Tampere University

AUT.360 Distributed Control and Optimization of Cyber-Physical Systems

Homework 5

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Problem 1

The dynamics of each agent is given below:

$$\dot{p}_i = v_i,$$
$$\dot{v}_i = u_i \quad (1)$$

where $u_i = \sum_{j \in N_i^{in}} (p_j - p_i) + \gamma(v_j - v_i)$, p_i is the position of Agent *i* and v_i is its velocity.

We are given 2 digraphs presented in figure 1 [1]. All agents in digraphs a and b have the dynamics in (1).

Figure 1: Leader-follower and directed ring network.

1. For the compact vector form we need to look at the dynamics in (1). From this we can create the compact vector form representation of the system dynamics. As both systems share their dynamics the compact vector form is the same.

   We can represent the dynamics as a function of the variables p_i, v_i and u_i directly from the given relations in (1):

$$\dot{p}_i = (0 * p_i + 1 * v_i) + 0 * u_i$$
$$\dot{v}_i = (0 * p_i + 0 * v_i) + 1 * u_i \quad (2)$$

   From this representation we can create the compact vector form of the dynamics easily:

$$\begin{bmatrix} \dot{p}_i \\ \dot{v}_i \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} p_i \\ v_i \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_i \quad (3)$$

2. For double integrator model, consensus is achieved if and only if matrix A has exactly two zero eigenvalues and all other have negative real parts. A matrix can be read from the overall dynamics of the system under consensus protocol:

$$\begin{bmatrix} \dot{p}_i \\ \dot{v}_i \end{bmatrix} = A \begin{bmatrix} p_i \\ v_i \end{bmatrix}, where\ A = \begin{bmatrix} 0_{n*n} & I_n \\ -L & -\gamma L \end{bmatrix} \tag{4}$$

For analysis of the system, we need to calculate the eigenvalues of A. By using determinates and the quadratic formula we end up with the following equation for A matrix eigenvalues:

$$\lambda_i(\mathrm{A}) = \frac{-\gamma\mu_i \pm \sqrt{\gamma^2 {\mu_i}^2 - 4\mu_i}}{2} \tag{5}$$

Where μ_i is the *i*-th eigenvalue of -L.

For computing A, we need L matrices for the graphs in Figure 1.

0	0	0	0	0
-1	1	0	0	0
0	-1	1	0	0
0	0	-1	1	0
0	0	0	-1	1

Table 1: L matrix for Figure 1(a)

1	0	0	0	-1
-1	1	0	0	0
0	-1	1	0	0
0	0	-1	1	0
0	0	0	-1	1

Table 2: L matrix for Figure 1(b)

From table 1 we can see that it is a lower triangular matrix and so its eigenvalues are in the diagonal of the matrix. Using equation 5, we can see we get two zero eigenvalues and other eigenvalues are real and negative for any $\gamma > 0$. As such consensus is achieved for any $\gamma > 0$.

For Figure1(b), that is directed spanning tree we need to calculate eigenvalues from equation 5:

Reaching consensus requires that the network contains a directed spanning tree. Both networks in figure 1 fulfil this necessary condition. Conditions for γ such as the second-order consensus is achieved.

$$\gamma^2 > \max_{i=\{2,\dots,n\}} \frac{(Im(\lambda_i(L)))^2}{Re(\lambda_i(L))|\lambda_i(L)|^2} \tag{5}$$

Firstly, we need to calculate the eigenvalues of L.

$$[0 \quad 0.6910 + 0.9511i \quad 0.6910 - 0.9511i \quad 1.809 + 0.5878i \quad 1.809 - 0.5878i]^T$$

From these with (5) we can calculate that we get the biggest value with 0.6910 – 0.9511i. We get that the gain γ need to be larger than 0.9732 for the system to reach consensus.

3. For the trajectories we will use the consensus dynamics in (4). We created the A matrix and used it to model the system dynamics as time goes on. The resulting trajectory plots for graphs a and b, from figure 1, are presented respectively in figures 2 and 3. We used gain γ = 2 for both simulations as that speeded the convergence of the agents.

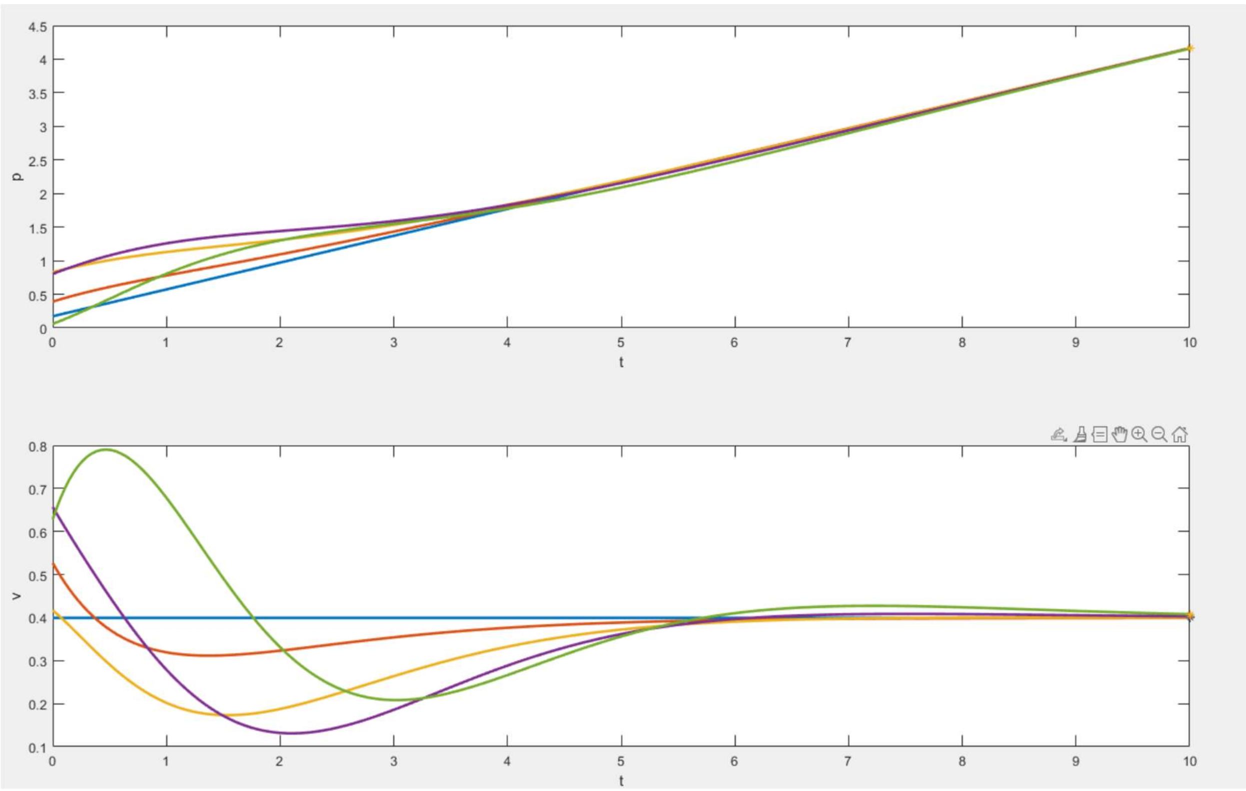

Figure 2: Trajectory plot for p and v for digraph a

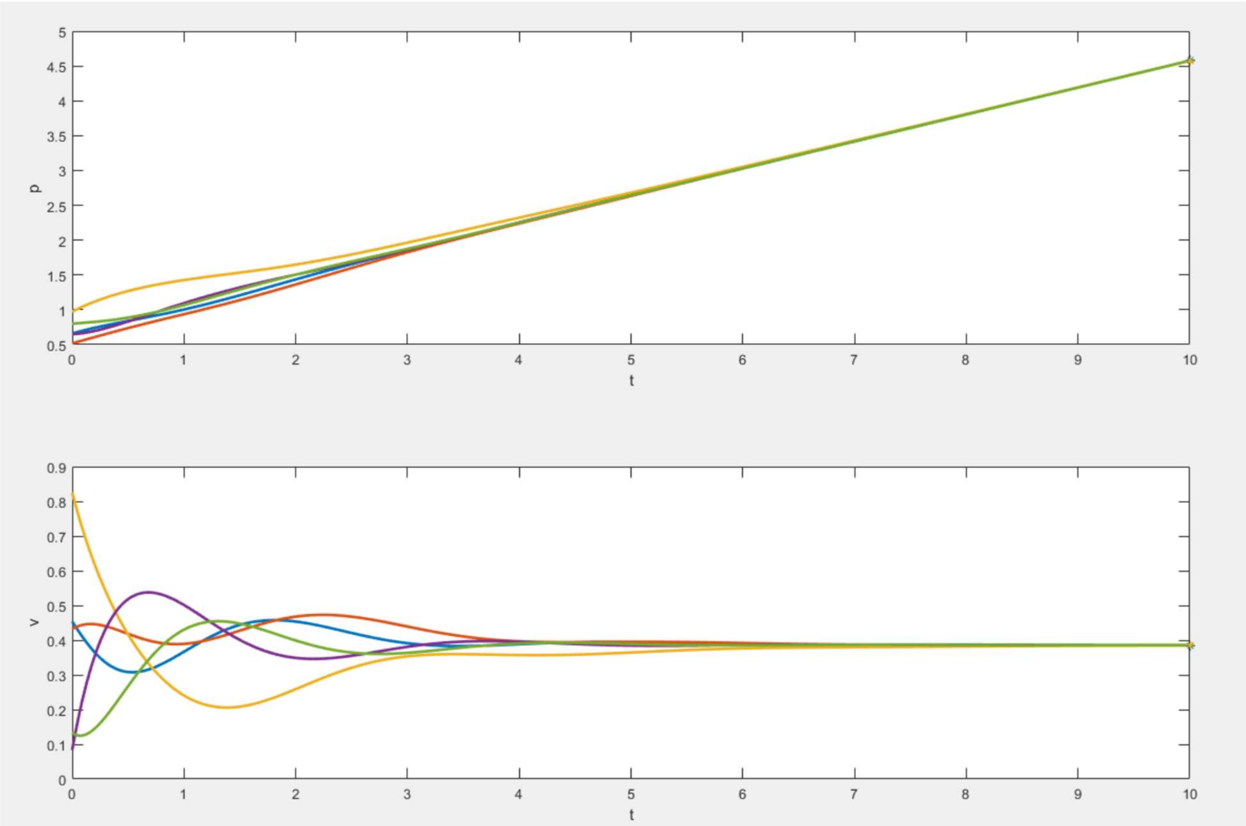

Figure 3: Trajectory plot for p and v for digraph b

Problem 2

Dynamics of five platooning vehicles are same as in equation 1 and Agent 1 is the leader of the network topology shown in Figure 3 [1].

Figure 4: Network with directed spanning tree.

1.

As the network is leader-follower type containing rooted-out branching we can choose a suitable γ for the parameter. Next, we need to design u_i for each agent so that they form a line with equal distance (d=2). We can see from the digraph in figure 4 that the agents 2, 3 and 4 are identical and will share the same input dynamics with each other. So, we will only need to make the input dynamics for agents 1, 2-4 and 5.

Agent 5 is a standard follower as it receives data from only one other node. This means that it can use the following input dynamics:

$$u_i = \sum_{N_i^{in}} (p_{i-1} - p_i - d) + \gamma(v_{i-1} - v_i) \tag{6}$$

Where d is equal to the desired gap of 2.

The agents 2-4 will need to follow some different input dynamics as they receive data from the agent ahead of them and the agent behind them. We can analyse the behaviour of the agents by examining the velocity error dynamics of the agents and even more specifically the part of the positions of the agents. If we would use the same dynamics as with agent 5, we would end up with:

$$u_i = (p_{i-1} - p_i - d) + (p_{i+1} - p_i - d)$$
$$u_i = (\bar{p}_{i-1} + p_{i-1}^* - \bar{p}_i - p_i^* - d) + (\bar{p}_{i+1} + p_{i+1}^* - \bar{p}_i - p_i^* - d)$$
$$u_i = (\bar{p}_{i-1} - \bar{p}_i + p_{i-1}^* - p_i^* - d) + (\bar{p}_{i+1} - \bar{p}_i + p_{i+1}^* - p_i^* - d)$$
$$u_i = (\bar{p}_{i-1} - \bar{p}_i + d - d) + (\bar{p}_{i+1} - \bar{p}_i - d - d)$$
$$u_i = (\bar{p}_{i-1} - \bar{p}_i) + (\bar{p}_{i+1} - \bar{p}_i - 2d)$$

We can now see that we end up with too much distance between the agents. But we can also notice that if we take away the added distance d altogether, we will end up with the correct dynamics for the system. And as such the dynamics for the agents 2-4 are:

$$u_i = \sum_{N_i^{in}} (p_{i-1} - p_i) + (p_{i+1} - p_i) + \gamma(v_{i-1} + v_{i+1} - 2v_i) \tag{7}$$

The agent 1 is the leader of the network and it follows its own dynamics without any influence from the other agents. As such

$$u_i = p_i + \gamma v_i \tag{8}$$

With these dynamics presented in 6, 7 and 8 the network should reach the correct line formation as time goes on.

2.

The trajectories for the network are plotted in figure 4. We can see from the positions trajectories that the vehicles form a line with equal spacing (d=2) and from the speed v plot we see that the other vehicles eventually reach the same constant speed as the leading vehicle.

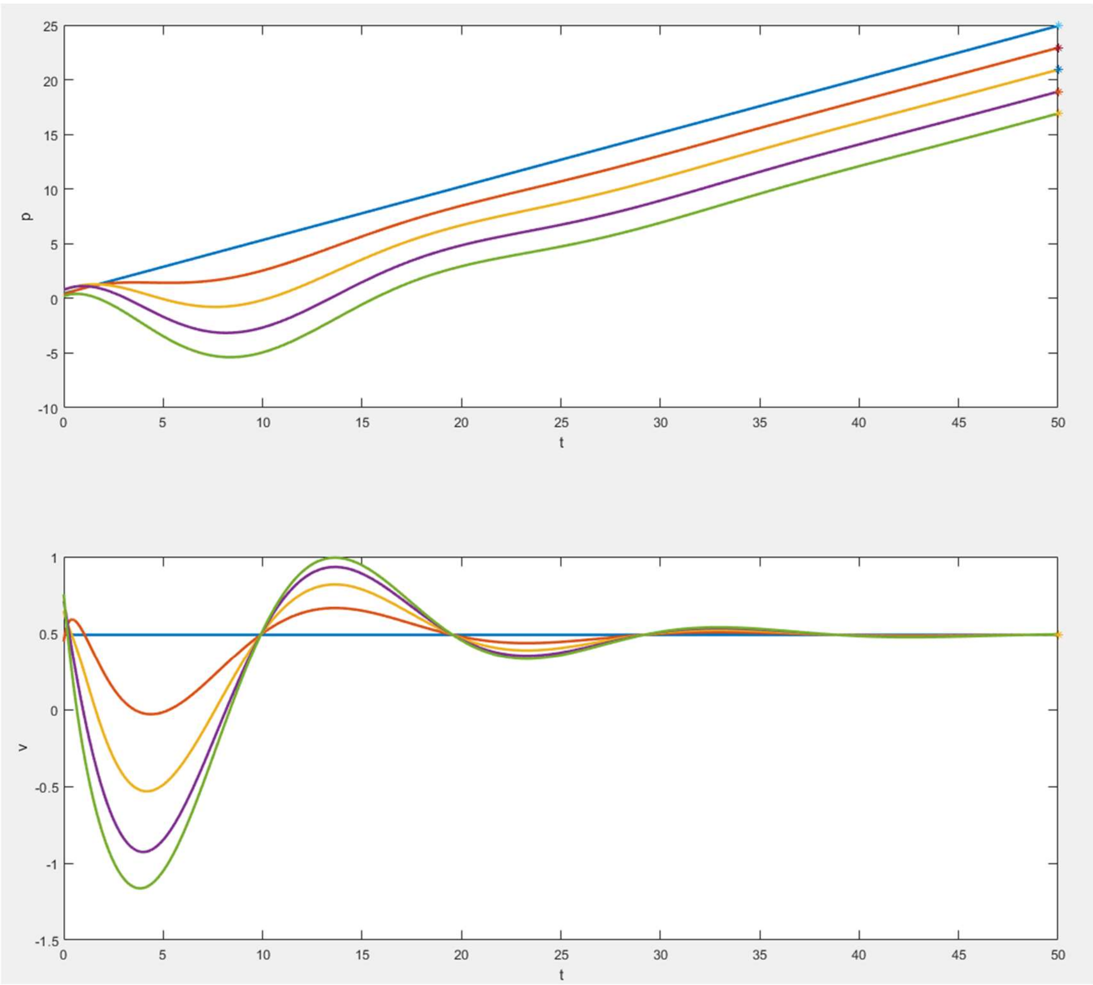

Figure 4: Trajectory plot for p and v for a platoon of five vehicles from figure 4.

Sources

(1) Homework 3, M. Iqbal, A. Gusrialdi, January 30, 2023. https://moodle.tuni.fi/pluginfile.php/3119479/mod_assign/introattachment/0/Homework5.pdf