

A Silicon Photonics Mach-Zehnder Interferometer

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I. INTRODUCTION

To be completed later.

II. THEORY

A. Compact waveguide model

The properties of a waveguide can be effectively described using a compact polynomial model for the waveguide index of refraction, as described in [1]. The index of refraction n is Taylor expanded as a function of wavelength λ (μm) as

$$n(\lambda) = n_1 + n_2(\lambda - \lambda_0) + n_3(\lambda - \lambda_0)^2. \quad (1)$$

The parameters n_1 (unit-less), n_2 (μm^{-1}), and n_3 (μm^{-2}) can be obtained through polynomial fitting (such as a least-squares fit method) once $n(\lambda)$ is calculated through a finite-difference eigenmode (FDE) or similar simulation of a waveguide.

As described in [1], they can also be converted to the more traditional parameters

$$n_{\text{eff}} = n_1 \quad (2)$$

$$n_g = n_1 - n_2 \cdot \lambda_0 \quad (3)$$

$$D = -2 \cdot \lambda_0 \cdot n_3/c \quad (4)$$

where n_{eff} is the waveguide effective index, n_g is the group index, and D is the waveguide dispersion.

B. Interferometer transfer function and free spectral range

The transfer function for a simple Mach-Zehnder interferometer is given by equation (4.19c) in [1] as

$$\frac{I_o}{I_i} = \frac{1}{2} [1 + \cos(\beta_1 L_1 - \beta_2 L_2)] \quad (5)$$

$$\beta = \frac{2\pi n(\lambda)}{\lambda} \quad (6)$$

Here β_1 and β_2 are the wavenumbers of the two waveguides and L_1 and L_2 are the lengths of the waveguides. This equation neglects overall loss and applies to single-mode waveguides where only the total field intensity is considered in each waveguide.

If the imbalance in the two interferometer arms is created by a waveguide length mismatch $\Delta L = L_2 - L_1$, the free spectral range (FSR) of an interferometer (in meters) is given by equation (4.20b) in [1]. It is

$$\text{FSR} = \frac{\lambda_0^2}{n_g \Delta L} \quad (7)$$

where λ_0 is the wavelength around which the FSR is measured.

In Sec. VI the group index of the fabricated waveguides will be determined based on the FSR found in the experimental

data. This will be easily obtained by solving Eq. 7 for n_g such that

$$n_g = \frac{\lambda^2}{\text{FSR} \cdot \Delta L}. \quad (8)$$

To extract n_g from the experimental data, the period of the interferometer fringes (the FSR) will first be determined numerically around a chosen wavelength λ (e.g. 1550 nm). The measured FSR, chosen λ , and the known ΔL from the layout will be substituted into Eq. 8 to determine the waveguide group index.

C. Free spectral range for a waveguide-width-mismatched interferometer

To determine the FSR for devices which have a mismatch in waveguide width (so that $n_1 \neq n_2$), but no mismatch in the waveguide length ($L_1 = L_2$), we start with Eq. 5. The period (FSR) of the transfer function is determined by the phase

$$\delta = (\beta_1 - \beta_2)L.$$

The FSR is the wavelength difference between adjacent peaks $\Delta\lambda = \lambda_{m+1} - \lambda_m$. The phase difference between adjacent peaks is always 2π , or in other words,

$$\begin{aligned} 2\pi &= \delta_m - \delta_{m+1} \\ &= (\beta_{1,m} - \beta_{2,m})L - (\beta_{1,m+1} - \beta_{2,m+1})L. \end{aligned}$$

Rearranging and writing

$$\Delta\beta_1 = \beta_{1,m} - \beta_{1,m+1}$$

$$\Delta\beta_2 = \beta_{2,m} - \beta_{2,m+1}$$

we have

$$\frac{2\pi}{L} = \Delta\beta_1 - \Delta\beta_2. \quad (9)$$

We can approximate that $\Delta\beta$ varies linearly with λ as

$$\Delta\beta = -\frac{d\beta}{d\lambda}\Big|_{\lambda_0} \Delta\lambda$$

and we can evaluate $\frac{d\beta}{d\lambda}$ at λ_0 since $\beta = \frac{2\pi n(\lambda)}{\lambda}$, which is

$$\frac{d\beta}{d\lambda}\Big|_{\lambda_0} = -\frac{2\pi}{\lambda_0^2} \left(n - \frac{dn}{d\lambda}\Big|_{\lambda_0} \lambda_0 \right) = -\frac{2\pi}{\lambda_0^2} n_g.$$

Plugging these into Eq. 9 we have an equation we can solve for $\Delta\lambda$ (the FSR):

$$\frac{2\pi}{L} = \frac{2\pi}{\lambda_0^2} (n_{1,g} - n_{2,g}) \Delta\lambda. \quad (10)$$

This results in

$$\text{FSR} = \Delta\lambda = \frac{\lambda_0^2}{L \Delta n_g}. \quad (11)$$

TABLE I
INTERFEROMETERS WITH LENGTH-MISMATCH

Device num	w	ΔL	n_{eff}	n_g	FSR
1	500 nm	100 μm	2.45	4.20	5.72 nm
2	500 nm	200 μm	2.45	4.20	2.86 nm
3	450 nm	100 μm	2.35	4.29	5.60 nm
4	550 nm	100 μm	2.52	4.12	5.83 nm
5	600 nm	100 μm	2.57	4.06	5.92 nm

TABLE II
INTERFEROMETERS WITH WIDTH-MISMATCH

Device num	w_1	w_2	L	FSR
6	500 nm	450 nm	900 μm	29.7 nm
7	500 nm	550 nm	900 μm	33.4 nm
8	500 nm	600 nm	900 μm	19.1 nm

D. Device parameter variations

In order to evaluate the effects of changing various parameters, the Mach-Zehnder interferometers will be designed, simulated, and fabricated across a sweep of waveguide *length* mismatches ΔL and a sweep of waveguide *width* mismatches. These parameters are shown in Tables I and II.

The devices in Table I have varying length mismatches between the two waveguides in the interferometer, but the widths of the two waveguides are identical. The FSR, n_{eff} , and n_g are evaluated at 1550 nm. The FSR is calculated using Eq. 7 while n_{eff} and n_g are taken from simulated data shown in Sec. III. The devices in Table II have varying width mismatches between the two waveguides, while the lengths of the two arms of the interferometer are identical. The FSR is evaluated at 1550 nm and is calculated using Eq. 11.

III. MODELING AND SIMULATION

Waveguides with widths 450 nm to 600 nm (as listed in Tables I and II) were simulated using Lumerical MODE™ software in order to obtain mode profiles and refractive indices. The simulation results presented here are for the TE mode.

Figs. 1 and 2 display the simulated values of n_{eff} and n_g as functions of wavelength from 1500 nm to 1600 nm. Note that while n_{eff} increases with increasing waveguide width, n_g decreases.

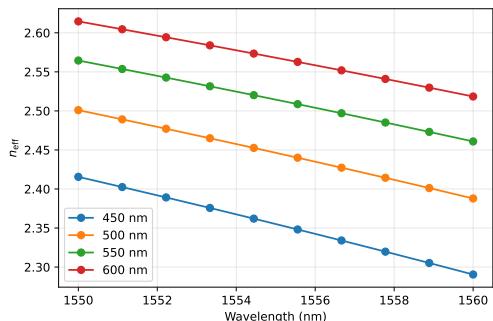


Fig. 1. Effective indices n_{eff} for waveguides of varying width.

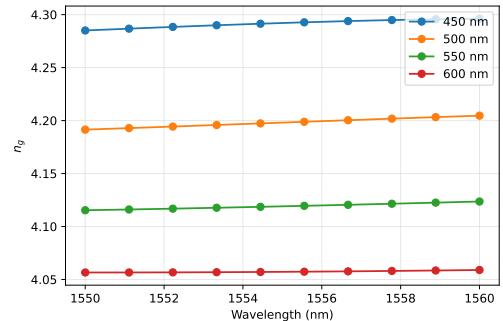


Fig. 2. Group indices n_g for waveguides of varying width.

TABLE III
WAVEGUIDE COMPACT MODEL PARAMETERS

Width	n_1	$n_2 (\mu\text{m}^{-1})$	$n_3 (\mu\text{m}^{-2})$
450 nm	2.353	-1.251	-0.037
500 nm	2.445	-1.131	-0.043
550 nm	2.513	-1.036	-0.027
600 nm	2.567	-0.962	-0.008

The values extracted from these simulations allow a compact model of the form of Eq. 1 to be written for these specific waveguides. The extracted values of n_1 , n_2 , and n_3 are listed in Table III.

Once a compact model is obtained, it can then be used to write and plot a specific form of the interferometer transfer function given in Eq. 5.

Simulated mode profiles are shown in Fig. 3. The geometries of the waveguides are shown superimposed as white rectangles. It can be seen that for the narrower waveguide there is more leakage of the electric field as an evanescent wave out the sides of the waveguide.

IV. FABRICATION

To be completed later (will include layout and details about fabrication).

V. EXPERIMENTAL DATA

To be completed later.

VI. ANALYSIS

To be completed later.

VII. CONCLUSION

To be completed later.

REFERENCES

- [1] L. Chrostowski and M. Hochberg, *Silicon Photonics Design*, 1st ed. Cambridge University Press, 2015.

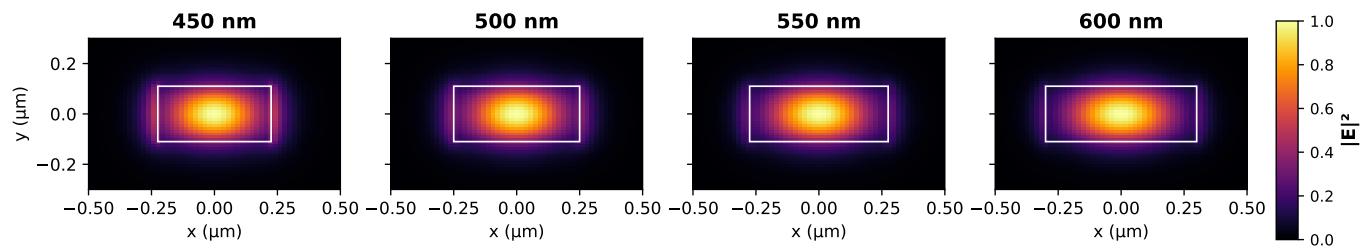


Fig. 3. Simulated mode profiles (electric field intensity $|\mathbf{E}|^2$) at varying waveguide widths.