# Linear Algebra Problems

**Question 1.** List all the axioms of a vector space. Verify whether the set of all $2 \times 2$ matrices with trace equal to 0 forms a vector space under standard matrix addition and scalar multiplication. Justify your answer carefully. (3 marks)

**Question 2.**Verify whether the set of all polynomials of degree exactly 3 (excluding lower degree polynomials) forms a vector space under standard polynomial addition and scalar multiplication. Justify your answer carefully. (3 marks)

**Question 3.** Determine if the set $S = \{(1, 2, -1), (2, 1, 3), (4, 5, 1)\}$ in $\mathbb{R}^3$ is linearly dependent or independent. Show your work.

**Question 4.** Determine if the set $S = \{(2, -1, 3, 1), (1, 0, 2, -1), (5, -2, 8, 1)\}$ in $\mathbb{R}^4$ is linearly dependent or independent. Show your work.

**Question 5.** Express the vector $\mathbf{v} = (7, 11, 15)$ as a linear combination of the vectors $\mathbf{u}_1 = (1, 2, 3)$, $\mathbf{u}_2 = (2, 3, 4)$, and $\mathbf{u}_3 = (1, 1, 1)$ if possible. If not possible, explain why.

**Question 6.** Consider the vectors $\mathbf{v}_1 = (1, -1, 2)$, $\mathbf{v}_2 = (2, 1, 3)$, and $\mathbf{v}_3 = (3, 0, 5)$ in $\mathbb{R}^3$.

(a) Determine whether these vectors are linearly dependent or independent.

(b) If they are linearly dependent, express one vector as a linear combination of the others.

**Question 7.** Let $S = \{(1, 0, 1, 0), (0, 1, 0, 1), (2, 1, 2, 1), (1, 1, 1, 1)\}$ be a set of vectors in $\mathbb{R}^4$.

(a) Determine the maximum number of linearly independent vectors in $S$.

(b) Find a basis for the span of $S$ by identifying a maximal linearly independent subset.

**Question 8.** Given vectors $\mathbf{v}_1 = (2, -3, 1)$, $\mathbf{v}_2 = (1, 4, -2)$, and $\mathbf{v}_3 = (k, 5, -3)$ in $\mathbb{R}^3$, where $k$ is a scalar.

(a) Find the value(s) of $k$ for which the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly dependent.

(b) Verify your answer by showing the linear dependence relation when $k$ takes the value(s) found in part (a).