# Design proposal of Mach-Zehnder Interferometer

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1. **INTRODUCTION**

   The Mach-Zehnder Interferometer (MZI) is an optical device that splits an incoming light wave into two separate paths, introduces a controlled phase difference between them, and then recombines the beams to produce interference at the output. The MZI are widely used in photonic systems due to their ability to precisely control optical phase and interference. Due to its simplicity, flexibility, and compatibility with photonic integration, the MZI has become a fundamental building block in modern optical systems.

   This work focuses on the design and characterization of the MZI. First, we perform a preliminary design and conduct optical simulations to evaluate device behavior. Next, we develop the circuit and physical layout for fabrication. Finally, we compare the simulated performance with measurement data obtained from the fabricated device.

2. **THEORY**

   The MZI consists of the key following components.
   1. Input splitter (Y-branch): Split the incoming optical power into two paths.
   2. Waveguides: Light travels with possibly different phase accumulations.
   3. Output combiner (Y-branch): Recombines the light and producing the interference.

   If the input electric field is $E_i$, and the splitter divides it equally, then electric field of each branch is $\frac{E_i}{\sqrt{2}}$

   a. **The propagation of light**

   For plane wave, the complex electric field is $E = E_0 e^{i(\omega t - \beta z)}$, where $E_0$ is a complex amplitude, $\omega$ is an optical frequency, and $\beta$ is a propagation constant.

   As light propagation through each branch, the propagation constant of light is $\beta = \frac{2\pi n}{\lambda}$, where n is the index of refraction and $\lambda$ is the wavelength.

   b. **The first half of MZI: Y-branch splitter and 2 waveguides**

   We define difference of the propagation constant of light of 2 waveguides. They also have different length, and the path length difference is $\Delta L = L1 - L2$.

   At the end of 2 waveguides, if there is no optical loss, the electric fields are

   $$E_{o1} = \frac{E_i}{\sqrt{2}} e^{-i\beta_1 L_1}, \ E_{o2} = \frac{E_i}{\sqrt{2}} e^{-i\beta_2 L_2}$$

   c. **The second half of MZI: 2 waveguides and Y-branch combiner**

   Light is combined in the Y-branch combiner and the output electric field is

   $$E_o = \frac{1}{\sqrt{2}}(E_{o1} + E_{o2}) = \frac{E_i}{\sqrt{2}}(e^{-i\beta_1 L_1} + e^{-i\beta_2 L_2})$$

Convert the electric field into intensity

$$I_o = \frac{I_i}{4}\left|e^{-i\beta_1 L_1} + e^{-i\beta_2 L_2}\right|^2 = \frac{I_i}{2}[1 + \cos(\beta_1 L_1 - \beta_2 L_2)]$$

For balance interferometer that lengths are same, the output intensity is

$$I_o = \frac{I_i}{2}[1 + \cos(\Delta\beta L)]$$

For imbalance interferometer with identical waveguides, the output intensity is

$$I_o = \frac{I_i}{2}[1 + \cos(\beta\Delta L)]$$

If we plot the transmission of the imbalance interferometer and wavelength, we can find the free spectral range from the spacing between peaks.

$$FSR = \frac{\lambda^2}{\Delta L n_g}, \text{ where group index}(n_g) = n - \lambda\frac{dn}{d\lambda}$$

3. **MODELLING AND SIMULATION**

   a. **Waveguide geometry**

   A silicon strip waveguide with a cross-sectional dimension of 500 nm × 220 nm (width × height) is designed, where silicon (Si) serves as the core material and silicon dioxide (SiO₂) acts as the cladding.
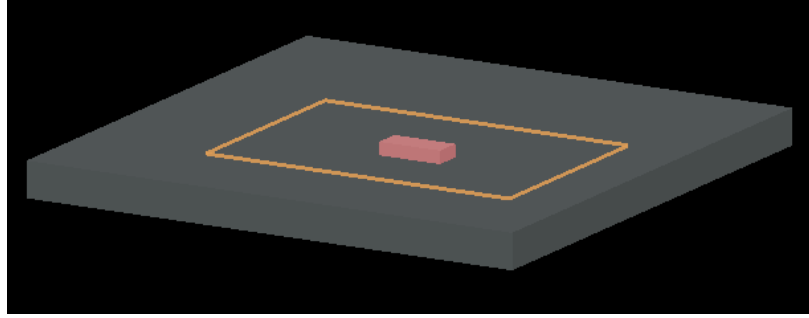


*Fig. 1 Design of waveguide geometry in Lumerical*

   b. **Simulated waveguide mode profile**

   The waveguide simulations are performed using the finite-difference eigenmode (FDE) solver in Lumerical. The results corresponding to a wavelength of 1550 nm are shown in the following figures.
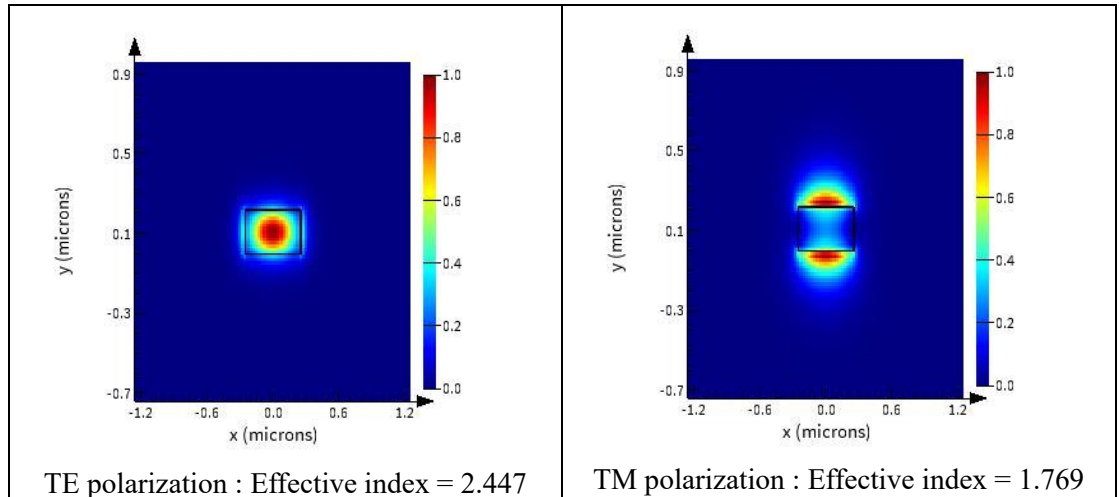


| TE polarization : Effective index = 2.447 | TM polarization : Effective index = 1.769 |

*Fig. 2 Simulated waveguide mode profile (E intensity) of TE01 and TM01 mode*

| TE polarization : Effective index vs. Wavelength | TE polarization : Group index vs. Wavelength |

*Fig. 3 Plot of effective index and group index of waveguide, versus wavelength of TE01 mode*



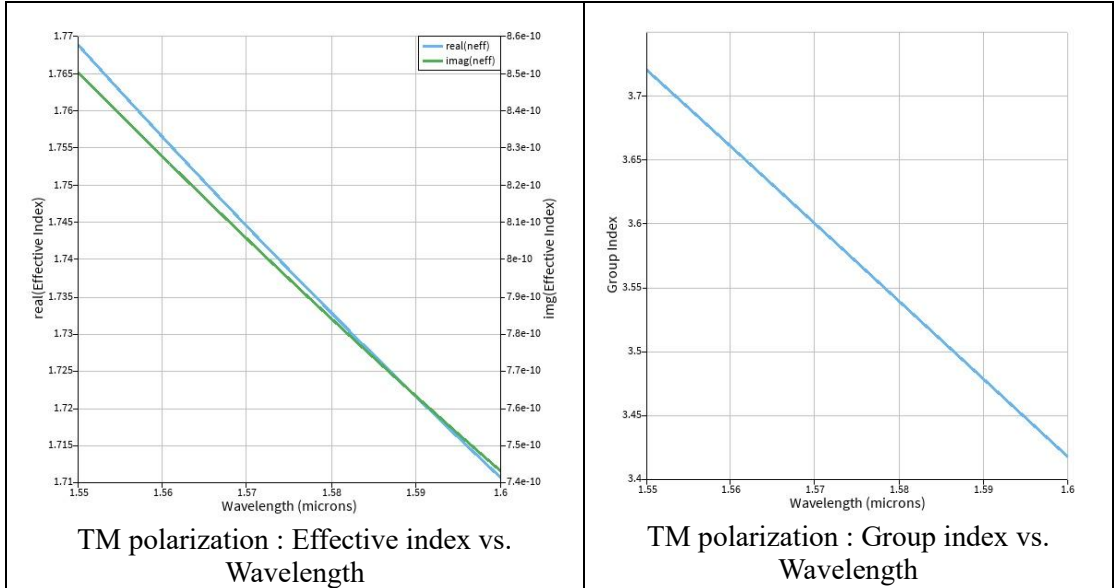| TM polarization : Effective index vs. Wavelength | TM polarization : Group index vs. Wavelength |

*Fig. 4 Plot of effective index and group index of waveguide, versus wavelength of TM01 mode*

### c. Compact model for the waveguide (polynomial expression)

The compact model for the waveguide is a second-order of polynomial, obtained using a Taylor expansion around the centre wavelength.

$$n_{eff}(\lambda) = n_1 + n_2(\lambda - \lambda_0) + n_3(\lambda - \lambda_0)^2$$

The data obtained from the wavelength sweep in Lumerical MODE are fitted using a script to extract the coefficients. A compact model of the waveguide is then obtained with a centre wavelength of $\lambda_0 = 1.55\mu m$, as given below

For TE polarization,

$$n_{eff}(\lambda) = 2.44682 - 1.13352(\lambda - 1.55) - 0.041065(\lambda - 1.55)^2$$

For TM polarization,

$$n_{eff}(\lambda) = 1.76882 - 1.25896(\lambda - 1.55) + 1.92378(\lambda - 1.55)^2$$

### d. Transfer function of the interferometer vs. wavelength

In this design, a Mach–Zehnder Interferometer (MZI) is utilized. The transfer functions of both the balanced and imbalanced MZI configurations are given below.

For balance MZI,

$$\frac{I_o}{I_i} = \frac{1}{2}[1 + \cos{(\Delta\beta L)}]$$

For imbalance MZI,

$$\frac{I_o}{I_i} = \frac{1}{2}[1 + \cos{(\beta\Delta L)}]$$

Where $I_i$ is the input light intensity, $I_o$ is the output light intensity, $L$ is length of the waveguide, and $\beta$ is the waveguide propagation constant.

### e. List of parameter variation and expected performance

The path length difference ($\Delta L$) is selected as the variable design parameter and is varied as described below.

| No. | $\Delta L$ (µm) |
|-----|-----------------|
| 1 | 50 |
| 2 | 100 |
| 3 | 150 |
| 4 | 200 |

Table 1 List of ΔL variation

According to the relationship $FSR = \frac{\lambda^2}{\Delta L n_g}$, an increasing in the path length difference ($\Delta L$) leads to reduction in the free spectral range (FSR) for an imbalanced MZI.

### f. Transmission spectrum of photonic circuit

Lumerical INTERCONNECT is used to calculate the transmission spectra of the imbalanced MZI for the TE mode, considering all path length differences ($\Delta L$) listed in Table 1.
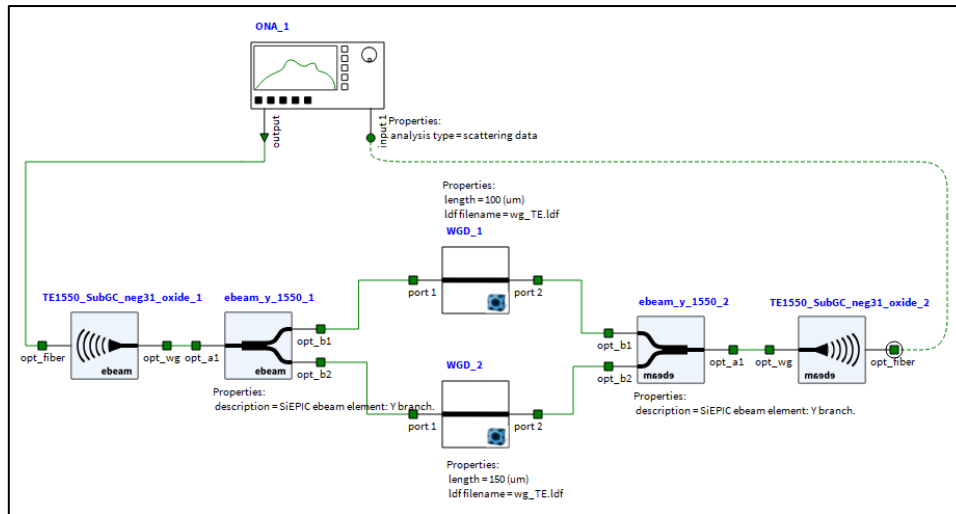


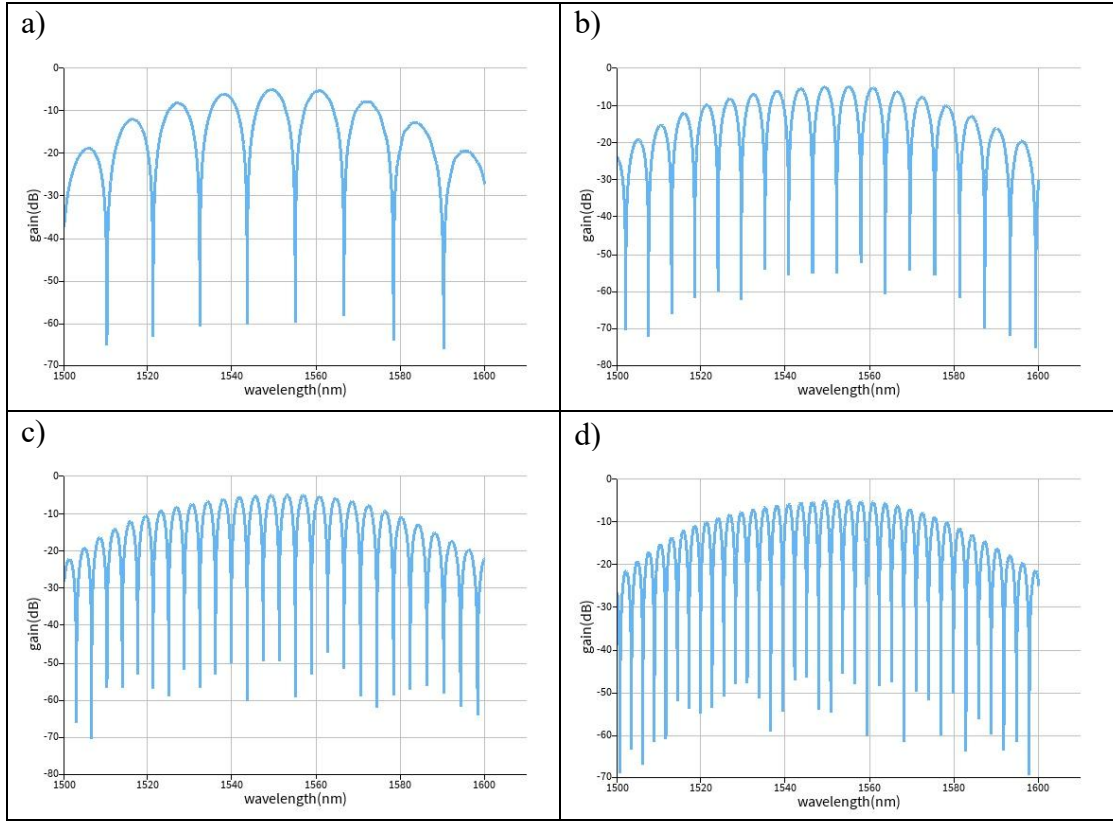Fig. 5 Imbalance MZI circuit for TE mode in Lumerical INTERCONNECT

*Fig. 6 Spectrum of the MZI for TE mode with path length difference a)50 µm, b)100µm, c)150µm and d)200µm*

The simulation results show that the free spectral range (FSR) decreases as the path length difference (ΔL) increases, consistent with expectations.

### g. Layout design

The physical layout of the Mach–Zehnder Interferometer (MZI) is designed using KLayout in conjunction with the SiEPIC-EBeam-PDK.
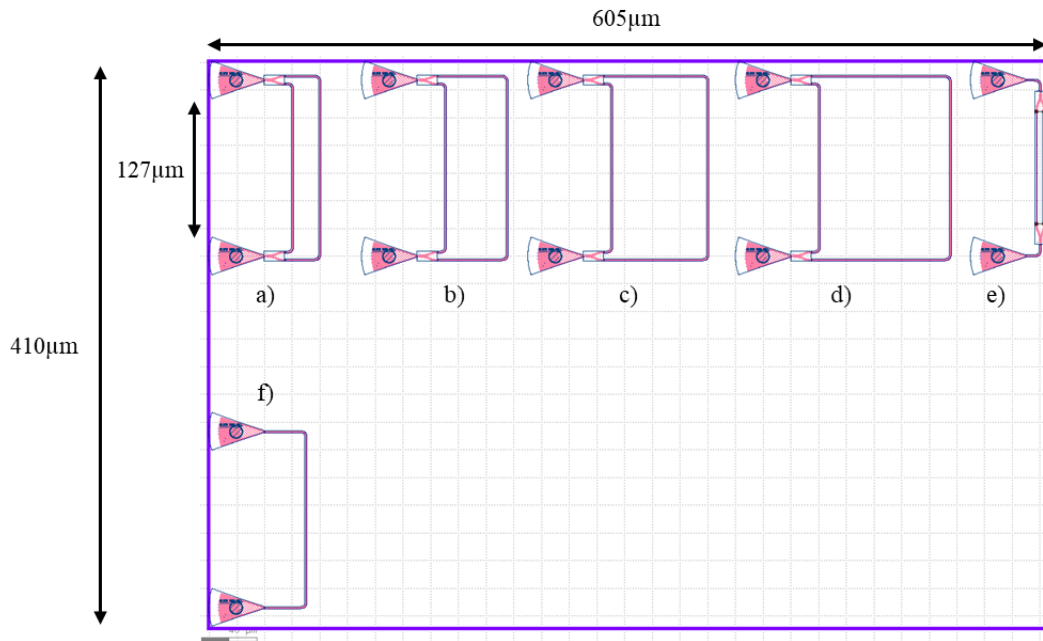


*Fig. 7 Layout of the MZI with path length difference a)50 µm, b)100µm, c)150µm, d)200µm, d)0µm and f)no MZI*