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A conceptual model of the soil water retention curve

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Abstract. A conceptual model based on the assumption that soil structure evolves from a uniform random fragmentation process is proposed to define the water retention function. The fragmentation process determines the particle size distribution of the soil. The transformation of particles volumes into pore volumes via a power function and the adoption of the capillarity equation lead to an expression for the water retention curve. This expression presents two fitting parameters only. The proposed model is tested on water retention data sets of 12 soils representing a wide range of soil textures, from sand to clay. The agreement between the fitted curves and the measured data is very good. The performances of the model are also compared with those of the two-parameter models of *van Genuchten* [1980] and *Russo* [1988] for the water retention function. In general, the proposed model exhibits increased flexibility and improves the fit at both the high and the low water contents range.

1. Introduction

The solution of the flow equation of water in soils requires the expression of two soil hydraulic characteristics, the water retention curve (WRC) and the hydraulic conductivity function (HCF). The WRC describes the relationship between the soil capillary head, ψ, and the volumetric water content, θ. The HCF describes the relationship between the unsaturated hydraulic conductivity, K, and θ. Different models permit defining the HCF in terms of the WRC [*Mualem*, 1986]. Therefore the WRC can be considered to be one of the most fundamental hydraulic characteristics of a soil. The experimental determination of the WRC is tedious and time consuming. Therefore the WRC is not always present in the usual data sets presenting the basic properties of soils. When it is available, it is a discrete representation of a limited number of volumetric water content–capillary head points within the range of the water matric tensions under interest. Consequently, intensive efforts were and are still invested in developing mathematical functions to be fitted to the available set of measured points in order to provide a continuous expression of the WRC. This study is a contribution to this effort. It assumes that the soil particle size distribution (PSD) stems from a fragmentation process and derives the void size distribution (VSD) from the PSD.

2. Approaches in Modeling the WRC

Different models which are in fact empirical curve fitting equations are available. A simple one-parameter model was proposed by *Tani* [1982]:

$$S_e(\psi) = (1 + \psi/\psi_o) \exp (\psi/\psi_o) \tag{1}$$

where ψ_o is the capillary head at the inflection point and S_e, the effective saturation degree, is defined as

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$$S_e = (\theta - \theta_r)/(\theta_s - \theta_r) \tag{2}$$

where θ_s and θ_r are the saturated and residual volumetric water contents, respectively. *Russo* [1988] suggested a two-parameter model which is a general form of Tani's equation:

$$S_e(\psi) = \{(1 + \beta|\psi|) \exp (-\beta|\psi|)\}^{2/(2+k)} \tag{3}$$

where β represents an empirical parameter and k is the parameter from *Mualem*'s [1976] model for the HCF.

Brooks and Corey [1964] have expressed the effective saturation S_e as a two-parameter power function of ψ:

$$\begin{aligned} S_e(\psi) &= (\psi/\psi_c)^{-\lambda} \qquad \psi < \psi_c \\ S_e(\psi) &= 1 \qquad \psi \geq \psi_c \end{aligned} \tag{4}$$

where ψ_c and λ are fitting parameters. *Campbell* [1974] also proposed an expression which was very similar to (4). The parameter ψ_c is the air entry value and is assumed to be related to the maximum size of pores forming a continuous network of flow paths within the soil. The parameter λ is dimensionless and is referred to as the pore size distribution index. *Milly* [1987] reported that the reference to a specific air entry value and the absence of an inflection point on the WRC cause discrepancies with field measured data. One of the shortcomings of the Brooks and Corey equation is the sharp discontinuity in the derivative at ψ_c. *Clapp and Hornberger* [1978] and *Hutson and Cass* [1987] suggested replacing the sharp corner with a parabolic curve, leading to a smoothly joined two-part retentivity equation. *Van Genuchten* [1980] has proposed the model

$$S_e(\psi) = 1/\{1 + \alpha|\psi|^n\}^m \tag{5}$$

where α and n represent empirical parameters that are reportedly related to the inverse of ψ_c and to λ ($n = \lambda + 1$), respectively. The parameter m is related to n by the equation $m = 1 - 1/n$. Generally, θ_r is regarded as the third empirical fitting parameter. *Tyler and Wheatcraft* [1989] have considered m as a fourth fitting parameter. This model does not account for the air entry value but does have an inflection point which allows better performance than the Brooks and Corey model for many soils, particularly for data near saturation [*van Genuchten and Nielsen*, 1985]. *Nimmo* [1991] and *Ross et al.* [1991] found that the van Genuchten model is successful at high and medium water contents but often gives poor results at low water contents. *Campbell and Shiozawa* [1992] have proposed a modification of the van Genuchten model for improving fits to dry range data.

Recently, *Zhang and van Genuchten* [1994] proposed two models of the WRC. The four fitting parameters model corresponds to a sigmoidal type WRC, while the five fitting parameters model leads to a bimodal type WRC.

A different approach was to develop expressions for the WRC starting from the particle size distribution (PSD) of the soil. *Arya and Paris* [1981] developed a model to predict the WRC of a soil from its PSD, bulk density, and particle density on the basis of the similarity in shape between the WRC and the PSD of a soil. They proposed the relationship between the mean pore radius, r, and the mean particle radius, R:

$$r = R[4en^{(1-\alpha)}/6]^{0.5} \tag{6}$$

where e is the void ratio, n is the number of spherical particles with radius r, and α is an empirical constant ranging from 1.35 to 1.4 that accounts for the departure from the assumption of sphericity in natural soil particles. The pore radii are then converted to volumetric water content and to equivalent soil water capillary head using the equation of capillarity. Model predictions for seven soil materials show close agreement with the experimental data. However, *Rouault and Assouline* [1998], in a study on the relationship between particle and void size distributions in multicomponent sphere packs, show that the linear relationship assumed between the particle and the pore size in (6) might not be adequate.

Haverkamp and Parlange [1986] have assumed also a linear relationship between the particle diameter, d, and the equivalent pore radius, r:

$$d = \gamma r \tag{7}$$

where γ, a packing parameter characteristic of the soil, is assumed to be constant. They adopt a power function to represent the cumulative PSD function $F(d)$:

$$F(d) = 1/[1 + (d_g/d)^n]^m \tag{8}$$

where d_g is a constant and $m = 1 - 1/n$. Combining this distribution with the similarity hypothesis in (7) and with the inverse relationship between ψ and r, and adopting the Brooks and Corey power function (equation (4)), they derived an analytical expression for $\theta(\psi)$. The application of the model to sandy soils with no organic matter provided good results.

The suitability of the power function to describe the particle/aggregate size distribution, at least for certain soil types, was experimentally shown [*Tyler and Wheatcraft*, 1989; *Young and Crawford*, 1991; *Bartoli et al.*, 1991]. When this kind of expression fits the soil under study, the soil is fractal-like and the factor of proportionality is similar to the fractal dimension, D [*Van Damme et al.*, 1988]. The application of fractal concepts to porous media was the basis of a new approach to WRC modeling [*de Gennes*, 1985; *Tyler and Wheatcraft*, 1990; *Rieu and Sposito*, 1991; *Pachepsky et al.*, 1995b; *Bird et al.*, 1996].

Tyler and Wheatcraft [1990] obtained a power law expression for the WRC similar to the Brooks and Corey model, where the power (2-D) is equivalent to λ in (4).

Rieu and Sposito [1991] have shown that power functions for the aggregate size distribution and the WRC directly stems from a fractal model of aggregate and pore space properties for structured soils.

However, if a fractal model leads to a power function for the WRC, the opposite is not necessarily correct. The fact that a power function could be fitted to the WRC does not prove that the soil is fractal [*Crawford et al.*, 1995].

Bird et al. [1996], in a review on water retention functions based upon fractal soil structures, concluded that the simple power law model does not represent a model of general validity in the context of fractal fabric and fractal pore surface models.

Pachepsky et al. [1995a] have shown that one reason for the deviations from the power law is the multifractal structure of soil porosity, which results in dependence of the fractal dimension on the radii. *Pachepsky et al.* [1995b] have developed a model for the WRC, assuming fractal self-similarity of pore volumes but adding a correcting factor accounting for the dependence on the radii. The factor chosen, $f(r)$, was a lognormal probability distribution function of the pore radii. The resulting expression for the $\theta(\psi)$ function is a three-parameter expression:

$$\theta = (\theta_o/2)\ \mathrm{erfc}\ [(1/\sigma\sqrt{2})\ \ln\ (\psi/\psi^*)] \tag{9}$$

where "erfc" denotes the complementary error function and θ_o, σ, and ψ^* are fitting parameters. *Kosugi* [1994] also proposed a WRC model that results from applying three-parameter lognormal distribution laws to the pore radius distribution function and to the pore capillary head distribution function. The resulting expression for the WRC is

$$S_e(\psi) = 0.5\ \mathrm{erfc}\ [(\ln\ \{(\psi_c - \psi)/(\psi_c - \psi_o)\} - \sigma^2)/2^{0.5}\sigma] \qquad \psi < \psi_c \tag{10}$$

$$S_e(\psi) = 1 \qquad \psi \geq \psi_c$$

This model presents four fitting parameters: θ_r, ψ_c, ψ_o, and a dimensionless parameter, σ, which is related to the width of the pore radius distribution. The model was tested for 50 sets of observed retention data for various soils and, according to the author, "it has produced acceptable results" and "performed as well as any existing empirical model" [*Kosugi*, 1994, p. 900].

In terms of the similarity hypothesis used, assuming a lognormal distribution of the pore size corresponds to assuming that the particle sizes are also lognormally distributed. Such an assumption was proposed by *Shirazi and Boersma* [1984], discussed by *Buchan* [1989], and validated by *Shiozawa and Campbell* [1991].

Conceptually, it can be assumed that the form of the particle/aggregate size distribution is the result of the fragmentation processes that shape it (by particle/aggregate we consider either elementary units to be sand grains or aggregates of fine particles). The lognormal distribution results from a fragmentation process where the probability of fragmentation is assumed to be independent of the particle size [*Tenchov and Yanev*, 1986]. It is more likely that the probability of fragmentation of a soil particle or aggregate is proportional to its size. The aim of this study is to develop such approach in order to derive a mathematical expression for the WRC.

Table 1. The Soils and the Curves Types That Constitute the First Data Set

Soil Type	Catalog Number	Curve Type	Reference
Sable de riviere	4118	main drying	*Vachaud* [1966]
Pachapa fine sandy clay	3503	main drying	*Gardner* [1959]
Rubicon sandy loam	3501	main drying	*Topp* [1969]
Pachapa loam	3403	main drying	*Jackson et al.* [1965]
Touchet silt loam	3304	first drying	*Jensen and Hanks* [1967]
Rideau clay loam	3101	main drying	*Topp* [1971]
Beit Netofa clay soil	1006	first drying	*Rawitz* [1965]

Soils and curve types from *Mualem* [1974].

3. Theory

Consider that initially, a soil is characterized by a uniform density distribution, $f_o(v'_a)$, of the particle/aggregate normalized volumes, v'_a:

$$f_o(v'_a) = 1 \qquad v'_a = (v_a - v_{a\min})/(v_{a\max} - v_{a\min}) \tag{11}$$

where $v_{a\max}$ is the largest aggregate volume and $v_{a\min}$ is the smallest. Accordingly, the probability distribution function, $F_o(v'_a)$, is given by

$$F_o(v'_a) = \int_0^{va'} f_o(x)\, dx = v'_a \tag{12}$$

Suppose now that the particle volume distribution of a natural soil is the result of a series of sequential fragmentation consequently to cycles of wetting and drying; physical, chemical, and biological effects; and cultivation practices. Assume that the fragmentation process is uniform and is subjected to the following characteristics: (1) the fragmentation is random; (2) the probability for a particle fragmentation is proportional to its volume and therefore is identical to $F_o(v'_a)$; and (3) the probability that the fragmentation of particles of volume w'_a yields particles of volume v'_a is independent of v'_a, so that the distributor of volumes w'_a to smaller volumes v'_a is $(1/w'_a)$.

For a large number of fragmentation events, N, the probability distribution function $F_N(v'_a)$ leads asymptotically to an exponential distribution which is a partial case of the Weibull distribution:

$$F(v'_a) = 1 - \exp(-\mu v'_a) \tag{13}$$

where μ is proportional to N [*Tenchov and Yanev*, 1986].

In order to transform the particle volume probability distribution, $F(v'_a)$, into the pore volume probability distribution, $F(v'_p)$, we propose the following relationship between v'_a and the pore volume, v'_p:

$$v'_a = \gamma v'^{u}_p \tag{14}$$

where γ and u are constants related to the packing and the respective shapes of the particles and the corresponding pores. This relationship was found to be appropriate for the case of multicomponent sphere packs for which exact expressions of the PSD and the VSD were available [*Assouline and Rouault*, 1997; *Rouault and Assouline*, 1998]; the results are shown in the appendix. Replacing (14) with (13) leads to the definition of the pore volume probability distribution function $F(v'_p)$:

$$F(v'_p) = 1 - \exp(-\omega v'^{u}_p) \qquad v'_p = (v_p - v_{p\min})/(v_{p\max} - v_{p\min}) \tag{15}$$

where $\omega = \mu\gamma$. Equation (15) is the general case of the Weibull distribution. For $v_p = v_{p\min}$, $F(v_p) = 0$. When $v_p = v_{p\max}$, $F(v_p) \rightarrow 1$ at a rate that depends on the value of ω. When all the pores, up to $v_{p\max}$, are filled with water, the soil water content is equal to θ_s, and $S_e = 1$. When all the pores are filled with air, the soil water content is equal to θ_r, and $S_e = 0$. Therefore (15) can also represent the effective saturation function:

$$S_e(v'_p) = 1 - \exp(-\omega v'^{u}_p) \qquad v'_p = (v_p - v_{p\min})/(v_{p\max} - v_{p\min}) \tag{16}$$

Every pore volume, v_p, is characterized by two specific capillary heads at which it drains or fills with water, ψ_d and ψ_w. In the following, we will consider only the drying process, but the approach is applicable also to the wetting phase in order to account with the hysteresis in the WRC. A relationship between v_p and ψ can be formulated when the equation of capillarity is applied to pores. Each pore volume can be represented by an equivalent pore radius, r:

$$v_p = ar^d \tag{17}$$

where a and d are parameters that depend on the shape of the pores. The radius, r, is related to the capillary head, ψ, by

$$r = -c\psi^{-1} \tag{18}$$

where c is a parameter depending on the liquid tension and the contact angle between liquid and solid. At a given ψ all the pores with an equivalent radius smaller than or equal to the corresponding r(ψ) value (equation (18)) are saturated with water, while all the pores with larger radii are empty. Therefore replacing (17) and (18) with (16) leads to the definition of the relationship between S_e and ψ:

$$S_e(\psi) = 1 - \exp\{-\omega[(\psi^{-d} - \psi_{\min}^{-d})/(\psi_{\max}^{-d} - \psi_{\min}^{-d})]^u\} \qquad \psi_{\max} \geq \psi \geq \psi_{\min} \tag{19}$$

The parameters in (19) can be evaluated using fitting procedures to experimental data. Generally, the number of measured points that are available to represent the WRC is limited. Therefore it is necessary to minimize the number of fitting parameters in order to obtain values that are statistically sound. We propose to adopt a simpler expression, very close to (19) without being identical, which has only two fitting parameters, ξ and η:

$$S_e(\psi) = 1 - \exp[-\xi(\psi^{-1} - \psi_{\min}^{-1})^\eta] \qquad 0 \geq \psi \geq \psi_{\min} \tag{20}$$

It can be assumed that $\psi_{\min} \rightarrow -\infty$. In that case the resulting expression from (20) is identical to that resulting from (20)

Table 2. The Soil Types, and Their Mechanical Analyses, for the Second Set of WRC Data

Soil Type	sand, %	silt, %	clay, %
Grignon	7.4	70.3	22.3
Thifontaine	62.9	5.9	31.2
Machelainville	2.0	60.8	37.2
Licheres	2.6	42.7	54.7
Chezal Benoit	2.6	4.5	92.9

with η = ud and $\xi = \omega\psi_{\max}^{\eta}$. However, in most of the practical cases involving the simulation of flow processes in soils, there is no need to consider the WRC beyond a capillary head, ψ_L, that corresponds to a very low water contents, θ_L, at which the hydraulic conductivity is negligible. Therefore the value of (θ_L, ψ_L) can be chosen according to the specific soil type under consideration. Alternatively, the suggestion of *van Genuchten* [1980] to truncate the WRC at the wilting point ($\psi_L = -1.5$ MPa) can be adopted. The saturated water content, θ_s, also can be available from measured data or evaluated in terms of the soil porosity [*Rogowski*, 1971]. Consequently, the two-parameter WRC function proposed herein is

$$\theta(\psi) = (\theta_s - \theta_L)\{1 - \exp[-\xi(\psi^{-1} - \psi_L^{-1})^{\eta}]\} + \theta_L \qquad (21)$$

$$0 \geq \psi \geq \psi_L$$

with the parameters ξ and η being determined by fitting procedures to measured data.

4. Material and Methods

Two different soil WRC data sets are used to test the proposed model. In all the cases, only drying curves are used. The first set is constituted from WRC data published in the catalog of *Mualem* [1974]. Seven soil types are chosen that represent a range of textures from sand to clay. The soils references are listed in Table 1.

The second set is constituted from WRC data measured from five soils from different regions of France. These soils present a large range of clay content, from 22% to 93%. The highest clay contents correspond to vertisols and related soils [*Tessier et al.*, 1991] or to B horizons of leached soils [*Bruand et al.*, 1988]. The mechanical analysis of these soils is presented in Table 2. Undisturbed soil samples of 500–2000 cm^3 were taken from the field at the end of the winter, at their higher water content. These samples were not let to dry, and stored at 5°C. Fragments of 10–20 cm^3 were taken from these samples and prealably brought to saturation. The WRCs within the range of ψ values between 0 and −1.6 MPa were measured on these fragments, using pressure membrane apparatus. Details on the method and the experimental setup are given by *Tessier and Berrier* [1979] and *Bruand* [1990]. The different pressure steps were −1.0, −3.2, −10.0, −31.6, −100.0, −316.0, −1000.0, and −1585.0 kPa. The volume of the samples was measured by kerosene displacement at every pressure step in order to account for the changes due to swelling or shrinking. The volumetric water content corresponding to each pressure step was thus determined considering the measured specific bulk density of each sample at every step.

For all the cases the value for θ_s was set equal to the reported volumetric water content at $\psi = 0$. The value of ψ_L was set equal to −1585.0 kPa. The corresponding value of θ_L was the value measured at ψ_L, or alternatively, the lowest water content measured where the hydraulic conductivity became negligible.

The values of the parameters ξ and η are determined using an iterative nonlinear regression procedure based on the Marquardt-Levenberg algorithm. This procedure finds the values of the fitting parameters that give the "best fit" between the model and the data, that is, that minimize the sum of the squared differences between the observed and predicted values of the dependent variable [*Glantz and Slinker*, 1990]. The square root of this sum is defined as the norm index, which is an indicator of the goodness of the fit reached. The lower the index is, the better the fit.

5. Results and Discussion

The values of the different parameters leading to the best fit of (21) to the measured data, and the corresponding norm index, are shown in Table 3 for each soil.

The fitted curves and the measured data for four of the soils of the first set are depicted in Figures 1a–1d. Although the measured WRCs present very different shapes, they are all well reproduced by the proposed model. The agreement between the fitted curves and the measured data is very good. The case of the sand (Figure 1a) illustrates the worst fit (the highest norm index is obtained). However, in this case too, the

Table 3. The Values of the Fitting Parameters, Corresponding to ψ, and the Resulting Norm Index

Soil Type	θ_s, m^3/m^3	θ_L, m^3/m^3	ξ	η	Norm
Sable de riviere	0.342	0.075	0.0008	4.480	0.042
Pachapa fine sandy clay	0.334	0.049	70.37	0.919	0.041
Rubicon sandy loam	0.381	0.166	0.506	3.943	0.038
Pachapa loam	0.456	0.075	418.7	1.163	0.030
Touchet silt loam	0.480	0.170	0.825	3.738	0.034
Rideau clay loam	0.416	0.286	1990.0	1.876	0.028
Beit Netofa clay soil	0.460	0.242	24.62	0.453	0.017
Grignon	0.386	0.163	29.04	0.509	0.019
Thifontaine	0.306	0.211	57.72	0.661	0.005
Machelainville	0.360	0.234	33.18	0.600	0.014
Licheres	0.409	0.288	12.06	0.486	0.003
Chezal Benoit	0.500	0.409	6.18	0.410	0.008

Values of ψ are in centimeters.

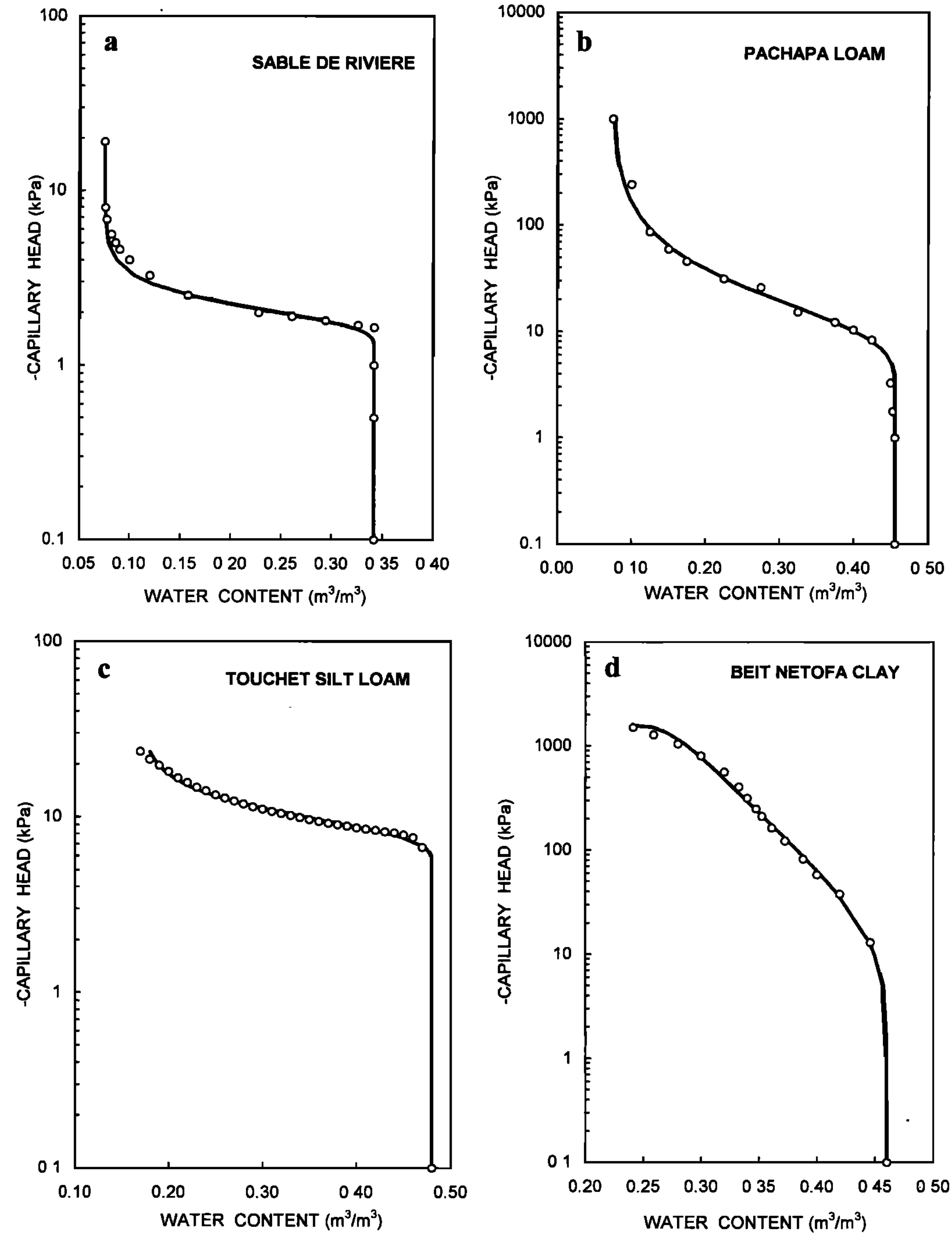

Figure 1. The fitted curves (solid lines) to the measured points (circles) for four soils from the first data set: (a) sable de riviere, (b) Pachapa loam, (c) Touchet silt loam, and (d) Beit Netofa clay soil.

proposed model fits the measured data well, and is flexible enough to reproduce even step-function-like shapes.

The fitted curves and the measured data for four of the soils of the second set are depicted in Figures 2a–2d. The data in this case represent undisturbed soil samples. Therefore the measured points reflect both inter and intra-aggregates pore space. Consequently, the WRCs might differ from the shapes expected when the mechanical analysis only is considered. It is the high water content range which is affected the most by the macroscopic soil structure. In fact, the measured air entry values for the heavier soil samples (Figures 2a–2d) are lower than the expected ones, considering the high clay contents, and apparently correspond to aggregated soil structures. For this data set too, the agreement between the fitted curves and the measured data is good. Compared to the first data set, the measured WRC of each soil in the second set is represented by fewer points. This situation is closer to the reality of available WRC data and requires WRC models with a minimum of fitting parameters. Therefore, in addition to its conceptual basis, the proposed expression presents also a practical interest since it seems flexible enough to be appropriate to a wide range of soil types while it requires only two fitting parameters.

In order to compare the performances of the proposed model to those of existing ones, equation (21) is compared to

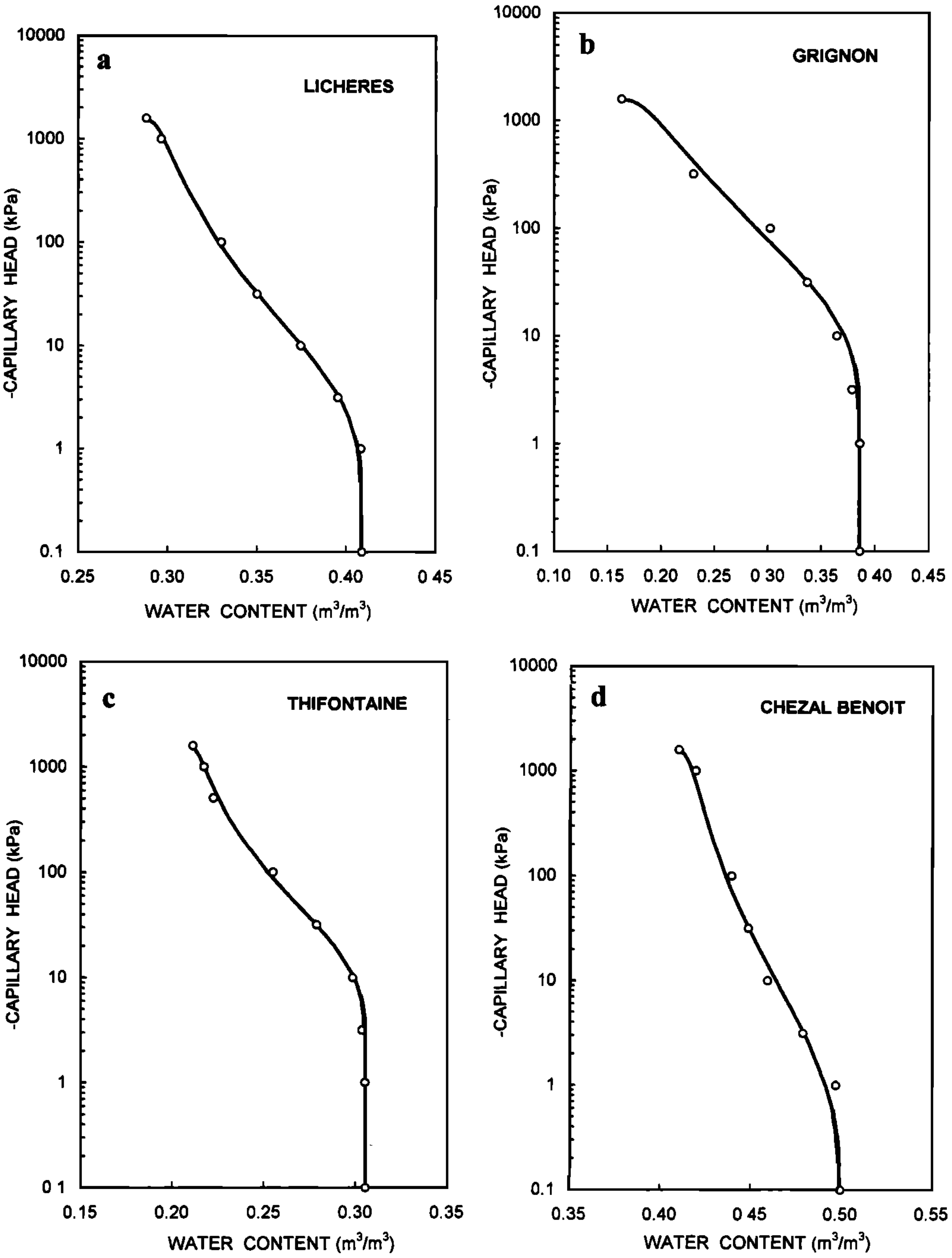

Figure 2. As in Figure 1, for four soils from the second data set: (a) Licheres, (b) Grignon, (c) Thifontaine, and (d) Chezal Benoit.

Table 4. The Values of the Fitting Parameters, Corresponding to ψ, and the Norm Index for the models of Russo and Van Genuchten for Four Different Soil WRCs and the Corresponding Norm Index of the Proposed Model

Soil Type	β*	k*	Norm*	α†	n†	Norm†	Norm‡
Rubicon sandy loam	2.87×10^{-4}	−1.99	0.131	2.54×10^{-9}	4.42	0.077	0.038
Pachapa fine sandy clay	4.65×10^{-1}	201.6	0.072	3.06×10^{-4}	1.86	0.039	0.041
Rideau clay loam	1.75×10^{-2}	−0.64	0.055	5.16×10^{-6}	3.00	0.036	0.028
Machelainville	2.35×10^{-1}	499.4	0.019	1.29×10^{-4}	1.66	0.017	0.014

Values of ψ in centimeters.
Russo [1988] (equation (3)).
†*Van Genuchten* [1980] (equation (5)).
‡Proposed model.

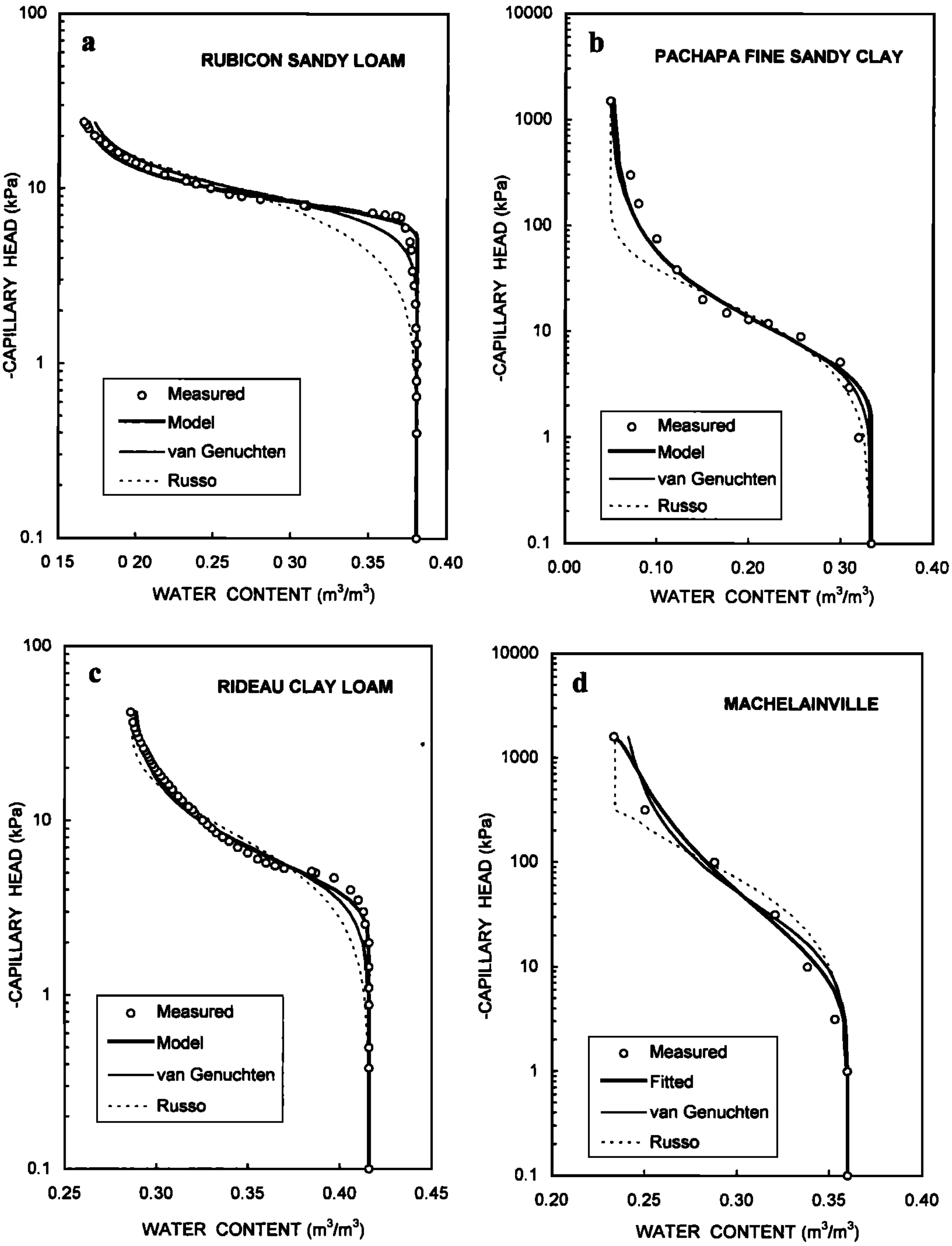

Figure 3. The fitted curves from the proposed model (equation (21); bold line) and from the models of *Russo* [1988] (equation (3); dotted line) and *van Genuchten* [1980] (equation (5); solid line) to the experimental data (circles) for four soils: (a) Rubicon sandy loam, (b) Pachapa fine sandy clay, (c) Rideau clay loam, and (d) Machelainville.

the two-parameter model proposed by *Russo* [1988] (equation (3)) and the two-parameter version of the van Genuchten model (equation (5)) where $m = 1 - 1/n$. In all the cases the same θ_s and θ_L values were used. Also, the same fitting procedure was used to evaluate the fitting parameters in each expression. Therefore the comparison will be based on the respective values of the norm index. The results for four different soils, three from the first set and one from the second set, are presented in Table 4. The fitted curves, corresponding to the three models, are depicted for every soil in Figures 3a–3d.

In all the cases the lowest performance is provided by *Russo*'s [1988] model. This expression hardly reproduces the exact shape of the measured WRCs and tends to neglect both the high and low water content ranges to the benefit of the narrow range of intermediate water content values. The proposed model performs the best in three of the four cases presented. In the case of the Pachapa fine sandy clay soil (Figure 3b) the van Genuchten expression gives better results than the proposed model and improves the fit for high water contents. Paradoxically, this is the principal range where the proposed model improves the fit in all the three other cases, compared to the van Genuchten expression. This is depicted clearly for the case of the Rubicon sandy loam soil (Figure 3a). The high water content part of the WRC is the most important part for many hydrological problems [*Clapp and Hornberger*, 1978]. This is also the part that power function models, like the Brooks and Corey expression (equation (4)) or expressions stemming from the fractal approach, fail to reproduce accurately [*Milly*, 1987]. Therefore the increased accuracy of (21) precisely in the high water content range is an additional advantage of the proposed model.

For the Rubicon sandy loam soil case (Figure 3a) and for the case of Machelainville soil (Figure 3d) the proposed model increases the fit goodness also in the range of low water contents. This is due to the fact that $\theta \to \theta_L$ when $\psi_{\min} \to \psi_L$ in the proposed expression, while $\theta \to \theta_L$ when $\psi_{\min} \to -\infty$ in the two other models.

6. Conclusion

The approach in this study describes a conceptual link between the soil structure and the WRC. The proposed model for the WRC stems from two assumptions: (1) The soil structure results from a uniform random fragmentation process where the probability of fragmentation of an aggregate is proportional to its size, and (2) a power function relates between the volume of the aggregates and the corresponding pore volumes. Tested on 12 different soil types that present a wide range of textures and mechanical analysis, the proposed model reproduces accurately the very different shapes of the respective WRCs, from the step-function shape of a sand to the sigmoidal almost linear shape of a clay. Also, it reproduces very well measured data in the whole range of water contents, from saturation to the water content at the wilting point. Compared to two existing two-parameter models [*van Genuchten*, 1980; *Russo*, 1988], the proposed expression exhibits increased flexibility and reproduces better the measured data both in the high and the low water content ranges. Therefore two main benefits can be attributed to the proposed model: (1) a gain in accuracy and flexibility with only two fitting parameters, valuable in a situation where most WRCs constitute in a limited number of measured points, and (2) a conceptual basis that can be the basis for further research towards a physically based link between soil structure and hydraulic properties.

Appendix

The relationship between particle and pore volumes proposed in (14) is validated for the multicomponent sphere-pack cases studied by *Rouault and Assouline* [1998]. Their study proposes a probabilistic approach in order to compute the VSD of a sphere pack given its PSD. Consequently, from the knowledge of the PSD and its corresponding VSD it is possible to find the expression that relates the sphere radii to the pore radii. The cases of Gaussian and lognormal PSDs and the resulting VSDs are used herein. These distributions are shown in Figure A1.

First, a Weibull distribution similar to (15) is fitted to the two PSDs expressed in terms of the normalized sphere radius r'_a:

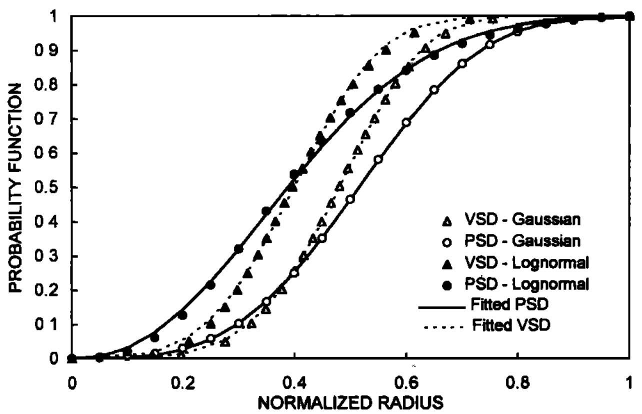

Figure A1. The Weibull distribution fitted to the particle size distribution (PSD) and to the corresponding void size distribution (VSD) in the case of multicomponent sphere packs with a Gaussian and a lognormal PSD studied by *Rouault and Assouline* [1998] and *Assouline and Rouault* [1997].

$$F(r'_a) = 1 - \exp(-ar'^b_a)$$
$$r'_a = (r_a - r_{a\min})/(r_{a\max} - r_{a\min}) \tag{A1}$$

For the Gaussian case the values of the fitting parameters are $a = 6.65$, $b = 3.42$, $r_{a\min} = 1$, and $r_{a\max} = 5$ (the radii are nondimensional). For the lognormal case the values are $a = 5.93$, $b = 2.27$, $r_{a\min} = 1$, and $r_{a\max} = 6$. The resulting curves are depicted by the solid lines in Figure A1. Then, the relationship suggested in eq. (14) between the normalized pore radius, r_p, and r_a is defined so that the expressions in (A1) can be transformed to fit the respective VSDs. For the Gaussian case the resulting relationship obtained is

$$r'_a = 1.37\ r'^{1.33}_p \tag{A2}$$

For the lognormal case the relationship is

$$r'_a = 1.72\ r'^{1.63}_p \tag{A3}$$

The resulting curves are depicted by the dashed lines in Figure A1. The goodness of the fits indicates that the relationship in (14) is valid for the case of sphere packs.

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