

# Nearest Smaller Distance Queue

## Problem Description

Given an integer array  $A$  of length  $N$ , for each index  $i$  (1-based), define the nearest strictly smaller element as the rightmost index  $j < i$  such that  $A_j < A_i$ . If such  $j$  exists, output the distance  $i - j$ ; otherwise, output  $-1$ .

Next, take all distance values that are not  $-1$  (in increasing order of their indices) and enqueue them into a queue. Finally, output the contents of this queue. If the queue is empty, output **EMPTY**.

You are expected to compute all distances in linear time using a monotonic increasing stack and perform queue operations in amortized  $O(1)$  time. Naïve solutions will time out.

## Input Format

The input consists of two lines:

- The first line contains an integer  $N$ .
- The second line contains  $N$  integers  $A_1, A_2, \dots, A_N$ .

## Output Format

- Line 1:  $N$  space-separated integers —the distance for each position.
- Line 2: the queue contents formed by enqueueing all distances that are not  $-1$  (space-separated). If the queue is empty, print **EMPTY**.

## Constraints

- $1 \leq N \leq 2 \times 10^5$  ◦
- $-10^9 \leq A_i \leq 10^9$  ◦
- Time Limit: 1 second ◦
- Memory Limit: 256 MB ◦

## Additional Notes

1. **Complexity Target:** Use a monotonic increasing stack to achieve overall  $O(N)$  time.

A brute-force scan to the left for every position is  $O(N^2)$  and will not pass within the time limit when  $N = 2 \times 10^5$ .

## Example Test Case

### Sample Input 1

```
6
2 1 4 3 5 6
```

### Explanation 1

- $i = 1$ : no elements to the left  $\rightarrow -1$ .
- $i = 2$ : left side [2] has nothing  $< 1 \rightarrow -1$ .
- $i = 3$ : nearest strictly smaller is  $A[2] = 1$ , distance  $3 - 2 = 1$ .
- $i = 4$ : nearest strictly smaller is  $A[2] = 1$ , distance  $4 - 2 = 2$ .
- $i = 5$ : nearest strictly smaller is  $A[4] = 3$ , distance 1.
- $i = 6$ : nearest strictly smaller is  $A[5] = 5$ , distance 1.

Queue of non-(-1) distances: 1 2 1 1.

### Sample Input 2

```
5
1 2 3 4 5
```

### Explanation 2

Strictly increasing sequence: for every position after the first, the nearest strictly smaller element is the immediate predecessor, so each distance is 1.

### Sample Input 3

```
5
5 4 4 4 1
```

### Explanation 3

We require strictly smaller. For each 4, the left side has 5 (not smaller) or equal 4, so all distances are  $-1$ ; thus the queue is empty.

### Sample Output 1

```
-1 -1 1 2 1 1
1 2 1 1
```

### Sample Output 2

```
-1 1 1 1 1
1 1 1 1
```

### Sample Output 3

```
-1 -1 -1 -1 -1
EMPTY
```