Homework 2 – Vanessa Crowe
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Accessing the fairness of the die

Possible Protocol 1 (PP1): roll once; if get 6, then conclude the dice is not fair; if roll any other number then conclude it is fair. Analyze PP1:
if the dice were fair, what is the probability it would be judged to be unfair?
Oppositely, if the dice were unfair, what is the probability that it would be judged to be fair?

Since the probability of rolling a 6 on a standard dice is 1/6, imposing the rule, there is a chance the dice will end up being not fair even if it was not tampered with since it states rolling 6 just once would conclude an unfair dice.

If the dice were unfair, the probability of rolling 6 would differ as it depends on the outcome of the tampered dice. For example, one pair was baked resulting in the dice landing on 6 14/20 times when it was rolled due to it melting in a way where it would only land on 6 or 1. This makes the probability the dice would be judged as fair greater because the changes of getting a 6 is either greater than 1/6 or less than 1/6.

PP2: Roll the dice 20 times (Each person should have done this beforehand.) Group can specify a decision rule to judge that dice is fair or unfair. Consider the stats question: if fair dice are rolled 20 times, what is likely number of 6 resulting? How unusual is it, to get 1 more or less than that? How unusual is it, to get 2 more or less? 3? Analyze PP2 including the question: If the dice were fair, what is the chance it could be judged as unfair?

Results of 6 appearing:
Vanessa: 14/20
Akash: 6/20
Mishal: 7/20
Computer (fair dice): 4/20

From our rolls, we determined that if the 6 shows up 3 times every 10 rolls, the dice are fair and more than 6/20 rolls, they are unfair.

PP3: roll 100 times and specify decision rules. Some cases are easy: If every single roll comes to 6 then might quickly conclude. But what about the edge cases? Is it fair to say that every conclusion has some level of confidence attached? Where do you set boundaries for decisions? Analyze PP3., what is the chance a fair dice would be judged unfair?

If all 100 rolls results in a 6, the dice is clearly not fair but judging dice for rolling 6 with a more spread-out distribution like 60-90 times is more difficult to determine fairness as it would also depend on the distribution of the other numbers. Since we know a standard die has the probability of 1/6, the probability of getting 6 in 100 rolls is 100*1/6 meaning approximately 17 6's is expected. If there are 100 6's rolled, the dice can for sure be determined unfair. If the dice

are rolled and result closer to the original probability (16.7%), which is attributed to the law of large numbers, that means with a more equal distribution we would be able to judge the dice as fair.