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CS 161
HW7

1. Prove

$Pr(\alpha 1, . . . , \alpha n \mid \beta) = Pr(\alpha 1 \mid \alpha 2, . . . , \alpha n, \beta) Pr(\alpha 2 \mid \alpha 3, . . . , \alpha n, \beta). . .Pr(\alpha n \mid \beta).$

Known:

$Pr(A|B) = Pr(A, B)/Pr(B)$

we can conclude that:

$Pr(A, B) = Pr(A|B) Pr(B)$

prove:

left hand side:

$Pr(\alpha 1, . . . , \alpha n \mid \beta)$
$= Pr(\alpha 1, . . . , \alpha n, \beta) / Pr(\beta)$
$= Pr(\alpha 1 \mid \alpha 2, ..., \alpha n, \beta) Pr(\alpha 2, ..., \alpha n, \beta) / Pr(\beta)$
$= Pr(\alpha 1 \mid \alpha 2, ..., \alpha n, \beta) Pr(\alpha 2 \mid \alpha 3 ..., \alpha n, \beta) Pr(\alpha 3, ... \alpha n, \beta) / Pr(\beta)$
= .
= .
= .
$= Pr(\alpha 1 \mid \alpha 2, ..., \alpha n, \beta) Pr(\alpha 2 \mid \alpha 3 ..., \alpha n, \beta) Pr(\alpha 3, ... \alpha n, \beta)...Pr(\alpha n, \beta) / Pr(\beta)$
$= Pr(\alpha 1 \mid \alpha 2, ..., \alpha n, \beta) Pr(\alpha 2 \mid \alpha 3 ..., \alpha n, \beta) Pr(\alpha 3, ... \alpha n, \beta)...Pr(\alpha n \mid \beta) Pr(\beta)/ Pr(\beta)$
$= Pr(\alpha 1 \mid \alpha 2, ..., \alpha n, \beta) Pr(\alpha 2 \mid \alpha 3 ..., \alpha n, \beta) Pr(\alpha 3, ... \alpha n, \beta)...Pr(\alpha n \mid \beta) * 1$
$= Pr(\alpha 1 \mid \alpha 2, ..., \alpha n, \beta) Pr(\alpha 2 \mid \alpha 3 ..., \alpha n, \beta) Pr(\alpha 3, ... \alpha n, \beta)...Pr(\alpha n \mid \beta)$
= right hand side

2.

Given:

Pr(Oil) = 0.5
Pr(~Oil) = 0.5
Pr(Gas) = 0.2
Pr(~Oil & ~Gas) = 0.3 //neither being present
Pr(positive|Oil) = 0.9 //oil is present and geological test is positive
Pr(positive|~Oil) = 0.1 // 1 - Pr(positive|Oil)
Pr(positive|Gas) = 0.3
Pr(positive|~Oil & ~Gas) = 0.1

Find:

Pr(Oil|positive) = ?

Ans:

According to Bayes Rule

Pr(Oil|positive)

= Pr(positive|Oil)*Pr(Oil) / Pr(positive)

Total Probability of being Positive:

Pr(positive) = Pr(positive, Oil) + Pr(positive, Gas) + Pr(positive, ~Oil & ~Gas)

= Pr(positive|Oil) Pr(Oil) + Pr(positive|Gas) Pr(Gas) + Pr(positive|~Oil&~Gas) Pr(~Oil&~Gas)

= 0.9 * 0.5 + 0.3 * 0.2 + 0.1 * 0.3

= 0.54

Pr(Oil|positive)

= Pr(positive|Oil)*Pr(Oil) / Pr(positive)

= 0.9 * 0.5 / 0.54

= 0.833

3.

Coin	Pr(Coin)
a	1/3
b	1/3
c	1/3

X1

Coin	Pr(Head)
a	0.2
b	0.4
c	0.8

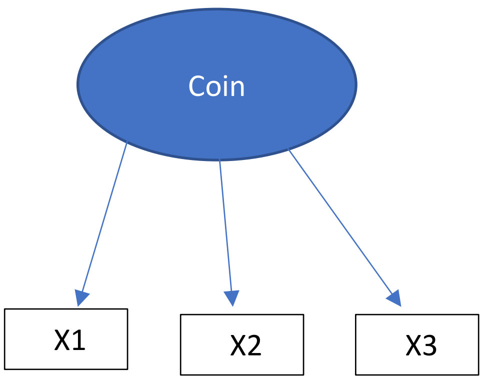
X2

Coin	Pr(Head)
a	0.2
b	0.4

c	0.8

X3

Coin	Pr(Head)
a	0.2
b	0.4
c	0.8



4.
(a)
I(A, {}, {B, E})
I(B, {}, {A, C})
I(C, A, {D, B, E})
I(D, {A, B}, {C, E})
I(E, B, {A, C, D, F, G})
I(F, {C,D}, {A,B,E})
I(G, F, {A,B,C,D,E,H})
I(H, {F,E}, {A,B,C,D,G})

(b)

d_separated (A, F, E) = False

ANS: In figure 1, we can clearly see that C is not known, and A→C→F is a sequential, thus the path A→C→F is not blocked. The path C→F→H is blocked since F is known. However, since we know F, the path C→F←D is not blocked since it is a convergent. F←D←B is also not blocked since D is not known and the path is sequential. Then if we look at the path D←B→E, the path is also not blocked. Based on the above, we are able to conclude that A is not independent of E by given F, and d_separated (A, F, E) is False.

d_separated (G, B, E) = True

ANS: From figure 1, we can see that there are three path from G to E. To prove d_separated(G, B, E) is true, we need to show that each of these paths from G to E is closed

(1) (G←F→H←E) Since F and H is not known, F→H←E is convergent and is closed. Thus the path is blocked.
(2) (G←F←C←A→D←B→E) Since D is not known, A→D←B is convergent and is closed. Thus the path is also blocked.
(3) (G←F←D←B→E) Since B is known, the path D←B→E is divergent, the path is closed.

According to (1), (2) and (3), we can conclude that d_separated (G, B, E) is True since none of path is smooth.

d_separated (AB, CDE, GH) = True

ANS: From figure 1, we can first conclude that A→D←B is smooth, G←F→H is also smooth. We can now treat the graph as two parts. If we want to prove AB and GH are independent by given CDE, we need to show that all path from A to (G and H) and B to (G and H) are closed.

(1) A→C→F is closed as it is a sequential and C is known. We no longer need to check the path goes with F since we already know that the path with G, F, H is smooth. In this path, we've proved that the path A→C … to G and H is closed.
(2) A→D→F is a sequential and is also closed since D is known. Based on (1), we also no longer need to check further about F. Therefore, combine with the result in (1), all paths goes out from A to G and H are closed.
(3) B→D→F is a sequential and D is already known, thus the path is closed. Same reason as (1) and (2), the path from B → D →…G or H are all closed.
(4) B→E→H is a sequential and E is known, thus the path is closed. Combine with solution in (3), all paths goes out from B to G and H are all closed.

According to (1), (2), (3) and (4), d_separated (AB, CDE, GH) is true since all paths from AB to GH are closed.

(c) Pr(a, b, c, d, e, f, g, h)
= Pr(a) Pr(b) Pr(c|a) Pr(d|a, b) Pr(e|b) Pr(f|c, d) Pr(g| f) Pr(h| e, f)

(d) A and B are independent to each other, therefore we can apply the following operation:

Pr(A=1, B=1) = Pr(A=1) * Pr(B=1) = 0.14

E and A are also independent, thus we will have the following operation:

$Pr(E=0 \mid A=0)$
$= Pr(E, A)/Pr(A)$
$= Pr(E=0)$
$= Pr(E=0|B=0)\, Pr(B=0) + Pr(E=0|B=1)\, Pr(B=1)$
$= 0.1 * 0.3 + 0.9 * 0.7$
$= 0.66$

5.
(a) models: w0 , w2, w3
(b) Pr(a) = Pr(w0, w2, w3) = 0.3 + 0.1 + 0.4 = 0.8
(c) Pr(A, B | a) = Pr(A, B, a) / Pr(a)

	A B	Pr(A, B)	Pr(A, B | a)
W0	T T	0.3	0.375
W2	F T	0.1	0.125
W3	F F	0.4	0.5

(d) $Pr(A \rightarrow \sim B|a) = (0.1+0.4) / 0.8 = 0.625$