AUT.360 Distributed Control and Optimization of Cyber-Physical Systems

Homework 3

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Problem 1

In problem 1 we were given the graph presented in figure 1. And asked some questions about it. Figure 1 has been taken directly from the homework assignment [1].

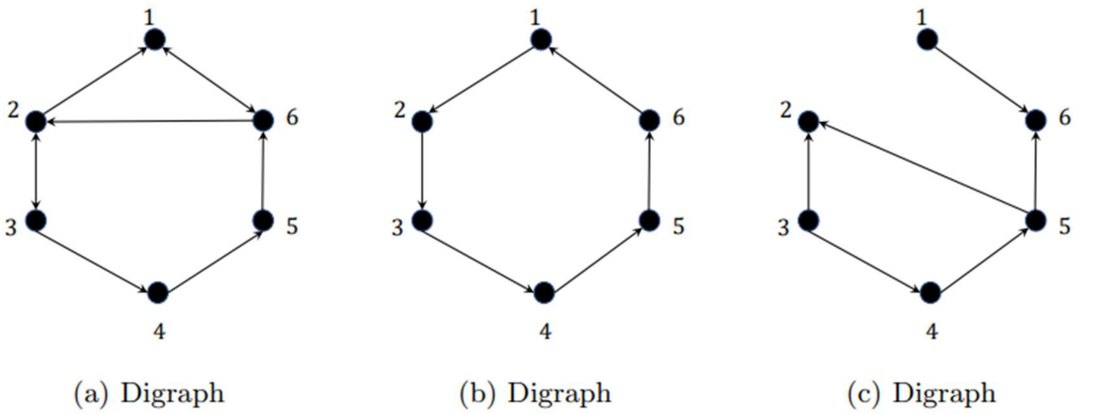

Figure 1: Three digraphs.

1. Finding the Laplacian matrices of the diagraphs in figure 1. For digraph a the Laplacian matrix is presented in table 1. For digraph b in table 2 and digraph c in table 3.
2.

2	-1	0	0	0	-1
0	2	-1	0	0	-1
0	-1	1	0	0	0
0	0	-1	1	0	0
0	0	0	-1	1	0
-1	0	0	0	-1	2

Table 1: Laplacian matrix for Diagraph a in Figure 1

1	0	0	0	0	-1
-1	1	0	0	0	0
0	-1	1	0	0	0
0	0	-1	1	0	0
0	0	0	-1	1	0
0	0	0	0	-1	1

Table 2: Laplacian matrix for Diagraph b in Figure 1

0	0	0	0	0	0
0	2	-1	0	-1	0
0	0	0	0	0	0
0	0	-1	1	0	0
0	0	0	-1	1	0
-1	0	0	0	-1	2

Table 3: Laplacian matrix for Diagraph c in Figure 1

3. Directed spanning tree contains rooted out-branching. In diagraph b node 1 can be viewed as a root and from there you can travel to any other node. Diagraph c doesn't contain directed spanning tree, because for example from node 1 you can only travel to node 6 but no further.

Problem 2

In problem 2 we are still focusing on the graphs given in figure 1. Let the states of the nodes be

$x_1(0) = -2, \quad x_2(0) = 1, \quad x_3(0) = -1, \quad x_4(0) = -2, \quad x_5(0) = 2, \quad x_6(0) = 3$

1. Considering that the dynamics for the *i*th agent are:

$$\dot{x}_i = \sum_{j \in N_i^{in}} (x_j - x_i), \; i \in V \tag{1}$$

We will plot the trajectories for each of the given graphs in figure 1 given that the agents follow the dynamics in equation 1. MATLAB will be used to plot these trajectories which are presented in the Figures 2-4.

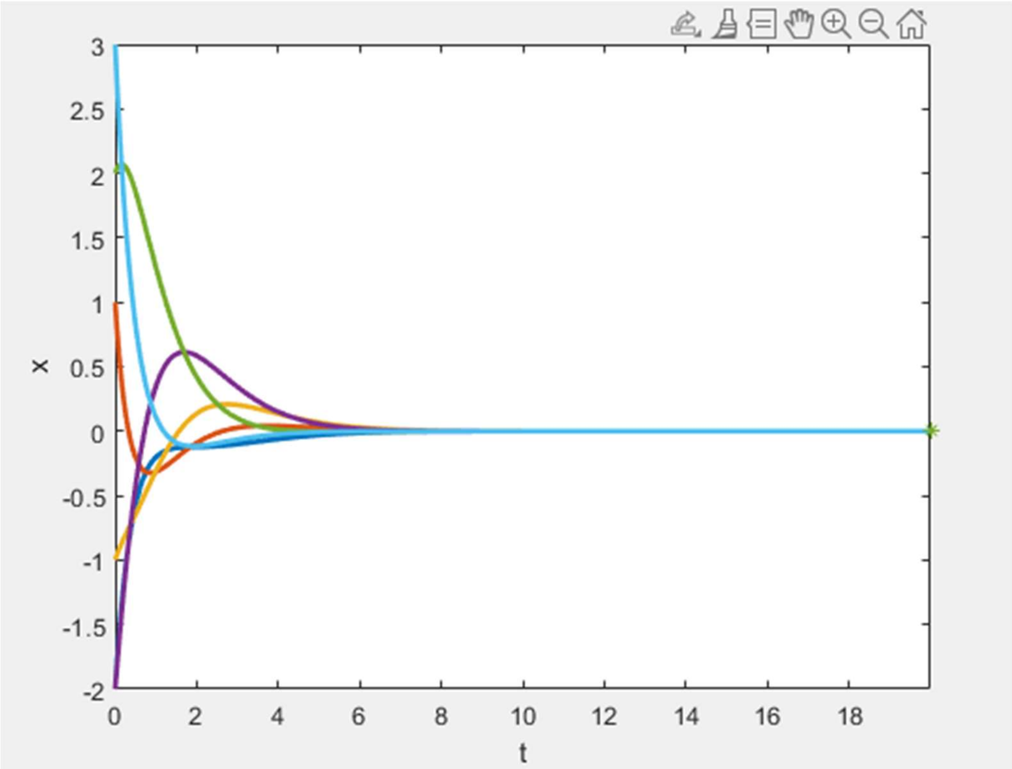

Figure 2: Trajectory plot for digraph a

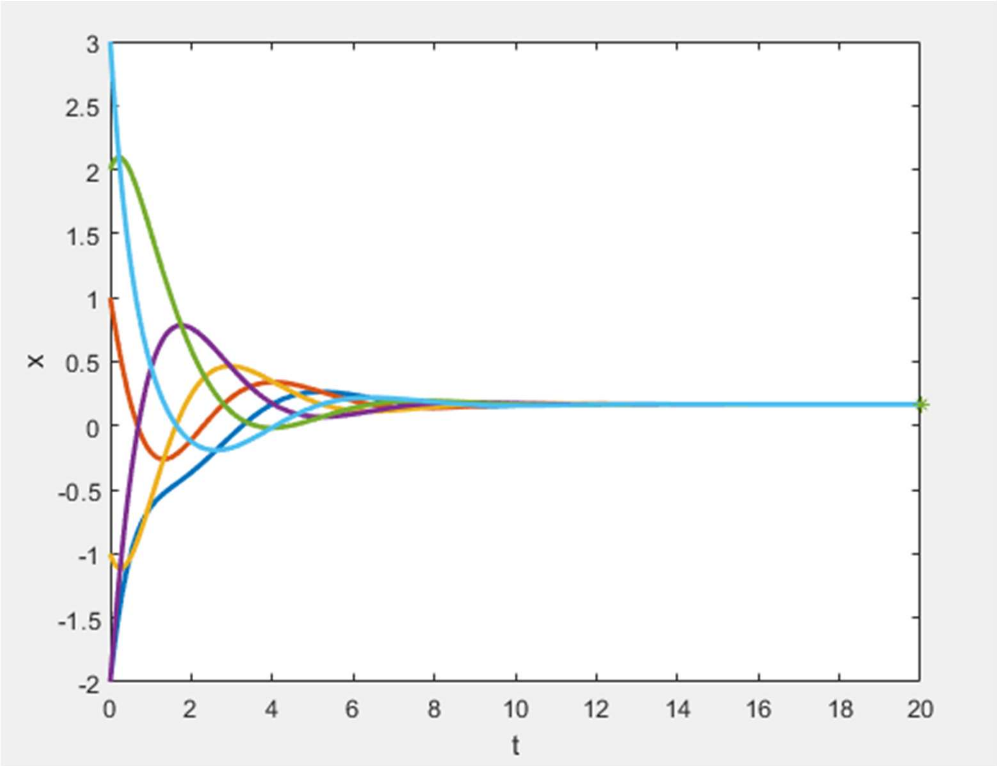

Figure 3: Trajectory plot for digraph b

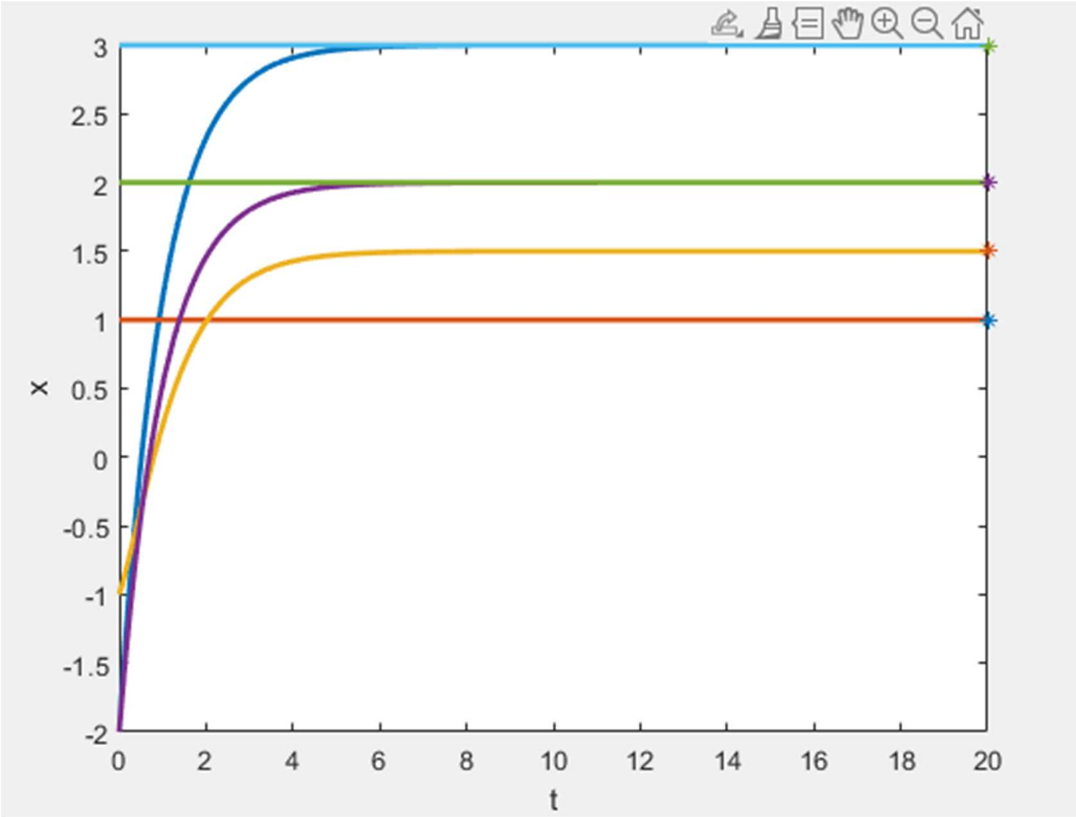

Figure 4: Trajectory plot for digraph c

2. From the Laplacian matrices of the graphs in Figure 1 we can calculate the eigenvalues of each graph. We want to compare the second eigenvalues of each graph to one another. Only the real parts are presented as they are the ones that matter in system dynamics.

$$\lambda_2(L_a) = 0.8213, \qquad \lambda_2(L_b) = 0.5, \qquad \lambda_2(L_c) = 0$$

From these eigenvalues we can see and confirm the plotted trajectories. The a graph reached consensus faster than the b graph which we can also see from the fact that the graph b:s second eigenvalue is smaller than that of graph a:s. And also, that graph c never reaches full consensus as it does not contain a rooted-out branching in it.

3. The left eigenvector for graph b in the figure 1. We get this from MATLAB from the Laplacian matrix of the graph b.

q_1^T = [0.4082 0.4082 0.4082 0.4082 0.4082 0.4082]

Which doesn't follow the formula that the elements of the eigenvector equal 1. Which we think should be the eigenvector so it would follow the expected outcome and fulfil the equation.

$$x_i = q_1^T x(0),\ with\ q_1^T \underline{1} = 1 \tag{2}$$

To fulfil this equation the vector should be

q_1^T = [0.1667 0.1667 0.1667 0.1667 0.1667 0.1667].

4. Networks a and b are converging to a consensus value. As they both contain a rooted-out branching. Network a:s topology gives nodes 2, 3 and 6 a little more power than the other nodes which have only 1 out neighbour. This has an effect on the consensus value as the network a isn't balanced and the final consensus value isn't the average of the starting positions of the nodes with them all ending up at 0 as shown in the trajectory plot in figure 2.
   Whereas network b is a balanced graph with all the nodes in the network equally connected to one another. Its consensus value is the average of the starting positions which is also clearly indicated in the trajectory plot of the network in figure 3. Where all the nodes end up at the consensus value of 0.1667.
   Network c on the other hand doesn't and isn't fully converging to a consensus value. Some of the nodes e.g., node 1 doesn't get any information and can't propagate its information to all the other nodes in the network. This means that the network can't ever reach consensus.

Problem 3

Considering the leader-follower network in figure 5, where initial states are

$$x_1(0) = 0,\ x_2(0) = 1,\ x_3(0) = 2,\ x_4(0) = 5,\ x_5(0) = -3.$$

Figure 5: Directed path.

1. Calculating Laplacian for network in figure 5

0	0	0	0	0
-1	1	0	0	0
0	-1	1	0	0
0	0	-1	1	0
0	0	0	-1	1

Table 4: Laplacian matrix for figure 5.

2. Considering that all the agents in the network presented in figure 5 have the dynamics presented in equation (1) we can use the same plotting as in problem 2 for this new network. The resulting trajectory plot is presented in figure 6.

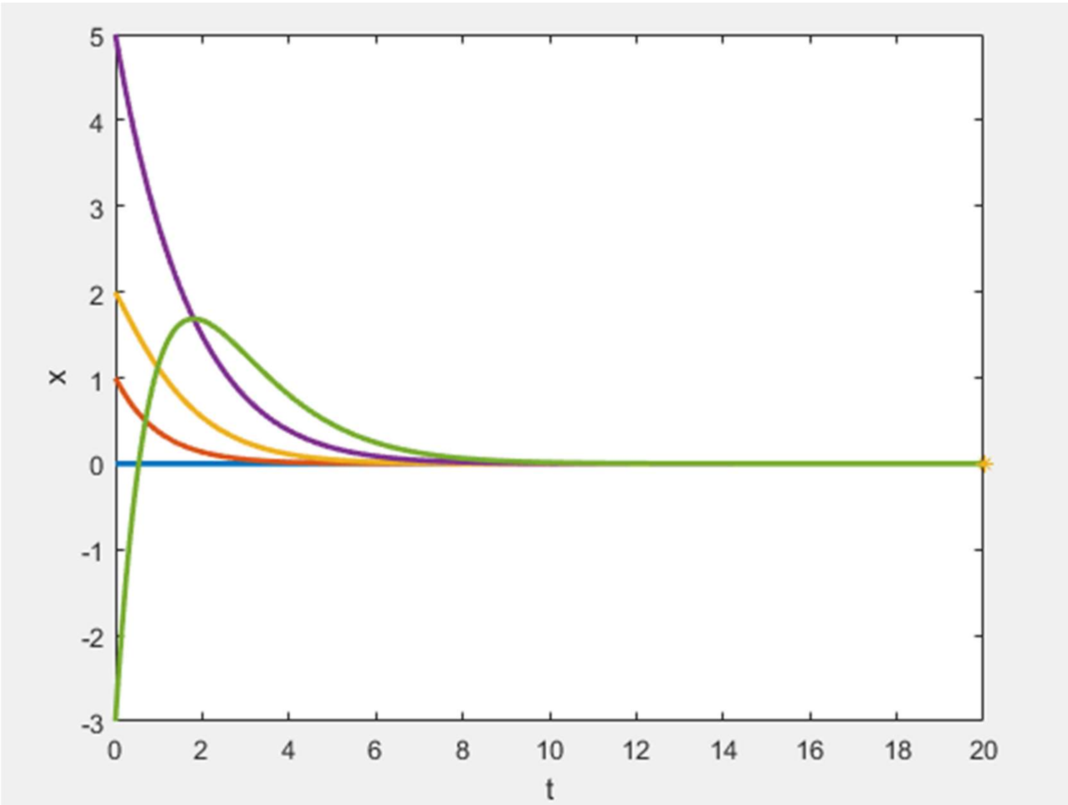

Figure 6: Trajectory plot for the graph in figure 5

3. From this trajectory plot we can see that all the other agents converge to the position of node 1 as time goes on, which is to be expected from the network graph as 1 doesn't

receive any information from any other nodes but shares its own to the rest of the network and works as a source node.

Sources

(1) Homework 3, M. Iqbal, A. Gusrialdi, January 30, 2023. https://moodle.tuni.fi/pluginfile.php/3070255/mod_resource/content/1/Homework_3.pdf