# A new model for the soil-water retention curve that solves the problem of residual water contents

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#### **Summary**

We present a new model for the soil-water retention curve,  $\theta(h_{\rm m})$ , which, in contrast to earlier models, anchors the curve at zero water content and does away with the unspecified residual water content. The proposed equation covers the complete retention curve, with the pressure head,  $h_{\rm m}$ , stretching over approximately seven orders of magnitude. We review the concept of pF from its origin in the papers of Schofield and discuss what Schofield meant by the 'free energy, F'. We deal with (historical) criticisms regarding the use of the log scale of the pressure head, which, unfortunately, led to the apparent demise of the pF. We espouse the advantages of using the log scale in a model for which the pF is the independent variable, and we present a method to deal with the problem of the saturated water content on the semi-log graph being located at a pF of minus infinity. Where a smaller range of the water retention is being considered, the model also gives an excellent fit on a linear scale using the pressure head,  $h_{\rm m}$ , itself as the independent variable. We applied the model to pF curves found in the literature for a great variety of soil textures ranging from dune-sand to river-basin clay. We found the equation for the model to be capable of fitting the pF curves with remarkable success over the complete range from saturation to oven dryness. However, because interest generally lies in the plant-available water range (i.e. saturation,  $\theta_s$ , to wilting point,  $\theta_{wp}$ ), the following relation, which can be plotted on a linear scale, is sufficient for most purposes:  $\theta(h_{\rm m}) = \theta_{\rm wp} + k_1 \{ \exp(-k_0/15\,000^n) - \exp(-k_0/h_{\rm m}^n) \}$ , where  $k_0$ ,  $k_1$  and nare adjustable fitting parameters.

# Introduction

During the past four decades many soil physicists have occupied themselves with the search for an equation to describe accurately the soil-water retention curve (e.g. Brooks & Corey, 1964; Visser, 1966; Campbell, 1974; van Genuchten, 1980; Hutson & Cass, 1987; Broadbridge & White, 1988). Such a fitting equation would be very useful to calculate the differential water capacity, which is needed to determine availability of water in soils, by the procedure described in Groenevelt *et al.* (2001). If a fitting equation is effectively anchored at both wet and dry ends, and its fitting parameters provide the correct inflection points, it can be used for valid mathematical interpolation. Most fitting equations now available, however, are anchored only at the saturated water content,  $\theta_s$ . Because they are not also anchored at the dry end, this causes what we refer to here as the 'problem of the residual water content',  $\theta_r$ .

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There is plenty of experimental evidence to show that most water retention curves, in which volumetric water content is plotted as ordinate against pF or negative water potential as abscissa, contain at least one inflection point. A satisfactory fitting equation must therefore contain at least three freely adjustable fitting parameters. As such, the equations of Brooks & Corey (1964) and of Campbell (1974) are unsatisfactory. In its original form, the equation of van Genuchten (1980) contained three such parameters,  $\alpha$ , N and M:

$$\theta(h_{\rm m}) = \theta_{\rm r} + (\theta_{\rm s} - \theta_{\rm r})(1 + \alpha h_{\rm m}^{\ N})^{-M},\tag{1}$$

where  $h_{\rm m}$  is the absolute value of the water potential (matric head, cm), and where the values of the saturated water content,  $\theta_{\rm s}$ , and the residual water content,  $\theta_{\rm r}$ , are assumed to be known. In most practical applications of Equation (1), however, it is assumed that  $M=1-N^{-1}$ , so much so that M is often no longer even mentioned (e.g. Minasny & McBratney, 2002). Because this assumption decreases the number of freely adjustable parameters by one, the residual water content,  $\theta_{\rm r}$ , is then given the role of such a parameter (e.g. Groenevelt *et al.*,

2001). In this situation, unbiased curve fitting can produce unrealistically large positive (or even negative) values for  $\theta_{\rm r}$ . For instance, unbiased curve fitting of the water-release curve for a Ragdale soil in the United Kingdom produced an excellent fitting result, but the value for the residual water content was negative  $30\,{\rm m}^3$  water  ${\rm m}^{-3}$  bulk (Groenevelt & Grant, 2001), which is ludicrous. Similarly, in the equation of Broadbridge & White (1988), their three coefficients must all be used as freely adjustable parameters to obtain the required inflection. In doing so, however, their residual water content also loses physical meaning. We contend here that this problem of residual water contents is unsatisfactory and needs to be resolved.

## Theory and methods

In contrast to all earlier attempts to fit the water-release curve, where a fit is sought for the relation between the matric potential and the water content on linear scales, we intend here to fit the complete curve from saturation to oven dry on a semi-logarithmic scale, which allows a better fit to the data, particularly at the extreme ends of the range. Our reasoning and the historical basis for this approach are outlined below.

### Anchoring the fitting equation at the dry end

By definition, the water content of a soil sample is agreed to be zero after it has been placed in an oven at 105°C and brought to equilibrium over some period, usually exceeding 24 hours. This, of course, does not mean that all water is removed from the soil under these conditions – it is only a practical convention to avoid loss of volatile components other than water so that the mass of the solid phase remains constant. Because water contents are usually calculated on this basis, we anchor our fitting equation at zero water content. This water content must be linked to a matric potential, and so the independent variable in any comprehensive model must span approximately seven orders of magnitude. Schofield (1935) dealt with this challenge by introducing the pF scale, which he defined as

$$pF \equiv \log_{10} h_{\rm m},\tag{2}$$

where  $h_{\rm m}\equiv$  the matric head expressed in cm, positive (i.e. the negative of the matric potential expressed in cm). Schofield (1935) explained: 'The pF is the logarithm of Buckingham's potential. By analogy with Sørensen's acidity scale the symbol "p" indicates its logarithmic character, while the symbol "F" is intended to remind us that by defining pF as the logarithm of the height in centimetres of the water column needed to give the suction in question, we are really using the logarithm of a free energy difference measured on a gravity scale. By basing our scale on free energy rather than pressure, we are not troubled in our calculations by the influence of pressure on the density of water'. Please note that Schofield did not specify the form of free energy, but from his reference to Bucking-

ham's potential one must conclude that he meant the pressure component of the Gibbs free energy.

We note that doubt has been cast in the literature over the concept of the pF (e.g. Rose, 1966, p. 143; Towner, 1989, p. 253), particularly in relation to the meaning of 'free energy' (and its measurement!), so we must be clear about the terms in which we define it. There is no ambiguity in the word 'potential' – it means 'free energy per unit mass'. Thus, if it is clear which potential (or component-potential) one wishes to deal with then it is automatically clear which form of 'free energy' one is dealing with. Obviously, following Schofield, we want to deal with the matric potential.

For the wetter part of the water content—potential relation, one measures the potential of the *soil solution*, which one accomplishes using a porous ceramic plate, sintered-glass funnel, or porous membrane, all of which separate the matrix from the equilibrium soil solution. Any particle that cannot pass through the plate or membrane is considered to be part of the soil matrix, and thus contributes to the matric potential. This, incidentally, applies to most techniques used to measure soilwater potential in the range available to plants. For these techniques, any particle that can pass freely through the membrane equilibrates independently of the matrix configuration and so does not contribute to the matric potential; it contributes only to the osmotic potential of the equilibrium soil solution.

For the drier part of the water content–potential relation, however, one measures the potential of the *water* ( $H_2O$ ) by equilibrating it with water vapour at a potential controlled by that of a saturated salt solution held at constant temperature. Here, the liquid–gas interface becomes the selecting mechanism by which only water molecules move freely. In this case, one measures the 'osmo-matric' potential of the water,  $h_{om}$ :

$$h_{\rm om} = h_{\rm o} + h_{\rm m},\tag{3}$$

where  $h_0$  is the osmotic head (the osmotic pressure, expressed in cm, and taken to be positive). To make the scale for the drier part consistent with that for the wetter part, as well as for the definition of pF shown in Equation (2), one must subtract  $h_0$  from the imposed  $h_{\rm om}$ .

The water content of zero is linked to a potential for the water that remains in the soil after it equilibrates at 105°C, which can be calculated from Schofield (1935):

$$pF = 6.6 + \log\{2 - \log(H_r)\},\tag{4}$$

where  $H_{\rm r}$  is the relative humidity in the soil atmosphere (expressed as a per cent). Note that Schofield used 6.5 instead of 6.6 in Equation (4). One can derive the fundamental equation that Schofield used by integrating the differential Gibbs equation in thermodynamics, after substituting the ideal gas law in the pressure term. The integration has to be carried out at constant temperature from the standard state to the equilibrium state in the oven. We therefore believe (as pointed out by one referee of this script) that  $T=378.16\,\rm K$  instead of 293.16 K, used by Schofield. Following the assumption of

Schofield, we propose to define zero water content as the water content present while the relative humidity in the soil pores is 1%. This can be achieved by setting the laboratory at say 20°C and 52% relative humidity, among many possible combinations. The pF of the water remaining in the soil pores can then be calculated from Equation (4) to be equal to 6.9.

For our standardized anchoring point, with the relative humidity,  $H_r$ , equal to 1% and pF = 6.913, the oven-imposed absolute value of  $h_{\rm om}$  is found to be  $h_{\rm om}(\theta=0)=8\,184\,648\,{\rm cm}$ . To evaluate the contribution of  $h_0$  to  $h_{om}$ , one could consider a situation where the original soil solution contains NaCl that gradually precipitates and generates a saturated salt solution during drying. In this situation, the maximum osmotic potential that could be generated is approximately  $|h_0| = 380\,000$  cm. Subtracting this from  $h_{om}$  would fix the anchor point for the dry end at pF( $\theta = 0$ ) = log(8 184 648 – 380 000) = 6.892. By rounding, the pF for the standardized oven-dry state would remain at 6.9. Thus the value of  $h_0$  in dry soils has only little effect on pF. In moister soils, at values of pF between (say) 4 and 6, removal of the osmotic contribution would be more significant and would cause the pF curve to bend slightly more steeply towards the pF axis. In the following work, we ignore any effects of  $h_0$  on the pF curves that we present because the samples were all taken from the Netherlands, where the soil has been repeatedly drained and leached of soluble salts in the wet climate. We consider any osmotic effects in this instance to be very small.

# General matching of pF curves

By adopting the point ( $\theta = 0$ , pF = 6.9) as the base from which all water contents are calculated, we can anchor all waterrelease curves at the dry end. By analogy with the constitutive shrinkage equation of Groenevelt & Bolt (1972) for unloaded soils (see Groenevelt & Grant, 2001), we propose the following fitting equation, which is well anchored at the dry end:

$$\theta(pF) = k_1 \left\{ \exp\left(\frac{-k_0}{6.9^n}\right) - \exp\left(\frac{-k_0}{pF^n}\right) \right\}. \tag{5}$$

Because of the logarithmic scale used in Equation (5), the saturated water content (i.e. zero matric head) is located at  $pF = -\infty$ . Equation (5) dictates that the fitted curve meets the ordinate perpendicularly, which for practical purposes means that the volumetric water content at a suction of 1 cm (i.e. pF = 0) must be equal to the saturated water content. This implies there can be no pores in the soil with a diameter > 3 mm. If there are larger pores and this anchoring at the wet end is not satisfactory, the following procedure of shifting the pF scale can be used until satisfaction is obtained.

## Anchoring the fitting equation at the wet end

If the value of the saturated water content,  $\theta_s$ , is not known (as in Koorevaar et al., 1983, p. 82), and if the measured pF curve clearly rises from the ordinate at an angle < 90 degrees then the pF can be re-defined as the logarithm of the water potential measured as height in mm of water instead of cm. This would make the new independent variable,  $\Delta pF$ , equal to pF + 1. The measured water contents would then be a function of  $\Delta pF$ , and the fitting equation would read

$$\theta(\Delta pF) = k_1 \left\{ \exp\left(\frac{-k_0}{(6.9+1)^n}\right) - \exp\left(\frac{-k_0}{(\Delta pF)^n}\right) \right\}.$$
 (6)

This, of course, requires that the newly fitted equation also meets perpendicularly at the original line, pF = -1. In practical terms, it allows the water content at 1 mm suction to be equal to the saturated water content – a more satisfactory situation accommodating soil pores up to a maximum of 3 cm diameter. If on rare occasion this still proves unsatisfactory, then the whole set of data can be shifted again and again by say  $\Delta$ units, such that the new independent variable,  $\Delta pF$ , would then be equal to  $pF + \Delta$ . The fitting equation would then read

$$\theta(\Delta pF) = k_1 \left\{ \exp\left(\frac{-k_0}{(6.9 + \Delta)^n}\right) - \exp\left(\frac{-k_0}{(\Delta pF)^n}\right) \right\}.$$
 (7)

If the value of the saturated water content,  $\theta_s$ , is known then the whole set of data should be shifted by  $\Delta$  units (not necessarily an integer) such that, on the original pF =  $-\Delta$  line, the fitted curve has the value  $\theta = \theta_s$ .

## Optimization and stability of curve fitting

To obtain the optimum value for the three fitting parameters,  $k_0$ ,  $k_1$  and n, resulting in the least sum of squared errors, one needs an optimization subroutine, such as in a MathCad solveblock (MathSoft, 1998). Such a routine asks for rough estimates to initialize the iteration. After the process is completed, the results are fed back into the routine as estimates, which should not change the results if the routine is stable. Occasionally, such stability is difficult to obtain, and the following stratagem might help to obtain stability with less effort. By anchoring the wet end of the curve at a water content,  $\theta_0$ , corresponding to pF = 0, one can eliminate  $k_1$  from Equation (5) and this decreases the number of adjustable parameters by one. From Equation (5) for pF = 0, one finds

$$k_1 = \theta_0 \exp\left(\frac{k_0}{6.9^n}\right). \tag{8}$$

Substitution of Equation (8) into Equation (5) gives

$$\theta(pF) = \theta_0 - \theta_0 \exp\{k_0(6.9^{-n} - pF^{-n})\}. \tag{9}$$

Our experience indicates stability for the optimization of Equation (9) is found more readily than for Equation (5).

Examples from Koorevaar et al. (1983)

Koorevaar et al. (1983, p. 82) presented schematic pF curves for nine Dutch soils ranging in texture from a dune-sand to a river-basin clay, and ranging in composition from calcareous to organic. At a water content of zero, all nine pF curves end at pF=7 instead of at pF=6.9, which we suggest above. To match their curves, we therefore temporarily adopt their limit at the dry end (pF=7,  $\theta$ =0). All their curves were based upon detailed measurements, but their lines are schematic in nature and show none of the individual data points. There are also no data shown for the saturated water contents – the wettest points occur at pF=0, corresponding to a suction of 1 cm. If, at pF=0, the curve is perpendicular to the ordinate, then one is probably safe to assume  $\theta_s$  is equal to  $\theta$ (pF=0). If, however, the curve leaves the ordinate at an acute angle then  $\theta_s$  is unknown.

For all nine curves of Koorevaar *et al.* (1983) we extracted values of  $\theta$  where the curves crossed pF=0, 1, 2, 3, 4, 5 and 6 (Appendix, Table A1). We used these points in a curve-fitting exercise to demonstrate both the flexibility of the proposed procedure as well as the technique of axis-shifting. All values for the curve-fitting parameters are listed in the Appendix, Table A2.

#### Results

In Figure 1 we present the results for soils 1 (dune-sand) and 2 (loamy sand) fitted to Equation (5). No axis-shifting was applied. This implies the  $\theta$  values, 0.417 and 0.457, at pF = 0 for each soil, represent the saturated water contents.

In Figure 2 we present the results for soils 4 (calcareous loam) and 6 (young oligotrophic peat). Because the water contents decreased rapidly between pF 1 and 2 (i.e. the curve does not meet the initial pF line perpendicularly), we surmised that the two soils were not quite saturated at pF = 0. It was

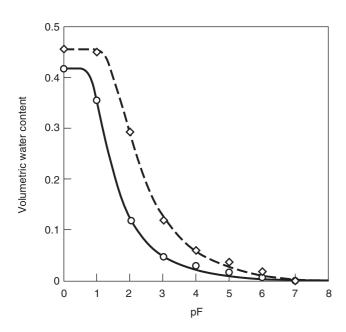
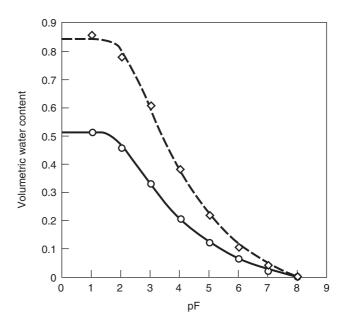


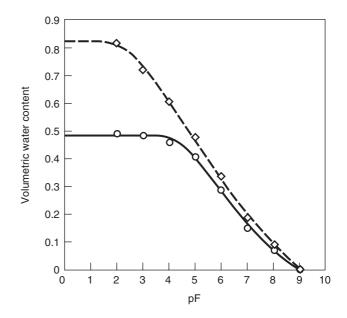
Figure 1 Water retention curves for Koorevaar's soil 1 (dune-sand,  $\bigcirc$ ), and soil 2 (loamy sand,  $\diamond$ ) using Equation (6).



**Figure 2** Water retention curves for Koorevaar's soil 4 (calcareous loam,  $\bigcirc$ ), and soil 6 (young oligotrophic peat,  $\diamond$ ) using Equation (7), with the axis shifted by  $\Delta = 1$ .

thus necessary to shift the base-line by one unit (i.e. one unit is added to all pF values including the anchoring point). The lines drawn through the points result from fitting Equation (6) to the data.

Figure 3 presents the results for soils 7 (marine clay) and 8 (eutrophic peat). For these soils two shifts were required. The lines drawn through the points result from fitting Equation (7) to the data with  $\Delta = 2$ .



**Figure 3** Water retention curves for Koorevaar's soil 7 (marine clay,  $\bigcirc$ ), and soil 8 (eutrophic peat,  $\diamondsuit$ ) using Equation (8) with an axis-shift of  $\Delta = 2$ .

Figure 4 shows the results for soils 3 (calcareous fine sandy loam) and 5 (silt loam), for which three shifts were used. The lines drawn through the points result from fitting Equation (7) to the data with  $\Delta = 3$ .

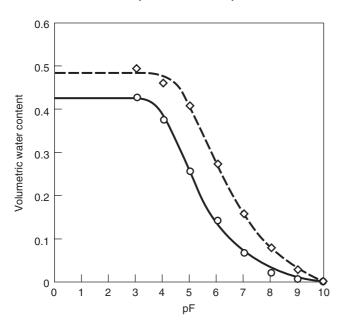
# Optimum shifting

Using Equation (7) for the curve fitting, one could introduce the number of shifts,  $\Delta$  (not necessarily an integer), as a freely adjustable parameter to find the optimum number of shifts required to provide the best fit of the equation to the data. The improvement in fitting, however, which is remarkably good after only one or two shifts, diminishes quickly, and after three integer-shifts there is evidently little improvement to be gained (N. Robinson, CSIRO Mathematical and Information Sciences, personal communication). For example, the optimal value of  $\Delta$  for soil 4 is 5.579; the difference between the fit found using  $\Delta=1$  in Figure 2 and that found using  $\Delta=5.579$  can be seen in Figure 5. While the optimum fit appears good, it is similar to that produced with only a single shift,  $\Delta=1$ .

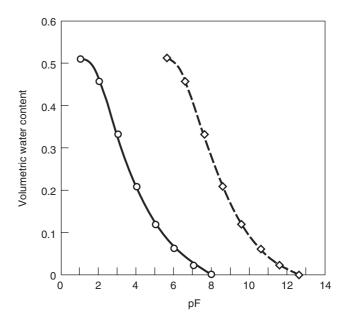
## Fitting soil 9

The last pF curve is for soil 9 (river-basin clay), which we present in Figure 6 together with the best possible fit of the transformed van Genuchten equation, which we first cast in the pF form as follows:

$$pF(\theta) = \frac{1}{N} log \left\{ \left( \frac{(\theta_s - \theta_r)}{(\theta - \theta_r)} \right)^{\frac{1}{M}} - 1 \right\} - log \alpha.$$
 (10)



**Figure 4** Water retention curves for Koorevaar's soil 3 (calcareous fine sandy loam,  $\bigcirc$ ), and soil 5 (silt loam,  $\diamond$ ) using Equation (8) with an axis-shift of  $\Delta = 3$ .

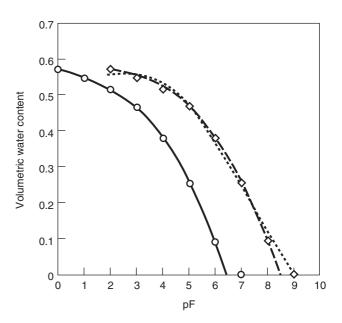


**Figure 5** Water retention curve for Koorevaar's soil 4 with an arbitrary shift of  $\Delta = 1$  (solid line) versus an optimal shift of  $\Delta = 5.579$  (broken line).

We then invert Equation (10) to get

$$\theta_{vG}(pF) = \theta_r + (\theta_s - \theta_r) \left\{ 1 + 10^{N(pF + \log \alpha)} \right\}^{-M}.$$
 (11)

We now use Equation (11) to find the best fit for soil 9, ignoring the oven-dry point. A stable solution and an



**Figure 6** Water retention curves for Koorevaar's soil 9 (river-basin clay) using Equation (9) with an axis-shift of  $\Delta = 2$  ( $\diamond$ ), using all eight points for the optimization (dotted line) and ignoring the oven-dry point (broken line). The undisplaced curve is the best possible fit of the transformed van Genuchten equation ( $\bigcirc$ ).

excellent fit was found with  $\theta_s = 0.605$ ,  $\alpha = 195.5 \times 10^{-9}$ , N=0.216, M=0.013 and  $\theta_r=-73.9$ . We emphasize here that the residual water content is negative and has no physical meaning. We also note that the widely used assumption that  $M = 1 - N^{-1}$  does not hold in this case (i.e.  $1 - N^{-1}$ -3.63 rather than 0.013). To produce the very close fit with the van Genuchten equation, four freely adjustable parameters had to be used, and the value for  $\theta_s$  had to be known.

Figure 6 shows the fitted result of Equation (7) with  $\Delta = 2$ . We found a stable solution using  $k_0 = 38.85$ ,  $k_1 = 1.816$ , and n = 1.589. Finally, to produce a closer comparison with the van Genuchten curve, we used Equation (7) with  $\Delta = 2$  again, but now also ignoring the oven-dry point and anchoring the curve at the point (pF = 6,  $\theta$  = 0.093) according to Equation

$$\theta(\Delta pF) = 0.093 + k_1 \left\{ \exp\left(\frac{-k_0}{(6+\Delta)^n}\right) - \exp\left(\frac{-k_0}{(\Delta pF)^n}\right) \right\}. \tag{12}$$

The resulting curve (Figure 6) is virtually identical to the van Genuchten curve.

#### Discussion and conclusions

The fitting equation presented in this paper for the soil-water retention curve uses three dimensionless parameters and can be anchored at any measured point. One valid anchor point is  $\theta = 0$  at the value of the pressure head equal to  $10^{6.9}$  cm – chosen here because water contents are generally calculated relative to this defined, universal reference. If one is interested only in the 'plant-available' range of soil water (i.e. nothing beyond pF = 4.18, the 15-bar 'wilting point'), the logarithmic scale should be avoided, and the curve should be anchored at the measured point,  $h_{\rm m} = 15\,000\,{\rm cm}$ ,  $\theta = \theta_{\rm wp}$  in the relation:

$$\theta(h) = \theta_{\rm wp} + k_1 \left\{ \exp\left(\frac{-k_0}{15\,000^n}\right) - \exp\left(\frac{-k_0}{h_{\rm m}^{\ n}}\right) \right\},$$
 (13)

where  $\theta_{\rm wp}$  is the measured water content at the wilting point. This relation, although completely different from the van Genuchten equation for soil-water retention, compares well with van Genuchten's model in the plant-available water range and does not suffer from what we call the 'problem of the residual water content'. The advent of powerful mathematical programs for personal computers makes the equations shown here very useful and flexible for many requirements in modelling. We do not yet claim that any of the three parameters has physical meaning – this has yet to be explored. It is possible, however, that they will predict some of the mechanical properties of soils along the lines explored by Dexter & Bird (2001) or Grant et al. (2002), or perhaps even some hydrodynamic properties of soils along the lines explored by Leij et al. (1992).

# Acknowledgements and historical note

We thank Dr N. Robinson, CSIRO Mathematical and Information Sciences, and Dr R.S. Murray, University of Adelaide, for constructive suggestions and discussions. We regret that we never had the opportunity to meet Dr R.K. Schofield, who, during the Third International Congress of Soil Science at Oxford in 1935, presented his formulation of the pF-scale, and who, in 1949, wrote the first scientific paper to appear in the Journal of Soil Science (Schofield, 1949). The paper immediately following his was written by Dr J.A. Prescott (Prescott, 1949), after whom the Prescott Building at the Waite Campus of the University of Adelaide was named, and in which the ideas for this paper were developed.

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## **Appendix**

Table A1 Data extracted from the water retention lines shown in Figure 3.12 of Koorevaar et al. (1983)

Soil no and name	pF							
	0	1	2	3	4	5	6	7
1: Dune-sand	0.417	0.354	0.117	0.048	0.029	0.017	0.007	0
2: Loamy sand	0.457	0.449	0.292	0.117	0.060	0.035	0.017	0
3: Calcareous fine sandy loam	0.427	0.375	0.255	0.142	0.067	0.020	0.006	0
4: Calcareous loam	0.513	0.458	0.333	0.209	0.120	0.061	0.022	0
5: Silt loam	0.492	0.460	0.407	0.273	0.157	0.079	0.029	0
6: Young oligotrophic peat	0.857	0.778	0.607	0.379	0.217	0.104	0.037	0
7: Marine clay	0.492	0.485	0.458	0.407	0.287	0.150	0.075	0
8: Eutrophic peat	0.815	0.717	0.604	0.475	0.333	0.186	0.072	0
9: River-basin clay	0.572	0.547	0.517	0.467	0.379	0.253	0.093	0

Table A2 Curve-fitting parameters for the water retention lines of Koorevaar et al. (1983) using Equation (7) for all soils, and Equation (11) for soil 9 only

	Parameters from Equation (7)					
Soil no and name	$k_0$	$k_1$	n	Δ		
1: Dune-sand	1.867	0.426	2.358	0		
2: Loamy sand	6.377	0.472	2.598	0		
3: Calcareous fine sandy loam	756.63	0.447	4.154	3		
4: Calcareous loam	9.574	0.620	1.874	1		
5: Silt loam	1062.04	0.547	3.906	3		
6: Young oligotrophic peat	13.402	1.015	2.056	1		
7: Marine clay	298.07	0.687	3.069	2		
8: Eutrophic peat	11.630	1.773	1.236	2		
9a: River-basin clay (dotted line, Figure 6)	38.85	1.816	1.589	2		
9b: River-basin clay (broken line, Figure 6)	24.02	$5.407 \times 10^6$	0.1884	2		
	Van Genuchten parameters from Equation (11) for soil 9 (solid line, circles in Figure 6)					
	$ heta_{ m s}$	α	N	M	$ heta_{ m r}$	
	0.605	$195.5 \times 10^{-9}$	0.216	0.013	-73.9	