Master UAB - *High Energy Physics, Astrophysics and Cosmology*

NSs, BHs and GWs

GRAVITATIONAL WAVE THEORY

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Recap I:

Covered last Thursday: General Relativity

- The **Einstein equations** are a set of tensor equations that completely control the **dynamics of gravitation**.
- The essence of the Einstein equations is that “Spacetime tells matter how to move, while matter tells spacetime how to curve.” as summarised by John Wheeler.
- In General Relativity, **gravitational phenomena** arise not from forces or fields but from the curvature of spacetime itself.

Recap II:

Covered last Friday: BHs

- All solutions of the Einstein-Maxwell equations are described by 3 parameters (mass, spin, charge) due to the **no-hair theorem.**
- **Kerr black holes** are those BHs that are relevant for astrophysical applications.

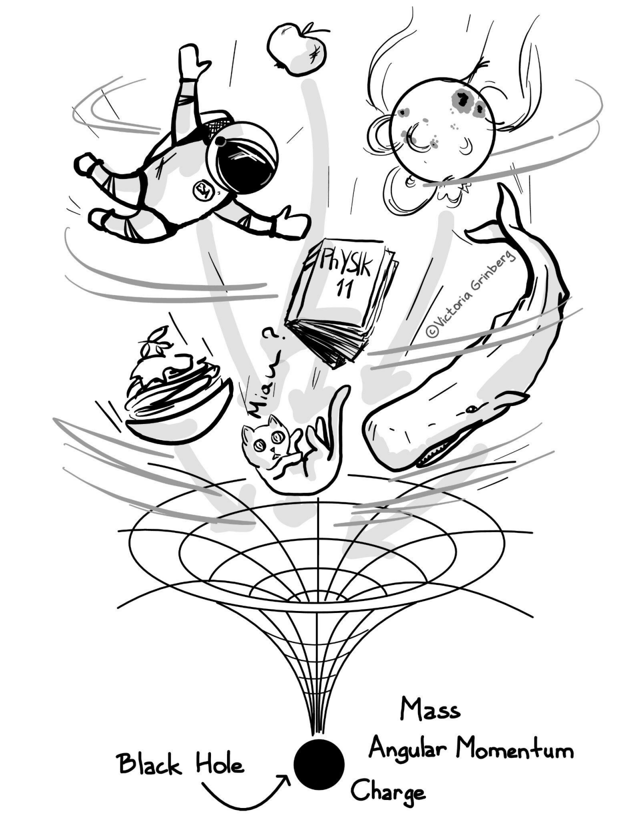

Image credit: Victoria Grinberg
@vicgrinberg

Overview:

Covered so far: special relativity, tensor calculus, equivalence principles, Einstein equations, black holes

1. A general introduction
2. Linear solution to the Einstein Equations
3. GWs from compact binaries

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1. **A general introduction**
2. **Linear solution to the Einstein Equations**
3. **GWs from compact binaries**

1.1 Video

- In today's lecture, we will start with a little video to provide a general introduction to the topic of gravitational waves. It will set the stage for our more mathematical discussion in the remainder of this lecture.
    - **Gravitational Waves Explained** by *PhDComics*

      https://tinyurl.com/5n7u37an

    - The video is about 3min long and we will have one round of questions after the video.

1.2 Questions https://tinyurl.com/5n7u37an

- Go to www.menti.com & enter 8802 0853.
  - 1. Every accelerating mass and/or energy distribution (e.g. all of us) generates gravitational waves.
    - Correct
    - Incorrect
  - 2. What is the crucial fact (constant) that we employ to actually detect the existence of gravitational waves? Please type out your answer in **1 or 2** sentences, but not more.

1.2 Questions https://tinyurl.com/5n7u37an

- Go to www.menti.com & enter 8802 0853.
  - 1. Every accelerating mass and/or energy distribution (e.g. all of us) generates gravitational waves.
    - Correct
    - Incorrect
  - 2. As any kind of ruler is also affected by the stretching/compression due to GWs, we cannot use them to measure spacetime distortions. Instead, we rely on the speed of light being constant and GWs affecting the travel time of light.

Overview:

Covered so far: special relativity, tensor calculus, equivalence principles, Einstein equations, black holes

1. A general introduction
2. **Linear solution to the Einstein Equations**
3. GWs from compact binaries

2.1 GWs - Linear ansatz

- In the following, we consider the case of **weak gravitational fields** but allow those to **vary in time** and set no restrictions on particle motion. The first assumption allows us to write

$$g_{ab} = \eta_{ab} + h_{ab}, \quad \text{where} \quad \eta_{ab} = \mathrm{diag}(1, -1, -1, -1) \quad \text{and} \quad |h_{ab}| \ll 1$$

- We **raise/lower indices** with η_{ab} and ignore all terms of second order or higher in h_{ab}. Using this and

$$\text{with} \quad h^{ab} \equiv \eta^{ac}\eta^{bd}h_{cd}, \quad \text{we have} \quad g^{ab} = \eta^{ab} - h^{ab}$$

2.2 GWs - Linearised equations

- With these metric tensors, the **Christoffel symbols**, the **Riemann/Ricci tensor** and the **Ricci scalar** become

$$\Gamma^a_{bc} \simeq \frac{1}{2}\eta^{ad}(h_{dc,\,b} + h_{db,\,c} - h_{bc,\,d})$$

$$R_{abcd} \simeq \frac{1}{2}(h_{ad,\,bc} + h_{bc,\,ad} - h_{ac,\,bd} - h_{bd,\,ac})$$

$$R_{ab} \simeq \frac{1}{2}({h^c}_{a,\,bc} + {h^c}_{b,\,ac} - {h_{ab,\,c}}^{,\,c} - {h^c}_{c,\,ab})$$

$$R \simeq \frac{1}{2}({h^{cd}}_{,\,bc} - {h^{c,\,d}}_{c,\,d})$$

- From this, we deduce the **linearised Einstein Equations**

$$G_{ab} = R_{ab} - \frac{1}{2}g_{ab}R$$

$$\simeq \frac{1}{2}({h^c}_{a,\,bc} + {h^c}_{b,\,ac} - {h_{ab,\,c}}^{,\,c} - {h^c}_{c,\,ab} - \eta_{ab}{h^{cd}}_{,\,bc} + \eta_{ab}{h^{c,\,d}}_{c,\,d}) = \kappa T_{ab}$$

2.3 GWs - Gauge freedom

$$g_{ab} = \eta_{ab} + h_{ab},$$

- Before solving the linearised EEs, we need to address the fact that our metric decomposition is **not unique**, i.e., there are other choices of coordinates, where this holds with h'_{ab}.
- Specifically, GR is invariant under **coordinate transformations** $x^a \rightarrow x'^a$ for which the full metric transforms as

$$g_{ab} \quad \rightarrow \quad g'_{ab}(x') = \frac{\partial x^c}{\partial x'^a}\frac{\partial x^d}{\partial x'^b}\, g_{cd}(x)$$

- We take advantage of this to recast the EEs in **simpler forms**.

2.4 GWs - Gauge transformations

- Let's consider an infinitesimal coordinate change of the form

$$x'^a = x^a + \xi^a(x), \quad \rightarrow \quad \frac{\partial x'^a}{\partial x^b} = \delta^a{}_b + \partial_b \xi^a, \quad \frac{\partial x^a}{\partial x'^b} = \delta^a{}_b - \partial_b \xi^a$$

- Using these, we obtain for the **transformed metric**

$$g'_{ab} = \frac{\partial x^c}{\partial x'^a}\frac{\partial x^d}{\partial x'^b} g_{cd}(x) \approx \eta_{ab} + h_{ab} - \partial_a \xi_b - \partial_b \xi_a = \eta_{ab} + h'_{ab}$$

with the following **gauge transformations** for the new metric perturbations:

$$h'_{ab} = h_{ab} - \partial_a \xi_b - \partial_b \xi_a$$

2.5 GWs - Wave equation

- Taking advantage of this, we can **define new variables** Ψ_{ab} to **simplify our linearised Einstein tensor** and obtain:

$$\Psi_{ab} \equiv h_{ab} - \tfrac{1}{2}\eta_{ab}h^{c}{}_{c}$$

$$G_{ab} \simeq \tfrac{1}{2}(\Psi^{c}{}_{a,\,bc} + \psi^{c}{}_{b,\,ac} - \Box\Psi_{ab} - \eta_{ab}\Psi^{cd}{}_{,\,cd})$$

with the **d'Alembert operator** $\Box = \eta^{ab}\partial_a\partial_b = \partial^a\partial_a = c^{-2}\partial_t^2 - \nabla^2$

- This form of G_{ab} suggests that the EEs reduce to a **wave equation** if $\Psi^{a}{}_{b,\,a}=0$ (or similarly $2h^{a}{}_{b,\,a}=h^{a}{}_{a,\,b}$). This **Hilbert** (Lorenz, Einstein, or Fock) gauge is equivalent to the Lorenz gauge in EM ($A^{a}{}_{,\,a}=0$) and gives

$$\Box\Psi_{ab} = -2\kappa T_{ab}$$

2.6 GWs - Transverse-traceless (TT) gauge

- The Hilbert gauge provides **4 conditions**, reducing the 10 independent components of the symmetric tensor h_{ab} to 6. Choosing the functions ξ^a, we impose **4** additional **constraints**.
- Specifically, we can choose ξ^0 so that $\Psi^a{}_a=0$ (**traceless**) giving also $\Psi^{ab}=h^{ab}$ and ξ^i so that $\Psi^i{}_0=0$. For b=0, the gauge condition thus gives

$$\Psi^0{}_{0,0} + \Psi^i{}_{0,i} = \Psi^0{}_{0,0} = 0 \quad \rightarrow \quad \Psi^0{}_0 = h^0{}_0 = \text{const}$$

- We saw before that for **weak fields**, $h^0{}_0=-2\phi$, with a Newtonian potential ϕ. The time-dependent GWs originate from the spatial components and do not 'care' about $h^0{}_0$. We can thus set $h^0{}_0=0$.

2.7 GWs - Vacuum solutions

- To study the **propagation of GWs** (and their interactions with test masses, i.e., detectors), we consider the behaviour **outside sources**, so that $T_{ab}=0$. In **vaccum**, the EEs read

$$\Box \Psi_{ab} = \Box h_{ab} = 0$$

- As GWs are **periodic changes of spacetime** and satisfy this wave equation, in the TT gauge we seek solutions of the form

$$h_{ab} = A_{ab} \cos(k_a x^a)$$

A_{ab} is the **symmetric polarisation tensor**, $k^a \equiv (k^0=\omega/c, k^i)$ the **wave vector**, ω the **GW frequency** and k^i/k its direction.

2.8 GWs - Properties

- From the gauge condition, we find an **orthogonality relation** $A_{ab}k^a = 0$, explaining the origin of *transverse* in the TT gauge.
- From the **wave equation** itself, we further obtain

$$\boxed{k_a k^a = 0} \qquad \boxed{\omega^2 = c^2|\vec{k}|^2 = c^2 k^2}$$

- This implies that the wave vector $\mathbf{k^a}$ is a **lightlike** (null) vector and **GWs propagate at the speed of light**. The group velocity ($\partial\omega/\partial k$) and phase velocity ($\omega/k$) are equal to c.
- GWs move along null geodesics and experience the phenomena associated with EM waves, e.g., Doppler shift, gravit. redshift.

2.9 GWs - Polarisation tensor

- We can look at a GW **propagating along the z-direction** so that $k^a = (\omega/c, 0, 0, \omega/c)$. Applying the **TT gauge**, we are left with **4 non-zero components** (A_{11}, A_{12}, A_{21}, A_{22}) for A_{ab} but only **2 are free** because A_{22}=-A_{11} and A_{21}=A_{12}. We thus write

$$h_{ab} = \left(h_+ e^+_{ab} + h_\times e^\times_{ab}\right) \cos[\omega(t - z/c)]$$

$$e^+_{ab} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad e^\times_{ab} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

2.10 Questions

- Go to www.menti.com & enter 39 43 69 9.
  - 1. We consider gravitational waves as perturbations to the flat (Minkowski) background metric.
    - Correct
    - Incorrect
  - 2. Which of the following relations is true for the TT gauge?
    - $h^a{}_0 = 0$ for a=0,1,2,3
    - $h^i{}_i = 0$ for i=1,2,3
    - $h^i{}_{j,i} = 0$ for j=1,2,3

2.10 Answers

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2.11 GWs - Effects on particles I

- We analyse effects of passing GWs by looking at the distance of **2 freely falling particles**. For a + polarised wave in z-direction, we get for particles at (x, y) = (-L, 0) & (L, 0) and (0, -L) & (0, L):

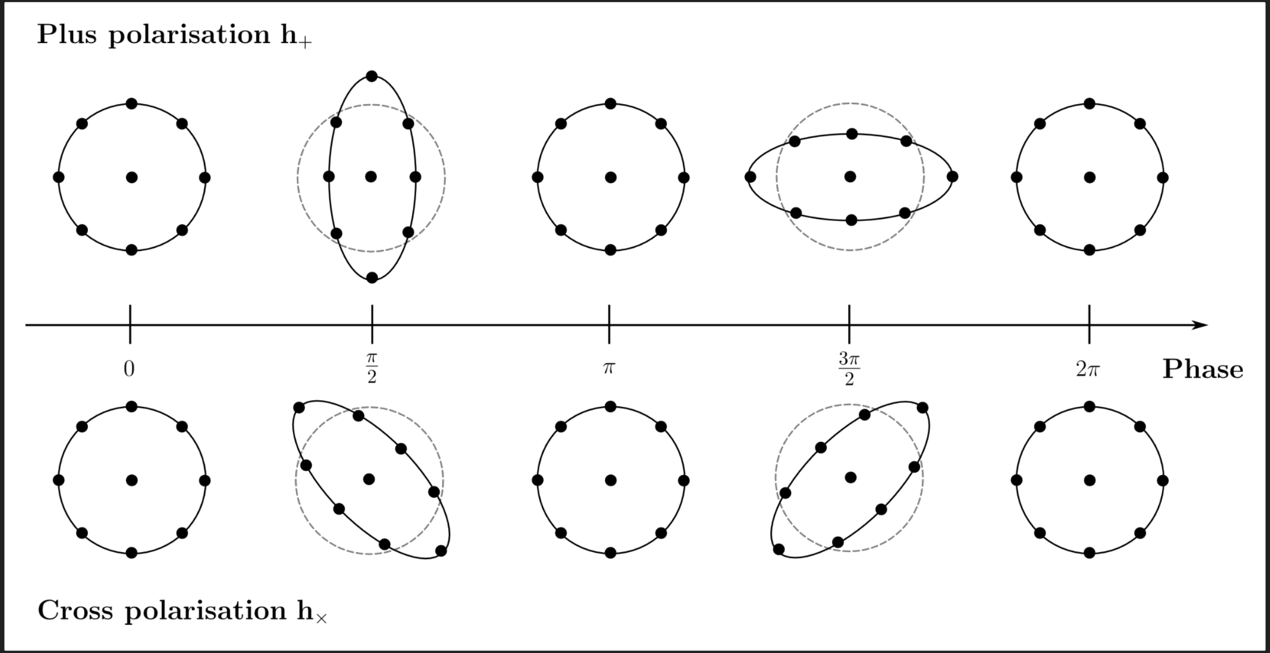
$$L_x(t) = \int \mathrm{d}s\Big|_{y=0} = \int_{-L}^{L} \mathrm{d}x\sqrt{-g_{xx}(t)} \simeq \int_{-L}^{L} \mathrm{d}x\sqrt{1-h_{xx}(t)}$$
$$\approx [1-\tfrac{1}{2}h_{xx}(t)]\int_{-L}^{L}\mathrm{d}x = 2L[1-\tfrac{1}{2}h_{+}\cos(\omega t)]$$

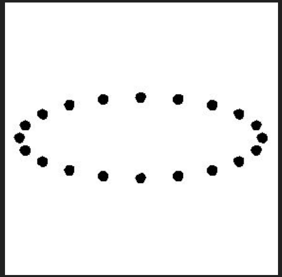
$$L_y(t) = 2L[1-\tfrac{1}{2}h_{yy}(t)] = 2L[1+\tfrac{1}{2}h_{+}\cos(\omega t)]$$

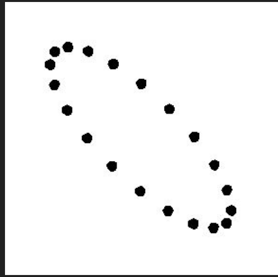
- The distances $L_x(t)$ and $L_y(t)$ **oscillate with opposite phase**.

2.12 GWs - Effects on particles II

- Make this even clearer by looking at **rings of test particles**:







2.13 GWs - Non-vacuum situation

$$\Box \Psi_{ab} = -2\kappa T_{ab}$$

- From the discussion of test particles, we know that GWs carry energy & momentum. How does the GW stress-energy tensor look like? Are GWs themselves sources of spacetime curvature?

- For $T_{ab} \neq 0$, we need to solve a wave equation with a **source term**. A general solution is obtained via **Green's functions**:

$$\Psi_{ab}(t, x^i) = \frac{2\kappa}{4\pi} \int \frac{T_{ab}(t - |x^i - x'^i|/c, x'^i)}{|x^i - x'^i|} \, \mathrm{d}^3 x'$$

- The disturbance at (t, x^i) is a sum of all influences from energy/momentum sources at $(t - |x^i - x''^i|/c, x^i - x''^i)$, i.e., from the past.

2.14 GWs - Quadrupole formula

$\Box\Psi_{ab} = -2\kappa T_{ab}$

- Let's look at the gravitational radiation emitted by an **isolated, slowly-moving source** of extension □R at a distance R far away from an observer, which implies $R \gg \Box R$ and $c/\omega \gg \Box R$.

- To **leading order** we can replace $|x^i - x'^i| = R$. Due to the **gauge condition**, we only need to consider spatial Ψ_{ab} components. Using the **conservation law** $\partial^a T_{ab} = 0$ (giving $\partial^2_t T_{00} = \partial_i \partial_j T_{ij}$) combined with integration by parts (twice), one arrives at

$$\Psi_{ij}(t, x^k) = \frac{\kappa}{4\pi}\frac{1}{R}\partial_t^2 \underbrace{\int x'_i x'_j T_{00}(t - R/c, x'^k)\, \mathrm{d}^3 x'}_{\equiv Q_{ij}(t-R/c)} = \frac{2G}{c^4 R}\partial_t^2 Q_{ij}(t - R/c)$$

2.15 GWs - Stress-energy tensor I

- To obtain the linearised Einstein Equations, we expanded G_{ab} to linear order in h_{ab}. Going to second order, we obtain

$$g_{ab} = \eta_{ab} + h^{(1)}_{ab} + h^{(2)}_{ab}, \quad \text{where} \quad G^{(1)}_{ab}[\eta + h^{(1)}] = 0$$

- The **second-order version of the EEs** consist of all terms either quadratic in $h^{(1)}_{ab}$ or linear in $h^{(2)}_{ab}$:

$$G^{(1)}_{ab}[\eta + h^{(2)}] + G^{(2)}_{ab}[\eta + h^{(1)}] = 0, \quad \rightarrow \quad G^{(1)}_{ab}[\eta + h^{(2)}] = \kappa t_{ab}$$

- Think of t_{ab} (which satisfies $\partial^a t_{ab}=0$) as the **energy-momentum tensor** of the **gravitational field** in the weak-field regime.

2.16 GWs - Stress-energy tensor II

- t_{ab} is invariant under global Lorentz transformations, but **not invariant** under general coordinate / gauge transformations. It is thus not a tensor, but referred to as a **pseudo tensor**.

- The stress-energy carried by GWs **cannot be localised** within a wavelength. Instead, we need to consider stress-energy contained within an **extended region of space** to obtain a gauge-invariant measure of the gravitational field. This implies

$$\langle t_{ab} \rangle = \frac{1}{\kappa} \langle G^{(2)}_{ab} \rangle \quad \rightarrow \quad t^{GW}_{ab} = \frac{1}{4\kappa} \langle (\partial_a h^{TT}_{ij})(\partial_b h^{ij}_{TT}) \rangle$$

2.17 GWs - Energy flux

- For the special case of a **plane wave** propagating along the z-direction, we can take advantage of our result from slide 2.9. In this case, t^{GW}_{ab} has only **3 non-zero components**

$$t_{00}^{GW} = t_{zz}^{GW} = -t_{0z}^{GW} = \frac{c^2\omega^2}{32\pi G}(h_+^2 + h_\times^2)$$

with **energy density** t^{GW}_{oo}, **momentum flux** t^{GW}_{zz} & **energy flux** c t^{GW}_{oz} (fluxes per unit area & unit time). We then deduce

$$L^{GW} = \frac{\mathrm{d}E}{\mathrm{d}t} = c\int_S t_{0a}^{GW}\hat{n}^a \mathrm{d}\Omega = \frac{G}{5c^5}\langle \partial_t^3 \bar{Q}_{ij}\partial_t^3\bar{Q}^{ij}\rangle\Big|_{t=t-R/c}$$

$$\bar{Q}_{ij} = Q_{ij} - \frac{1}{3}\delta_{ij}\delta^{kl}Q_{kl}$$

2.18 Questions

- Go to www.menti.com & enter 5586 0441.
  - 3. As a GW passes through a ring of test particles, the separation of any two particles oscillates periodically in time.
    - Incorrect
    - Correct
  - 4. Gravitational wave energy can be measured locally, i.e., at a single point in spacetime.
    - Incorrect
    - Correct

2.18 Answers

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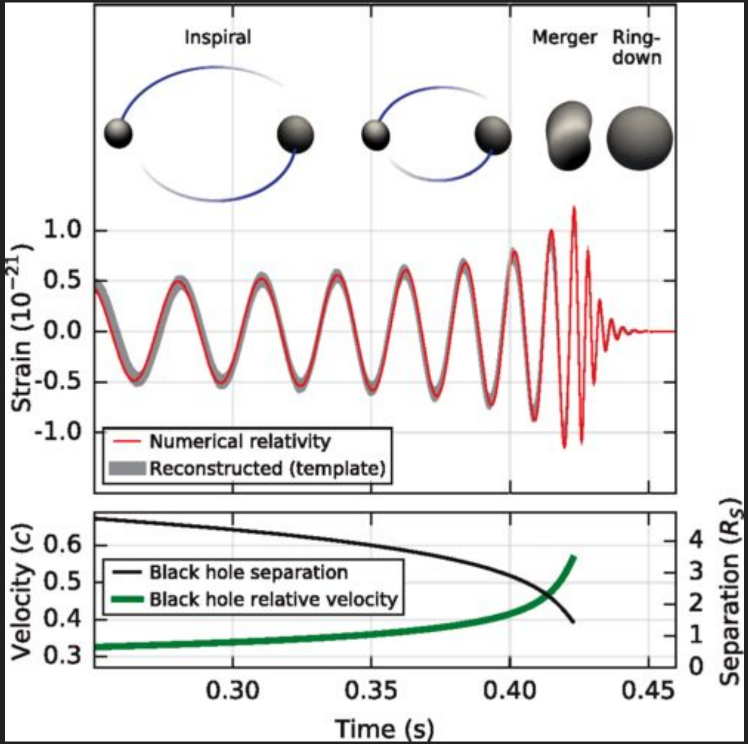
Overview:

Covered so far: special relativity, tensor calculus, equivalence principles, Einstein equations, black holes

1. A general introduction
2. Linear solution to the Einstein Equations
3. **GWs from compact binaries (CBs)**

3.1 CBs - Inspiral

- GWs are generated by **coherent bulk motion** of matter. The more compact the matter and the faster the motion, the larger the ripples. GWs thus allow **probing of regions of strong gravity**.



- The fiducial example of this is the GW signal from **compact object binaries**, e.g., two BHs. The **time evolution** of the inspiral is determined by the emission of GWs.

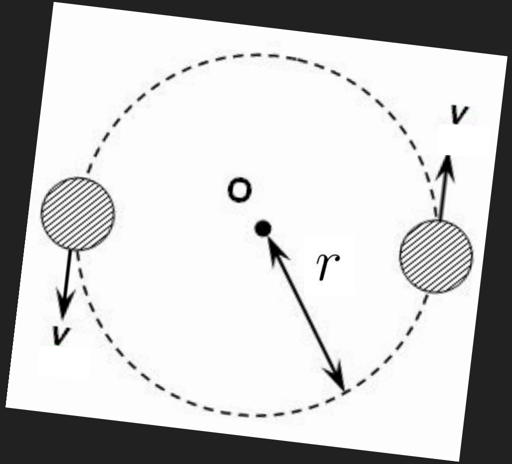
3.2 CBs - Orbits

- Orbits can be treated within the **Newtonian approximation**, just like in classical celestial mechanics. For **circular orbits**, we equate gravity with the outward centrifugal force to obtain

$$\frac{GM^2}{(2r)^2} = \frac{Mv^2}{r}, \quad \to \quad v = \sqrt{\frac{GM}{4r}}$$

- We deduce the **orbital angular frequency**

$$T = \frac{2\pi r}{v}, \qquad \Omega = \frac{2\pi}{T} = \sqrt{\frac{GM}{4r^3}}$$



3.3 CBs - Quadrupole moment

- Employing Ω, we can specify the **paths of both masses** as

$$(x_A^1, x_A^2) = (r\cos\Omega t, r\sin\Omega t), \quad (x_B^1, x_B^2) = (-r\cos\Omega t, -r\sin\Omega t)$$

and thus the **energy density** of the system is

$$T^{00}(t, x^i) = M\delta(x^3)\left[\delta(x^1 - x_A^1)\delta(x^2 - x_A^2) + \delta(x^1 - x_B^1)\delta(x^2 - x_B^2)\right]$$

- Using the definition of the **quadrupole moment**, we obtain

$$\begin{aligned} q_{11} &= 2Mr^2\cos^2\Omega t = Mr^2(1 + \cos 2\Omega t) \\ q_{22} &= 2Mr^2\sin^2\Omega t = Mr^2(1 - \cos 2\Omega t) \\ q_{12} &= q_{21} = 2Mr^2(\cos\Omega t)(\sin\Omega t) = Mr^2\sin 2\Omega t \end{aligned}$$

3.4 CBs - Metric perturbation

- Applying the **quadrupole formula** (i.e., taking 2 time derivatives of the quadrupole tensor), gives the **spatial components** of the **trace-reversed metric perturbations** Ψ_{ij}

$$\Psi_{ij}(t, x^k) = \frac{8GM}{c^4 R} \Omega^2 r^2 \begin{pmatrix} -\cos 2\Omega t_r & -\sin 2\Omega t_r & 0 \\ -\sin 2\Omega t_r & \cos 2\Omega t_r & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

where t_r is the retarded time, i.e., $t_r = t - R/c$, and thus $\boldsymbol{\omega = 2\Omega}$.

- It is possible to generalise this to **unequal masses**, where the prefactor reads $4G\mu S^2\Omega^2/c^4R$, where S is the separation and $\mu = m_1 m_2/(m_1+m_2)$ the **reduced mass** of the system.

3.5 CBs - GW energy loss I

- For our equal-mass system, we can also determine the **traceless** part of the **quadrupole moment**

$$\bar{Q}_{ij} = \frac{Mr^2}{3}\begin{pmatrix} (1+3\cos 2\Omega t) & 3\sin 2\Omega t & 0 \\ 3\sin 2\Omega t & (1-3\cos 2\Omega t) & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

- The **third time derivative** is then given by

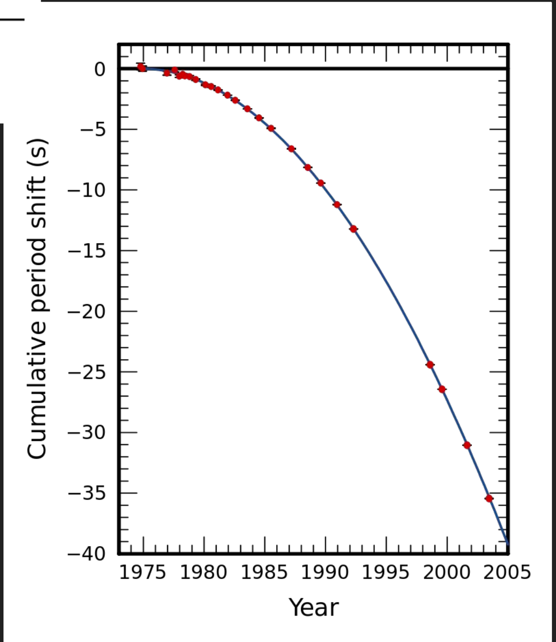
$$\partial_t^3 \bar{Q}_{ij} = 8M\Omega^3 r^2 \begin{pmatrix} \sin 2\Omega t & -\cos 2\Omega t & 0 \\ -\cos 2\Omega t & -\sin 2\Omega t & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

3.6 CBs - GW energy loss II

- The resulting power radiated by the binary is thus equal to

$$L^{GW} = \frac{128 G M^2 \Omega^6 r^4}{5c^5} \underbrace{\langle \sin^2(2\Omega t) + \cos^2(2\Omega t) \rangle}_{=1} = \frac{2G^4 M^5}{5c^5 r^5}$$

- This was confirmed in 1974, when **Hulse & Taylor** discovered a **double neutron star binary** (PSR1913+16). As one is a **pulsar**, we can accurately measure the evolution of the orbit. The changes in the orbit are exactly as predicted by GR: first **indirect GW detection!!**



3.7 CBs - Frequency evolution

- Using Keplerian physics and orbital energy conservation, we find with $E_{bin}=E_{kin}+E_{pot}$ the **frequency evolution** of an emitted GW:

$$\frac{\mathrm{d}}{\mathrm{d}t}(E_{bin}+E_{GW})=0 \quad \rightarrow \quad \frac{\mathrm{d}r}{\mathrm{d}t}=\frac{4r^2L^{GW}}{GM^2} \quad \rightarrow \quad \frac{\mathrm{d}\omega}{\mathrm{d}t}=\frac{12G^2M^2}{5c^5r}\omega^3$$

- Keeping in mind that $r^3=GM/\omega^2$, we can solve this ODE for ω:

$$\omega(t)=\frac{1}{4(G\mathcal{M}_c)^{5/8}}\left(\frac{5}{t_c-t}\right)^{3/8} \qquad t_c-t=\frac{5r^4}{32(GM)^3}$$

with the **chirp mass** $\mathcal{M}_c=(2M)^{2/5}\mu^{3/5}$ and **coalescence time** t_c.

3.8 Questions

- Go to www.menti.com & enter 5835 4399.
  - 1. We can treat the orbits of two inspiralling compact objects within the Newtonian approximation.
    - Incorrect
    - Correct
  - 2. What is the frequency of gravitational waves emitted by the inspiral of two compact objects assuming the orbital angular frequency is equal to Ω?
    - $\omega=\Omega$
    - $\omega=2\Omega$

$$\begin{pmatrix} -\cos 2\Omega t_r & -\sin 2\Omega t_r & 0 \\ -\sin 2\Omega t_r & \cos 2\Omega t_r & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

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    - ω=Ω
    - ω=2Ω

$$\begin{pmatrix} -\cos 2\Omega t_r & -\sin 2\Omega t_r & 0 \\ -\sin 2\Omega t_r & \cos 2\Omega t_r & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Summary I:

Covered today: linearised solution to the EEs

- We derived linearised EEs in the weak-field limit by decomposing the metric into the **flat metric plus** a **perturbation.**
- This decomposition was **not unique** but we can use the gauge freedom to rewrite the lin. EEs as a **wave equation**, which in vacuum permits **plane waves** with 2 free parameters h_+ & h_x.
- We looked at the EEs coupled to matter and derived the **quadrupole formula**, dictating that time-varying Q_{ij} sources GW. Finally, we derived t_{ab} & a relation for the **GW luminosity**.

Summary II:

Covered today: GWs from compact binaries

- GWs are generated by **coherent bulk motion** of matter. The more compact the matter and the faster the motion, the larger the ripples. GWs allow us to **probe strong gravity systems.**
- We looked at the fiducial case of two inspiralling compact objects, where the orbits can be approximated in a classical way but the actual evolution of the orbit is determined by GR.
- A measurement of the shrinking orbit in a double neutron star binary allowed the first indirect detection of GWs.

Questions?

- Are there any more **questions from your side** on what we covered so far?