

Welcome to Webinar 1

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Demystifying Machine Learning

Understand The Fundamentals Of

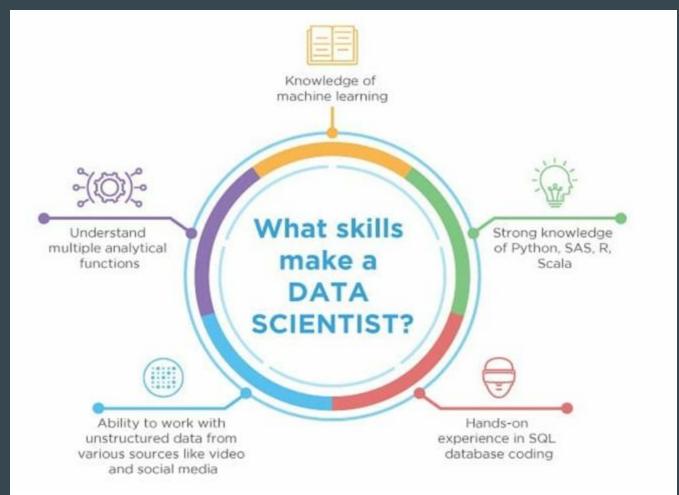
Mathematics for Data Science

The purpose of this webinar is to give you an introduction of how to be a successful data scientist.

Required Skills:

- 1. Understanding the Mathematics.
- 2. The Basic Terminology.
- 3. Algorithms and Coding.
- 4. The Cloud and Devops.
- 5. How to communicate your Findings.

Mathematics is wrapping it



Why Mathematics !!!?

- Helps with modeling a process
- Constructing a hypothesis
- Understanding the abstract logic behind it
- Understand the limitations of a model

Agenda

- Basics of Calculus
- Basics of Set Theory
- Basics of Linear Algebra

Things to Remember

- At the end of this presentation, we sources you to refer to practice these skills.

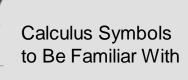


Seminar 1: Mathematics (Calculus)

- 1. Derivatives
- 2. Chain Rule
- 3. Gradients
- 4. Integrals



Calculus



Symbol Name	Meaning / definition	Example
limit	limit value of a function	
epsilon	represents a very small number, near zero	$\varepsilon \to 0$
e constant / Euler's number	e = 2.718281828	$e = \lim_{x \to \infty} (1+1/x)^x$,
derivative	derivative - Leibniz's notation	$(3x^3)' = 9x^2$
second derivative	derivative of derivative	$(3x^3)'' = 18x$
nth derivative	n times derivation	$(3x^3)^{(3)} = 18$
derivative	derivative - Lagrange's notation	$d(3x^3)/dx = 9x^2$
second derivative	derivative of derivative	$d^2(3x^3)/dx^2=18x$
nth derivative	n times derivation	
time derivative	derivative by time - Newton notation	
time second derivative partial derivative	derivative of derivative	$\partial (x^2+y^2)/\partial x = 2x$
double integral	integration of function of 2 variables	
triple integral	integration of function of 3 variables	
closed contour / line integral		
closed surface integral		
closed volume integral closed interval	$[a,b] = \{x \mid a \le x \le b\}$	
open interval	$(a,b) = \{x \mid a < x < b\}$	
imaginary unit	$t = \sqrt{-1}$	z = 3 + 2t
complex conjugate	$z = \alpha + bt \rightarrow z^* = \alpha - bt$	$z^* = 3 + 2i$
complex conjugate	$z = a+bi \rightarrow \overline{z} = a-bi$	$\overline{z} = 3 + 2i$
nabla / del	gradient / divergence	$\nabla f(x,y,z)$
	limit epsilon e constant / Euler's number derivative second derivative nth derivative derivative second derivative time derivative time derivative time second derivative partial derivative integral double integral triple integral closed contour / line integral closed surface integral closed volume integral closed interval open interval imaginary unit complex conjugate	limit value of a function represents a very small number, near zero e constant / Euler's number $e = 2.718281828$ derivative derivative - Leibniz's notation second derivative in times derivative of derivative nth derivative in times derivative of derivative derivative of derivative in times derivative in time derivative in derivative by time - Newton notation derivative integral derivative integral opposite to derivative integral closed contour / line integral closed contour / line integral closed surface integral closed surface integral closed interval $[a,b] = \{x \mid a \le x \le b\}$ open interval $[a,b] = \{x \mid a < x < b\}$ imaginary unit $[a,b] = \{x \mid a < x < b\}$ imaginary unit $[a,b] = \{x \mid a < x < b\}$ imaginary unit $[a,b] = \{x \mid a < x < b\}$ imaginary unit $[a,b] = \{x \mid a < x < b\}$ complex conjugate $[a,b] = [a,b] = [a,b]$ imaginary unit $[a,b] = [a,b]$ imaginary unit $[a,b]$

Calculus

- Mathematical study of the continuous rate of change.
- We use calculus in machine learning to formulate the function used to train algorithms to reach their objective known by loss, cost, and objective functions.

More in Depth A Future Seminar:

Loss Function - evaluates how well specific algorithms model the given data.

Cost Function - They are used to estimate how badly models are performing

Objective Function - The most general term for any function you optimize during training.

Derivatives

- 1. Rate of Change
- 2. Slope of a line at a specific point

$\frac{dx}{dy} \qquad \frac{d^2y}{dx^2}$

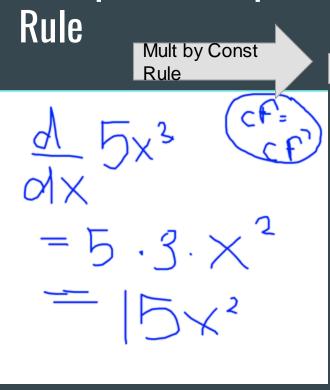
So why do we need this in Machine Learning?

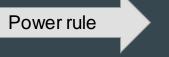
- In Machine Learning we use derivatives in optimization problems
- Optimization problems like gradient descent use derivatives to decide whether to increase or decrease weights or to maximize or minimize the objective (model accuracy or error function (model performance))
- Also help in approximating nonlinear functions as linear functions

Derivative Rules

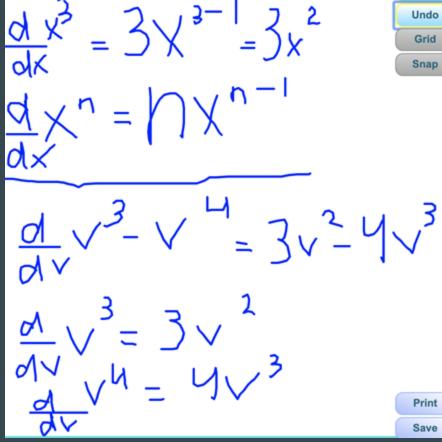
Rules	Function	Derivative
Multiplication by constant	cf	cf'
Power Rule	xn	nx ⁿ⁻¹
Sum Rule	f + g	f' + g'
Difference Rule	f - g	f' – g'
Product Rule	fg	f g' + f' g
Quotient Rule	f/g	$(f'g - g'f)/g^2$
Reciprocal Rule	1/f	-f'/f ²
Chain Rule (as <u>"Composition of Functions")</u>	f ° g	(f' ° g) × g'
Chain Rule (using ')	f(g(x))	f'(g(x))g'(x)
Chain Rule (using $\frac{d}{dx}$)	_	dy du dx

Examples: Multiplication By Const & Power Rule, Difference





Difference Rule



Example: Product Rule

Product Rule

Example: What is the derivative of cos(x)sin(x)?

The Product Rule says:

the derivative of fg = f g' + f' g

In our case:

- f = cos
- g = sin

We know (from the table above):

- $\frac{d}{dx}\cos(x) = -\sin(x)$
- $\frac{d}{dx}\sin(x) = \cos(x)$

So:

the derivative of cos(x)sin(x) = cos(x)cos(x) - sin(x)sin(x)

$$= \cos^2(x) - \sin^2(x)$$

Reciprocal Rule

$$\frac{d}{dx}\left(\frac{1}{x}\right)$$

$$=\frac{1}{x} = -\frac{1}{x^2}$$

$$f(x) = \frac{1}{x}$$

$$f(x) = \frac{1}{x^2}$$

Quotient Rule

The derivative of $(4x-2)/(x^2+1)$ is:

$$egin{aligned} rac{d}{dx} \left[rac{(4x-2)}{x^2+1}
ight] &= rac{(x^2+1)(4)-(4x-2)(2x)}{(x^2+1)^2} \ &= rac{(4x^2+4)-(8x^2-4x)}{(x^2+1)^2} \ &= rac{-4x^2+4x+4}{(x^2+1)^2} \end{aligned}$$

Derivatives of quotients (Quotient Rule)

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$$

Example: Chain Rule - the derivative of f(g(x)) = f'(g(x))g'(x)

$$\frac{d}{dx} \sin(x^2)$$

$$\frac{d}{dx} \sin$$

Gradients

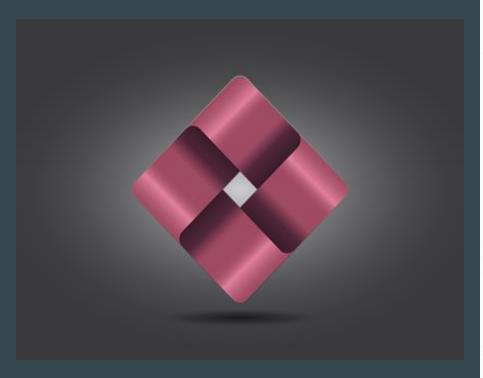
- -It is a vector <>, that stores the partial derivative of multivariable functions.
- -It helps us calculate the calculate the slope at a specific point on a curve for functions with multiple independent variables.
- They can be used with partial derivatives, as we store partial derivatives in them. Which represents the full derivative of the multivariate function.
- It will always point in the direction of greatest increase of a function (Understanding Pythagorean Distance and the Gradient)

Why Gradients For Machine Learning?

- They will help us understand Gradient Descent (Future Seminar)
- We use them in optimization algorithms
- Use to analyze variants of signal in multiple dimensions
- Use it in image processing to detect pixel direction

- Is zero at a local max or min?

Gradient Example



COILC Ullate derivative W/ respect x, z

$$\frac{df}{dx} = 4xz^3$$

$$\frac{dF}{dz} = 6z^2x^2$$

store in gradient

$$\nabla F(X,Z) = \begin{bmatrix} \frac{\partial F}{\partial x} \\ \frac{\partial F}{\partial y} \end{bmatrix} = \begin{bmatrix} \frac{4 \times 2^3}{6 \times 2^2 \times 2^2} \end{bmatrix}$$

Integrals- (It can be used to find areas,central points, & other things)

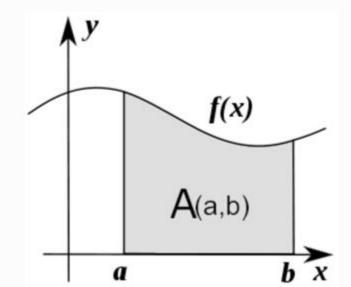
The integrals f(x) corresponds to the computation of the area under the graph of f(x)

The under f(x) between points x=a and x=b

Application

- Computing probabilities
- Expected value
- Variance

$$A(a,b) = \int_a^b f(x) \, dx.$$



The Integral of Many Functions Are Well Known.

We can use rules to work out the more complicated functions.

Common Functions	Function	Integral
Constant	∫a dx	ax + C
Variable	∫x dx	$x^2/2 + C$
Square	$\int x^2 dx$	$x^3/3 + C$
Reciprocal	$\int (1/x) dx$	In x + C
Exponential	∫e ^x dx	e ^x + C
	∫a ^x dx	a ^x /ln(a) + C
	$\int \ln(x) dx$	$x \ln(x) - x + C$
Trigonometry (x in radians)	$\int \cos(x) dx$	sin(x) + C
	∫sin(x) dx	-cos(x) + C
	$\int sec^2(x) dx$	tan(x) + C
Rules	Function	Integral
Multiplication by constant	∫cf(x) dx	c∫f(x) dx
Power Rule (n≠-1)	∫x ⁿ dx	$\frac{x^{n+1}}{n+1} + C$
Sum Rule	$\int (f+g) dx$	$\int f dx + \int g dx$
Difference Rule	$\int (f - g) dx$	∫f dx - ∫g dx
Integration by Parts	See Integration by Parts	
	See Integration by Substitution	

Integration

Based on the Fundamental
Theorem of Calculus - links the
concept of differentiating a function with
the concept of integrating a function

Suppose f(x) is a continuous function on [a,b] and also suppose that F(x) is any anti-derivative for f(x). Then,

$$\int_{a}^{b} f(x) dx = F(x)|_{a}^{b} = F(b) - F(a)$$

Integral Symbol

differential

Function we want to integrate (integrand)

Constant of Integration

Indefinite Integral- an integral expressed without limits, and so containing an arbitrary constant

Definite Integral- has start and end values.

Integral Symbol- used to denote integrals and antiderivatives

Differential-Indicates the variable of integration

Constant of Integration - The indefinite integral of a given function (i.e the set of all antiderivatives) on a connected domain is defined up to an additive to an constant. The constant expresses an ambiguity inherent in the construction of anti-derivatives.

Basic Integration Example Indefinite Integral :

Which Rule would we use from our toolkit?

STXdx
J X 1/2 dx
$=\frac{1.5}{1.5}$

Common Functions	Function	Integral
Constant	∫a dx	ax + C
Variable	∫× d×	$x^2/2 + C$
Square	∫x² dx	$x^3/3 + C$
Reciprocal	$\int (1/x) dx$	Injxj + C
Exponential	∫e [×] dx	e ^x + C
	∫a× dx	$a^{\times}/ln(a) + C$
	∫ln(x) dx	$x \ln(x) - x + C$
Trigonometry (x in radians)	∫cos(x) dx	sin(x) + C
	∫sin(x) dx	-cos(x) + C
	∫sec²(x) dx	tan(x) + C
Rules	Function	Integral
Multiplication by constant	∫cf(x) dx	c∫f(x) dx
Power Rule (n≠-1)	∫x ⁿ dx	$\frac{x^{n+1}}{n+1} + C$
Sum Rule	$\int (f+g) dx$	∫f dx + ∫g dx
Difference Rule	∫(f - g) dx	∫f dx - ∫g dx
Integration by Parts	See Integration by Parts	
Substitution Rule	See Integration by Substitution	

A More Complicated Example:

Integration By Substitution: The Reverse Chain Rule

"Integration by Substitution" (also called "u-Substitution" or "The Reverse Chain Rule") is a method to find an integral, but only when it can be set up in a special way.

The first and most vital step is to be able to write our integral in this form:

$$\int f(g(x)) g'(x) dx$$

Note that we have g(x) and its <u>derivative</u> g'(x)

How we know its U Substitution

$$f = U_3$$

$$\begin{cases} f(x) & dv \\ f(x)$$

$$S(x+1)^{2} dx$$

$$= \int u^{2} du$$

$$= \int u^{3} + C$$

$$= (x+1)^{3} + C$$

$$= 3$$

Basic Integration Example: Definite Integral

$$\int_{0.5}^{3} \cos(x) dx$$

$$= \sin(3) - \sin(\frac{11}{2})$$

$$= \sin(3) - \sin(\frac{11}{2})$$

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$$= \cos(3$$

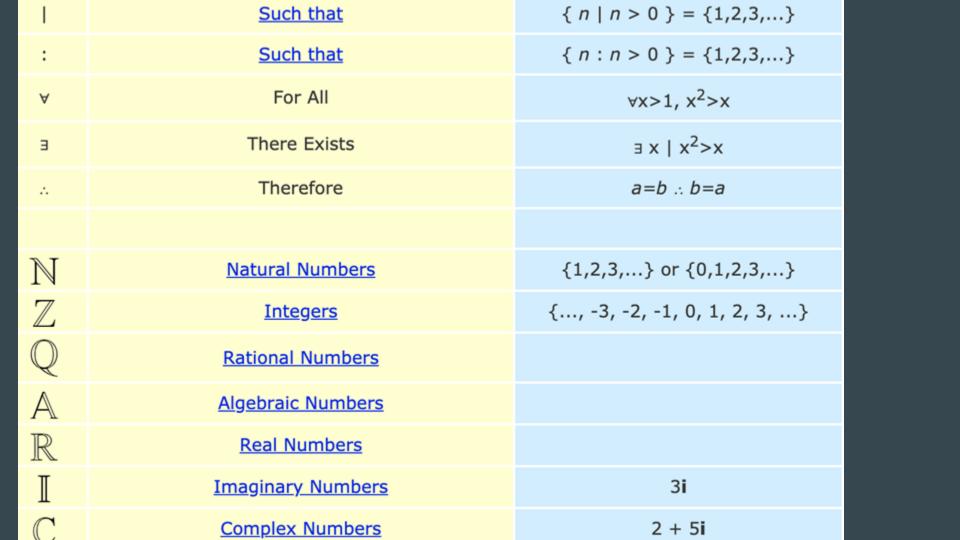
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Set Theory

- Branch of mathematics that informally deals with the collections of objects.
- It is essential to understand the basics of set theory in machine learning, if you want to understand the theory behind it rather than just the algorithm

Symbol	Meaning	Example
{ }	Set: a collection of elements	{1,2,3,4}
A ∪ B	Union: in A or B (or both)	$C \cup D = \{1,2,3,4,5\}$
$A \cap B$	Intersection: in both A and B	C ∩ D = {3,4}
A ⊆ B	Subset: A has some (or all) elements of B	{3,4,5} ⊆ D
$A \subset B$	Proper Subset: A has some elements of B	{3,5} ⊂ D
A⊄B	Not a Subset: A is not a subset of B	{1,6} ⊄ C
A ⊇ B	Superset: A has same elements as B, or more	{1,2,3} ⊇ {1,2,3}
$A \supset B$	Proper Superset: A has B's elements and more	{1,2,3,4} ⊃ {1,2,3}
A⊅B	Not a Superset: A is not a superset of B	{1,2,6} ⊅ {1,9}

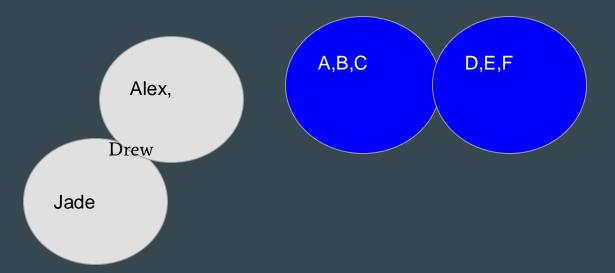
A ^c	Complement: elements not in A	$D^{c} = \{1,2,6,7\}$ When $U = \{1,2,3,4,5,6,7\}$
A – B	<u>Difference</u> : in A but not in B	$\{1,2,3,4\} - \{3,4\} = \{1,2\}$
a∈A	Element of: a is in A	3 ∈ {1,2,3,4}
b∉A	Not element of: b is not in A	6 ∉ {1,2,3,4}
Ø	Empty set = {}	$\{1,2\} \cap \{3,4\} = \emptyset$
U	Universal Set: set of all possible values (in the area of interest)	
P(A)	Power Set: all subsets of A	$P({1,2}) = { {}, {1}, {2}, {1,2} }$
A = B	Equality: both sets have the same members	{3,4,5} = {5,3,4}
A×B	Cartesian Product (set of ordered pairs from A and B)	$\{1,2\} \times \{3,4\}$ = $\{(1,3), (1,4), (2,3), (2,4)\}$
A	Cardinality: the number of elements of set A	{3,4} = 2



Venn Diagram

What is it?

 It shows sets and which elements belong to the set by drawing regions around them,



Set Theory Example:

Set A= {1,2,3}

Set B={3,A,B,C,D,E}

 \cap intersection $\{1,2\}$

U union {1,2,3,A,B,C,D,E}

Cardinality A= 3

Cardinality B = 6

Cardinality C = 2

A - B {1,2}

Symmetric Difference= {1,2,A,B,C,D,E}

Linear Algebra

- Scalars, vectors, matrices, and tensors

Scalar - is a single number

Vector - is an array of numbers

Matrix - 2-D array

Tensor - n dimensional array n > 2

It is a branch of mathematics that concerns linear equations linear functions such as and their representations through matrices and vector spaces.

Principal Component Analysis - (helps with reducing dimensions)

- -Used in deep learning (as with neural networks we store weights in matrices)
- Many ML algorithms are tied together with Linear Algebra techniques

Definition to Remember:

Scalar - An element of a field which is used to define a vector space.

Vectors

- They are one dimensional arrays of numbers or terms.
- A vector with more than one dimension is a matrix

Vectors why do we need them in machine learning?

- Dimensions (attributes taken for analysis)
- Embeddings (mapping from a discrete objects to vectors of real numbers)

$$V = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Vectors, Scalar Operations

- They involve a vector and a number.
- You modify the vector in place by adding, subtracting, or multiplying the number from all values in the vector

Scalar Operations

Vector + number

$$\begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix} + 1 = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$$

Vectors, Elementwise Operations

- We are able to combine operations like addition, subtraction, division, values that correspond potentially to a produce a new vector.

$$\begin{bmatrix}
a_1 \\
b_2
\end{bmatrix} + \begin{bmatrix}
b_1 \\
b_2
\end{bmatrix} = \begin{bmatrix}
a_1 + b_1 \\
a_2 + b_2
\end{bmatrix}$$

Dot Product

$$\frac{DoT}{A}$$
 Product 2 vectors
 $\frac{A}{A}$ DoT Product 2 vectors
 $\frac{A}{A}$ SCOILUIV.
 $\frac{A}{A}$ $\frac{A}{A}$

Example with Dot Product using Calculus

The gradient will tell us direction we are traveling, while the directional derivative will help nus find the slope if we move in a direction different from the specified gradient.

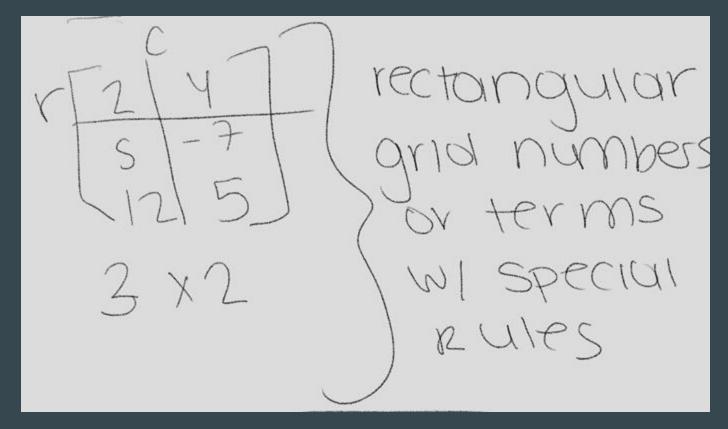
torke directional derivative compute it by taking present direction.

Directional
Derivative- is the rate at which the function changes at a point in the direction

ex: f(x, y, z) compute directional devivative aion a

Directional Derivative Example: dot product of avachent Vy f = 2 of + 3 of 14

What is a matrix



Element Wise Operations

Basic Matrix Operations

$$\begin{bmatrix} 2 & 1 \\ 7 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 7 & 11 \end{bmatrix}$$
$$A_{2x2} + B_{2x2} = C_{2x2}$$

$$\begin{bmatrix} 2 & 1 \\ 7 & 9 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 5 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 7 & 9 \end{bmatrix} \mathbf{x} \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ 26 & 27 \end{bmatrix}$$
$$A_{2x,2} \mathbf{x} B_{2x,2} = C_{2x,2}$$

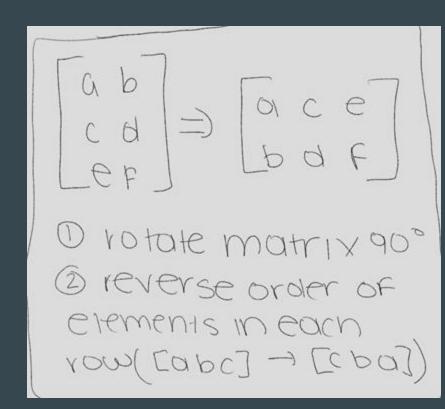
$$\frac{1}{8} \begin{bmatrix} 2 & 4 \\ 6 & 1 \end{bmatrix} = \begin{bmatrix} 1/4 & 1/2 \\ 3/4 & 1/8 \end{bmatrix}$$

Matrix Transpose

- Often denoted as 'T' M^T provides a way to rotate on of the matrices that the operation works with the multiplication requirements.

When would we use this?

 When we use neural networks as often we do not meet the requirements of matrix multiplication (When processing weights and inputs of different sizes)



Questions to Think About & Discuss

Calculate product
$$A = \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix} V = \begin{bmatrix} 8 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$

Calculate product (inner)

A= \[\frac{3}{2} \]

B= \[\frac{8}{2} \]

A \cdot B what is A Tr

 $4 + f(x) = \frac{x^3}{5000}$ for $0 \le x \le 10$ 4 + f(x) = 0

6 (110W+9W3+SIN(X) 01X

@ (8x-12) (4x2-12x) 4 dx

a) derivative, e1-cos(x)

OACC (TorT) actineoru A= \$1,4,7,63 B= {112,3,4,5° @ | A1, [B], C= {2,4,6,8} power set 2112,33 what is ax cartesian prod

Things to think about for future webinars.

- 1. What is a neural network?
- 2. What are inputs and weights in the context of a neural network?
- 3. What are the types of elements we can have in dataset.
- 4. What are the advantages of using Machine learning and data science?
- 5. What is the difference between Business Intelligence and Data Science?
- 6. What is the process in machine learning development?

Sources to Look at

- 1. http://ee263.stanford.edu/notes/matrix-primer-lect2.pdf (Linear Algebra, Matrix Operations)
- 2. http://tutorial.math.lamar.edu/Classes/CalcI/ComputingDefiniteIntegrals.aspx (Paul's Notes, Calculus I)
- 3. <u>https://www.mathsisfun.com/calculus/integration-rules.html</u> (Integration Practice)

Sources

- 1. https://www.mathsisfun.com/sets/symbols.html
- 2. https://www.pinterest.com/pin/788833690953621119/
- 3. https://ml-cheatsheet.readthedocs.io/en/latest/calculus.html#gradients
- 4. http://tutorial.math.lamar.edu/Classes/Calcl/ComputingDefiniteIntegrals.aspx
- 5. https://en.wikipedia.org/wiki/Constant_of_integration
- **6.** https://www.mathsisfun.com
- **1.** https://en.wikibooks.org/wiki/Calculus/Quotient_Rule
- 8. https://ml-cheatsheet.readthedocs.io/en/latest/linear_algebra.html#matrix-transpose

Thank You

For All you Questions please post it to the https://forum.clouderizer.com/. Under the header Seminar 1.

https://clouderizer.com/

https://forum.clouderizer.com/

Seminar 2: Statistics, Probability, and Solutions of questions.

 If you have suggestions for seminar 2 please post in the forum.

Tentative Date Seminar 2: August 23