For this problem, I adopted dynamic programming method. First I did manual method as explained below.

For example there are only 1p, 2p, 5p, 10p available and want to find number of ways to make 10p with unlimited combinations from available coins.

Manual method:

Value

Number of different ways

Coins available

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1p | 2p | 3p | 4p | 5p | 6p | 7p | 8p | 9p | 10p |
| 1p | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2p | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 6 |
| 5p | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 10 |
| 10p | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 11 |

*Table1. Showing manual method of filling the table*

If we have 1p and want the value of 1p there will be only one way and it is similar for other values. So first row is 1 in each case.

If we have 2p and want to make 1p it is possible (so I made initial value of value =1), which is true for 5p and 10p so the first column is set to 1.

If we want value 2p and we have 1p and 2p then the number of ways to get that will be {2}, {1,1} so 2 ways to make 2p.

If we want to make the value of 3p with available 1p and 2p, the number of ways will be: {2, 1}, {1, 1, 1} (two ways)

If we want to make the value of 8p with the available 1p, 2p, 5p, the number of ways will be : {5,2, 1}, {2,2,2}, {2,2,1,2,1}, {1,1,2,1,2,1},{1,1,1,1,1,2,1}, {1,1,1,1,1,,1,1,1}, {1,1,2,2} (7 ways).

This way I filled in the whole table (Table1) shown above. The answer for number of different ways to get 10p with available 1,2, 5, 10p is 11.

Finding solution using Matlab program

As explained in dynamic programming, I can see the pattern in each row/column in the table above (Table1).

Number of different ways

Value

Coins

Number of coins is represented as i

Value is represented as j

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1p | 2p | 3p | 4p | 5p | 6p | 7p | 8p | 9p | 10p |
| 1p | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2p | 1 | If j value is equal to i value, then just add the above row in the same column and add 1 1+1 =2 | 2 | 3 | 3 | If j value is greater than I value, then: the ways are calculated as add the above value + j-ways of j-coins(i)= here it is 1+ways(j-coin(i)= 1+ways(4p)=1+3 4 | 4 | 5 | 5 | 6 |
| 5p | 1 | If the j value is lesser than i value, then copy the above row of the same column value. In this case it is = 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 10 |
| 10p | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 11 |

*Table2. Showing automatic method of filling the table*

**Initial values are highlighted in yellow.** The pattern observed:

Let’s assume coins are represented by ‘i’ and values are represented by ‘j’ and the table the calculates the number of ways is T

1. If ‘j’ is less than coins(i) then the number of ways is T(i,j)=T(i-1,j);
2. If ‘j’ is equal to coins (i), then the number of ways is T(i,j) = T(i-1,j)+1;
3. If ‘j’ is greater than coins(i) the number of ways is T(i,j)=T(i-1,j)+ T(i,j-coins(i));