

DESIGN PROPOSAL FOR A MACH-ZEHNDER INTERFEROMETER

LUIGI TALARICO luta.vig@hotmail.it

INTRODUCTION

The **Mach-Zehnder Interferometer** (abbr. **MZI**) is one of the most important tools in Silicon Photonics and represent one of the most fundamental building blocks of this industry and the first step in the ladder of object normally used.

The MZI allows to split an input light in two path and by shifting the phase of the light in one path it recombines and through constructive and destructive interference shift the intensity of the final output light.

The MZI is important because it enables control of the intensity of the output light and therefore modulate, perform wavelength filtering and create, when combined into meshes, optical switches in complex circuits.

This is used for on-chips optical interconnects, which are essential in data centers, where high data transfer rates and synchronization between multiple GPUs are crucial and must increasingly achieved using fiber optic links with high bandwidth and near speed of light propagation.

In other application Silicon Photonics Interferometers promise low power consumption, higher speed and Wavelength Division Multiplexing (abbr. WDM) for combining and separating light within a single fiber to increase the total information volume.

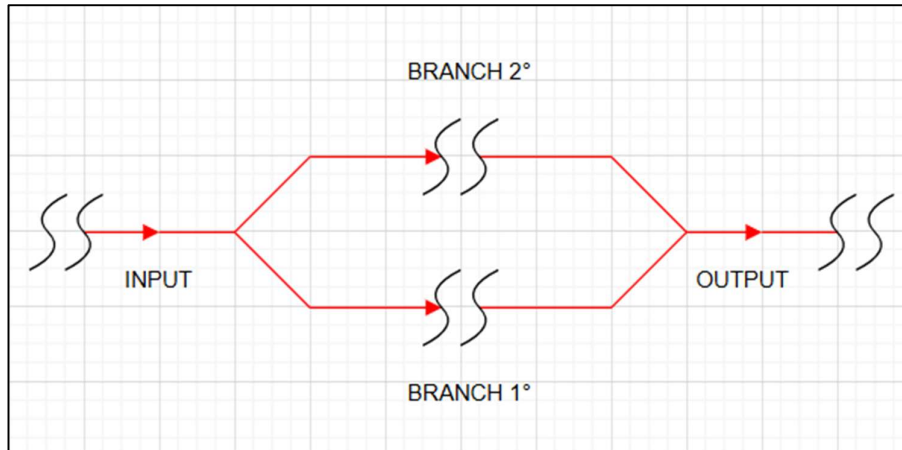
The following report aims to carry out silicon photonics design, calculations, simulations, and fabrication steps required to produce a Mach-Zehnder Interferometer, and to compare the theoretically predicted result with the measured result.

We will also examine the effects and differences caused by real propagation loss and production tolerance on the predicted results.

THEORY

To work this tool, it requires to calculate and account for the relative phase shift between the two beams of light that are created from the split input. In the present chapter we discuss the theory behind this a Mach-Zehnder Interferometer.

We start from the geometry, a MZI in its simplest form has an input branch which splits in two and then combine in one single output branch.



The power of the MZI derived from the difference in length between branch 1° and branch 2°. The light which propagates at the same speed travel different path length and therefore the wavelength peak recombines with shifted phase causing constructive and destructive interference.

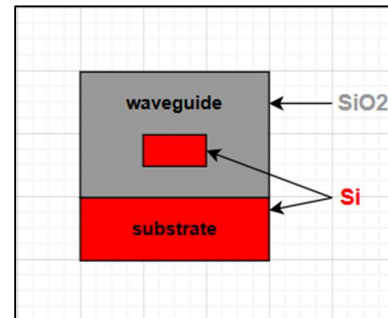
In a passive MZI the path length difference is fixed and embedded in the design produce while in an active MZI the path length can be adjusted using thermal dilatation of the material. It must be noted that to obtain a 180° phase shift, only minimum length difference, in the order of nanometer, are require, therefore:

- **Production** is difficult because it needs to respect very low dimensional tolerances to obtain the desire results.
- **Thermal dilatation** effect is more than enough to control active interferometer and phase shift of output light, as it usually ranges in the order of microns.

In photonics the light beam is constrain within was it called a waveguide which is constituted of a high refractive index material cladded inside a low refractive index material. The high contrast is required to have tight confinement of light, and therefore more circuit per area.

In silicon photonics the silicon (i.e.: Si) is the waveguide while Silicon dioxide (i.e.: SiO_2) is the cladding. The shape of this waveguide for production reason is a rectangular beam.

Material	Effective refractive index n_{eff}
Si	3.47
SiO2	1.44



This is different from a fiber optics cable where the index contrast is usually very low to guarantee ultra-low loss, while coupling and high bend radius are in the order of micron and not nanometer like silicon photonics.

Light is defined by two related, specific value, which are intensity, written as I and measure in [mW], and electric field E , measure in [Vm].

These values follow specific formula.

Intensity:

$$(1) I = \frac{1}{2} \cdot n \cdot \epsilon_0 \cdot c \cdot |E|^2 \Leftrightarrow I = \frac{E}{\sqrt{2}} \cdot n \cdot \epsilon_0 \cdot c$$

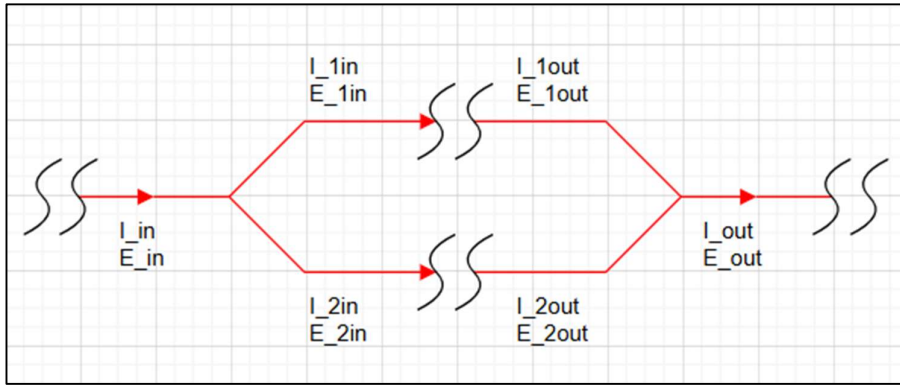
Electric field for plane wave:

$$(2) E = E_0 \cdot e^{i(\omega t - \beta L - \frac{\alpha}{2} L)}$$

Where the propagation constant β and attenuation constant α are:

$$(3) \beta = \frac{2\pi \cdot n}{\lambda} ; \alpha \propto material$$

To get the output Intensity and electric relative to the input and the path length it is useful to define all the light in 6 different regions in an MZI



The intensity and electric field of the **input** splits equally:

$$(4) I_{1in} = \frac{I_{in}}{2} ; E_{1in} = \frac{E_{in}}{\sqrt{2}}$$

$$(5) I_{2in} = \frac{I_{in}}{2} ; E_{2i} = \frac{E_{in}}{\sqrt{2}}$$

The electric field of the **output** is the sum of the two electric field:

$$(6) E_{out} = \frac{E_{2out} + E_{1out}}{\sqrt{2}}$$

Following the relation stated in (1) we can relate the input and output electric field branch:

$$(7) E_{1out} = E_{1in} \cdot e^{i(\omega t - \beta_1 L_1 - \frac{\alpha_1}{2} L)}$$

$$(8) E_{2out} = E_{2in} \cdot e^{i(\omega t - \beta_2 L_2 - \frac{\alpha_2}{2} L)}$$

Combining equation (6) (7) and (8):

$$E_{out} = \frac{E_{2in} \cdot e^{i(\omega t - \beta_2 L_2 - \frac{\alpha_2}{2} L)} + E_{1in} \cdot e^{i(\omega t - \beta_1 L_1 - \frac{\alpha_1}{2} L)}}{\sqrt{2}}$$

Using also equation (4) and (5):

$$E_{out} = \frac{E_{in}/\sqrt{2} \cdot e^{i(\omega t - \beta_2 L_2 - \frac{\alpha_2}{2} L)} + E_{in}/\sqrt{2} \cdot e^{i(\omega t - \beta_1 L_1 - \frac{\alpha_1}{2} L)}}{\sqrt{2}}$$

$$E_{out} = \frac{E_{in}}{2} \cdot \left[e^{i(\omega t - \beta_2 L_2 - \frac{\alpha_2}{2} L)} + e^{i(\omega t - \beta_1 L_1 - \frac{\alpha_1}{2} L)} \right]$$

Finally with equation (1) we can now get the output intensity as a function of t :

$$I_{out} = \frac{I_{in}}{2} \cdot \left[e^{i(\omega t - \beta_2 L_2 - \frac{\alpha_2}{2} L)} + e^{i(\omega t - \beta_1 L_1 - \frac{\alpha_1}{2} L)} \right]^2$$

If we assume zero attenuation loss for both branch $\alpha = 0$, and check for the peak intensity of the wave $t = 0$ we get a more simplified formula:

$$I_{out} = \frac{I_{in}}{2} \cdot \left[e^{i(-\beta_2 L_2)} + e^{i(-\beta_1 L_1)} \right]^2$$

$$I_{out} = \frac{I_{in}}{2} \cdot \left[2 \cos \left(\frac{\beta_2 L_2 - \beta_1 L_1}{2} \right) \right]^2$$

$$I_{out} = I_{in} \cdot \left[\cos \left(\frac{\beta_2 L_2 - \beta_1 L_1}{2} \right) \right]^2$$

$$I_{out} = I_{in} \cdot [1 + \cos(\beta_2 L_2 - \beta_1 L_1)]$$

Now if we assume an identical waveguide with the same propagation loss $\beta = \beta_1 = \beta_2$ we can see that the output intensity is dependent on the path length difference and therefore it can be controlled by it:

$$I_{out} = I_{in} \cdot [1 + \cos(\beta \cdot (L_2 - L_1))]$$

If the **Free Spectral Range** (abbr. **FSR**) is the space between adjacent peaks of the same light wave

$$FSR = \Delta\lambda = \lambda_2 - \lambda_1$$

If we assume that $\Delta\lambda$ is a variation linear variation of β we can write:

$$\frac{\Delta\lambda}{d\lambda} \approx -\frac{\Delta\beta}{d\beta} \Leftrightarrow \Delta\lambda \approx -\Delta\beta \cdot \left(\frac{d\beta}{d\lambda} \right)^{-1} = \frac{2\pi}{\Delta L} \cdot \left(\frac{d}{d\lambda} \cdot \frac{2\pi n}{\lambda} \right)^{-1} = \frac{1}{\Delta L} \cdot (-\lambda^{-2})^{-1} \frac{dn}{d\lambda} = \frac{\lambda^2}{\Delta L \cdot n_g}$$

MODELLING AND SIMULATION

Waveguide modelling

At first, we simulate the light confinement inside the waveguide cross-section to find the spatial field distribution of the TE (i.e.: Transverse Electric) and TM (i.e.: Transverse Magnetic) modes. To do this we use the LUMERICAL MODE software. We will use for production purpose only strip-type waveguide and no Ridge-type.

The cross-section is usually $h = 220$ [nm] height and $b = 500$ [nm] width in combination with light of $\lambda = 1550$ [nm] wavelength which is the most commonly used value in the industry because it is easier to maintain a quasi-single mode and has the smallest footprint.

From the mode graph it can be seen how the field is mostly contained within the waveguide and how some discontinuity at the boundary is present.

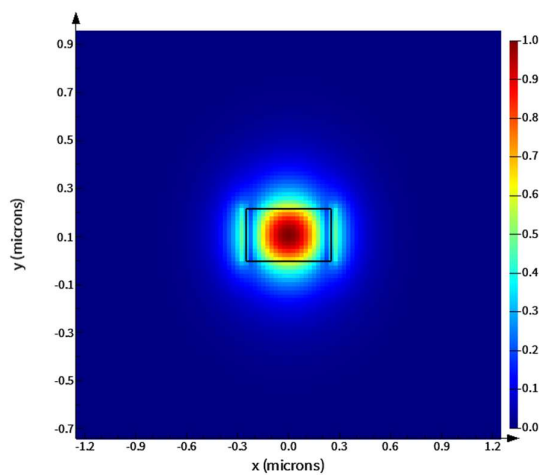


Figure 4: Electric field E_x intensity of TE (dominant) mode in the waveguide 220x500[nm]

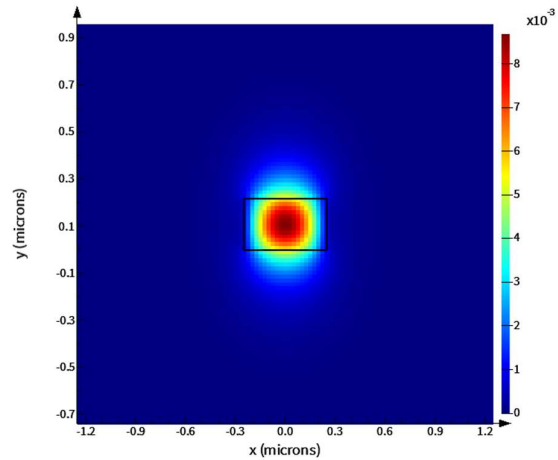


Figure 1: Electric field H_y intensity of TE (dominant) mode in the waveguide 220x500[nm]

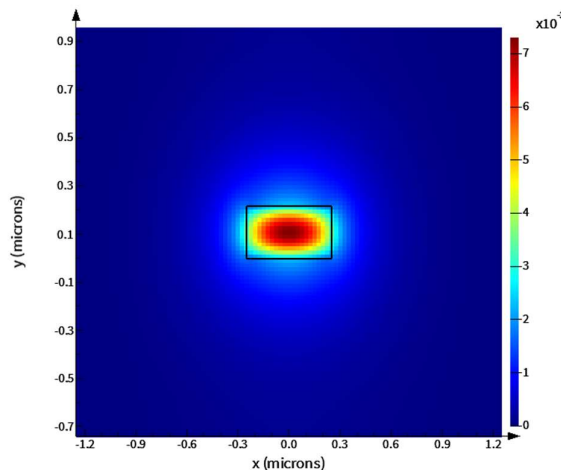


Figure 3: Electric field H_x intensity of TM (dominant) mode in the waveguide 220x500[nm]

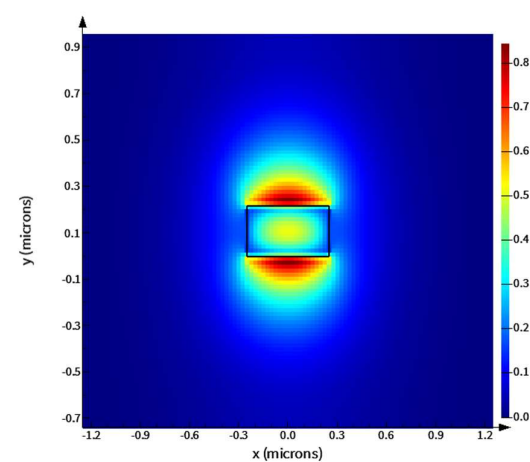


Figure 2: Electric field E_y intensity of TM (dominant) mode in the waveguide 220x500[nm]

A compact model for the waveguide effective index can be found in the form of a Taylor expansion of the second order for both the TE and TM mode.

$$n_{eff}(\lambda) = n_1 + n_2 \cdot (\lambda - \lambda_0) + n_3 \cdot (\lambda - \lambda_0)^2$$

Where n_1, n_2 and n_3 are the polynomial parameter while λ_0 is the center wavelength around which we approximate the function of the effective refractive index

Using Lumerical MODE software first we do a frequency sweep and simulate the refractive and group index with changing frequency between 1500[nm] and 1600[nm].

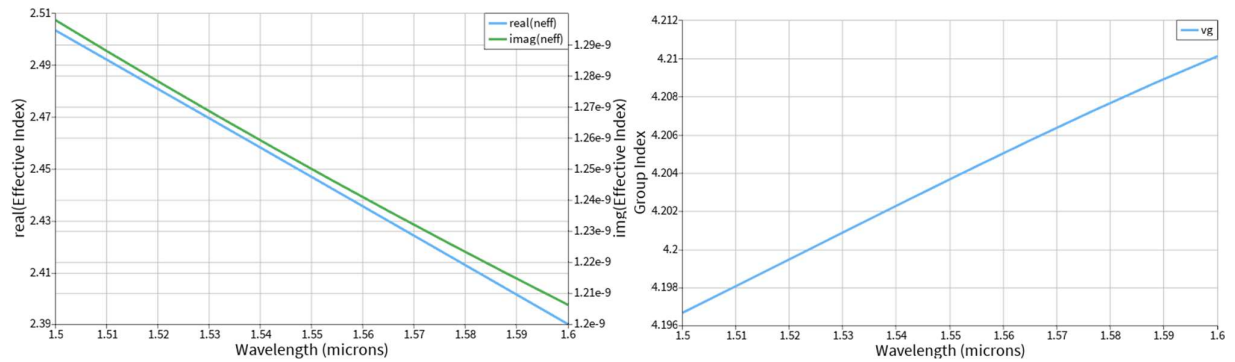


Figure 6: Effective index and group index of TE (dominant) mode

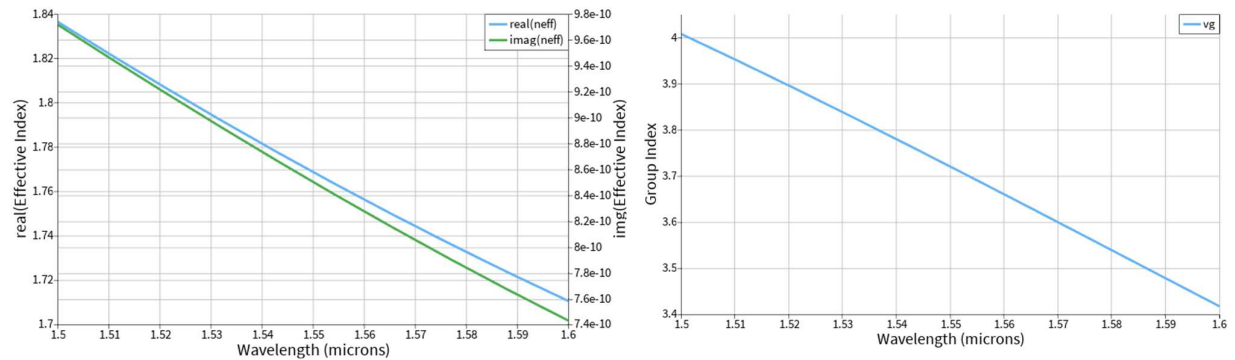


Figure 5: Effective index and group index of TM (dominant) mode

We can of course consider the imaginary component to be negligible and derive the Taylor expansion:

$$n_{eff-}(\lambda) = 2.44682 - 1.13339 \cdot (\lambda - 1.55) - 0.0439366 \cdot (\lambda - 1.55)^2$$

$$n_{eff-T}(\lambda) = 1.76882 - 1.25867 \cdot (\lambda - 1.55) + 1.91348 \cdot (\lambda - 1.55)^2$$

Mach-Zehnder Interferometer

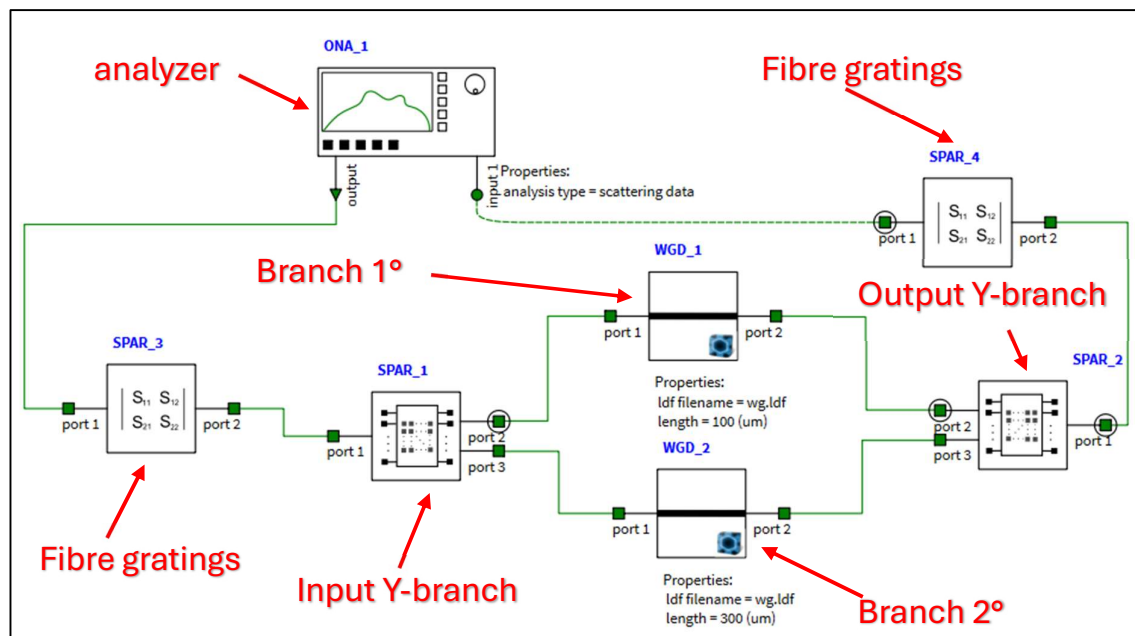
With the simulation and data from the waveguide mode simulation we can now simulate the circuit itself. To do this simulation we use Lumerical INTERCONNECT.

Using the formula of the second chapter for the FSR we calculate some theoretical value:

$$FSR = \frac{\lambda^2}{\Delta L \cdot n_g} = \frac{1.550^2}{\Delta L \cdot 4.204}$$

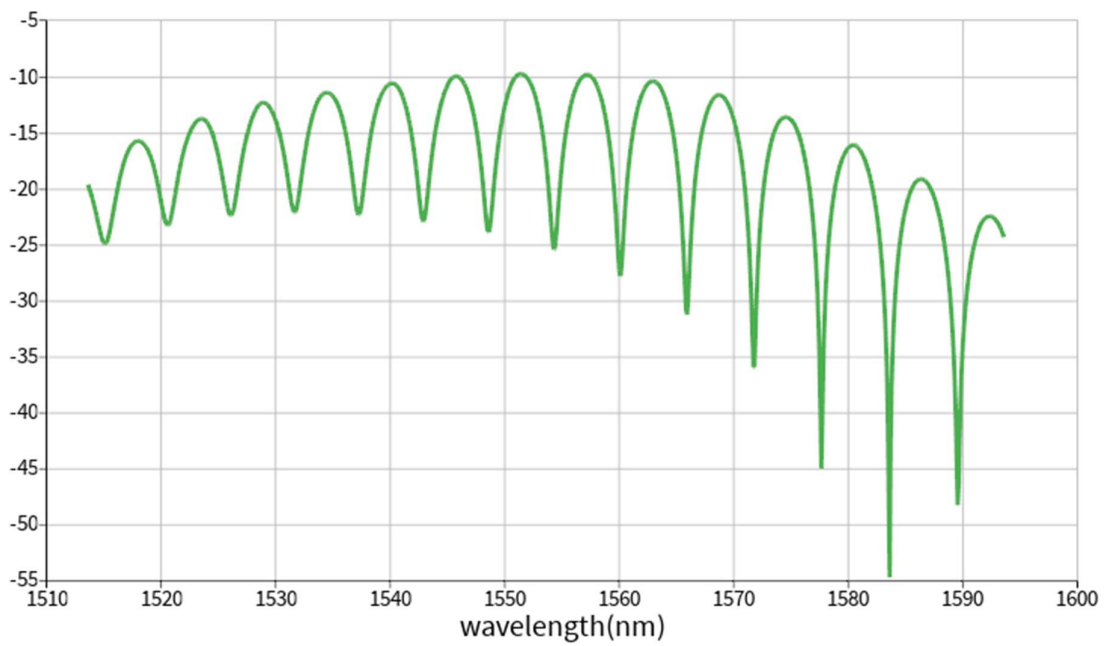
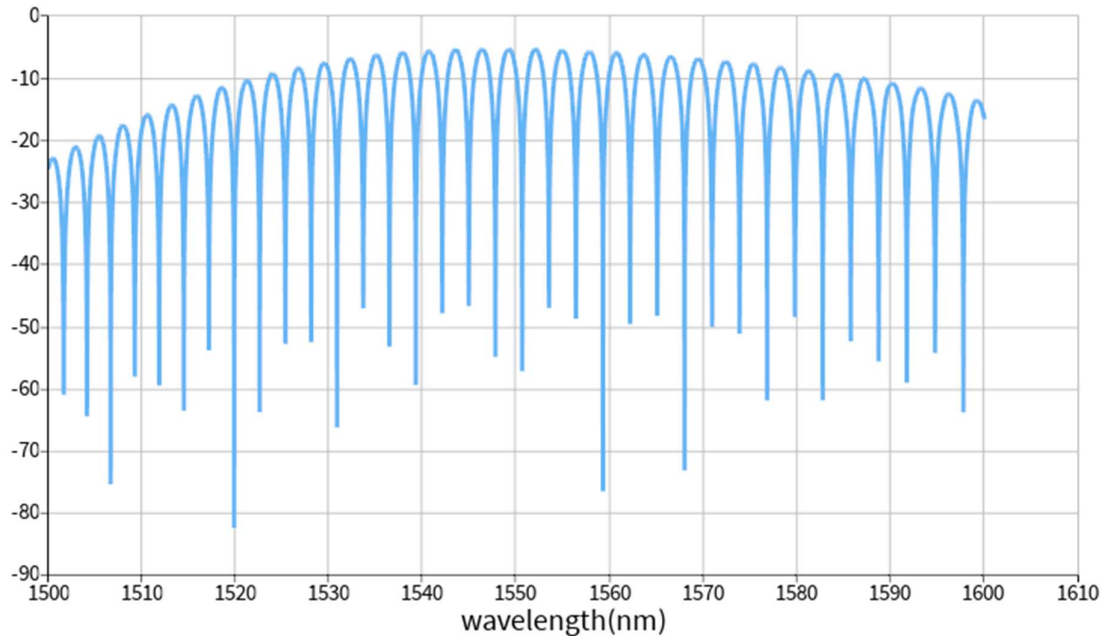
$\Delta L [\mu m]$	$FSR [\mu m]$
100	0,005714795
150	0,003809864
200	0,002857398
250	0,002285918
300	0,001904932

Let's build the MZI within the INTERCONNECT software:



We can get the gain graph for the different ΔL .

---imagine simulazione---



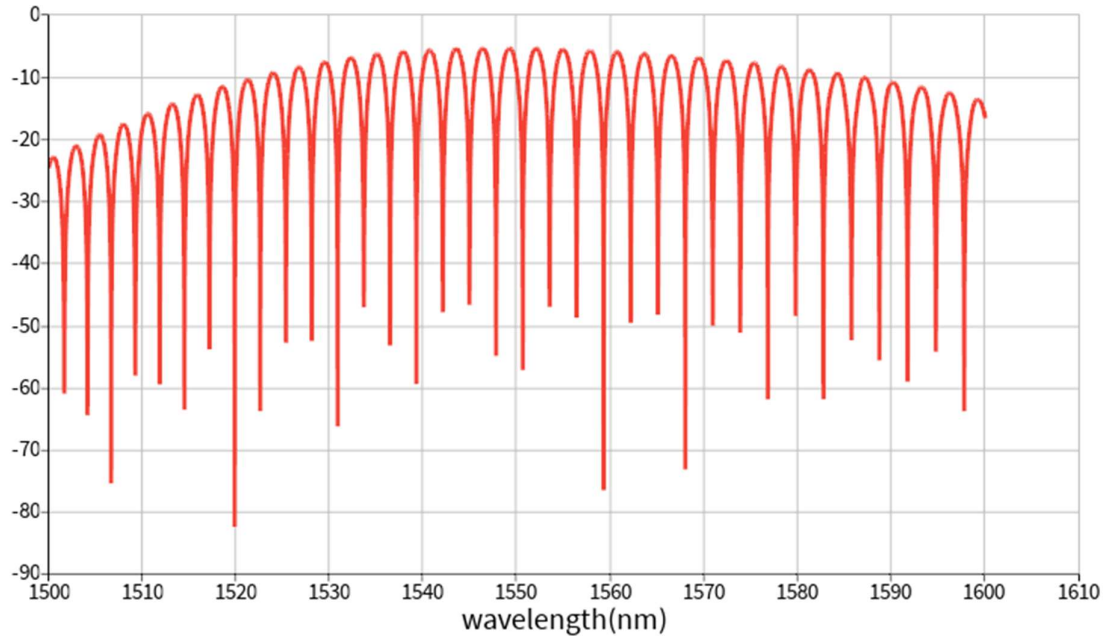


Figure 10: gain $\Delta L=200$

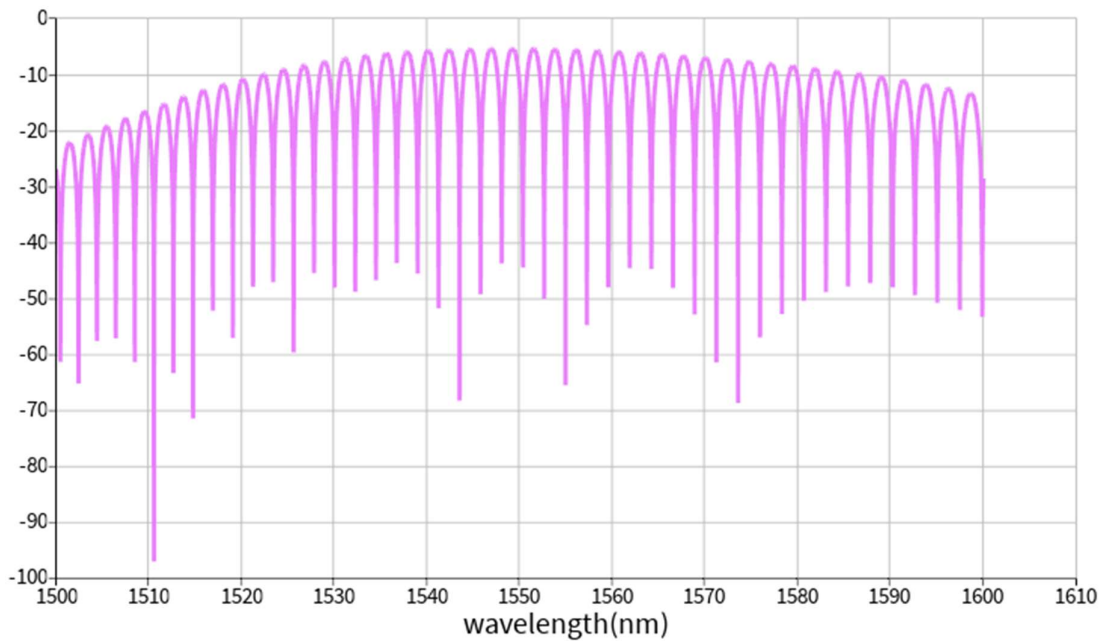


Figure 9: gain $\Delta L=250\text{micron}$

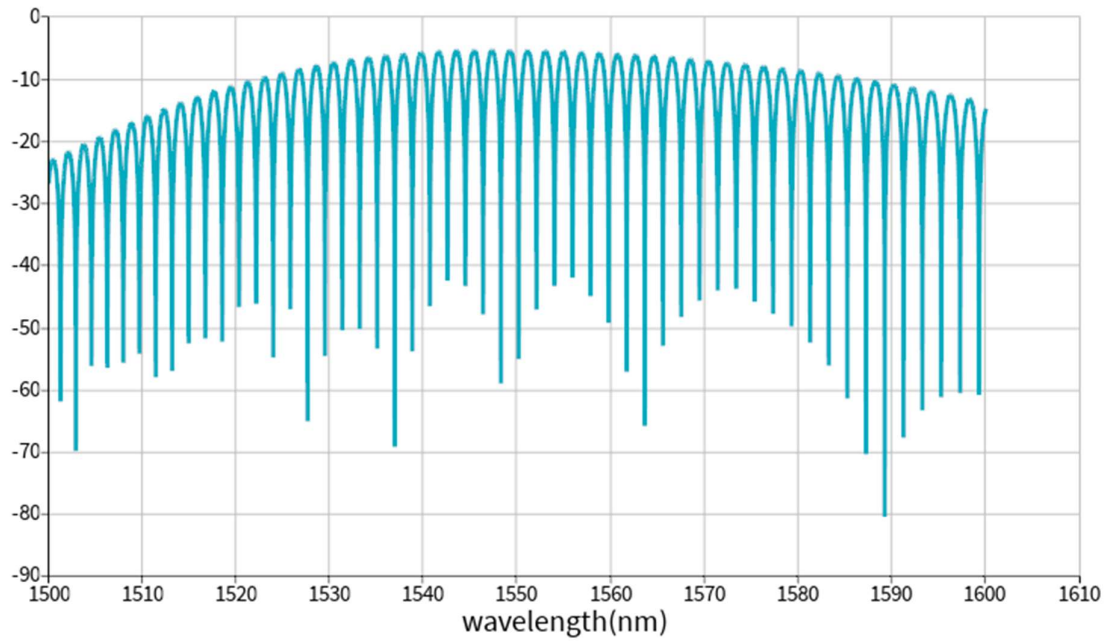


Figure 11: gain $\Delta L=300\mu\text{m}$

FABRICATION

to be completed later to include your layout and details about fabrication

EXPERIMENT DATA

to be completed later

ANALYSIS

to be completed later

CONCLUSION

to be completed later

REFERENCE