

## Solutions to Homework06

1. Given a relation schema ABCDEFG satisfying the following functional dependencies, find all keys.

- $A \rightarrow I$
- $AB \rightarrow C$
- $AE \rightarrow GH$
- $BE \rightarrow DF$
- $H \rightarrow A$

Solution:

Attributes on the left: BE

Attributes on the right: CDFG

Attributes on both sides: AH

Attributes never appears: N/A

- (a) Every key needs to include B and E. First We compute  $(BE)^+ = BDEF$
- (b) Compute  $(ABE)^+ = ABCDEFGHI = R$   
Then ABE is a key.
- (c) Compute  $(BEH)^+ = ABCDEFGHI = R$   
Then BEH is a key.
2. Given a relation schema ABCDEFGH, show that the given functional dependencies is a minimal cover.

- $A \rightarrow B$
- $ADE \rightarrow C$
- $ADF \rightarrow G$
- $CF \rightarrow GH$

Solution:

- (a) 1.  $A \rightarrow B$
2.  $DE \rightarrow C$  (Try LHS simplification and remove A)
3.  $ADF \rightarrow G$
4.  $CF \rightarrow GH$
- (a) could only be stronger, so we compute the closure of DE using the original set of FDs, getting DE. As C is not included, we proved non-equivalence. We cannot replace the original FDs with this set (a).
- (b) 5.  $A \rightarrow B$
6.  $AE \rightarrow C$  (Try LHS simplification and removes D)

35           7.  $ADF \rightarrow G$

36           8.  $CF \rightarrow GH$

37       (b) could only be stronger, so we compute the closure of AE using the original set of FDs,  
38       getting AEB. As C is not included, we proved non-equivalence. We cannot replace the  
39       original FDs with this set (b).

40   (c)   9.  $A \rightarrow B$

41       10.  $AD \rightarrow C$  (Try LHS simplification and removes E)

42       11.  $ADF \rightarrow G$

43       12.  $CF \rightarrow GH$

44       (c) could only be stronger, so we compute the closure of AD using the original set of FDs,  
45       getting ADB. As C is not included, we proved non-equivalence. We cannot replace the  
46       original FDs with this set (c).

47   (d)   13.  $A \rightarrow B$

48       14.  $ADE \rightarrow C$

49       15.  $DF \rightarrow G$  (Try LHS simplification and removes A)

50       16.  $CF \rightarrow GH$

51       (d) could only be stronger, so we compute the closure of DF using the original set of FDs,  
52       getting DF. As G is not included, we proved non-equivalence. We cannot replace the  
53       original FDs with this set (d).

54   (e)   17.  $A \rightarrow B$

55       18.  $ADE \rightarrow C$

56       19.  $AF \rightarrow G$  (Try LHS simplification and removes D)

57       20.  $CF \rightarrow GH$

58       (e) could only be stronger, so we compute the closure of AF using the original set of FDs,  
59       getting ABF. As G is not included, we proved non-equivalence. We cannot replace the  
60       original FDs with this set (e).

61   (f)   21.  $A \rightarrow B$

62       22.  $ADE \rightarrow C$

63       23.  $AD \rightarrow G$  (Try LHS simplification and removes F)

64       24.  $CF \rightarrow GH$

65       (f) could only be stronger, so we compute the closure of AD using the original set of FDs,  
66       getting ABD. As G is not included, we proved non-equivalence. We cannot replace the  
67       original FDs with this set (f).

68   (g)   25.  $A \rightarrow B$

69           26.  $ADE \rightarrow C$

70           27.  $ADF \rightarrow G$

71           28.  $CF \rightarrow H$  (Try RHS simplification and removes G)

72           (g) could only be weaker, so we compute the closure of CF using (g), getting CFH. As G

73           is not included, we proved non-equivalence. We cannot replace the original FDs with this

74           set (g).

75       (h) 29.  $A \rightarrow B$

76           30.  $ADE \rightarrow C$

77           31.  $ADF \rightarrow G$

78           32.  $CF \rightarrow G$  (Try RHS simplification and removes H)

79           (h) could only be weaker, so we compute the closure of CF using (h), getting CFG. As H

80           is not included, we proved non-equivalence. We cannot replace the original FDs with this

81           set (h).

82       (i) 33.  $A \rightarrow B$

83           34.  $ADE \rightarrow C$

84           35.  $ADF \rightarrow G$

85           36.  $F \rightarrow GH$  (Try LHS simplification and removes C)

86           (i) could only be stronger, so we compute the closure of F using the original set of FDs,

87           getting F. As GH is not included, we proved non-equivalence. We cannot replace the

88           original FDs with this set (i).

89       (j) 37.  $A \rightarrow B$

90           38.  $ADE \rightarrow C$

91           39.  $ADF \rightarrow G$

92           40.  $C \rightarrow GH$  (Try LHS simplification and removes F)

93           (j) could only be stronger, so we compute the closure of C using the original set of FDs,

94           getting C. As GH is not included, we proved non-equivalence. We cannot replace the

95           original FDs with this set (j).

96       Therefore, no simplification can be made and the given set of functional dependencies is a

97       minimal cover.

98       3. Given a relation schema ABCDEFGH satisfying the following functional dependencies, find a

99       minimal cover.

100           •  $A \rightarrow BC$

101           •  $AB \rightarrow D$

102           •  $B \rightarrow C$

103 Solution:

104 (a) •  $A \rightarrow B$  (Try RHS simplification and removes C)

105 •  $AB \rightarrow D$

106 •  $B \rightarrow C$

107 (a) could only be weaker, so we compute the closure of A using (a), getting ABCD. As C

108 is included, we proved equivalence. We can replace the original set of FDs with (a).

109 (b) •  $A \rightarrow B$

110 •  $A \rightarrow D$  (Try LHS simplification and removes B)

111 •  $B \rightarrow C$

112 (b) could only be stronger. We compute the closure of A using (a), getting ABDC. As D

113 is included, we proved equivalence. We can replace (a) with (b).

114 (c) 1.  $A \rightarrow BD$  (Union Rule)

115 2.  $B \rightarrow C$

116 No further LHS simplification or RHS simplification can be done. We obtain the minimal

117 cover.

118  $A \rightarrow BD$

119  $B \rightarrow C$

120

121 4. Given a relation schema ABCDEFGH satisfying the following functional dependencies, find a

122 minimal cover.

123 •  $A \rightarrow HI$

124 •  $AB \rightarrow CD$

125 •  $CD \rightarrow EF$

126 •  $E \rightarrow F$

127 •  $G \rightarrow AD$

128 •  $H \rightarrow B$

129 •  $I \rightarrow AG$

130 Solution:

131 (a) There is no union rule can be applied for now.

132 (b) 1.  $A \rightarrow I$  (Try RHS simplification and removes H)

133 2.  $AB \rightarrow CD$

134 3.  $CD \rightarrow EF$

135 4.  $E \rightarrow F$

136           5.  $G \rightarrow AD$

137           6.  $H \rightarrow B$

138           7.  $I \rightarrow AG$

139       (b) could only be weaker. We compute the closure of A using (b), getting AIGD. As H is  
140       not included, we proved non-equivalence. We cannot replace the original FDs with (b).

141   (c)   8.  $A \rightarrow H$  (Try RHS simplification and removes I)

142           9.  $AB \rightarrow CD$

143          10.  $CD \rightarrow EF$

144          11.  $E \rightarrow F$

145          12.  $G \rightarrow AD$

146          13.  $H \rightarrow B$

147          14.  $I \rightarrow AG$

148       (c) could only be weaker, so we compute the closure of A using (c), getting AHBCDEF  
149       . As I is not included, we proved non-equivalence. We cannot replace the original FDs  
150       with (c).

151   (d)   15.  $A \rightarrow HI$

152           16.  $AB \rightarrow C$  (Try RHS simplification and removes D)

153           17.  $CD \rightarrow EF$

154           18.  $E \rightarrow F$

155           19.  $G \rightarrow AD$

156           20.  $H \rightarrow B$

157           21.  $I \rightarrow AG$

158       (d) could only be weaker, so we compute the closure of AB using (d), getting  
159       ABCHIBGDEF . As D is included, we proved equivalence. We can replace the original  
160       FDs with (d).

161   (e)   22.  $A \rightarrow HI$

162           23.  $A \rightarrow C$  (Try LHS simplification and removes B)

163           24.  $CD \rightarrow EF$

164           25.  $E \rightarrow F$

165           26.  $G \rightarrow AD$

166           27.  $H \rightarrow B$

167 28.  $I \rightarrow AG$

168 (e) could only be stronger, so we compute the closure of A using (d), getting AHIBCGDEF

169 . As B is included, we proved equivalence. We can replace (d) with (e).

170 (f) 29.  $A \rightarrow CHI$  (union rule)

171 30.  $CD \rightarrow EF$

172 31.  $E \rightarrow F$

173 32.  $G \rightarrow AD$

174 33.  $H \rightarrow B$

175 34.  $I \rightarrow AG$

176 (f) union rule

177 (g) 35.  $A \rightarrow CHI$

178 36.  $CD \rightarrow E$  (Try RHS simplification and removes F)

179 37.  $E \rightarrow F$

180 38.  $G \rightarrow AD$

181 39.  $H \rightarrow B$

182 40.  $I \rightarrow AG$

183 (g) could only be weaker. We compute the closure of CD using the (g), getting CDEF.

184 As F is included, we proved equivalence. We can replace the (f) with (g).

185 (h) 41.  $A \rightarrow CHI$

186 42.  $C \rightarrow E$  (Try LHS simplification and removes D)

187 43.  $E \rightarrow F$

188 44.  $G \rightarrow AD$

189 45.  $H \rightarrow B$

190 46.  $I \rightarrow AG$

191 (h) could only be stronger, so we compute the closure of C using (g), getting C . As E

192 is not included, we proved non-equivalence. We cannot replace (g) with (h). Same for

193 removing C in  $CD \rightarrow E$ .

194 (i) 47.  $A \rightarrow CHI$

195 48.  $CD \rightarrow E$

196 49.  $E \rightarrow F$

197 50.  $G \rightarrow D$  (Try RHS simplification and removes A)

198 51.  $H \rightarrow B$

199 52.  $I \rightarrow AG$

200 (i) could only be weaker. We compute the closure of G using the (i), getting GD. As A is

201 not included, we proved non-equivalence. We cannot replace the (h) with (i).

202 (j) 53.  $A \rightarrow CHI$

203 54.  $CD \rightarrow E$

204 55.  $E \rightarrow F$

205 56.  $G \rightarrow A$  (Try RHS simplification and removes D)

206 57.  $H \rightarrow B$

207 58.  $I \rightarrow AG$

208 (j) could only be weaker. We compute the closure of G using the (j), getting GACHIB.

209 As D is not included, we proved non-equivalence. We cannot replace the (h) with (j).

210 (k) 59.  $A \rightarrow CHI$

211 60.  $CD \rightarrow E$

212 61.  $E \rightarrow F$

213 62.  $G \rightarrow AD$

214 63.  $H \rightarrow B$

215 64.  $I \rightarrow G$  (Try RHS simplification and removes A)

216 (k) could only be weaker. We compute the closure of I using the (k), getting IGADCHEFB.

217 As A is included, we proved equivalence. We can replace the (h) with (k).

218 No further LHS simplification or RHS simplification can be done. We obtain the minimal

219 cover.

220  $A \rightarrow CHI$

221  $CD \rightarrow E$

222  $E \rightarrow F$

223  $G \rightarrow AD$

224  $H \rightarrow B$

225  $I \rightarrow G$

226 5. Given the following minimal cover, create a 3NF decomposition.

227 •  $ABD \rightarrow G$

228 •  $AG \rightarrow E$

229 •  $BD \rightarrow C$

230 •  $CF \rightarrow A$

231 •  $G \rightarrow B$

232 Solution:

233 (a) Creating tables from this minimal cover:  
 234 We get:

- 235 1. Table ABDG (ABD key)
- 236 2. Table AGE (AG key)
- 237 3. Table BDC (BD key)
- 238 4. Table CFA (CF key)
- 239 5. Table GB (G key)

240 (b) Check redundancy of tables and there is a redundant table of GB because table ABDG  
 241 already contains GB. So, remove Table GB.

242 (c) Computing global key:  
 243 Attributes on the left: DF  
 244 Attributes on the right: E  
 245 Attributes on both sides: ABCG  
 246 Attributes never appears: N/A  
 247 Start from  $(DF)^+ = DF$   
 248  $(ADF)^+ = ADF$   
 249  $(BDF)^+ = ABCDEFG = R$   
 250 Therefore, BDF is the global key and we need to create a table to store BDF. GDF is  
 251 also a global key, so table GDF is acceptable as well.

252 (d) The 3NF decomposition is:

- 253 1. Table ABDG (ABD key)
- 254 2. Table AGE (AG key)
- 255 3. Table BDC (BD key)
- 256 4. Table CFA (CF key)
- 257 5. Table BDF (BDF key)

258 (e) If with the original version of question 5, Given relation schema  $R = ABCDEFGH$   
 259 Computing global key:  
 260 Attributes on the left: DF  
 261 Attributes on the right: E  
 262 Attributes on both sides: ABCG  
 263 Attributes never appears: H  
 264 Start from  $(DFH)^+ = DFH$   
 265  $(ADFH)^+ = ADFH$   
 266  $(BDFH)^+ = ABCDEFGH = R$   
 267 Therefore, BDFH is a global key and we need to create a table to store BDFH. GDFH is  
 268 a global key as well. Table GDFH is acceptable as well.

269 (f) If with the original version of question 5 with relation schema  $ABCDEFGH$   
 270 The 3NF decomposition is:



271           1. Table ABDG (ABD key)

272           2. Table AGE (AG key)

273           3. Table BDC (BD key)

274           4. Table CFA (CF key)

275           5. Table BDFH (BDFH key)

276 6. Given a relation schema ABCDEFG and the following minimal cover, create a 3NF decompo-

277       sition.

278       •  $A \rightarrow B$

279       •  $B \rightarrow DE$

280       •  $CF \rightarrow DE$

281       •  $DG \rightarrow CF$

282 Solution:

283 (a) Creating tables from this minimal cover:

284       We get:

285       1. Table AB (A key)

286       2. Table BDE (B key)

287       3. Table CFDE (CF key)

288       4. Table DGCF (DG key)

289 (b) There is no redundant table.

290 (c) Check if there is at least one table that stores global key.

291 (d) Computing key:

292       Attributes on the left: AG

293       Attributes on the right: E

294       Attributes on both sides: BCDF

295       Attributes never appears: N/A

296       Start from  $(AG)^+ = ABCDEFG = R$

297       Therefore, AG is the global key and we need to create a table to store AG.

298 (e) The 3NF decomposition is:

299       1. Table AB (A key)

300       2. Table BDE (B key)

301       3. Table CFDE (CF key)

302       4. Table DGCF (DG key)

303       5. Table AG (AG key)

304 (f) If with the original version of question 6: Given a relation schema ABCDEFGH  
 305 The first two steps will be the same until we computing the key.  
 306 Computing key:  
 307 Attributes on the left: AG  
 308 Attributes on the right: E  
 309 Attributes on both sides: BCDF  
 310 Attributes never appears: H  
 311 Start from  $(AGH)^+ = ABCDEFGH = R$   
 312 Therefore, AGH is the global key and we need to create a table to store AGH.

313 (g) The original version of question 6 with relation schema ABCDEFGH  
 314 The 3NF decomposition is:

- 315 1. Table AB (A key)
- 316 2. Table BDE (B key)
- 317 3. Table CFDE (CF key)
- 318 4. Table DGCF (DG key)
- 319 5. Table AGH (AGH key)

320