC18 PC19 PC20  
## [1,] -4.543433e-01 6.035175e-01 7.590846e-01 1.119596e-01 3.381261e-01  
## [2,] 9.340803e-02 -6.358546e-02 -8.526216e-02 5.229235e-02 -1.878685e-01  
## [3,] 2.130191e-01 -6.753931e-01 -8.296893e-02 2.427603e-02 -1.807642e-01  
## [4,] -4.197639e-01 8.923368e-01 -2.516532e-01 -8.910667e-01 1.408124e+00  
## [5,] 1.362643e-01 -2.415022e-01 -1.078004e+00 -1.009029e-01 3.603374e-01  
## [6,] -4.320876e-01 4.502787e-01 1.139092e+00 -3.186078e-01 -1.444887e-01  
## [7,] -3.620128e-01 7.107700e-01 3.585918e-01 -1.295903e-01 -1.514853e-01  
## [8,] -3.387932e-01 1.241490e+00 -1.305356e+00 -3.282554e+00 -1.308430e+01  
## [9,] -3.574079e-01 2.238009e+00 -6.923491e-01 3.020916e+00 -1.076879e+00  
## [10,] -1.950680e-01 1.784572e-01 8.082767e-01 1.801272e-02 -3.481575e-01  
## [11,] 1.572898e-01 -4.641650e-01 -7.127222e-01 -1.296234e-01 3.652002e-01  
## [12,] 6.012226e-01 -9.562653e-01 -6.844959e-01 1.474855e-02 5.210529e-01  
## [13,] 9.078936e-02 -2.089871e-02 -4.962077e-01 4.204871e-02 9.475303e-02  
## [14,] -5.027881e-01 4.666954e-01 -1.210361e-01 -4.504299e-01 -1.668977e-01  
## [15,] 5.826051e-02 1.747567e-02 2.336067e-02 9.982191e-02 -3.455754e-01  
## [16,] 2.190541e-02 3.647197e-01 -1.038966e+00 1.788677e-01 6.410551e-02  
## [17,] 6.034596e-01 6.856641e-01 3.957704e-01 -1.818032e-02 -3.177377e-01  
## [18,] -1.029316e-02 -4.376011e-02 -1.211923e+00 5.252034e-02 4.508261e-01  
## [19,] -7.202031e-02 2.465419e-01 -1.179978e+00 -1.680039e-01 6.216816e-01  
## [20,] -1.694452e-01 6.797308e-01 1.544126e-01 3.044621e-01 -2.388944e-01  
## [21,] -1.724621e-01 7.067788e-01 1.579500e-01 4.279492e-01 -2.240564e-01  
## PC21 PC22 PC23 PC24 PC25  
## [1,] 3.888129e-01 -4.352068e-01 4.816230e-02 1.807458e-01 -6.063616e-02  
## [2,] -3.073772e-01 5.952714e-01 -5.498925e-02 -1.678502e-01 -7.079074e-01  
## [3,] 6.024562e-02 3.487673e-01 2.699528e-01 4.590656e-01 5.511879e-01  
## [4,] 1.798589e+00 8.323111e-01 -6.812369e-01 1.142331e+00 7.818321e-01  
## [5,] 2.381219e-01 1.953171e-01 5.368636e-01 -3.780682e-01 5.850901e-02  
## [6,] -2.787953e-01 -6.328304e-01 -4.281123e-01 4.456621e-01 -3.920309e-02  
## [7,] -2.789139e-01 -2.323602e-01 -8.408091e-02 3.582110e-03 4.546705e-02  
## [8,] 9.477970e+00 -3.563691e+00 3.596318e+00 -7.418338e-01 -1.555252e+00  
## [9,] 2.019355e-01 2.408970e+00 6.134953e-01 -1.362084e+00 1.401157e+00  
## [10,] -4.012266e-01 -4.230598e-01 -2.564743e-01 4.640486e-01 1.397833e-01  
## [11,] 2.761721e-01 -8.420716e-02 9.106836e-01 6.077485e-01 8.242816e-01  
## [12,] 6.823884e-01 1.909564e-01 -9.613054e-02 9.693707e-02 -1.338936e-01  
## [13,] -2.228157e-01 -1.065167e-01 2.994188e-01 -1.067406e-01 -1.272808e-01  
## [14,] -4.920325e-01 4.690306e-02 2.220963e-02 3.527730e-02 2.691766e-01  
## [15,] -1.822443e-01 -2.429769e-01 -2.738308e-01 1.589088e-01 -2.132210e-01  
## [16,] -1.382683e-01 5.007671e-01 8.332477e-02 -8.156394e-01 -3.919407e-01  
## [17,] -6.073038e-01 -1.863466e-01 6.309876e-03 2.837017e-01 3.012827e-02  
## [18,] -1.852953e-02 -2.769055e-02 1.594193e+00 8.805549e-01 1.338389e+00  
## [19,] 3.246194e-01 2.649022e-01 6.057414e-01 7.662986e-01 8.120964e-01  
## [20,] -1.810949e-01 3.937515e-01 -8.735887e-02 -6.049010e-01 -3.721119e-02  
## [21,] -2.782131e-01 -5.190565e-02 5.543637e-02 1.484759e-01 3.243032e-01  
## PC26 PC27 PC28 PC29 PC30  
## [1,] -2.592738e-01 6.005619e-01 3.837402e-01 2.612310e-01 -0.2725792593  
## [2,] 1.621010e-02 -1.520228e+00 7.423093e-01 1.081101e+00 0.1297330509  
## [3,] -1.560486e-01 2.960688e-01 1.193491e-01 -1.310970e-01 0.3712989113  
## [4,] -7.417342e-01 -2.603565e-01 -2.909234e-01 2.044967e-01 -0.9630866667  
## [5,] -2.750681e-01 -2.582198e-01 -3.914077e-01 5.932265e-02 -0.2326775612  
## [6,] 4.522337e-01 1.898457e-01 2.866063e-01 5.294238e-01 0.4845214570  
## [7,] 1.279416e-01 1.733526e-01 1.461810e-01 -1.400261e-01 0.6923809849  
## [8,] 5.354751e-01 3.007059e-01 -4.966256e-01 1.311209e-01 0.1927066126  
## [9,] -6.873897e-02 -9.448125e-01 -2.317712e-01 -8.787779e-01 -0.2012025524  
## [10,] 2.306881e-01 6.095677e-01 2.723170e-01 -5.379798e-01 0.2967410720  
## [11,] -6.975614e-01 2.179328e-01 -1.766845e-01 -3.175358e-01 -0.3915368455  
## [12,] -2.453577e-01 -4.211757e-01 -2.404426e-01 3.404056e-01 -1.4217310304  
## [13,] -1.334004e-01 1.990754e-01 -5.179131e-02 -3.776629e-01 0.2702174573  
## [14,] -4.917648e-01 -4.536313e-01 4.852175e-01 4.467104e-01 1.0292030666  
## [15,] 9.977436e-02 -7.185988e-02 2.412261e-01 6.208545e-01 -0.0275736779  
## [16,] 3.106082e-01 -1.068467e+00 1.854525e-02 1.631078e+00 0.4963861873  
## [17,] 1.015509e-02 1.870972e-01 1.123274e-01 -3.488166e-01 -0.8036691436  
## [18,] -8.642884e-01 -1.584348e-01 1.181733e-01 5.222737e-01 0.1536993351  
## [19,] -4.176122e-01 9.552638e-03 4.117420e-02 2.173687e-01 -0.1437191007  
## [20,] 3.117071e-01 -4.481206e-01 -2.654392e-01 2.227041e-01 -1.0352149758  
## [21,] -7.591765e-02 2.066203e-01 3.209735e-02 -1.387229e-01 -0.4623849918  
## PC31 PC32 PC33 PC34 PC35  
## [1,] 5.992170e-02 3.945590e-01 8.494272e-02 1.684039e-01 -5.943083e-01  
## [2,] -2.455254e-01 1.939029e-01 -1.050701e-01 -6.073976e-02 -3.891187e-02  
## [3,] 1.484244e-01 1.374988e-01 -1.580152e-01 -9.666443e-02 -1.318982e-01  
## [4,] 3.203436e-03 -4.283417e-03 6.131561e-01 7.369829e-02 3.082872e-02  
## [5,] -6.589848e-01 8.080180e-02 7.611812e-02 -1.607551e-02 -2.706808e-02  
## [6,] 7.857180e-02 -2.137292e-03 -1.179459e-01 1.785303e-02 5.001483e-02  
## [7,] 1.979010e-01 3.829558e-02 -7.483879e-02 -7.819494e-03 2.587674e-02  
## [8,] 7.251087e-02 1.030110e-01 4.005313e-01 1.450970e-01 8.660155e-02  
## [9,] 3.480818e-01 5.300855e-02 -2.265019e-01 -2.642807e-01 -3.320893e-01  
## [10,] 2.018920e-01 -1.539103e-02 1.423196e-01 6.274790e-02 6.054695e-02  
## [11,] 4.144079e-01 6.724671e-02 1.467396e-01 2.058628e-02 5.117064e-02  
## [12,] -9.973454e-01 -1.545009e-02 1.176749e-01 1.058264e-02 -5.988309e-02  
## [13,] 1.189174e-01 -1.413806e-01 2.600471e-01 6.833502e-02 5.608357e-02  
## [14,] -3.717993e-02 1.121464e-01 -6.494547e-01 7.795960e-01 6.470908e-01  
## [15,] 1.257656e-01 -4.102144e-02 3.090503e-01 4.234222e-01 2.751815e-01  
## [16,] -1.874987e-02 1.089186e-01 3.923745e-01 9.453977e-02 3.582054e-02  
## [17,] -1.396269e-01 -1.187743e-01 -4.343540e-02 3.025247e-02 2.399771e-02  
## [18,] -9.653444e-02 7.301514e-02 5.365492e-01 1.443572e-01 4.397139e-02  
## [19,] -5.332740e-02 6.653743e-02 5.157870e-01 1.371435e-01 8.636794e-02  
## [20,] 7.227616e-03 -2.256851e-02 -3.659167e-04 -5.106241e-02 4.464404e-04  
## [21,] 4.076469e-02 -9.320861e-02 2.334588e-01 2.493957e-01 2.018840e-02  
## PC36 PC37 PC38 PC39 PC40  
## [1,] 8.204035e-01 -3.072695e-01 1.466168e-02 2.186202e-01 -2.001187e-01  
## [2,] -3.591770e-02 1.149235e-01 -5.764065e-02 4.913375e-02 5.902539e-02  
## [3,] 2.708922e-01 -9.466684e-02 -1.475684e-01 -2.425614e-01 2.125450e-02  
## [4,] -3.563503e-01 3.197768e-01 8.215113e-02 -1.296179e-01 -1.918730e-01  
## [5,] -5.972594e-02 -2.052888e-01 5.989323e-01 -3.975686e-01 1.228212e-01  
## [6,] -1.561983e-01 7.746031e-02 3.024239e-01 3.820722e-01 -3.863666e-02  
## [7,] -1.135435e-01 3.399233e-02 1.938819e-02 -9.050365e-02 -5.916043e-02  
## [8,] 3.532166e-02 1.131271e-01 -7.317699e-02 -1.250710e-01 -1.186694e-02  
## [9,] 4.568504e-03 -2.749260e-01 3.472291e-01 2.193961e-01 -8.931227e-02  
## [10,] -6.922680e-02 1.091182e-01 1.819010e-01 9.261699e-02 -8.174618e-02  
## [11,] -8.038338e-02 -5.195302e-02 6.650362e-02 -2.242242e-01 3.208093e-02  
## [12,] 4.667981e-02 -4.032715e-02 -3.239940e-01 9.518901e-02 -6.310490e-03  
## [13,] -2.183395e-03 8.521011e-02 1.413072e-02 1.516909e-02 6.571127e-02  
## [14,] 7.659073e-01 -5.198201e-01 -2.253851e-01 2.351383e-01 4.160582e-01  
## [15,] -1.036032e-01 -1.075926e-01 6.950891e-02 1.284144e-01 3.274479e-02  
## [16,] -4.918518e-02 1.018003e-01 -9.886341e-02 1.839843e-02 2.570504e-02  
## [17,] -4.307987e-01 -3.966730e-01 -1.585581e-01 -2.302642e-01 -3.531971e-02  
## [18,] -1.717414e-02 -1.020516e-02 -1.218168e-01 -8.528470e-02 -4.357873e-02  
## [19,] -1.917930e-01 9.474198e-02 -4.770903e-02 -7.634133e-02 -2.499888e-02  
## [20,] -6.273670e-02 5.252725e-02 1.957691e-01 2.425143e-01 1.591961e-02  
## [21,] 3.132876e-04 -3.373443e-01 -3.743402e-02 -5.056594e-02 -3.844133e-01  
## PC41 PC42 PC43 PC44 PC45  
## [1,] 1.426002e-01 3.266993e-01 7.063886e-02 1.018711e-01 -1.613408e-01  
## [2,] -2.283959e-02 7.299500e-02 -2.919505e-02 -1.460775e-02 2.774218e-02  
## [3,] 3.144990e-02 1.044013e-02 2.386787e-03 1.157630e-02 -7.497335e-03  
## [4,] -2.203801e-02 -7.206577e-02 -3.187518e-02 1.818423e-02 2.665681e-02  
## [5,] -1.444054e-01 1.353920e-02 -1.332183e-02 -2.765266e-02 9.264939e-03  
## [6,] 1.269299e-02 -4.256480e-02 -9.396453e-03 2.214019e-02 -5.136123e-03  
## [7,] 3.092660e-02 -8.336865e-03 6.590445e-04 2.855208e-03 -1.463391e-03  
## [8,] -3.436386e-02 -4.450873e-02 5.286834e-03 -3.439035e-02 2.153245e-02  
## [9,] -1.081641e-01 1.290717e-02 -2.671637e-02 -1.069461e-02 -2.762709e-02  
## [10,] 1.121648e-02 3.470076e-03 -3.524038e-04 2.199234e-02 3.202469e-03  
## [11,] 6.225759e-02 -4.117372e-02 5.907434e-03 9.577420e-03 -9.571642e-03  
## [12,] -1.496251e-01 -2.409902e-02 1.785309e-02 4.879525e-03 -3.992889e-02  
## [13,] -1.757155e-03 3.689970e-02 9.524682e-03 8.631094e-04 9.033165e-03  
## [14,] 2.881545e-02 8.870709e-02 7.132051e-02 -6.367785e-02 -2.999242e-02  
## [15,] 1.231915e-02 3.040042e-02 -1.203335e-02 -1.706734e-04 1.237022e-02  
## [16,] -2.244259e-02 3.050932e-02 7.660844e-04 -4.359304e-03 2.157691e-02  
## [17,] 4.267014e-02 -3.084915e-02 -3.407108e-02 -9.326506e-02 3.465410e-02  
## [18,] 2.282235e-02 7.212464e-02 6.114840e-03 1.941671e-02 -2.140598e-03  
## [19,] -4.192397e-03 7.003040e-02 -4.144967e-03 3.102884e-03 2.215856e-02  
## [20,] 7.670117e-03 3.739940e-02 5.501026e-03 2.115846e-02 8.515412e-04  
## [21,] 1.002042e-01 5.912568e-03 -1.006964e-01 1.146422e-02 -3.680481e-02  
## PC46  
## [1,] -3.898960e-02  
## [2,] 1.710154e-02  
## [3,] 3.468831e-03  
## [4,] -2.859361e-03  
## [5,] -4.959767e-02  
## [6,] 3.045958e-03  
## [7,] 6.410876e-03  
## [8,] 2.799559e-04  
## [9,] 2.574829e-03  
## [10,] -1.583896e-03  
## [11,] 2.790757e-02  
## [12,] 9.640121e-02  
## [13,] 1.848787e-03  
## [14,] 1.909516e-02  
## [15,] 9.832876e-04  
## [16,] 1.646108e-03  
## [17,] 6.118869e-03  
## [18,] -9.339273e-03  
## [19,] -2.052633e-03  
## [20,] 2.339601e-03  
## [21,] -8.631774e-03  
## [ reached getOption("max.print") -- omitted 9979 rows ]

Next, form a feature vector, that is, a matrix with the chosen principal components you want to keep. You can analyze just the feature vectors, or you can go further and use the feature vector to reorient the original data and perform further analyses. Kaiser’s Rule retains principal components with eigenvalues greater than 1. If a certain percentage of the explained variability within the data is the goal, then that cumulative variance percent can be used to identify which components to keep by viewing the eigenvalue table.The scree plot can be used as another way to determine which components to keep (those left of the elbow-where a significant drop is shown). For this example the first 14 components were chosen to form the feature vector using Kaiser’s Rule.

## Summary

The first principal component accounts for 23.1% of the explained variability in the data while the second principal component accounts for 8.7%. Looking at the biplot and the loading values in PC1 we see that expAB, psfMag, and modelMag (u, g, r, i, z) have the largest contributions to the first principal component, while PC2 is represented primarily by modelFlux measures.

The same measurements with different filters tend to load together across PC1 and PC2. This indicates that these PCs are associated with different measurement types opposed to the different filters. Furthermore, variables of similar measurement type cluster together in the initial PCs. For example, the variables u, g, r, i, z and psfMag\_u, psfMag\_g, psfMag\_r, psfMag\_i and psfMag\_z cluster together as they are all magnitude measures of brightness in the celestial object.

## References

Outside Reference 1: <https://online.stat.psu.edu/stat505/lesson/11>

Outside Reference 2: <https://towardsdatascience.com/principal-component-analysis-pca-explained-visually-with-zero-math-1cbf392b9e7d>

Outside Reference 3: <https://www.researchgate.net/publication/316652806_Principal_Component_Analysis>

Outside Reference 4: <https://www.cs.cmu.edu/~elaw/papers/pca.pdf>

Outside Reference 5: <https://mathvoices.ams.org/featurecolumn/2021/08/01/principal-component-analysis/>

Outside Reference 6: <https://royalsocietypublishing.org/doi/10.1098/rsta.2015.0202>

Outside Reference 7: <https://uw.pressbooks.pub/appliedmultivariatestatistics/chapter/pca/>

Outside Reference 8 (R reference): <https://bookdown.org/brian_nguyen0305/Multivariate_Statistical_Analysis_with_R/>

Outside Reference 9 (R reference): <https://www.statology.org/principal-components-analysis-in-r/>

Outside Reference 10 (R reference): <http://faculty.concordia.ca/pperesne/BIOL_422_680/tutorial-12-pca-and-rda.html>