CSCI-GA.3033-004

Graphics Processing Units (GPUs): Architecture and Programming

Lecture : Parallel Patterns

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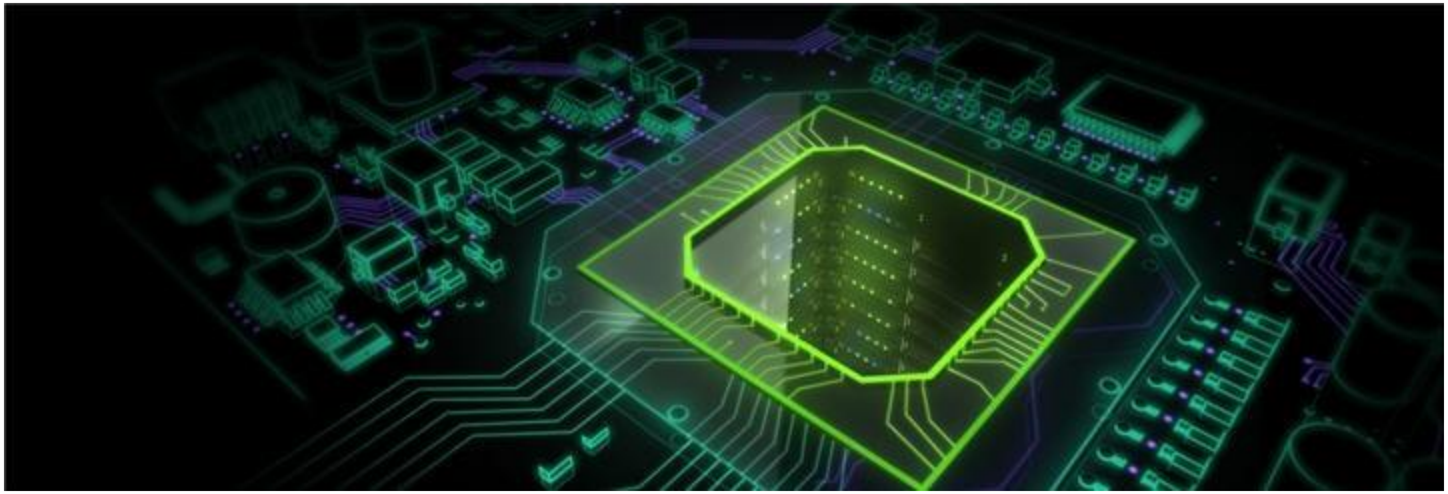
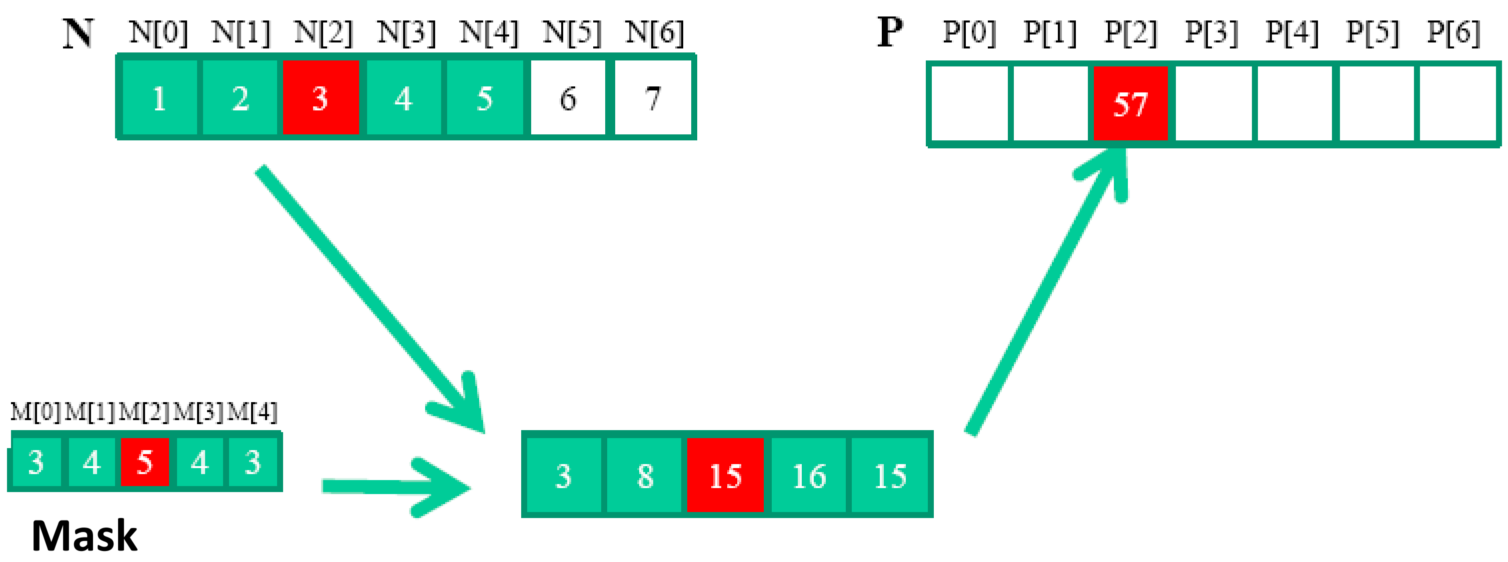


Convolution

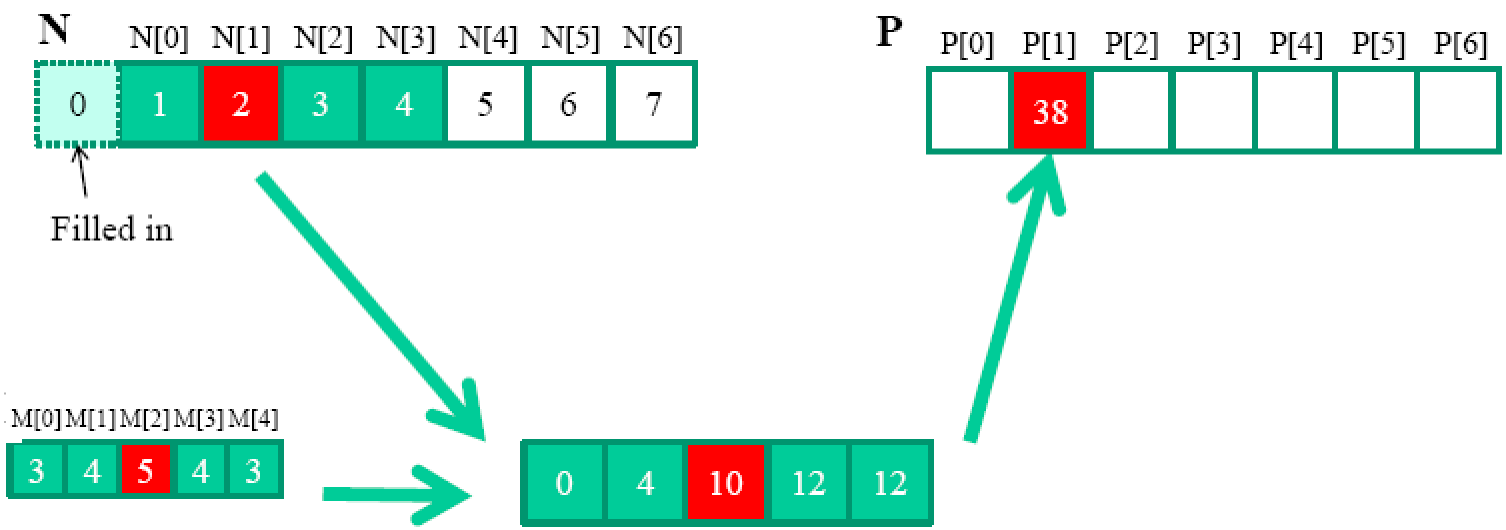
Convolution

- An Array operation
- Output data element = weighted sum of a collection of neighboring input elements.
- The weights are defined by an input mask array.
- Usually used as filters to transform signals (or pixels or …) into more desirable form.

Convolution



Convolution



Convolution can also be 2D.

Convolution

N

1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	5	6
5	6	7	8	5	6	7
6	7	8	9	0	1	2
7	8	9	0	1	2	3

P

		321				

M

1	2	3	2	1
2	3	4	3	2
3	4	5	4	3
2	3	4	3	2
1	2	3	2	1

1	4	9	8	5
4	9	16	15	12
9	16	25	24	21
8	15	24	21	16
5	12	21	16	5

Convolution

```
__global__ void convolution_1D_basic_kernel(float *N, float *M, float *P,
  int Mask_Width, int Width) {

  int i = blockIdx.x*blockDim.x + threadIdx.x;

  float Pvalue = 0;
  int N_start_point = i - (Mask_Width/2);
  for (int j = 0; j < Mask_Width; j++) {
    if (N_start_point + j >= 0 && N_start_point + j < Width) {
      Pvalue += N[N_start_point + j]*M[j];
    }
  }
  P[i] = Pvalue;

}
```

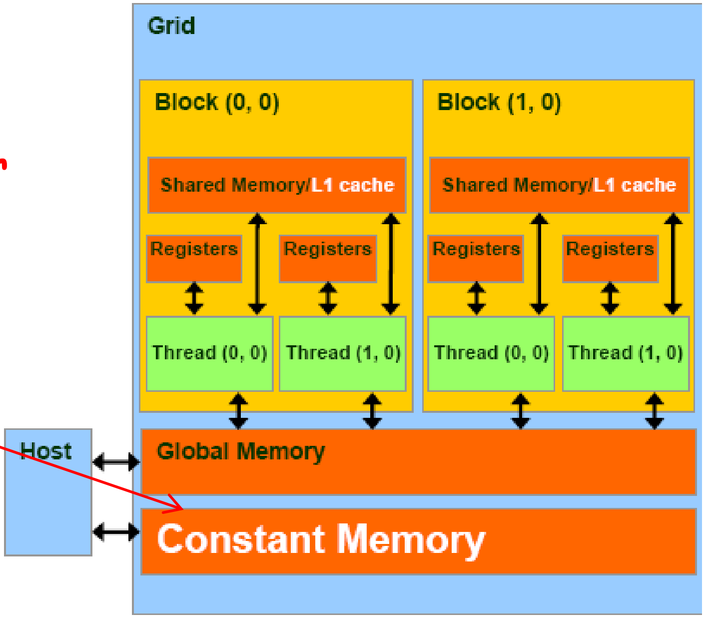
The 1D Version

- Thread organized as 1D grid.
- Pvalue allows intermediate values to be accumulated in registers to save DRAM bw.
- We assume *ghost* values are 0.
- There will be control flow divergence (due to ghost elements).
- Ratio of floating point arithmetic calculation to global memory access is ~ 1.0 → What can we do??

Regarding Mask M

- Size of M is typically small.
- The contents of M do not change during execution.
- All threads need to access M and in the same order.

Doesn't this make M a good candidate for constant memory>



Constant Memory

- Constant memory variables are visible to all thread blocks.
- Constant memory variables cannot be changed during kernel execution.
- The size of constant memory can vary from device to device.

How to Use Constant Memory

- Host code allocates, initializes variables the same way as any other variables that need to be copied to the device
- Use **cudaMemcpyToSymbol(dest, src, size)** to copy the variable into the device memory
- This copy function tells the device that the variable will not be modified by the kernel and can be safely cached.

Mask M and Constant Memory

- In host:
  - *#define MASK_WIDTH 10*
    __constant__ float M[MASK_WIDTH]
  - Allocate and initialize a mask h_M
  - cudaMemcpyToSymbol(M, h_M, MASK_WIDTH * sizeof(float), offset, kind);
- Kernel functions
  - access constant memory variables as global variables → no need to pass pointers of these variables to the kernel as parameter.

Question: Isn't the constant memory also in DRAM? Why is it assumed faster than global memory?

Answer:

- CUDA runtime knows that constant memory variables are not modified.
- It directs the hardware to aggressively cache them during kernel execution.

Reduction Trees

What? And Why?

- Arguably the most widely used parallel computation pattern.
- A commonly used strategy for processing large input data sets
  - There is no required order of processing elements in a data set (associative and commutative)
  - Partition the data set into smaller chunks
  - Have each thread to process a chunk
  - Use a reduction tree to summarize the results from each chunk into the final answer
- Google and Hadoop MapReduce frameworks are examples of this pattern

Reduction enables other techniques

- Reduction is also needed to clean up after some commonly used parallelizing transformations

- Example: Privatization
  - Multiple threads write into an output location
  - Replicate the output location so that each thread has a private output location
  - Use a reduction tree to combine the values of private locations into the original output location

What is a reduction computation

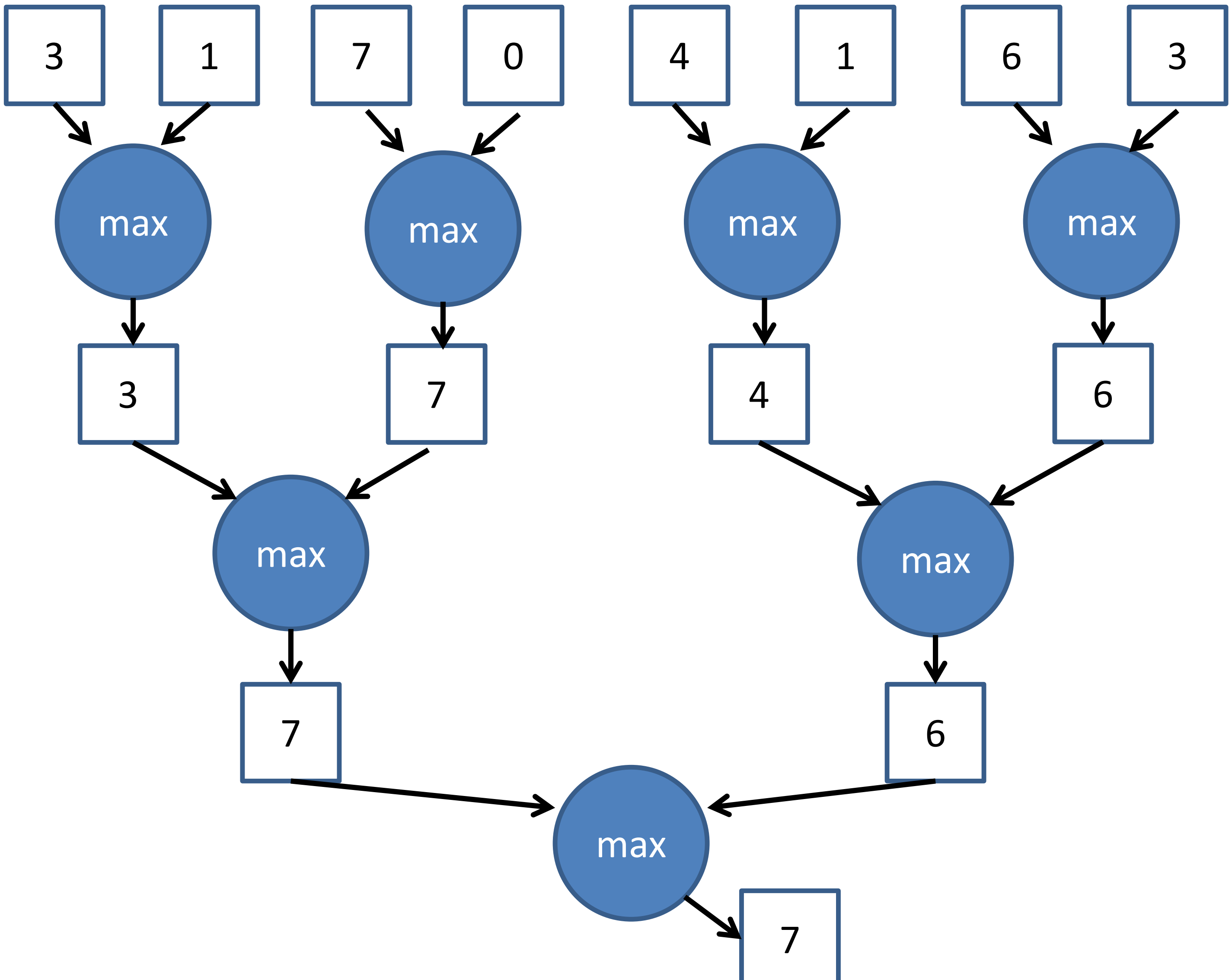
- Summarize a set of input values into one value using a “reduction operation”
  - Max
  - Min
  - Sum
  - Product
  - Often with user defined reduction operation function as long as the operation
    - Is associative and commutative
    - Has a well-defined identity value (e.g., 0 for sum)

An efficient sequential reduction algorithm performs N operations in O(N)

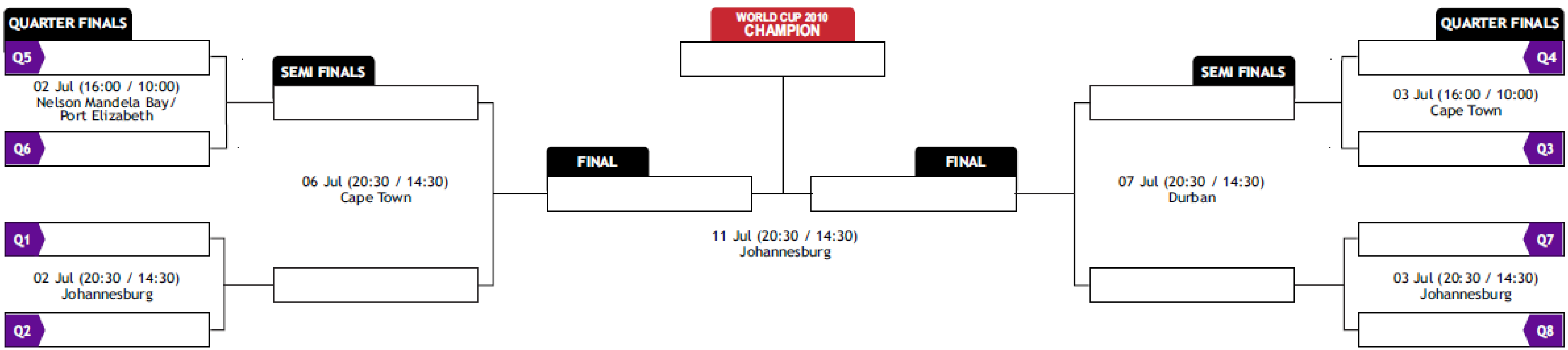
- Initialize the result as an identity value for the reduction operation
  - Smallest possible value for max reduction
  - Largest possible value for min reduction
  - 0 for sum reduction
  - 1 for product reduction

- Scan through the input and perform the reduction operation between the result value and the current input value

A parallel reduction tree algorithm performs N-1 Operations in log(N) steps



A tournament is a reduction tree with “max” operation



A more artful rendition of the reduction tree.

A Quick Analysis

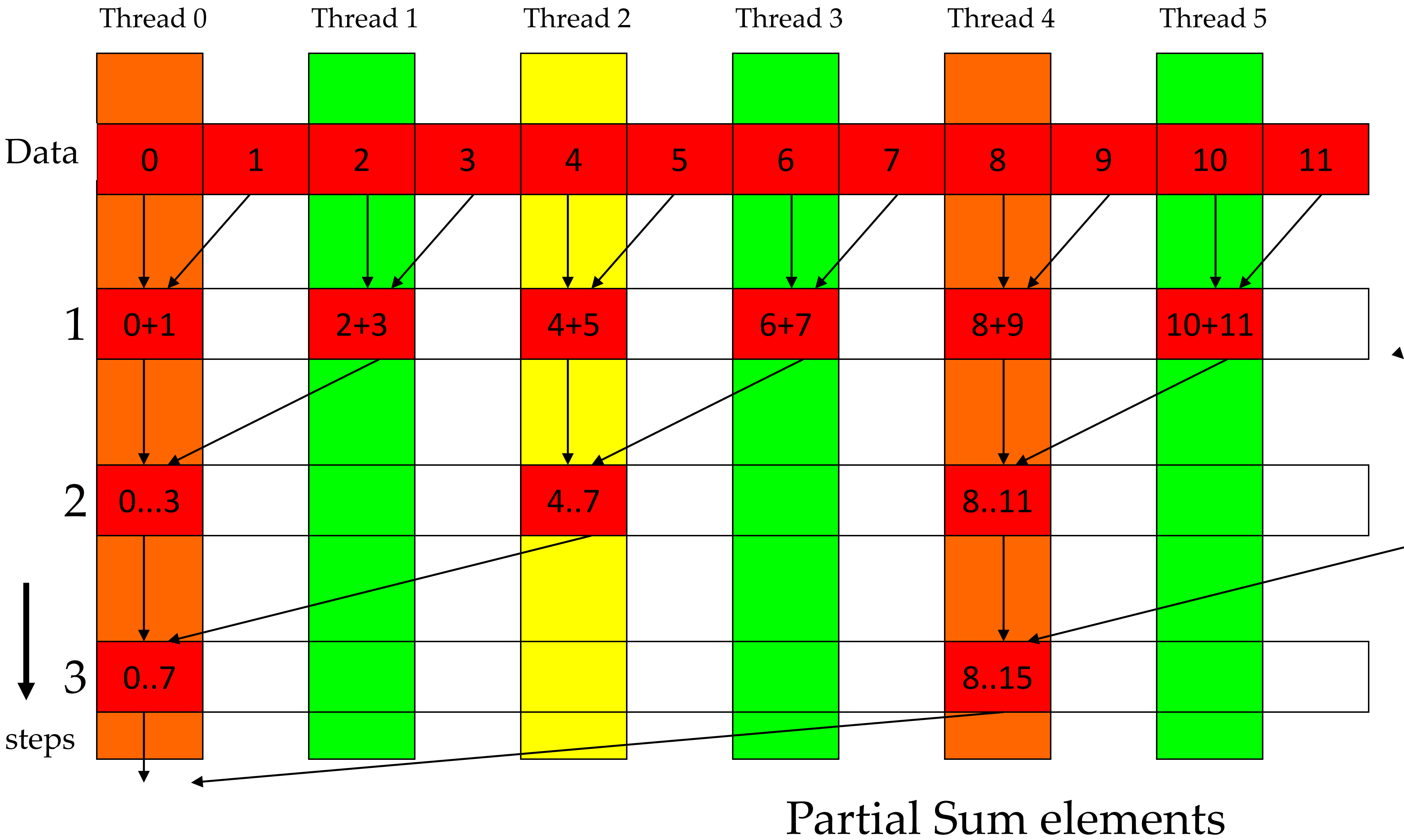
- For N input values, the reduction tree performs
  - $(1/2)N + (1/4)N + (1/8)N + \ldots (1/N) = (1- (1/N))N = N-1$ operations
  - In Log (N) steps – 1,000,000 input values take 20 steps
    - Assuming that we have enough execution resources
  - Average Parallelism (N-1)/Log(N))
    - For N = 1,000,000, average parallelism is 50,000
    - However, peak resource requirement is 500,000!
- This is a work-efficient parallel algorithm
  - The amount of work done is comparable to sequential
  - Many parallel algorithms are not work efficient

A Sum Reduction Example

- Parallel implementation:
  - Recursively halve the # of threads, add two values per thread in each step
  - Takes log(n) steps for n elements, requires n/2 threads

- Assume an in-place reduction using shared memory
  - The original vector is in device global memory
  - The shared memory is used to hold a partial sum vector
  - Each step brings the partial sum vector closer to the sum
  - The final sum will be in element 0
  - Reduces global memory traffic due to partial sum values

Vector Reduction with Branch Divergence



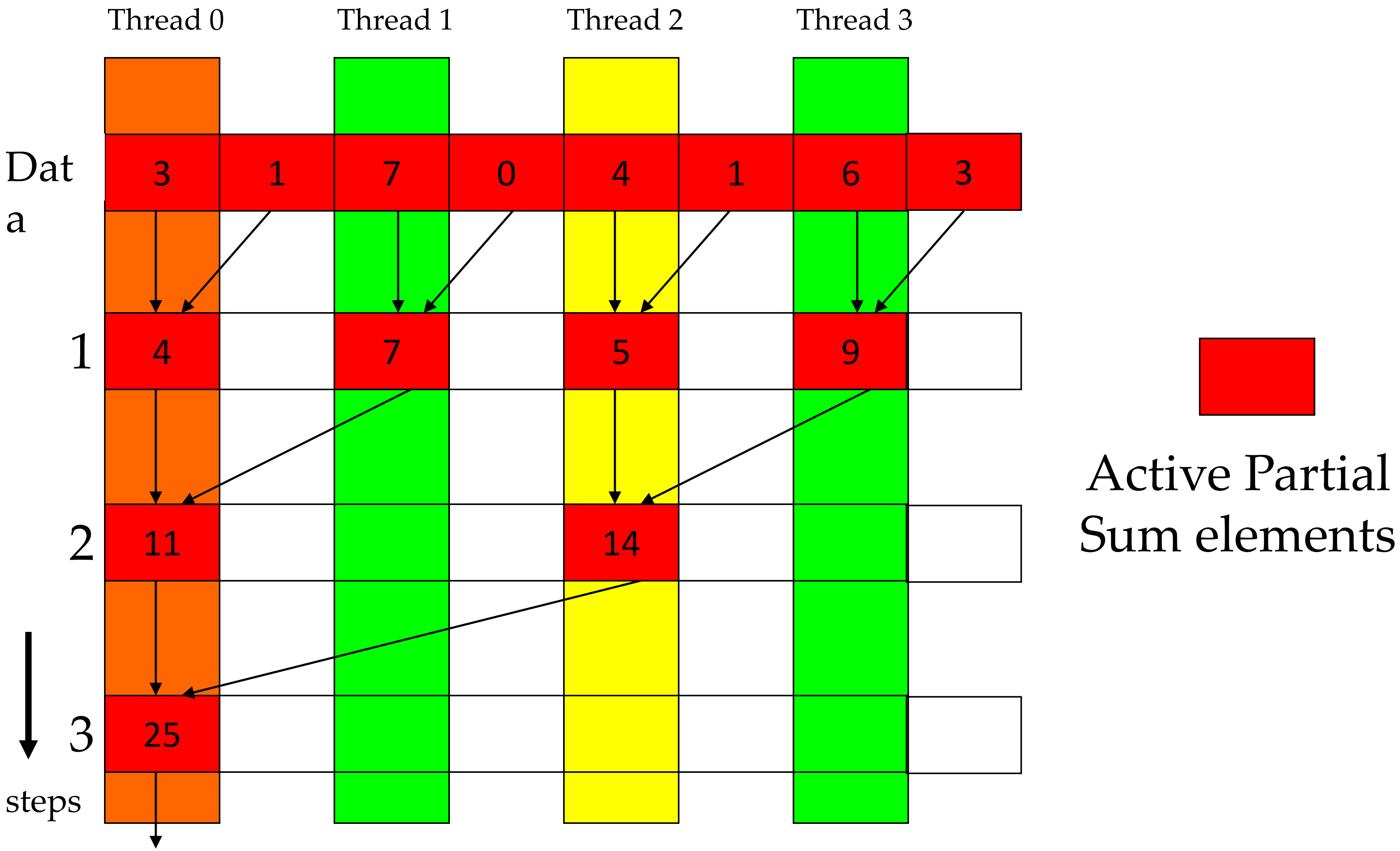
Simple Thread Index to Data Mapping

- Each thread is responsible of an even-index location of the partial sum vector
  - locations: 0, 2, 4, 6, … hold sum of 0+1, 2+3, 4+5, …
- After each step, half of the threads are no longer needed
- In each step, one of the inputs comes from an increasing distance away

Optimizing Reduction Trees

- Performance factors of a reduction kernel
  - Memory coalescing
  - Control divergence
  - Thread utilization

A Sum Example (review)



The Reduction Steps

```
for (unsigned int stride = 1;
    stride <= blockDim.x;  stride *= 2)
{
  __syncthreads();
  if (t % stride == 0)//t is thread ID
    partialSum[2*t]+=
  partialSum[2*t+stride];
}
```

Why do we need syncthreads()?

Barrier Synchronization

- __syncthreads() are needed to ensure that all elements of each version of partial sums have been generated before we proceed to the next step

- Why not another __syncthread() at the end of the reduction loop?

Back to the Global Picture

- At the end of the kernel execution, thread 0 in each block writes the sum of the block (stored in partialSum[0]) into a vector indexed by the value of **blockIdx.x**
- There can be a large number of such sums if the original vector is very large
  - The host code may iterate and launch another kernel
- If there are only a small number of sums, the host can simply transfer the data back and add them together.

Some Observations

- In each iteration, two control flow paths will be sequentially traversed for each warp
  - Threads that perform addition and threads that do not
  - Threads that do not perform addition still consume execution resources
- No more than half of threads will be executing after the first step
  - All odd-index threads are disabled after first step
  - After the 5^{th} step, entire warps in each block will fail the if-condition, poor resource utilization but no divergence.
    - This can go on for a while, up to 5 more steps ($1024/32=16= 2^5$), where each active warp only has one productive thread until all warps in a block retire

Thread Index Usage Matters

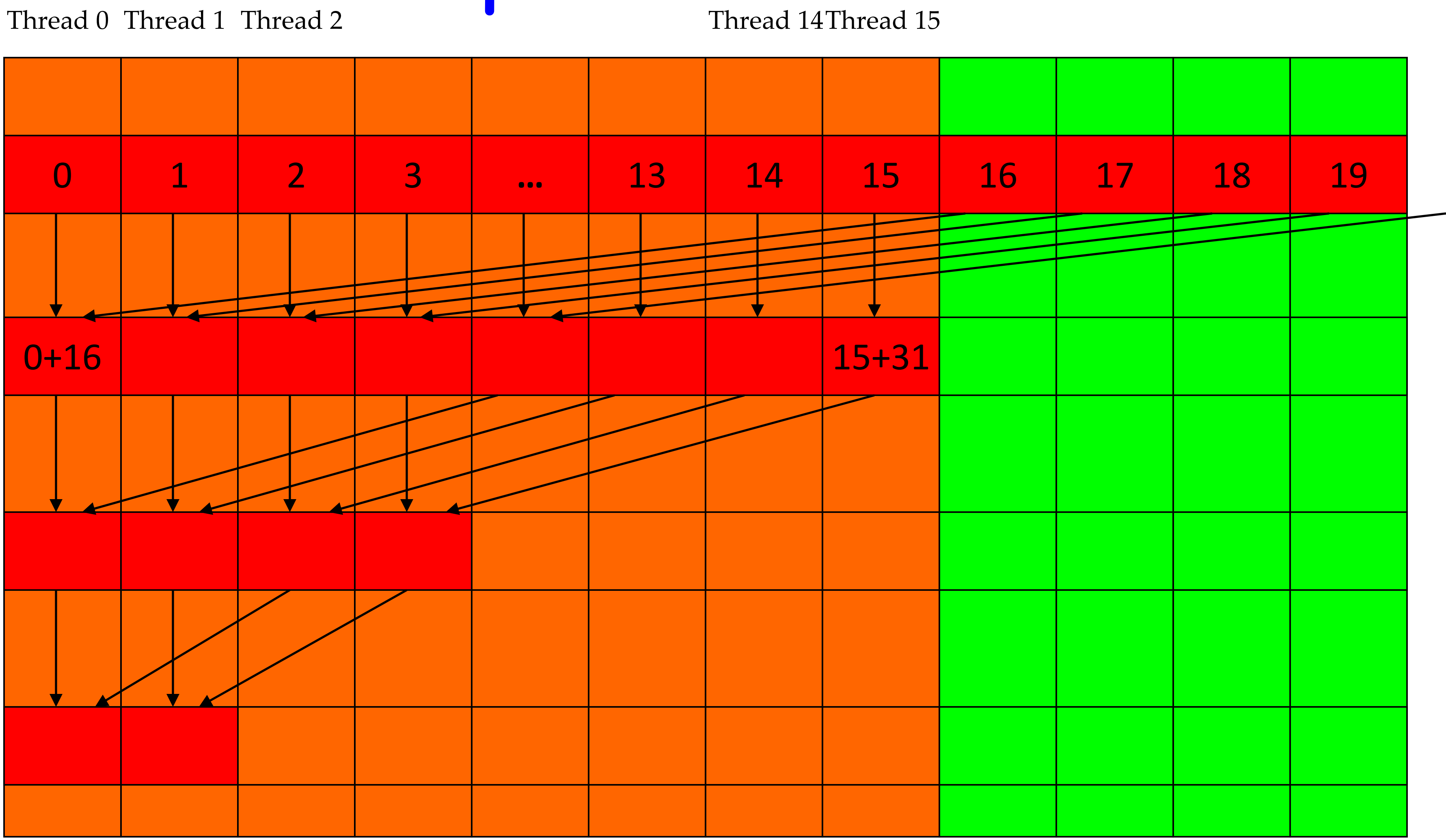
- In some algorithms, one can shift the index usage to improve the divergence behavior
  - Commutative and associative operators
  - At the end, the performance of many CUDA kernels depends on clever indexing.

- Reduction satisfies this criterion.

A Better Strategy

- Always compact the partial sums into the first locations in the partialSum[] array
- Keep the active threads consecutive

An Example of 16 threads



A Better Reduction Kernel

```
for (unsigned int stride =
  blockDim.x;
    stride >= 1;  stride /= 2)
{
  __syncthreads();
  if (t < stride) // t is thread ID
    partialSum[t] +=
  partialSum[t+stride];
}
```

A Quick Analysis

- For a 1024 thread block
  - No divergence in the first 5 steps
  - 1024, 512, 256, 128, 64, 32 consecutive threads are active in each step
  - The final 5 steps will still have divergence

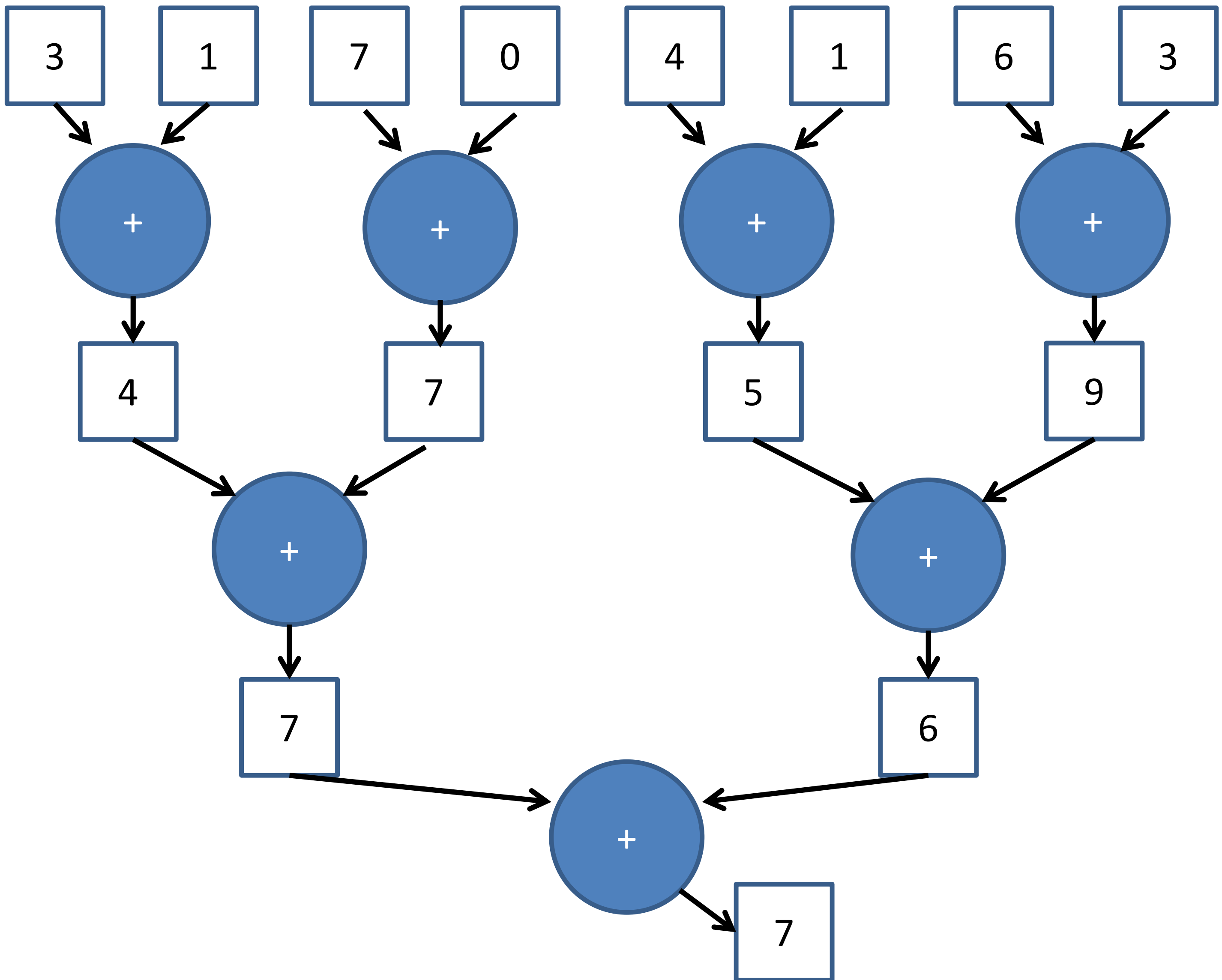
Parallel Algorithm Overhead

```
__shared__ float partialSum[2*BLOCK_SIZE];

unsigned int t = threadIdx.x;
unsigned int start = 2*blockIdx.x*blockDim.x;
partialSum[t] = input[start + t];
partialSum[blockDim+t] = input[start+ blockDim.x+t];
 for (unsigned int stride = blockDim.x;
      stride >= 1;  stride >>= 1)
 {
   __syncthreads();
   if (t < stride)
      partialSum[t] += partialSum[t+stride];
 }
```

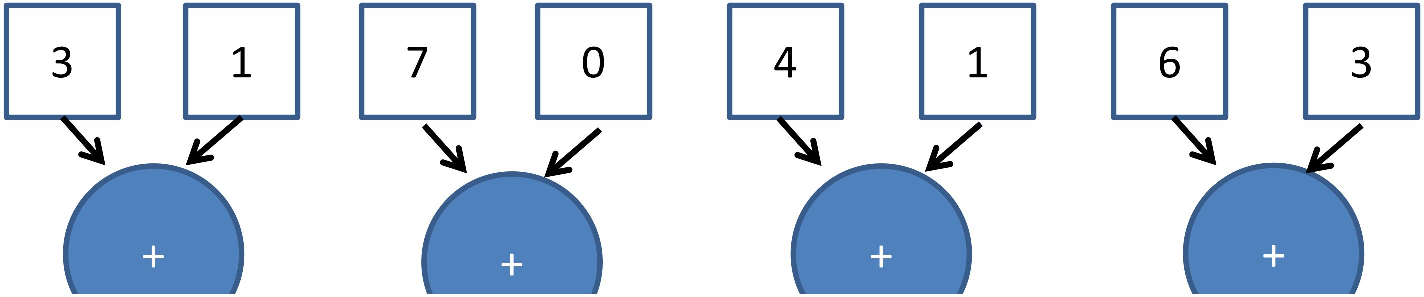
Parallel Algorithm Overhead

```
__shared__ float partialSum[2*BLOCK_SIZE];

unsigned int t = threadIdx.x;
unsigned int start = 2*blockIdx.x*blockDim.x;
partialSum[t] = input[start + t];
partialSum[blockDim+t] = input[start+ blockDim.x+t];
 for (unsigned int stride = blockDim.x;
        stride >= 1;  stride >>= 1)
 {
   __syncthreads();
   if (t < stride)
      partialSum[t] += partialSum[t+stride];
 }
```

Parallel Execution Overhead

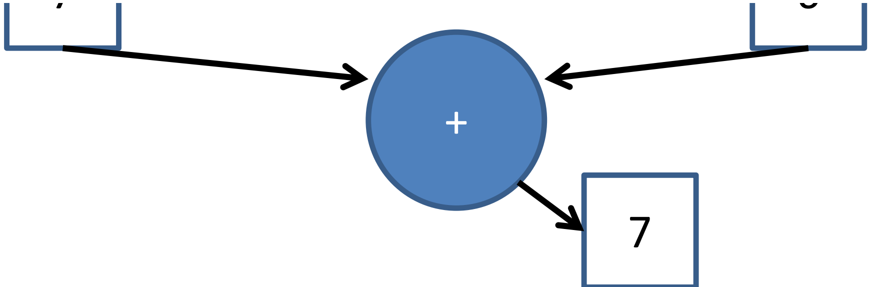


Parallel Execution Overhead



Although the number of “operations” is N, each “operation involves much more complex address calculation and intermediate result manipulation.

If the parallel code is executed on a single-thread hardware, it would be significantly slower than the code based on the original sequential algorithm.



Parallel Scan (Prefix Sum)

What? Why?

- Frequently used for parallel work assignment and resource allocation
- A key primitive in many parallel algorithms to convert serial computation into parallel computation
  - Based on reduction tree and reverse reduction tree

(Inclusive) Scan (Prefix-Sum) Definition

Definition: *The scan operation takes a binary associative operator* $\oplus$*, and an array of n elements*

$$[x_0, x_1, \ldots, x_{n-1}],$$

and returns the prefix-sum array

$$[x_0, (x_0 \oplus x_1), \ldots, (x_0 \oplus x_1 \oplus \ldots \oplus x_{n-1})].$$

Example: If $\oplus$ is addition, then the scan operation on the array [3 1 7 0 4 1 6 3] would return [3 4 11 11 15 16 22 25]

A Inclusive Scan Application Example

- Assume that we have a 100-inch bread to feed 10
- We know how much each person wants in inches
  - [3 5 2 7 28 4 3 0 8 1]
- How do we cut the bread quickly?
- How much will be left?

- Method 1: cut the sections sequentially: 3 inches first, 5 inches second, 2 inches third, etc.

- Method 2: calculate prefix-sum array
  - [3, 8, 10, 17, 45, 49, 52, 52, 60, 61] (39 inches left)
  - You can make 10 cuts in parallel at the above 10 cut points

Typical Applications of Scan

- Scan is a simple and useful parallel building block
  - Convert recurrences from sequential :

```
for(j=1;j<n;j++) out[j] = out[j-1] + f(j);
```

  - into parallel:

```
forall(j) { temp[j] = f(j) };
scan(out, temp);
```

- Useful for many parallel algorithms:

  - radix sort
  - quicksort
  - String comparison
  - Lexical analysis
  - Stream compaction
  - Polynomial evaluation
  - Solving recurrences
  - Tree operations
  - Histograms
  - …

Other Applications

- Assigning camp slots
- Assigning farmer market space
- Allocating memory to parallel threads
- Allocating memory buffer to communication channels
- …

An Inclusive Sequential Scan

Given a sequence $[x_0, x_1, x_2, \dots]$
Calculate output $[y_0, y_1, y_2, \dots]$

Such that
$$y_0 = x_0$$
$$y_1 = x_0 + x_1$$
$$y_2 = x_0 + x_1 + x_2$$
$$\dots$$

Using a recursive definition
$$y_i = y_{i-1} + x_i$$

A Sequential C Implementation

```
y[0] = x[0];
for (i = 1; i < Max_i; i++) y[i] = y [i-1] + x[i];
```

Computationally efficient:

N additions needed for N elements → O(N)

A Naïve Inclusive Parallel Scan

- Assign one thread to calculate each y element
- Have every thread to add up all x elements needed for the y element

$$y_0 = x_0$$
$$y_1 = x_0 + x_1$$
$$y_2 = x_0 + x_1 + x_2$$

"Parallel programming is easy as long as you do not care about performance."

Parallel Inclusive Scan using Reduction Trees

- Calculate each output element as the reduction of all previous elements
  - Some reduction partial sums will be shared among the calculation of output elements
  - Based on design by Peter Kogge and Harold Stone at IBM in the 1970s – Kogge-Stone Trees

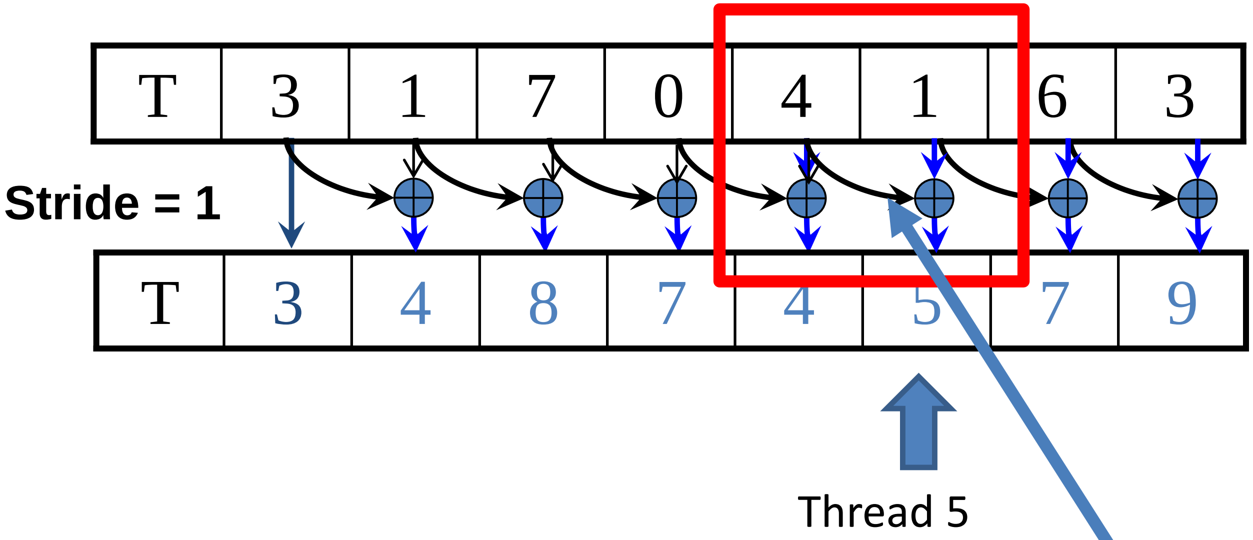
A Slightly Better Parallel Inclusive Scan Algorithm

T	3	1	7	0	4	1	6	3

1. Load input from global memory into shared memory array T

Each thread loads one value from the input (global memory) array into shared memory array T.

A Kogge-Stone Parallel Scan Algorithm

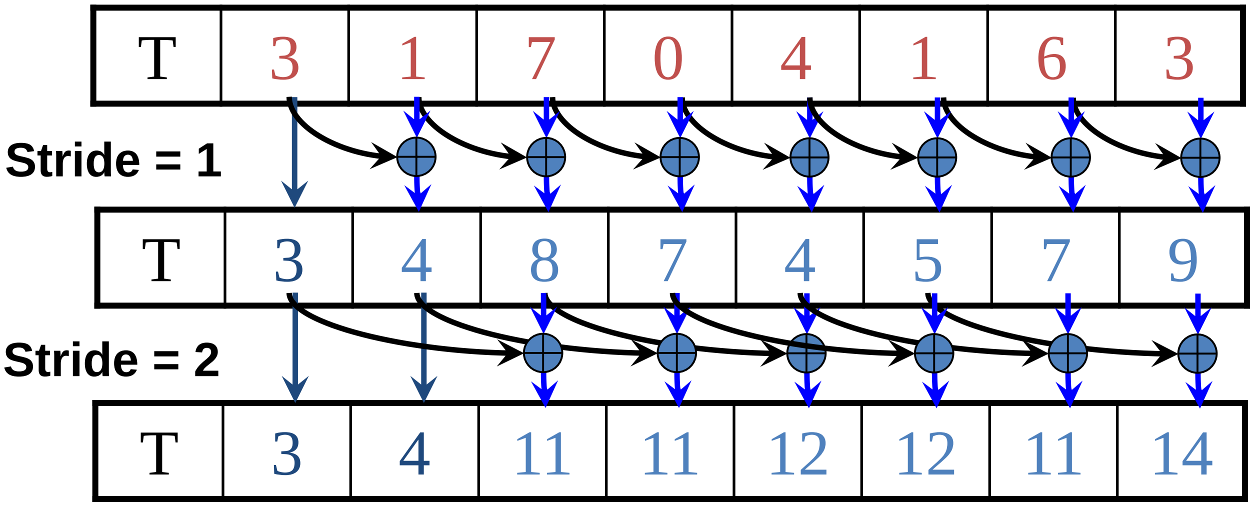


1. (previous slide)
2. Iterate log(n) times, stride from 1 to ceil(n/2.0). Threads from *stride* to *n-1* *are* active: add pairs of elements that are *stride* elements apart.

- Active threads: *stride* to *n*-1 (*n-stride* threads)
- Thread *j* adds elements *j* and *j-stride* from T and writes result into shared memory buffer T
- Each iteration requires two syncthreads
  - syncthreads(); // make sure that input is in place
  - float temp = T[j] + T[j - stride];
  - syncthreads(); // make sure that previous output has been consumed
  - T[j] = temp;

Iteration #1
Stride = 1

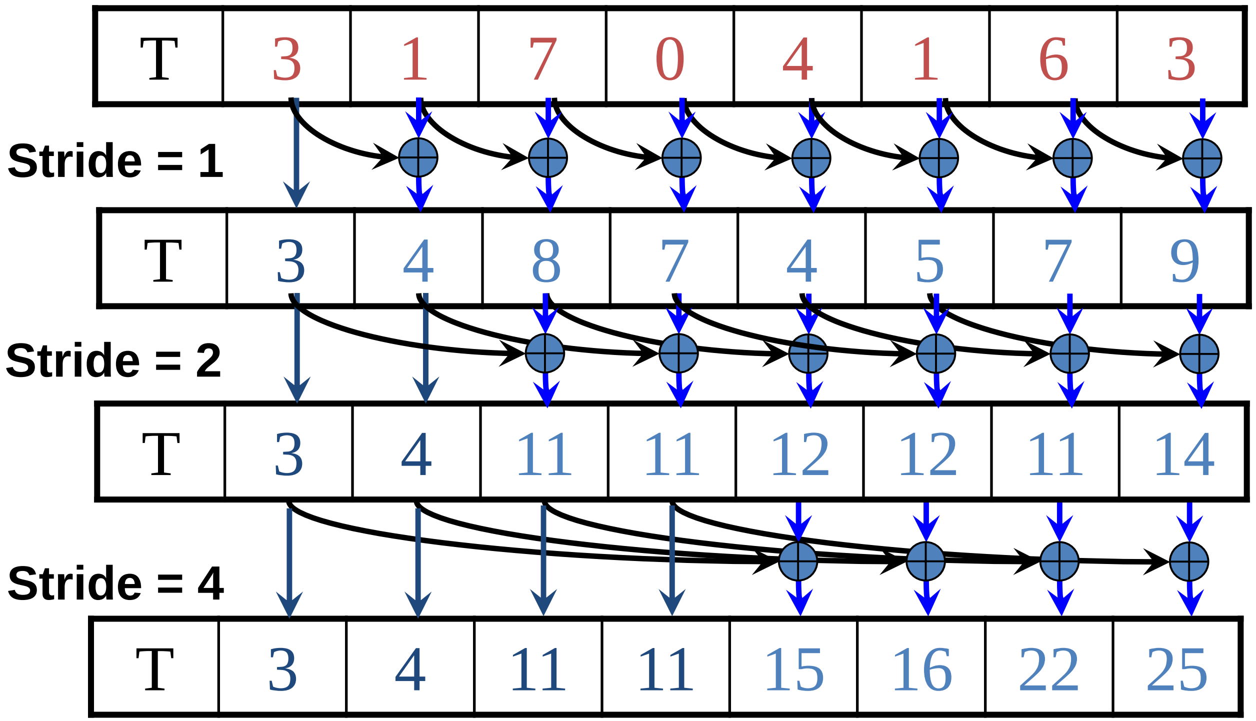
A Kogge-Stone Parallel Scan Algorithm



1. …
2. Iterate log(n) times, stride from 1 to ceil(n/2.0). Threads *stride* to *n-1 active:* add pairs of elements that are *stride* elements apart.

Iteration #2
Stride = 2

A Kogge-Stone Parallel Scan Algorithm



1. Load input from global memory to shared memory.
2. Iterate log(n) times, stride from 1 to ceil(n/2.0). Threads *stride* to *n-1 active:* add pairs of elements that are *stride* elements apart.
3. Write output from shared memory to device memory

Iteration #3
Stride = 4

Enhancement: Double Buffering

- Use two copies of data T0 and T1
- Start by using T0 as input and T1 as output
- Switch input/output roles after each iteration
  - Iteration 0: T0 as input and T1 as output
  - Iteration 1: T1 as input and T0 and output
  - Iteration 2: T0 as input and T1 as output
- This is typically implemented with two pointers, source and destination that swap their contents from one iteration to the next
- This eliminates the need for the second syncthreads

Work Efficiency Analysis

- A Kogge-Stone scan kernel executes log(n) parallel iterations
  - The steps do (n-1), (n-2), (n-4),..(n- n/2) add operations each
  - Total # of add operations: n * log(n) - (n-1) → O(n*log(n)) work
- This scan algorithm is not very work efficient
  - Sequential scan algorithm does *n* adds
  - A factor of log(n) hurts: 20x for 1,000,000 elements!
  - Typically used within each block, where n ≤ 1,024
- A parallel algorithm can be slow when execution resources are saturated due to low work efficiency

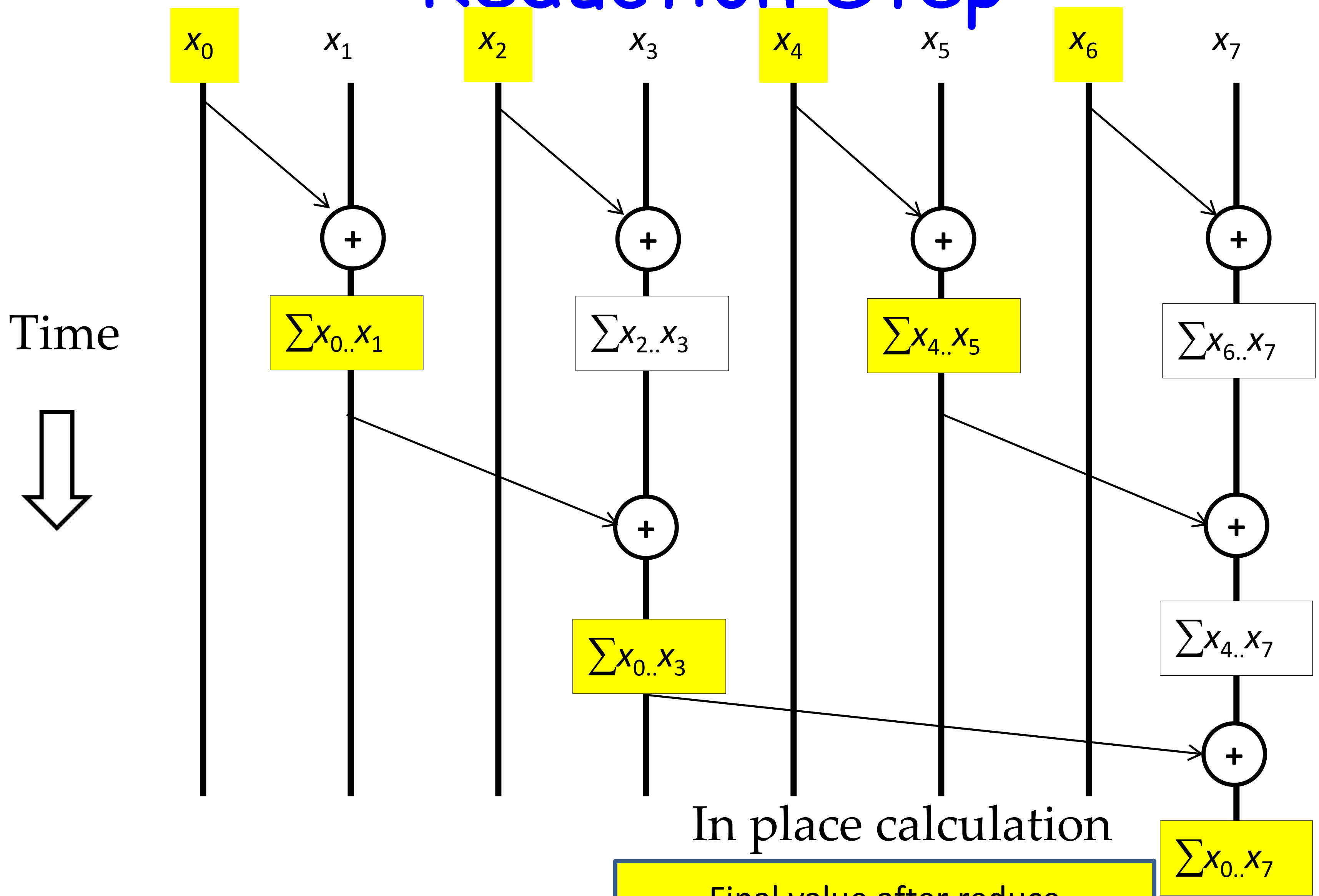
Improving Efficiency

- A common parallel algorithm pattern:

  Balanced Trees
  - Build a balanced binary tree on the input data and sweep it to and from the root
  - Tree is not an actual data structure, but a concept to determine what each thread does at each step

- For scan:
  1. Traverse down from leaves to root building partial sums at internal nodes in the tree
     - Root holds sum of all leaves
  2. Traverse back up the tree building the scan from the partial sums

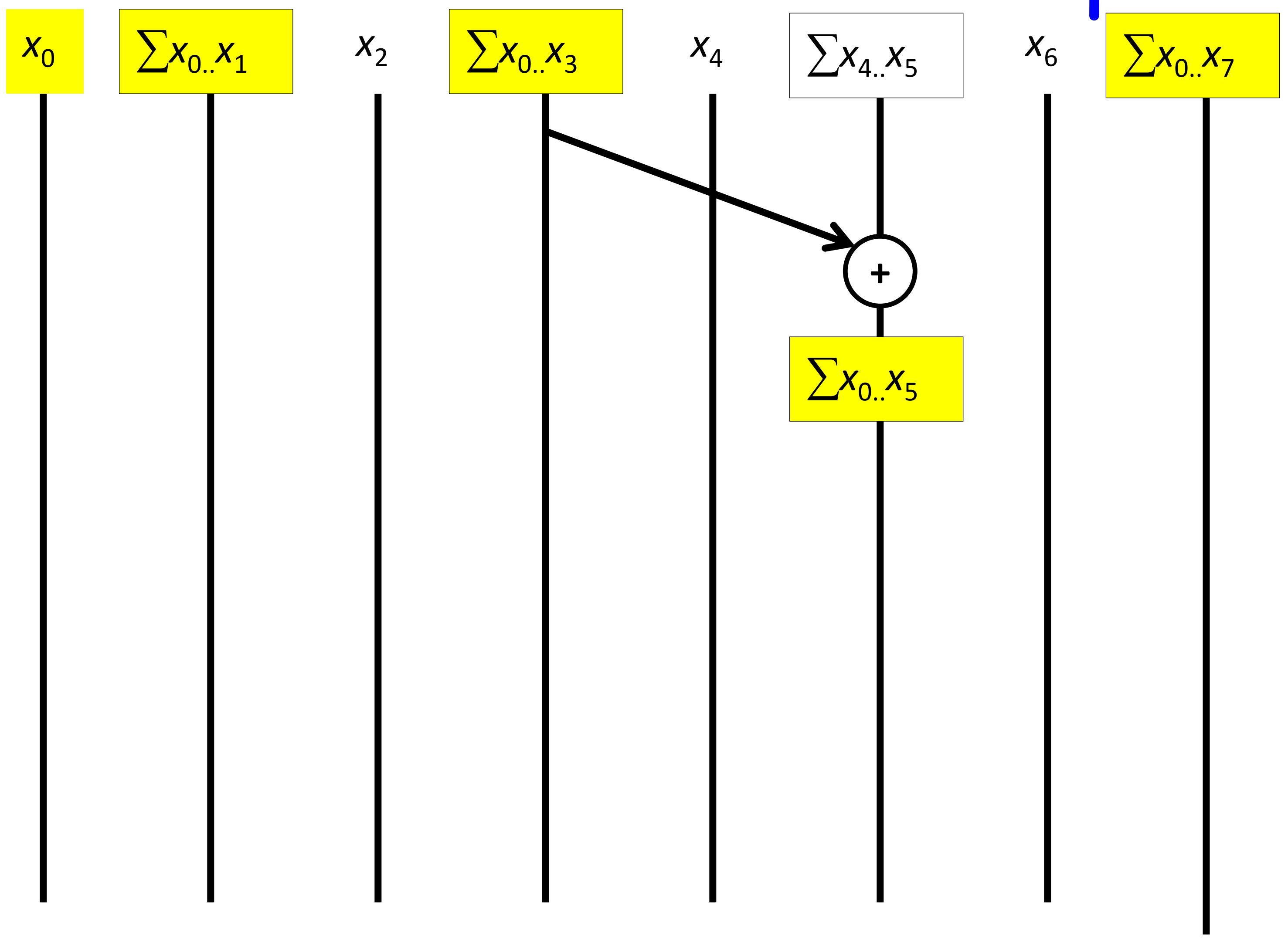
Brent-Kung Parallel Scan - Reduction Step



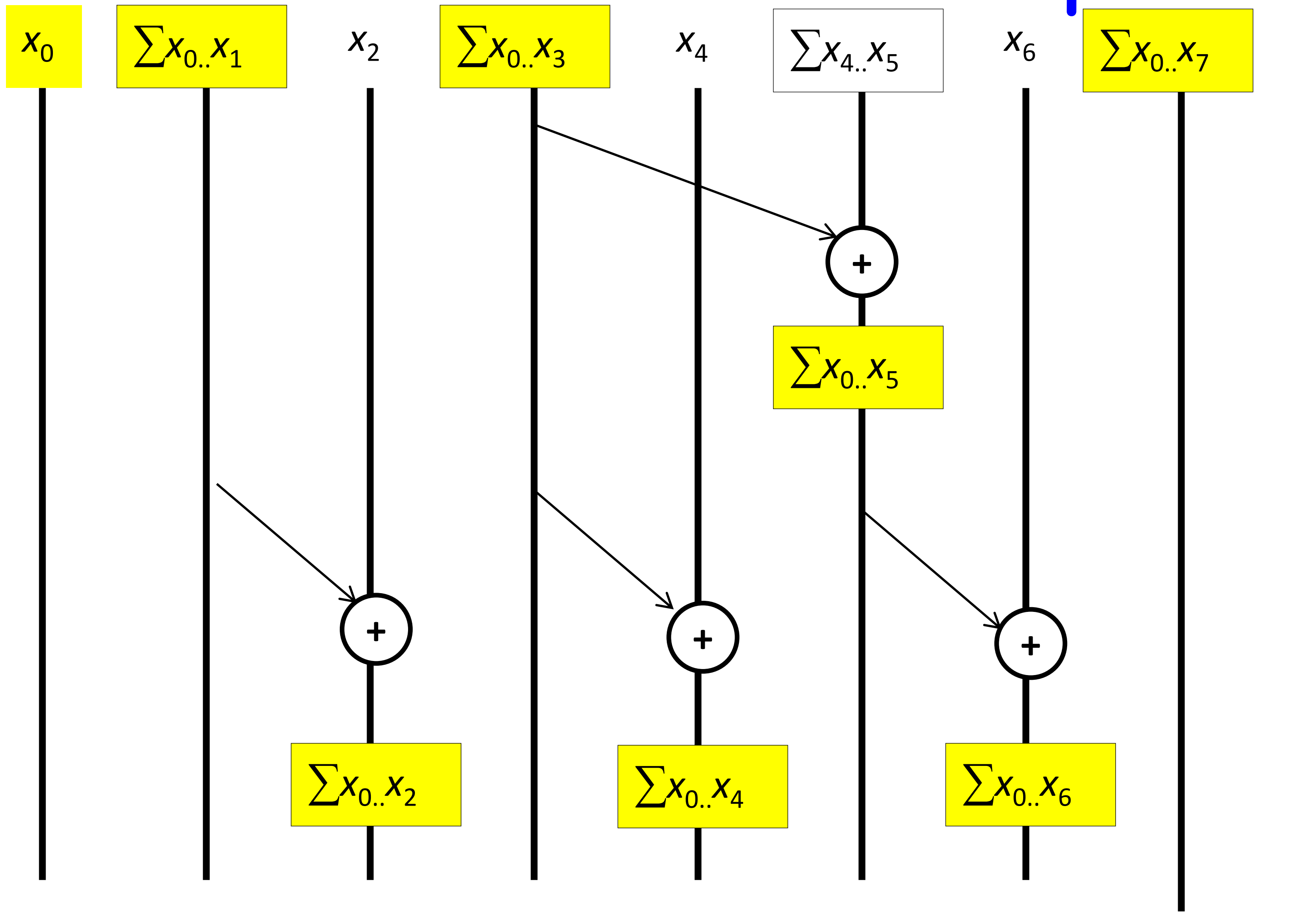
In place calculation

Final value after reduce

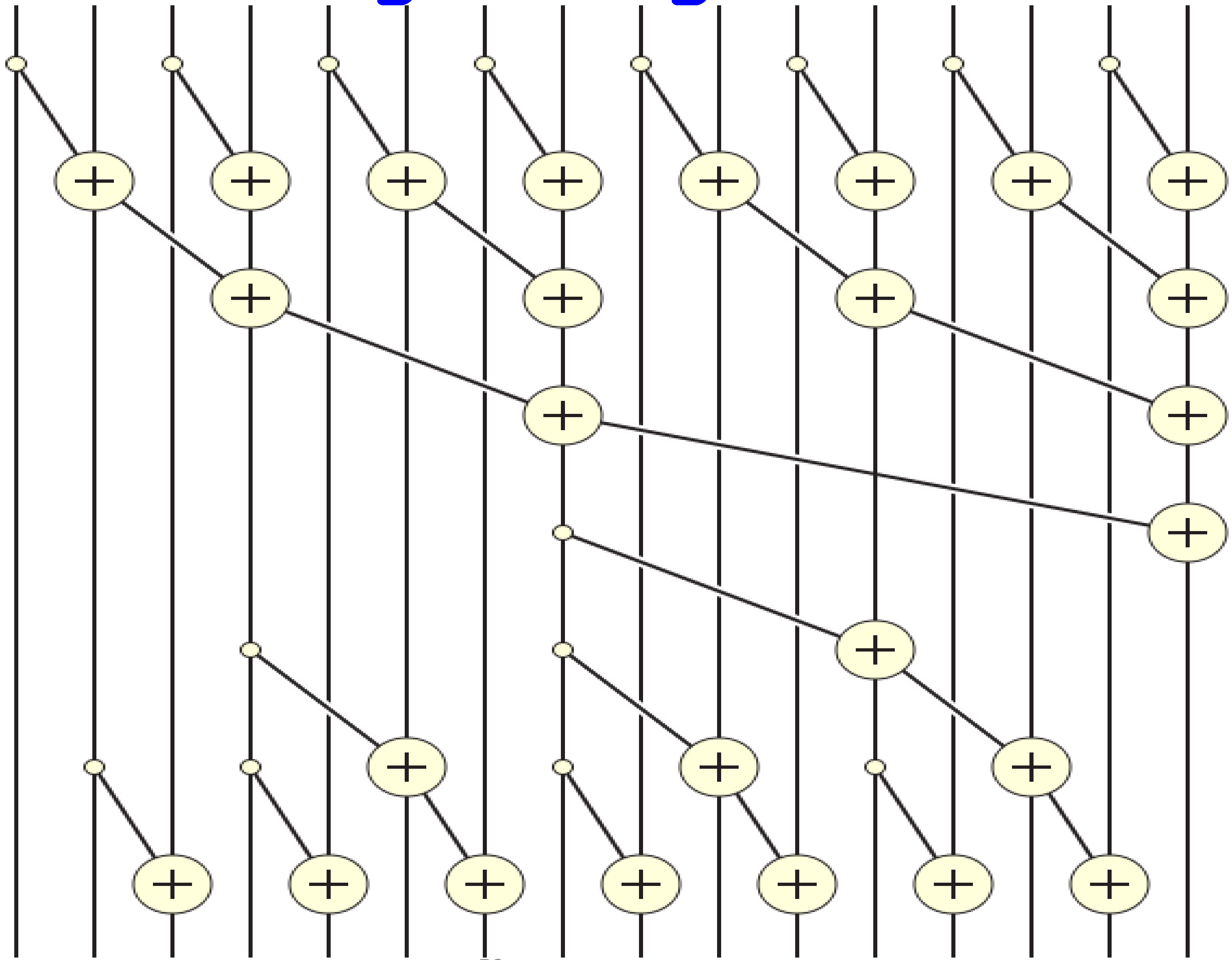
Inclusive Post Scan Step



Inclusive Post Scan Step



Putting it Together



Reduction Step Kernel Code

```
// float T[BLOCK_SIZE] is in shared memory

int stride = 1;
while(stride < BLOCK_SIZE)
{
    int index = (threadIdx.x+1)*stride*2 - 1;
    if(index < BLOCK_SIZE)
        T[index] += T[index-stride];
    stride = stride*2;

    __syncthreads();
  }
```

Post Scan Step

```
int stride = BLOCK_SIZE/2;
while(stride > 0)
  {
      int index = (threadIdx.x+1)*stride*2 - 1;
      if(index < BLOCK_SIZE)
      {
         T[index+stride] += T[index];
      }
      stride = stride / 2;
      __syncthreads();
  }
```

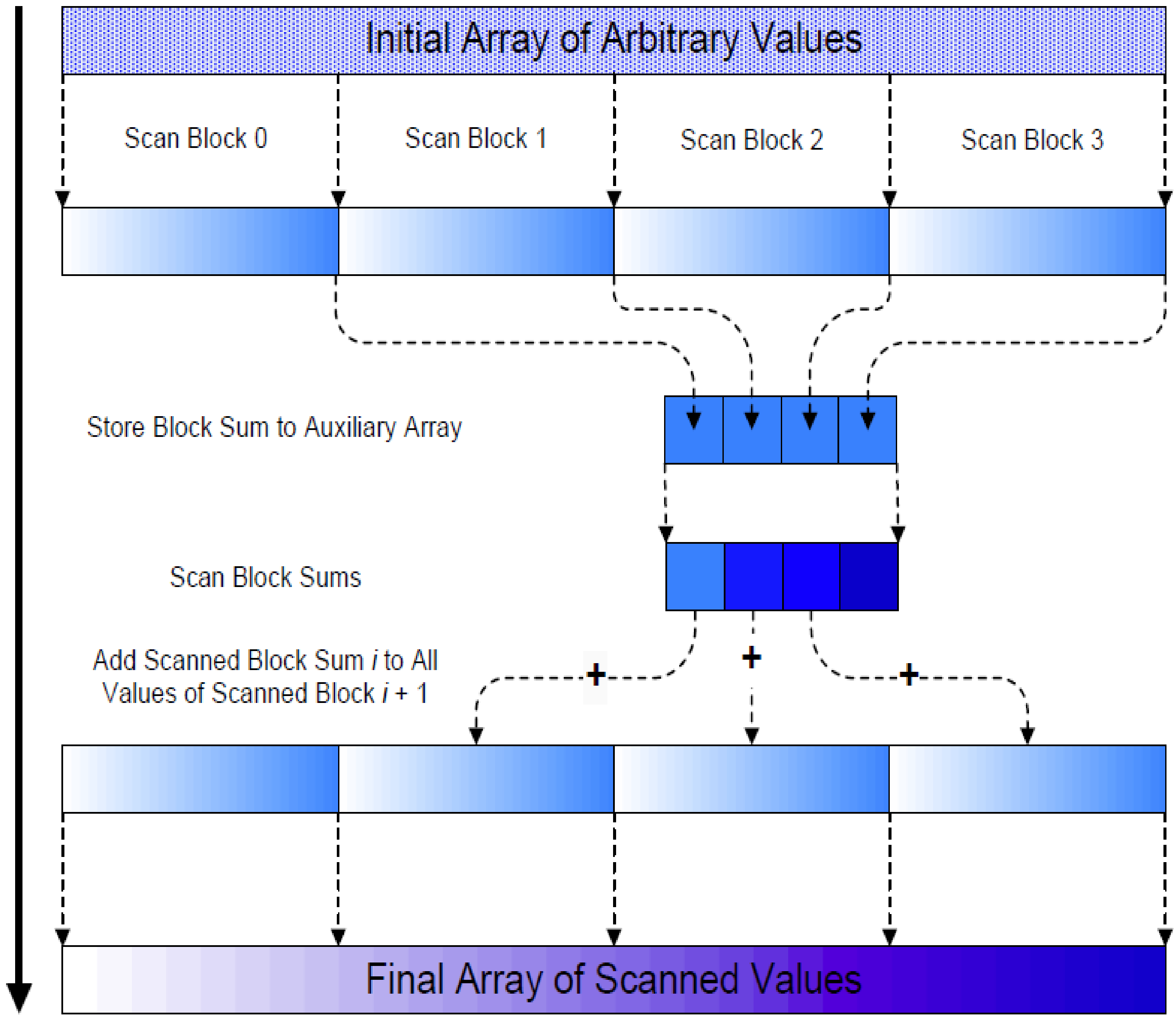
Work Analysis

- The parallel Inclusive Scan executes 2* log(n) parallel iterations
  - log(n) in reduction and log(n) in post scan
  - The iterations do n/2, n/4,..1, 1, …., n/4. n/2 adds
  - Total adds: 2* (n-1) → O(n) work
- The total number of adds is no more than twice of that done in the efficient sequential algorithm
  - The benefit of parallelism can easily overcome the 2X work when there is sufficient hardware

A couple of details

- Brent-Kung uses half the number of threads compared to Kogge-Stone
  - Each thread should load two elements into the shared memory
- Brent-Kung takes twice the number of steps compared to Kogge-Stone
  - Kogge-Stone is more popular for parallel scan with blocks in GPUs

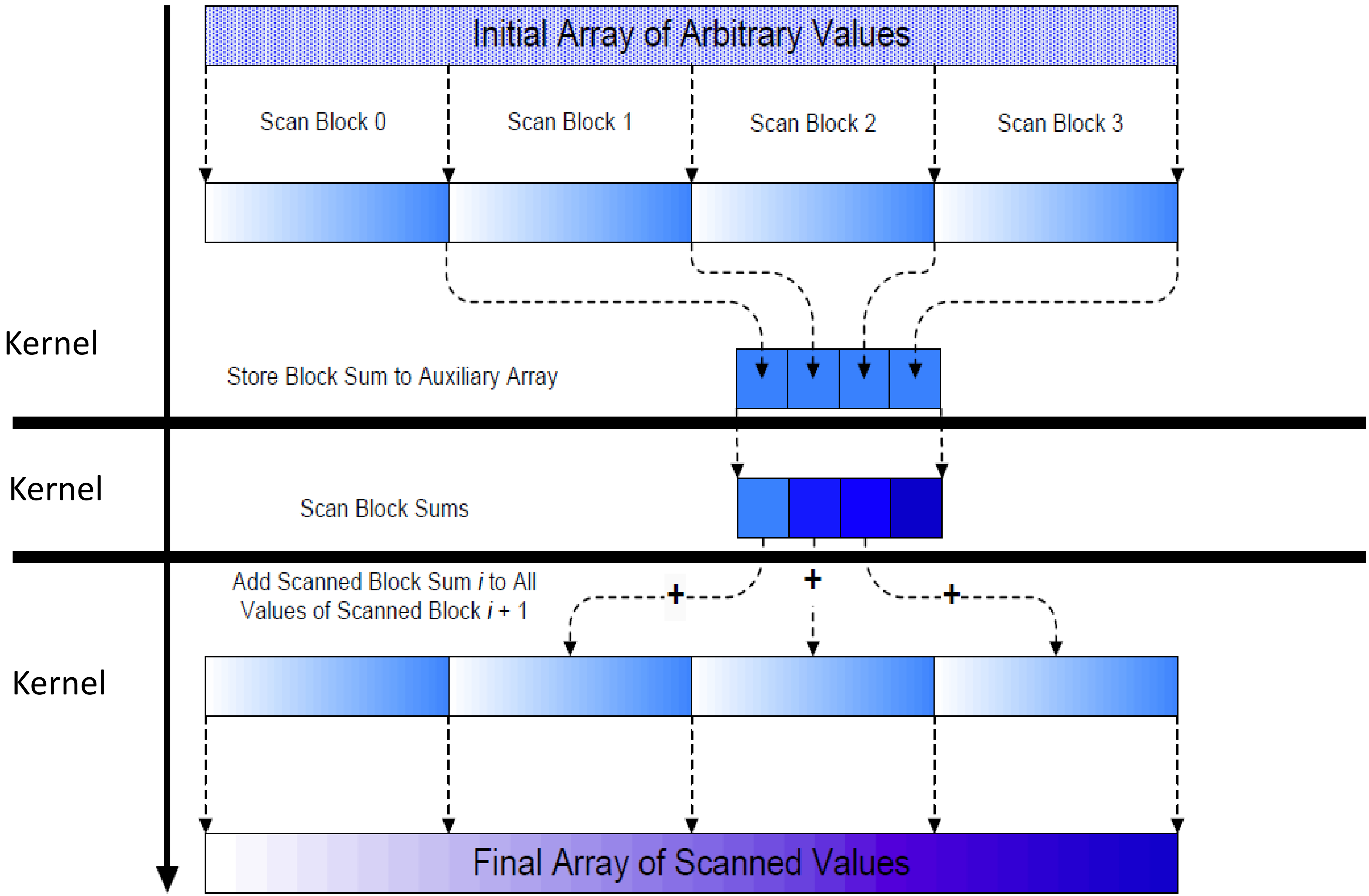
Overall Flow of Complete Scan
A Hierarchical Approach



Using Global Memory Contents in CUDA

- Data in registers and shared memory of one thread block are not visible to other blocks
- To make data visible, the data has to be written into global memory
- However, any data written to the global memory are not visible until a memory fence. This is typically done by terminating the kernel execution
- Launch another kernel to continue the execution. The global memory writes done by the terminated kernels are visible to all thread blocks.

Overall Flow of Complete Scan
A Hierarchical Approach



(Exclusive) Scan Definition

Definition: *The exclusive* scan *operation takes a binary associative operator* $\oplus$*, and an array of n elements*

$$[x_0, x_1, \ldots, x_{n-1}]$$

and returns the array

$$[0, x_0, (x_0 \oplus x_1), \ldots, (x_0 \oplus x_1 \oplus \ldots \oplus x_{n-2})].$$

Example: If $\oplus$ is addition, then the exclusive scan operation on [3 1 7 0 4 1 6 3]
would return [0 3 4 11 11 15 16 22]

Why Exclusive Scan

- To find the beginning address of allocated buffers
- Inclusive and Exclusive scans can be easily derived from each other; it is a matter of convenience

	[3 1 7 0 4 1 6 3]
Exclusive	[0 3 4 11 11 15 16 22]
Inclusive	[3 4 11 11 15 16 22 25]

Conclusions

- We have reviewed several useful parallel patterns that you can use in your own GPU programming:
  - Convolution and tiled convolution
  - Reduction trees
  - Prefix scan (inclusive and exclusive)
- Parallel version must be work efficient
- Then we apply different GPU optimizations from our bag of tricks (coalescing, shared memory usage, …).