

Joint mixed-effects models to account for unmeasured contextual confounding

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Load dependencies and source script

```
library(boot)
library(tidyverse)
library(igraph)
library(data.table)
library(pbapply)
library(lme4)
library(MASS)
library(statnet)
library(intergraph)
library(statmod)
library(fastGHQuad)
library(geex)
library(parallel)
library(here)
library(numDeriv)
library(rjags)
library(ggplot2)
library(dplyr)
# New package
library(Matrix)
library(mvtnorm)
source(url("https://github.com/vanessamcnealis/Joint-Mixed-Effects-Models-Network-Interference/blob/main/"))
```

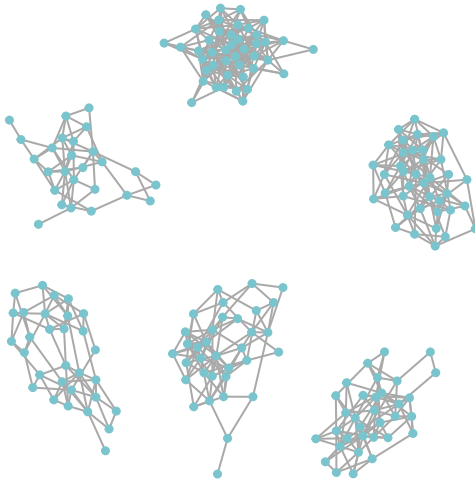
Data preparation

The data at hand `dat` consists of one simulated dataset used for the simulation studies presented in the paper entitled “Joint mixed-effects models for causal inference in clustered network-based observational studies”. The data generating scheme for the assigned treatment Z , the outcome Y , and the covariates X_1 , X_2 and X_3 is described in the manuscript. Note that a SAR structure with $\tau = 0.1$ was used to generate the random errors in the outcome model.

```
load(url("https://github.com/vanessamcnealis/Joint-Mixed-Effects-Models-Network-Interference/blob/main/"))
load(url("https://github.com/vanessamcnealis/Joint-Mixed-Effects-Models-Network-Interference/blob/main/"))
```

Network visualization

```
V(network)$component.id <- sim_data$component.id
subg <- subgraph(network, v=V(network)[V(network)$component.id %in% c(1, 4, 5, 10, 15, 25)])
plot(subg, vertex.size=3, vertex.label=NA, vertex.color="cadetblue3",
      vertex.frame.color="cadetblue3")
```



Prepare the data for modeling

```
# Total number of nodes
N <- nrow(sim_data)

#Number of disjoint subgraphs
m <- length(unique(sim_data$component.id))

# Adjacency matrix associated with the graph
A <- as.matrix(as_adjacency_matrix(network))

# Create a variable indicating the degree of a node
sim_data$degree <- as.numeric(A %*% rep(1, N))

# Create a variable indicating the number of treated neighbours for each node
sim_data$k_treated <-
```

```

as.numeric(A %*% matrix(sim_data$Z, ncol = 1, nrow = nrow(sim_data)))

# Create a variable indicated the proportion of treated neighbours for each node
sim_data$prop_treated <- ifelse(sim_data$degree == 0, 0, sim_data$k_treated/sim_data$degree)

# Change this vector for different alpha values
alpha.vec <- alpha.vec <- seq(0.1, 0.9, by=0.1)

```

Modeling

In this section, we will perform posterior inference for the average potential outcomes and associated causal contrasts under the proposed joint mixed effects model (JMM). The average potential outcome for a unit depends on an individual exposure/treatment z and treatment coverage $\alpha \in (0, 1)$, where the treatment coverage represents the counterfactual probability that first-order neighbours receive the exposure/treatment. For each treatment coverage α value, the R program will output the **posterior means, posterior variances, and 95% credible intervals for average potential outcomes** $\mu_{0\alpha}, \mu_{1\alpha}, \mu_\alpha$. Also, the R program will output the these quantities for different causal contrasts of interest, presented in the main paper.

Equations for calculating the four causal effect estimates are:

- $\hat{DE}(\alpha) = \hat{\mu}_{1\alpha} - \hat{\mu}_{0\alpha}$
- $\hat{IE}(\alpha_0, \alpha_1) = \hat{\mu}_{0\alpha_1} - \hat{\mu}_{0\alpha_0}$
- $\hat{TE}(\alpha_0, \alpha_1) = \hat{\mu}_{1\alpha_1} - \hat{\mu}_{0\alpha_0}$
- $\hat{OE}(\alpha_0, \alpha_1) = \hat{\mu}_{\alpha_1} - \hat{\mu}_{\alpha_0}$,

where α_0 and α_1 represent distinct treatment coverages with $\alpha_0 \neq \alpha_1$.

We first specify the models for the outcome and treatment propensity (note that these correspond to the oracle models). In the model for the treatment propensity, we include a random effect term for the subgraph. Note that these models are fitted **jointly** in the MCMC procedure, with a SAR prior for the model errors to account for residual network autocorrelation. Please refer to the paper for details.

```

outcome_formula <- Y ~ Z + prop_treated + prop_treated*Z + abs(X1) + X2 + abs(X1)*X2 + (1|component.id)
propensity_formula <- Z ~ abs(X1) + I(X2*abs(X1)) + X3 + (1|component.id)

```

Joint mixed-effects model (JMM) estimates

Here, we fit the JMM to the simulated data using MCMC with weakly informative priors described in the paper. The procedure is implemented in **rjags**. Posterior inference is performed using four independent chains of 30,000 samples after a burn-in period of 10,000 iterations and with a thinning parameter of 50.

```

# Uncomment these lines to run the fitting procedure (which might take a
# substantial amount of time)

#jmm_estimates <- jmm_fit(data=sim_data,
#                           outcome_formula = outcome_formula,
#                           propensity_formula = propensity_formula,
#                           A=A,
#                           alpha.vec = alpha.vec)

```

```
load(url("https://github.com/vanessamcnealis/Joint-Mixed-Effects-Models-Network-Interference/blob/main/
jmm_estimates
```

```
## $mu.0.hat
##      0.1      0.2      0.3      0.4      0.5      0.6      0.7      0.8
## 0.7547423 0.8476936 0.9406450 1.0335963 1.1265476 1.2194989 1.3124502 1.4054016
##      0.9
## 1.4983529
##
## $mu.1.hat
##      0.1      0.2      0.3      0.4      0.5      0.6      0.7      0.8
## 2.903652 3.093957 3.284262 3.474567 3.664872 3.855177 4.045482 4.235788
##      0.9
## 4.426093
##
## $mu.hat
##      0.1      0.2      0.3      0.4      0.5      0.6      0.7      0.8
## 0.9696333 1.2969463 1.6437301 2.0099847 2.3957100 2.8009060 3.2255728 3.6697103
##      0.9
## 4.1333186
##
## $DE.hat
##      0.1      0.2      0.3      0.4      0.5      0.6      0.7      0.8
## 2.148910 2.246263 2.343617 2.440971 2.538325 2.635678 2.733032 2.830386
##      0.9
## 2.927740
##
## $IE.hat
##      0.1      0.2      0.3      0.4      0.5      0.6
## 0.1 0.00000000 -0.09295132 -0.18590264 -0.27885396 -0.37180528 -0.46475660
## 0.2 0.09295132  0.00000000 -0.09295132 -0.18590264 -0.27885396 -0.37180528
## 0.3 0.18590264  0.09295132  0.00000000 -0.09295132 -0.18590264 -0.27885396
## 0.4 0.27885396  0.18590264  0.09295132  0.00000000 -0.09295132 -0.18590264
## 0.5 0.37180528  0.27885396  0.18590264  0.09295132  0.00000000 -0.09295132
## 0.6 0.46475660  0.37180528  0.27885396  0.18590264  0.09295132  0.00000000
## 0.7 0.55770793  0.46475660  0.37180528  0.27885396  0.18590264  0.09295132
## 0.8 0.65065925  0.55770793  0.46475660  0.37180528  0.27885396  0.18590264
## 0.9 0.74361057  0.65065925  0.55770793  0.46475660  0.37180528  0.27885396
##      0.7      0.8      0.9
## 0.1 -0.55770793 -0.65065925 -0.74361057
## 0.2 -0.46475660 -0.55770793 -0.65065925
## 0.3 -0.37180528 -0.46475660 -0.55770793
## 0.4 -0.27885396 -0.37180528 -0.46475660
## 0.5 -0.18590264 -0.27885396 -0.37180528
## 0.6 -0.09295132 -0.18590264 -0.27885396
## 0.7  0.00000000 -0.09295132 -0.18590264
## 0.8  0.09295132  0.00000000 -0.09295132
## 0.9  0.18590264  0.09295132  0.00000000
##
## $OE.hat
##      0.1      0.2      0.3      0.4      0.5      0.6      0.7
## 0.1 0.00000000 -0.3273130 -0.6740968 -1.0403514 -1.4260767 -1.8312727 -2.2559395
## 0.2 0.3273130  0.0000000 -0.3467838 -0.7130383 -1.0987636 -1.5039597 -1.9286264
```

```

## 0.3 0.6740968 0.3467838 0.0000000 -0.3662545 -0.7519798 -1.1571759 -1.5818427
## 0.4 1.0403514 0.7130383 0.3662545 0.0000000 -0.3857253 -0.7909213 -1.2155881
## 0.5 1.4260767 1.0987636 0.7519798 0.3857253 0.0000000 -0.4051960 -0.8298628
## 0.6 1.8312727 1.5039597 1.1571759 0.7909213 0.4051960 0.0000000 -0.4246668
## 0.7 2.2559395 1.9286264 1.5818427 1.2155881 0.8298628 0.4246668 0.0000000
## 0.8 2.7000770 2.3727640 2.0259802 1.6597257 1.2740004 0.8688043 0.4441375
## 0.9 3.1636853 2.8363723 2.4895885 2.1233339 1.7376086 1.3324126 0.9077458
##      0.8      0.9
## 0.1 -2.7000770 -3.1636853
## 0.2 -2.3727640 -2.8363723
## 0.3 -2.0259802 -2.4895885
## 0.4 -1.6597257 -2.1233339
## 0.5 -1.2740004 -1.7376086
## 0.6 -0.8688043 -1.3324126
## 0.7 -0.4441375 -0.9077458
## 0.8 0.0000000 -0.4636083
## 0.9 0.4636083 0.0000000
##
## $TE.hat
##      0.1      0.2      0.3      0.4      0.5      0.6      0.7      0.8
## 0.1 2.148910 2.055958 1.963007 1.870056 1.777104 1.684153 1.591202 1.498251
## 0.2 2.339215 2.246263 2.153312 2.060361 1.967410 1.874458 1.781507 1.688556
## 0.3 2.529520 2.436569 2.343617 2.250666 2.157715 2.064763 1.971812 1.878861
## 0.4 2.719825 2.626874 2.533922 2.440971 2.348020 2.255068 2.162117 2.069166
## 0.5 2.910130 2.817179 2.724227 2.631276 2.538325 2.445373 2.352422 2.259471
## 0.6 3.100435 3.007484 2.914532 2.821581 2.728630 2.635678 2.542727 2.449776
## 0.7 3.290740 3.197789 3.104837 3.011886 2.918935 2.825984 2.733032 2.640081
## 0.8 3.481045 3.388094 3.295143 3.202191 3.109240 3.016289 2.923337 2.830386
## 0.9 3.671350 3.578399 3.485448 3.392496 3.299545 3.206594 3.113642 3.020691
##      0.9
## 0.1 1.405299
## 0.2 1.595604
## 0.3 1.785909
## 0.4 1.976214
## 0.5 2.166519
## 0.6 2.356824
## 0.7 2.547130
## 0.8 2.737435
## 0.9 2.927740
##
## $var.mu.0
##      0.1      0.2      0.3      0.4      0.5      0.6      0.7
## 0.02953432 0.02827780 0.02744951 0.02704947 0.02707765 0.02753407 0.02841873
##      0.8      0.9
## 0.02973162 0.03147275
##
## $var.mu.1
##      0.1      0.2      0.3      0.4      0.5      0.6      0.7
## 0.03065527 0.02907905 0.02804313 0.02754749 0.02759215 0.02817710 0.02930234
##      0.8      0.9
## 0.03096788 0.03317370
##
## $var.mu
##      0.1      0.2      0.3      0.4      0.5      0.6      0.7

```

```

## 0.02873795 0.02723942 0.02641881 0.02607552 0.02611305 0.02653893 0.02746481
##      0.8      0.9
## 0.02910638 0.03178343
##
## $var.DE
##      0.1      0.2      0.3      0.4      0.5      0.6
## 0.010094056 0.007491455 0.005756149 0.004888137 0.004887419 0.005753996
##      0.7      0.8      0.9
## 0.007487867 0.010089033 0.013557494
##
## $var.IE
##      0.1      0.2      0.3      0.4      0.5
## 0.1 0.0000000000 0.0002141176 0.0008564704 0.0019270584 0.0034258817
## 0.2 0.0002141176 0.0000000000 0.0002141176 0.0008564704 0.0019270584
## 0.3 0.0008564704 0.0002141176 0.0000000000 0.0002141176 0.0008564704
## 0.4 0.0019270584 0.0008564704 0.0002141176 0.0000000000 0.0002141176
## 0.5 0.0034258817 0.0019270584 0.0008564704 0.0002141176 0.0000000000
## 0.6 0.0053529401 0.0034258817 0.0019270584 0.0008564704 0.0002141176
## 0.7 0.0077082337 0.0053529401 0.0034258817 0.0019270584 0.0008564704
## 0.8 0.0104917626 0.0077082337 0.0053529401 0.0034258817 0.0019270584
## 0.9 0.0137035267 0.0104917626 0.0077082337 0.0053529401 0.0034258817
##      0.6      0.7      0.8      0.9
## 0.1 0.0053529401 0.0077082337 0.0104917626 0.0137035267
## 0.2 0.0034258817 0.0053529401 0.0077082337 0.0104917626
## 0.3 0.0019270584 0.0034258817 0.0053529401 0.0077082337
## 0.4 0.0008564704 0.0019270584 0.0034258817 0.0053529401
## 0.5 0.0002141176 0.0008564704 0.0019270584 0.0034258817
## 0.6 0.0000000000 0.0002141176 0.0008564704 0.0019270584
## 0.7 0.0002141176 0.0000000000 0.0002141176 0.0008564704
## 0.8 0.0008564704 0.0002141176 0.0000000000 0.0002141176
## 0.9 0.0019270584 0.0008564704 0.0002141176 0.0000000000
##
## $var.OE
##      0.1      0.2      0.3      0.4      0.5
## 0.1 0.0000000000 0.0003732319 0.0013071597 0.0026011879 0.0041587964
## 0.2 0.0003732319 0.0000000000 0.0002890209 0.0010396991 0.0021555145
## 0.3 0.0013071597 0.0002890209 0.0000000000 0.0002395016 0.0009110056
## 0.4 0.0026011879 0.0010396991 0.0002395016 0.0000000000 0.0002246741
## 0.5 0.0041587964 0.0021555145 0.0009110056 0.0002246741 0.0000000000
## 0.6 0.0059875402 0.0036440222 0.0020220670 0.0009210791 0.0002445384
## 0.7 0.0081990500 0.0056168528 0.0036843165 0.0022008456 0.0010699198
## 0.8 0.0110090316 0.0082897122 0.0061134600 0.0042796793 0.0026918502
## 0.9 0.0147372661 0.0119823815 0.0096292785 0.0074773616 0.0054301106
##      0.6      0.7      0.8      0.9
## 0.1 0.0059875402 0.0081990500 0.0110090316 0.0147372661
## 0.2 0.0036440222 0.0056168528 0.0082897122 0.0119823815
## 0.3 0.0020220670 0.0036843165 0.0061134600 0.0096292785
## 0.4 0.0009210791 0.0022008456 0.0042796793 0.0074773616
## 0.5 0.0002445384 0.0010699198 0.0026918502 0.0054301106
## 0.6 0.0000000000 0.0002990945 0.0013575276 0.0034950808
## 0.7 0.0002990945 0.0000000000 0.0003883423 0.0017839026
## 0.8 0.0013575276 0.0003883423 0.0000000000 0.0005122819
## 0.9 0.0034950808 0.0017839026 0.0005122819 0.0000000000
##

```

```

## $var.TE
##          0.1          0.2          0.3          0.4          0.5          0.6
## 0.1 0.010094056 0.008537408 0.007408994 0.006708816 0.006436874 0.006593166
## 0.2 0.009098720 0.007491455 0.006312425 0.005561631 0.005239072 0.005344748
## 0.3 0.008643676 0.006985795 0.005756149 0.004954738 0.004581562 0.004636622
## 0.4 0.008728924 0.007020426 0.005740164 0.004888137 0.004464344 0.004468788
## 0.5 0.009354464 0.007595350 0.006264471 0.005361828 0.004887419 0.004841246
## 0.6 0.010520297 0.008710566 0.007329071 0.006375811 0.005850786 0.005753996
## 0.7 0.012226421 0.010366074 0.008933962 0.007930086 0.007354444 0.007207038
## 0.8 0.014472838 0.012561874 0.011079146 0.010024653 0.009398395 0.009200373
## 0.9 0.017259546 0.015297966 0.013764622 0.012659512 0.011982638 0.011733999
##          0.7          0.8          0.9
## 0.1 0.007177694 0.008190456 0.009631454
## 0.2 0.005878659 0.006840805 0.008231187
## 0.3 0.005119916 0.006031446 0.007371212
## 0.4 0.004901466 0.005762380 0.007051528
## 0.5 0.005223308 0.006033605 0.007272137
## 0.6 0.006085441 0.006845122 0.008033038
## 0.7 0.007487867 0.008196932 0.009334231
## 0.8 0.009430585 0.010089033 0.011175716
## 0.9 0.011913596 0.012521427 0.013557494
##
## $CrI.mu.0
##          0.1          0.2          0.3          0.4          0.5          0.6          0.7
## 2.5% 0.413873 0.5178401 0.6197421 0.7164376 0.8113408 0.8966862 0.9801676
## 97.5% 1.098608 1.1840239 1.2662503 1.3535272 1.4429629 1.5366555 1.6321068
##          0.8          0.9
## 2.5% 1.068171 1.152909
## 97.5% 1.736658 1.836308
##
## $CrI.mu.1
##          0.1          0.2          0.3          0.4          0.5          0.6          0.7          0.8
## 2.5% 2.558726 2.759509 2.957449 3.148789 3.336098 3.528184 3.710268 3.885882
## 97.5% 3.243478 3.424168 3.606561 3.790254 3.979518 4.181656 4.381621 4.587014
##          0.9
## 2.5% 4.068328
## 97.5% 4.790445
##
## $CrI.mu
##          0.1          0.2          0.3          0.4          0.5          0.6          0.7          0.8
## 2.5% 0.6408188 0.9774267 1.327591 1.699335 2.088187 2.488530 2.904857 3.340094
## 97.5% 1.3058240 1.6204042 1.960385 2.323113 2.708712 3.116902 3.543188 4.003184
##          0.9
## 2.5% 3.782731
## 97.5% 4.488417
##
## $CrI.DE
##          0.1          0.2          0.3          0.4          0.5          0.6          0.7          0.8
## 2.5% 1.954008 2.072365 2.189866 2.304174 2.400895 2.478752 2.555642 2.622233
## 97.5% 2.346296 2.415555 2.488974 2.574305 2.666162 2.775731 2.896557 3.025938
##          0.9
## 2.5% 2.687498
## 97.5% 3.151157
##

```

```

## $CrI.IE.lb
##          0.1          0.2          0.3          0.4          0.5          0.6
## 0.1 0.00000000 -0.12253358 -0.24506716 -0.36760074 -0.49013432 -0.61266790
## 0.2 0.06371798 0.00000000 -0.12253358 -0.24506716 -0.36760074 -0.49013432
## 0.3 0.12743595 0.06371798 0.00000000 -0.12253358 -0.24506716 -0.36760074
## 0.4 0.19115393 0.12743595 0.06371798 0.00000000 -0.12253358 -0.24506716
## 0.5 0.25487191 0.19115393 0.12743595 0.06371798 0.00000000 -0.12253358
## 0.6 0.31858988 0.25487191 0.19115393 0.12743595 0.06371798 0.00000000
## 0.7 0.38230786 0.31858988 0.25487191 0.19115393 0.12743595 0.06371798
## 0.8 0.44602584 0.38230786 0.31858988 0.25487191 0.19115393 0.12743595
## 0.9 0.50974381 0.44602584 0.38230786 0.31858988 0.25487191 0.19115393
##          0.7          0.8          0.9
## 0.1 -0.73520147 -0.85773505 -0.9802686
## 0.2 -0.61266790 -0.73520147 -0.8577351
## 0.3 -0.49013432 -0.61266790 -0.7352015
## 0.4 -0.36760074 -0.49013432 -0.6126679
## 0.5 -0.24506716 -0.36760074 -0.4901343
## 0.6 -0.12253358 -0.24506716 -0.3676007
## 0.7 0.00000000 -0.12253358 -0.2450672
## 0.8 0.06371798 0.00000000 -0.1225336
## 0.9 0.12743595 0.06371798 0.0000000
##
## $CrI.IE.ub
##          0.1          0.2          0.3          0.4          0.5          0.6
## 0.1 0.00000000 -0.06371798 -0.12743595 -0.19115393 -0.25487191 -0.31858988
## 0.2 0.1225336 0.00000000 -0.06371798 -0.12743595 -0.19115393 -0.25487191
## 0.3 0.2450672 0.12253358 0.00000000 -0.06371798 -0.12743595 -0.19115393
## 0.4 0.3676007 0.24506716 0.12253358 0.00000000 -0.06371798 -0.12743595
## 0.5 0.4901343 0.36760074 0.24506716 0.12253358 0.00000000 -0.06371798
## 0.6 0.6126679 0.49013432 0.36760074 0.24506716 0.12253358 0.00000000
## 0.7 0.7352015 0.61266790 0.49013432 0.36760074 0.24506716 0.12253358
## 0.8 0.8577351 0.73520147 0.61266790 0.49013432 0.36760074 0.24506716
## 0.9 0.9802686 0.85773505 0.73520147 0.61266790 0.49013432 0.36760074
##          0.7          0.8          0.9
## 0.1 -0.38230786 -0.44602584 -0.50974381
## 0.2 -0.31858988 -0.38230786 -0.44602584
## 0.3 -0.25487191 -0.31858988 -0.38230786
## 0.4 -0.19115393 -0.25487191 -0.31858988
## 0.5 -0.12743595 -0.19115393 -0.25487191
## 0.6 -0.06371798 -0.12743595 -0.19115393
## 0.7 0.00000000 -0.06371798 -0.12743595
## 0.8 0.12253358 0.00000000 -0.06371798
## 0.9 0.24506716 0.12253358 0.00000000
##
## $CrI.OE.lb
##          0.1          0.2          0.3          0.4          0.5          0.6          0.7
## 0.1 0.00000000 -0.3655753 -0.7471689 -1.1429884 -1.5545461 -1.9857441 -2.4350145
## 0.2 0.2905607 0.00000000 -0.3809961 -0.7772730 -1.1914465 -1.6233430 -2.0792239
## 0.3 0.6046869 0.3139313 0.00000000 -0.3971488 -0.8116715 -1.2475344 -1.7058404
## 0.4 0.9417938 0.6490012 0.3355320 0.00000000 -0.4158448 -0.8529202 -1.3108542
## 0.5 1.2980023 1.0065959 0.6924097 0.3555073 0.00000000 -0.4369514 -0.8967713
## 0.6 1.6776598 1.3848194 1.0665219 0.7297017 0.3735384 0.00000000 -0.4596838
## 0.7 2.0772291 1.7775365 1.4594033 1.1206151 0.7642203 0.3903380 0.00000000
## 0.8 2.4885512 2.1891050 1.8676919 1.5284405 1.1710140 0.7949997 0.4045680

```



```

## 0.9 2.9188067 2.6147686 2.2926608 1.9516900 1.5899995 1.2137039 0.8225597
##          0.8          0.9
## 0.1 -2.9109135 -3.4116808
## 0.2 -2.5587606 -3.0586597
## 0.3 -2.1847569 -2.6903140
## 0.4 -1.7935426 -2.2984190
## 0.5 -1.3790514 -1.8848024
## 0.6 -0.9424012 -1.4476225
## 0.7 -0.4825408 -0.9895920
## 0.8 0.0000000 -0.5073355
## 0.9 0.4183318 0.0000000
##
## $CrI.OE.ub
##          0.1          0.2          0.3          0.4          0.5          0.6          0.7
## 0.1 0.0000000 -0.2905607 -0.6046869 -0.9417938 -1.2980023 -1.6776598 -2.0772291
## 0.2 0.3655753 0.0000000 -0.3139313 -0.6490012 -1.0065959 -1.3848194 -1.7775365
## 0.3 0.7471689 0.3809961 0.0000000 -0.3355320 -0.6924097 -1.0665219 -1.4594033
## 0.4 1.1429884 0.7772730 0.3971488 0.0000000 -0.3555073 -0.7297017 -1.1206151
## 0.5 1.5545461 1.1914465 0.8116715 0.4158448 0.0000000 -0.3735384 -0.7642203
## 0.6 1.9857441 1.6233430 1.2475344 0.8529202 0.4369514 0.0000000 -0.3903380
## 0.7 2.4350145 2.0792239 1.7058404 1.3108542 0.8967713 0.4596838 0.0000000
## 0.8 2.9109135 2.5587606 2.1847569 1.7935426 1.3790514 0.9424012 0.4825408
## 0.9 3.4116808 3.0586597 2.6903140 2.2984190 1.8848024 1.4476225 0.9895920
##          0.8          0.9
## 0.1 -2.4885512 -2.9188067
## 0.2 -2.1891050 -2.6147686
## 0.3 -1.8676919 -2.2926608
## 0.4 -1.5284405 -1.9516900
## 0.5 -1.1710140 -1.5899995
## 0.6 -0.7949997 -1.2137039
## 0.7 -0.4045680 -0.8225597
## 0.8 0.0000000 -0.4183318
## 0.9 0.5073355 0.0000000
##
## $CrI.TE.lb
##          0.1          0.2          0.3          0.4          0.5          0.6          0.7          0.8
## 0.1 1.954008 1.875405 1.791192 1.707783 1.616561 1.523474 1.419930 1.319117
## 0.2 2.152678 2.072365 1.991395 1.912544 1.825699 1.730846 1.629283 1.528321
## 0.3 2.346520 2.268282 2.189866 2.110835 2.025848 1.934057 1.833105 1.727125
## 0.4 2.535388 2.462615 2.383402 2.304174 2.215016 2.123990 2.022766 1.918458
## 0.5 2.719737 2.648338 2.569375 2.487703 2.400895 2.303397 2.205955 2.099562
## 0.6 2.899474 2.826391 2.741927 2.660245 2.574938 2.478752 2.381194 2.276410
## 0.7 3.071390 2.994397 2.915856 2.829681 2.740322 2.652478 2.555642 2.449562
## 0.8 3.238880 3.162532 3.085341 3.000617 2.907972 2.820065 2.724169 2.622233
## 0.9 3.408197 3.331227 3.249811 3.164165 3.076886 2.984721 2.889961 2.789523
##          0.9
## 0.1 1.211625
## 0.2 1.417835
## 0.3 1.618605
## 0.4 1.810644
## 0.5 1.992131
## 0.6 2.169565
## 0.7 2.346131
## 0.8 2.516825

```

```

## 0.9 2.687498
##
## $CrI.TE.ub
##      0.1      0.2      0.3      0.4      0.5      0.6      0.7      0.8
## 0.1 2.346296 2.239142 2.133175 2.034748 1.935941 1.841857 1.755190 1.669788
## 0.2 2.526196 2.415555 2.309859 2.209698 2.111784 2.017624 1.927456 1.848664
## 0.3 2.708344 2.600013 2.488974 2.388313 2.290119 2.197117 2.111641 2.031531
## 0.4 2.902763 2.786195 2.677595 2.574305 2.473730 2.384177 2.296822 2.215383
## 0.5 3.093849 2.982181 2.873761 2.765892 2.666162 2.577473 2.486986 2.405892
## 0.6 3.295248 3.184498 3.076627 2.972424 2.870061 2.775731 2.690222 2.606826
## 0.7 3.505108 3.395859 3.285624 3.181817 3.077237 2.985823 2.896557 2.812879
## 0.8 3.721627 3.612248 3.502687 3.398252 3.296256 3.201900 3.110748 3.025938
## 0.9 3.938330 3.828649 3.722477 3.615687 3.514079 3.419962 3.323588 3.240753
##      0.9
## 0.1 1.592194
## 0.2 1.769069
## 0.3 1.953820
## 0.4 2.138313
## 0.5 2.323627
## 0.6 2.524724
## 0.7 2.731035
## 0.8 2.939643
## 0.9 3.151157

```

The object above is a list containing the following components:

- `mu.0.hat` is the vector of posterior means for $\mu_{0\alpha}$ for values of α contained in `alpha.vec`;
- `mu.1.hat` is the vector of posterior means for $\mu_{1\alpha}$;
- `mu.hat` is the vector of posterior means for $\mu_\alpha = \alpha\mu_{1\alpha} + (1 - \alpha)\mu_{0\alpha}$;
- `DE.hat` is the vector of posterior means for $DE(\alpha)$;
- `IE.hat`, `OE.hat`, and `TE.hat` are the matrices of posterior means for $IE(\alpha_0, \alpha_1)$, $OE(\alpha_0, \alpha_1)$, and $TE(\alpha_0, \alpha_1)$, respectively;
- `var.mu.0` is the vector of posterior variances for $\mu_{0\alpha}$ for values of α contained in `alpha.vec`;
- `var.mu.1` is the vector of posterior variances for $\mu_{1\alpha}$;
- `var.mu` is the vector of posterior variances for μ_α ;
- `var.DE` is the vector of posterior variances for $DE(\alpha)$;
- `var.IE`, `var.OE`, and `var.TE` are the matrices of posterior variances for $IE(\alpha_0, \alpha_1)$, $OE(\alpha_0, \alpha_1)$, and $TE(\alpha_0, \alpha_1)$, respectively;
- `CrI.mu.0` is a matrix containing 95% credible intervals for $\mu_{0\alpha}$, with the first row containing the lower bounds and the second row containing the upper bounds, for values of α contained in `alpha.vec`;
- `CrI.mu.1` is a matrix containing 95% credible intervals for $\mu_{1\alpha}$;
- `CrI.mu` is a matrix containing 95% credible intervals for μ_α ;
- `CrI.DE` is a matrix containing 95% credible intervals for $DE(\alpha)$;
- `CrI.IE.lb`, `CrI.OE.lb`, and `CrI.TE.lb` are the matrices containing the lower bounds of 95% credible intervals for $IE(\alpha_0, \alpha_1)$, $OE(\alpha_0, \alpha_1)$, and $TE(\alpha_0, \alpha_1)$, respectively;

- `CrI.IE.ub`, `CrI.OE.ub`, and `CrI.TE.ub` are the matrices containing the upper bounds of 95% credible intervals for $IE(\alpha_0, \alpha_1)$, $OE(\alpha_0, \alpha_1)$, and $TE(\alpha_0, \alpha_1)$, respectively.

We can plot the estimated direct $DE(\alpha)$, indirect $IE(\alpha, 0.3)$, overall $OE(\alpha, 0.3)$, and total $TE(\alpha, 0.3)$ effects as follows.

```
estimates <- c(jmm_estimates$DE.hat,
               jmm_estimates$IE.hat[,3],
               jmm_estimates$OE.hat[,3],
               jmm_estimates$TE.hat[,3])

lower <- c(jmm_estimates$CrI.DE[1,],
           jmm_estimates$CrI.IE.lb[,3],
           jmm_estimates$CrI.OE.lb[,3],
           jmm_estimates$CrI.TE.lb[,3])

upper <- c(jmm_estimates$CrI.DE[2,],
           jmm_estimates$CrI.IE.ub[,3],
           jmm_estimates$CrI.OE.ub[,3],
           jmm_estimates$CrI.TE.ub[,3])

df <- data.frame(
  alpha = alpha.vec,
  estimate = estimates,
  lower = lower,
  upper = upper,
  type = rep(c("DE(alpha)", "IE(alpha,0.3)", "OE(alpha,0.3)", "TE(alpha,0.3)"), each = length(alpha.vec))
)

facet_labels <- c(
  "DE(alpha)" = expression(DE(alpha)),
  "IE(alpha,0.3)" = expression(IE(alpha, 0.3)),
  "TE(alpha,0.3)" = expression(TE(alpha, 0.3)),
  "OE(alpha,0.3)" = expression(OE(alpha, 0.3))
)

# Plot
ggplot(df, aes(x = alpha, y = estimate)) +
  geom_ribbon(aes(ymin = lower, ymax = upper), alpha = 0.15, colour = NA) +
  geom_line(linewidth = 0.5) +
  facet_wrap(~ type, ncol = 2, scales = "free_y") +
  labs(
    x = expression("Treatment coverage " * alpha),
    y = "Estimate"
  ) +
  theme_classic(base_size = 14) +
  theme(
    legend.position = "top",
    legend.title = element_text(size = 13),
    legend.text = element_text(size = 12),
    strip.background = element_blank(),
    strip.text = element_text(size = 13, face = "bold"),
    panel.spacing = unit(1, "lines")
  )
```

)

