Probabilistic Robotics Course

Discrete Filtering: Localization

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Discrete filtering: recap

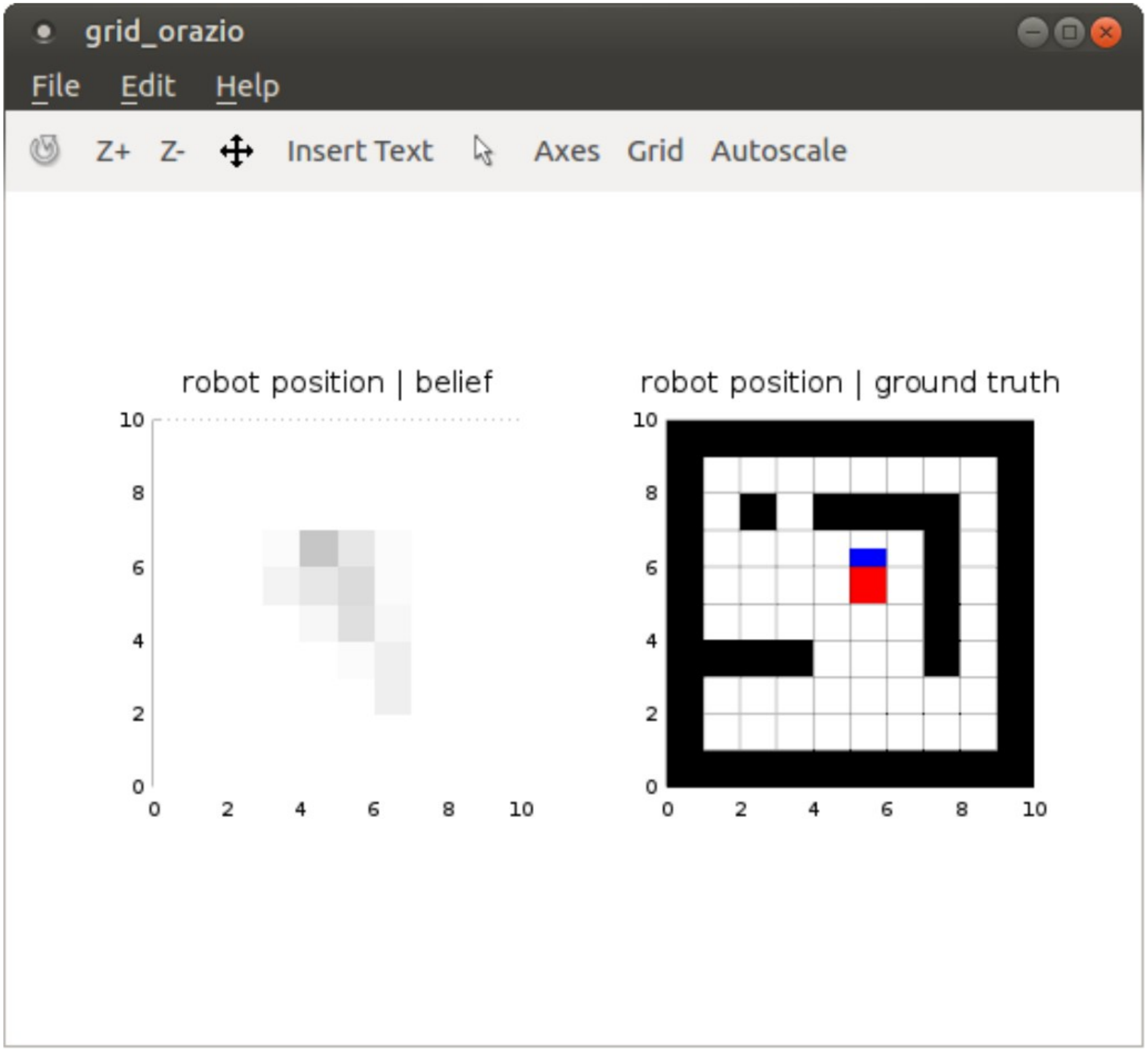
- Estimate the current state belief given
  - A) Previous state belief
  - B) Sequence of observations $z_{0:t}$
  - C) Sequence of controls $u_{0:t-1}$
  - D) Transition model
  - E) Observation model

$$\boxed{b(\mathbf{x}_t)} = \eta_t \boxed{p(\mathbf{z}_t \mid \mathbf{x}_t)} \sum_{\mathbf{x}_{t-1}} \boxed{p(\mathbf{x}_t \mid \mathbf{x}_{t-1}, \mathbf{u}_{t-1})} \boxed{b(\mathbf{x}_{t-1})}$$

Implementing a Bayes Filter

- Choose how to represent the state
- Choose how to represent the controls
- Choose how to represent the observations
- Implement a transition model
- Implement an observation model

Implementing a Bayes Filter



Implementation Outline

- Scenario: The map, grid-orazio

- Modeling the problem
  - A) Transition model $p(\mathbf{x}_t \mid \mathbf{x}_{t-1}, \mathbf{u}_{t-1})$
    - 1) without noise
    - 2) with noise
  - B) Observation model $p(\mathbf{z}_t \mid \mathbf{x}_t)$

- Building the filter
  - C) **Predict** belief $p(\mathbf{x}_t | \mathbf{u}_{1:t-1}, \mathbf{z}_{1:t-1})$
  - D) **Update** belief $p(\mathbf{x}_t | u_{1:t-1}, \mathbf{z}_{1:t})$

Scenario: The map

In this problem we will use some prior knowledge: a 2D *map* (maps/map.txt).

```
1 1 1 1 1 1 1 1 1 1
1 0 0 0 0 0 0 0 0 1
1 0 1 0 1 1 1 1 0 1
1 0 0 0 0 0 0 1 0 1
1 0 0 0 0 0 0 1 0 1
1 0 0 0 0 0 0 1 0 1
1 1 1 1 0 0 0 1 0 1
1 0 0 0 0 0 0 0 0 1
1 0 0 0 0 0 0 0 0 1
1 1 1 1 1 1 1 1 1 1
```

cell at [row: 7, col: 10]

Our map is a grid, represented by a matrix with the convention:

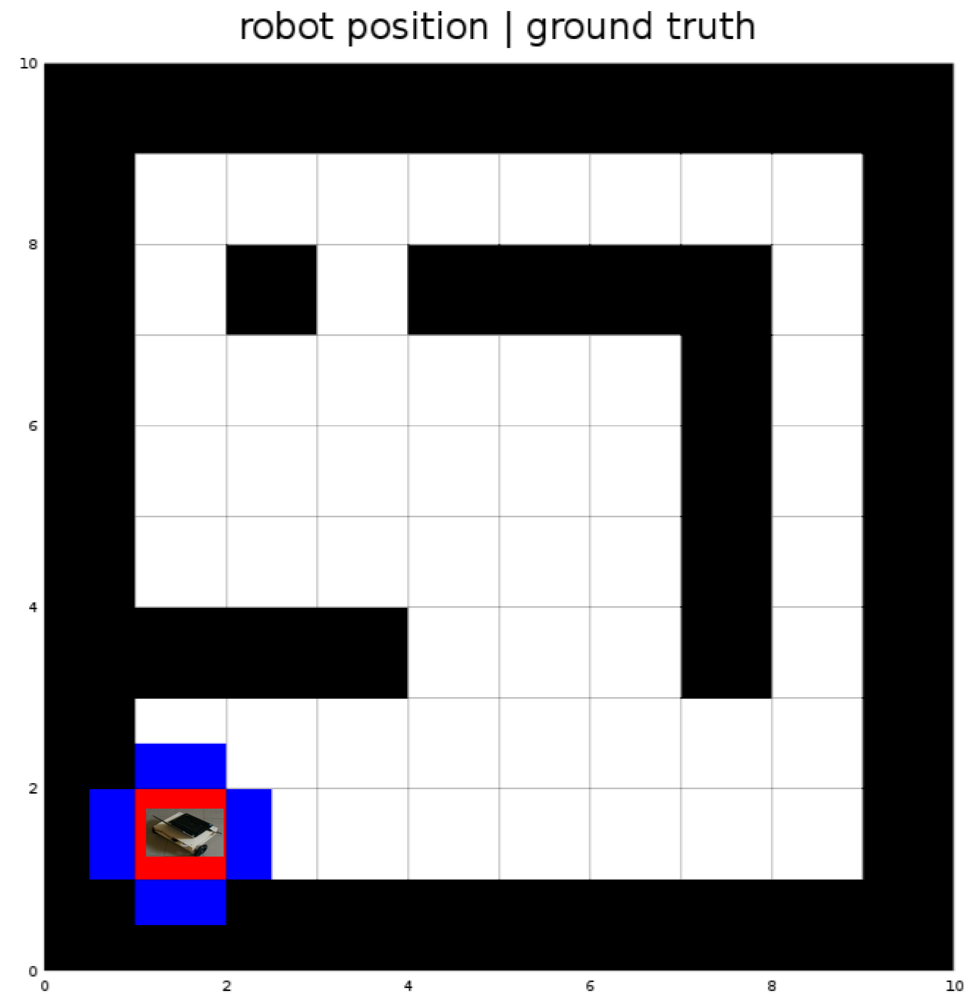
- a cell having value 0 is free
- a cell having value 1 is occupied

Scenario: grid-orazio

grid-orazio (red) lives in a grid world. The cells of this world are either free (white) or occupied (black)

At each point in time, grid-orazio can receive one of the 4 commands to move: UP/DOWN/LEFT/RIGHT

grid-orazio senses the state around it with 4 bumpers (blue) mounted at its 4 sides



Note: **I did not have time to fix the y axis. It should be inverted** (keep in mind when MOVE_UP will be explained)

Scenario: Our program

```
#retrieve new robot position according to our transition model
state_ground_truth = getNextState(map, state_ground_truth, control_input);

#obtain current observations according to our observation model
observations = getObservations(map, state_ground_truth(1), state_ground_truth(2));

#PREDICT robot position belief
state_belief_previous = state_belief;
state_belief = zeros(map_rows, map_cols);
for row = 1:map_rows
  for col = 1:map_cols
    state_belief += %TODO
  endfor
endfor

#UPDATE robot position belief and COMPUTE the normalizer
inverse_normalizer = 0;
for row = 1:map_rows
  for col = 1:map_cols
    state_belief(row, col) *= %TODO
    inverse_normalizer     += %TODO
  endfor
endfor

#NORMALIZE the belief probabilities to [0, 1]
normalizer =  %TODO
state_belief *= %TODO
```

A) Transition Model (transitionModel.m)

How to move grid-orazio? We need to implement a function in the form:

```
function transition_probability_matrix = transitionModel(map_,
                                                          row_from_,
                                                          col_from_,
                                                          control_input_)
```

that given:

- a start state [row_from_, col_from_]
- a control input

```
#available robot controls
global MOVE_UP    = 119; # W
global MOVE_DOWN  = 115; # S
global MOVE_LEFT  = 97;  # A
global MOVE_RIGHT = 100; # D
```

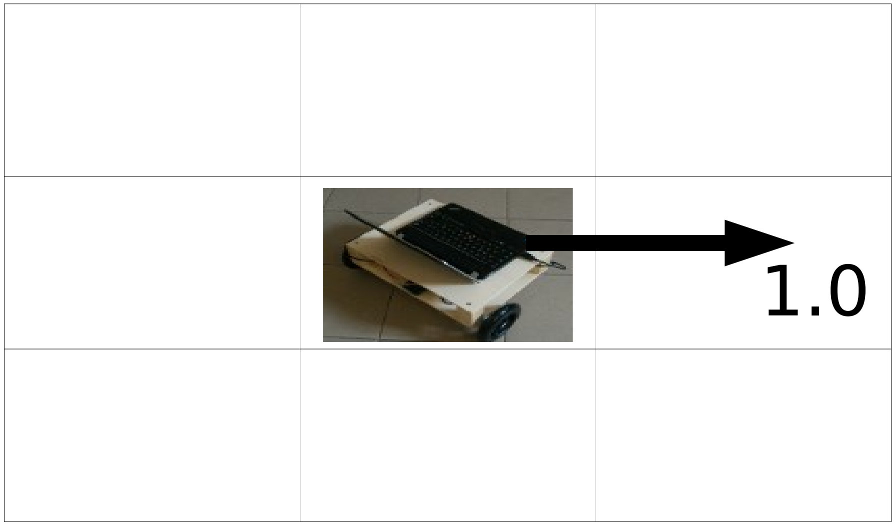
returns the probability of moving to any cell in the map from the start state

A1) Transition Model: Without noise

We assume that the controls we issue to grid-orazio, have a deterministic effect (**no noise**).

To a control MOVE_RIGHT, the robot will respond by moving right with probability 1.0.

MOVE_RIGHT:

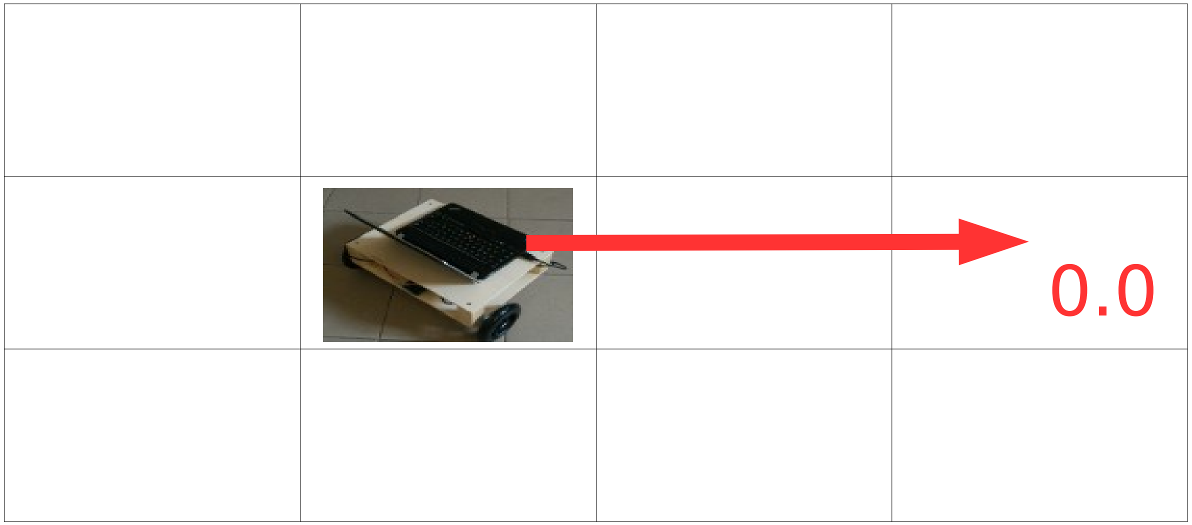


The behavior is symmetric for all 4 controls

A1) Transition Model: Motion constraint

The robot can move only to adjacent cells, for farther cells the transition probability becomes 0.0.

MOVE_RIGHT:

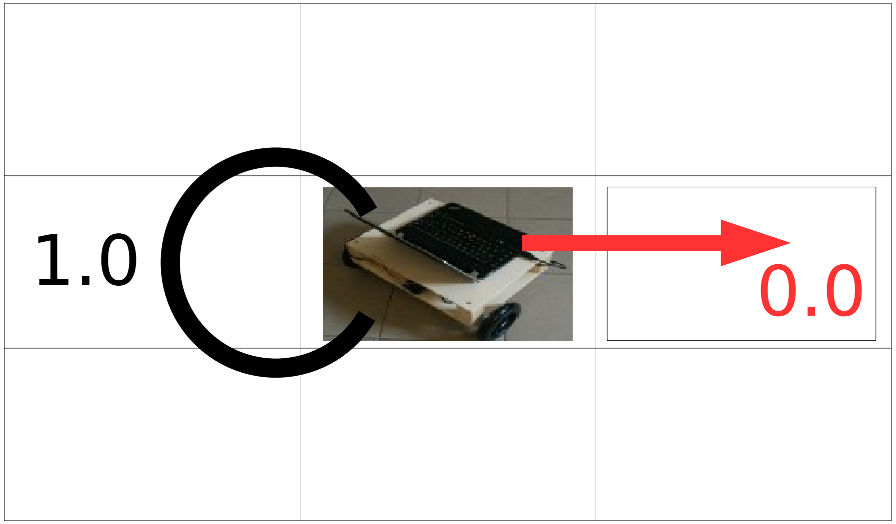


The behavior is symmetric for all 4 controls

A1) Transition Model: Motion feasibility

If the target cell (**noise free** transition) is occupied, the robot will stay where it is with probability 1.0.

MOVE_RIGHT:



The behavior is symmetric for all 4 controls

A1) Transition Model: Motion constraint

Loop over the rows and columns of the map

The robot can move only to adjacent cells:

```
#compute resulting position difference
translation_rows = row_to - row_from_;
translation_cols = col_to - col_from_;

#allow only unit motions (1 cell): check if we have a bigger motion
if(abs(translation_rows) > 1 || abs(translation_cols) > 1)
  continue;
endif
```

If the two cells are farther away than 1, the transition probability remains 0.

A1) Transition Model: Next state

Retrieve the *noise free* next state based on the control input:

```
#compute target robot position according to input
target_row = row_from_;
target_col = col_from_;

switch (control_input_)
        case MOVE_UP
                target_row++;
        case MOVE_DOWN
                target_row--;
        case MOVE_LEFT
                target_col--;
        case MOVE_RIGHT
                target_col++;
        otherwise
                return;
endswitch
```

A1) Transition Model: Motion feasibility

We have to check if the next state is feasible on our map (i.e. the cell is not occupied and we're not going over the border):

```
#check if the desired motion is infeasible
invalid_motion = false;
if (target_row < 1 || target_row > map_rows || target_col < 1 || target_col > map_cols) #if we're going over the border
  invalid_motion = true;
elseif (map_(target_row, target_col) == 1 || map_(row_to, col_to) == 1) #obstacle in the goal cell
  invalid_motion = true;
endif
if (invalid_motion)

  #if the desired translation is zero
  if (translation_rows == 0 && translation_cols == 0)
    transition_probability_matrix(row_to, col_to) = 1; #we stay with 100% probability (no motion has full confidence)
    continue;
  else
    continue; #we cannot move
  endif
endif
```

A1) Transition Model: Without noise

Set the probability of moving to a cell depending on the control input:

```
#our motion is feasible - compute resulting transition
switch (control_input_)
  case MOVE_UP
    if (translation_rows     ==  1 && translation_cols ==  0) transition_probability_matrix(row_to, col_to) = 1.0;
    endif;
  case MOVE_DOWN
    if (translation_rows     == -1 && translation_cols ==  0) transition_probability_matrix(row_to, col_to) = 1.0;
    endif;
  case MOVE_LEFT
    if (translation_rows     ==  0 && translation_cols == -1) transition_probability_matrix(row_to, col_to) = 1.0;
    endif;
  case MOVE_RIGHT
    if (translation_rows     ==  0 && translation_cols ==  1) transition_probability_matrix(row_to, col_to) = 1.0;
    endif;
endswitch
```

Since we're assuming no uncertainty, the probability for moving to the next state is maximal. And 0 for all other states.

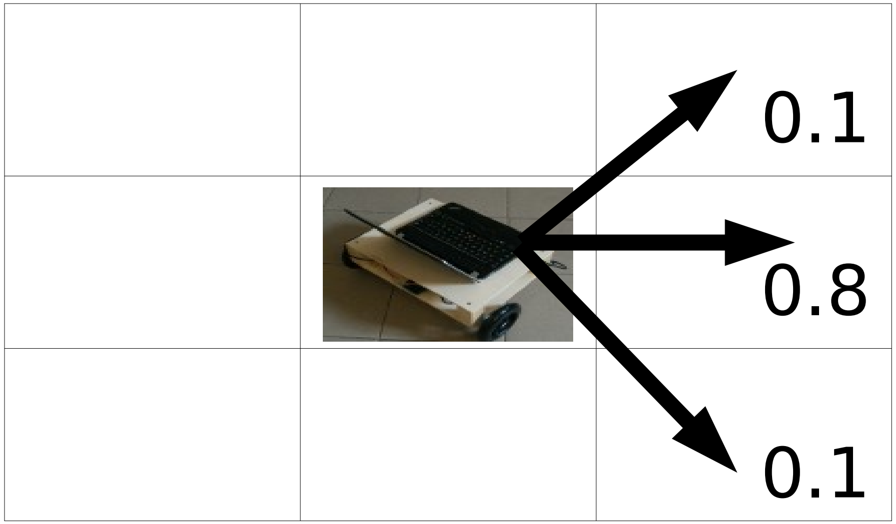
A2) Transition Model: Modeling the Controls

The controls we issue to grid-orazio, do not have a deterministic effect anymore (because we have **noise**)

To a control MOVE_RIGHT, the robot will respond by moving

- right with prob. 0.8
- top-right with prob. 0.1
- bottom-right with prob. 0.1

MOVE_RIGHT:



The behavior is symmetric for all 4 controls

A2) Transition Model: Modeling the Controls

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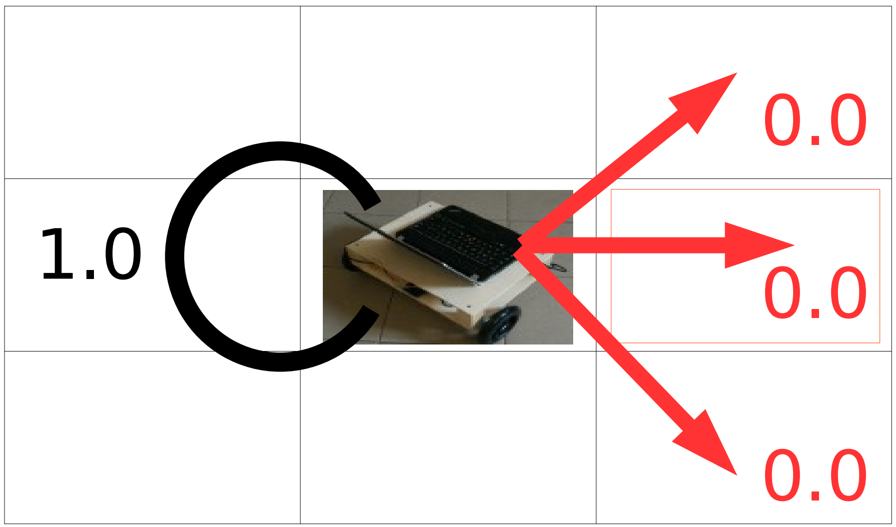
MOVE_RIGHT:



If the target (noise free) cell is occupied,
the robot will stay where it is with probability 1.0.

A2) Transition Model: With noise

Introduce noise into the transition:

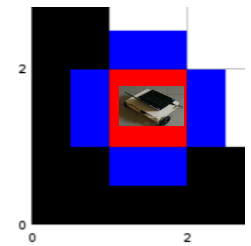
```
#our motion is feasible - compute resulting transition
switch (control_input_)
  case MOVE_UP
    if (translation_rows     == -1 && translation_cols ==  0) transition_probability_matrix(row_to, col_to) = 1.0;
     elseif (translation_rows == -1 && translation_cols ==  1) transition_probability_matrix(row_to, col_to) = TODO
     elseif (translation_rows == -1 && translation_cols == -1) transition_probability_matrix(row_to, col_to) = TODO
    endif;
  case MOVE_DOWN
    if (translation_rows     ==  1 && translation_cols ==  0) transition_probability_matrix(row_to, col_to) = 1.0;
     elseif (translation_rows ==  1 && translation_cols ==  1) transition_probability_matrix(row_to, col_to) = TODO
     elseif (translation_rows ==  1 && translation_cols == -1) transition_probability_matrix(row_to, col_to) = TODO
    endif;
  case MOVE_LEFT
    if (translation_rows     ==  0 && translation_cols == -1) transition_probability_matrix(row_to, col_to) = 1.0;
     elseif (translation_rows ==  1 && translation_cols == -1) transition_probability_matrix(row_to, col_to) = TODO
     elseif (translation_rows == -1 && translation_cols == -1) transition_probability_matrix(row_to, col_to) = TODO
    endif;
  case MOVE_RIGHT
    if (translation_rows     ==  0 && translation_cols ==  1) transition_probability_matrix(row_to, col_to) = 1.0;
     elseif (translation_rows ==  1 && translation_cols ==  1) transition_probability_matrix(row_to, col_to) = TODO
     elseif (translation_rows == -1 && translation_cols ==  1) transition_probability_matrix(row_to, col_to) = TODO
    endif;
endswitch
```

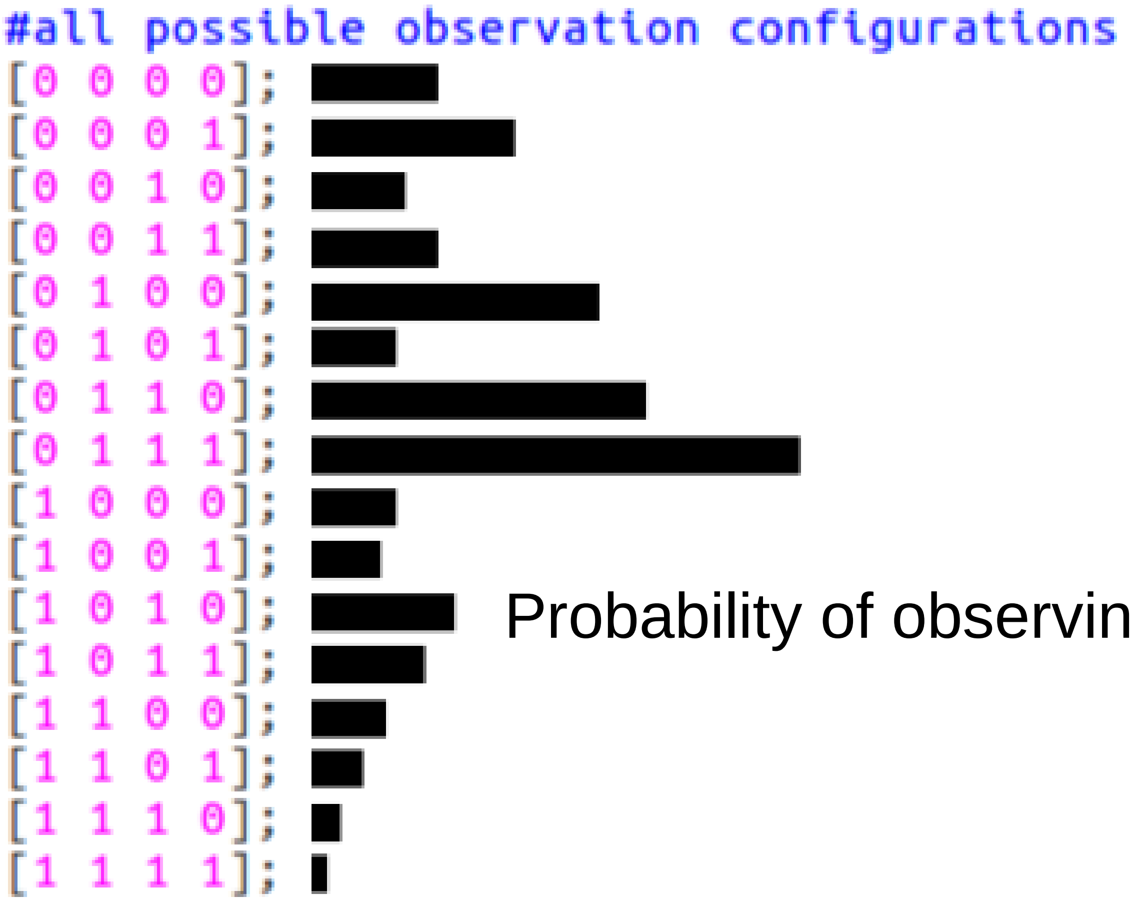
Now we cannot be certain that grid-orazio is moving into the desired direction.

B) Observation Model

Each bumper can be toggled or not.

4 bumpers result in 16 possible configurations:





Probability of observing a configuration

B) Observation Model (observationModel.m)

To retrieve the 4 observations around grid-orazio we use our observation model:

```
current_probability = observationModel(map_,
                                        state_ground_truth_(1),
                                        state_ground_truth_(2),
                                        current_observations);
```

that given:

- a start state
- a observation sample (4 values)

returns the probability of observing the current observation sample

B) Observation Model: Modeling the Bumper

Given the location, each of the 4 bumpers is independent

A bumper gives a wrong measurement with probability 0.2

In this situation

p($\mathbf{z}_{RIGHT}$=toggled)=0.8

During the synthesis of an observation model you **assume** you know **both** state and the measurement. The observation model tells you how likely the measurement is in the state

B) Observation Model: With noise

We have:

```
#update probability depending on observations
if (cell_up_occupied == observations_(1))
%TODO
endif
if (cell_down_occupied == observations_(2))
%TODO
endif
if (cell_left_occupied == observations_(3))
%TODO
endif
if (cell_right_occupied == observations_(4))
%TODO
endif
```

Hence we might obtain bumper readings for cells which are actually not occupied.

Localizing grid-orazio

We have knowledge of:

- the controls grid-orazio receives (MOVE_..)
- 0-4 observations from the bumpers

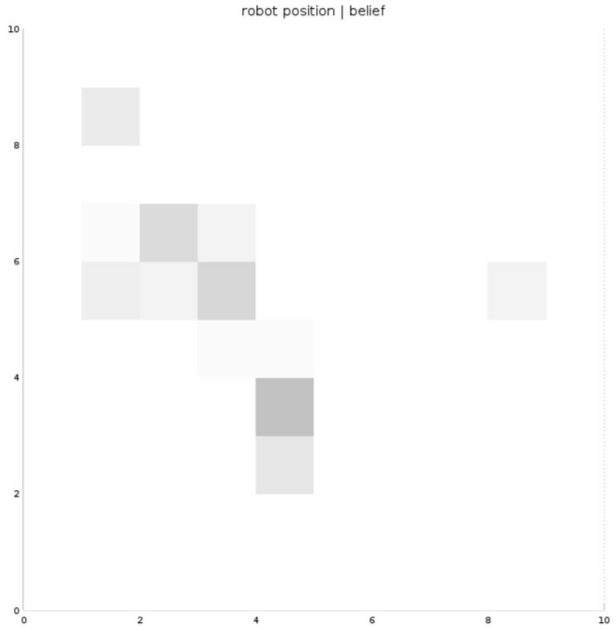
We want to determine the distribution over all possible locations on the map using this information.

Belief

The belief should contain a probability value for each state.

```
#initialize state_belief over the complete grid
number_of_free_cells = rows(map)*columns(map);
belief_initial_value = 1/(number_of_free_cells);

state_belief = ones(rows(map), columns(map))*belief_initial_value;
```



C) Predict belief

We have:

$$\underbrace{p(\mathbf{x}_t|\mathbf{u}_{1:t-1},\mathbf{z}_{1:t-1})}_{b_{t|t-1}} = \sum_{\mathbf{x}_{t-1}} p(\mathbf{x}_t,\mathbf{x}_{t-1}|\mathbf{u}_{1:t-1},\mathbf{z}_{1:t-1})$$

and:

$$p(\mathbf{x}_t,\mathbf{x}_{t-1}|\mathbf{u}_{1:t-1},\mathbf{z}_{1:t-1}) = p(\mathbf{x}_t|\mathbf{x}_{t-1},\mathbf{u}_{t-1})\underbrace{p(\mathbf{x}_{t-1}|\mathbf{u}_{1:t-2},\mathbf{z}_{1:t-1})}_{b_{t-1}}$$

```
#PREDICT robot position belief
state_belief_previous = state_belief;
state_belief = zeros(map_rows, map_cols);
for row = 1:map_rows
  for col = 1:map_cols
    state_belief += %TODO
  endfor
endfor
```

D) Update belief

We have:

$$b(\mathbf{x}_t) = \eta_t p(\mathbf{z}_t \mid \mathbf{x}_t) \sum_{\mathbf{x}_{t-1}} p(\mathbf{x}_t \mid \mathbf{x}_{t-1}, \mathbf{u}_{t-1}) b(\mathbf{x}_{t-1})$$

with:

$$\eta_t = \frac{1}{\sum_{\mathbf{x}_t} p(\mathbf{x}_t | \mathbf{x}_{t-1}, \mathbf{u}_{t-1}) b(\mathbf{x}_{t-1})}$$

Straightforward implementation:

```
#UPDATE robot position belief and COMPUTE the normalizer
inverse_normalizer = 0;
for row = 1:map_rows
  for col = 1:map_cols
    state_belief(row, col) *= %TODO
    inverse_normalizer     += %TODO
  endfor
endfor
```

Test it

- In a console run:

```
octave-cli grid_orazio.m <map_file.txt>
```

- grid-orazio can be controlled with the keys: W,A,S,D

You will notice that issuing a motion command has non-deterministic effects.

Observe the “belief” window, and see the probability mass changing as grid-orazio explores the map.

Exercise

What if grid-orazio has also an *orientation* and its available controls change to:

- MOVE_FORWARD
- MOVE_BACKWARD
- ROTATE_LEFT
- ROTATE_RIGHT

How does the state change?

What about the observation and transition model?