Project Proposal: Waveguide Compact Model and Unbalanced MZI for Group Index Extraction

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edX Public Username: a-chernyadiev

1 Introduction

Silicon photonics enables the integration of optical waveguides and interferometric circuits on a CMOS-compatible silicon-on-insulator (SOI) platform, supporting compact footprint and large-scale photonic integrated circuits for communications, sensing, and signal processing (Chrostowski & Hochberg, 2015; Capmany & Pérez, 2020). A common building block is the Mach–Zehnder interferometer (MZI), whose spectral fringes can be used to characterize waveguide dispersion via the free spectral range (FSR). The objective of this project is to (i) design and simulate a single-mode SOI strip waveguide, (ii) extract a compact polynomial model for its effective index, and (iii) use an unbalanced MZI to relate the measured/simulated FSR to the waveguide group index.

Key deliverables for this proposal include the waveguide geometry and mode profile, effective/group index versus wavelength, a waveguide compact model, the MZI transfer function, parameter sweeps of path-length difference ΔL and expected FSR, simulated transmission spectra, and a derivation and validation of the group-index extraction method.

3 Theory

3.1 Waveguide effective and group indices

For the fundamental TE mode, the propagation constant is $\beta\left(\lambda\right)=\dfrac{2\pi n_{eff}}{\lambda}$. The group index is defined as $n_g\left(\lambda\right)=n_{eff}\left(\lambda\right)-\lambda\dfrac{dn_{eff}}{d\lambda}$, which captures first-order dispersion and determines the spectral period of interferometric fringes.

3.2 Unbalanced Mach–Zehnder interferometer (MZI) transfer function

An ideal 2×2-coupler-based MZI with equal couplers (nominally 3 dB) and an arm path-length difference ΔL has two complementary output intensities (normalized to input) that vary with wavelength as:

$$\frac{I_{out,1}(\lambda)}{I_{in}(\lambda)} = \frac{1}{2}\left(1+\cos\left(\beta(\lambda)\Delta L\right)\right),$$

$$\frac{I_{out,2}(\lambda)}{I_{in}(\lambda)} = \frac{1}{2}\left(1-\cos\left(\beta(\lambda)\Delta L\right)\right).$$

Maxima occur when $\beta(\lambda_m)\Delta L = 2\pi m$, for integer m.

3.3 Free spectral range (FSR) and relation to group index

The FSR is the wavelength spacing between adjacent maxima (or minima). For an unbalanced interferometer and small spacing around a center wavelength λ_0, the FSR can be approximated by:

$$FSR_\lambda \approx \frac{\lambda_0^2}{n_g(\lambda_0)\Delta L}.$$

Rearranging gives a practical extraction formula for group index:

$$n_g(\lambda_0) \approx \frac{\lambda_0^2}{FSR_\lambda \Delta L}.$$

4 Modelling and Simulation

4.1 Waveguide geometry and polarization

Waveguide platform: SOI strip waveguide.
Geometry: width w = 500 nm, height (Si thickness) h = 220 nm.
Polarization: fundamental TE mode.
Center wavelength for reporting: λ_0 = 1550 nm.

TE fundamental mode at λ = 1550 nm. Waveguide size w = 500 nm, thickness = 220 nm.

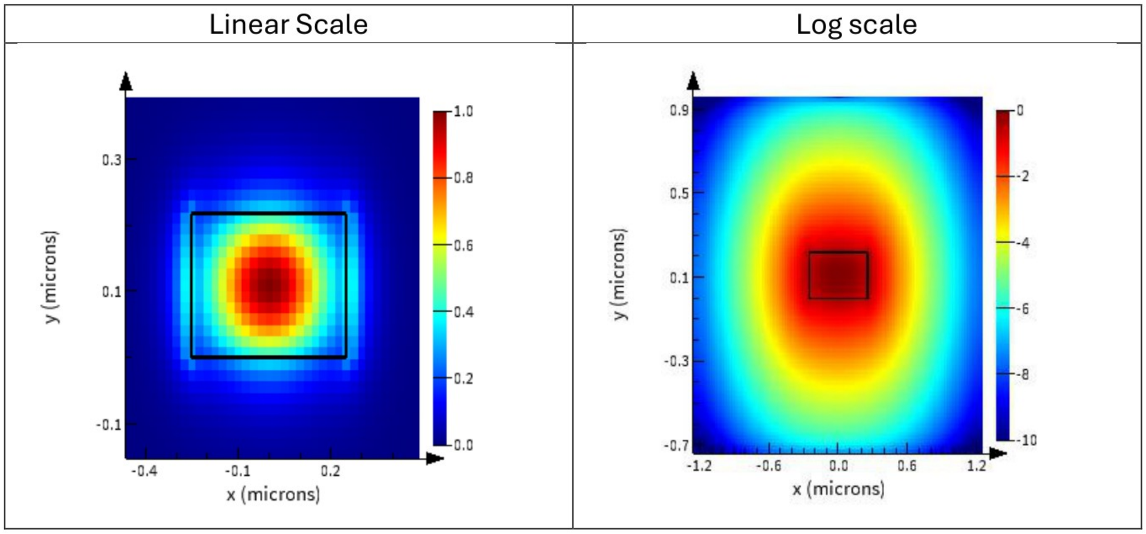

Figure 1. Simulated TE fundamental mode profile at λ = 1550 nm (linear and log scale).

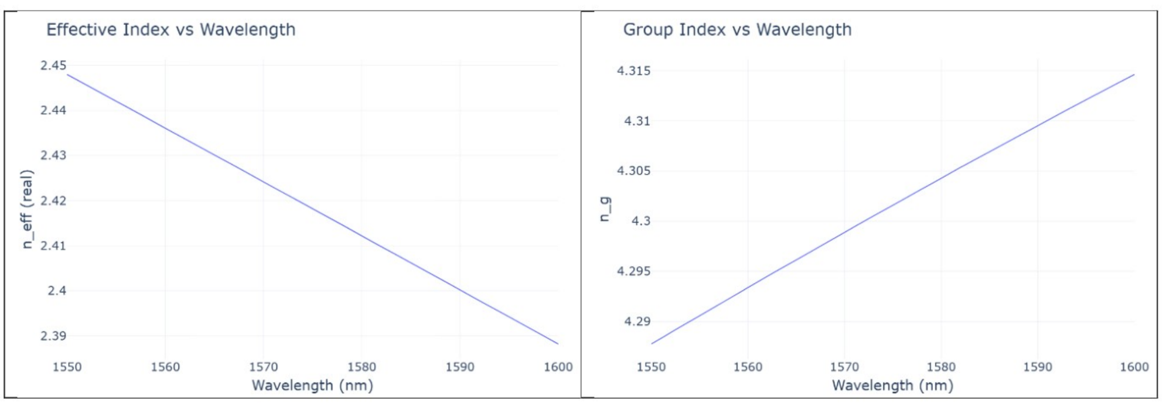

Figure 2. Simulated effective index (n_eff) and group index (n_g) versus wavelength.

4.2 Waveguide compact model

A second-order polynomial compact model is used for $n_{eff}(\lambda)$ around λ_0:

$$n_{eff}\left(\lambda\right) = n_1 + n_2\left(\lambda - \lambda_0\right) + n_3\left(\lambda - \lambda_0\right)^2,$$

with λ expressed in micrometers (μm).

From the fitted model:

$n1 = 2.44798$

$n2 = -1.1872$ [μm^{-1}]

$n3 = 0.170881$ [μm^{-2}]

Using $n_g(\lambda) = n_{eff}(\lambda) - \lambda \frac{dn_{eff}}{d\lambda}$, the group index at λ_0 is:
$n_g(\lambda_0) = n_1 - n_2 \lambda_0$.

With $\lambda_0 = 1.55$ μm, this gives $n_g(\lambda_0) \approx 4.2881$.

4.3 Parameter variations and expected FSR

A sweep of path-length difference ΔL is used to verify the expected inverse relationship between FSR and ΔL. Using $\lambda_0 = 1550$ nm and $n_g(\lambda_0) \approx 4.29$, the expected FSR values are listed below.

Case	ΔL (μm)	Expected FSR (nm) @ 1550 nm
1	50	11.20
2	75	7.47
3	100	5.60
4	125	4.48
5	150	3.74

4.4 Simulated MZI transmission spectrum

The simulated transmission spectrum shows periodic fringes whose spacing decreases as ΔL increases, consistent with the FSR expression.

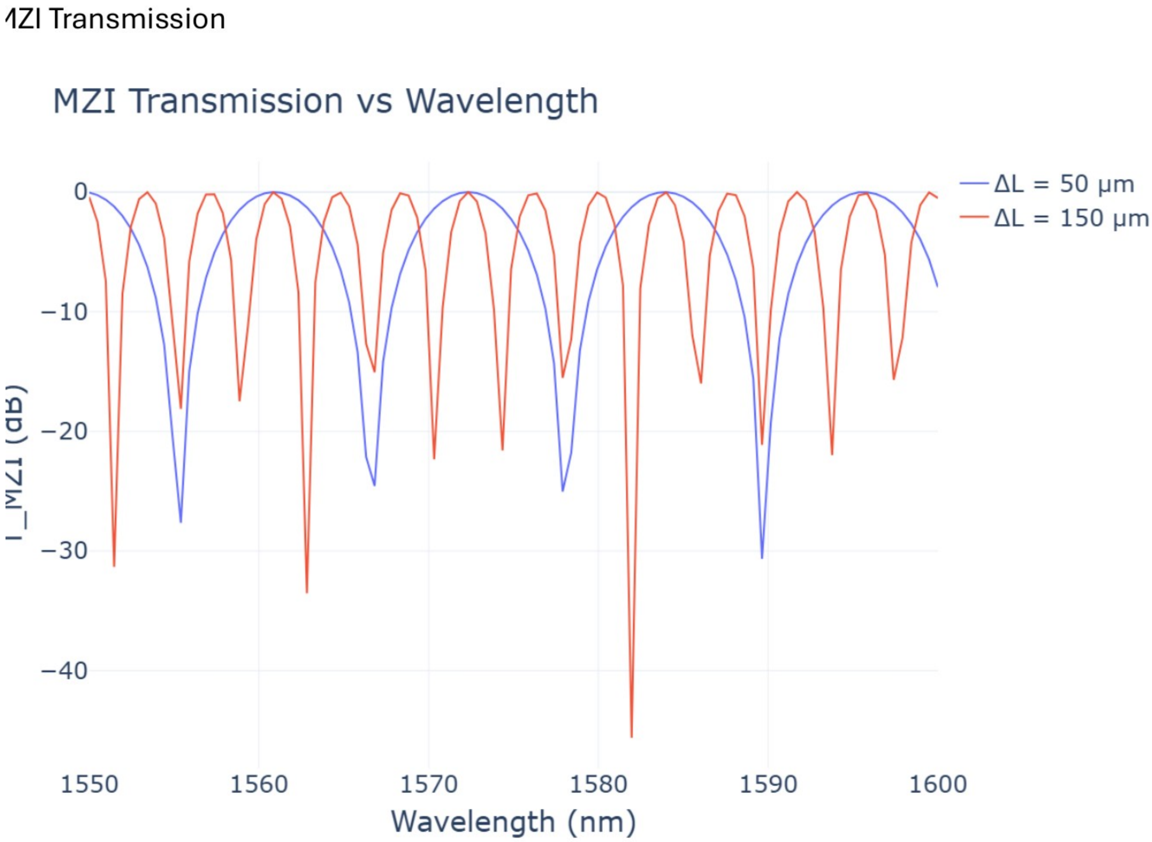

Figure 3. MZI transmission versus wavelength for ΔL = 50 µm and ΔL = 150 µm.

4.5 Lumerical INTERCONNECT circuit simulation

To incorporate realistic component models (e.g., couplers and waveguide dispersion), an unbalanced MZI was also simulated in Lumerical INTERCONNECT using SiEPIC/ebeam library elements. The arm lengths were set to 50 µm and 150 µm (ΔL = 100 µm) with waveguide width 0.5 µm, and the transmission was computed using an optical network analyzer (scattering-data analysis).

Schematic of an MZI in Lumerical Interconnect.

ΔL = 100 um.

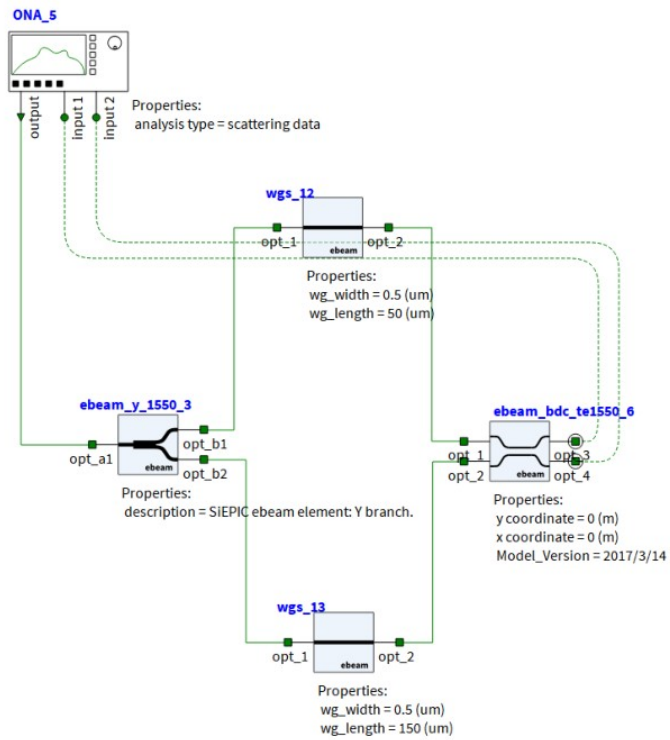

Figure 4. Lumerical INTERCONNECT schematic of the unbalanced MZI (ΔL = 100 µm).

Figure 5 shows the simulated transmission at the two output ports. As expected from the ideal MZI transfer functions (Section 3.2), the outputs are complementary (out of phase), so that when one output is at a maximum the other is at a minimum.

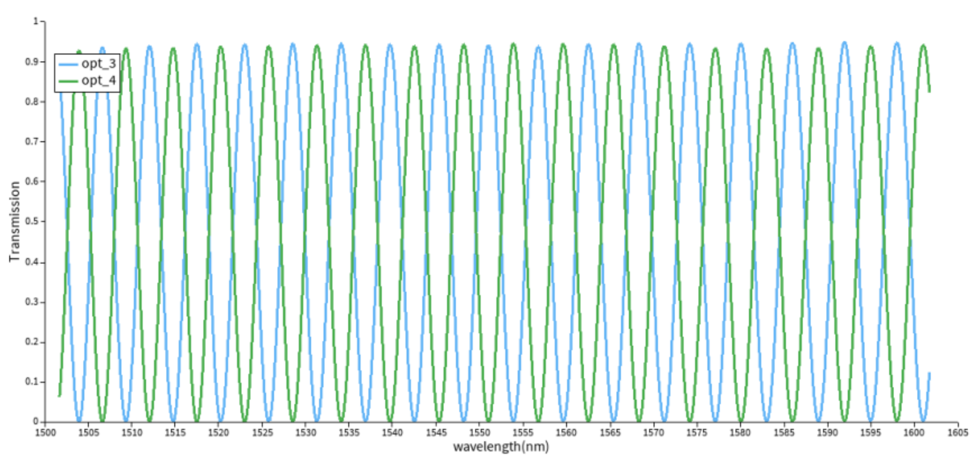

Figure 5. INTERCONNECT simulated transmission of the two complementary MZI outputs (opt_3 and opt_4).

From the INTERCONNECT output spectrum, the fringe spacing (FSR) can be measured versus wavelength. Figure 6 plots the extracted FSR as a function of wavelength, with FSR(λ = 1550 nm) = 5.68 nm for this ΔL = 100 µm design.

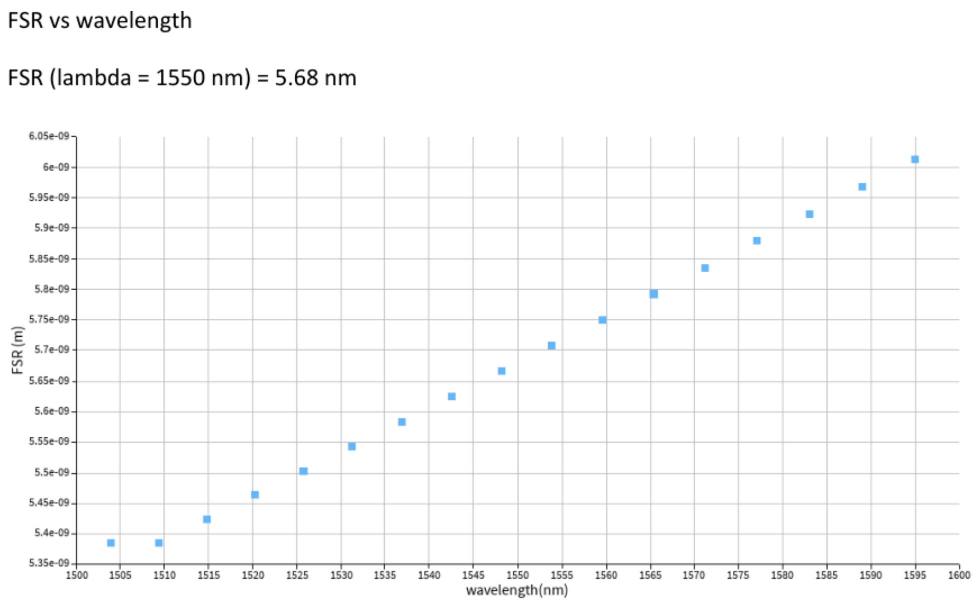

Figure 6. INTERCONNECT extracted free spectral range (FSR) versus wavelength for ΔL = 100 µm.

4.6 Derivation and validation of group-index extraction from FSR

Starting from the constructive interference condition for maxima:

$$\beta\left(\lambda_m\right)\Delta L = 2\pi m.$$

For the adjacent maximum at m+1:

$$\beta\left(\lambda_{m+1}\right)\Delta L = 2\pi\left(m+1\right).$$

Subtracting gives:

$$\left[\beta\left(\lambda_{m+1}\right) - \beta\left(\lambda_m\right)\right]\Delta L = 2\pi.$$

Approximating $\beta(\lambda_{m+1}) - \beta(\lambda_m) \approx \left(\frac{d\beta}{d\lambda}\right) FSR_\lambda$ and using

$$\frac{d\beta}{d\lambda} = -\left(\frac{2\pi}{\lambda^2}\right) n_g(\lambda),$$

yields the wavelength-domain approximation:

$$FSR_\lambda \approx \frac{\lambda^2}{n_g(\lambda)\,\Delta L},$$

and therefore $n_g \approx \frac{\lambda^2}{FSR_\lambda \Delta L}$.

Using the expected FSR values above at λ_0 = 1550 nm (consistency check), the extracted group index values are listed below. In addition, using the INTERCONNECT-simulated value FSR(1550 nm) = 5.68 nm for ΔL = 100 µm (Figure 6) gives $n_g \approx 1550^2 / (5.68 \cdot 100000) \approx 4.23$, which is within ~1–2% of the polynomial-model value and illustrates sensitivity to the effective ΔL and component phase contributions.

ΔL (µm)	FSR (nm)	Extracted n_g	Δn_g vs. 4.288
50	11.20	4.2902	+0.0020
75	7.47	4.2883	+0.0001
100	5.60	4.2902	+0.0020
125	4.48	4.2902	+0.0020
150	3.74	4.2825	-0.0056

The extracted n_g values cluster near 4.29, in agreement with the group index obtained directly from the polynomial compact model.

4.8 How n_g will be obtained from experimental data

Experimentally, an unbalanced MZI with known lithographic ΔL can be measured by sweeping a tunable laser over a wavelength window (e.g., 1500–1600 nm) while recording the transmitted power at one output. The FSR is obtained by measuring the wavelength spacing between successive maxima/minima (or via an FFT-based fringe-frequency estimate). The group index is then computed at each wavelength using:

$$n_g\left(\lambda\right) \approx \frac{\lambda^2}{FSR_\lambda \Delta L}.$$

To reduce noise, FSR can be averaged over multiple fringe periods and evaluated locally versus wavelength to capture dispersion. Major uncertainty contributors are ΔL fabrication error, wavelength calibration of the laser, and spectral ripple from couplers/grating couplers.

References

Chrostowski, L., and Hochberg, M., Silicon Photonics Design: From Devices to Systems, Cambridge University Press, 2015.

Capmany, J., and Pérez, D., Programmable Integrated Photonics, Oxford University Press, 2020.