

1.

(a)  $P(A, B, C), P(x, y, z)$

Unifier:  $\{x/A, y/B, z/C\}$

(b)  $Q(y, G(B, A), D), Q(G(x, x), y, D)$ .

The unifier does not exist because  $x$  cannot bind to both  $A$  and  $B$

(c)  $R(x, z, A), R(A, z, y)$

$\{x/A, y/A, z/B\}$

(d)  $\text{Older}(\text{Father}(y), \text{John}), \text{Older}(\text{Father}(x), x)$ .

$\{x/\text{John}, y/\text{John}\}$

(e)  $\text{Knows}(y, y), \text{Knows}(\text{Father}(x), x)$ .

The unifier does not exist since we cannot force  $y/\text{Father}(y)$

2.

- John likes all kinds of food.
- Apples are food.
- Chicken is food.
- Anything someone eats and isn't killed by is food.
- If you are killed by something, you are not alive.
- Bill eats peanuts and is still alive. \*
- Sue eats everything Bill eats.

1. (a) Translate these sentences into formulas in first-order logic.

- a.  $(\forall x) (\text{Food}(x) \Rightarrow \text{Likes}(\text{John}, x))$
- b.  $\text{Food}(\text{Apples})$
- c.  $\text{Food}(\text{Chicken})$
- d.  $((\forall a \forall b) (\text{Eat}(a, b) \ \& \ (\sim \text{Kill}(b, a))) \Rightarrow \text{Food}(b))$

- e.  $((\forall q \forall r) \text{Kills}(q, r) \Rightarrow (\neg \text{Alive}(r)))$
- f.  $\text{Eats}(\text{Bill}, \text{Peanuts}) \wedge \text{Alive}(\text{Bill})$
- g.  $(\forall z) ((\text{Eats}(\text{Bill}, z)) \Rightarrow \text{Eats}(\text{Sue}, z))$

(b) Convert the formulas of part (a) into CNF (also called clausal form).

- a.  $\neg \text{Food}(x) \vee \text{Likes}(\text{John}, x)$
- b.  $\text{Food}(\text{Apples})$
- c.  $\text{Food}(\text{Chicken})$
- d.  $\neg \text{Eat}(a, b) \vee \text{Kill}(b, a) \vee \text{Food}(b)$
- e.  $\neg \text{Kills}(q, r) \vee \neg \text{Alive}(r)$
- f.  $\text{Eats}(\text{Bill}, \text{Peanut}) \wedge \text{Alive}(\text{Bill})$
- g.  $\neg \text{Eats}(\text{Bill}, z) \vee \text{Eats}(\text{Sue}, z)$

(c) Prove that John likes peanuts using resolution.

- a.  $\neg \text{Food}(x) \vee \text{Likes}(\text{John}, x)$
- b.  $\text{Food}(\text{Apples})$
- c.  $\text{Food}(\text{Chicken})$
- d.  $\neg \text{Eat}(a, b) \vee \text{Kill}(b, a) \vee \text{Food}(b)$
- e.  $\neg \text{Kills}(q, r) \vee \neg \text{Alive}(r)$
- f.  $\text{Eats}(\text{Bill}, \text{Peanut}) \wedge \text{Alive}(\text{Bill})$
- g.  $\neg \text{Eats}(\text{Bill}, z) \vee \text{Eats}(\text{Sue}, z)$

Prove: h.  $\neg \text{Likes}(\text{John}, \text{peanuts})$

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- |   |                         |
|---|-------------------------|
| i. $\neg \text{Food}(\text{Peanuts})$   | a, h                    |
| j. $\neg \text{Eats}(\text{Bill}, \text{Peanuts}) \vee \text{Kills}(\text{Peanuts}, \text{Bill})$ | d, i, a/Bill, b/Peanuts |
| k. $\text{Eats}(\text{Bill}, \text{Peanuts})$   | f                       |
| l. $\text{Kills}(\text{Peanuts}, \text{Bill})$  | j, k                    |

m.  $\sim \text{Alive}(\text{Bill})$

r/Bill, e, l

n.  $\text{Alive}(\text{Bill})$

f

o. False

contradiction between m and n. Thus,  $\sim \text{Like}(\text{John}, \text{peanuts})$  is false.  $\text{Likes}(\text{John}, \text{peanuts})$  has been proved.

(d) Use resolution to answer the question, “What food does Sue eat?”

a.  $\sim \text{Food}(x) \mid \text{Likes}(\text{John}, x)$

b.  $\text{Food}(\text{Apples})$

c.  $\text{Food}(\text{Chicken})$

d.  $\sim \text{Eat}(a, b) \mid \text{Kill}(b, a) \mid \text{Food}(b)$

e.  $\sim \text{Kills}(q, r) \mid \sim \text{Alive}(r)$

f.  $\text{Eats}(\text{Bill}, \text{Peanut}) \ \& \ \text{Alive}(\text{Bill})$

g.  $\sim \text{Eats}(\text{Bill}, z) \mid \text{Eats}(\text{Sue}, z)$

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$\text{Eats}(\text{Sue}, \text{peanut})$

f. g. z/Peanut

(e)

(CNF)

1.  $\text{Eats}(x, y) \mid \text{Die}(x)$

2.  $\sim \text{Die}(x) \mid \sim \text{Alive}(x)$

3.  $\text{Alive}(\text{Bill})$

Ans:

a.  $\sim \text{Food}(x) \mid \text{Likes}(\text{John}, x)$

b.  $\text{Food}(\text{Apples})$

c.  $\text{Food}(\text{Chicken})$

d.  $\sim \text{Eat}(a, b) \mid \text{Kill}(b, a) \mid \text{Food}(b)$

e.  $\sim \text{Kills}(q, r) \mid \sim \text{Alive}(r)$

g.  $\sim \text{Eats}(\text{Bill}, z) \mid \text{Eats}(\text{Sue}, z)$

1.  $\text{Eats}(o, q) \mid \text{Die}(o)$

2.  $\sim \text{Die}(w) \mid \sim \text{Alive}(w)$

3.  $\text{Alive}(\text{Bill})$

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4.  $\sim \text{Die}(\text{Bill})$  x/Bill

5.  $\text{Eats}(\text{Bill}, q)$  o/Bill

6.  $\text{Eats}(\text{Bill}, M)$  q/M

7.  $\sim \text{Eats}(\text{Bill}, M) \mid \text{Eats}(\text{Sue}, M)$  z/M

8.  $\text{Eats}(\text{Sue}, M)$  6, 7